Weekly Homework 32

Math Gecs

October 13, 2024

Exercise 1

Let S be a square with the side length 20 and let M be the set of points formed with the vertices of S and another 1999 points lying inside S. Prove that there exists a triangle with vertices in M and with area at most equal with $\frac{1}{10}$.

Source: 1999 JBMO Problem 3

Solution. *Triangulate S into triangles with vertices being the vertices of S and the members of M. There are $2*(1999+1) = 4000$ triangles thusly formed, so by the pigeonhole principle, at least one of the holes has to have area at most $\frac{20^2}{4000} = \frac{1}{10}$, and we are done.*