Problem 5

October 18, 2019

1 5.1

We consider a b-compliment number system with base 5 and 4 "digits".
For the representation of -1 in this number system: First we consider the representation of 1. Which is:
$1(5^0)$:

0001

We choose the negative such that when the two numbers are added, the last 4 "digits" represent a 0 and we ignore the overflow. In this case, the number that satisfies this condition for -1 is:

4444

We use the same process for the representation of -8.
Representation of 8:

$1(5^1) + 3(5^0)$

0013

Number which causes an overflow equivalent to (-8):

4432

When adding the two numbers:

4444
4432

14431

We only take 4 of the least significant bits and ignore the overflow giving us:

4431

Converting the number into decimal:

4431 (base 5)

$4(5^3) + 4(5^2) + 3(5^1) + 1(5^0)$

616 (base 10)

2 5.2

We are working with a 32 bit format of floating point numbers. This means that we have a 23 bit mantissa to store numbers. Thus we can store 24 bits in total after normalization.
The decimal number we want to convert to a floating point binary is -273.15
Our first step is to set the "signed bit" to 1 because our number is negative.
Then we convert the digits before decimal to binary:
273
$(2^8) + (2^4) + (2^0)$
100010001

Now we use an algorithm for the decimal part of our number:
Since we already need 8 bits to represent the previous binary, we are left with 15 bits
$0.15- > 0.3$ (0)
$0.3 - > 0.6$ (00)
$0.6 - > 1.2$ (001)
$0.2 - > 0.4$ (0010)
$0.4 - > 0.8$ (00100)
$0.8 - > 1.6$ (001001)
$0.6 - > 1.2$ (0010011)
From this point on, the pattern is recursive. Thus our binary approximation will be:
001001100110011
We stop at the 15th bit.
The binary conversion of 273.15 is:

100010001.001001100110011

After normalization we get the mantissa:

00010001001001100110011

And the exponent: 2^8
But we add 127 to the exponent and store it as binary giving us:

10000111

Thus the final 32 bit floating point representation is:

11000011100010001001001100110011

For the approximation of the decimal fraction stored:

001001100110011

$(2^{-3}) + (2^{-6}) + (2^{-7}) + (2^{-10}) + (2^{-11}) + (2^{-14}) + (2^{-15})$

0.1499939