Weekly Homework 15

Math Gecs

May 07, 2024

Exercise 1

For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

Source: 2015 AMC 12A Problem 22

Answer. $\boxed{8}$

Solution. *We can start off by finding patterns in $S(n)$. When we calculate a few values we realize either from performing the calculation or because the calculation was performed in the exact same way that $S(n) = 2^n - 2((n_4) - (n_5) \dots (n_n))$. Rearranging the expression we realize that the terms aside from 2^{2015} are congruent to 0 mod 12(Just put the equation in terms of 2^{2015} and the four combinations excluded and calculate the combinations mod 12). Using patterns we can see that 2^{2015} is congruent to 8 mod 12. Therefore $\boxed{8}$ is our answer. Very minor edit in LaTeX by get-rickrolled*