Weekly Homework 43

Math Gecs

December 30, 2024

Exercise 1

A moving particle starts at the point $(4, 4)$ and moves until it hits one of the coordinate axes for the first time. When the particle is at the point (a, b), it moves at random to one of the points $(a-1, b)$, $(a, b-1)$, or $(a-1, b-1)$, each with probability $\frac{1}{3}$, independently of its previous moves. The probability that it will hit the coordinate axes at $(0, 0)$ is $\frac{m}{3^n}$, where m and n are positive integers such that m is not divisible by 3. Find $m + n$.

Source: 2019 AIME I Problem 5

Solution. *One could recursively compute the probabilities of reaching* $(0, 0)$ *as the first axes point from any point* (x, y) *as*

$$P(x, y) = \frac{1}{3}P(x-1, y) + \frac{1}{3}P(x, y-1) + \frac{1}{3}P(x-1, y-1)$$

for $x, y \geq 1$*, and the base cases are* $P(0, 0) = 1, P(x, 0) = P(y, 0) = 0$ *for any* x, y *not equal to zero. We then recursively find* $P(4, 4) = \frac{245}{2187}$ *so the answer is* $245 + 7 = \boxed{252}$.