Finite Automata & Regular Expressions

https://xkcd.com/208/

What is a Computer?

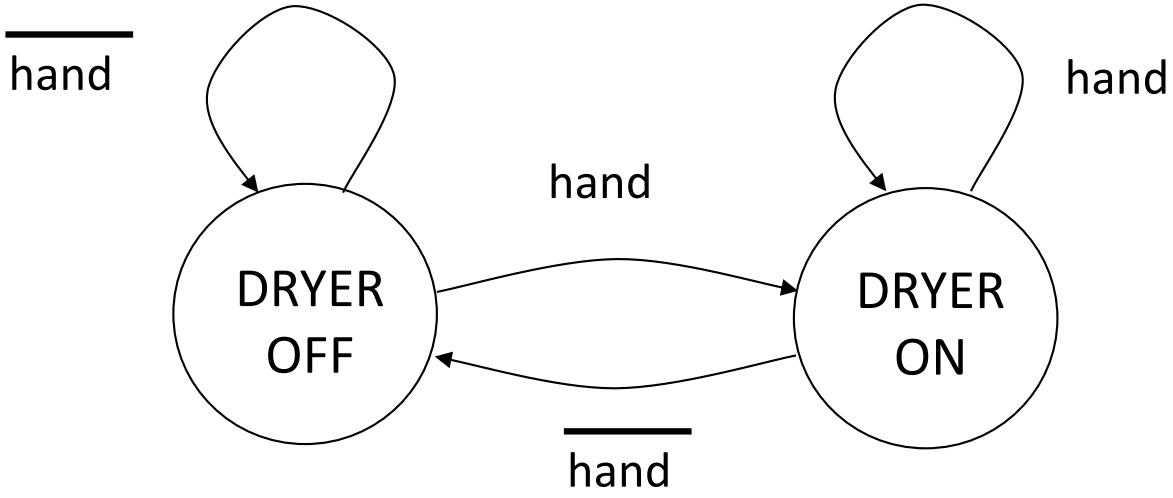
- Theory of computation:
  - What are the fundamental capabilities and limitations of computers?
  - Complexity theory: what makes some problems computationally hard and others easy?
  - Computability theory: what makes some problems solvable and others unsolvable?
  - Automata theory: definitions and properties of mathematical models of computation

Finite Automata

- Also known as finite state automata (FSA) or finite state machine (FSM)
- A simple mathematical model of a computer
- Applications: hardware design, compiler design, text processing

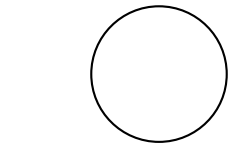
A First Example

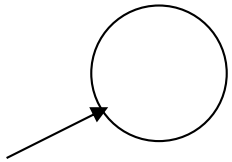
- Touch-less hand-dryer

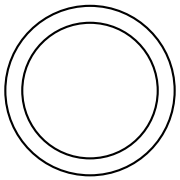


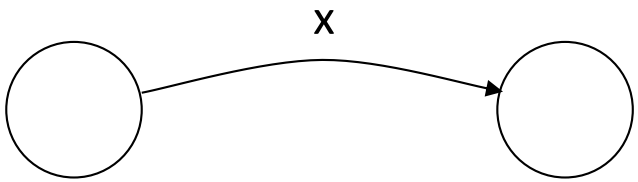
Finite Automata State Graphs

- A state
- The start state
- An accepting state
- A transition







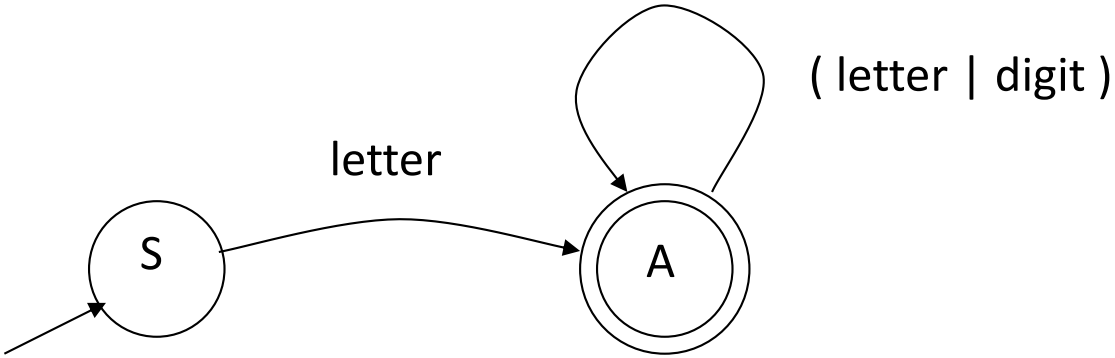


Finite Automata

- Transition: $s_1 \rightarrow^x s_2$
  - In state s_1 on input "x" go to state s_2
- At end of input
  - If in accepting state => *accept*
  - Else => *reject*
- If no transition possible => reject

Language of a FA

- Language of finite automaton M: set of all strings accepted by M
- Example:

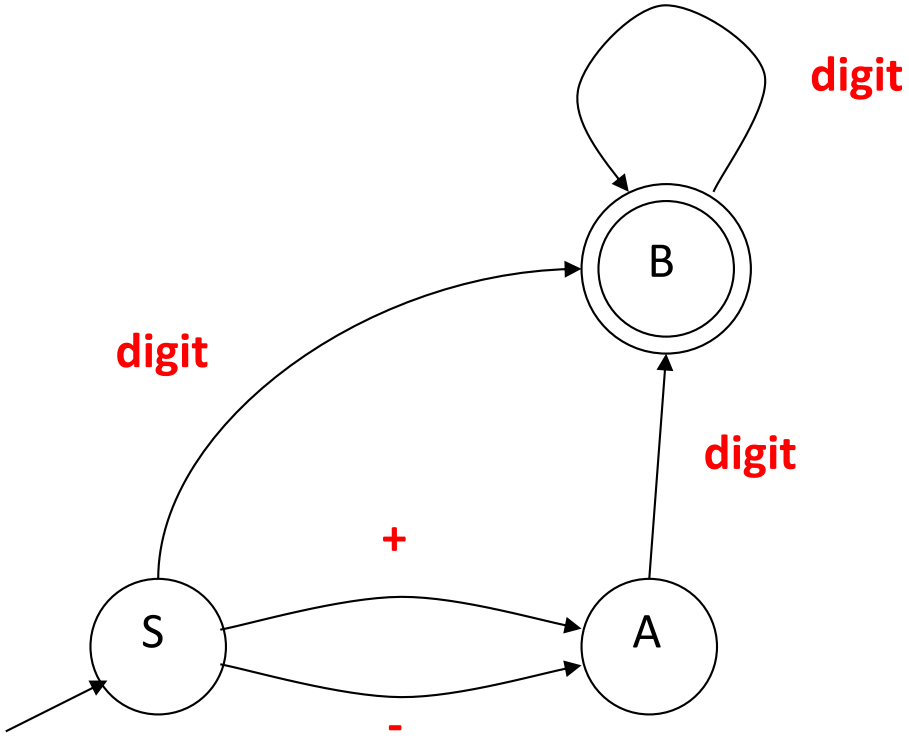


- Which of the following are in the language?
  - x, tmp2, 123, a?, 2apples

- A language is called a regular language if it is recognized by some finite automaton

Example

- What is the language of this FA?

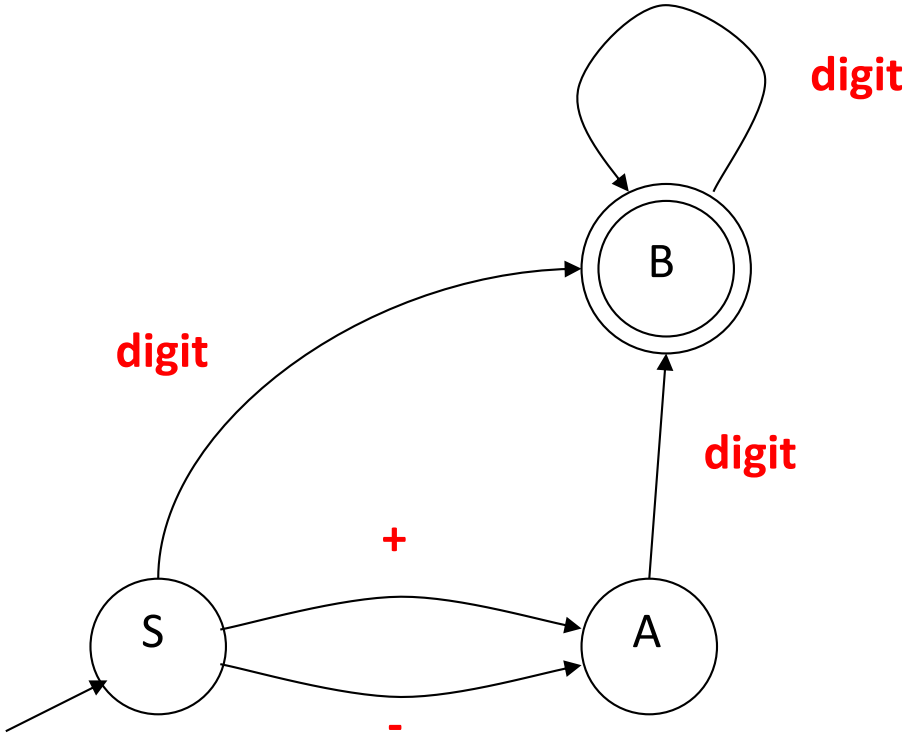


Regular Expressions

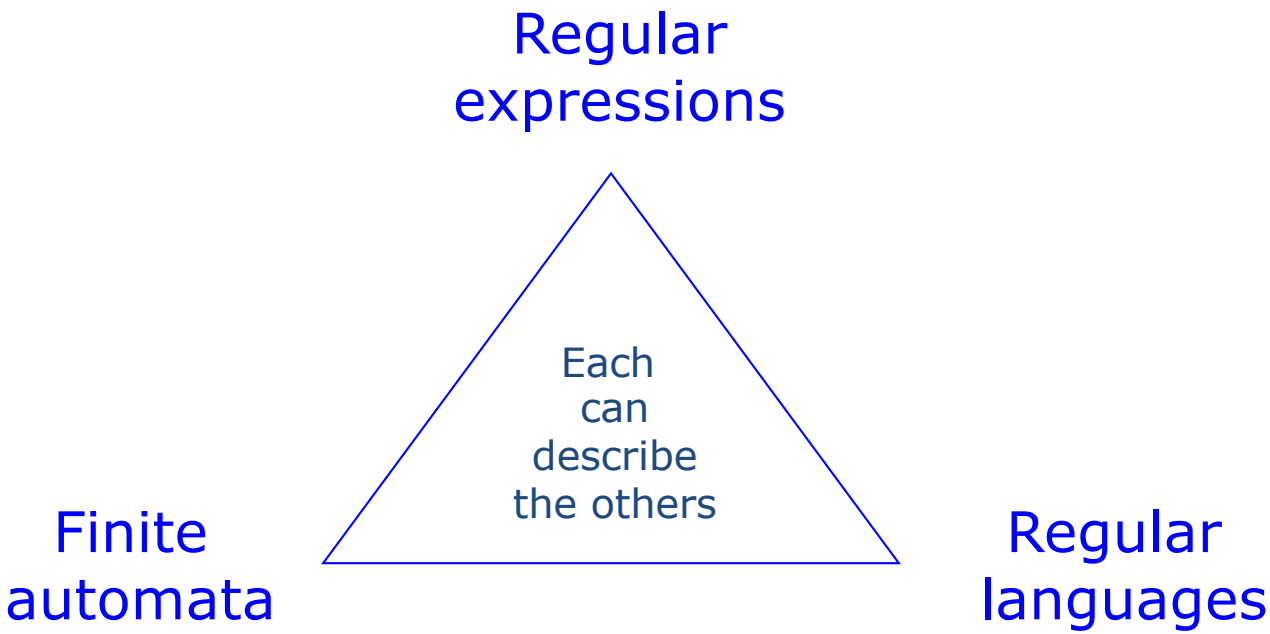
- Regular expressions (regex) are used to describe regular languages
- Arithmetic expression example: (8+2)*3
- Regular expression example: (\+|-)?[0-9]+

Example

- What is the language of this FA?
- Regular expression: (\+|-)?[0-9]+



Three Equivalent Representations



Regex Rules

- Goal: match patterns
- String of characters matches the same string

woodchuck	‘how much wood does a woodchuck chuck?’
e	‘you are a programmer’
206	‘INFO206 consists of 206A and 206B.’
!	‘Keep it to yourself!’

- . Wildcard matches any character at that position

p.nt	‘pant, pint, paint, print’

Regex Rules

- ? Zero or one occurrences of the preceding character/regex

woodchucks?	'how much wood does a woodchuck chuck?'
behaviou?r	'behaviour is the British spelling of behavior'

- * Zero or more occurrences of the preceding character/regex

baa*	ba, baa, baaa, baaaa …
ba*	b, ba, baa, baaa, baaaa …
[ab]*	ε, a, b, ab, ba, baaa, aaabbb, …
[0-9][0-9]*	any positive integer, or zero
cat.*cat	A string where 'cat' appears twice anywhere

- + One or more occurrences of the preceding character/regex

ba+	ba, baa, baaa, baaaa …

- {n} Exactly n occurrences of the preceding character/regex

ba{3}	baaa

Regex Rules

- * is greedy:

  <.*> Home

- Lazy (non-greedy) quantifier:

  <.*?> Home

  Similarly, +? is the non-greedy quantifier for +, and ?? is the non-greedy quantifier for ?

Regex Rules

- [ ] Disjunction (Union)

  [wW]ood ‘how much wood does a Woodchuck chuck?’

  [aeiou]* ‘you are a programmer’

  [A-Za-z0-9] (any letter or digit)

  [A-Za-z]* (any letter sequence)

- | Disjunction (Union)

  (cats?|dogs?)+ ‘It is raining cats and a dog.’

- ( ) Grouping

  (gupp(y|ies))* ‘His guppy is the king of guppies.’

Regex Rules

- ^ $ \b Anchors (start/end of input string; word boundary)

^The	‘The cat in the hat.’
^The end\.$	‘The end.’
^The .* end\.$	‘The bitter end.’
(the)*	‘I saw him the other day.’
(\bthe\b)*	‘I saw him the other day.’

- Special rule: when ^ is FIRST WITHIN BRACKETS it means NOT

[^A-Z]*	(anything **not** an upper case letter)

Regex Rules

- \ Escape characters

  \. ‘The + and \ characters are missing.’

  \+ ‘The + and \ characters are missing.’

  \\ ‘The + and \ characters are missing.’

  and so on

Operator Precedence

Operator	Precedence
()	highest
* + ? {}	
sequences, anchors	
|	lowest

- What is the difference?
  - [a-z][a-z]|[0-9]*
  - [a-z]([a-z]|[0-9])*