**Isle Royale Worksheet 4**

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# Review of logistic growth

Before the wolves arrived on Isle Royale, the moose population grew to around 500 and stayed there. This was because the death rate due to starvation equaled the birth rate. The moose were eating balsam fir and other plants just as fast as those plants could grow. You may recall that our assumptions for this ecosystem were:

* The only way the moose population changed was via births and deaths.
* The moose births were proportional to the moose population. In particular, the number of moose born each year as 0.2 times the moose population. That is, the moose birth rate was a constant 0.2.
* The moose deaths were proportional to the moose population.
* However, the moose death rate was not constant. It starts near zero and rises to the moose birth rate as the number of moose approaches the carrying capacity of the ecosystem. In particular,  
  moose death rate = moose birth rate\*(moose population / carrying capacity)

This kind of capacity-limited growth is called logistic growth. The logistic function, like the exponential function or the sine function, is a well-studied mathematical function with many interesting properties. However, we won’t bother with discussing the logistic function here. It suffices that you know that this type of curve (like a squashed S) is called a logistic curve.

# Competition for a resource

Suppose that it wasn’t wolves that immigrated across the ice to Isle Royale, but instead some deer came over. The deer and moose eat similar food, so they are competing for a resource. Intuitively, the two populations interact, but not like the wolves and moose interacted. Thus, we need a new kind of model.

If you think about the equation for moose death rate, the term *moose population / carrying capacity* represents how close the moose are to exhausting the carrying capacity. The more moose, the closer this ratio is to 1. When it finally hits 1, then the moose population stabilizes because moose are dying as fast as they are being born. Now if the deer are also using up the same resource, then perhaps the ratio should *be (moose population + deer population)/carrying capacity*. That is, when the number of animals that eat balsam fir etc. reaches the carrying capacity then the moose are starving just enough to stabilize the moose population. The deer population should be limited in the same way. That is, when the ratio *(moose population + deer population)/carrying capacity* reaches 1, then the deer deaths should equal the deer births, and the deer population should stabilized.

This is all fine, but it assumes that moose and deer eat the same amount and that starvation has the same effect on both of them. That assumption is rather implausible. Because we don’t know exactly how the deer population effects moose starvation, nor do we know exactly how the moose population effects deer starvation, let us just add parameters (lets call them just P1 and P2 for now) to the model that allow us to try different values for them until we get the model’s curves to match the actual system behavior. In particular, let us assume:

*Moose death rate = moose birth rate\*((moose population + P1\*deer population)/carrying capacity)*

*Deer death rate = deer birth rate\*((deer population + P2\*moose population)/carrying capacity)*

The parameters P1 and P2 are positive numbers, but it isn’t clear in advance what they should be, so you should find values by trial and error so that the model’s predictions fit the “observed” system behavior shown below:



This assumes that in 1900, 20 moose and 2 deer immigrated to Isle Royale.

* Build a model for the moose-deer ecosystem