Problem 3

October 3, 2019

1 3.1

Prove the following statement by induction: The number of elements in the power set $P(S)$ of a finite set S with n elements is 2^n

Let us assume a set K where:

$K = \{\forall n, n \in N, |S| = n : |P(S)| = 2^n\}$

Base Case : $n = 0$ $|P(S)| = 2^0 = 1$ because the only subset of S is an empty set.

let the new set be $Q = S \cup \{a\}$ thus $|Q| = n + 1$ for Q there are two types of subsets, $P(S)$ and $R \cup a, R \in P(S)$ since there are $|P(S)|$ amount of R we can concider, $|P(Q)| = |P(S)| + |R \cup a|$ $|P(Q)| = 2^n + 2^n$ $|P(Q)| = 2^n(1 + 1)$ $|P(Q)| = 2^n.2$ $|P(Q)| = 2^{n+1}$

Since the assumption holds for $n + 1$, K is inductive. This the assumption holds for all values of n.

2 3.2

a) $R = \{(a, b)|a, b \in Z \wedge a \neq b\}$

For reflexivity : $\forall a \in A, (a, a) \notin R$

Since a and b are never equal.

For symmetry: $\forall a, b \in A, (a, b) \in R \Rightarrow (b, a) \notin R$

Because if $(a, b) \in R \Rightarrow (b, a) \in R$ it implies $a = b$ which is not true for this relation.

For transitivity: $\forall a, b, c \in A, ((a, b) \in R \wedge (b, c) \in R) \Rightarrow (a, c) \notin R$

Because if $a, b, c \in A, ((a, b) \in R \wedge (b, c) \in R) \Rightarrow (a, c) \in R$ it implies $a = b$ which is not true for this relation.

b) $R = \{(a, b)|a, b \in Z \wedge |a - b| \leq 3\}$

For reflexivity: $\forall a \in A, (a, a) \in R$

Because the distance between same numbers is zero, which satisfies the condition.

For symmetry: $\forall a, b \in A, (a, b) \in R \Rightarrow (b, a) \in R$

Because the operation $|a - b|$ is commutative thus the relation is symmetric as long as the distance between a and b is less than 3.

For transitivity: $\forall a, b, c \in A, ((a, b) \in R \wedge (b, c) \in R) \Rightarrow (a, c) \notin R$

Because even if (a, b) and (b, c) satisfy the relation, it does not imply that (a, c) satisfies the condition. Example: $(1, 3) = 2$ $(3, 5) = 2$ $(1, 5) = 4$ which does not satisfy the relation.

c) $R = \{(a, b)|a, b \in Z \wedge (amod10) = (bmod10)\}$

For reflexivity: $\forall a \in A, (a, a) \in R$

Because $(amod10) = (amod10)$ is true.

For symmetry: $\forall a, b \in A, (a, b) \in R \Rightarrow (b, a) \in R$

Because the operation $(amod10) = (bmod10)$ is commutative.

For transitivity: $\forall a, b, c \in A, ((a, b) \in R \wedge (b, c) \in R) \Rightarrow (a, c) \in R$

Because (a, b) and (b, c) satisfy the relation, it implies that (a, c) satisfies the condition.