# Weekly Homework 13

Math Gecs

April 22, 2024

## Exercise 1

If $a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ and $a_1 + a_2 + \cdots + a_n = 1$, then prove:

$$a_1^2 + 3a_2^2 + 5a_3^2 + \cdots + (2n-1)a_n^2 \leq 1$$

Source: 2002 Pan African MO Problem 6

**Solution.** *Note that* $1 = (a_1 + a_2 + \cdots + a_n)^2 = \sum_{i=1}^{n}(a_i^2) + 2a_1a_2 + 2a_1a_3 + \cdots 2a_{n-1}a_n$. *Additionally, if* $i \leq j$, *then* $a_i \geq a_j$ *and* $2a_ia_j \geq 2a_j^2$.

*For a given value* $j$, *there are* $j-1$ *terms in the form* $2a_ia_j$. *Thus,*

$$(\sum_{i=1}^{n} a_i)^2 \geq \sum_{i=1}^{n}(a_i^2) + 2a_2^2 + 4a_3^2 + \cdots + (2n-2)a_n^2$$
$$1 \geq a_1^2 + 3a_2^2 + 5a_3^2 + \cdots + (2n-1)a_n^2.$$