Weekly Homework 19

Math Gecs

June 08, 2024

Exercise 1

The nine horizontal and nine vertical lines on an 8×8 checkerboard form r rectangles, of which s are squares. The number s/r can be written in the form m/n, where m and n are relatively prime positive integers. Find $m + n$.

Source: 1997 AIME Problem 2

Answer. $\boxed{125}$

Solution. *To determine the two horizontal sides of a rectangle, we have to pick two of the horizontal lines of the checkerboard, or* $\binom{9}{2} = 36$. *Similarly, there are* $\binom{9}{2}$ *ways to pick the vertical sides, giving us* $r = 1296$ *rectangles.*

For s, *there are* 8^2 *unit squares,* 7^2 *of the* 2×2 *squares, and so on until* 1^2 *of the* 8×8 *squares. Using the sum of squares formula, that gives us* $s = 1^2 + 2^2 + \cdots + 8^2 = \dfrac{(8)(8+1)(2 \cdot 8 + 1)}{6} = 12 * 17 = 204$.

Thus $\frac{s}{r} = \dfrac{204}{1296} = \dfrac{17}{108}$, *and* $m + n = \boxed{125}$.