



# Simultaneous Measurement of absolute distance and displacement using self-mixing interferometry.

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### Summary:

This project focusses on the simulation of a low-cost simultaneous self-mixing interferometry system (SMI), using a simulation software package called MATLAB. This system consists of a laser, micro-lens, and an external target. The light from the laser will reflect off of the moving target and re-enter the laser cavity, modulating the amplitude and frequency of the laser power. The simulation requires the parameters of the distance and displacement to be pre-determined before the simulation of measurement occurs. The scripts that allow the calculation of the external round trip phase for both the absolute distance and the displacement, must be combined before being called into a self-mixing method. A power variation graph will display the resulting waveform providing the information to discover and calculate the measurement values required to complete the objective of this research project. The methods of calculation will need to take into consideration the wavelength of the laser to be able to determine the final measured values that had been simulated.

### Acknowledgements:

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I would also like to thank my supervisors, Yuanlong Fan and Sivagunalan Sivanathan for giving me the guidance needed to develop my understanding of the subject and pointing me in the right direction with patience when I hit several mental brick walls.

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### Introduction:

With an increase of technology and preventative measures for infrastructure, laser measurements are a cheap and highly accurate form of vibration and distance measurement. Just as A.J. Bougard and B.R. Ellis have done in 2000 [13] while studying the behaviours of buildings, the area of measurement being out of reach. Otherwise more unorthodox methods would have to be used while measuring in dangerous environments.

The use of self-mixing interferometry to measure the distance or the displacement of an object in motion is a common, effective, and cheap method to achieve these goals. The earlier version of an interferometry system is called the Michelson system that had a complex setup where the user had to manually operate the machine to measure displacement.

Then when the 1980s came along, it brought with it the concept of a self-mixing signal using only a laser, lens, and a target. A paper from 2014 [6] shows the concept of using an injection current and a sinusoidal motion from a target to calculate the absolute distance and displacement of said target. The paper also provides a concise and easy to understand list of scripts that perform these objectives individually.

It explores the methods in which a sinusoidal motion waveform and an injection triangle current can be constructed in a simulated environment to artificially create these modulation methods. Modulation methods that are required to calculate the absolute distance and displacement values. Simultaneously mixing these two signals, in theory, would allow the user to calculate both the absolute distance and displacement simultaneously using the self-mixing method.

Simultaneous self-mixing the absolute distance and displacement should be able to produce a power variation graph capable of containing the calculations for these measurements, allowing for the multi-calculation of said measurements. As the goal of this project is to find a method in which to do so.

This project aims will have the focus of creating a MATLAB program that simulates a low-cost self-mixing interferometry (SMI) system. A simulation model that consists of a laser and an external target, the external target being simulated as a speaker emitting a constant frequency. The objectives are to review the measuring methods for the distance and displacement, modelling the system by solving the phase equation. Developing different methods of simultaneous measurement will then be conducted before producing a series of tests to analyse the effect the given method had produced. Future discussions and improvements will then be discussed along with the final conclusion.

### Background Research:

As a baseline for this type of subject, a simple setup of an SMI, or self-mixing interferometry system, will include an LD, laser diode, component that will emit a laser beam from the diode cavity that will then become a concentrated beam of light with the aid of a focusing lens, before finally having the laser reach its target object. The laser will then reflect off of the target back into the laser diode cavity, causing a self-mixing event to occur. Typically, a target object that is in motion. The diagram for this type of setup can be seen in a diagram for figure 1, on page 25, in the 2003 paper titled 'Self-mixing laser diode vibrometer' [1][2]. The reason for requiring the target to be in motion is because there needs to be a slight change in distance for the coherent laser signal to be reflected and to interfere with the light, which the amplitude can then be observed on an oscilloscope. This can then be used to calculate the displacement motion of the object.

To observe the absolute distance of the object, an injection current will need to be injected into the laser diode component before the signal gets emitted. Adding this periodic injection current will induce a change in the wavelength of the light being emitted, simulating a pseudo motion effect instead of requiring the target to be in motion, instead the target can be at a standstill.

This type of setup was used in the 1980 paper, first experimenting, and researching the optical feedback effects on injection laser properties [11]. This paper discusses the use of an injection current to induce the effects described above and then observing the results. It noted that the spectral changes made is reflective on if the external mirror distance is an equal to an integral multiple of the effective laser diode cavity length or not. The definition of an integral multiple being the integer multiples of a value of  $x$ .

The Optical feedback level is an important factor for creating a clear modulating signal as this will, in turn, dictate the loss of 'fringes' representing half-wavelength displacements, which can also be seen as the 'rings' in the projection of a 'Michelson interferometry' system. The optical feedback variable is represented by 'C'. When the value of C is  $\ll 1$  then the modulated signal will retain its number of 'fringes'. [5.1] However, even if the C value is more than 1 by a small amount, the signal can still retain all of its intended fringes up to a certain point. During this, the value of  $F(\phi)$  is mimicking the shape of the motion sinusoidal waveform of the target with the modulation index increasing "for an increasing level of optical feedback" [3]. When  $C \approx 1$ , then a small signal distortion and loss of 'fringes' could be seen, and at  $C > 1$  then the signal starts to represent a sine wave and manifesting signs of hysteresis. [3] [1] From this it can be inferred that the higher the feedback value is the higher the loss of 'fringes' there will be, essentially replicating the original signal. [5.2]

Early Model Systems of Interferometry: -

The earlier models for an interferometry system were the Michelson interferometer invented in 1852 – 1931 [12]. A system that uses multiple mirrors and lenses and a laser diode and laser detector. The set up would essentially have the laser beam be directed to a splitter lens, or half mirror, that will divide the laser, splitting off to the fixed reference mirror and a movable mirror. The light will then be reflected off of the mirrors and converge back through the splitting lens. This convergence will then modulate the wavelengths of the converging lasers, which can then be projected onto a surface. This projection will be displayed in the form of a series of looping rings. These rings each represent half or 180 degrees of a waveform. If the distance of the movable mirror target is altered, then there will be the observation of the ring fringes beginning to cycle every 180 degrees. By counting these number of fringes as they cycle, then it is possible to calculate the change of distance when multiplying the number of 360-degree changes by the wavelength of the chosen laser. These wavelengths can approximately range from 850nm to 630nm, which is largely dependent on the colour of the chosen laser.

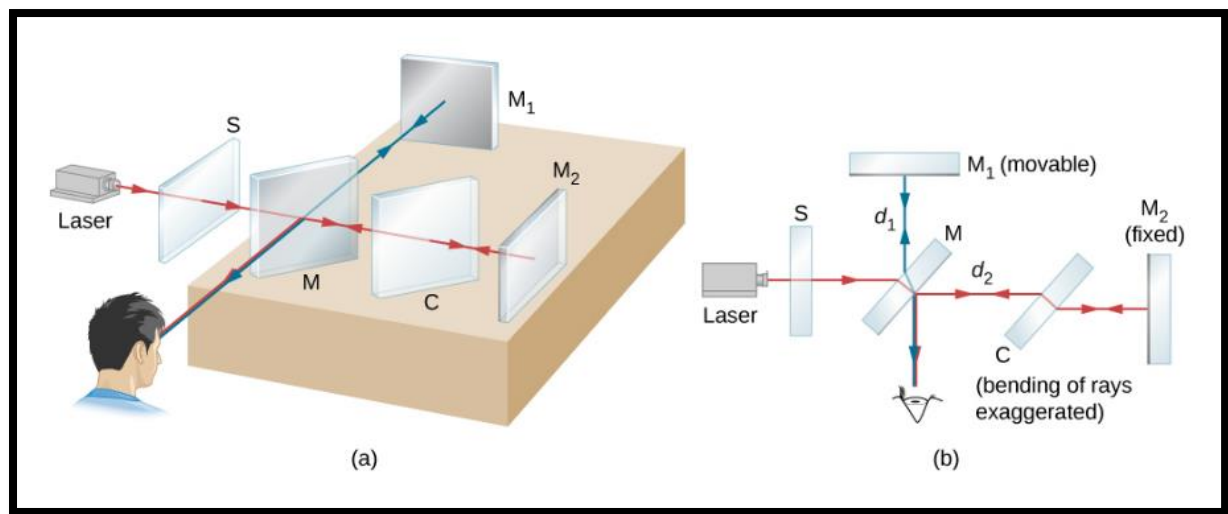


Figure 1 setup system diagram for a Michelson interferometer

The fact that the two beams originated from the same laser source means that they are coherent and are able to interfere with each other, creating this effect. It can also be noted that one beam crosses through the splitter mirror/lens more often than the other beam, so a secondary lens is used as a compensator, lens C in the diagram. Lens C is the same thickness as the lens splitter, lens M. The reason for the inclusion of lens C is to compensate for the phase difference that occurs when the one divergent beam does not normally pass-through M the same number of times, compared to the other beam that is being directed towards the movable mirror target.

This was then improved upon when the study of self-mixing using a single laser was first being experimented with. The inclusion of an injection current signal drives a slight change in the colour of the laser beam. The change in colour would change the wavelength of the laser, which can be seen in the data sheet for an Osram Opto PL 520\_B1\_2 Green Laser Diode, 515nm 30mW [14]. This method of wavelength sweeping will allow the laser to acquire the characteristics to measure distances of an object. [2] [4]

### Project Specifications:

For this particular project, the only requirement to conduct the experiments is by doing so mathematically using the simulation program MATLAB. MATLAB will conduct the mathematical equations as specified in the pre-written program given to it. However, it needs to be given specified values for variables that are included in said equations, which can easily be achieved by creating a second script declaring the variables and calling the secondary script as a class method. This means that the calculations could then be achieved by triggering just this one script and then the rest would follow. MATLAB would also give the ability of generating a graph to preview the results from, allowing for a clear representation of the calculated modulated waveform.

After simulating the process of a self-mixing interferometry system, using scripts in MATLAB, a way of combining the displacement and absolute distance measurements must then be discovered before calculating their values using a power variation graph.

### Intended Method:

Using the scripts from the 2014 paper [6], Alterations could then be made to accommodate for the goal of simultaneously measuring the absolute distance and displacement of a self-mixing system. Looking through both scripts it would be easy to simply take integral elements from each script and place them into a single script where they could then easily interact with each other.

It would be possible to call each script from a separate class file but placing all the required calculations into one class would greatly improve the efficiency of this application. It would also simplify the project in doing so.

This script will be easily implemented using the MATLAB programming package. The software package that is used to simulate complex mathematical simulations with the instructions of a programming language.

Looking towards the behaviours of past research work and their findings may prove to be useful when thinking on the next and integral following step.

Once the groundworks for the simulation have been completed then a series of tests must be conducted with the intent of finding a method of simultaneous measurement. As the waveforms are combined there must be a way to be able to distinguish between them in foresight. It is possible that the presented final combined waveform might make it difficult to distinguish between the two sets of fringes for the absolute distance and displacement.

Self-mixing Theory and simulation algorithm:

An SMI system, or an injection laser system, was researched in 1980 by Roy Lang and Kohroh Kobayashi. Knowing and even observing that an external feedback can make an injection laser multi-stable, causing a hysteresis effect that showcases a display of fringes. This hysteresis effect has been shown to occur when induced by a large external feedback value. Without this feedback value being used, the output undulation would be displayed as a flat linear incline with no distinguishable fringes being shown. This can be clearly seen in the graph below from this same 1980 paper by Roy Lang and Kohroh Kobayashi.

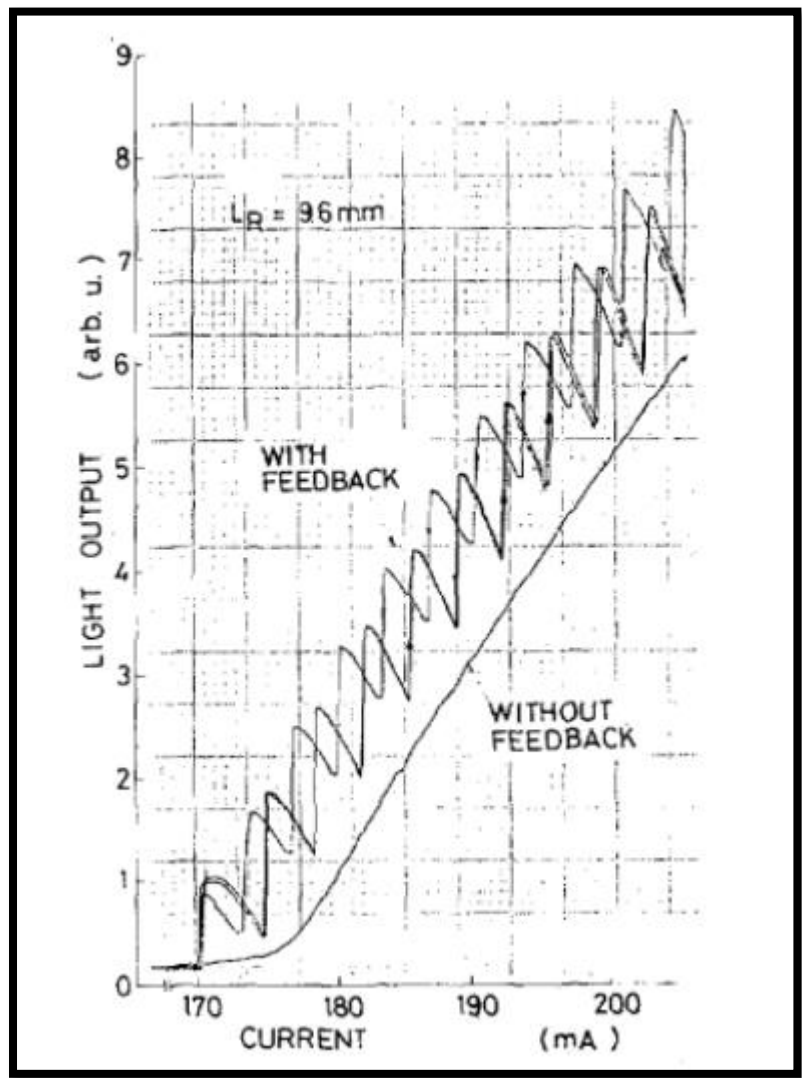


Figure 2 light output versus current curves for both the external feedback and without the external feedback

[11]

Complications with producing visible fringes can occur however, when the distance of the external mirror distance,  $L_R$ , is not an integral multiple of the effective diode cavity length,  $L_D$ . This is because, different longitudinal modes of the diode cavity are competing for lasing with the slightest change in the interference condition of the laser. An example of this effecting the fringes can be seen in the following figures...

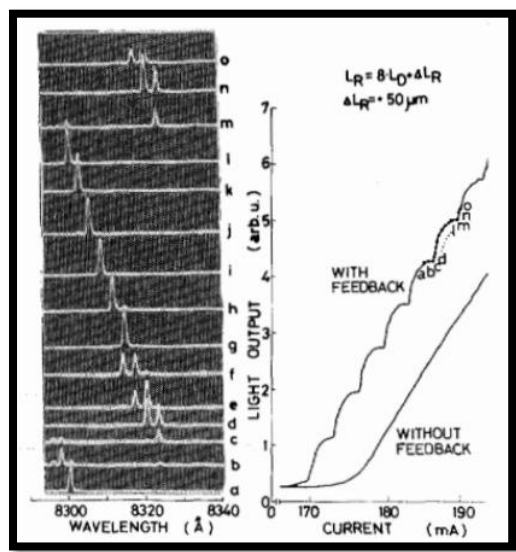


Figure 3 spectral change and output undulation with a current increase, when  $L_R$  is greater than  $50\mu m$  than the nearest integral multiple of  $L_D$ .

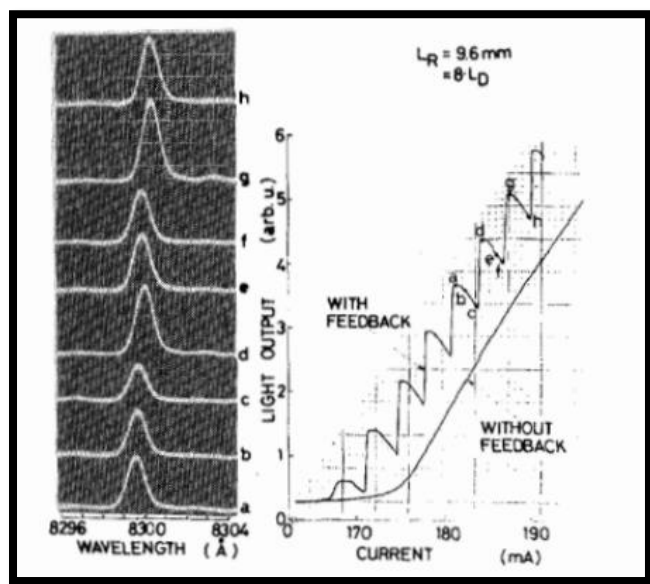


Figure 4 When  $L_R$  equal an integral multiple of  $L_D$ .

[11]

The difference is clear and easy to distinguish when  $L_R$  is and is not an integral multiple of the laser cavity length,  $L_D$  as the current increases over time. Even as the current increases, the change in the wavelength is continuous and smooth between points a to c on the light output against current graph, (L-I graph), before making a fast linear jump from c to d by around  $0.36\text{\AA}$ .

Using the series of demo simulations [6] of a self-mixing interferometry system. This SMI algorithm demonstrates how calculations for a self-mixing interferometry modulation system will work when applied with distance and displacement, external round trip phase modulated, waveforms.

The self-mixing power class called '*selmixpower.m*' is going to be the main function that shall be called during operation of the simulation. The purpose of this class is to simulate a self-mixing interferometer



system, using external round trip phase values from another class that would calculate the absolute distance or displacement self-mixed waveform for this simulation. For example, calculating absolute distance simultaneously with the displacement of a target.

Looking at the code in MATLAB, it calculates the power level at the sampled time calling variables from the specified task class, which I will refer to as the secondary class. The software would then be able to have the capability to check the provided feedback value, '**C**', from the previous class, '**selmixpower.m**' was called from, to ascertain if it were less than or equal to 1. If it is found to be equal, then it will call the '**boundsweak**' method which will calculate the **minimum** and **maximum** bounds by subtracting or adding the feedback '**C**' from the value of '**Phi0**'.

The '**else**' condition will be able to check to ascertain if '**C**' is higher than 1. And if it is, then the program jumps to the '**boundsstrong**' method. This method will make the value '**m**' persistent and checks to see if it has a value. If it does not have a value, then it is given the value of '**0**' by default to account for this. The '**persistent**' declaration basically means that the value for '**m**' will be stored between calls to the function. What follows is the calculation for the values '**mlower**' and '**mupper**' using the formula...

$$\frac{(Phi0 + atan(alpha) + acos\left(\frac{1}{C}\right) - \sqrt{(C \times C - 1)})}{(2 \times \pi) - 1.5}$$

... for the lower and...

$$\frac{(Phi0 + atan(alpha) - acos\left(\frac{1}{C}\right) + \sqrt{(C \times C - 1)})}{(2 \times \pi) - 0.5}$$

... for the upper.

'**If**' statements would then check to see if '**m**' were smaller than '**mlower**' and if '**m**' was higher than '**mupper**'. If either one is true, then the value of '**m**' would then become equal to the carriage that it was being compared to. After which '**m**' would then be used to calculate the value for '**phimin**' and '**phimax**' using the formulas...

$$(2 \times m + 1) \times \pi + acos\left(\frac{1}{C}\right) - atan(alpha)$$

... for '**phimin**' and...

$$(2 \times m + 3) \times \pi + acos\left(\frac{1}{C}\right) - atan(alpha)$$

... before returning to the power method.

A handle called '**excessphase**' is then given the formula...

$$@ (x) x - phi0 + C \times \sin(x + atan(alpha))$$

Another series of '**if**' statements would then happen to check if '**excessphase(phimin)**' would be more than '**0**' else if '**excessphase(phimax)**' is less than '**0**', then the value of '**phi**' would be equal to '**phimin**' or '**phimax**' respectively. If neither of these cases are true then the final '**else**' statement will then make...

$$phi = fzero(excessphase, [phimin, phimax])$$

... so, then it can find a point at which '**excessphase(x)**' is equal to '**0**'.

The value of '**power**' is then calculated and will be given by the formula '**cos (phi)**' and return the result to the original function it was called from.

This class is the base layer of the entire simulation and is repeatedly called for, in a loop, every required sample calculation because of this.

The next step is to be able to recreate this process that can be used, over and over, for the purposes of being able to calculate the absolute distance and displacement of a simultaneously self-mixing signal. This would require that the foundation class, that holds the external round trip phase of both the absolute distance and displacement, will be able to give a conclusive result that would lead to finding these calculated values.

### Literature reviews about Self-Mixing signals:

#### Absolute distance:

I have read a number of articles which discuss the theoretical conclusion to calculate the absolute distance and displacement as totally separate entities. To give an example of this, the use of a linear frequency sweep could be used to find the absolute distance [6] as stated in the 2014 paper by Russell Kliese. The distance from the laser to a particular stationary target can be calculated by the means of frequency modulation injecting an ostensibly triangular current waveform of the laser diode, inducing an optical frequency shift [6][8][9]. The purpose of the modulated injection current is to obtain variations of optical power when there are changes in the phase. These phase changes are typically caused by changes in the optical path length, depending on the value of L (distance) and d (displacement). This triangle waveform will help to induce an optical frequency shift, performing a linear frequency sweep. In a practical setting, external variables would create distortions in the ostensibly triangular waveform. Thermals and other external effects would distort the injected current triangle waveform. The triangle waveform is typically distorted from any thermal build-up from the cavity of the laser diode, causing a property change of the cavity length, index of refraction of the cavity and a change in mirror reflectivity for VCSELs components [9]. But, in simulations the triangle waveform could typically be pre-distorted to conform to, and reflect, the real-world conditions. This can be observed in the spacing of the peaks and the Fast Fourier Transform (FFT) methods. Adjusting the injection current modulation waveform can eliminate the differences between the average spacing of the peak in the processed optical signal [9]. This different average peak spacing is seen as two peaks in the FFT method in the frequency spectrum. Sometimes a single wide peak can be the product of multiple peaks being too close to each other. A higher spectrum resolution increase or an intentional introduction of thermal effects can then make obvious the separation of the multiple peaks. An example of this can be shown from reference [9] along the frequency domain.

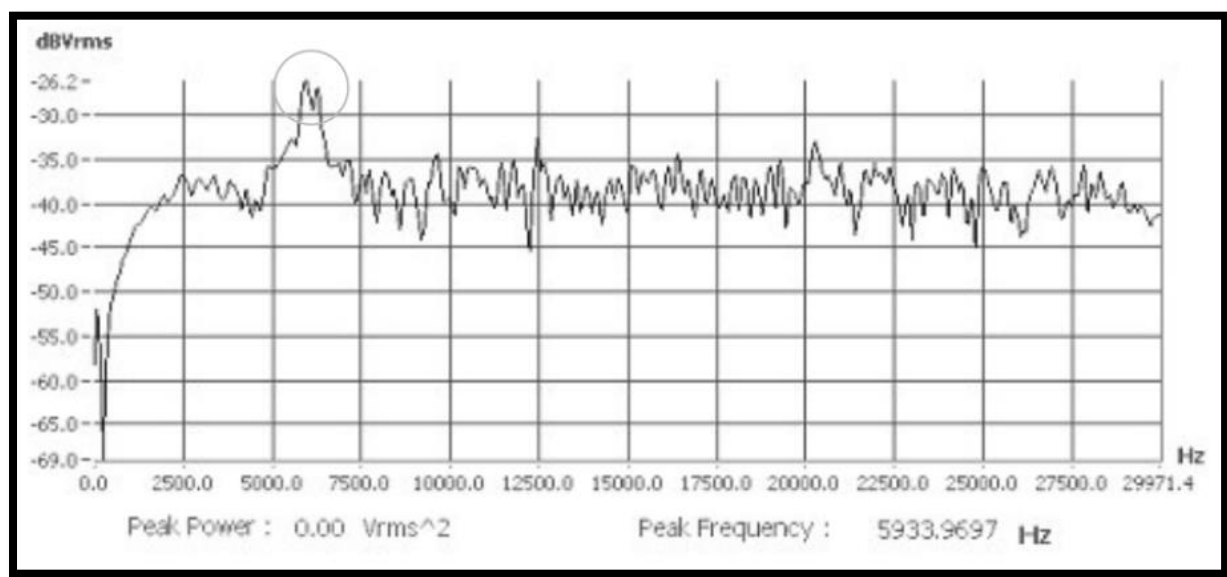


Figure 5 the observation of dual peaks because of thermal effects

[9]

From this graph it can be seen that the spacing between the high, top, peaks are shorter than the spacing between the low, bottom, peaks. The peaks make the most occurrence in the higher transition compared to the low transition. So, “In the frequency domain this means that the average peak spacing in the upward slope transition has a lower peak frequency in the FFT spectrum.” [9]. Thus, telling us that the upward slope is the dominant peak, between it and the downward slope peak.

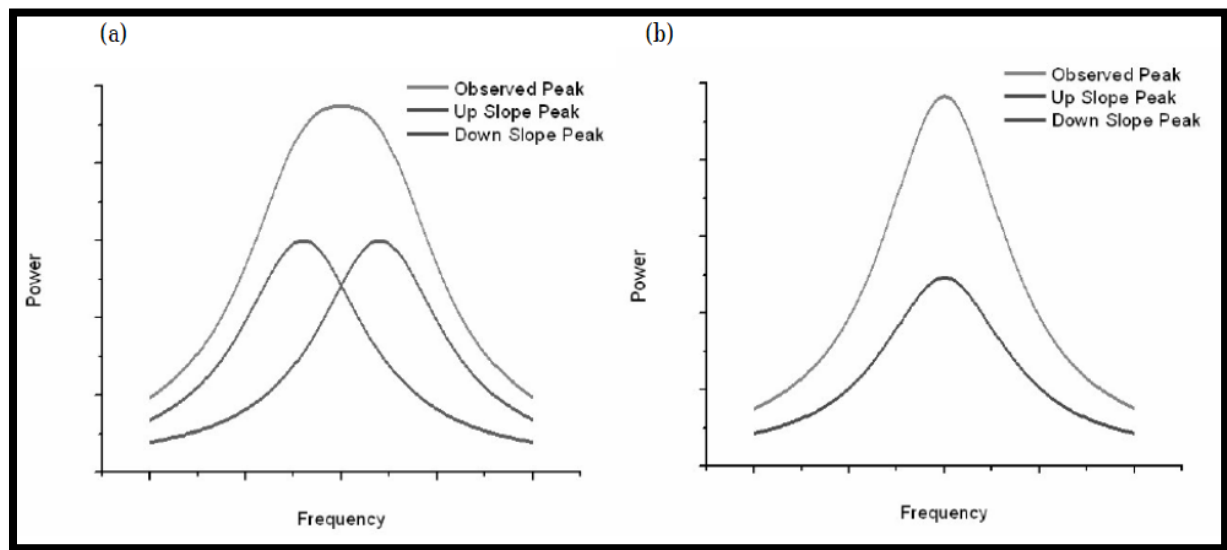


Figure 6 a) simplified effect of triangle wave modulation b) simplified effect of reshaped modulation

The figure above, from the “Optimum Injection Current Waveform for a Laser Range Finder Based on the Self-Mixing Effect” research paper by **Karl Bertling** and **J. R. Tucker** [9], shows non-reshaped, a), and reshaped, b), modulations respectively, showcasing the effects that it has. The intended purpose to reshape the modulation is to reduce the spectral width of the FFT spectrum, or to shift the centre frequency of the involved downward slope peak to line up with the upward slope peak.

The formula for a modulated waveform is shown below...

$$\Delta v(t_n) = \Delta F \times Tri(t_n)$$

" $(t_n)$ " representing the current sample being calculated out of an  $x$  number of samples. By multiplying the triangle function,  $Tri$ , and the frequency modulation coefficient,  $\Delta F$ , will calculate the frequency modulation waveform.  $v'_0$  can be used to express the laser frequency that is unmodulated and without feedback. When this is used in tandem with the frequency modulated waveform,  $\Delta v(t_n)$ , then the following formula occurs...

$$v_0(t_n) = v'_0 + \Delta v(t_n) = v'_0 + (\Delta F \times Tri(t_n))$$

In order for this to be used to achieve a power variation graph, the external phase must be obtained from this formula. To obtain this external phase, combining the formula above with,  $\phi_0 = 2\pi v_0 \tau_{ext}$ , will give the formula of...

$$\phi_0(t_n) = \frac{4\pi[v'_0 + (\Delta F \times Tri(t_n))]L}{c}$$

Which can then be further developed using  $c = v'_0 \times \lambda_0$  to rewrite the formula as...

$$\phi_0(t_n) = 4\pi L \left[ \frac{1}{\lambda_0} + \frac{(\Delta F \times Tri(t_n))}{c} \right]$$

This modulation method will also affect the modulation of the laser power output. Adding the power modulation multiplied by the triangle waveform,  $Tri(t_n)$ , to the produced self-mixing power variation signal. The amplitude of the power variation signal must also be equal to the value of the set power modulation coefficient. The power modulation coefficient can be represented using  $\rho$ , Rho. But can also be alternatively labelled as  $\beta$ , beta. This can be expressed as the formula...

$$p(t_n)' = p(t_n) + (\rho \times Tri(t_n))$$

The last two equations will be used in order to calculate the final absolute distance signal required, giving a power against time graph. This graph will show a number of rising and falling steps in the shape of the generated triangle waveform,  $Tri(t_n)$ .

From the example in the 2014 paper titled 'solving self-mixing equations for arbitrary feedback levels: a concise algorithm', an example of this generated power waveform can be shown in the example below...

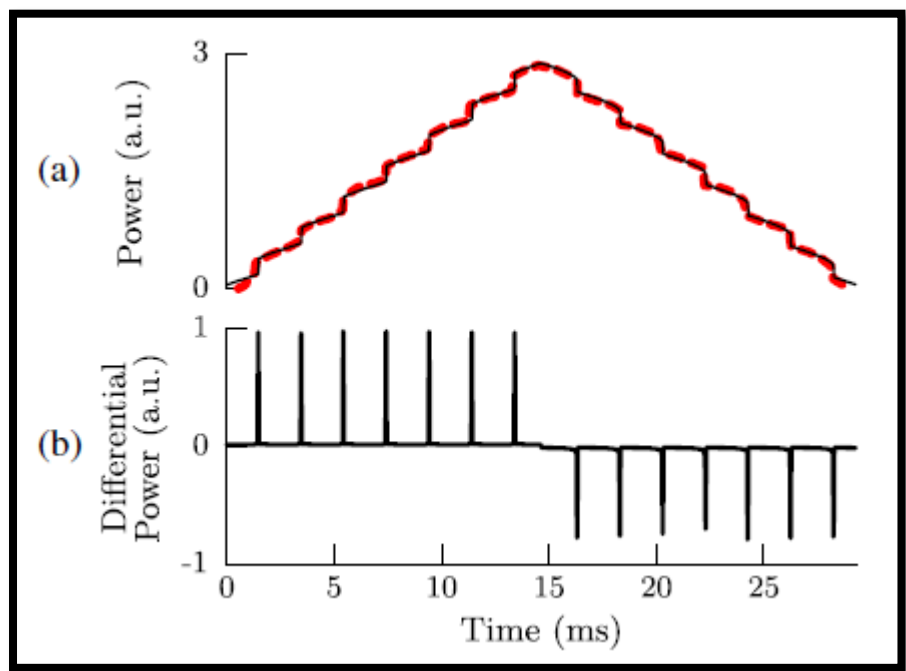


Figure 7 power variation diagram example from 2014 paper ‘solving self-mixing equations for arbitrary feedback levels: a concise algorithm’,

In this example, the target distance,  $L$ , from the laser source is 24mm. The laser frequency sweep set at a range of 46GHz with a base frequency that can be calculated using,  $v'_0 = \frac{c}{\lambda_0}$ .

Graph (a) shows the expected simulation graph and that step triangle shape that was discussed. The number of steps, or fringes, on one side of the modulated triangle waveform can be counted, and inserted into a formula that will calculate an approximate absolute distance...

$$L = N_f \times \frac{c}{2 \times \Delta F}$$

C being the speed of light,  $N_f$  being the number of fringes and  $\Delta F$  being the frequency modulation coefficient. The number of fringes in this example equaling to  $N_f = 7$ , so when this is put into the formula above...

$$L = 7 \times \frac{299792458}{2 \times 46 \times 10^9} = 0.0228 \text{ or } 22.8\text{mm} \pm 3.3\text{mm}$$

The resulting answer is  $22.8 \pm 3.3\text{mm}$ . the offset of  $\pm 3.3\text{mm}$  is used as a range since the expected result of 24mm lies within that range from a 22.8mm calculated approximate value given.

Interestingly enough, the complex refractive index of a stationary target, subjected to the laser of a self-mixing machine, can be found with the use of linear frequency modulation. As a quick explanation into what a refractive index is, it is essentially the index reduction in speed at which light passes through an object. Light travels the fastest in vacuum with the highest speed of  $3\text{e}+8 \text{ ms}^{-1}$ . It is only marginally less in air, but it is considerably reduced in glass or water.

This can be further explored in the “Sensing and imaging using laser feedback interferometry with quantum cascade lasers” 2019 paper under the “Applications: Refractive index measurement” heading. [10]

### Displacement:

Displacement, or harmonic motion, can be simulated and calculated through a simple, widely popular, method. Calculating the displacement of a target is one of the most popular and easy tasks to achieve. Hence why so many papers use it as an example and discuss it quite often.

Firstly, to stimulate a reaction to generate a waveform that shows the displacement distance, there must be motion of the target. This can be easily achieved in a practical example using a speaker and setting that speaker to emit a set frequency at a variable amplitude. The calculation of the displacement will then be able to calculate and give an approximate value for the distance of the vibration or movement that the speaker will travel during this time of operation. The movement of the speaker can be used to simulate the displacement using a manufactured periodic waveform labeled as “displacement samples”.

$$d(t_n) = A \times \cos(2\pi f t_n)$$

The formula above is the manufactured waveform that the speaker will emit.  $A$  is the set amplitude of the waveform,  $f$  is the frequency being generated and emitted from the speaker and  $t_n$  is the time at each sample element. The displacement sample result can be substituted into a formula that will calculate the proportional change in phase labelled as,  $\phi_0$ . This formula can be shown as...

$$\phi_0(t_n) = \frac{4\pi v_o}{c} [L_0 + d(t_n)]$$

This can then be altered further with the relationship,  $c = v_o \times \lambda_0$ , so the resulting formula could then be used to calculate the phase...

$$\phi_0(t_n) = \frac{4\pi}{\lambda_0} [L_0 + d(t_n)]$$

The reason for this change is that now the laser wavelength,  $\lambda_0$ , can now be used in this formula instead of the modulated laser frequency without feedback value since this information is not available for this application.

Now that the proportional change in phase can be calculated, when the signal becomes self-mixed, there will be a power against time graph shown as a frequency modulated power signal...

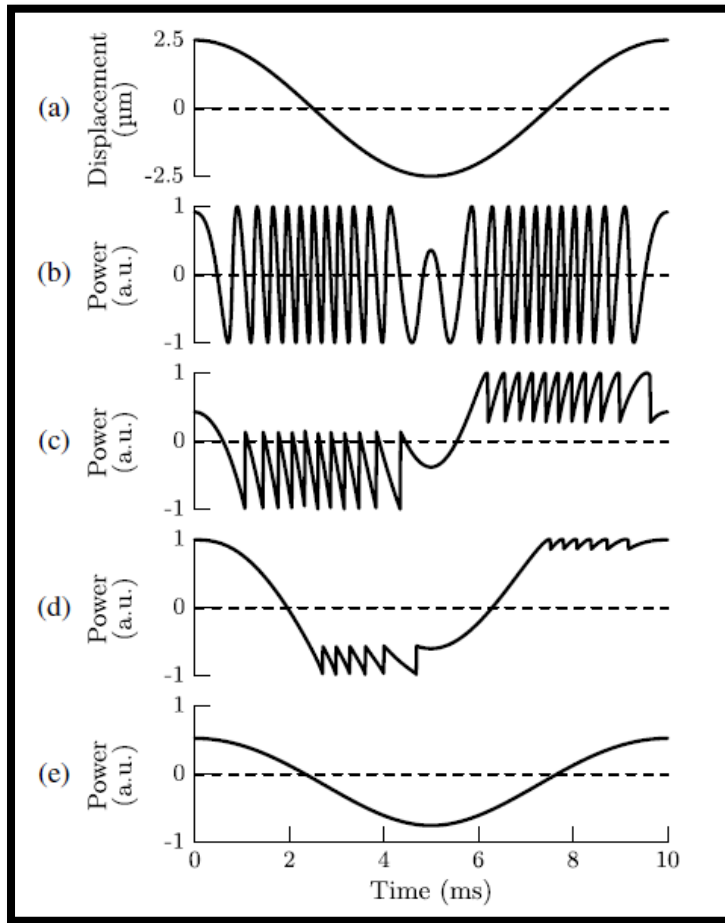


Figure 8 Displacement self-mixing result when the feedback  $C$  ranges from 0.5, translucent surface, to 60, a very rough surface

The first graph, a), shows the generated displacement motion signal set to 100Hz at  $2.5\mu m$  amplitude with the second graph, b), showing the self-mixed signal that has been simulated. The wavelength for this,  $\lambda_0$ , equals 850nm with a physical harmonic motion of  $2.5\mu m$  at 100Hz frequency. So, in theory, the expected target result of this simulation should be approximately the peak-to-peak value of the  $2.5\mu m$  amplitude of the displacement sample generation. To calculate the displacement value from the resulting waveform, the number of top peaks must be counted before then being halved to account for the whole 360 degrees range of motion. This is because the distance between each top peaks are only 180 degrees. Multiplying this number by the wavelength of the laser,  $\lambda_0$ , will then provide the approximation of the motion of displacement.

In this example there are 11 peaks, which would be 5.5 when halved. Multiply that by the 850nm wavelength and the resulting displacement value is calculated as...

$$d \approx 5.5 \times 850 \times 10^{-9} \approx 4.675 \times 10^{-6} \gg 4.7\mu m$$

Giving an approximate displacement value of  $4.7\mu m$ , which is to be believed to be an accurate calculation given.

In examples c) to e) the feedback value,  $C$ , is given a varying range of values in each example showing the effects it has on the resulting waveform. From b) to e) the feedback value given to these waveforms go in the order of, 0.5, 5, 20 and 60. In b) the waveform is clear and consistent with no distortion of the waveform, giving a value of  $4.7\mu m$ .

As the feedback value increases down the list however, there is a change in the appearance of the waveform as it begins to become distorted from hysteresis. The beginning of this can be seen in c) and if

the displacement were to be calculated then the value of  $4.7\mu m$  would remain the same as the number of top peaks are still consistent at a feedback value of  $C = 5$ .

When the feedback  $C = 20$  however, the higher feedback will then begin to have an effect on the number of peaks being shown on the waveform. This would be the result of distortion from an extreme feedback value producing a loss in information required to make the necessary calculations. In graph d) the number of peaks shown is 6 halved to 3, multiplied by the wavelength would provide the result of...

$$d \approx 3 \times 850 \times 10^{-9} \approx 2.55 \times 10^{-6} \gg 2.55\mu m$$

Which is not close to the expected value required. As the feedback increases finally to the point of  $C = 60$ , then the loss of fringes would come to a point when the modulation catalyst for the simulation would be replicated.

#### Described method:

Individually, the distance and displacement can be calculated through the same algorithm but will become self-mixed separately. Requiring their own series of loops through the self-mixing algorithm to display a graph for each.

However, for this assignment the requirement is the simultaneous self-mixing of the absolute distance and displacement values required to accurately calculate each. To do this there will obviously need to be a way to combine both these waveforms before self-mixing occurs.

First it must be broken down that the displacement is modulated by the generated displacement sample, and the absolute distance is modulated by the ostensibly triangle waveform injection current at the input of the laser.

In both cases, the displacement and absolute distance self-mixed waveforms function in a similar way, in the sense that they require a constant change in amplitude to be driven and stimulated. Considering that  $\rho \times Tri(t_n)$  is not necessary in the calculation of the displacement and absolute distance. However, it will help to provide a clear graph showing the number of steps being made over the period of the triangle waveform with the power modulation,  $\rho$ , used to increase the amplitude of the resulting stair step fringe waveform. In this instance it would make a small difference to include the displacement waveform, adding it to absolute distance waveform before self-mixing them both simultaneously. This way, in theory, the number of steps on final self-mixing graph should have stair step fringes that show the values required to calculate both the absolute distance and displacement.

When in the testing phase of the project, it will also be useful to include the use of the 'subplot' function so that the final simultaneous self-mixing signal, the displacement, and absolute distance power variation graphs can be displayed simultaneously, in that order.

#### Implemented method in MATLAB:

The method implemented would utilise the self-mixing power algorithm found in a Kaliese 2014 paper discussing the topic, '**Solving self-mixing equations for arbitrary feedback Levels**' This paper would go on to discuss the methodologies of calculating the displacement and absolute distance of a target using a self-mixing signal. This is discussed further under the header '**literature reviews about self-mixing signals**'. These papers would go on to talk about how to appropriately stimulate the self-mixed signal to calculate the displacement and absolute distance.

The displacement calculation only requires that to self-mix the interferometry signal, there must be a stimulus to modulate the reflected signal with. This stimulus would come in the form of a vibrating or moving target. The target in this instance would be a simulation of a speaker, so displacement samples in



the form of a sine waveform would be used to simulate this. However, the reflectivity of the laser on a rough surface of a moving speaker would not be accounted for in this simulation and instead would be ignored.

This would be simulated in the simulation software package, MATLAB, by constructing the sine wave that represents the frequency of the vibration of the speaker and its distance of movement as its amplitude.

$$\varphi_0 = \frac{4 \times \pi}{\lambda_0 \times (L + d)}$$

This would be called the displacement samples and would then be used in the formula above to modulate the signal.  $\lambda_0$  is the laser wavelength. The measurement in nano-meters of one cycle. This will then be used to calculate the displacement value later on when looking at the power variation graph results.

$(L + d)$  is the distance of the target from the laser source being added to the amplitude of the displacement samples. This can be seen below in a simulated graph from the simulation software package, MATLAB...

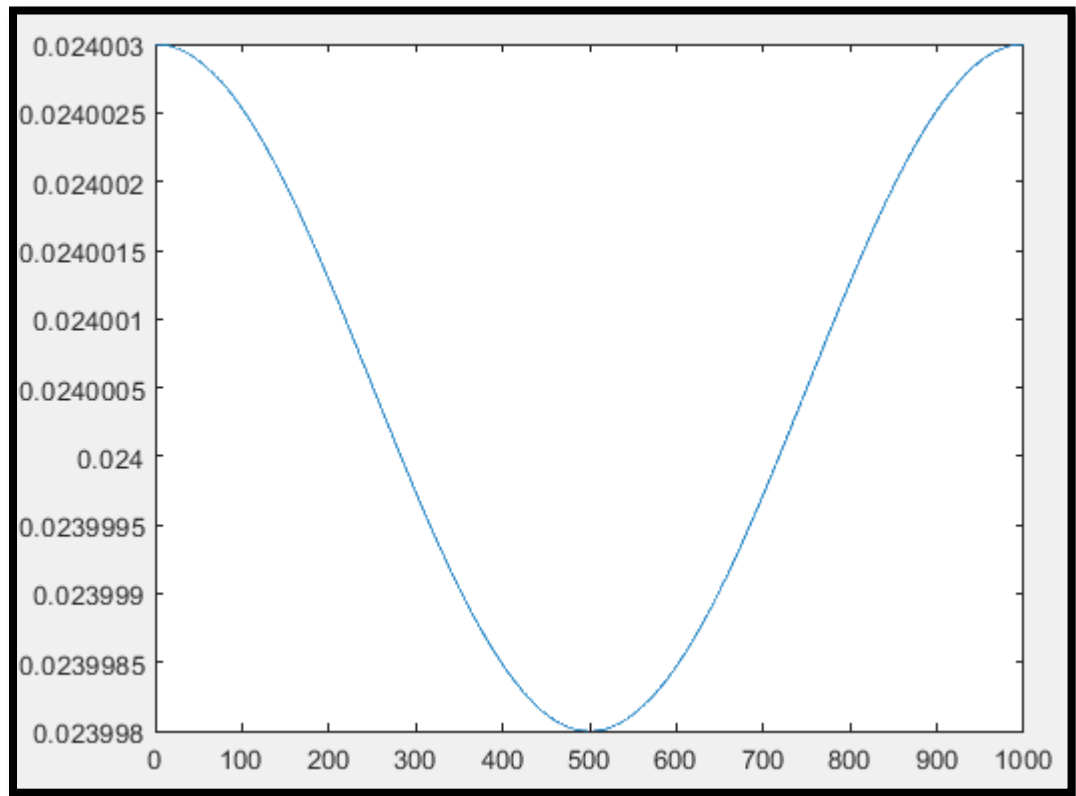


Figure 9 Graph displaying  $(L+d)$  simulated waveform

The absolute distance requires a different approach but one that is similar in a single way. The signal also requires a stimulation of some description to modulate. In this case the stimulation is an injection current as a triangle waveform. For simulation purposes this will basically be a triangle step response. The idea behind this is that in a practical sense, this would provide a slight change in the LD’s laser colour and in turn its laser wavelength.

The triangle waveform had to be constructed with the simulation time in mind for this application.

### Simulation results:

For my final simulation results, A graph showing a number of modulated steps across a triangle waveform would be shown, representing the total target absolute distance and displacement waveform of power variation over time. The values for this test result have a self-limiting, maximum simulation time of 100Hz, which the triangle frequency is required to match the motion frequency of the displacement samples, 'd' waveform. The amplitude of the displacement waveform is...

$$d = 2.5e^{-6} \times \cos(2\pi \times f \times t)$$

... with the amplitude being set to a default value of,  $A = 2.5e^{-6}$ , 't' being an element for each number of 1000 samples and 'f' being the varying motion frequency of the target speaker. The default target distance for the target speaker within the simulation is also set to a value of,  $L = 0.024 + 0.5e^{-6}$ .

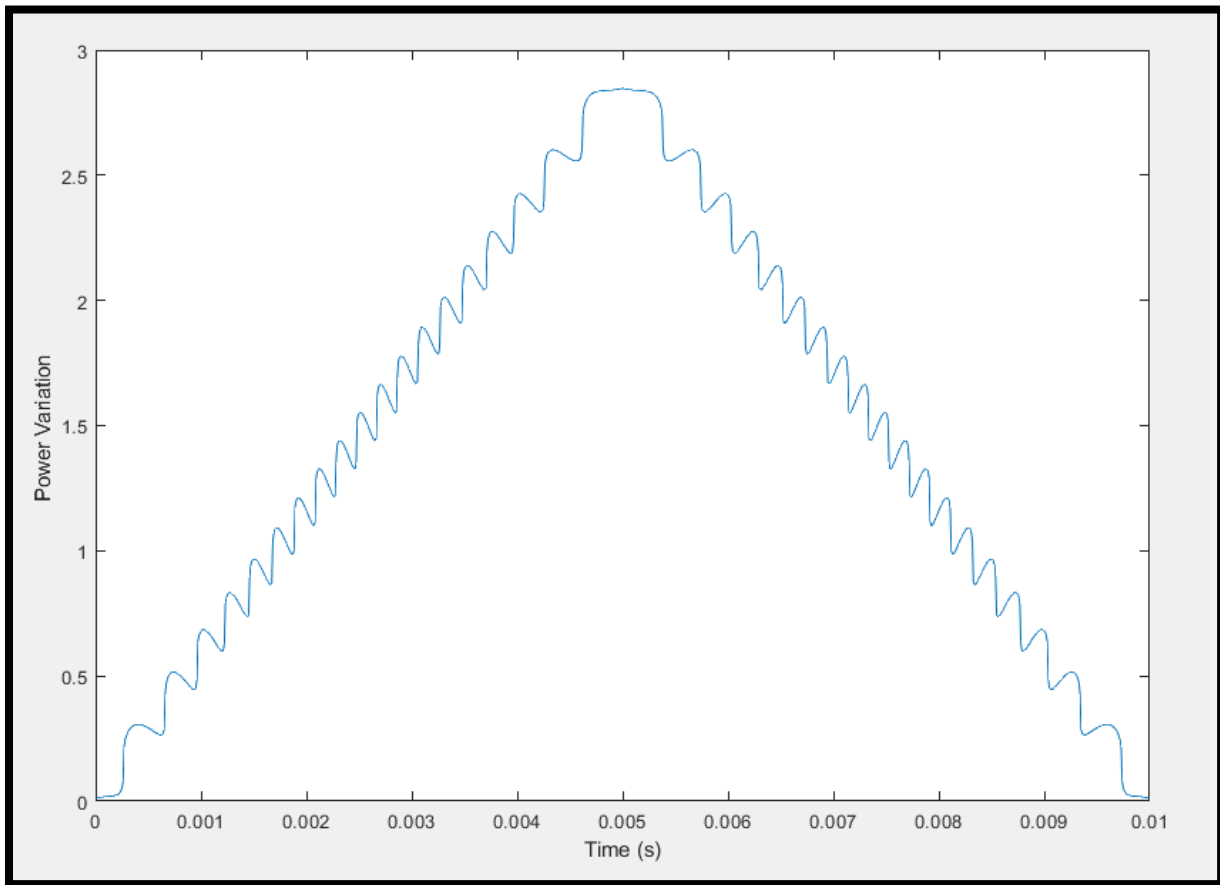


Figure 10 example of displacement fringes on a power variation graph

Usually, a power variation graph, of both the absolute distance and the displacement, separately would produce their own graphs with their own set number of fringes or steps. If the number of fringes for both are counted individually and then the simultaneous power variation graph steps are counted, the finding is that the added number of steps for the independent steps are equal to the steps counted in the simultaneous calculation. One theory is that by finding the ratio between the number of fringes, that rule can then be applicable to any varying degree of fluctuating distances of the target and the displacement of the speaker in this type of application.

Another avenue, my supervisor advised that I explored, would be to observe the effects of the self-mixing simulation when the motion frequency and the triangle frequency were not in synch, or in a 1 to 1 ratio, with each other. Exploring this could show interesting results on the behaviour of the fringes and could be manipulated to help fulfil the target of this project of being able to simultaneously self-mix the absolute distance and displacement and be able to calculate those final values.

By exploring these experimental avenues, the behaviour of self-mixing interferometry can be observed and used to achieve the final goal. These experiments will now be explained further below in the next paragraph.

For the first test, an exploration between the number of fringes for the absolute distance and the displacement in the self-mixing graph can be observed in the form of a ratio. This ratio, if it can be calculated, can then be used as a sort of rule of thumb to distinguish and separate the absolute distance fringes with the displacement fringes. And for this new test, a new set of results would then be used to calculate the percentage difference between the fringes, which would then be able to produce an approximated distinguishment between the two sets of combined fringes. In theory, for this to work, the ratio of this ‘rule of thumb’ should stay consistent despite if any of the distance’s changes.

This can be done by recording the fringes on the absolute distance, displacement, and final self-mixing signal while at a fixed triangle frequency and motion frequency. Meanwhile, the values of the absolute distance and displacement will start at 24mm and 2.5µm. both increasing by 1mm and 0.1µm until 50mm and 5µm has been achieved and recorded.

Using these recorded fringes of the absolute distance, displacement and self-mixing signal, the self-mixing signal fringe number can be used to find the ratio by using the formula...

$$\frac{\text{Displacement – or – absolute distance fringes}}{\text{SelfMixing fringes}} = \text{ratio}\%$$

**Table 1 – Distance change from 24mm to 50mm**

Test number	Change in absolute distance (mm)	Displacement step number	Absolute distance step number	Number of steps
1	24	12 (63%)	7 (37%)	19
2	25	12 (60%)	8 (40%)	20
3	26	12 (58%)	8 (42%)	19
4	27	12 (55%)	9 (45%)	20
5	28	12 (57%)	9 (43%)	21
6	29	12 (57%)	9 (43%)	21
7	30	12 (57%)	9 (43%)	21
8	31	12 (57%)	9 (43%)	21
9	32	12 (55%)	10 (45%)	22
10	33	12 (55%)	10 (45%)	22
11	34	12 (50%)	11 (50%)	22
12	35	12 (52%)	11 (48%)	23
13	36	12 (48%)	12 (52%)	23
14	37	12 (50%)	12 (50%)	24
15	38	12 (52%)	11 (48%)	23
16	39	12 (50%)	12 (50%)	24
17	40	12 (50%)	12 (50%)	24
18	41	12 (46%)	13 (54%)	24
19	42	12 (48%)	13 (52%)	25
20	43	12 (42%)	14 (58%)	25
21	44	12 (46%)	14 (54%)	26
22	45	12 (42%)	14 (58%)	25
23	46	12 (46%)	14 (54%)	26

24	47	12 (48%)	14 (52%)	27
25	48	12 (44%)	15 (56%)	27
26	49	12 (44%)	15 (56%)	27
27	50	12 (41%)	16 (59%)	27

Table 2 – Displacement change from 2.5µm to 5.0µm

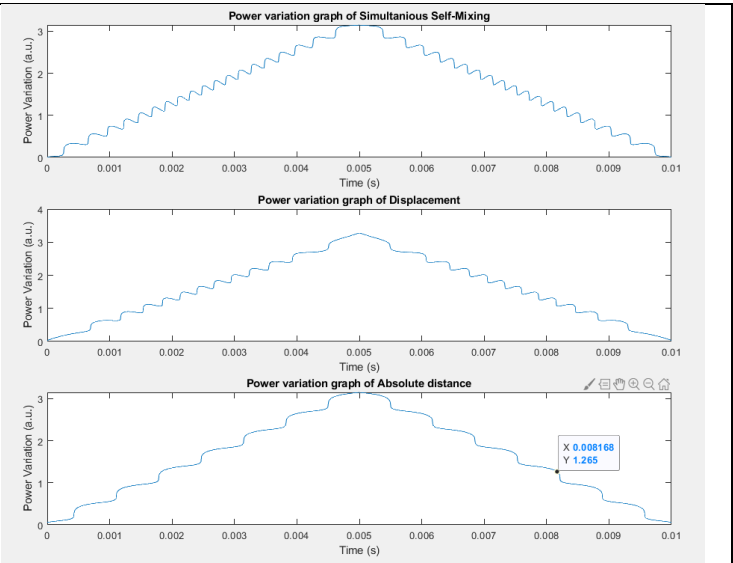
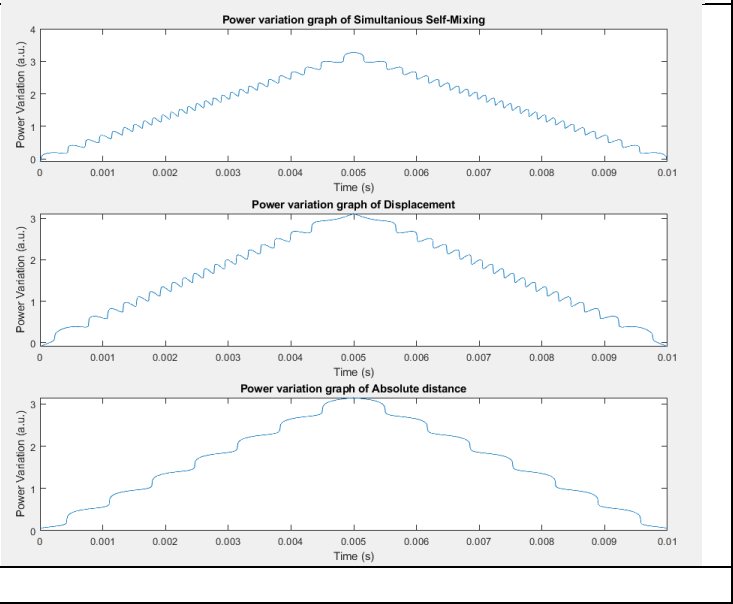
Test number	Change in displacement (µm)	Displacement step number	Absolute distance step number	Number of steps
1	2.5	12 (63%)	7 (37%)	19
2	2.6	12 (63%)	7 (37%)	19
3	2.7	13 (62%)	7 (38%)	21
4	2.8	13 (62%)	7 (38%)	21
5	2.9	14 (67%)	7 (33%)	21
6	3.0	14 (67%)	7 (33%)	21
7	3.1	14 (64%)	7 (36%)	22
8	3.2	15 (65%)	7 (35%)	23
9	3.3	16 (70%)	7 (30%)	23
10	3.4	16 (70%)	7 (30%)	23
11	3.5	16 (67%)	7 (33%)	24
12	3.6	17 (68%)	7 (32%)	25
13	3.7	18 (72%)	7 (28%)	25
14	3.8	18 (72%)	7 (28%)	25
15	3.9	18 (69%)	7 (31%)	26
16	4.0	19 (70%)	7 (30%)	27
17	4.1	20 (74%)	7 (26%)	27
18	4.2	20 (74%)	7 (26%)	27
19	4.3	20 (74%)	7 (26%)	27
20	4.4	21 (75%)	7 (25%)	28
21	4.5	22 (76%)	7 (24%)	29
22	4.6	22 (76%)	7 (24%)	29
23	4.7	22 (76%)	7 (24%)	29
24	4.8	23 (77%)	7 (23%)	30
25	4.9	23 (74%)	7 (26%)	31
26	5.0	24 (77%)	7 (23%)	31

Observation table for change in distance		
Test number – Table 1	comment	Graph result

No.1	The starting default power variation graphs for self-mixed signal, the displacement and absolute distance in that order. Here the absolute distance is the value to change.	<div><div>Power variation graph of Simultaneous Self-Mixing</div><div>Power variation graph of Displacement</div><div>Power variation graph of Absolute distance</div></div>
No. 7	Changes to the absolute distance can cause slight changes in the power variation waveform for the displacement since the target distance is apart of the formula to calculate the displacement. This, however, does not affect the number of fringes that the displacement power graph displays.	<div><div>Power variation graph of Simultaneous Self-Mixing</div><div>Power variation graph of Displacement</div><div>Power variation graph of Absolute distance</div></div>
No. 22	The number of fringes for the displacement power graph remains the same throughout this table of the experiment.	<div><div>Power variation graph of Simultaneous Self-Mixing</div><div>Power variation graph of Displacement</div><div>Power variation graph of Absolute distance</div></div>

**Observation table for change in Displacement**

Test number – Table 2	comment	Graph result
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No. 1	The default graphs before displacement starts to increase.	
No. 16	The number of fringes for the absolute distance graph remains the same when the displacement value changes.	

The second test would include having a fixed starting value for the motion frequency for one set of test results and a fixed triangle waveform frequency for another. During these tests, with a starting value of 1e-3 or 10Hz for the fixed motion frequency and fixed triangle frequency in that order, these values will be increased by intervals of +2e-3 and +2Hz until the final values of 100e-3 and 100Hz have been achieved.

During this time, the number of fringes for the displacement, absolute distance and the final self-mixing signal graphs will be recorded for each interval. While doing this, the effects this will have on the number of fringes will be observed and recorded as they change throughout the test. There will also be selective screenshots in a separate table when a visual observation has been made, with a comment on how this has affected the overall result.

Table 3 – change in injection current frequency

Test number	Test Values at 10Hz	Displacement step number	Absolute distance step number	Number of steps
1	1e-3	0	7	7
2	3e-3	0	7	7
3	5e-3	0	7	7

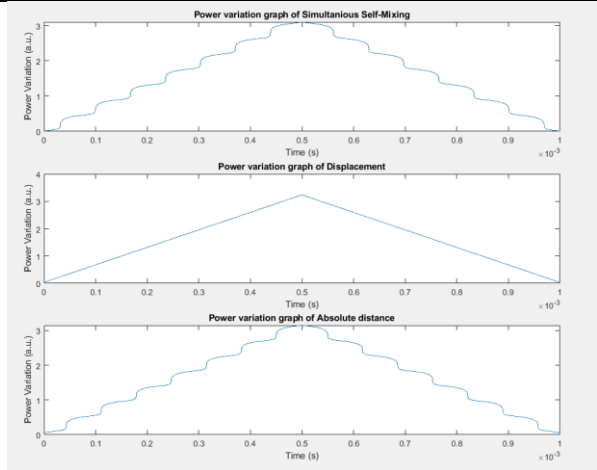
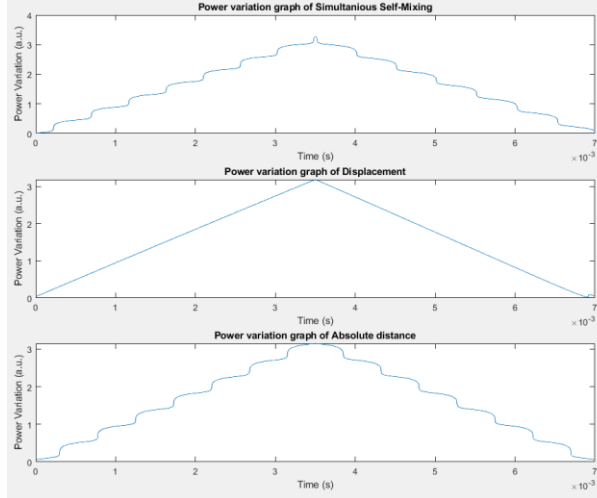
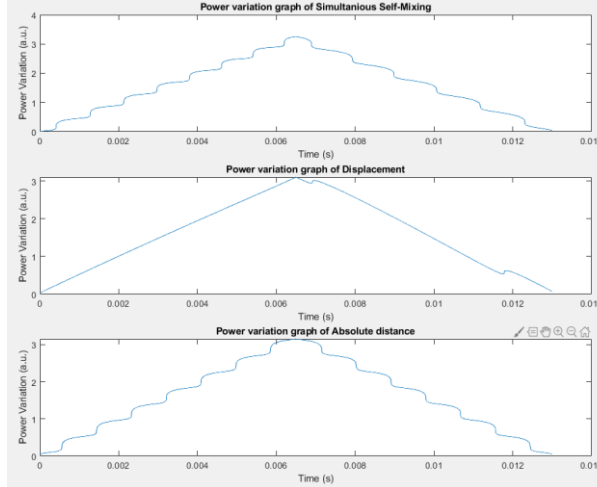
4	7e-3	0	7	7
5	9e-3	0	7	7
6	11e-3	0	7	7
7	13e-3	0	7	7
8	15e-3	1	7	8
9	17e-3	1	7	8
10	19e-3	1	7	8
11	21e-3	1	7	8
12	23e-3	1	7	8
13	25e-3	2	7	9
14	27e-3	2	7	9
15	29e-3	2	7	9
16	31e-3	2	7	9
17	33e-3	3	7	10
18	35e-3	3	7	10
19	37e-3	3	7	10
20	39e-3	4	7	11
21	41e-3	4	7	11
22	43e-3	4	7	11
23	45e-3	5	7	12
24	47e-3	5	7	12
25	49e-3	6	7	13
26	51e-3	6	7	13
27	53e-3	6	7	13
28	55e-3	7	7	14
29	57e-3	7	7	14
30	59e-3	7	7	14
31	61e-3	8	7	15
32	63e-3	8	7	15
33	65e-3	8	7	15
34	67e-3	9	7	16
35	69e-3	9	7	16
36	71e-3	9	7	16
37	73e-3	10	7	17
38	75e-3	10	7	17
39	77e-3	10	7	17
40	79e-3	10	7	17
41	81e-3	11	7	18
42	83e-3	11	7	18
43	85e-3	11	7	18
44	87e-3	11	7	18
45	89e-3	11	7	18
46	91e-3	11	7	18
47	93e-3	12	7	19
48	95e-3	12	7	19
49	97e-3	12	7	19
50	99e-3	12	7	19

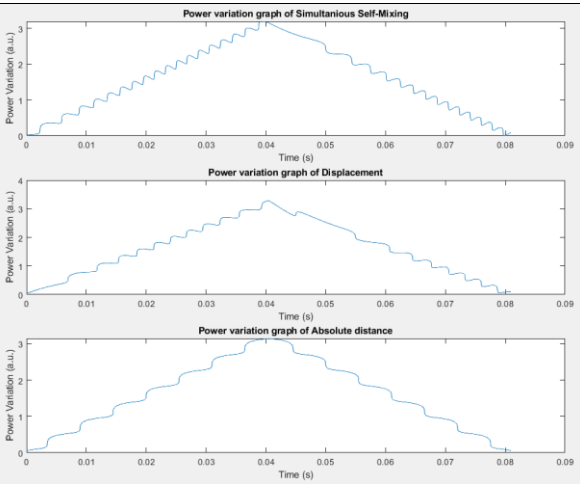
**Table 4 – change in displacement sample motion frequency.**

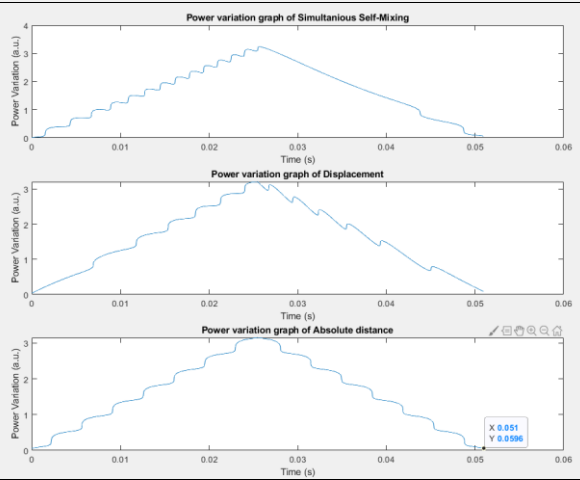
Test number	Test Values at T = 10e-3	Displacement fringe step number	Absolute distance fringe step number	Number of fringe steps
1	10	0	7	7

2	12	0	7	7
3	14	0	7	7
4	16	1	7	8
5	18	1	7	8
6	20	1	7	8
7	22	1	7	8
8	24	1	7	8
9	26	2	7	9
10	28	2	7	9
11	30	2	7	9
12	32	3	7	10
13	34	3	7	10
14	36	3	7	10
15	38	4	7	11
16	40	4	7	11
17	42	4	7	11
18	44	5	7	12
19	46	5	7	12
20	48	6	7	13
21	50	6	7	13
22	52	6	7	13
23	54	7	7	14
24	56	7	7	14
25	58	7	7	14
26	60	8	7	15
27	62	8	7	15
28	64	8	7	15
29	66	9	7	16
30	68	9	7	16
31	70	9	7	16
32	72	10	7	17
33	74	10	7	17
34	76	10	7	17
35	78	10	7	17
36	80	11	7	18
37	82	11	7	18
38	84	11	7	18
39	86	11	7	18
40	88	11	7	18
41	90	11	7	18
42	92	12	7	19
43	94	12	7	19
44	96	12	7	19
45	98	12	7	19
46	100	12	7	19

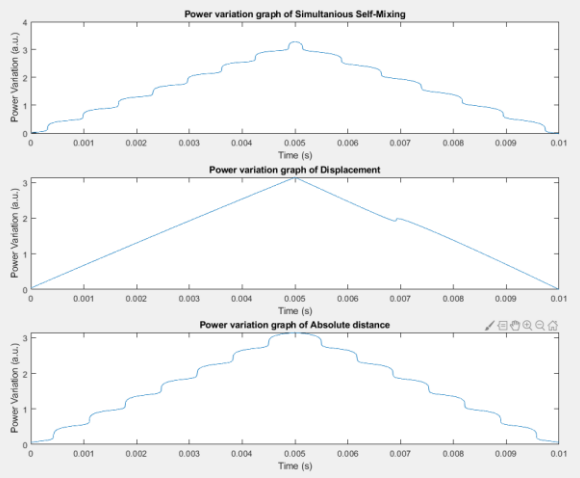


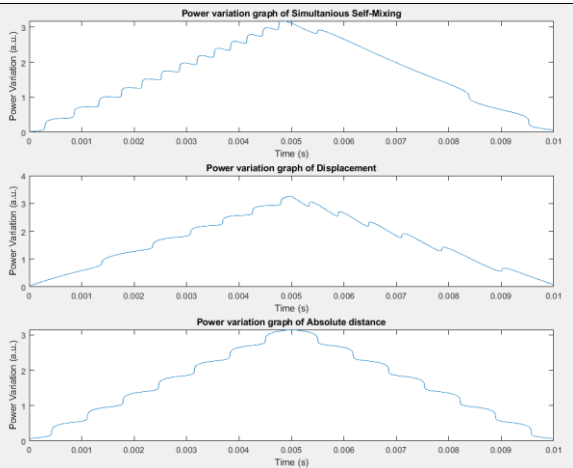
Observation table for change in injection current frequency		
Test number – Table 3	comment	Graph result
No.1	There are no steps along this triangle waveform	
No.4	A small fringe along the bottom right of the triangle waveform	
No.7	A second fringe has appeared, and they seem to be climbing the triangle waveform as the time period of the triangle waveform increases, meaning the frequency of it decreases. The frequency of the triangle waveform being 77Hz.	

No.41	At a certain point it becomes clear that the fringes are sliding from the right to the left of the triangle waveform. This is at a triangle waveform frequency of 12.3Hz	
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No.26	At a triangle frequency of double the motion frequency, the one half of the sliding total fringes, or 180 degrees of the waveform, can be seen with the final value of 12 fringes seen.	
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**Observation table for change in motion frequency**

Test number – Table 4	comment	Graph result
No. 1	In the second table where the motion frequency is varying, the same effect of the fringe sliding across the triangle waveform	

No. 21	When the motion frequency is half of the triangle waveform frequency, then all the fringes required to count the displacement can be seen spread across the triangle waveform.	 <p>The figure consists of three vertically stacked line graphs, each showing 'Power Variation (a.u.)' on the y-axis and 'Time (s)' on the x-axis. The x-axis for all graphs ranges from 0 to 0.01 seconds. The top graph is titled 'Power variation graph of Simultaneous Self-Mixing' and shows a sawtooth-like pattern with a peak around 0.005s. The middle graph is titled 'Power variation graph of Displacement' and shows a similar sawtooth pattern with a peak around 0.005s. The bottom graph is titled 'Power variation graph of Absolute distance' and shows a sawtooth-like pattern with a peak around 0.005s. All three graphs show a similar trend of increasing power followed by a decrease, with some minor fluctuations.</p>

Conclusion:

The first experiment into the possibility of a ratio rule of thumb being observed showed a very conclusive results of the findings. In the first table, table 1, the values for the triangle waveform frequency and the motion frequency were fixed to values of 10e-3 and 100Hz, respectively. While this was the case, the value of the target distance was incrementing by 2mm while the displacement was fixed to a value of 2.5µm. As the target distance increased, the number of fringes for the absolute distance power variation graph also increased. This has led to a clear widening gap between the ratio of the absolute distance and the displacement fringes, as the number of displacement fringes had not changed at all. However, the displacement power variation graph had shown signs of slight changes at the midpoint of the waveform, but this had no overall effect on the number of displacement fringes. This slight alteration could also be predicted as the formula to simulate displacement modulation,  $\phi_0(t_n) = \frac{4\pi}{\lambda_0} [L_0 + d(t_n)]$ , is shown to include the addition of the target distance to the equation before the occurrence of self-mixing.

It is a similar story for the results in table 2 with the fixed target distance of 24mm and the starting motion amplitude of 2.5µm, incrementing by 0.1µm up to the final value of 5.0µm. This time, however, the absolute distance fringe number value is the result that remains the same throughout this table of simulation results. The number of fringes for the displacement in this experiment begins to increase, overtaking the overall percentage value of the absolute distance fringes, creating a more lopsided ratio value.

It can be concluded from these results that the number of fringes is too independent for there to be a ratio rule that can be abused for this type of application when a varying array of distance and displacement values can be used. It can be noted, however, that every 2 to 4 increments shows that there is an increase of fringes by 1, with some varying number of fluctuations between those 2 to 4 increments.

From the second experimentation into the effects of independently varying the motion frequency and the triangle function waveform frequency, the modulation of the displacement self-mixing signal appears to be shown sliding across the triangle function as the motion frequency is less than the frequency of the triangle waveform.

Looking at table 3, the set motion frequency is at 10Hz and the triangle function time / frequency starting at 1e-3 or 1KHz, increasing the time by steps of '2e-3' until the peak of 100e-3, or 10Hz has been achieved.

Starting at the first test in table 3 at T = 1e-3 or 1KHz, there are no appearances of any fringes in the simulated result graph. However, as the time went on, making the frequency of the triangle function

decrease in turn, then the fringes begin to slowly start shifting from the right side of the triangle waveform to the left. This would continue until the frequency of the triangle waveform was a 1 to 1 match with the motion frequency. At this point the modulated waveform would include all the usable fringes that would be required to calculate the displacement of the speaker motion.

When simulating this with my focus on the absolute distance power graph, as expected, the number of fringes remained the same with no change whatsoever. From this it can be trusted that as long as the distance remains the same, then the number of fringes will remain the same. Meaning that the only focus should then be on the displacement fringes, as they are the most effected by the change in triangle function frequency in relation to the motion frequency value, compared to the absolute distance.

These will then show up on the final self-mixing graph as the standard 7 fringes to begin with before slowly increasing in size as the triangle frequency and the motion frequency begin to align. This will continue until the number of fringes is an addition of the displacement and absolute distance fringes. There does not appear to be any distinguishing factors that separate the absolute distance and the displacement fringes, so finding the exactly correct number of fringes will be difficult to do in this situation. Possibly knowing the exact value of the displacement fringes would help to distinguish between the two sets of fringes but would require prior knowledge of the displacement distance before-hand. This, however, would not be viable for this project considering the aim of the project is to simultaneously self-mix and find the absolute distance and displacement.

Looking at table 4, the set triangle function time / frequency is set at  $10e-3$  and the set motion frequency starts at 10Hz, increasing the time by steps of 2Hz until the peak of 100Hz have been achieved.

This will similarly follow the formula of the displacement fringes sliding across the triangle waveform as the triangle frequency and the motion frequency become closer to equal in value. Also, as before the change in these frequencies will have no effect on the absolute distance fringes. Because of this, the situation is similar to the test before, producing the same results. Being unable to distinguish between the absolute distance and displacement fringes being the main result from this.

### Future improvements:

Seeing the results from the previous experiments, it can be concluded that I was able to, with the help of my highly experienced supervisor, successfully, simultaneously, self-mix the absolute distance and displacement using an interferometry system. The trouble occurs when the result of the self-mixed simultaneous signal is required to calculate the absolute distance and displacement, using their specific number of fringes. There were virtually no distinguishing factors between the two sets of fringes when simultaneously self-mixed. They were a similar amplitude and time period to each other.

The two tested methods of calculating a ratio rule of thumb and the observation of the effects on the displacement, when the motion frequency and the triangle frequency are not 1 to 1, have shown to hold very little influence on the final calculation of the final distance and displacement values.

After observing the effect of the displacement fringe shift however, this has sparked an idea that possibly one improvement I could make would be to go past that 1 to 1 limit and see how that might affect the resulting self-mixing waveform then. Perhaps, when the frequency of the triangle waveform is multiplied by the number of absolute distance fringes, then the displacement fringes would become easier to see. This is taking into consideration that the user would know how many fringes there are for the absolute distance measurement before beginning the simultaneous self-mixing simulation.

Another improvement I could make would be to conduct research into seeing if there is a way to negate the fringes on the one side of the self-mixed signal for either the absolute distance or the displacement. By doing this, it might be possible to completely isolate the displacement and absolute distance fringes to

either side of the triangle waveform. This would possibly need to be performed before the self-mixing simulation period, or a type of feedback loop could be used to perform this.

Throughout the project, I had tried to follow the steps set out for myself from the design chart and the Gantt chart created during the interim report stage of this experiment. When looking at the software side of the design chart, it states the plan to make multiple classes for the self-mixing algorithm, the displacement calculation, and the absolute distance calculation. In the case of the latter two, they would be required to be written in the same class so that the initial modulated wavelengths could be combined before the occurrence of self-mixing.

The mathematical calculation side had included me to find the formulas for the external round trip phase and the power variation calculations, so that I might be able to calculate backwards from the self-mixed waveform result. This, however, would produce a wavelength that would be very noisy and difficult to read from. This was my initial impression of how the project worked before I was able to fully comprehend the final outcome of these experiments.

The design section also discusses the operating distance range and operating frequency range. The operating distance of 1 meter was my initial impression, however, as the project moved forward, I could see that the set distance of 24mm would make this design choice irrelevant, especially considering the operating distance is not factored in for this project and is therefore irrelevant. The operating frequency range is theoretically a non-major factor for the simulation side, but in a real-life scenario, the limitations of the hardware and the environment could start to create problems.

As for following the Gantt chart along the entire length of this project, I was able to follow through, keeping on schedule with the meetings with my supervisor consistently and on time. Because of being on time, I was able to fully utilise the help and guidance given to me by my supervisor during this project. However, when discussing the other targets of the Gantt chart, I was being very liberal with the amount of time given to myself due to my initial lack of understanding for the topic.

In the future, the structure and contents of the design chart and Gantt chart would be changed to reflect a better understanding of the subject and aims of the project. As this was an entirely new subject matter for me, I had initially struggled to grasp the vision of the final goal and misunderstood the title of the project. But, with the guiding help of my supervisor, I was able to understand the project in more depth later into the year.

#### Reference:

[1] :- Figure 1, page 25, 2003 Self-mixing laser diode vibrometer, institute of physics publishing, Meas. Sci. Technol. 14 (2003) 24-32, 11/09/2020

[2] :- 4.3 Precision and accuracy, page 29, 2003 Self-mixing laser diode vibrometer, institute of physics publishing, Meas. Sci. Technol. 14 (2003) 24-32, 11/09/2020

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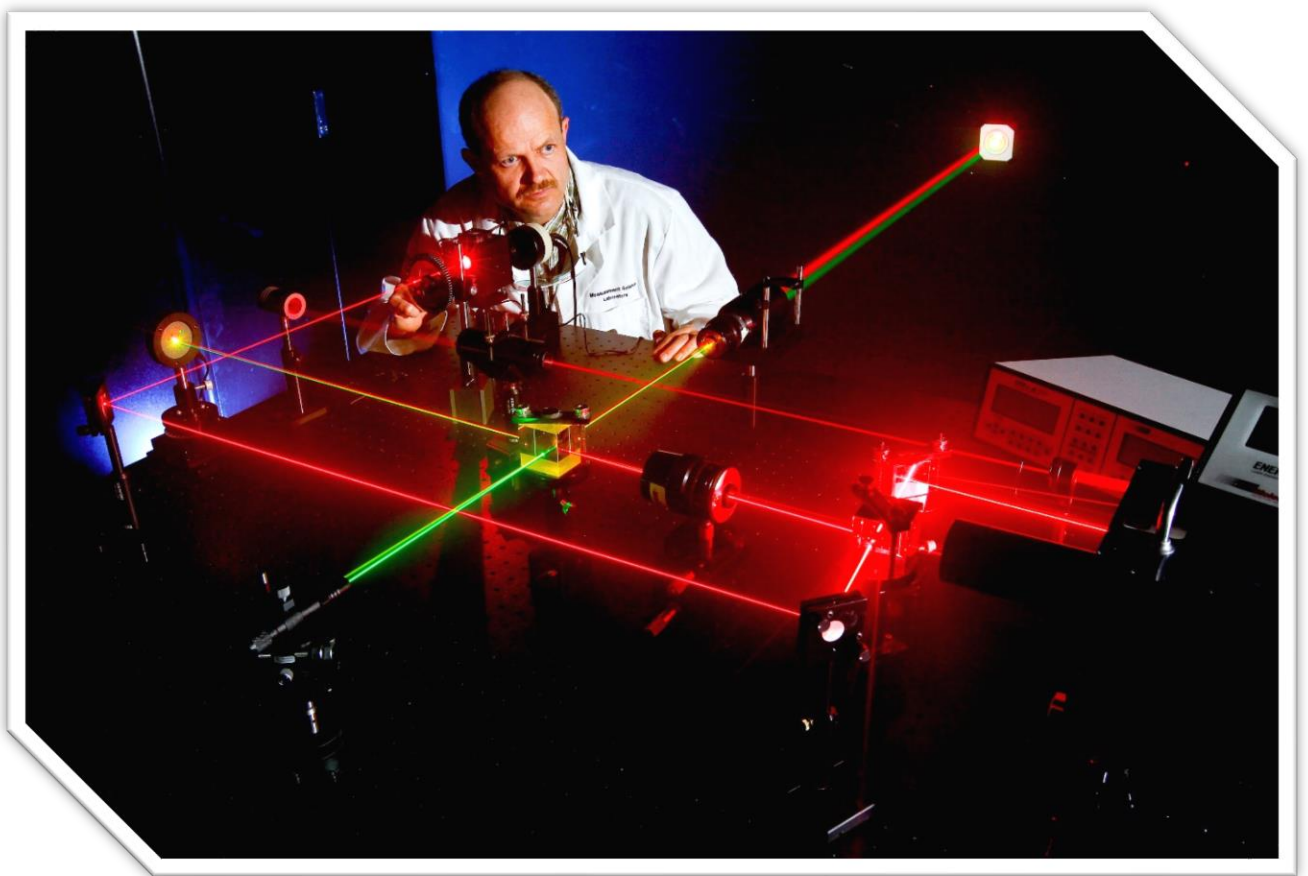
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Appendix 1 – Interim report:

## Simultaneous Measurement of absolute distance and displacement using self-mixing interferometry.



[7]

**Supervisor name:** *Yuanlong Fan*

**Name:** *Benjamin Samuel Edwards*

**Student number:** 18026826



### Aims:

- To simultaneously measure absolute distance and displacement of a target using a simple and low-cost self-mixing interferometry, also called an (SMI).
- To be able to simulate this process using both MATLAB and Simulink software packages to prove mathematically the concept.
- The basic SMI must consist of only laser diode, a micro lens and an external target. A speaker in this case would be the external target.

### Objectives:

- Review of measuring method for absolute distance and displacement
- Modelling self-mixing interferometry by solving the phase equation of the laser with optical feedback system using MATLAB.
- Develop methods for simultaneously measuring absolute distance and displacement from the self-mixing signals simulated from the above work.
- Numerically test the developed methods using MATLAB.
- Results, analysis, discussions, and suggestions for future work

### Pre-research summary:

The pre-research for this project can be summarised as doing extensive research into the performance of the device and the physics theory behind it. For instance, an SMI system will direct light from a LD (laser diode) source, which will be directed towards a target device to reflect off of, which would be a speaker in this instance. So, when the speaker is emitting a set frequency, this can then reflect the light back at the LD source which will have a sensor to detect the received laser signal. This can be seen in the diagram for figure 1, on page 25, in the 2003 paper titled 'Self-mixing laser diode vibrometer' [1]. The movement from the speaker when emitting a frequency, will then reflect the laser with the emitted frequency modulated to the laser source.

However, with the system setup like this, with just a LD source and speaker with nothing in-between, the range of the laser will be severely limited to an extremely short distance. To rectify this problem the use of a focussing lens will be used to concentrate the laser, significantly increasing the range for this application. [2] This can also mean, to an extent, that if the distance between the objective lens and the target remains the same then the distance between the LD and the objective lens is irrelevant. This is because the configuration does this by using LD wavelength sweeping, which also coincidentally gives it its characteristic that allows it to measure distance. [2] [4] In the 1992 Paper titled, 'Compact and high-precision range finder with wide dynamic range and its application' the value of  $T_m$  is sampled every cycle to calculate the current range.

To reduce the error of the calculation of the range measurement, several samples would need to be averaged to give one datum of measured range. Averaging these samples can reduce the noise being produced by the lasers FM signal. [4] This however is necessary only for physical testing. Since I am exclusively doing simulation work however, I won't need to be concerned with this, but it is something to make note of for the future.

The Optical feedback level is an important factor for creating a clear modulating signal as this will dictate the loss of 'fringes' representing half-wavelength displacements, which can also be seen as the 'rings' in the projection of a 'Michelson interferometry' system. The optical feedback variable is represented by 'C'. When the value of C is  $\ll 1$  then the modulated signal will retain its number of 'fringes'. [5.1] During this, the value of  $F(\phi)$  is very close to the shape of a sine wave with the modulation index increasing "for an increasing level of optical feedback" [3]. When  $C \approx 1$ , then a small signal distortion and loss of 'fringes' could be seen, and at  $C > 1$  then the signal starts to represent a sine wave and manifesting signs of hysteresis. [3] [1] From this it can be inferred that the higher the feedback value is the higher the loss of 'fringes' there will be, essentially replicating the original signal. [5.2]



Project Specifications:

For this project the only requirement to conduct the experiments is by doing so mathematically using the simulation program MATLAB. MATLAB will conduct the mathematical equations as specified in the pre-written program given to it. However, it needs to be given specified values for variables that are included in said equations, which can easily be achieved by creating a second script declaring the variables and calling the secondary script as a class method. This means that the calculations could then be achieved by triggering just this one script and then the rest would follow. MATLAB would also give the ability of generating a graph to preview the results from, allowing for a clear representation of the calculated modulated waveform.

When it comes to simulating the complete self-mixing system, the package for MATLAB, called Simulink, can then be used to create a block diagram of the feedback system without needing to write lines of script and calling back to other functions. This can be described as visual scripting. Using this visual scripting would then allow for the creation of a visual representation of the program that can then be shown as a control system diagram. The formulas in the block diagram would then need to be altered from the default product to replicate the formula required for the SMI system.

Intended Method:

The intended method of operation during this project is to first carry out a period of background research on the subject. Learn the basic principles for this application and read several papers on this subject to get a better intellectual understanding of how the system mathematically operates so then that can be used for simulation purposes in MATLAB. When simulating the system, altering the values of variables will then affect the system in a minor or major way. So, for this it would be interesting to see how this would affect the results from the simulation if drastic changes were made to a variable. Then the observed result could then be compared to the predicted result that was reviewed in the research papers from earlier in the project development.

Gantt Chart:



**Gantt Chart**

receiving project specification

initial pre-emptive research

Have meetings with supervisor every week

make Gantt chart

Interim report

continued research into mathematics

test simulations on matlab

write out formula to calculate phase displacement

write out formula to calculate absolute distance

write program to calculate self-mixing power, absolute distance and phase displacement

Test the program

create a series of simulation tests to challenge the program with specific calculations

Write the final project report

prepare presentation

Risk no.	Description of risk	Probability of the risk	Effect on the project	Contingencies/actions
1	A license for the Matlab program could become void or lost/stolen	Extremely low	Would halt the progress of the simulation and testing	Can request another license from the university technicians
2	Access to the internet could become lost	Low	Would stop any more research into the subject	Can contact the university accommodation to see when I can be sorted and can use mobile data instead.
3	The storage device that all the files are stored on can fail losing all the progress made.	Low	Would completely halt my progress and may even cause me to have to redo large portions of the work again.	Can have a second storage device that a backup of the files can be stored on and updated regularly.

### Current Work to Date:

Currently, I have been provided with a demo simulation [6] of a self-mixing interferometry system that demonstrates how calculations for a self-mixing interferometry modulation system works. The self-mixing power class called 'selmixpower.m' is going to be the main function that is going to be called during operation. The purpose of this class is to simulate a self-mixing interferometer system, using values from another class that would calculate a specific task for the simulation. For example, calculating absolute distance.

Looking at the MATLAB code, it calculates the power level at the sampled time calling variables from the specified task class which I will refer to as the secondary class. It would then check the feedback value, 'C', to see if it was less than or equal to 1. If it is then it will call the boundsweak method which will calculate the minimum and maximum bounds by subtracting/adding the feedback 'C' from the value of 'phi0'.

The else condition checks to see if 'C' is higher than 1 and if it is then the program jumps to the boundsstrong method. This method will make the value 'm' persistent and checks to see if it has a value. If it doesn't then it is given the value of 0. Persistent basically means that the value for m will be stored between calls to the function. What follows is the calculation for the values 'mlower' and 'mupper' using the formula  $(\phi_0 + \tan^{-1}(\alpha) + \cos^{-1}(1/C) - \sqrt{C^2 - 1})/(2\pi) - 1.5$  for the lower and  $(\phi_0 + \tan^{-1}(\alpha) - \cos^{-1}(1/C) + \sqrt{C^2 - 1})/(2\pi) - 0.5$  for the upper.

If statements would then check to see if m was smaller than 'mlower' and if m was higher than 'mupper'. If either is true, then the value of 'm' would be equal to the variable that it was being compared to. After which 'm' would then be used to calculate the value for 'phimin' and 'phimax' using the formulas  $(2m+1)\pi + \cos^{-1}(1/C) - \tan^{-1}(\alpha)$  for 'phimin' and  $(2m+3)\pi - \cos^{-1}(1/C) - \tan^{-1}(\alpha)$  before returning to the power method.

A handle called excessphase is then given the formula  $(x) - \phi_0 + C \sin(x + \tan^{-1}(\alpha))$ . Another series of if statements would then happen to check if excessphase(phimin) would be  $> 0$  else if excessphase(phimax)  $< 0$ , then the value of 'phi' would be equal to 'phimin' or 'phimax' respectively. If neither these cases are true then the final else will then make 'phi' = fzero(excessphase, [phimin, phimax]) so then it can find a point at which 'excessphase(x) = 0'.

The value of 'power' is then calculated and given by the formula  $\cos(\phi)$  and return the result to the function it was called from. This class is the base layer of the entire simulation and is repeatedly called for every required calculation because of this.

The next step is to recreate this process that can be used for the purposes of this project. This would require that the foundation class is able to give a result that would lead to finding the calculated value of absolute distance and displacement.

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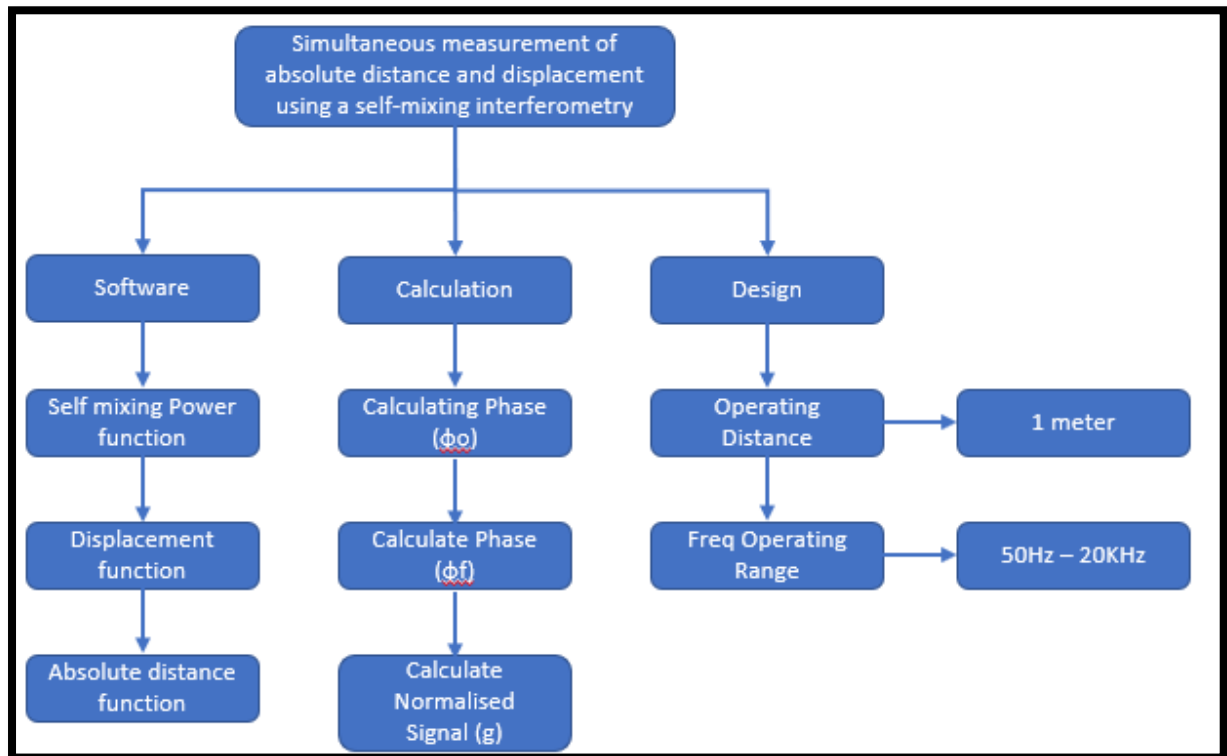
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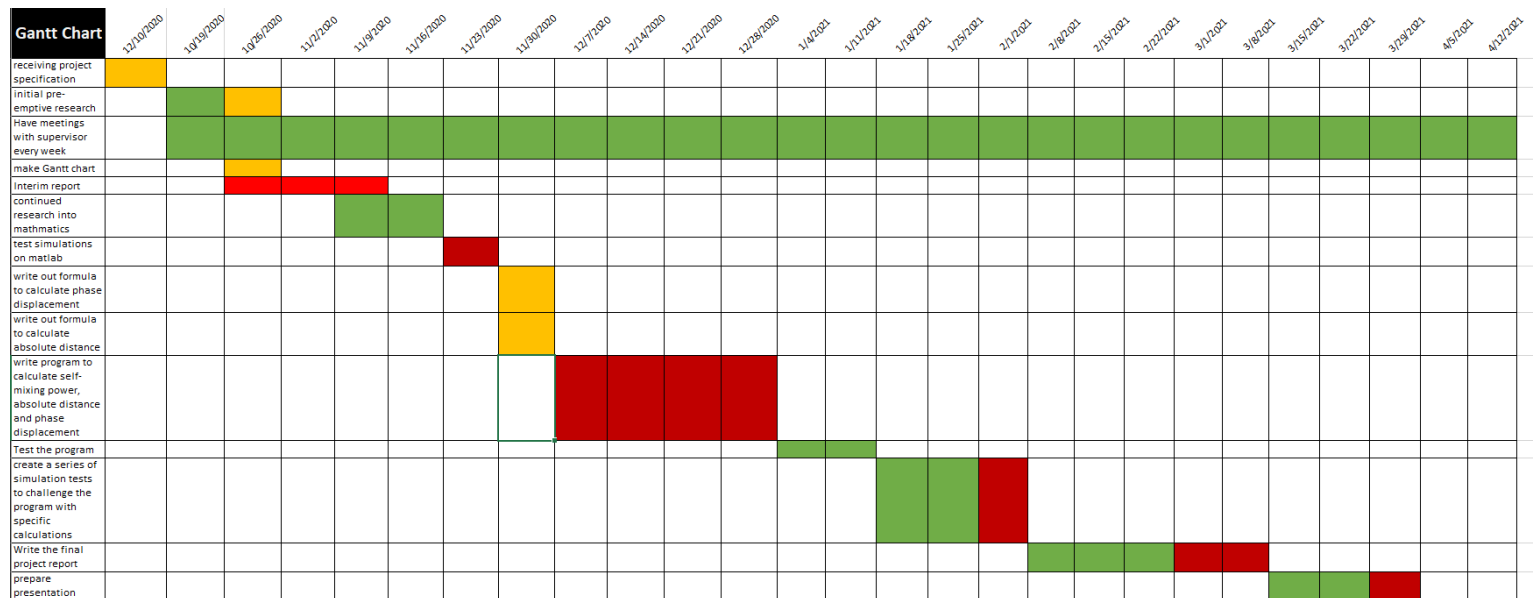
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## Appendix 2 – Project Plan:



**Gantt Chart**

receiving project specification
initial pre-emptive research
Have meetings with supervisor every week
make Gantt chart
Interim report
continued research into mathematics
test simulations on matlab
write out formula to calculate phase displacement
write out formula to calculate absolute distance
write program to calculate self-mixing power, absolute distance and phase displacement
Test the program
create a series of simulation tests to challenge the program with specific calculations
Write the final project report
prepare presentation



**Green** = can be easily rescheduled for another time.

**Yellow** = can be rescheduled but will affect the overall schedule structure.

**Red** = very difficult to rearrange and should be targeted to be completed within the time given.

Risk no.	Description of risk	Probability of the risk	Effect on the project	Contingencies/actions
1	A license for the Matlab program could become void or lost/stolen	Extremely low	Would halt the progress of the simulation and testing	Can request another license from the university technicians
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3	The storage device that all the files are stored on can fail losing all the progress made.	Low	Would completely halt my progress and may even cause me to have to redo large portions of the work again.	Can have a second storage device that a backup of the files can be stored on and updated regularly.

## MATLAB Code:

[AbsoluteDistanceAndDisplacementModulation.m](#)

```
c = 299792458; % Speed of light in vacuum (m/s)

C = 1; % Feedback parameter

alpha = 4.6; % Linewidth enhancement factor (4.6)

% T = 29.3e-3; % Simulation time (s)

T = 10e-3; % Simulation time (s)

N = 1000; % Number of samples

beta = 0.1; % Laser power modulation coefficient

lambda0 = 845e-9; % Laser wavelength (m)


L = 0.024 + 0.5e-6; % Target distance (m)


A = 2.5e-6; % Motion amplitude (m)


f = 50; % Motion frequency (Hz)


deltaf = - 46e9; % Frequency modulation coefficient (Hz)

pm = 3.2; % Power modulation coefficient


t = 0:T/(N - 1):T; % Sample times


d = A*cos (2* pi*f*t); % Displacement samples


triper = T; % Triangle period (s)

tri = 1 + sign (mod (t/triper, 1) - 0.5).*(1 - 2* mod (t/triper, 1)); % Triangle waveform


phi0 = 4*pi*L*(1/lambda0 + deltaf * tri/c) + 4* pi/lambda0 *(L + d); % Round—trip phase samples
phi0D = 4* pi/lambda0 *(L + d); % Round—trip phase samples displacement
phi0AD = 4*pi*L*(1/lambda0 + deltaf * tri/c); % Round—trip phase samples absolute distance


p = zeros (1, N); % Self—mixing signal samples simultaniuse
pD = zeros (1, N); % Self—mixing signal samples displacement
pAD = zeros (1, N); % Self—mixing signal samples absolute distance


for i = 1 : N % Generate the synthetic self—mixing signal
p(i) = beta * selmixpower (C, phi0 (i), alpha) + pm * tri(i);
pD(i) = beta * selmixpower (C, phi0D (i), alpha) + pm * tri(i);
pAD(i) = beta * selmixpower (C, phi0AD (i), alpha) + pm * tri(i);
```



end

subplot(3,1,1);

plot (t, p); xlabel ('Time (s)'); ylabel ('Power Variation (a.u.)'); % Plot the results

title('Power variation graph of Simultaneous Self-Mixing');

subplot(3,1,2);

plot (t, pD); xlabel ('Time (s)'); ylabel ('Power Variation (a.u.)'); % Plot of Displacement

title('Power variation graph of Displacement');

subplot(3,1,3);

plot (t, pAD); xlabel ('Time (s)'); ylabel ('Power Variation (a.u.)'); % Plot of Absolute distance

title('Power variation graph of Absolute distance');

### [SelmixPower.m](#)

% Functions for solving self—mixing equations—Kliese et al., 2014

function power = selmixpower (C, phi0, alpha) % Power level at a sample in time

if (C <= 1.0)

[phimin, phimax] = boundsweak (C, phi0);

else

[phimin, phimax] = boundsstrong (C, phi0, alpha);

end

excessphase = @(x)x - phi0 + C\*sin(x + atan (alpha));

% If the value at the left bound positive, then it will be very close to the solution.

% If the value at the upper bound is negative, it will be very close to the solution.

if (excessphase (phimin) > 0)

excessphase (phimin)

phi = phimin;

elseif (excessphase (phimax) < 0)

excessphase (phimax)

phi = phimax;

else

phi = fzero (excessphase, [phimin, phimax]);

end

power = cos (phi);

end

function [phimin, phimax] = boundsweak (C, phi0) % Find search region when C <= 1

phimin = phi0 - C;

phimax = phi0 + C;

end

function [phimin, phimax] = boundsstrong (C, phi0, alpha) % Find search region when  $C \geq 1$

persistent m; % Solution region number

if isempty (m); m = 0; end

% Calculate upper & lower values of m where solutions exist then ensure m is between them

mlower = ceil ((phi0 + atan (alpha) + acos (1/C) - sqrt (C\*C - 1))/(2\*pi) - 1.5);

mupper = floor ((phi0 + atan (alpha) - acos (1/C) + sqrt (C\*C - 1))/(2\*pi) - 0.5);

if (m < mlower); m = mlower; end

if (m > mupper); m = mupper; end

phimin = (2\*m+1)\*pi + acos (1/C) - atan (alpha); % Trough

phimax = (2\*m+3)\* pi - acos (1/C) - atan (alpha); % Peak

end