UMD CS Computer Graphics

Distribution Ray Tracing

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Outline

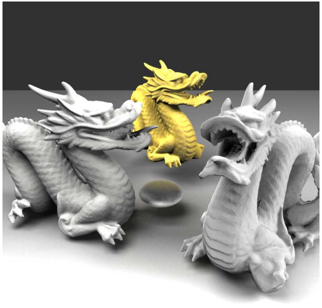
Objective

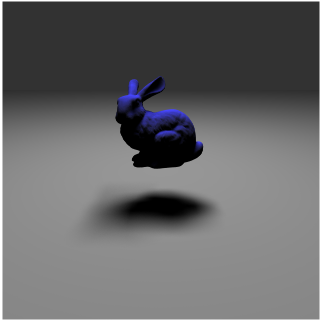
Develop a Cook-style ray tracer that provides more appropriate sampling strategies with an accelerated intersection data structure.

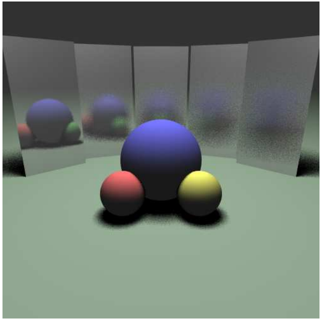
Outcomes for this section of course

- Instancing of renderable objects
- Spatial data structures for accelerating ray tracers
- Loading more complicated objects
- Develop better sampling mechanisms for ray tracing effects

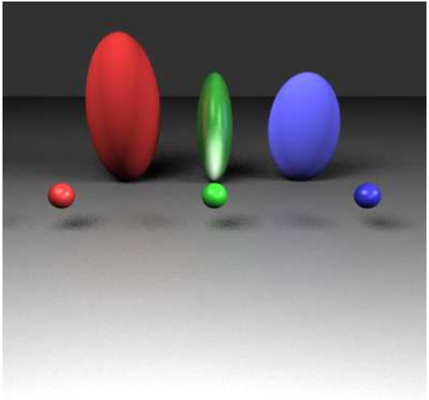
Examples





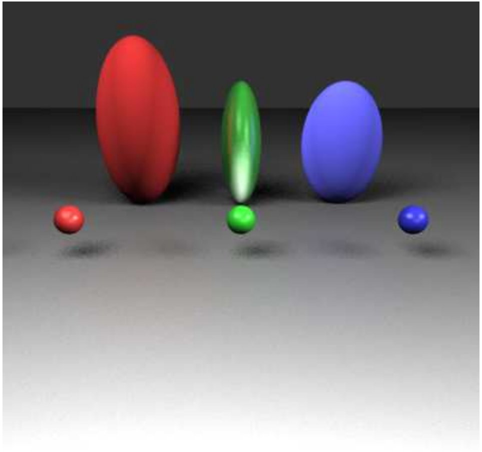


Object instancing



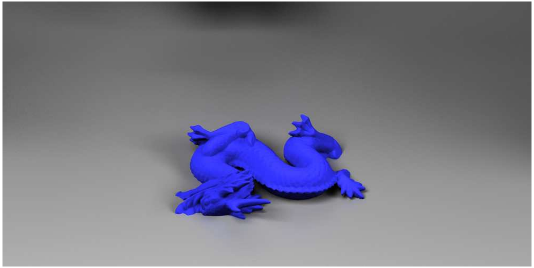
- Conserve memory footprint - reuse object geometry
- Create interesting scenes - Manipulate and transform objects

Focus on ellipsoid spheres



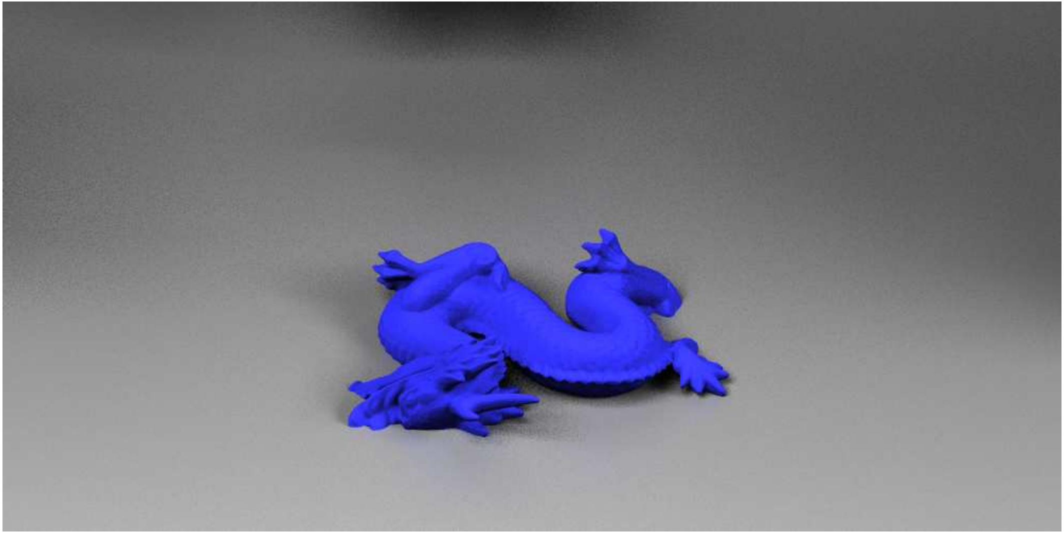
How can these be produced?

Another example to consider:



What is the blue dragon's local coordinate system? (*Hint: You can assume Cartesian Coordinates are as I'd draw them on the board*)

Rotate object about X axis by -90 degrees



Rotate object about X axis by -90 degrees

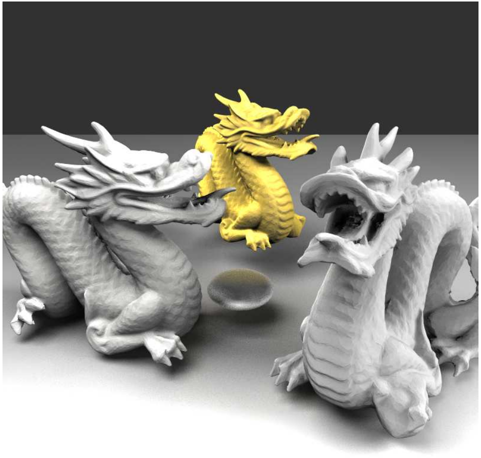
Rotate object about X axis by -90 degrees

Translate in +Y

Translate in +Y

Translate in $+Y$

Translate in +Y



How about other dragons?

- Perform additional transformations to create other instances of dragons
- Scale, translate, rotate

Object Instancing Requirements

- New class in renderable object hierarchy - *InstancedObject*
- Support for matrix transformations of object geometry

Object Instancing Requirements

class InstancedObject

- New class understands instancing and transformations
- Maintains a pointer to the base renderable object that was loaded only once (!!)
- Contains a matrix that describes how to transform the base object geometry
- Works seamlessly with other objects and ray tracer codebase

Object Instancing Requirements

class InstancedObject

- New class understands instancing and transformations
- Maintains a pointer to the base renderable object that was loaded only once (!!)
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- Works seamlessly with other objects and ray tracer codebase

- Will need a new list of *special* objects so that the base models are only loaded once!

3D Linear Transformations

We will focus on 3D transformations:

- Scale - s_x, s_y, s_z - change the size of an object
- Rotate-Z - θ - rotate about the Z axis
- Rotate-Y - θ - rotate about the Y axis
- Rotate-X - θ - rotate about the X axis

3D Linear Transformations

$$\text{Scale}(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix}$$

3D Linear Transformations

Rotations

$$\text{Rotate-X}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

$$\text{Rotate-Y}(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

$$\text{Rotate-Z}(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What properties do these matrices all have?

3D Linear Transformations

Rotations

$$\text{Rotate-X}(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\theta & -sin\theta \\ 0 & sin\theta & cos\theta \end{bmatrix}$$

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$$\text{Rotate-Z}(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What properties do these matrices all have?

- Rows are orthogonal to each other
- Columns are orthogonal to each other
- Represent the basis vectors of some rotation

Arbitrary Rotations

General form for all rotations:

$$R_{uvw} = \begin{bmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{bmatrix}$$

These are the components of a set of orthogonal (basis) vectors!

- $\vec{u} = x_u\vec{x} + y_u\vec{y} + z_u\vec{z}$
- $\vec{v} = x_v\vec{y} + y_v\vec{y} + z_v\vec{z}$
- $\vec{w} = x_w\vec{z} + y_w\vec{y} + z_w\vec{z}$

So, with basis vectors being orthogonal,

- $\vec{u} \cdot \vec{u} = \vec{v} \cdot \vec{v} = \vec{w} \cdot \vec{w} = 1$
- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{u} = 0$

Arbitrary Rotation Matrices - Meaning

So, what does that mean. Start by multiplying $\vec{u}$ by R_{uvw}. What happens?

$$R_{uvw}\vec{u} = \begin{bmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{bmatrix} \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix} = \begin{bmatrix} x_u x_u + y_u y_u + z_u z_u \\ x_v x_u + y_v y_u + z_v z_u \\ x_w x_u + y_w y_u + z_w z_u \end{bmatrix}$$

Well, for starters, $R_{uvw}\vec{u}$ is really the dot product between the rows and $\vec{u}$:

$$R_{uvw}\vec{u} = \begin{bmatrix} \vec{u} \cdot \vec{u} \\ \vec{v} \cdot \vec{u} \\ \vec{w} \cdot \vec{u} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \vec{x}$$

Similarly, $R_{uvw}\vec{v} = \vec{y}$ and $R_{uvw}\vec{w} = \vec{z}$!

Arbitrary Rotation Matrices - Meaning

Thus,

- R_{uvw} takes the basis $\vec{u}\vec{v}\vec{w}$ to the Cartesian coordinate system via a rotation operation.

How do you go back from the Cartesian coordinate system to the $\vec{u}\vec{v}\vec{w}$ basis?

Arbitrary Rotation Matrices - Meaning

Thus,

- R_{uvw} takes the basis $\vec{u}\vec{v}\vec{w}$ to the Cartesian coordinate system via a rotation operation.

How do you go back from the Cartesian coordinate system to the $\vec{u}\vec{v}\vec{w}$ basis?

By the inverse of R_{uvw}

Arbitrary Rotations - From XYZ to UVW

To go from the Cartesian coordinate system to the uvw system, we use R_{uvw}^{-1}

Inverse of R_{uvw} is R_{uvw}^{T}, or the transpose of R_{uvw}

- If R_{uvw} is a rotation matrix with orthogonal rows, the R_{uvw}^{T} is a rotation matrix with orthogonal columns
- Inverse of an orthogonal matrix is always its transpose
- And, R_{uvw}^{T} is in fact R_{uvw}^{-1}

Thus,

$$R_{uvw}^{T}\vec{x} = \begin{bmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix}$$

Similarly, $R_{uvw}^{T}\vec{y} = \vec{v}$ and $R_{uvw}^{T}\vec{z} = \vec{w}$!

Rotating about Arbitrary Axis

You now have enough foundation to rotate an object about an arbitrary axis not just the Cartesian axis!

1. You must create an orthonormal basis about the arbitrary axis (you know how to do that!)
2. Rotate the basis to the Cartesian coordinate system, R_{uvw}
3. Apply your rotation about the Z-axis
4. Rotate back to the the original basis frame from the Cartesian coordinate system, R_{uvw}^T

$$M = \begin{bmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{bmatrix}$$

Affine Transformations and Translation

Thus far, all discussion has been with transforms that have the form:

$$x' = a_{11}x + a_{12}y$$

$$y' = a_{21}x + a_{22}y$$

These transforms can only scale and rotate objects and cannot move them!

What we need is to be able to perform

$$x' = x + x_t$$

$$y' = y + y_t$$

However, it is not possible to add that translation to a 2x2 matrix!

Affine Transformations and Translation

To achieve what we want, we will use 3x3 matrices (for 2D transformations) and represent point (x, y) as $[xy1]^T$.

$$\begin{bmatrix} a_{11} & a_{12} & x_t \\ a_{21} & a_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix}$$

All vectors must now have a 1 in the last place!

$$\begin{bmatrix} x\prime \\ y\prime \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & x_t \\ a_{21} & a_{22} & y_t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11}x + a_{12}y + x_t \\ a_{21}x + a_{22}y + y_t \\ 1 \end{bmatrix}$$

These are called *affine transformations* using *homogeneous coordinates*!

Affine Transformations

Some issues:

- Transformations were for points!
- What about offsets, displacements, or directions?

For locations, last coordinate will be 1.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

For directions, last coordinate will be 0.

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Affine Transformations in 3D

Works fine in 3D:

$$\begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + x_t \\ y + y_t \\ z + z_t \\ 1 \end{bmatrix}$$

Homogeneous Transform

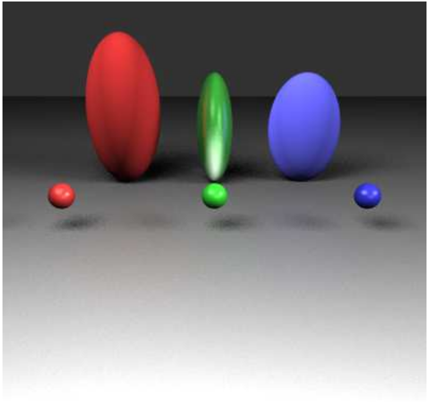
This allows us to define a 4x4 Matrix that holds a rotation and a translation (with rotation happening first):

$$\begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & x_t \\ a_{21} & a_{22} & a_{23} & y_t \\ a_{31} & a_{32} & a_{33} & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

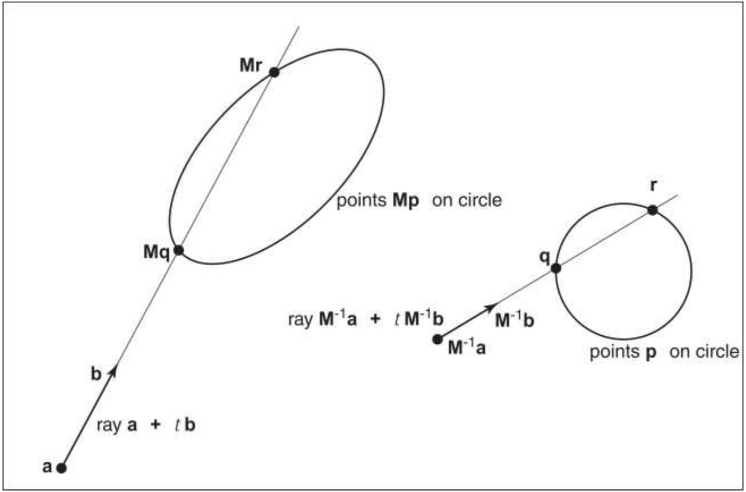
Instancing Overview

Benefits of Instancing Objects

- Achieves object complexity by manipulating base objects
- Saves memory footprint by re-using objects as necessary
- Forces your code to deal with transformations



Instancing Overview



Instancing Algorithm

InstanceObject::intersect(Ray r, t_{min}, t_{max}, hitRecord rec)

Ray $r' \leftarrow \mathbf{M}^{-1} r.\vec{origin} + t\mathbf{M}^{-1} r.\vec{direction}$
if (baseObjectPtr→intersect(r', t_{min}, t_{max}, rec)) **then**
  rec.$\vec{n} \leftarrow (\mathbf{M}^{-1})^T$ rec.$\vec{n}$
  return true;
else
  return false;
end if

Instancing

Take note that

- Only requires adding a new class (InstancedObject) to handle the manipulations of the Rays
- Object pointers to the actual instanced objects are maintained (and likely reused amonst several InstancedObjects)

Take special note that

- The normal is transformed too!!
- Why?

Transforming Normal Vectors

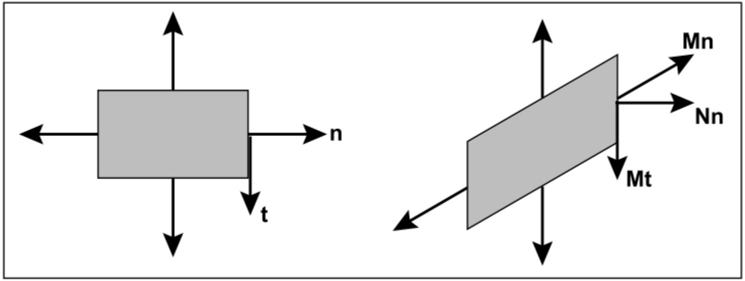
- What happens when you transform objects in the scene?

Transforming Normal Vectors

- What happens when you transform objects in the scene?
- Does it make sense to transform their normal vectors? What happens?

Transforming Normal Vectors

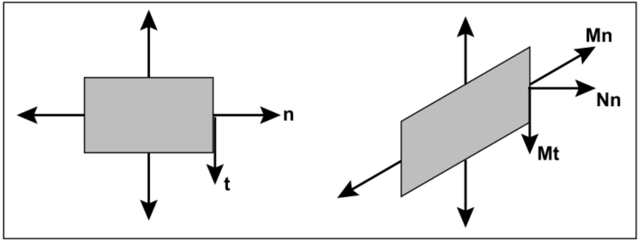
- What happens when you transform objects in the scene?
- Does it make sense to transform their normal vectors? What happens?
- Tranforming normal vectors with the transformation matrices:



- What's wrong?

Transforming Normal Vectors

We need to find a **N** such that normals are correctly transformed!



Need to maintain:

$$\vec{n}^T \vec{t} = 0$$

and need to find **N** such that

$$\vec{t}_M = \mathbf{M}\vec{t}$$

$$\vec{n}_N = \mathbf{N}\vec{n}$$

Transforming Normal Vectors

Recall that $\mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$, so

$$\vec{n}^T\vec{t}$$

$$= \vec{n}^T\mathbf{I}\vec{t}$$

$$= \vec{n}^T\mathbf{M}^{-1}\mathbf{M}\vec{t} = 0$$

Transforming Normal Vectors

Recall that $\mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$, so

$$\vec{n}^T \vec{t}$$

$$= \vec{n}^T \mathbf{I} \vec{t}$$

$$= \vec{n}^T \mathbf{M}^{-1} \mathbf{M} \vec{t} = 0$$

and now, let's try to see this as dot products:

$$(\vec{n}^T \mathbf{M}^{-1})(\mathbf{M}\vec{t}) = 0$$

Transforming Normal Vectors

Recall that $\mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$, so

$$\begin{aligned} &\vec{n}^T\vec{t} \\ &= \vec{n}^T\mathbf{I}\vec{t} \\ &= \vec{n}^T\mathbf{M}^{-1}\mathbf{M}\vec{t} = 0 \end{aligned}$$

and now, let's try to see this as dot products:

$$(\vec{n}^T\mathbf{M}^{-1})(\mathbf{M}\vec{t}) = 0$$

$$(\vec{n}^T\mathbf{M}^{-1})\vec{t}_M = 0$$

Transforming Normal Vectors

Thus, the left part of this expression is the vector that is perpendicular to $\vec{t}_M$:

$$(\vec{n}^T \mathbf{M}^{-1})\vec{t}_M = 0$$

This must be a row vector so we use the transpose to represent a column vector (our standard vectors) as a row vector:

$$\vec{n}_N^T = \vec{n}^T \mathbf{M}^{-1}$$

Transforming Normal Vectors

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thus,

$$\vec{n}_N = (\vec{n}^T \mathbf{M}^{-1})^T$$
$$\vec{n}_N = (\mathbf{M}^{-1})^T \vec{n}$$

and so,

$$N = (\mathbf{M}^{-1})^T$$

Transforming Normal Vectors

Make sure that you tranform your normal vectors appropriately, when instancing objects!

$$N = (\mathbf{M}^{-1})^T$$

Accelerating the Ray Tracer

Objective

Reduce the overall complexity by focusing on being more efficient about which intersection tests to perform.

Efficiency by limiting the intersection tests

- What is the complexity of your ray tracers, thus far?

Accelerating the Ray Tracer

Objective

Reduce the overall complexity by focusing on being more efficient about which intersection tests to perform.

Efficiency by limiting the intersection tests

- What is the complexity of your ray tracers, thus far?
- Focus is on complexity of a single intersection test
- $O(N)$, in which a ray must be checked with N objects in the scene
- Essentially, linear search.

Reducing the complexity

- Need to bring complexity down to sub-linear time
- We'll start by find a simpler intersection test to perform

Bounding Boxes

Axis-aligned box that surrounds your objects. Uses the minimum and maximum extents in the three dimensions to form the box around an object. Can be specified with two 3D vectors.

Bounding Boxes

- Different than intersection tests for spheres and triangles
- Why?

Bounding Boxes

- Different than intersection tests for spheres and triangles
- Why?
- With spheres and triangles (or other objects), intersection test computes where the ray hits the object (*i.e.* $\vec{p}(t) = \vec{r}_o + t\vec{r}_d$
- With bounding boxes, we only need to know **IF** the ray hits the box, not where

Bounding Box Intersection Tests

Start by examining the 2D version of the bounding box.

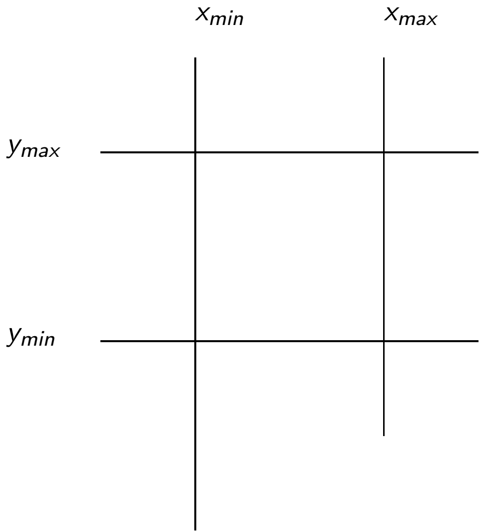
2D Bounding Box Definition

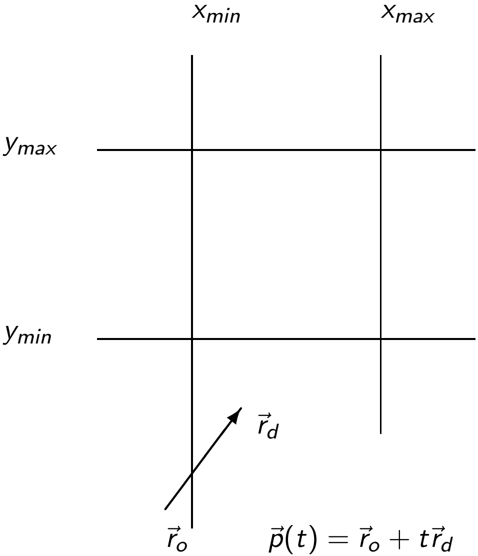
Bounding box is defined by the boundaries:

$$x_{min}, x_{max}, y_{min}, y_{max}$$

A point (x, y) is in the bounding box if

$$(x, y) \in [x_{min}, x_{max}] \times [y_{min}, y_{max}]$$


x_{min}
x_{max}
y_{max}
y_{min}



$$\vec{p}(t) = \vec{r}_o + t\vec{r}_d$$

Compute intersections of Bounding Box

Recall that our ray is defined by

$$\vec{p}(t) = \vec{r}_o + t\vec{r}_d$$

That's a vector equation, but we can also think of it in terms of the sub-components. For instance,

$$x_{min} = r_{x_o} + tr_{x_d}$$

$$x_{max} = r_{x_o} + tr_{x_d}$$

$$y_{min} = r_{y_o} + tr_{y_d}$$

$$y_{max} = r_{y_o} + tr_{y_d}$$

Compute intersections of Bounding Box

$$x_{min} = r_{x_o} + tr_{x_d}$$

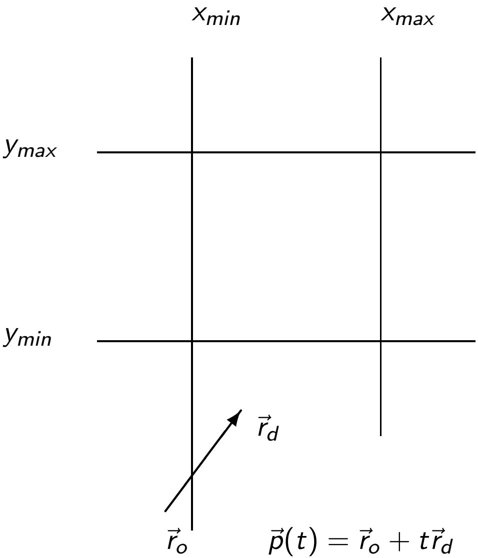
$$x_{max} = r_{x_o} + tr_{x_d}$$

$$y_{min} = r_{y_o} + tr_{y_d}$$

$$y_{max} = r_{y_o} + tr_{y_d}$$

So, solve for the t values associated with each of these parameters:

$$t_{x_{min}} = \frac{x_{min} - r_{x_o}}{r_{x_d}}$$



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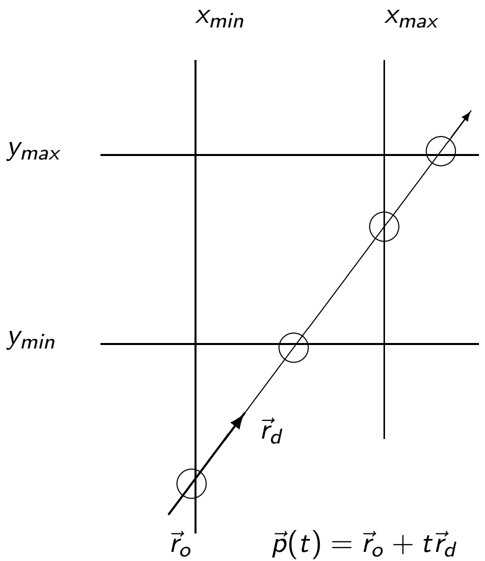
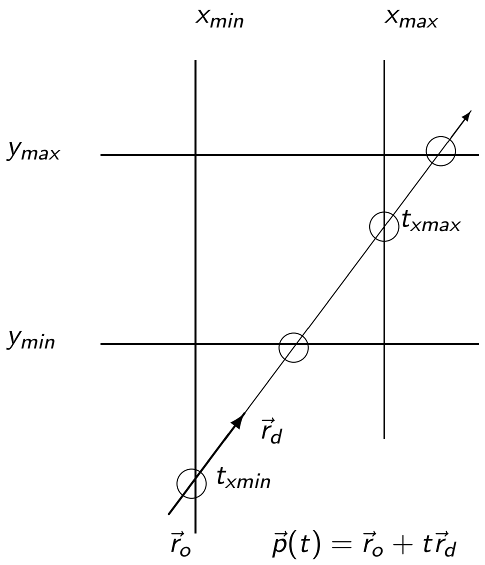


$$\vec{p}(t) = \vec{r}_o + t\vec{r}_d$$


x_{min}
x_{max}
y_{max}
y_{min}
t_{xmax}
$\vec{r}_d$
t_{xmin}
$\vec{r}_o$
$\vec{p}(t) = \vec{r}_o + t\vec{r}_d$

Bounding Box Computation

$$t_{x_{min}} = (x_{min} - r_{o_x})/r_{d_x}$$
$$t_{x_{max}} = (x_{max} - r_{o_x})/r_{d_x}$$
$$t_{y_{min}} = (y_{min} - r_{o_y})/r_{d_y}$$
$$t_{y_{max}} = (y_{max} - r_{o_y})/r_{d_y}$$

if $(t_{x_{min}} > t_{y_{max}})$ or $(t_{y_{min}} > t_{x_{max}})$ **then**
  return false;
else
  return true;
end if

Bounding Box Computation

What happens when r_{d_x} or r_{d_y} is negative?

Bounding Box Computation

What happens when r_{d_x} or r_{d_y} is negative?

- The ray is basically coming from a different side
- Take into account when calculating the t values
- For instance, for x:

if $r_{d_x} \geq 0$ **then**

$$t_{x_{min}} = (x_{min} - r_{o_x})/r_{d_x}$$
$$t_{x_{max}} = (x_{max} - r_{o_x})/r_{d_x}$$

else

$$t_{x_{min}} = (x_{max} - r_{o_x})/r_{d_x}$$
$$t_{x_{max}} = (x_{min} - r_{o_x})/r_{d_x}$$

end if

You'll need to adapt for Y, and Z, of course.

Changes to the Scene Structure

To support the next assignment, we will need

- Transforms
- Mesh objects
- Instanced objects

The XML file will support them as follows:

Transforms

```
<transform name="xform1">
  <translate>12 13 14</translate>
  <rotate axis="X">90</rotate>
  <rotate axis="Y">120</rotate>
  <rotate axis="Z">10</rotate>
  <scale>2 2 2</scale>
</transform>
```