Weekly Homework 33

Math Gecs

October 20, 2024

Exercise 1

Let γ be circle and let P be a point outside γ. Let PA and PB be the tangents from P to γ (where $A, B \in \gamma$). A line passing through P intersects γ at points Q and R. Let S be a point on γ such that $BS \parallel QR$. Prove that SA bisects QR.

Source: 2000 Pan African MO Problem 5

Solution. *There is a projective transformation which maps γ to a circle and that maps the midpoint of QR to its center (EXPAND); therefore, we may assume without loss of generality that the midpoint of QR is the center of γ. But then B is the reflection of A across QR, so that S is the antipode of A on γ, and we are done.*