Weekly Homework 40

Math Gecs

December 10, 2024

Exercise 1

Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible for every natural number n.)

Source: 1959 IMO Problem 1

Solution. *Denoting the greatest common divisor of a, b as (a, b), we use the Euclidean algorithm:*

$$(21n + 4, 14n + 3) = (7n + 1, 14n + 3) = (7n + 1, 1) = 1$$

Their greatest common divisor is 1, so $\frac{21n+4}{14n+3}$ is irreducible. Q.E.D.