Weekly Homework 36

Math Gecs

November 12, 2024

Exercise 1

Show that, for all integers n, $n^2 + 2n + 12$ is not a multiple of 121.

Source: 1971 Canadian MO Problem 6

Answer. *type your answer here.*

Solution. *Notice* $n^2 + 2n + 12 = (n + 1)^2 + 11$. *For this expression to be equal to a multiple of 121,* $(n + 1)^2 + 11$ *would have to equal a number in the form* $121x$. *Now we have the equation* $(n + 1)^2 + 11 = 121x$. *Subtracting* 11 *from both sides and then factoring out* 11 *on the right hand side results in* $(n + 1)^2 = 11(11x - 1)$. *Now we can say* $(n + 1) = 11$ *and* $(n + 1) = 11x - 1$. *Solving the first equation results in* $n = 10$. *Plugging in* $n = 10$ *in the second equation and solving for* x, $x = 12/11$. *Since* $12/11$ *121 *is clearly not a multiple of 121,* $n^2 + 2n + 12$ *can never be a multiple of* 121.

Solution. *In order for* 121 *to divide* $n^2 + 2n + 12$, 11 *must also divide* $n^2 + 2n + 12$.

Plugging in all numbers modulo 11*:*

$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, *or* $0, 1, 2, 3, 4, 5, (-5), (-4), (-3), (-2), (-1)$ *to make computations easier,*

reveals that only 10 *satisfy the condition* $n^2 + 2n + 12 \equiv 0 \pmod{11}$.

Plugging 10 *into* $n^2 + 2n + 12$ *shows that it is not divisible by* 121.

Thus, there are no integers n *such that* $n^2 + 2n + 12$ *is divisible by* 121.