FoSAP Exercise 1

Group 3

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Introduction

Welcome To this special endeavour, where I write every single thing in LaTeX How is any of this gonna work?
Simple: With patience and a format definition. Therefore, please let me define the format of this.

THE FORMAT

- Every file will be named similar to the sections in here, so `2.1-stack_exercise.c` is Exercise 2, section 1.
- Every Solution **WILL** be in this pdf along with everything predefined by the exercise.
- Any explanation will be both in this PDF as well as in each file.
- This explanation will be in each PDF, in case someone who doesn't know the format tries to correct the exercises

1 H1

`Exercise:` Let $v, w \in \Sigma^*$ with $vw = w^R v$ and $|w| \geq |v|$.Prove $(vw)^R = vw$
Because $(vw)^R = vw \Leftrightarrow$ vw is a palindrome, we only have to show that vw is a palindrome when $vw = w^R v$. Now we have the following cases:

$|v| = |w|$: $v = w^R \land w = v$
$\Rightarrow w = v \land w = w^R$
Therefore w and v must be a palindrome, and thus vw is a palindrome as well

$|v| = 0$: then $vw = w^R = w$ because $v = \epsilon$, therefore vw is a palindrome

$1 \leq |v| < |w|$:

$$\begin{aligned} \text{let } v &= v_1, \ldots, v_n \\ \text{then } w^R &= v_1, \ldots, v_n, w'_n, \ldots, w'_1 \\ \text{and } w &= w'_1, \ldots, w'_n, v_n, \ldots, v_1 \\ \Rightarrow vw &= v_1, \ldots, v_n, w'_1, \ldots, w'_n, v_n, \ldots, v_1 \\ \text{therefore: } w^R v &= v_1, \ldots, v_n, w'_n, \ldots, w'_1, v_n, \ldots, v_1 \end{aligned}$$

Because $vw = w^R v$, $w'_1, \ldots, w'_n = w'_n, \ldots, w'_1$ applies and therefore $w'_1, \ldots, w'_n$ must be a palindrome. Furthermore, $v_1, \ldots, v_n = v_n, \ldots, v_1$, making v a palindrome as well. Since w' and v, which can be found on both sides of w', are both palindromes, vw must be a palindrome as well, Proving the statement $vw = w^R v$.

2 H2

Solution: $((((AB)^+A)((((CB)^+ + (C(BA)^+)^+)^+ + ((CA^+) + (C(AB)^+)^+)^+)^*C))^+$

Explanation:

$$\begin{aligned} ((AB)^+A) &: \text{ Every option before a C} \\ ((CB)^+ + (C(BA)^+)^+)^+ &: \text{ Every transition } C \to B \\ ((CA^+) + (C(AB)^+)^+ &: \text{ Every transition } C \to A \\ C &: \text{ Makes sure that we always go through before ending} \end{aligned}$$

3 H3