ICS Problem sheet 8

November 8, 2019

1 Prime Implicants

Number of 1	Minterm	Binary	Used		Used	Size 2 Implicants		Size 4 Implicants
0	m0	0 0 0 0 0	*	m(0,2)	*	0 0 0 _ 0	m(0,2,4,6)	0 0 _ _ 0
				m(0,4)	*	0 0 _ 0 0	m(0,4,2,6)	0 0 _ _ 0
				m(0,16)		_ 0 0 0 0		
1	m2	0 0 0 1 0	*	m(2,6)	*	0 0 _ 1 0	m(2,6,10,14)	0 _ _ 1 0
				m(2,10)	*	0 _ 0 1 0		
	m4	0 0 1 0 0	*	m(4,6)	*	0 0 1 _ 0		
	m16	1 0 0 0 0	*	m(16,17)		1 0 0 0 _		
2	m6	0 0 1 1 0	*	m(6,14)	*	0 _ 1 1 0		
	m9	0 1 0 0 1	*	m(9,13)		0 1 _ 0 1		
	m10	0 1 0 1 0	*	m(10,14)	*	0 1 _ 1 0	m(10,14,26,30)	_ 1 _ 1 0
				m(10,26)	*	_ 1 0 1 0		
	m17	1 0 0 0 1	*	m(17,21)		1 0 _ 0 1		
3	m13	0 1 1 0 1	*	m(13,15)		0 1 1 _ 1		
	m14	0 1 1 1 0	*	m(14,15)	*	0 1 1 1 _	m(14,15,30,31)	_ 1 1 1 _
				m(14,30)	*	_ 1 1 1 0		
	m21	1 0 1 0 1	*					
	m26	1 1 0 1 0	*	m(26,30)	*	1 1 _ 1 0		
	m28	1 1 1 0 0	*	m(28,30)		1 1 1 _ 0		
4	m15	0 1 1 1 1	*	m(15,31)	*	_ 1 1 1 1		
	m30	1 1 1 1 0	*	m(30,31)	*	1 1 1 1 _		
5	m31	1 1 1 1 1	*					

The prime implicants are all the ones which cannot be further be minimized:

$m(0, 16) : (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$

$m(16, 17) : (A \wedge \neg B \wedge \neg C \wedge \neg D)$

$m(9, 13) : (\neg A \wedge B \wedge \neg D \wedge E)$

$m(17, 21) : (A \wedge \neg B \wedge \neg D \wedge E)$

$m(13, 15) : (\neg A \wedge B \wedge C \wedge \neg E)$

$m(28, 30) : (A \wedge B \wedge C \wedge \neg E)$

$m(0, 2, 4, 6) : (\neg A \wedge \neg B \wedge \neg E)$

$m(2, 6, 10, 14) : (\neg A \wedge D \wedge \neg E)$

$m(10, 14, 26, 30) : (B \wedge D \wedge \neg E)$

$m(14, 15, 30, 31) : (B \wedge C \wedge D)$

2 Essential Prime Implicants

Minterms	0	2	4	6	9	10	13	14	15	16	17	21	26	28	30	31
M(0,16)	*	-	-	-	-	-	-	-	-	*	-	-	-	-	-	-
M(16,17)	-	-	-	-	-	-	-	-	-	*	*	-	-	-	-	-
M(9,13)	-	-	-	-	*	-	*	-	-	-	-	-	-	-	-	-
M(17,21)	-	-	-	-	-	-	-	-	-	-	*	*	-	-	-	-
M(13,15)	-	-	-	-	-	-	*	-	*	-	-	-	-	-	-	-
M(28,30)	-	-	-	-	-	-	-	-	-	-	-	-	-	*	*	-
M(0,2,4,6)	*	*	*	*	-	-	-	-	-	-	-	-	-	-	-	-
M(2,6,10,14)	-	*	-	*	-	*	-	*	-	-	-	-	-	-	-	-
M(10,14,26,30)	-	-	-	-	-	*	-	*	-	-	-	-	*	-	*	-
M(14,15,30,31)	-	-	-	-	-	-	-	*	*	-	-	-	-	-	*	*

From the table we derive the essential prime implicants:

$m(0,2,4,6) : (\neg A \wedge \neg B \wedge \neg E)$

$m(9,13) : (\neg A \wedge B \wedge \neg D \wedge E)$

$m(17,21) : (A \wedge \neg B \wedge \neg D \wedge E)$

$m(28,30) : (A \wedge B \wedge C \wedge \neg E)$

$m(10,14,26,30) : (B \wedge D \wedge \neg E)$

$m(14,15,30,31) : (B \wedge C \wedge D)$

3 Minimal boolean expressions

The essential prime implicants do not cover m_{16}. The minimal expressions is the sum of all the essential prime implicants and the prime implicants that cover the missing conditions.

$(\neg A \wedge \neg B \wedge \neg E) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (B \wedge D \neg E) \vee (B \wedge C \wedge D) \vee (A \wedge B \wedge C \wedge \neg E) \vee (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$

$(\neg A \wedge \neg B \wedge \neg E) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (A \wedge \neg B \wedge \neg D \wedge E) \vee (B \wedge D \neg E) \vee (B \wedge C \wedge D) \vee (A \wedge B \wedge C \wedge \neg E) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$