Weekly Homework 28

Math Gecs

September 15, 2024

Exercise 1

α, β, and γ are the roots of $x(x-200)(4x+1) = 1$. Let

$$\omega = \tan^{-1}(\alpha) + \tan^{-1}(\beta) + \tan^{-1}(\gamma).$$

The value of $\tan(\omega)$ can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Determine the value of $m+n$.

Source: Mock AIME 2 Pre 2005 Problem 11

Answer. $\boxed{167}$

Solution. *We know that* α, β, γ *are the roots of* $x(x-200)(x+1/4) - 1/4 = x^3 - \frac{799}{4}x^2 - 50x - \frac{1}{4}$. *By Vieta's formulas, we have:*
$\alpha + \beta + \gamma = \frac{799}{4}$
$\alpha\beta + \beta\gamma + \gamma\alpha = -50$
$\alpha\beta\gamma = \frac{1}{4}$
Now, by tangent addition formulas, we have $\tan(\omega) = \frac{\alpha+\beta+\gamma-\alpha\beta\gamma}{1-\alpha\beta-\beta\gamma-\gamma\alpha}$. *Substituting Vieta's formulas, we obtain* $\tan(\omega) = \frac{\frac{799}{4}-\frac{1}{4}}{1-(-50)} = \frac{\frac{798}{4}}{51} = \frac{133}{34}$. *Therefore, our answer is* $133+34 = \boxed{167}$ *and we are done.*