ADS4

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1 $T(n) = 36T(n/6) + 2n$

Here,

$n^{log_b(a)} = n^6$

$2n$ is polynomially smaller than n^6

Therefore:

$$\Theta(n) = n^6$$

2 $T(n) = 5T(n/3) + 17n^{1.2}$

$n^{log_b(a)} \approx n^{1.5}$ (Rounded up for worst case)

$n^{1.2}$ is polynomially smaller than $n^{1.5}$

Therefore:

$$\Theta(n) = n^{1.5}$$

3 $T(n) = 12T(n/2) + n^2 \cdot log_2(n)$

The effect of $log_2(n)$ on n^2 can be neglected.

Thus

$f(n) = n^2$

$n^{log_b(a)} \approx n^3.6$ (Rounded up for worst case)

n^2 is polynomially smaller than $n^{3.6}$

Therefore:

$$\Theta(n) = n^{3.6}$$

4 $T(n) = 3T(n/5) + T(n/2) + 2^n$

We use recursion tree method for this recursive definition.

Initial: 2^n

First Iteration: $3(2^{n/5}) + 2^{n/2}$

Second Iteration: $4(3 \cdot 2^{n/25}) + 6(2^{n/10}) + 2^{n/4}$

Third Iteration: $4^2(3 \cdot 2^{n/125}) + 6(4 \cdot 2^{n/50}) + 4(2^{n/20}) + 2^{n/8}$ We can see that after each iteration, the expressions get closer to a constant value.($2^n \to 1$ as $n \to 0$) Therefore:

$$\Theta(n) = 2^n$$

5 $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$

We can try to find the bounds of the equation.

Lower bound is when $\Theta(n) > n^{log_{5/2}(3)}$ because we take $T(3n/5) \approx T(2n/5)$

Upper bound is when $\Theta(n) < n^{log_{5/3}(3)}$ because we take $T(3n/5) \approx T(2n/5)$