Problemsheet 7

November 1, 2019

1 7.1

Prove two elementary function $\rightarrow$ and $\neg$ are universal.
We know that the functions $\wedge$ $\vee$ $\neg$ are universal functions.
If $\rightarrow$ and $\neg$ can express any of the above boolean functions, (mainly $\wedge$ and $\vee$) it is universal.
Truth table for A $\rightarrow$ B:

A	B	A $\rightarrow$ B	A $\wedge$ B	A $\vee$ B
0	0	1	0	0
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

for $\wedge$: $\neg(A \rightarrow \neg B)$ gives the same result in the truth table.
for $\vee$: $\neg B \rightarrow A$ gives the same result in the truth table.
Since $\wedge$ and $\vee$ can both be expressed by $\rightarrow$ and $\neg$

2 2

Boolean Expression:
$\varphi(P, Q, R, S) = (\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$

2.1 a

φ gives 1 when $(\neg P \vee Q)$, $(\neg Q \vee R)$, $(\neg R \vee S)$, $(\neg S \vee P)$ The only conditions for which this is true are:
P = 0, Q = 0, R = 0, S = 0
P = 1, Q = 1, R = 1, S = 1

2.2 b

From the cases where the expression is true, we can drive the DNF as:
$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$

2.3 c

Given CNF:

$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$

For $(\neg P \vee Q) \wedge (\neg Q \vee R)$

$\neg P \wedge (\neg Q \vee R) \vee Q \wedge (\neg Q \vee R)$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

For $(\neg R \vee S) \wedge (\neg S \vee P)$

$\neg R \wedge (\neg S \vee P) \vee S \wedge (\neg S \vee P)$

$(\neg R \wedge \neg S) \vee (\neg R \wedge P) \vee (S \wedge P)$

Equating both of the equations:

$\{(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R)\} \wedge \{(\neg R \wedge \neg S) \vee (\neg R \wedge P) \vee (S \wedge P)\}$

For $(\neg P \wedge \neg Q) \wedge \{(\neg R \wedge \neg S) \vee (\neg R \wedge P) \vee (S \wedge P)\}$

$\{(\neg P \wedge \neg Q) \wedge (\neg R \wedge \neg S)\} \vee \{(\neg P \wedge \neg Q) \wedge (\neg R \wedge P)\} \vee \{(\neg P \wedge \neg Q) \wedge (S \wedge P)\}$

$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S)$

For $(\neg P \wedge R) \wedge \{(\neg R \wedge \neg S) \vee (\neg R \wedge P) \vee (S \wedge P)\}$

$\{(\neg P \wedge R) \wedge (\neg R \wedge \neg S)\} \vee \{(\neg P \wedge R) \wedge (\neg R \wedge P)\} \vee \{(\neg P \wedge R) \wedge (S \wedge P)\}$

$0 \vee 0 \vee 0$

0

For $(Q \wedge R) \wedge \{(\neg R \wedge \neg S) \vee (\neg R \wedge P) \vee (S \wedge P)\}$

$\{(Q \wedge R) \wedge (\neg R \wedge \neg S)\} \vee \{(Q \wedge R) \wedge (\neg R \wedge P)\} \vee \{(Q \wedge R) \wedge (S \wedge P)\}$

$0 \vee 0 \vee (P \wedge Q \wedge R \wedge S)$

$(P \wedge Q \wedge R \wedge S)$

Equating all:

$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee 0 \vee (P \wedge Q \wedge R \wedge S)$

$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$