Contents

Remarks:

- Write your answer to 2 decimal places, unless otherwise instructed.
- Some solutions will contain complex components.

1 Creating Matrices

Recall how to compute matrices, using MATLAB.

```
X = [1 2 3 4 6 4 8 4 5]
X =

     1     2     3     4     6     4     8     4     5

Y = [1,3,6;2,7,8;0,3,9]
Y =
  1    3    6
  2    7    8
  0    3    9
```

1.1 Basic Matrix Operations

For the matrix Y described in the above code, answer the following questions.

`trace(Y)` Compute the ***trace*** of matrix Y

`rank(Y)` Compute the ***rank*** of matrix Y

`inv(Y)` Computes the inverse of matrix Y

`det(Y)` Computes the determinant of matrix Y

$Y * Y$ Computes Y^2.

$Y * Y * Y$ Computes Y^3.

2 Determining Eigen-values

In the first instance the command `eig()` is used to compute the eigen-values of a matrix.The `poly` function generates a vector containing the coefficients of the characteristic polynomial. Recall that the characteristic polynomial of a matrix A is defined as:

$$det(\lambda I - A)$$

i) determine the characteristic polynomial of matrix Z. (The full equation, not just the coefficients)

ii) compute W, the inverse of Z.

iii) compute the eigenvalues of W.(four decimal places)

```
Z = [1 2 0; 2 5 -1; 4 10 -1];
```

3 Eigen-decomposition

The command `eig()` can be used to perform the command ***eigen-decomposition***, when used in the manner described below. The command `[V,D]=eig(A)` produces the V matrix, whose columns are eigenvectors, and the diagonal matrix D whose values are eigenvalues of the matrix A.

3.1 Example

```
A = [5 3 2; 1 4 6; 9 7 2];
[V,D]=eig(A);
```

```
V =
  0.7217   0.1918   0.7680
  0.5557 - 0.7773 - 0.6388
  0.4127   0.5992   0.0459

D =

Diagonal Matrix

 -3.2847   0         0
  0        1.7486    0
  0        0        12.5361
```

i) Compute the characteristic equation of the following matrix.

ii) Compute the eigenvalues of the matrix.

iii) Compute the eigenvectors of the matrix.

$$\begin{pmatrix} 5 & 9 & 13 \\ 11 & 7 & 1 \\ 8 & 9 & 3 \end{pmatrix}$$

4 Solving linear systems of equations

The MATLAB command sequence `A\b` is used to solve the equation $\boldsymbol{Ax=b}$.

Solve the following system of linear equations,using your own names for the matrices.

$$\begin{pmatrix} 5 & 9 & 13 \\ 11 & 7 & 1 \\ 8 & 9 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$$

5 Using Commands

Given the array A = [2 7 9 7 ; 3 1 5 6 ; 8 1 2 5], explain the results of the following commands:

```
A = [2 7 9 7 ; 3 1 5 6 ; 8 1 2 5];
```

i) `A'`

ii) `A(:,[1 4])`

iii) `A([1 2],[1 2])`

iv) `A([2 3],[1 2])`

v) `A([2 3],[1 3])`

vi) `A([2 3],[3 1])`

vii) `reshape(A,2,6)`

viii) `size(A)`

ix) `flipud(A)`

x) `fliplr(A)`

xi) `A(end,:)`

xii) `[A ; A(end,:)]`

xiii) `A(1:2,:)`

xiv) `A'(1:3,:)`

xv) `[A ; A(1:2,:)]`

xvi) `sum(A)`

xvii) `sum(A')`

xviii) `sum(A,1)` and `sum(A,2)`

xiv) `[ [ A ; sum(A) ] [ sum(A,2) ; sum(A(:))  ]  ]`

xv) `A(:)`

6 More Exercises

Question 1

Consider the matrices A and B, given as:

$$A = \begin{pmatrix} 2.4 \\ 1.4 \\ 1.2 \end{pmatrix}$$

$$B = \begin{pmatrix} 2.2 & 1.3 & 1.2 \end{pmatrix}$$

Determine the following matrices.

i. $C = A \times B$

ii. $D = B \times A$

iii. $E = C^{-1}$

Question 2

i. Determine the rank of the following matrix F.

$$\mathbf{F} = \begin{bmatrix} 9 & 13 & 5 \\ 1 & 11 & 7 \\ 3 & 7 & 2 \\ 6 & 0 & 7 \end{bmatrix}$$

Question 3

compute the determinant of matrix G.

$$G = \begin{pmatrix} 1 & 5 & 9 & 13 \\ 2 & 11 & 7 & 1 \\ 4 & 8 & 10 & 3 \\ 6 & 15 & 9 & 8 \end{pmatrix}$$

C= [1 3 9; 6 7 2; 8 -1 -2];