Problem 2

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1 Problem 2.1

If a natural number n is not divisible by 3, then the same is true for 15.

Predicate A: $\frac{n}{3}$ is a natural number
Predicate B: $\frac{n}{15}$ is a natural number
Theorem: If A is not true, it implies B is also not true.
Proof: We prove the theorem by contra positive i.e.
If $\neg B \longrightarrow \neg A$
$y = \frac{n}{15}\ y \in N$
$y = \frac{n}{3*5}$
We can see that if $\neg B$ is true, $\neg A$ is also true.
Thus $A \longrightarrow B$

2 Problem 2.2

If $n \in N, n \leq 1$ prove that:

$1^2 + 3^2 + 5^2 + ...(2n-1)^2 = \frac{2n(2n+1)(2n-1)}{6}$
Theorem:
$1^2 + 3^2 + 5^2 + ...(2n-1)^2 = \frac{2n(2n+1)(2n-1)}{6} \forall n, n \leq 1, n \in N$
Proof:
Let us take a set of values K such that:
$K : \{\forall n, n \leq 1, n \in N \mid 1^2 + 3^2 + 5^2 + ...(2n-1)^2 = \frac{2n(2n+1)(2n-1)}{6}\}$
For K to represent all N , it must be inductive. i.e.
$n \in K \wedge s(n) \in K$
$s(n) = n + 1$
for $s(n)$,
$1^2 + 3^2 + 5^2 + ...(2n-1)^2 + (2(n+1)-1)^2 = \frac{2(n+1)(2(n+1)+1)(2(n+1)-1)}{6}$
From our initial assumption of n,
$\frac{2n(2n+1)(2n-1)}{6} + (2(n+1)-1)^2 = \frac{2(n+1)(2(n+1)+1)(2(n+1)-1)}{6}$
$\frac{2n(2n+1)(2n-1)+6(2n+1)(2n+1)}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$
$\frac{2(2n+1)\{n(2n-1)+3(2n+1)\}}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$
$\frac{2(2n+1)(2n^2+5n+3)}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$
$\frac{2(2n+1)\{2n(n+1)+3(n+1)\}}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$

$\frac{2(2n+1)(n+1)(2n+3)}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$
$\frac{(2n+2)(2n+3)(2n+1)}{6} = \frac{2n+2)(2n+3)(2n+1)}{6}$
Thus $n \in K \land s(n) \in K,\ K \subseteq N$
Since K is inductive, the theorem holds for all N