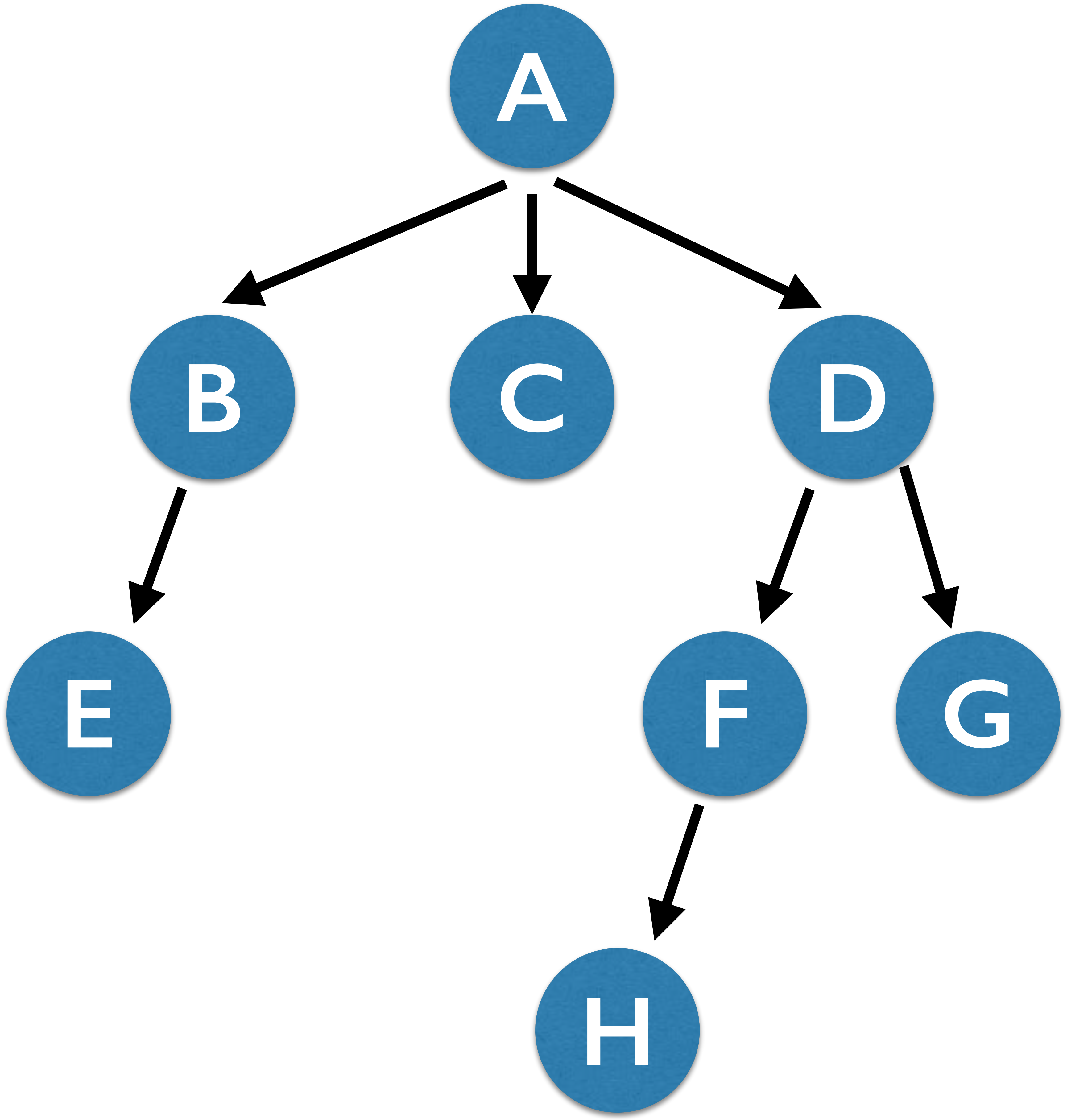
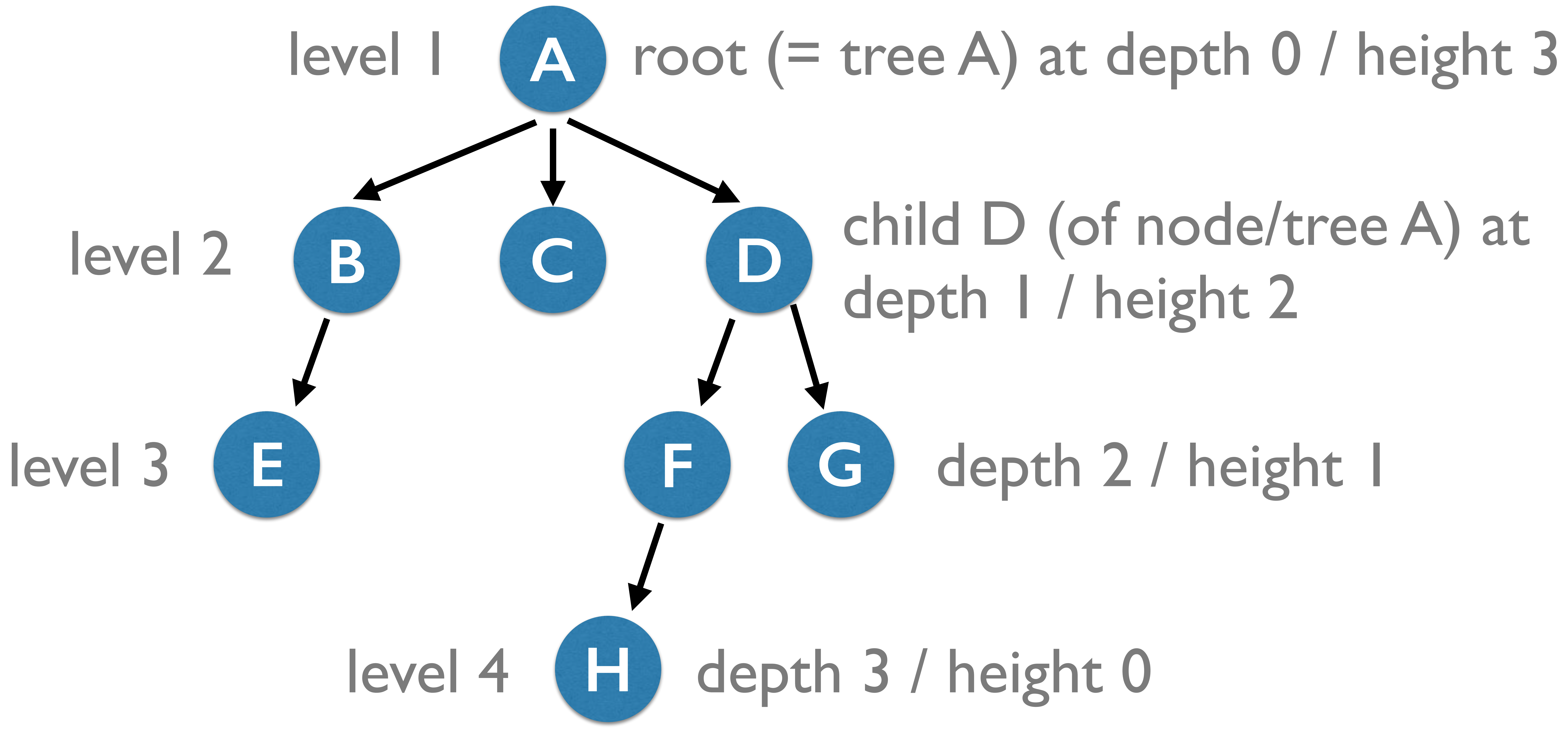
Trees

The Tree ADT

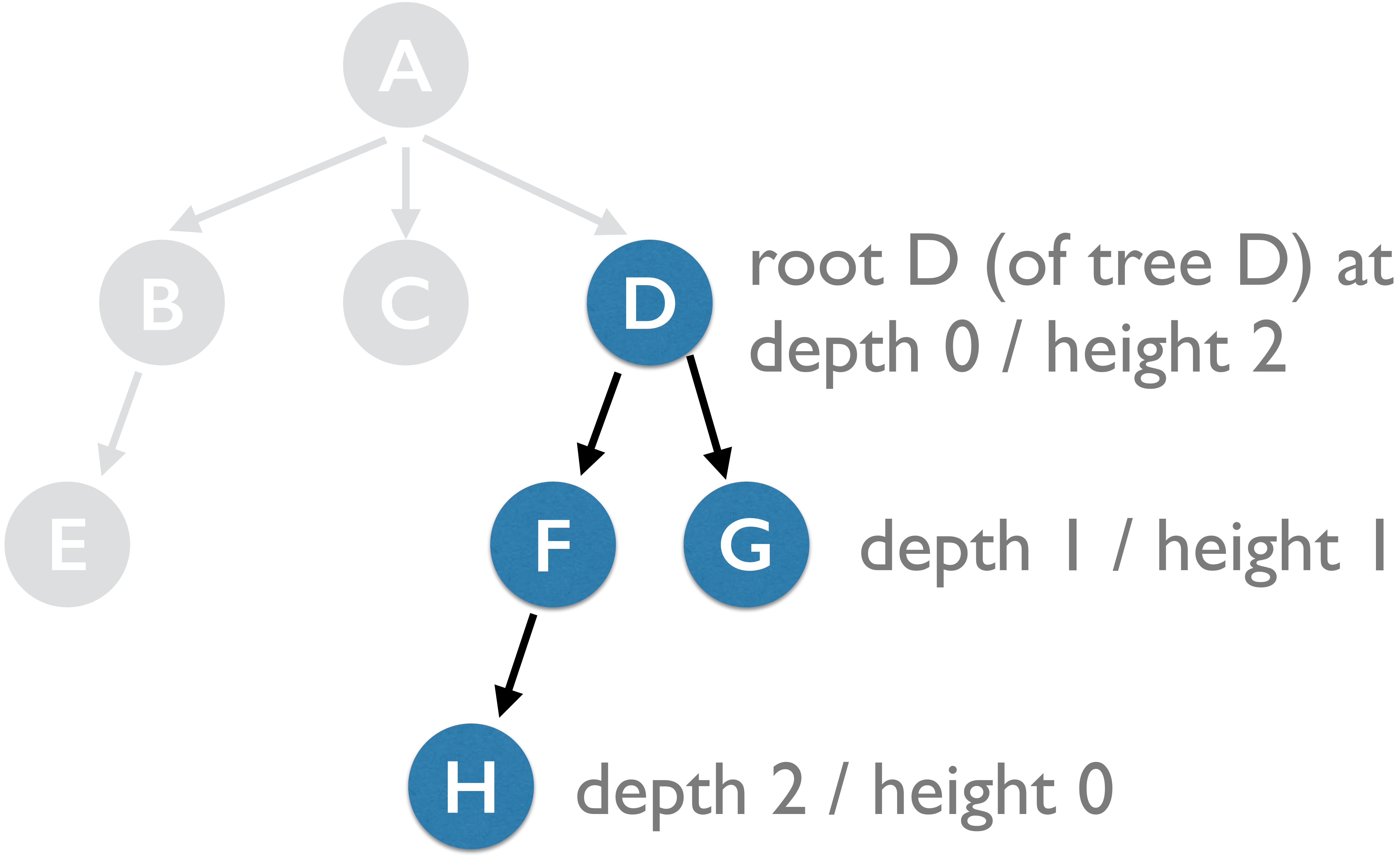


- Nodes contain value(s)
- A primary "root" node
- Children are subtrees (recursive!)
- No duplicated children (cycles); trees can *branch* but never *converge*
- Final nodes called "leaves"
- Height of tree = longest path to leaf
- Level = 1 + number of jumps to root
- Depth = inverse of height

The Tree ADT

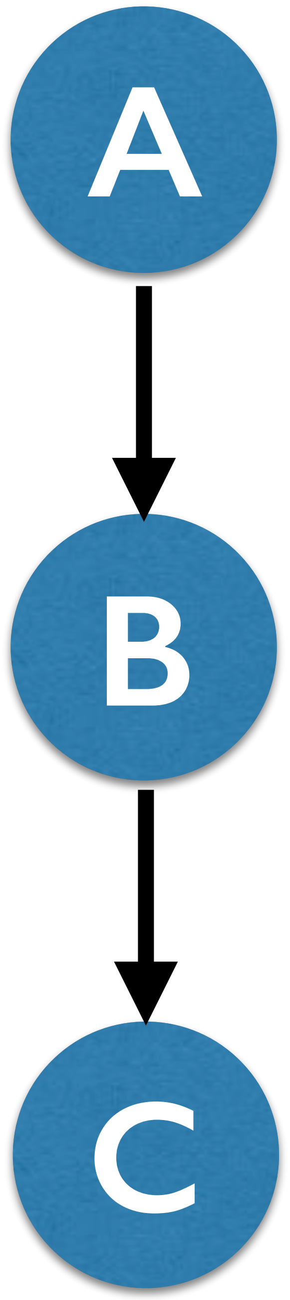

level 1
A
root (= tree A) at depth 0 / height 3
level 2
B
C
D
child D (of node/tree A) at
depth 1 / height 2
level 3
E
F
G
depth 2 / height 1
level 4
H
depth 3 / height 0

Every node is the root of a tree.
You might even say a node *represents* a tree.



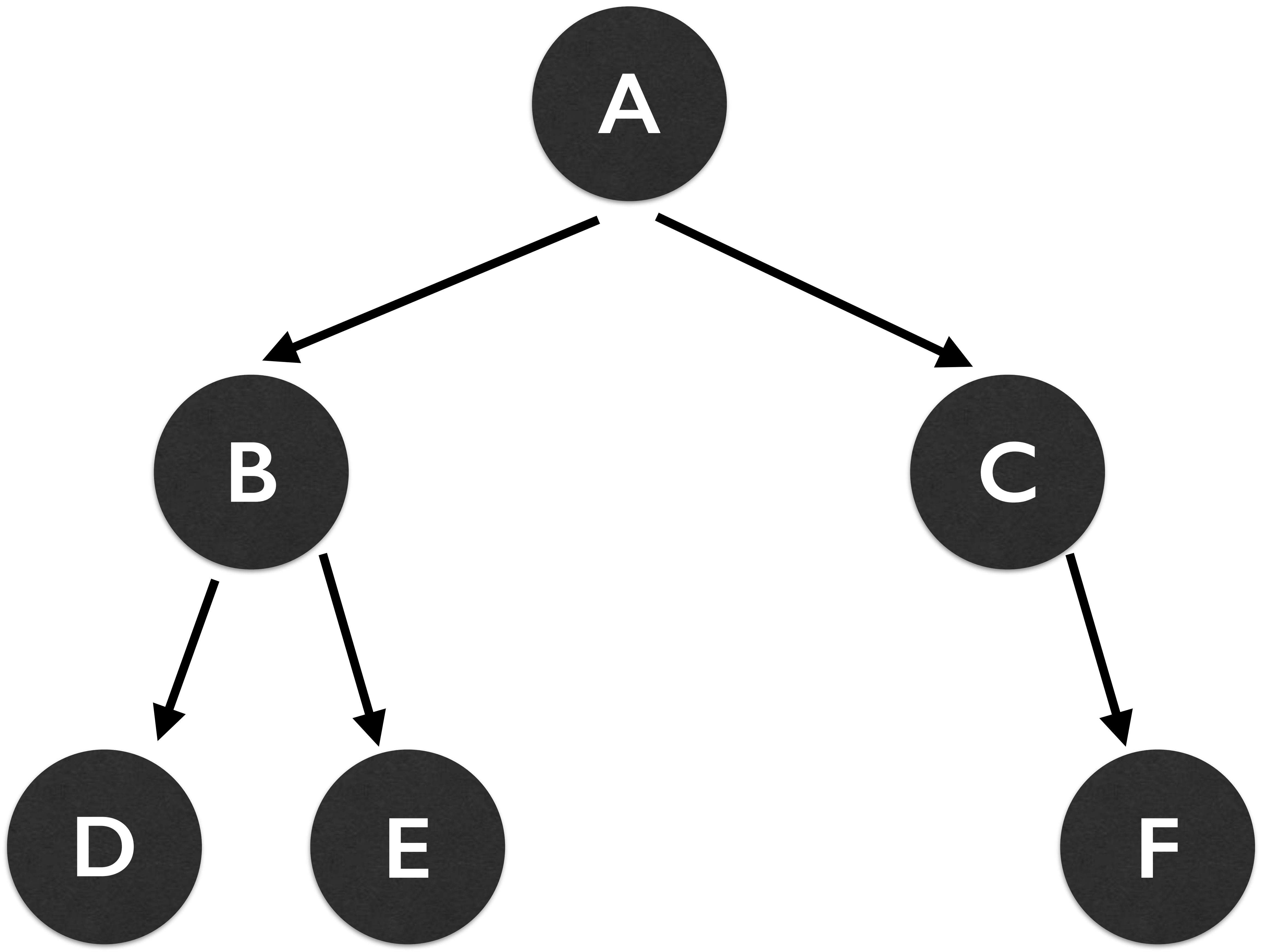
"Degenerate" trees are still trees

A tree of one node

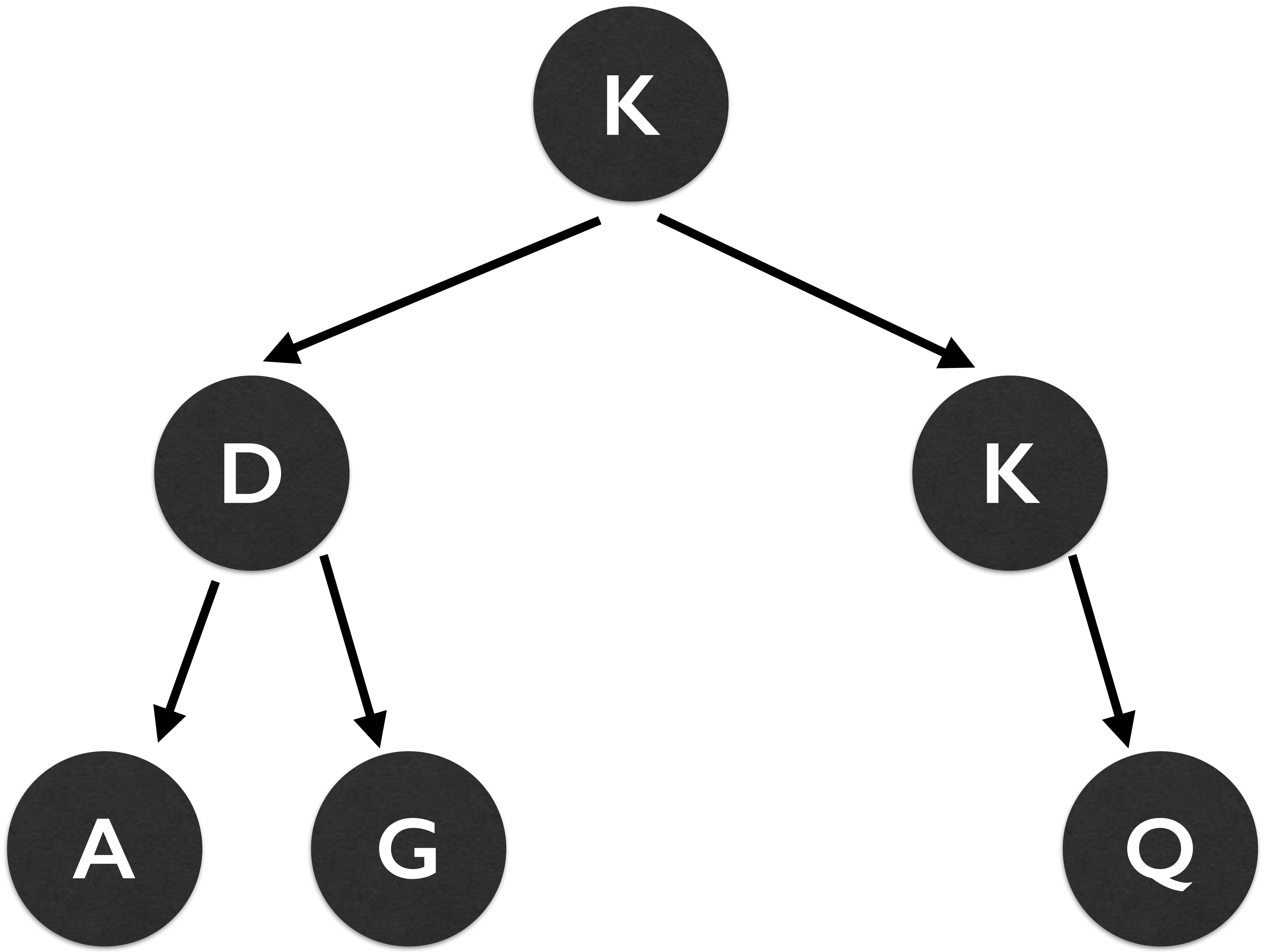


A tree of three nodes

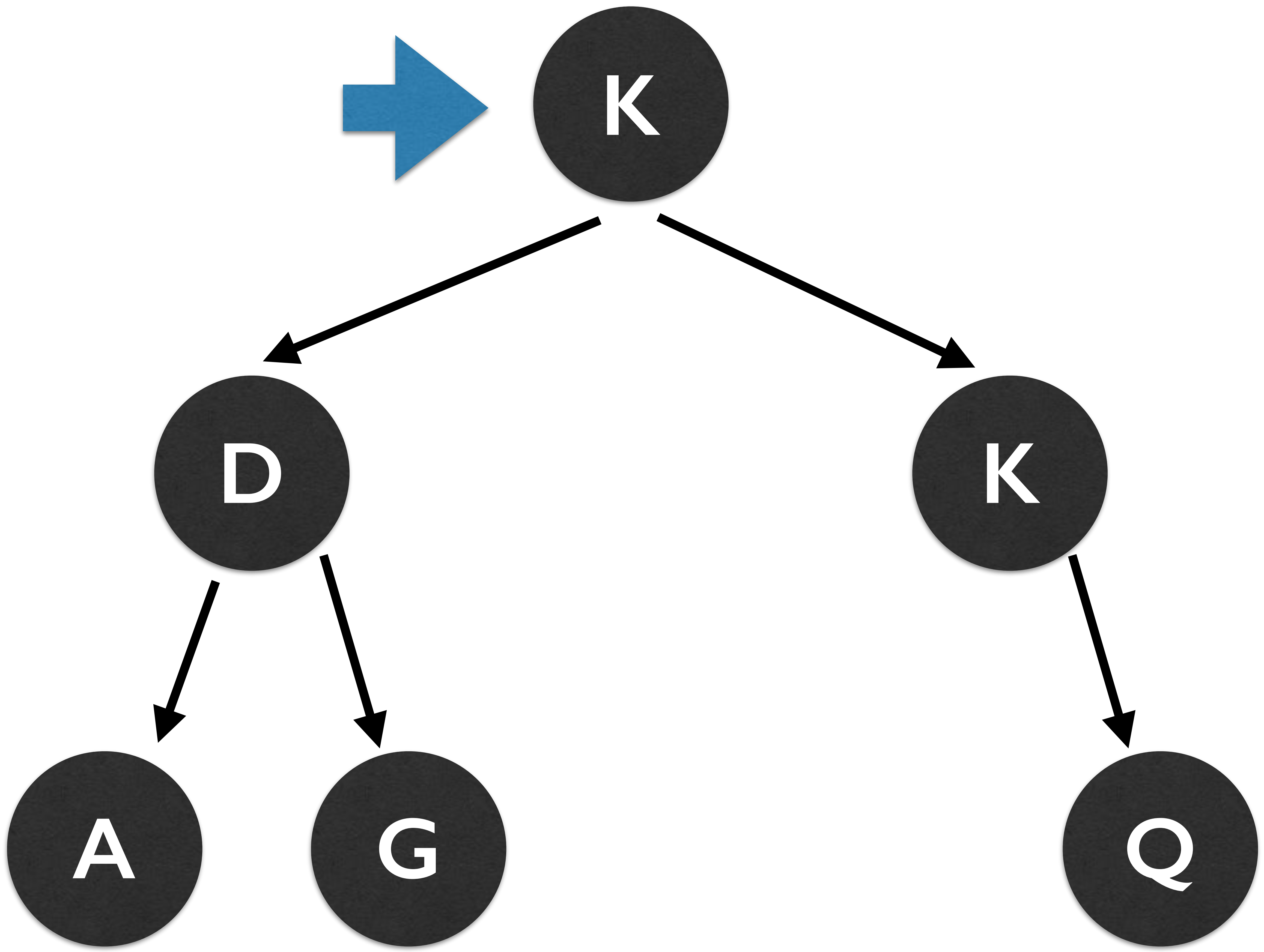
Binary Tree



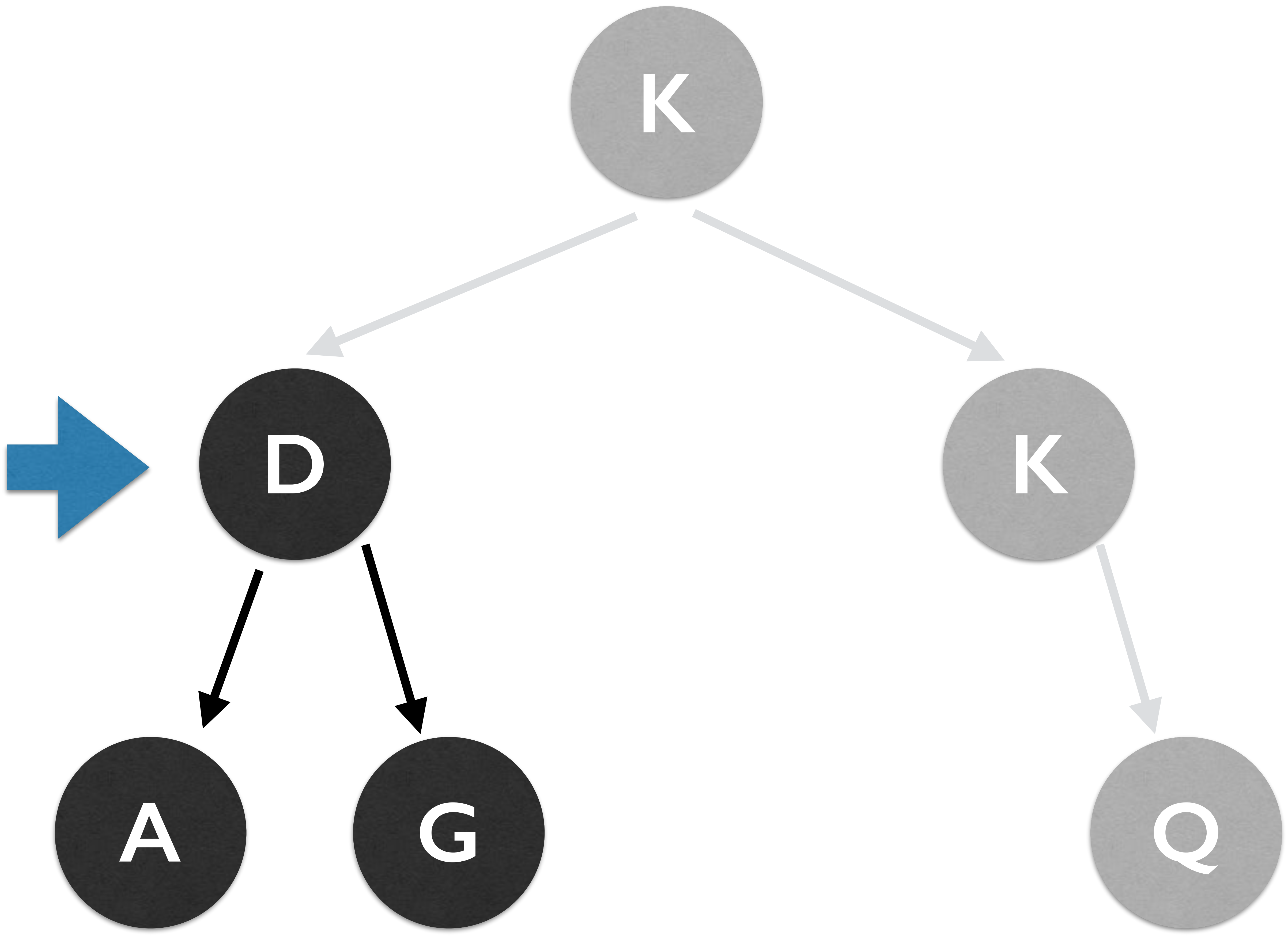
Binary *Search* Tree



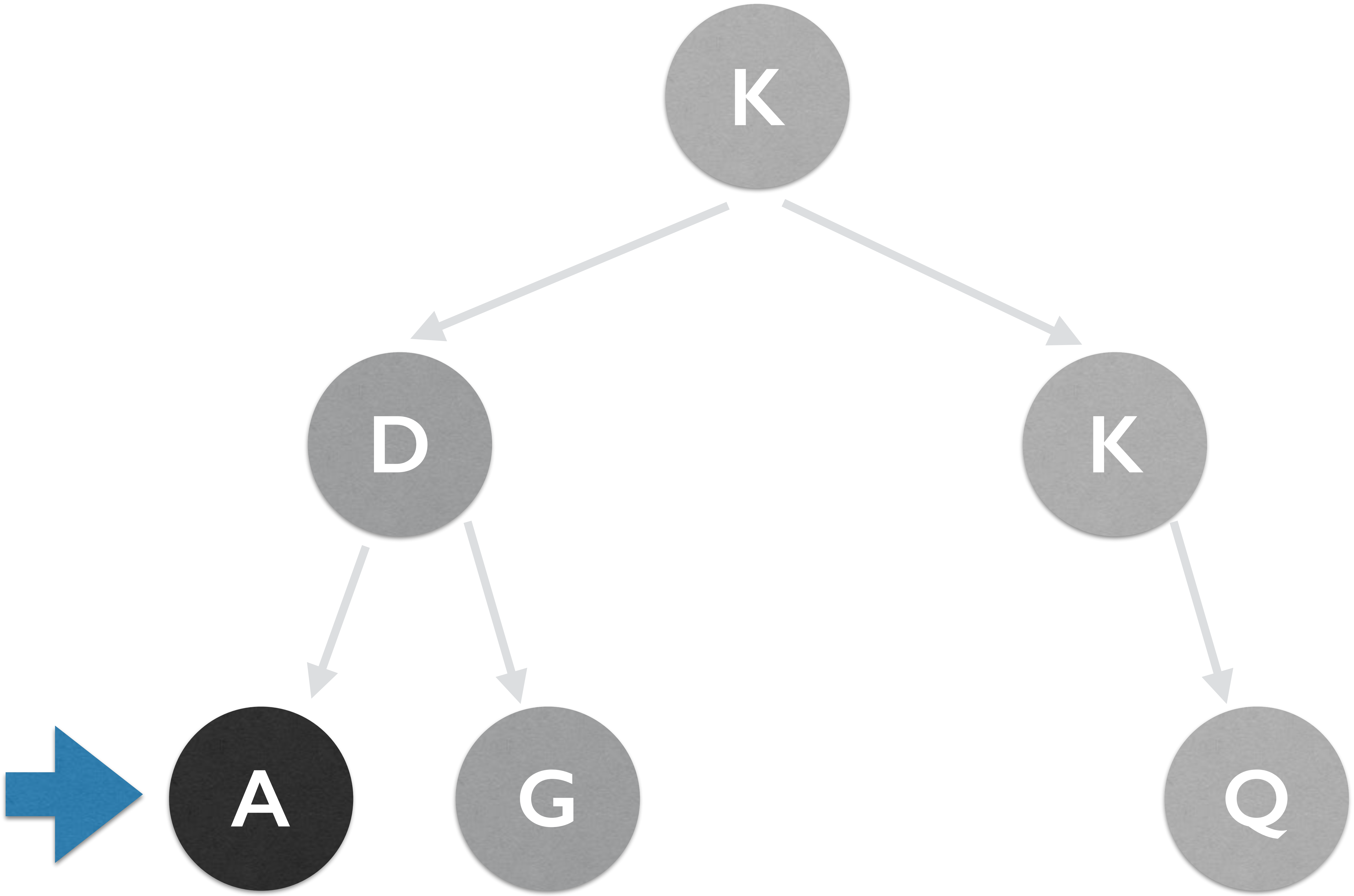
Binary Search Tree: Min Value?



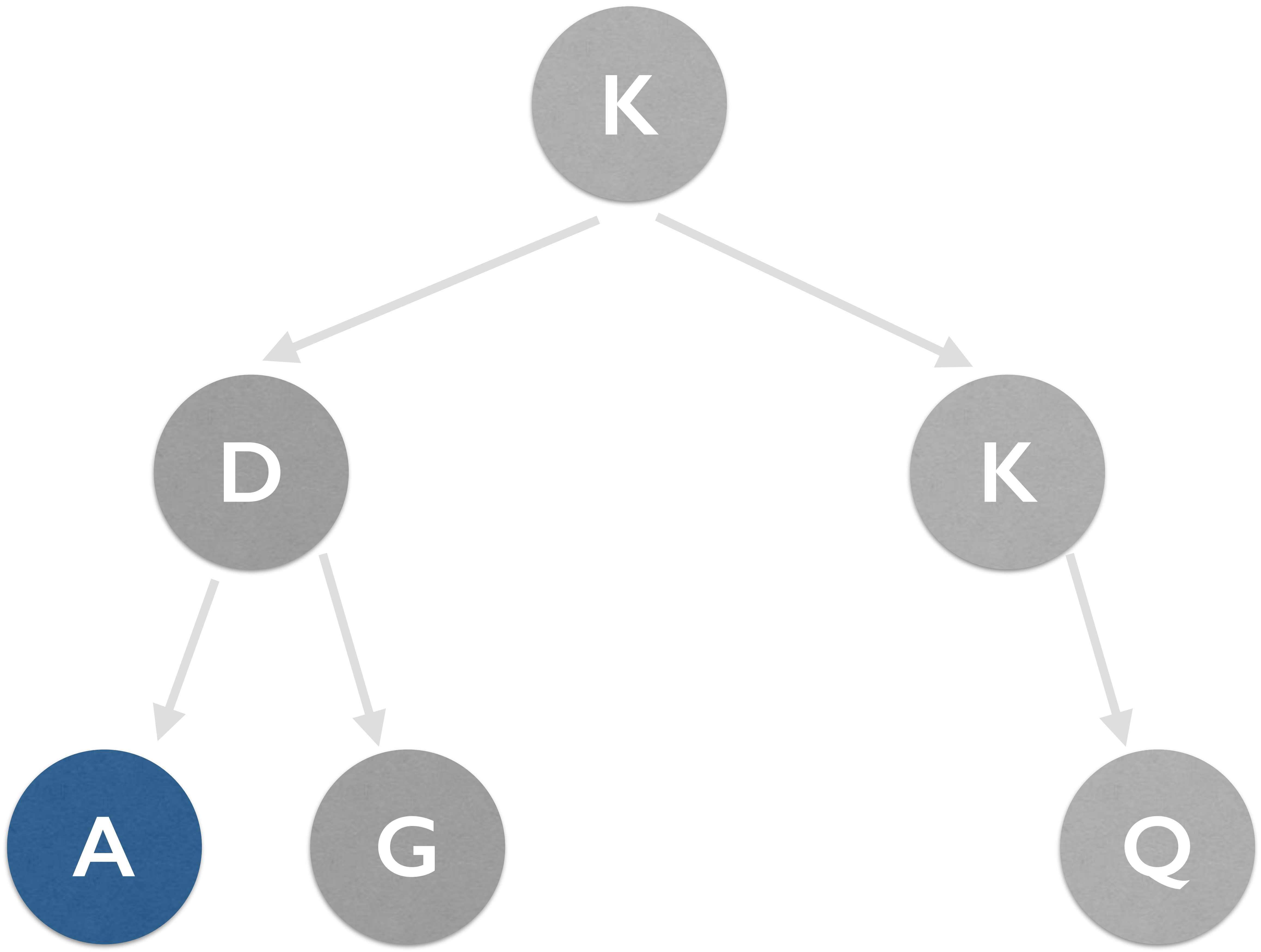
Binary Search Tree: Min Value?



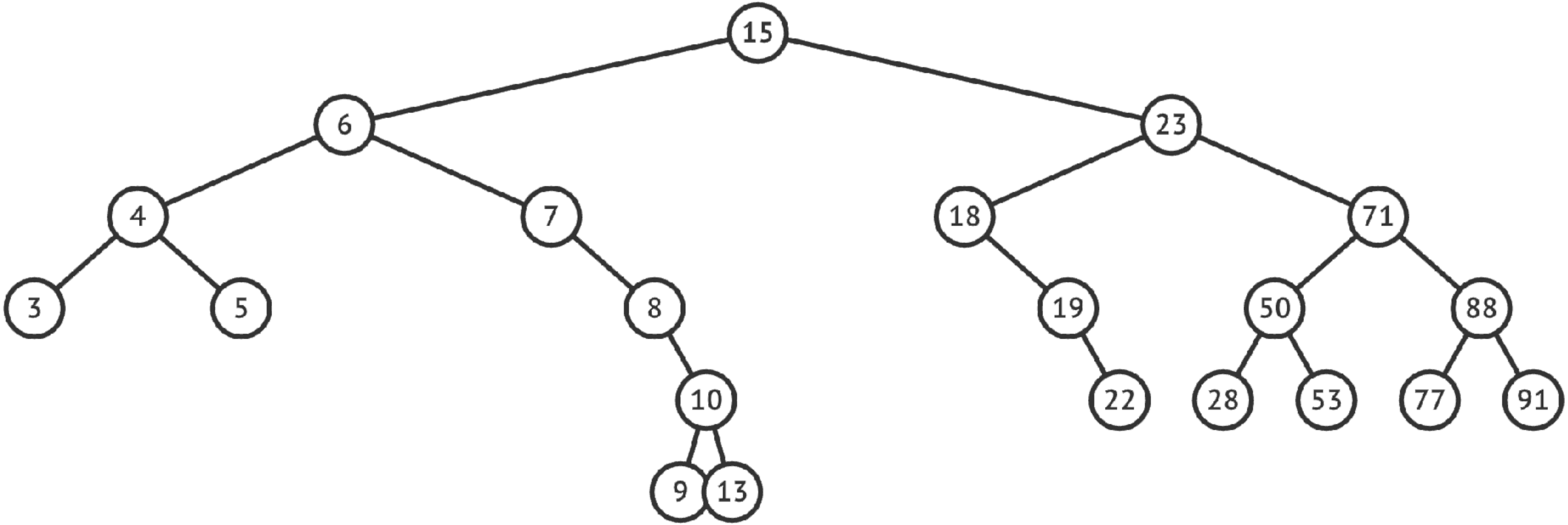
Binary Search Tree: Min Value?



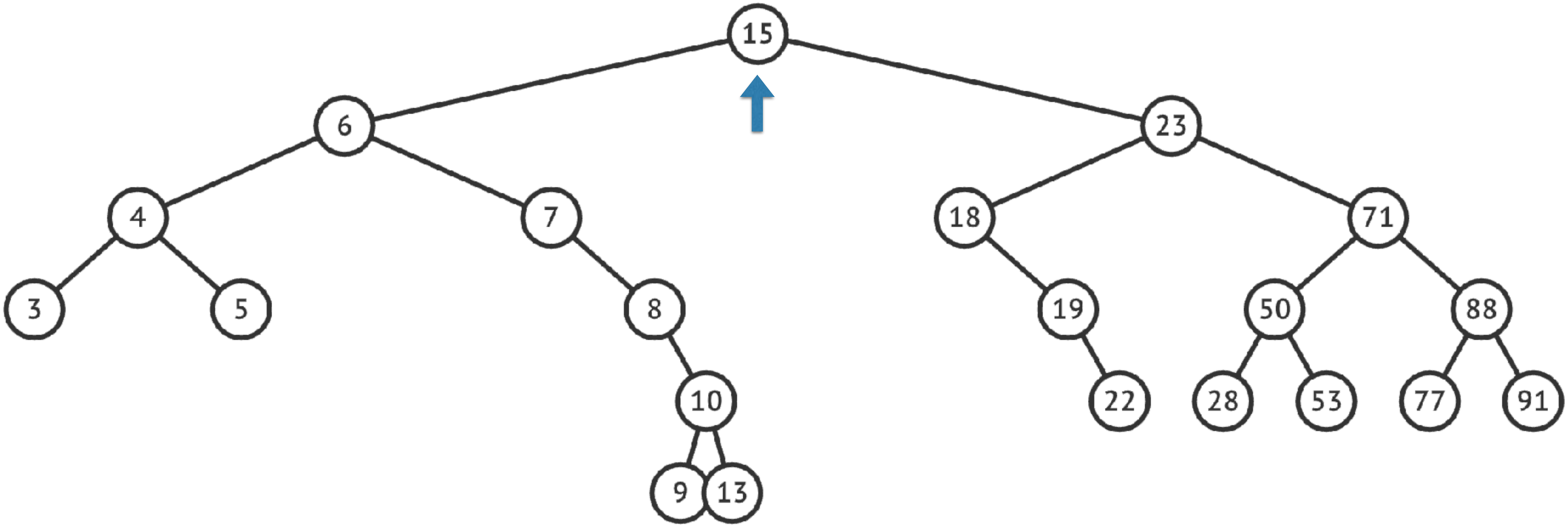
Binary Search Tree: Min Value?



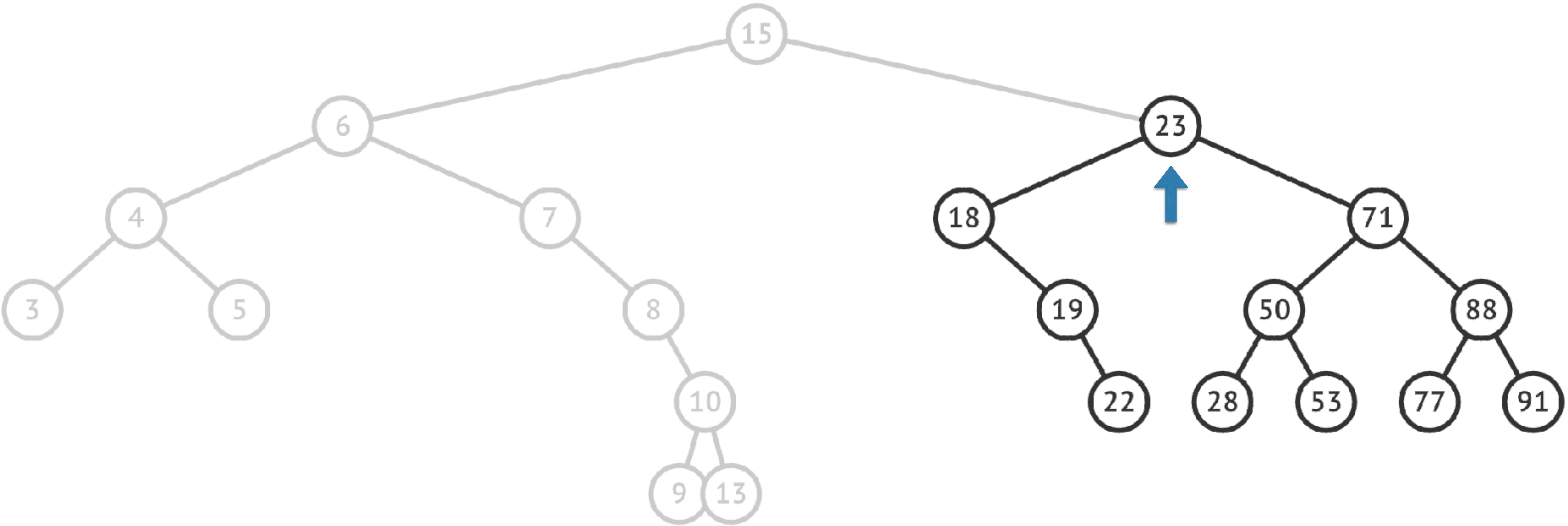
Does this BST contain the value 28?



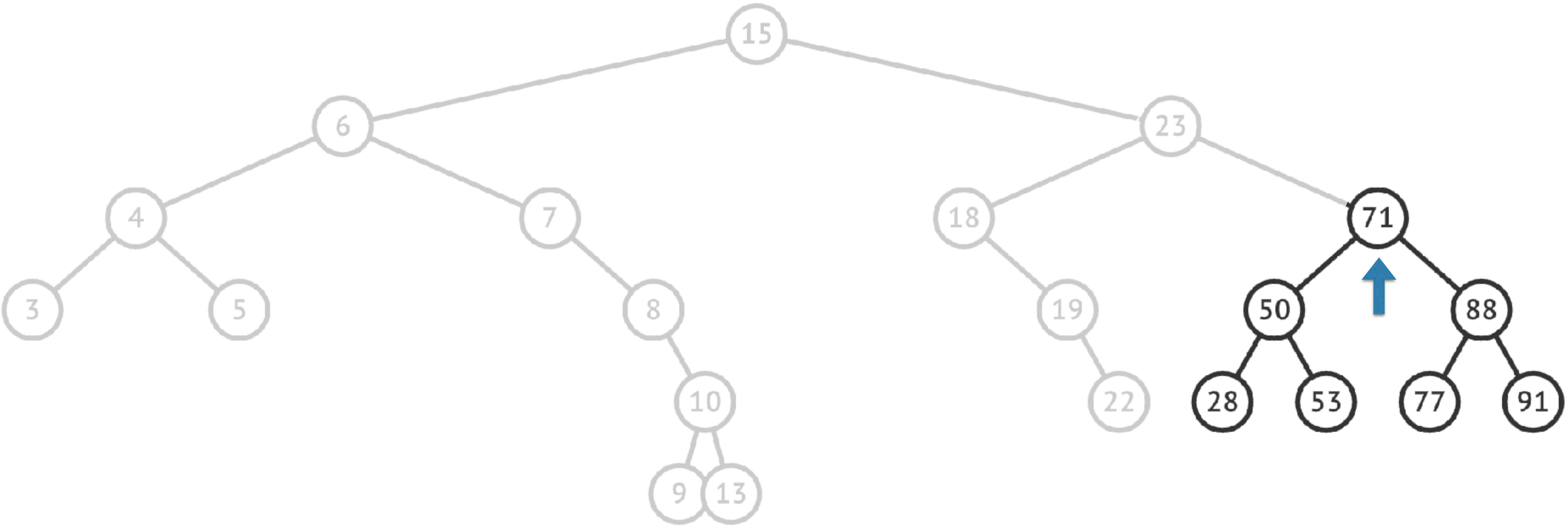
Does this BST contain the value 28?



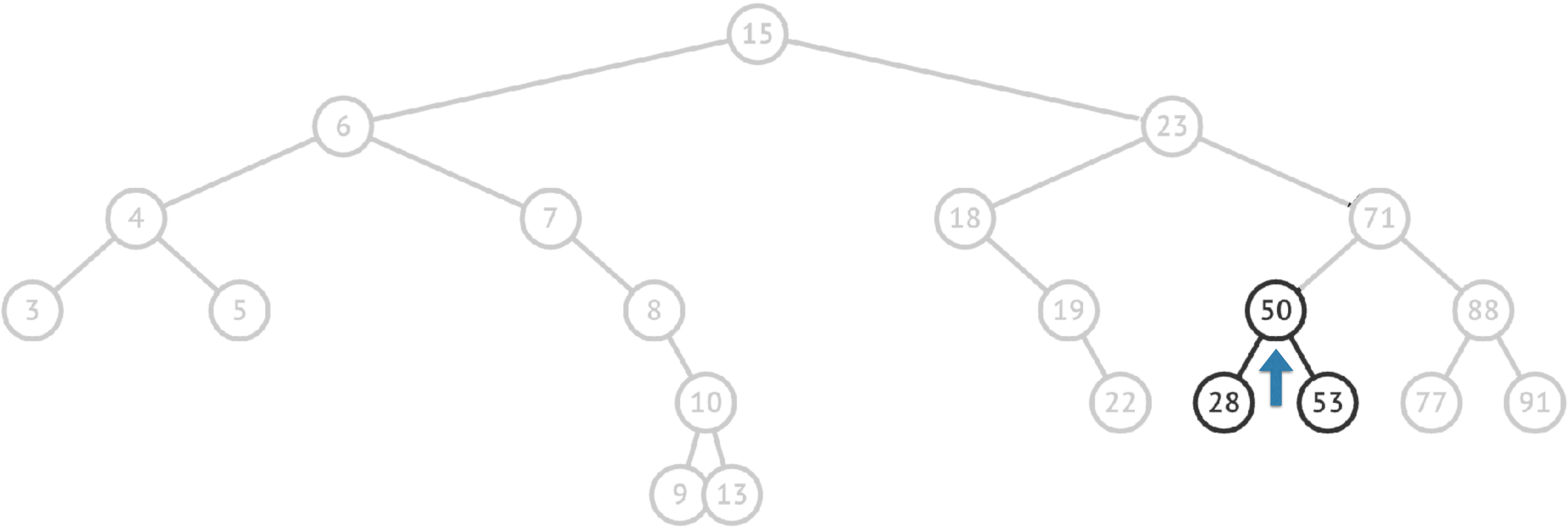
Does this BST contain the value 28?



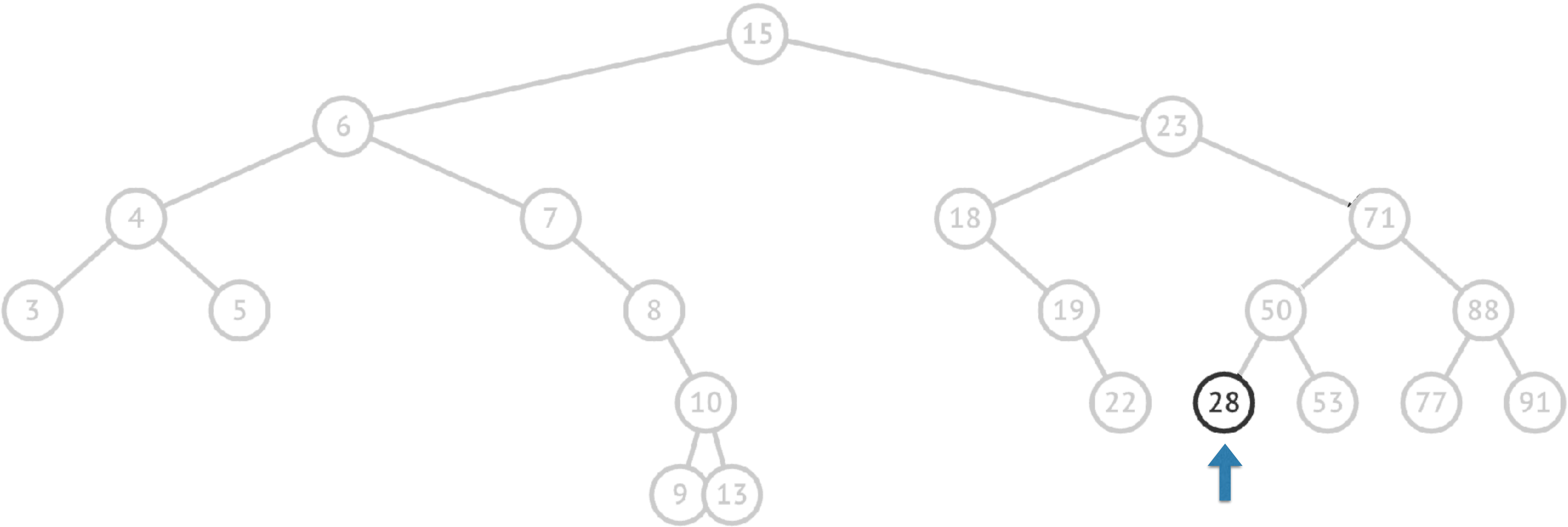
Does this BST contain the value 28?



Does this BST contain the value 28?



Does this BST contain the value 28?



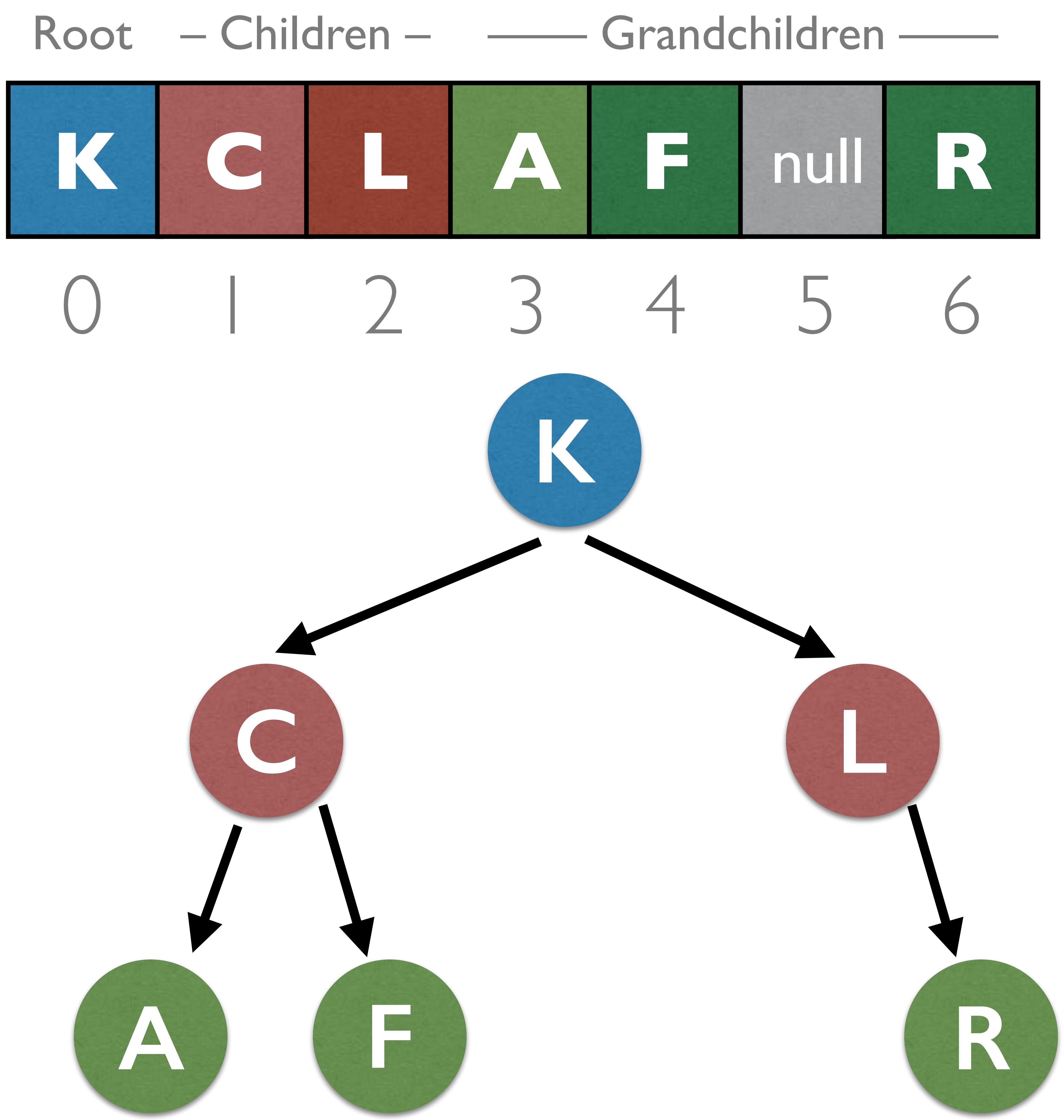
BST ADT

- **Root node satisfies ordering principle**
  - Left descendants < root value <= right descendants
- **Both children are BSTs (recursive definition)**
- Operations
  - **Insert** new values, *respecting the ordering principle*
  - **Find** existing values (takes advantage of ordering)
  - **Delete** values (tricky, skipped in workshop)

How to implement this ADT?

...Maybe an array (seriously)?

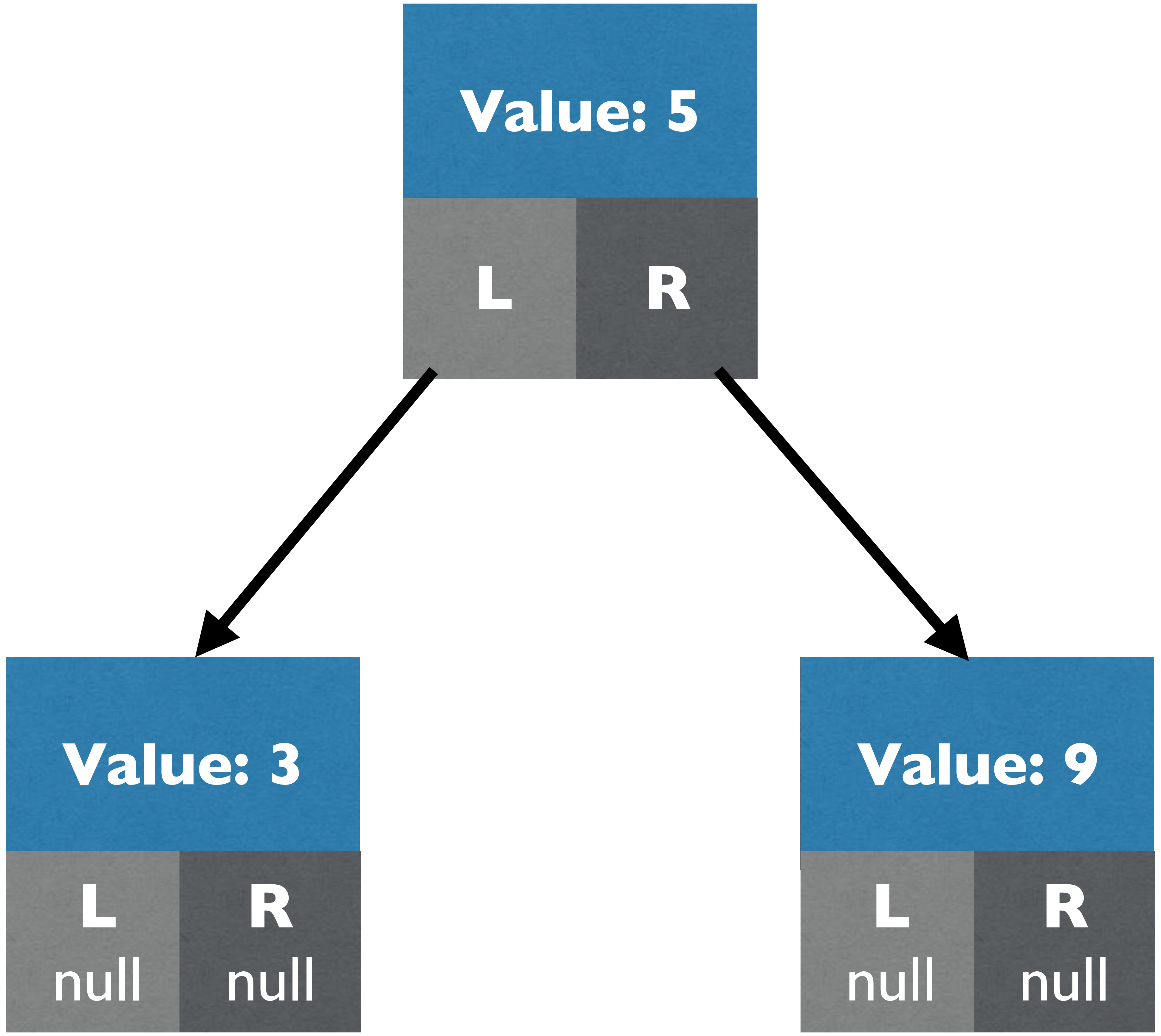
- The Tree ADT, with its talk of "nodes" and "references," seems so obviously to describe a data *structure* that it is perhaps confusing to tell the two apart.
- In fact, a tree can be stored in a few different ways. For example, if you knew your tree nodes always had at most two children, you could store the tree in an array!



The Linked Tree Data Structure

- However, the concept of *nodes* with *values* and *children* maps so well to the concrete case of *memory structs* with *fields* and *references* that the most common DS used to implement the Tree ADT is…

  …the Linked Tree DS.



Tree traversal

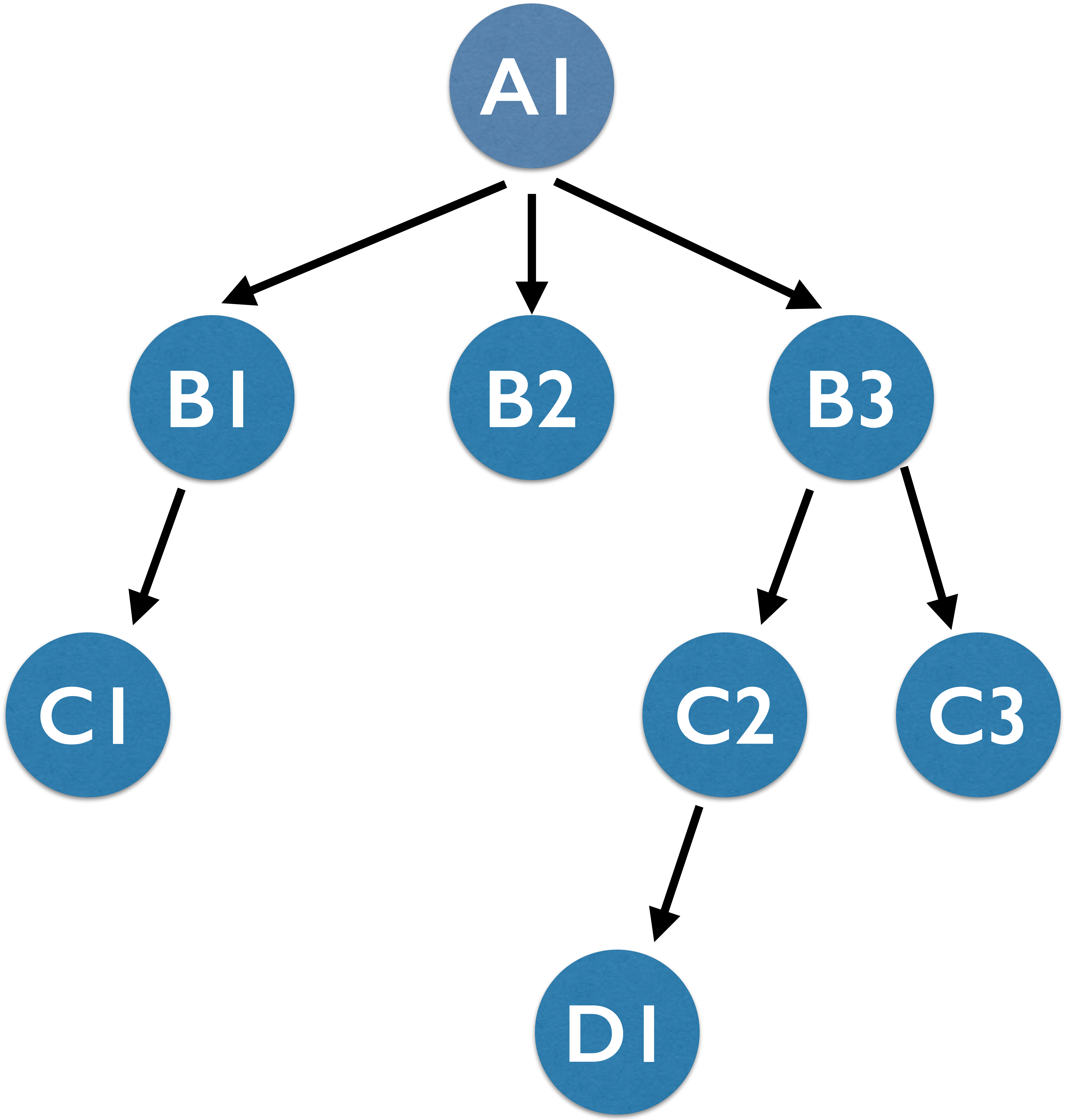
Traversal: visiting every node

- **Breadth-first search (level by level)**

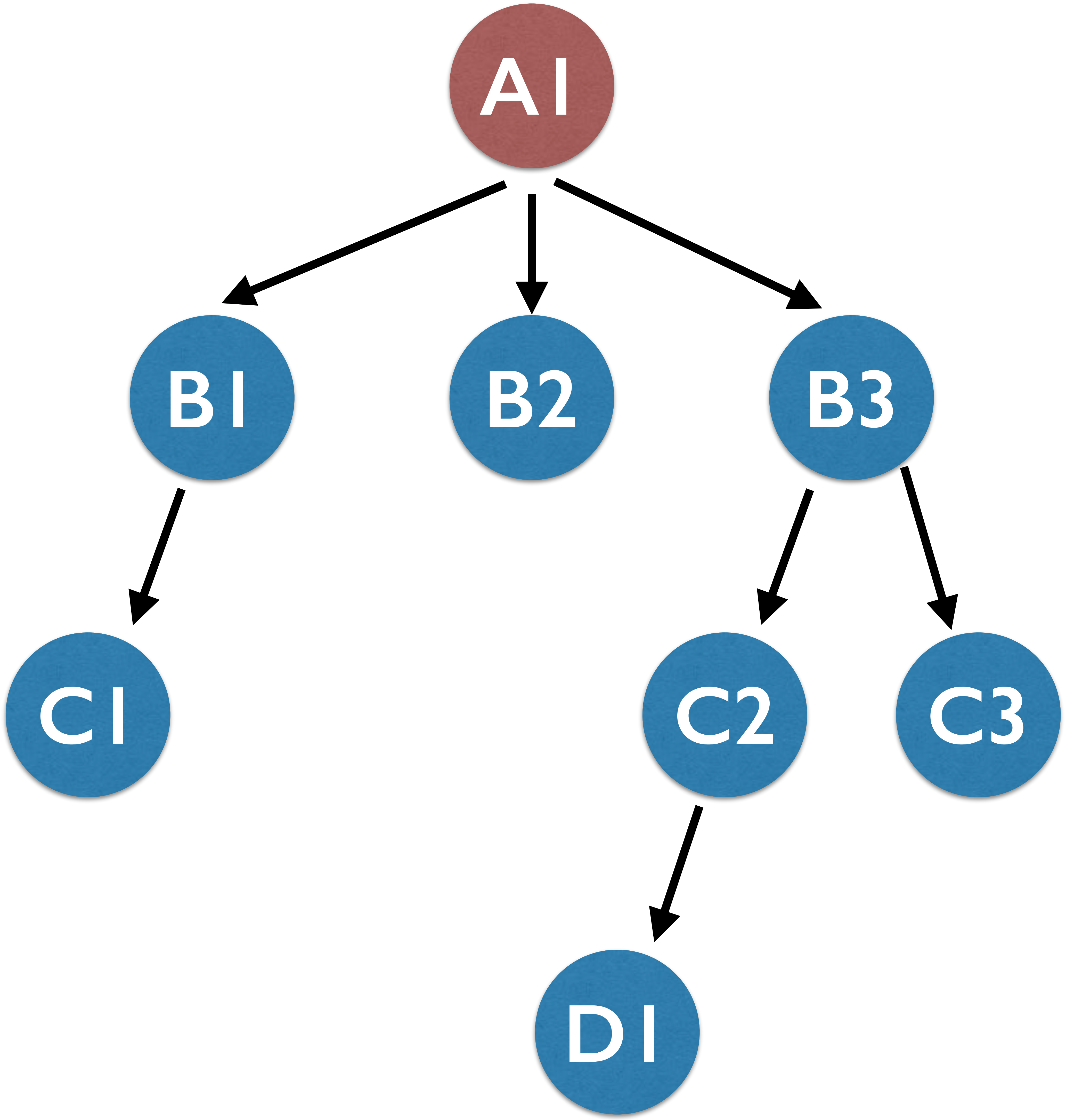
- **Depth-first search (branch by branch)**
  - Pre-order: process **root node**, process **left subtree**, process **right subtree**
  - **In-order**: process **left subtree**, process **root node**, process **right subtree**
  - Post-order: process **left subtree**, process **right subtree**, process **root node**

Breadth-First

BFS

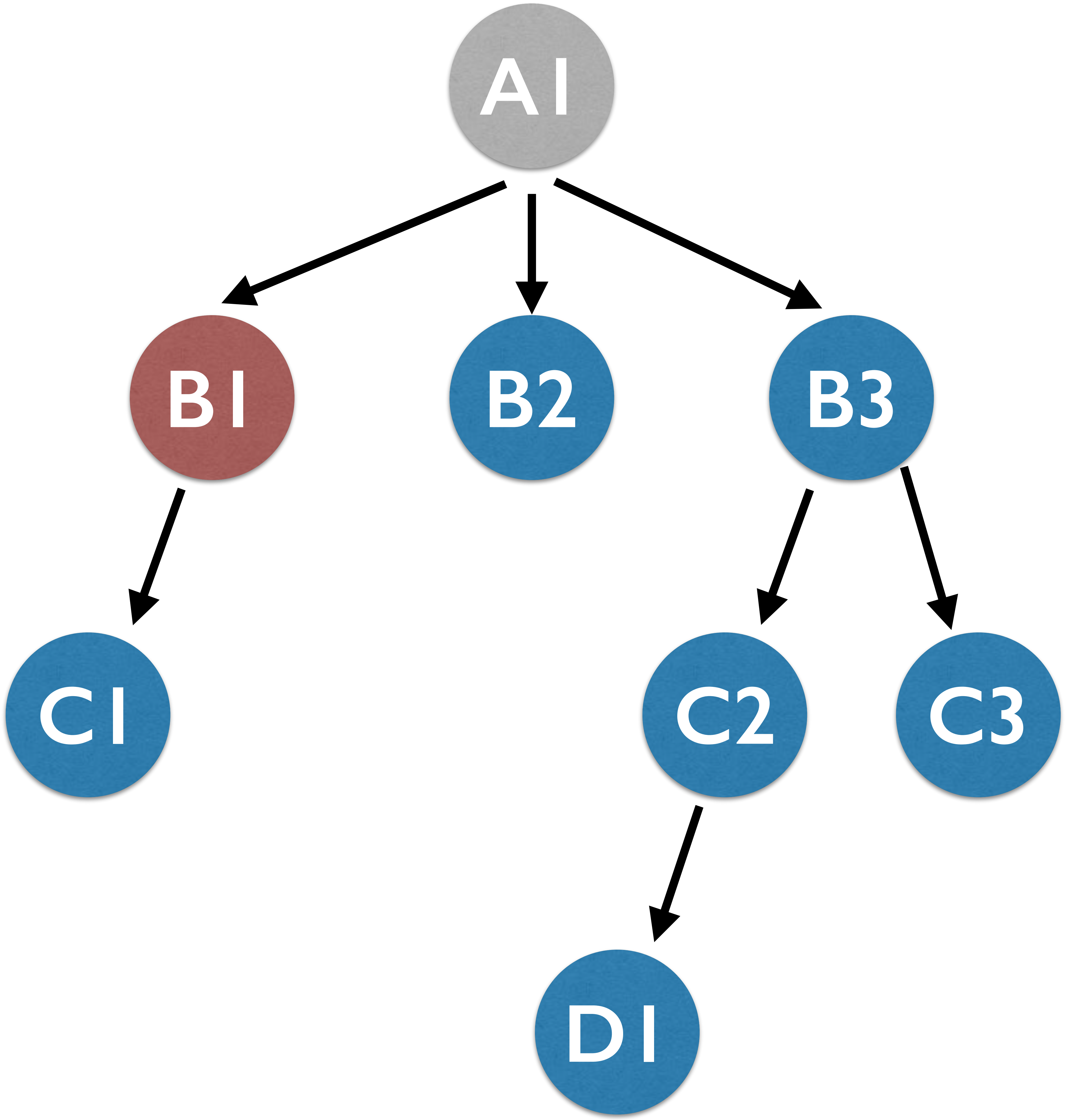


BFS



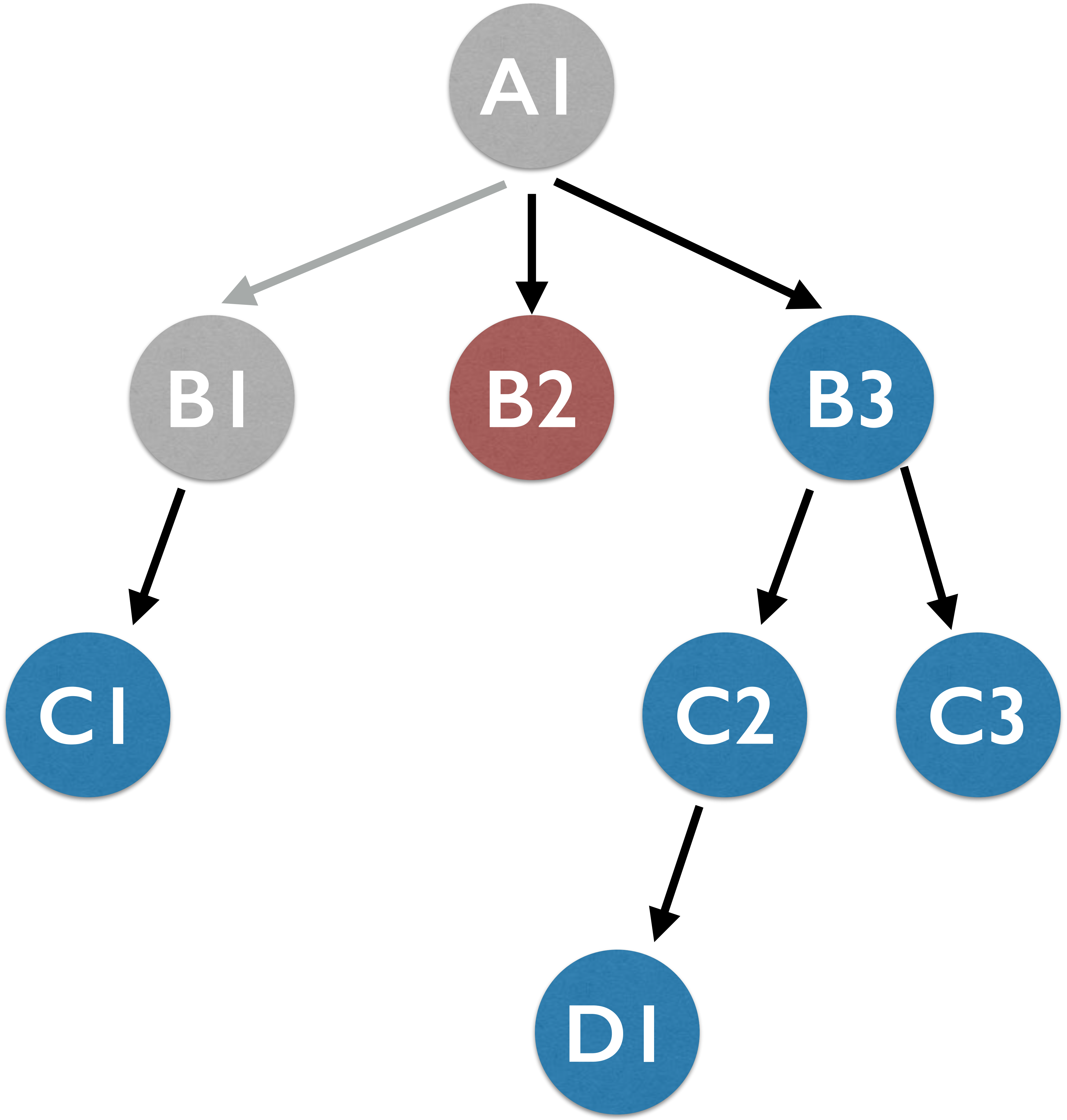
A1

BFS



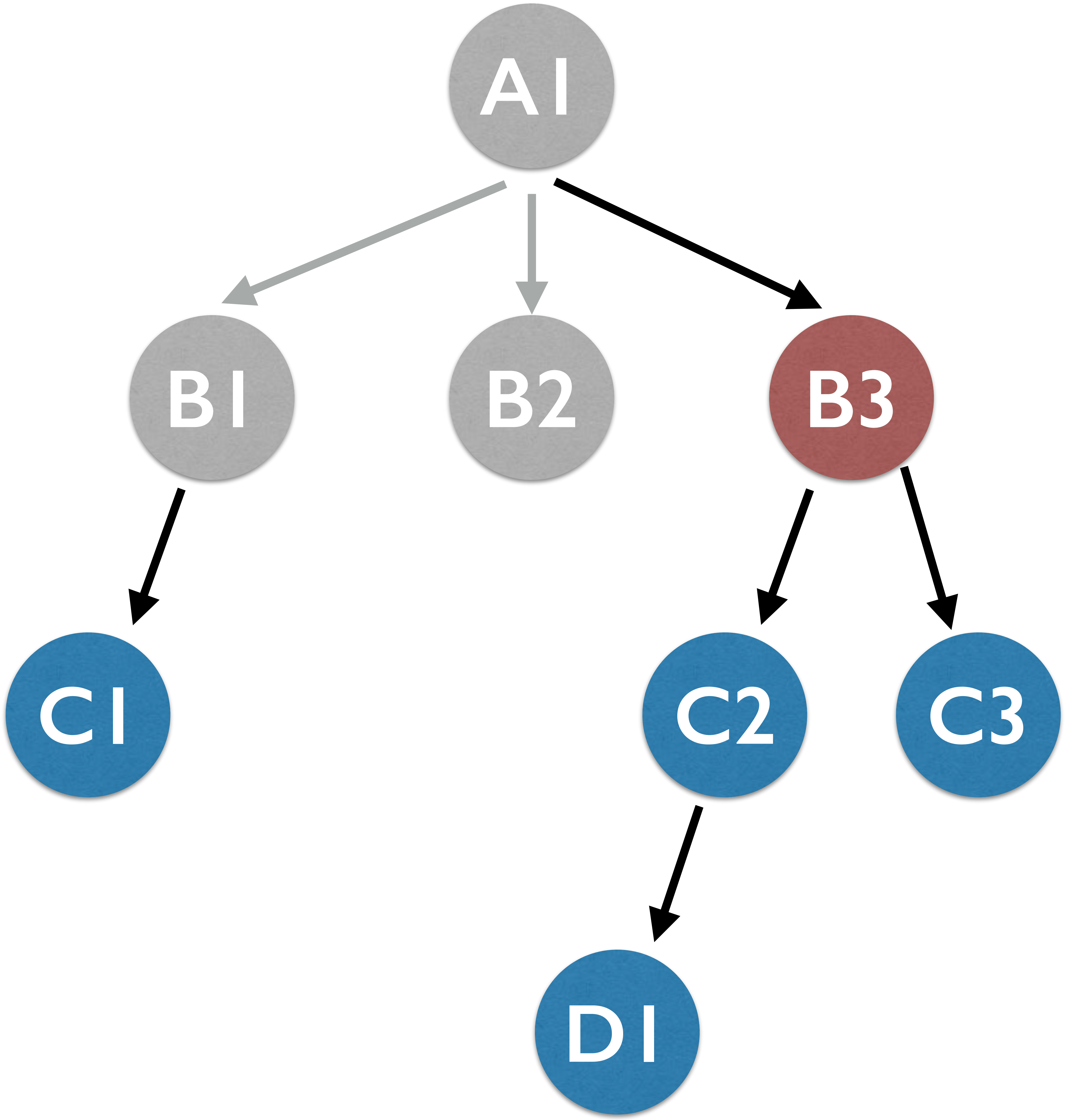
A1, B1

BFS



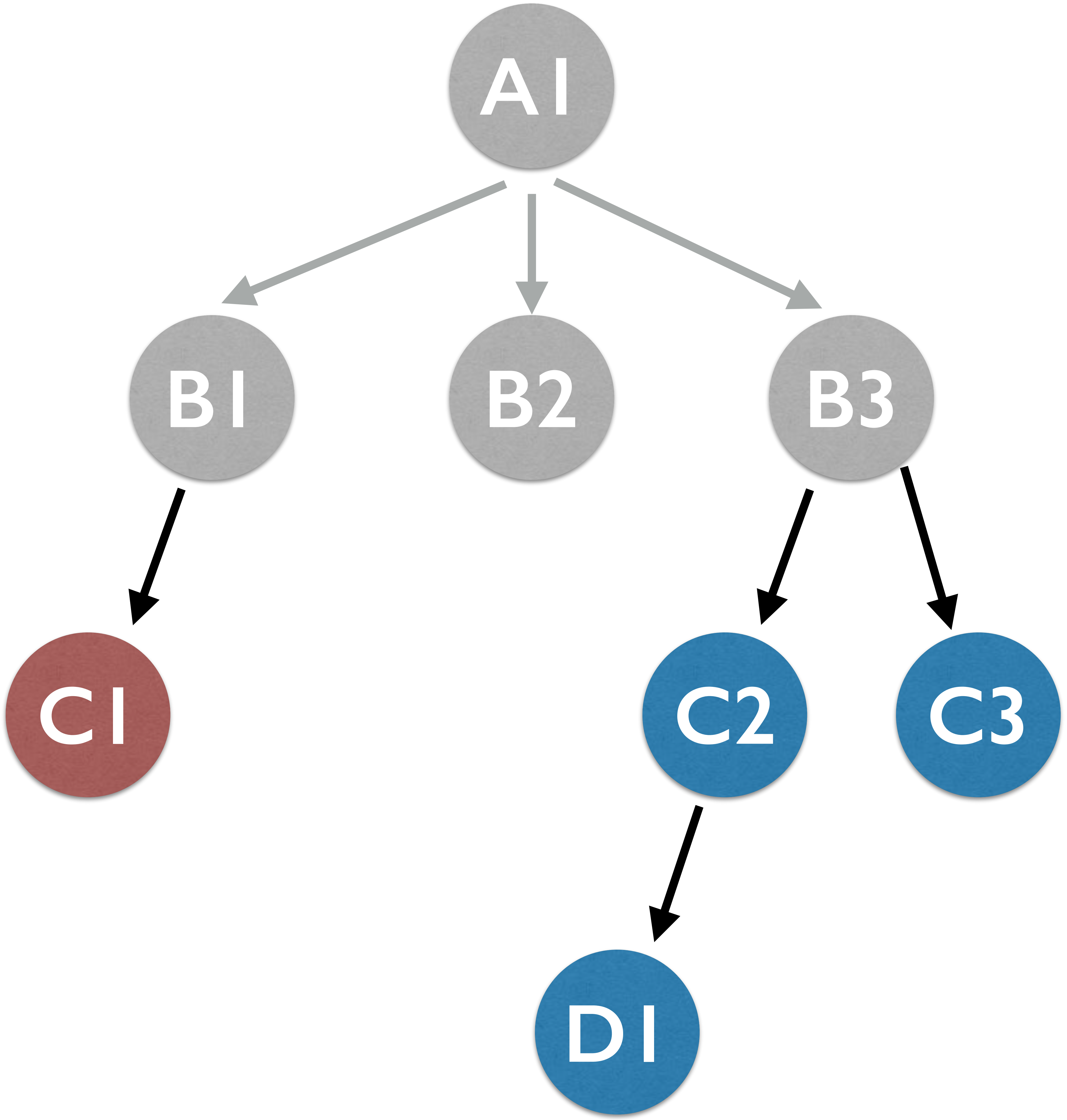
A1, B1, B2

BFS



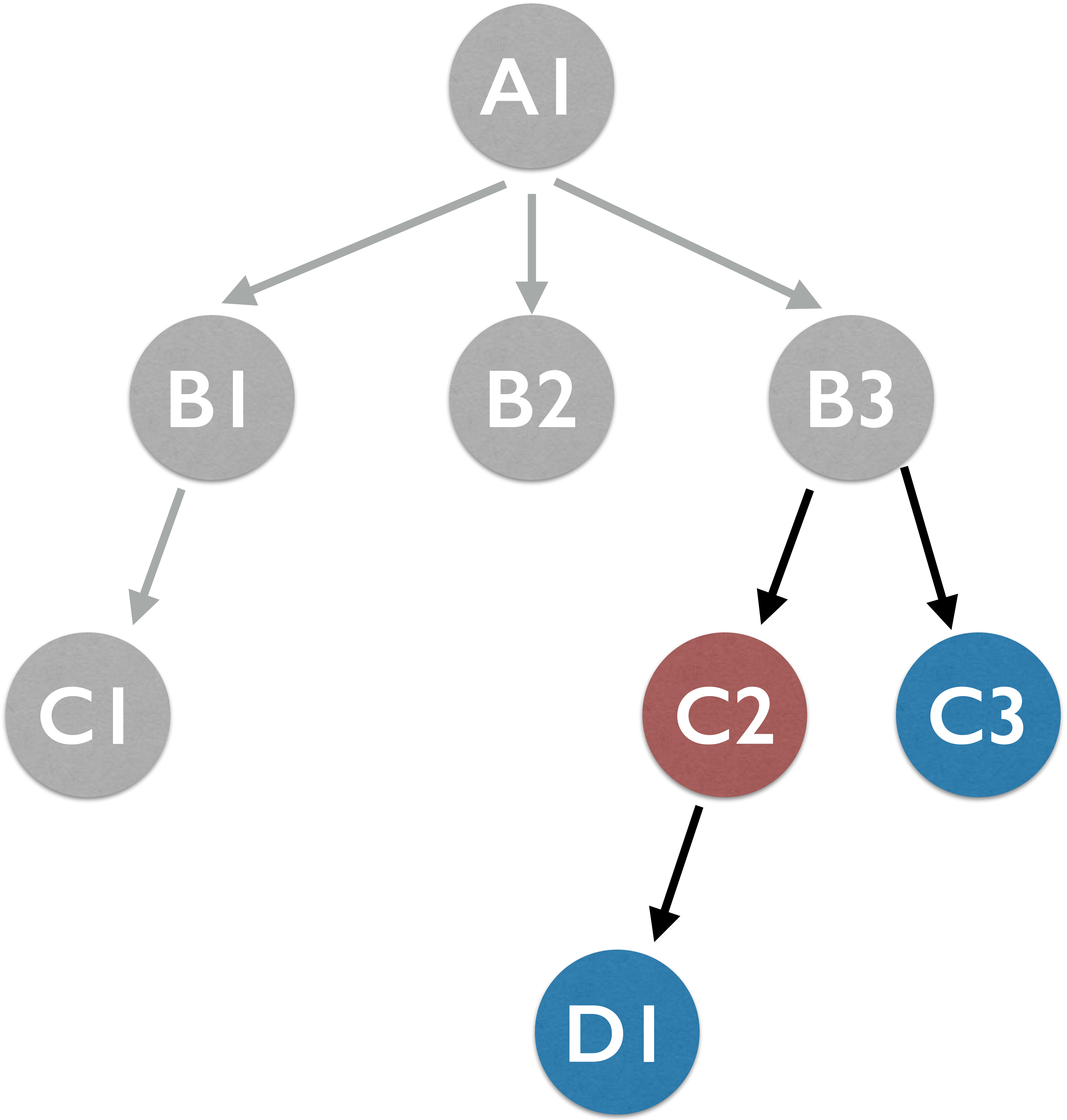
A1, B1, B2, B3

BFS



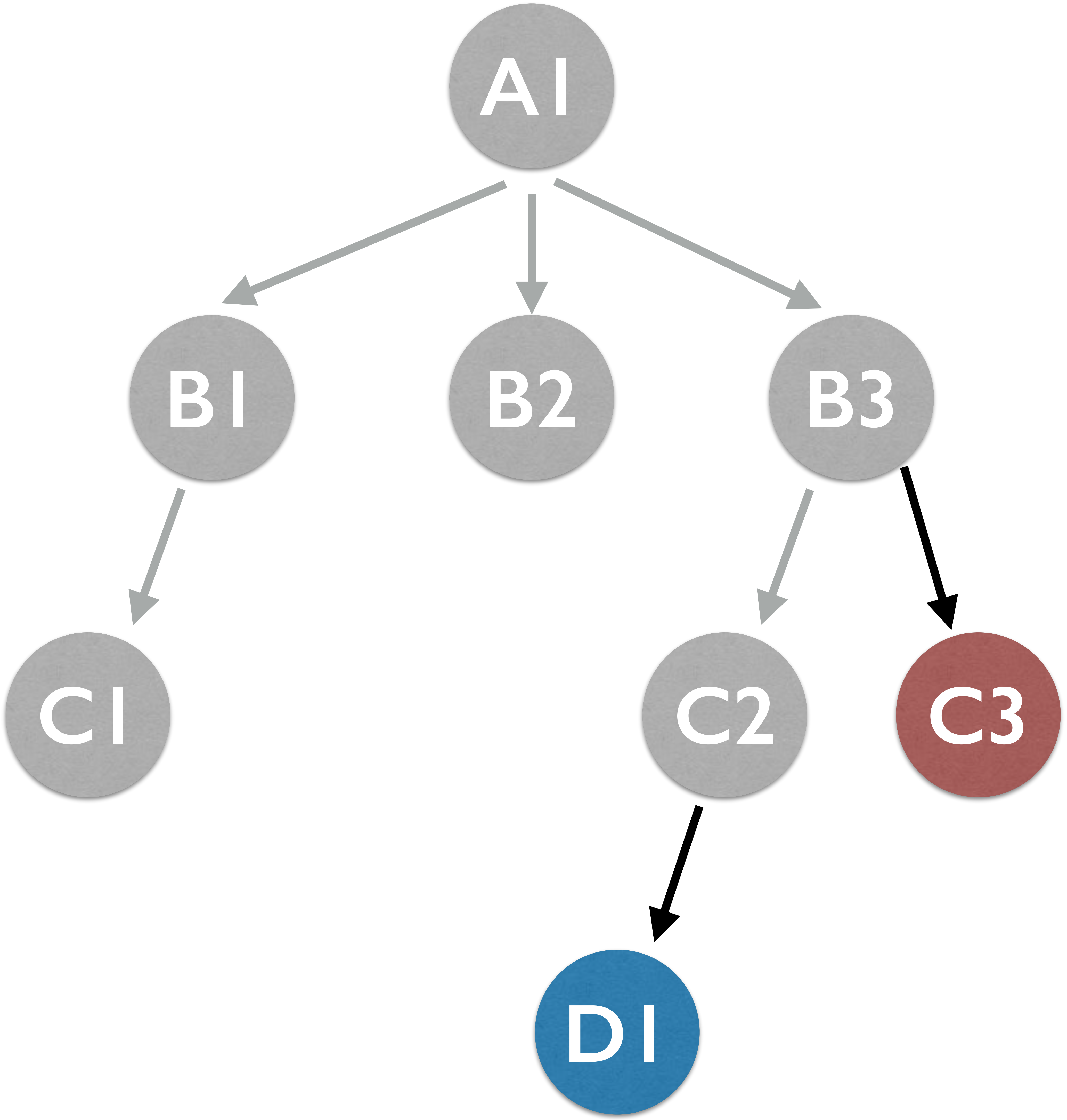
A1, B1, B2, B3, C1

BFS



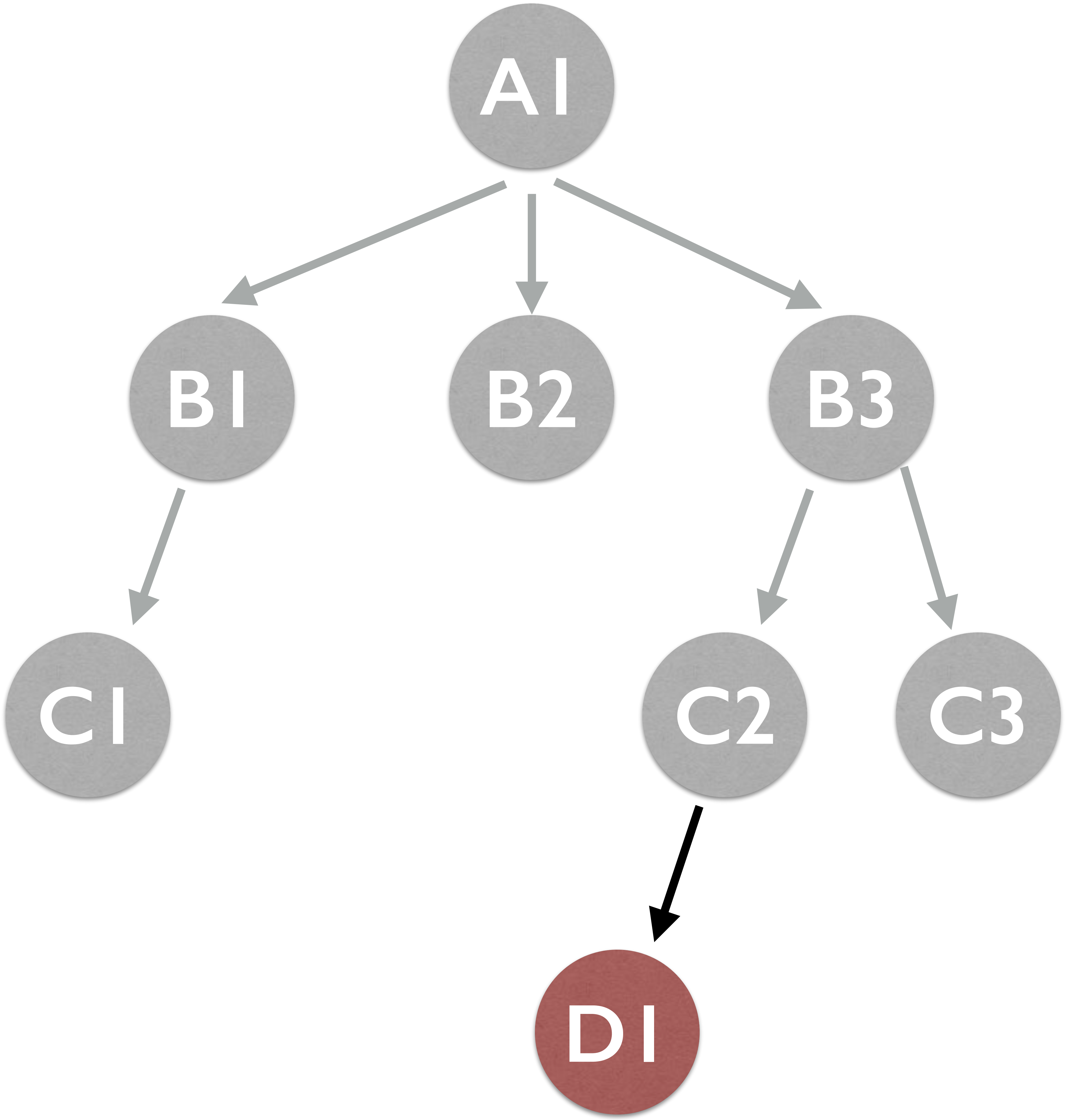
A1, B1, B2, B3, C1, C2

BFS



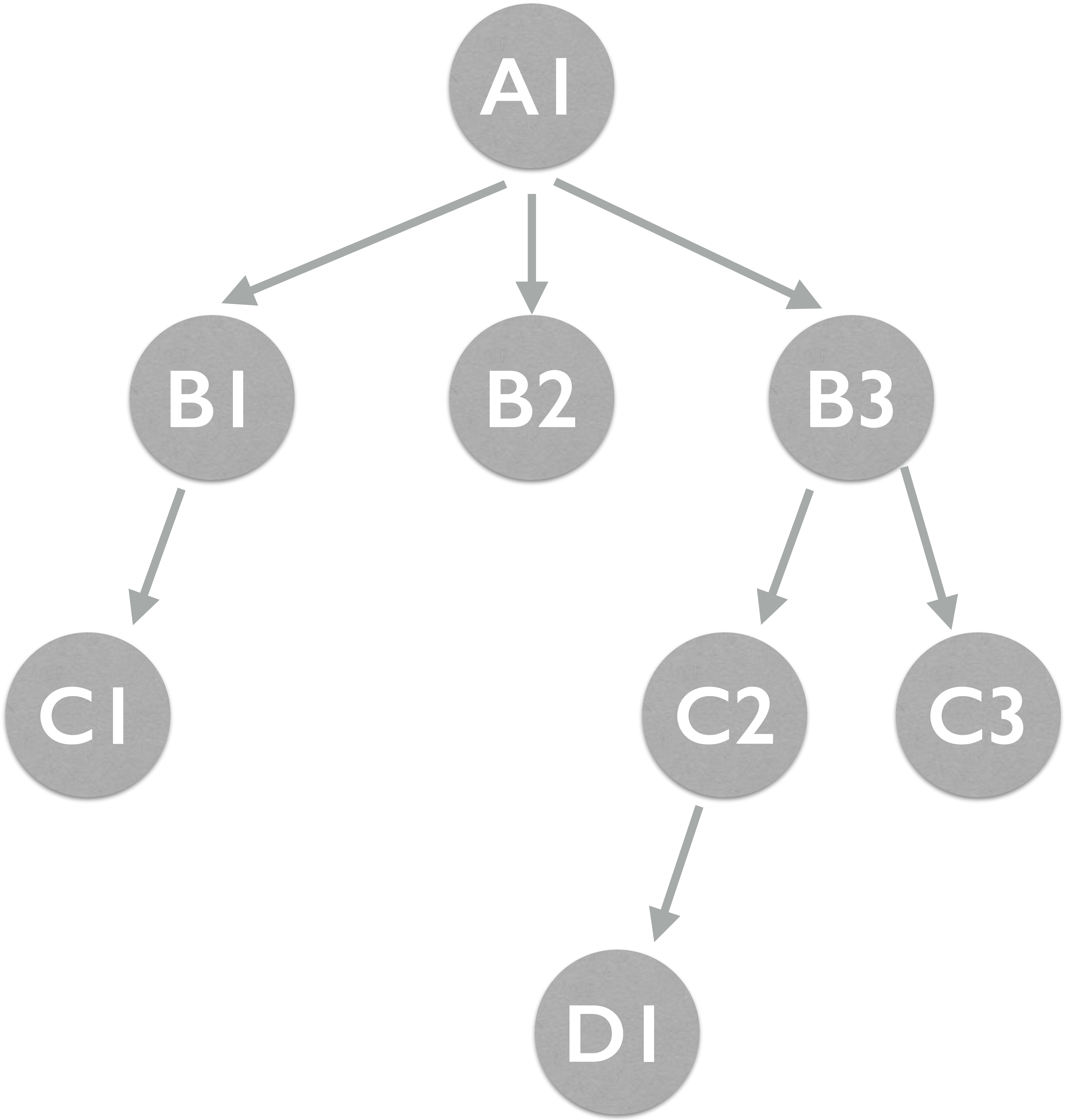
A1, B1, B2, B3, C1, C2, C3

BFS



A1, B1, B2, B3, C1, C2, C3, D1

BFS

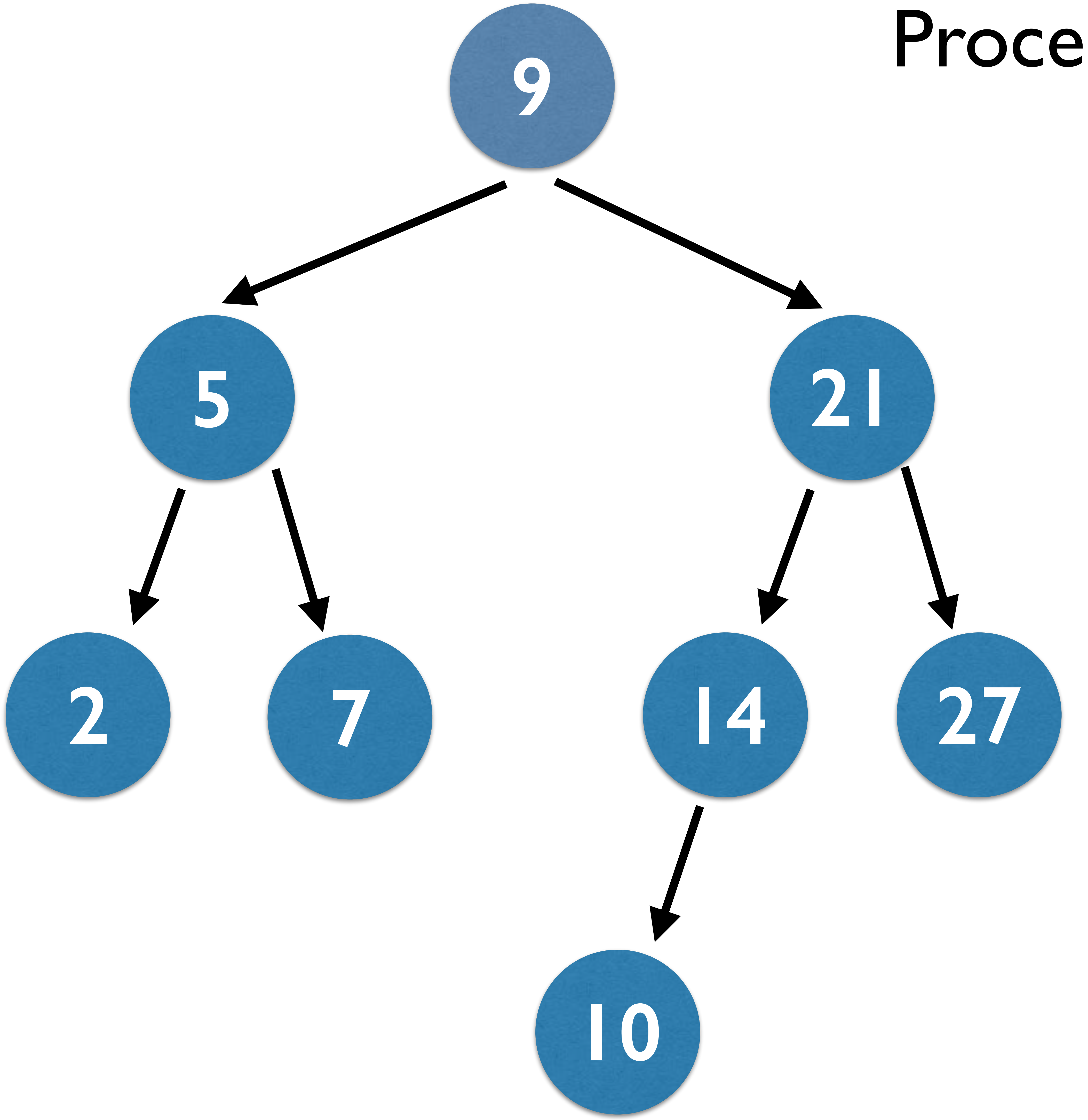


A1, B1, B2, B3, C1, C2, C3, D1

- Looks easy… but with a tree data structure it actually requires an elegant trick to do correctly.
- No full solution here, but a BIG hint: you'll need a queue!

Depth First: In-Order

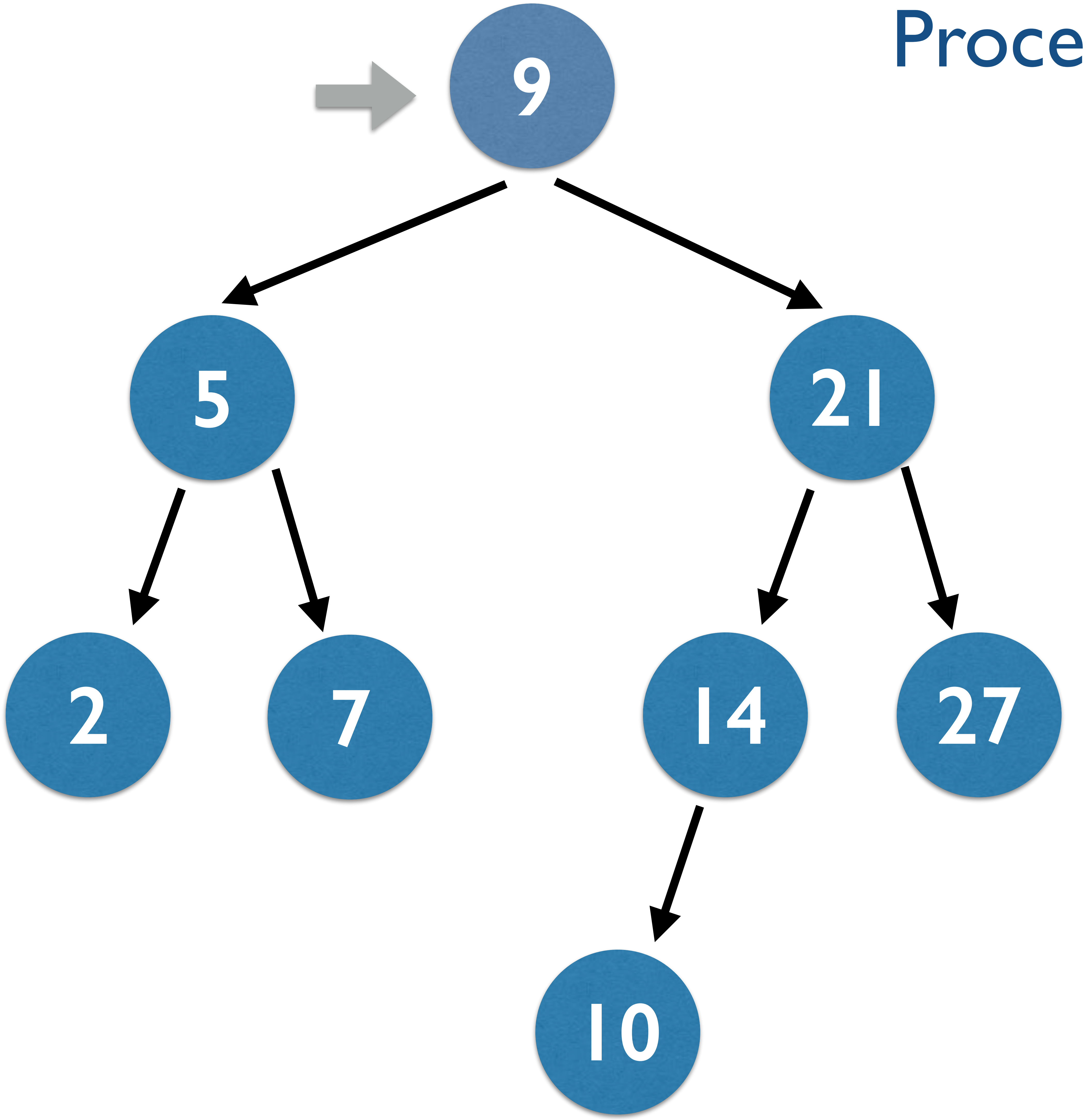
DFS: IN



Process left · Process root · Process right

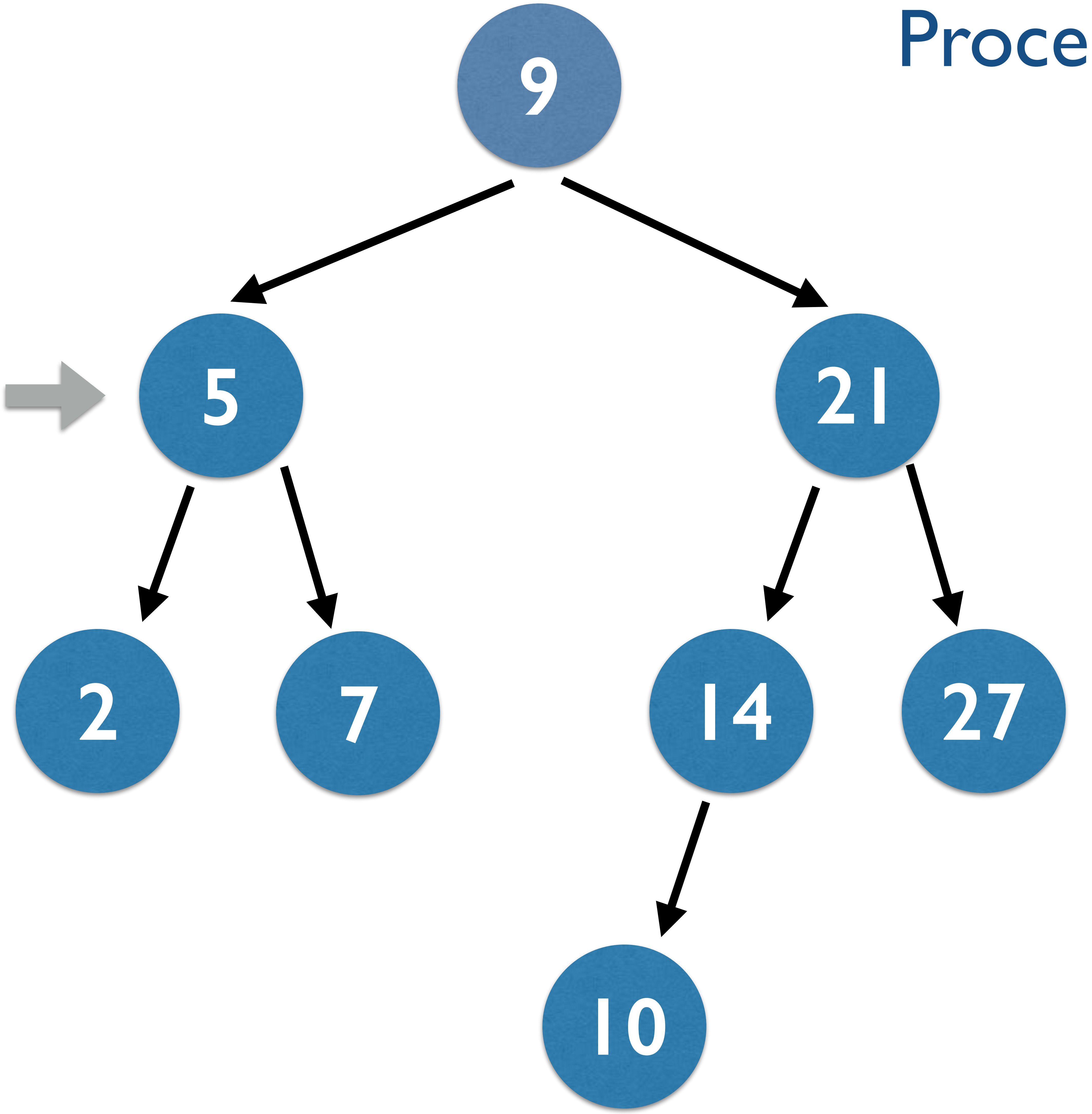
DFS: IN

Process left · Process root · Process right



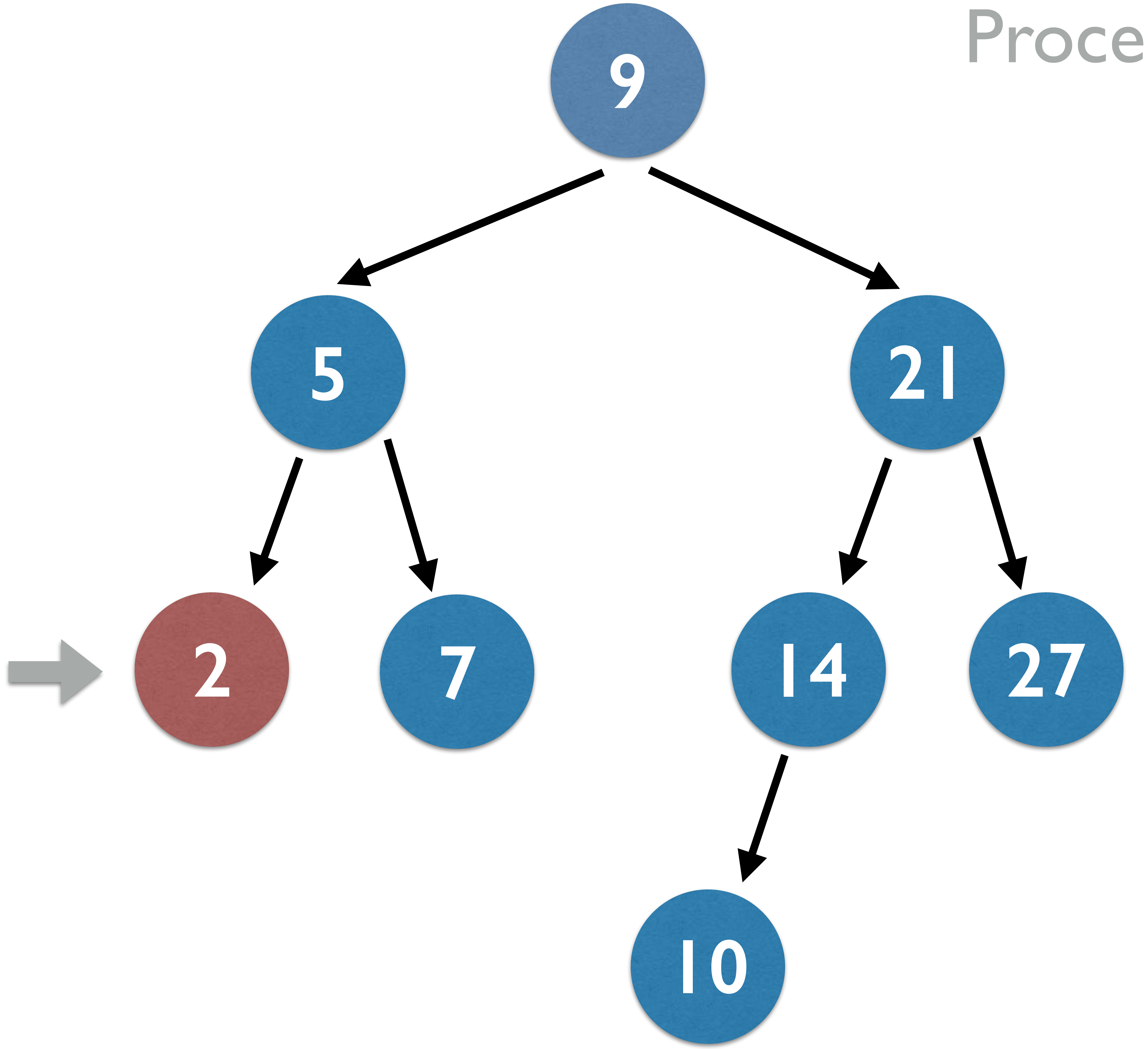
DFS: IN

Process left · Process root · Process right



DFS: IN

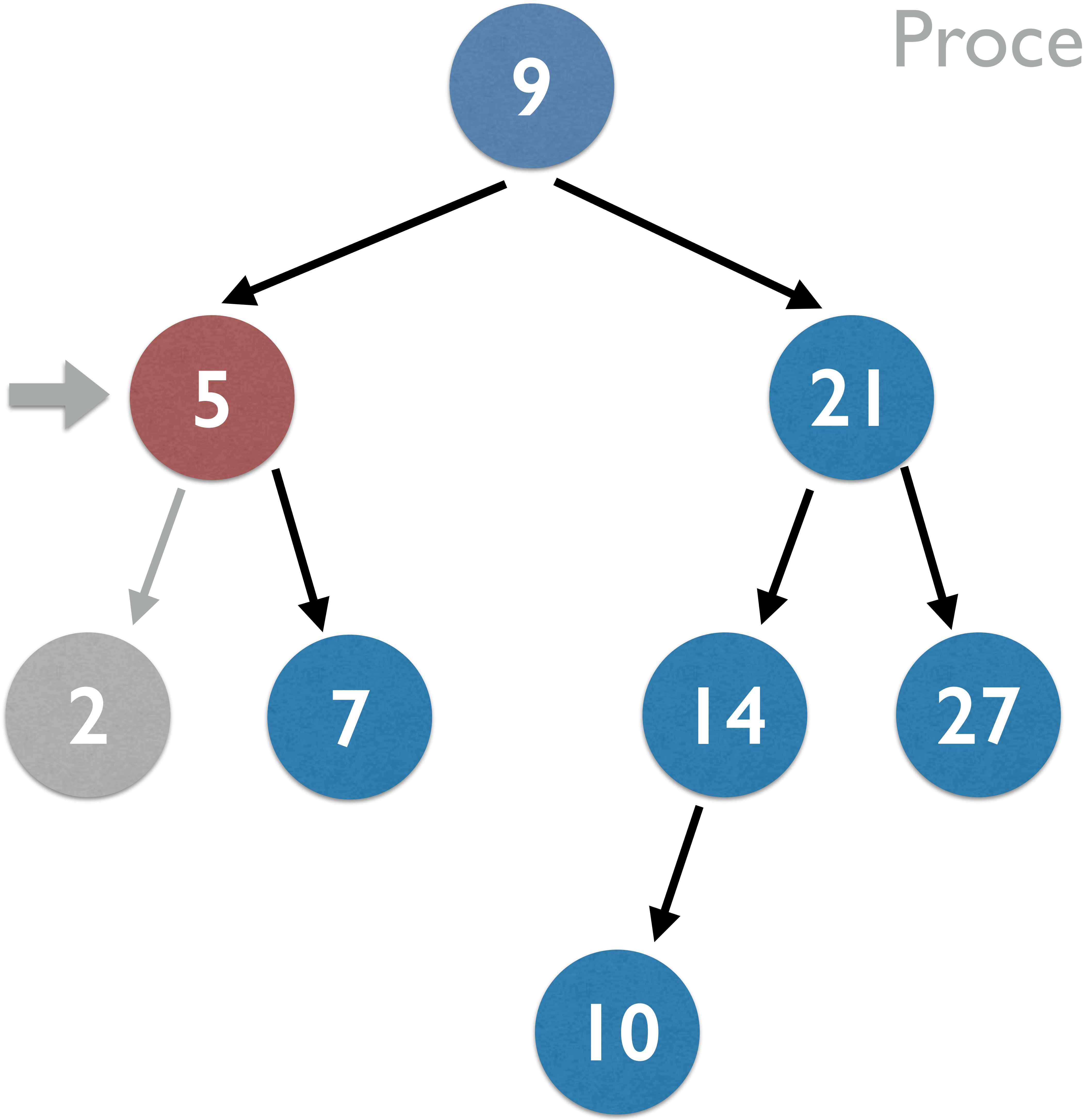
Process left · Process root · Process right



2

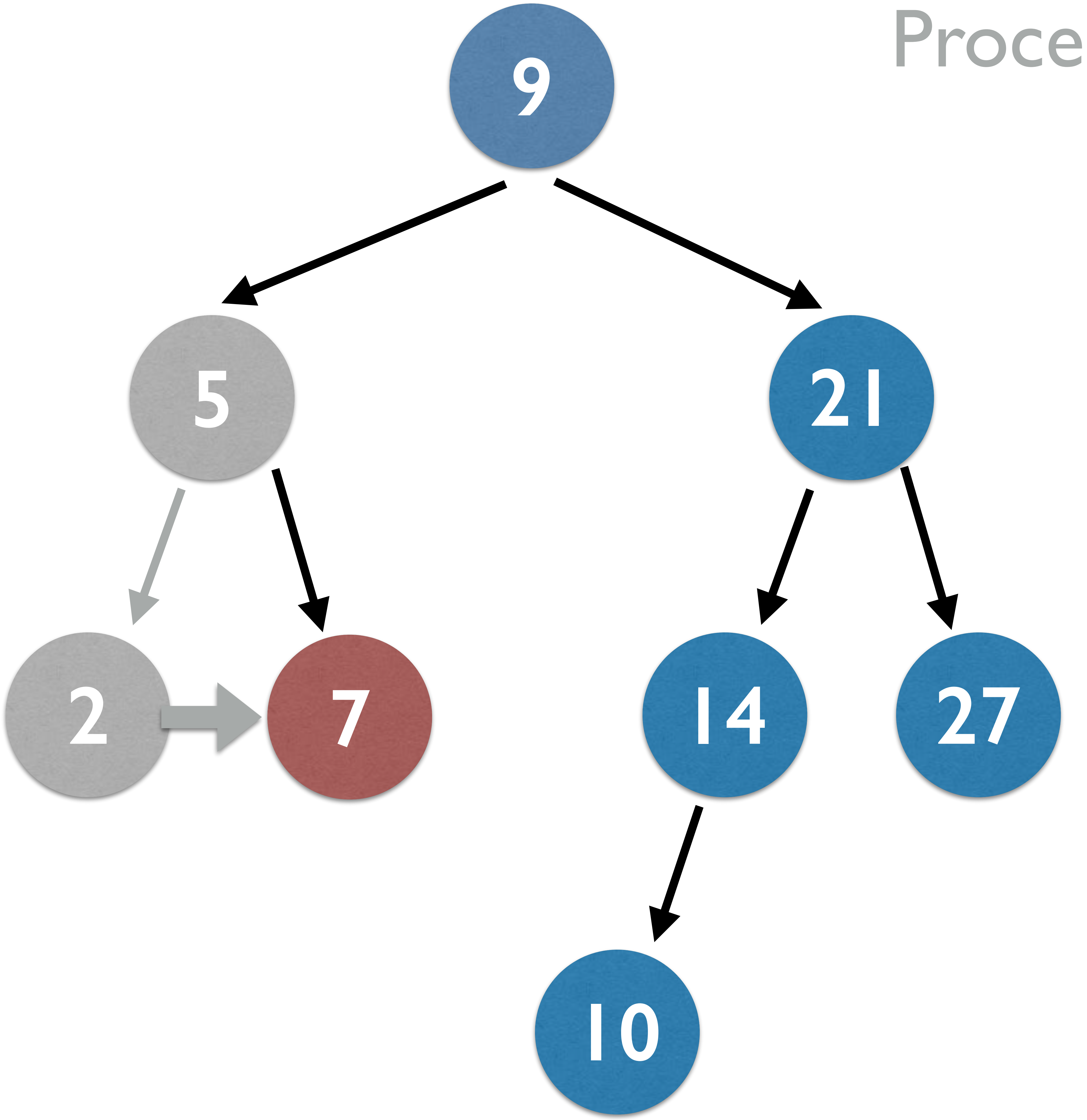
DFS: IN

Process left · Process root · Process right



2, 5

DFS: IN

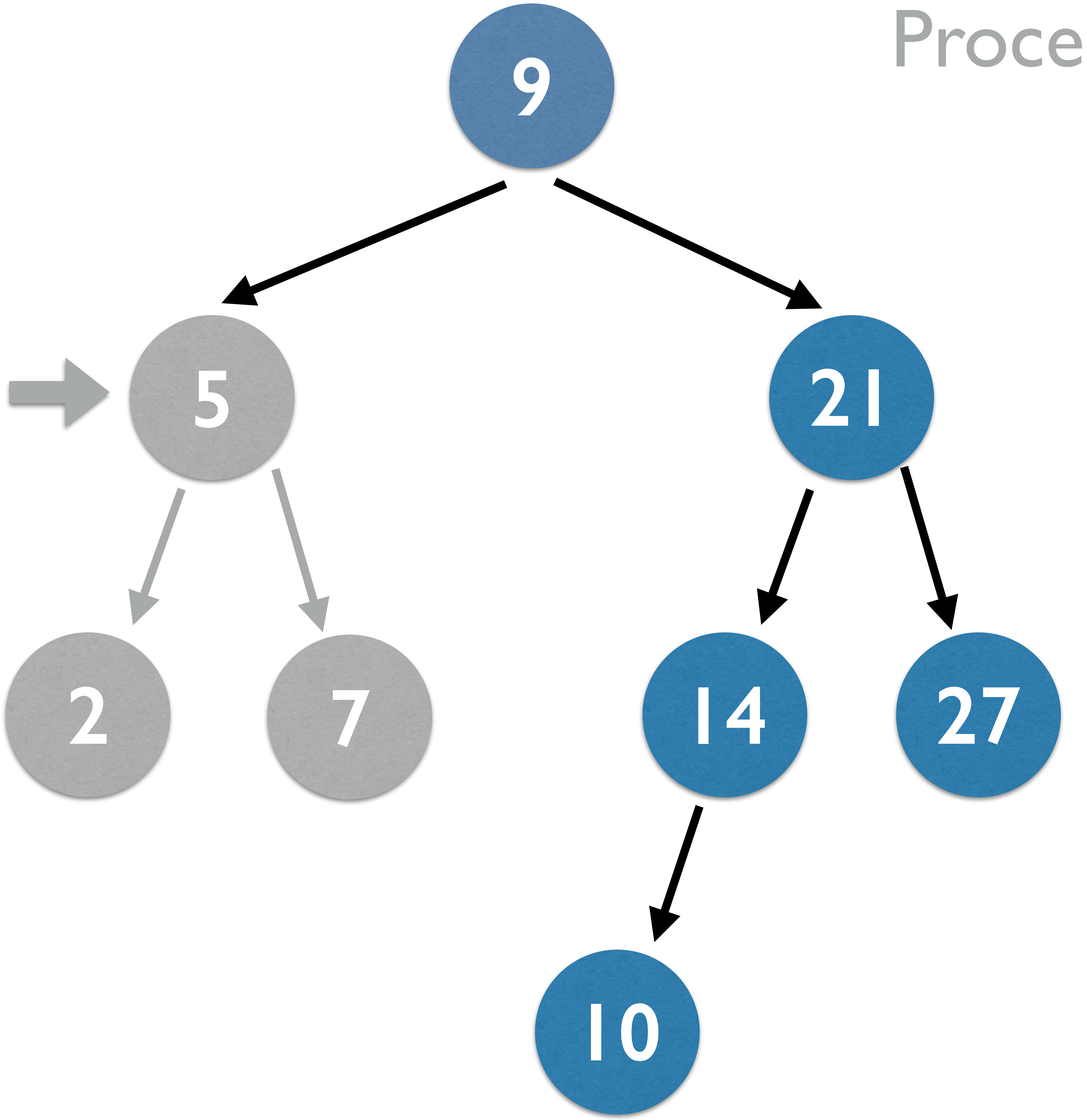


Process left · Process root · Process right

2, 5, 7

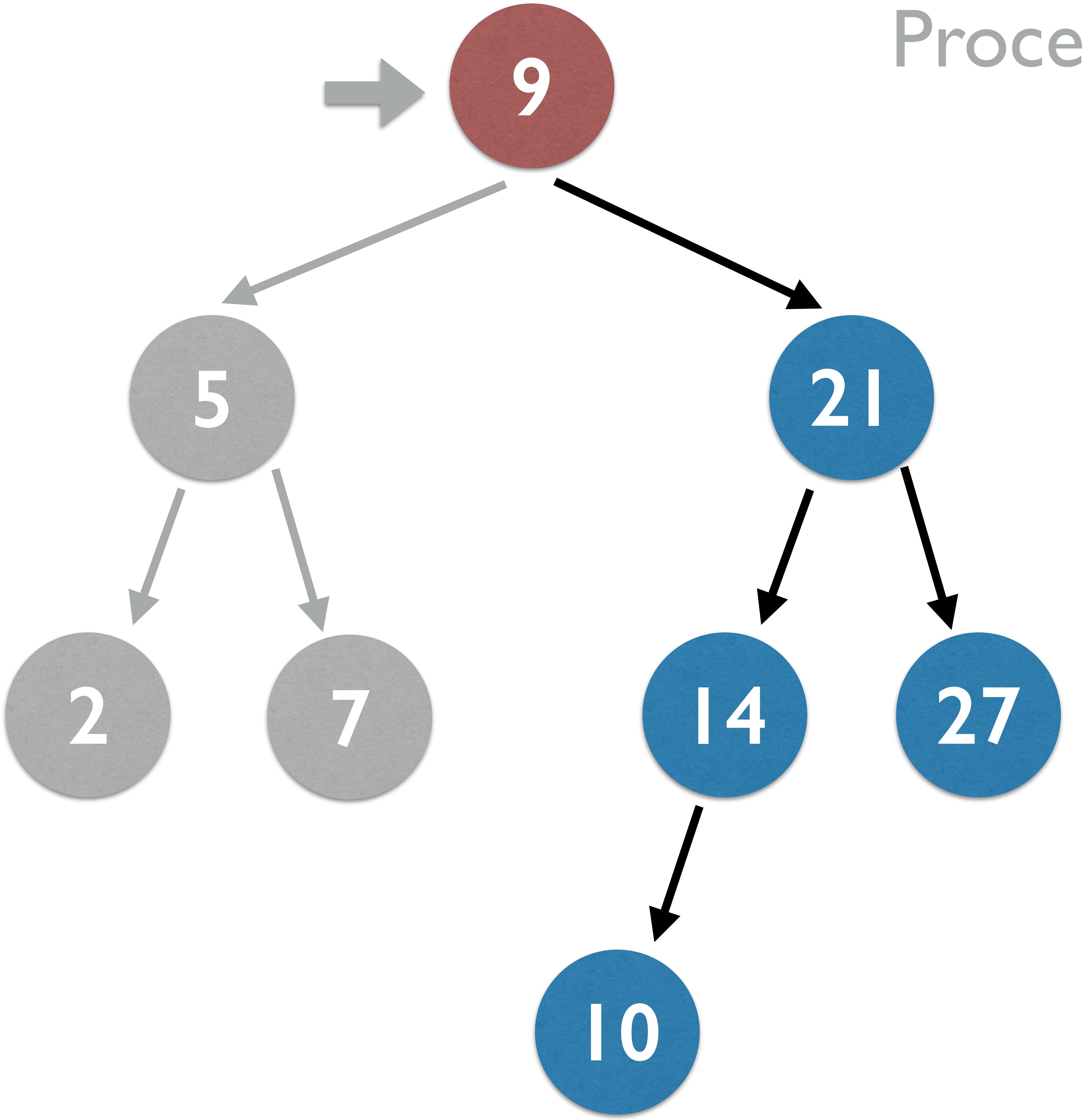
DFS: IN

Process left · Process root · Process right



2, 5, 7

DFS: IN

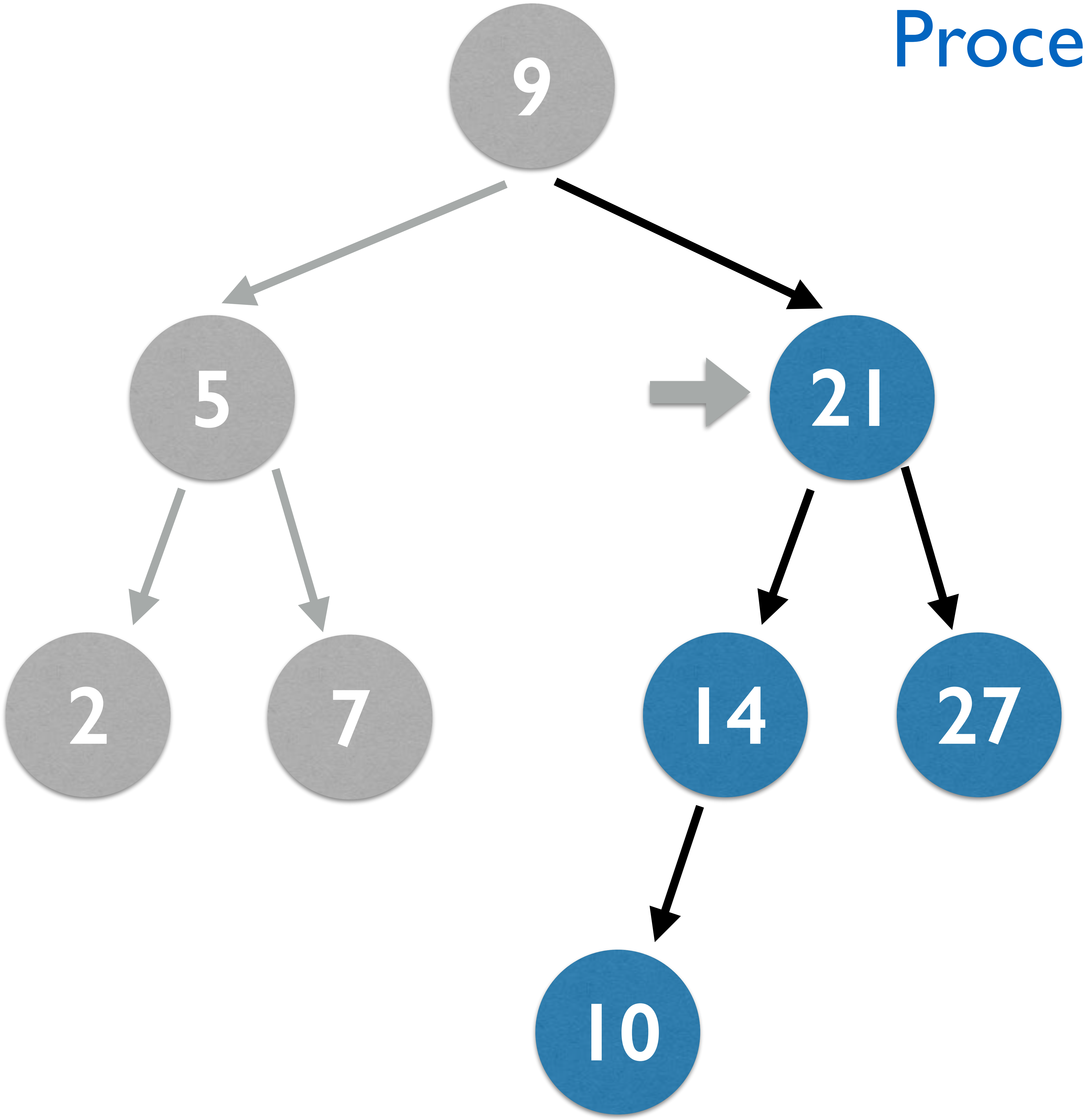


Process left · Process root · Process right

2, 5, 7, 9

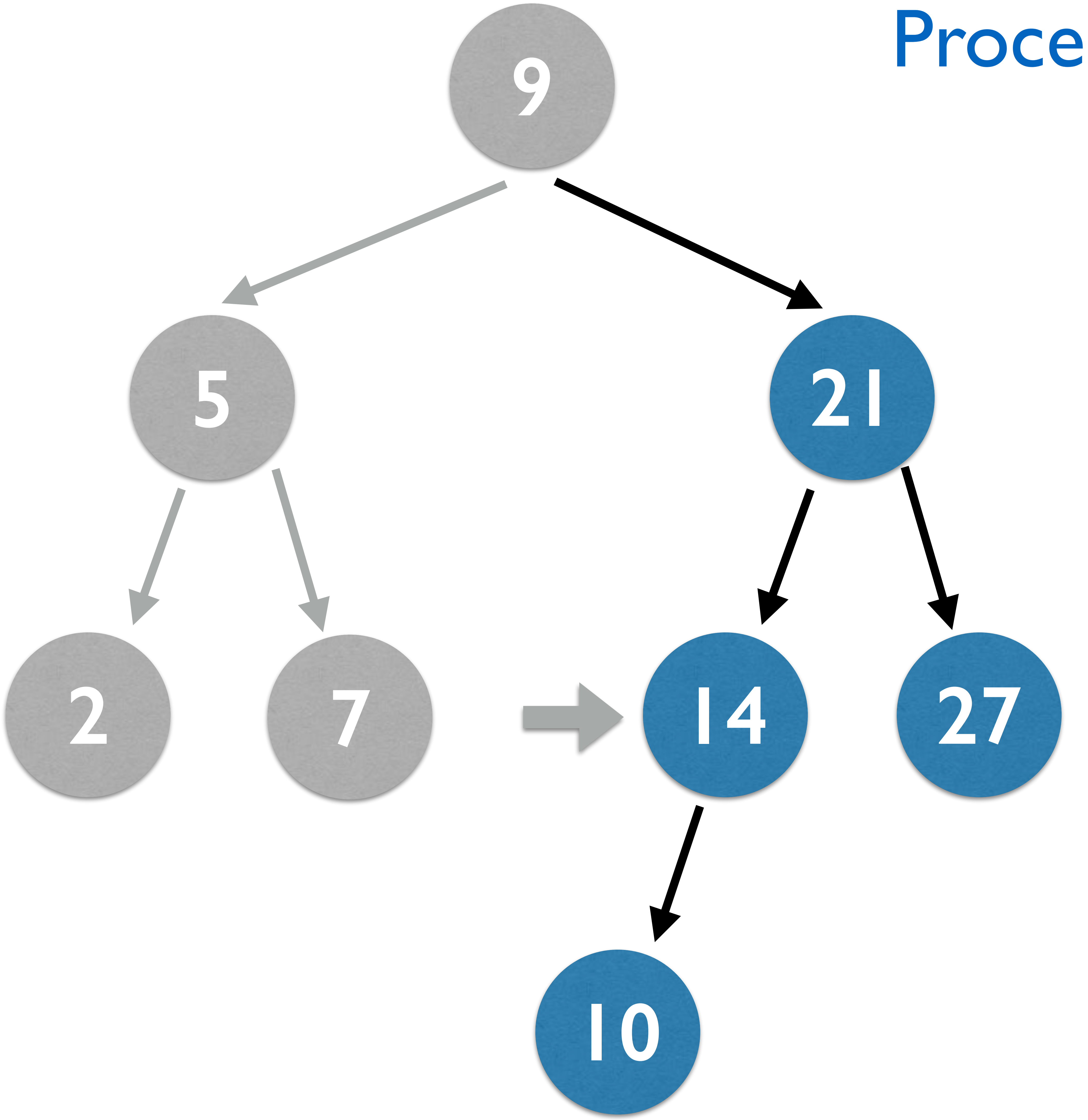
DFS: IN

Process left · Process root · Process right



2, 5, 7, 9

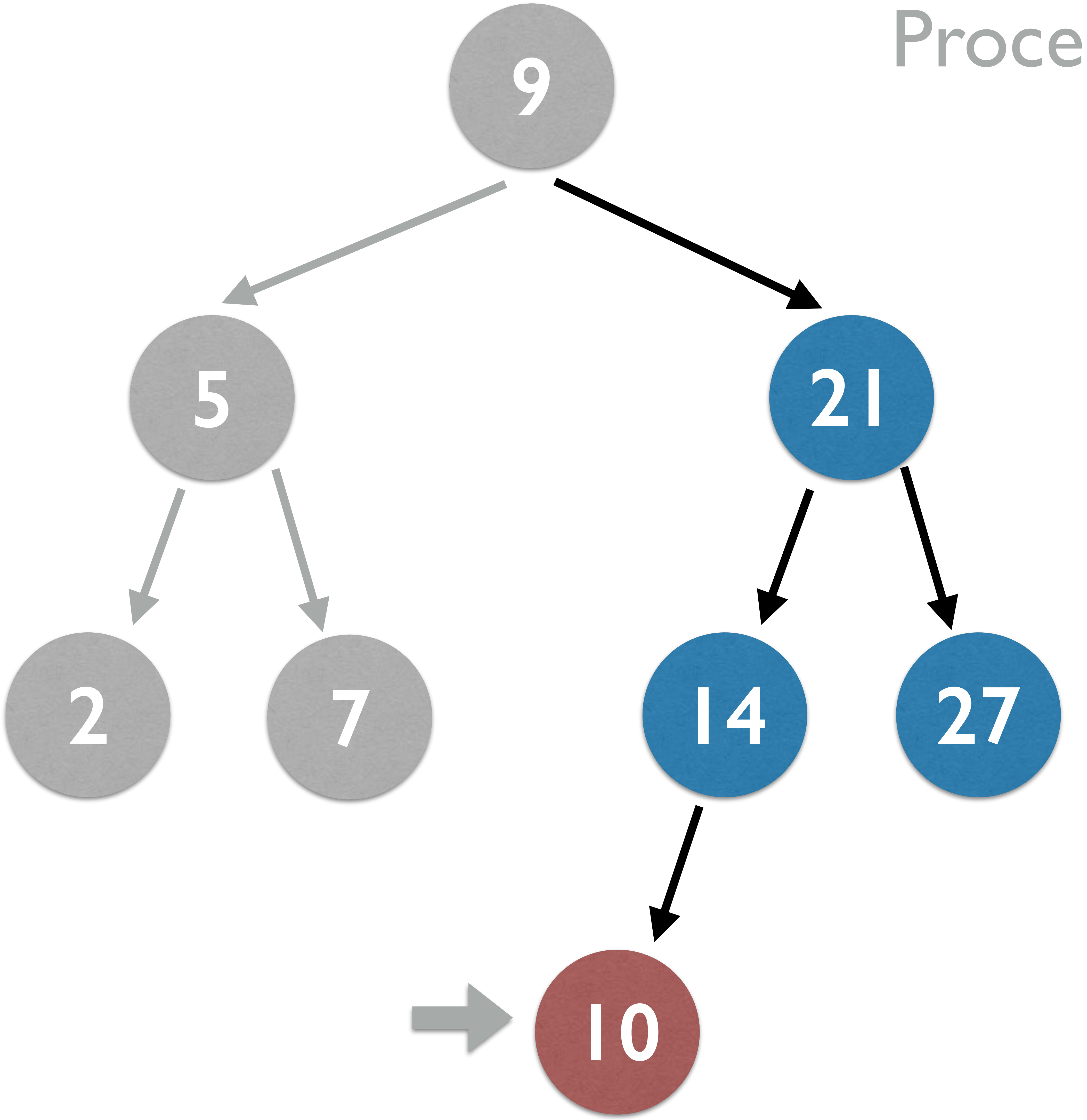
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9

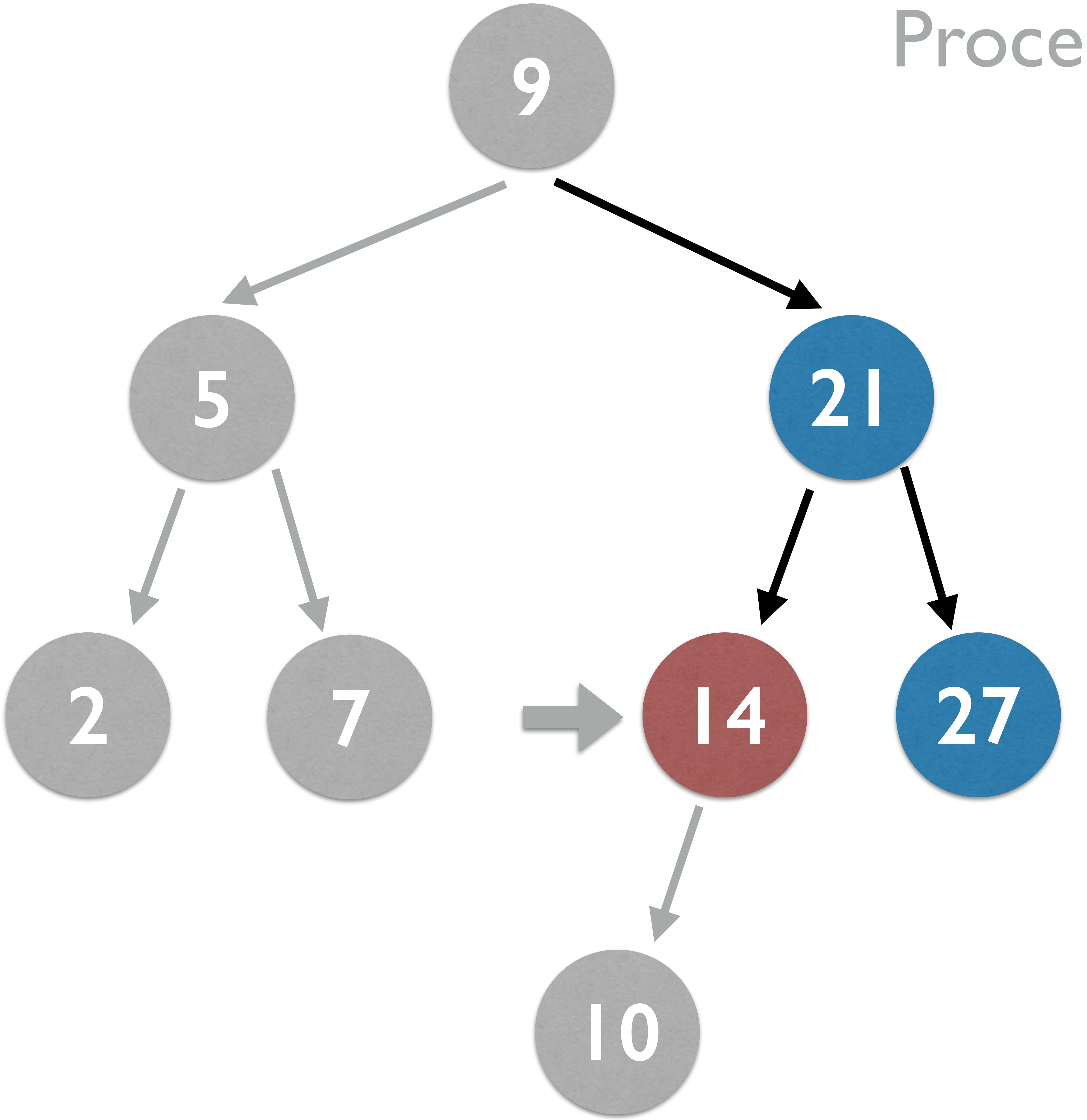
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10

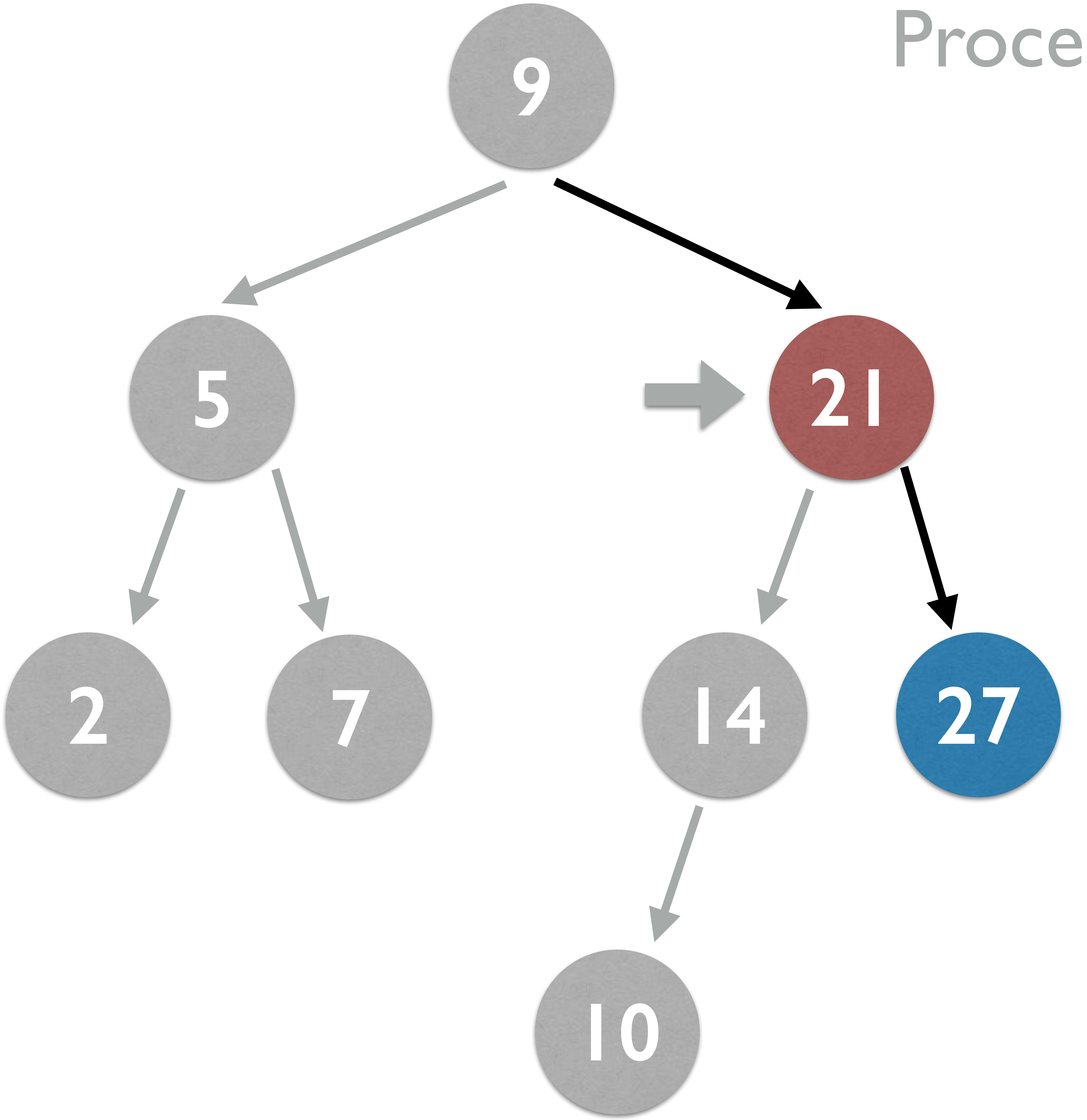
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14

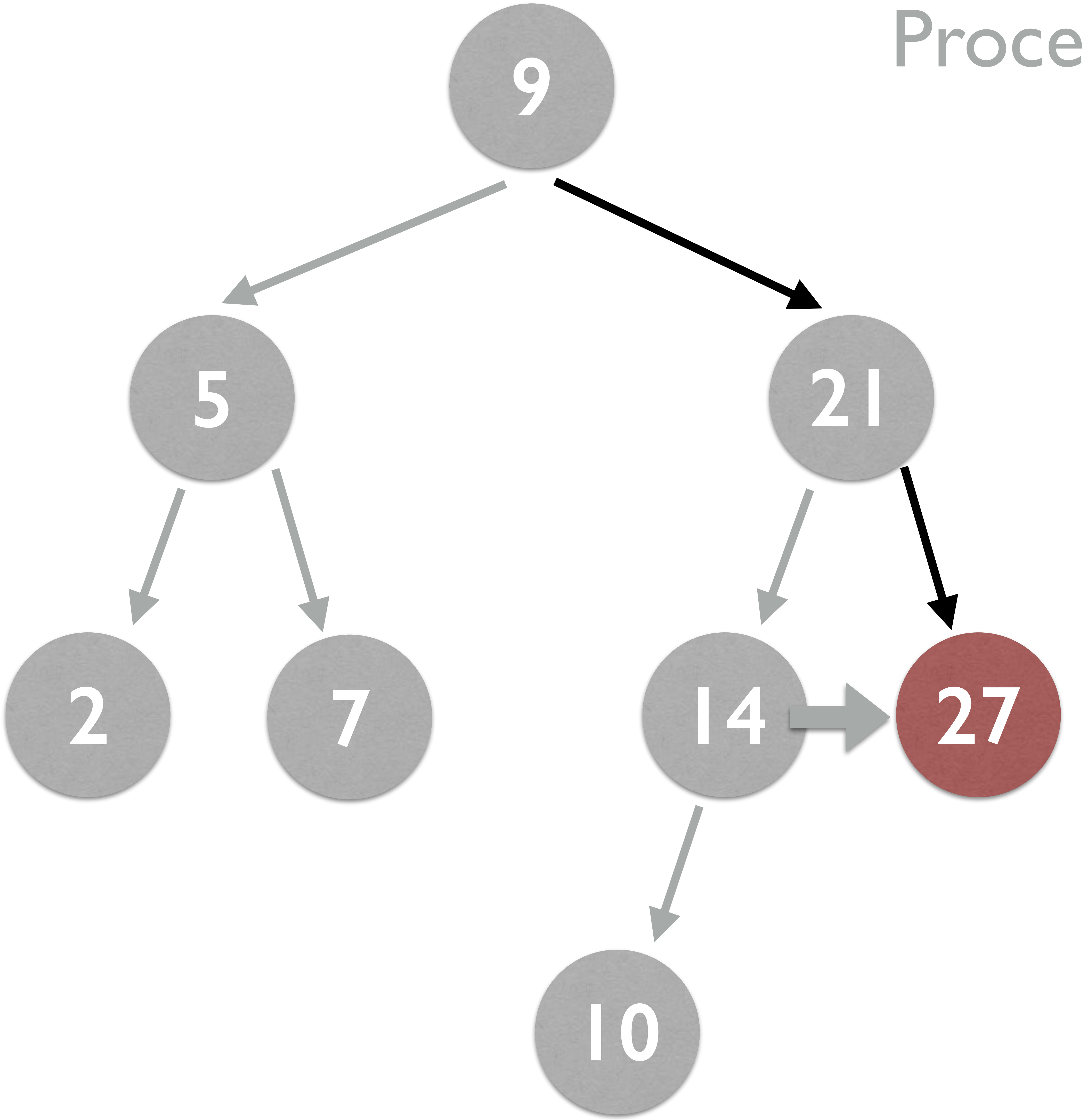
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21

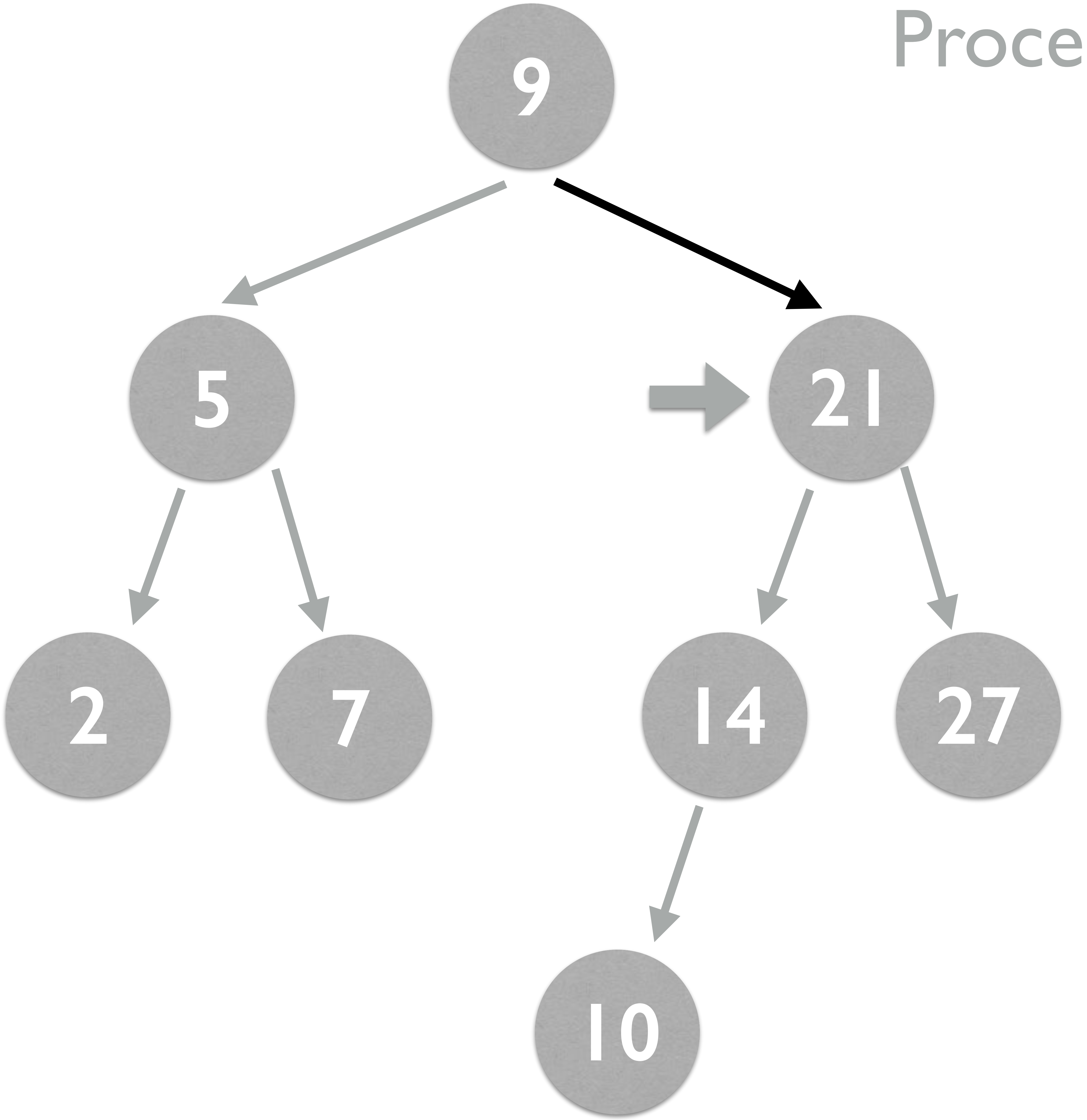
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21, 27

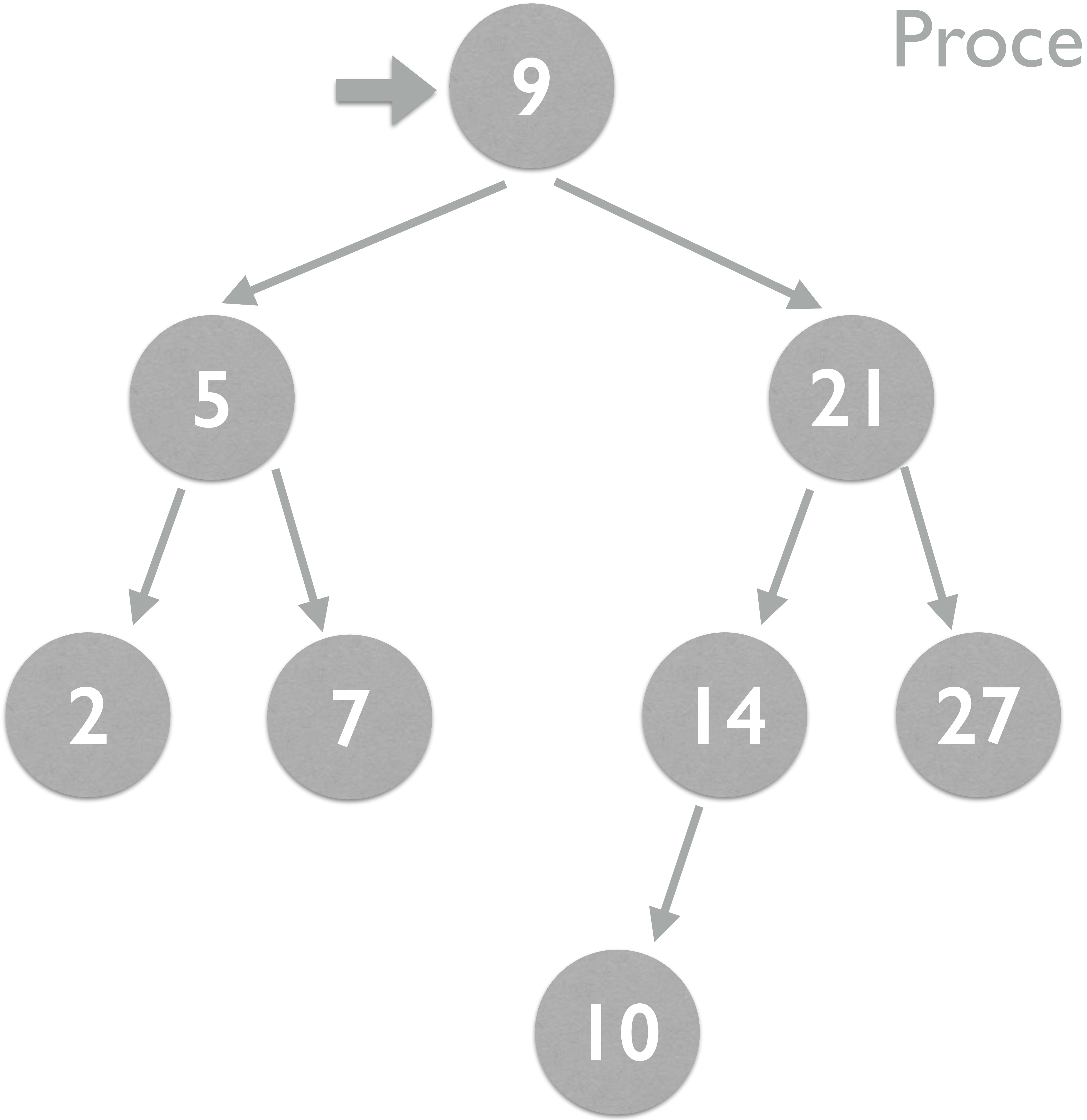
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21, 27

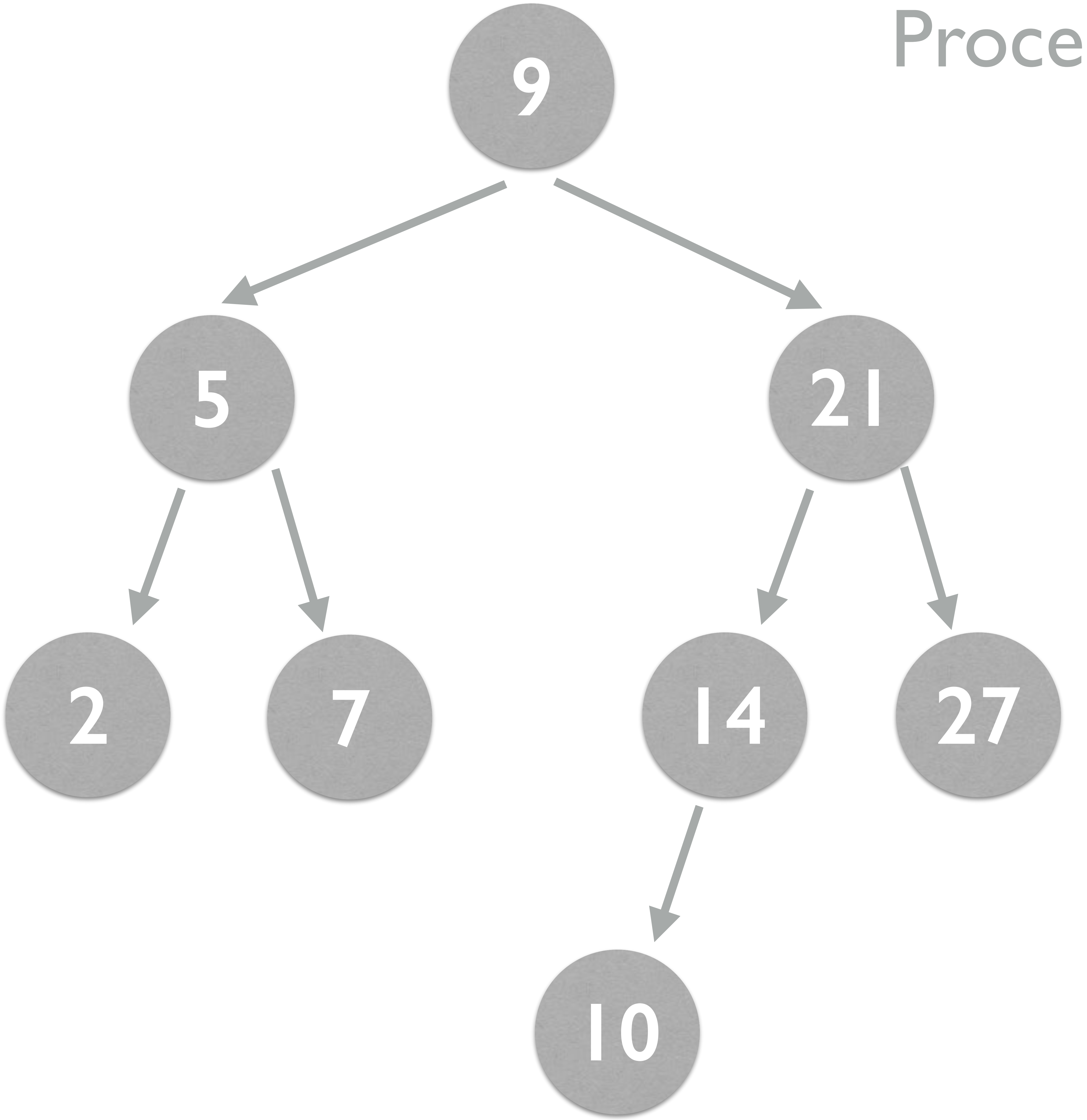
DFS: IN



Process left · Process root · Process right

2, 5, 7, 9, 10, 14, 21, 27

DFS: IN



Process left · Process root · Process right

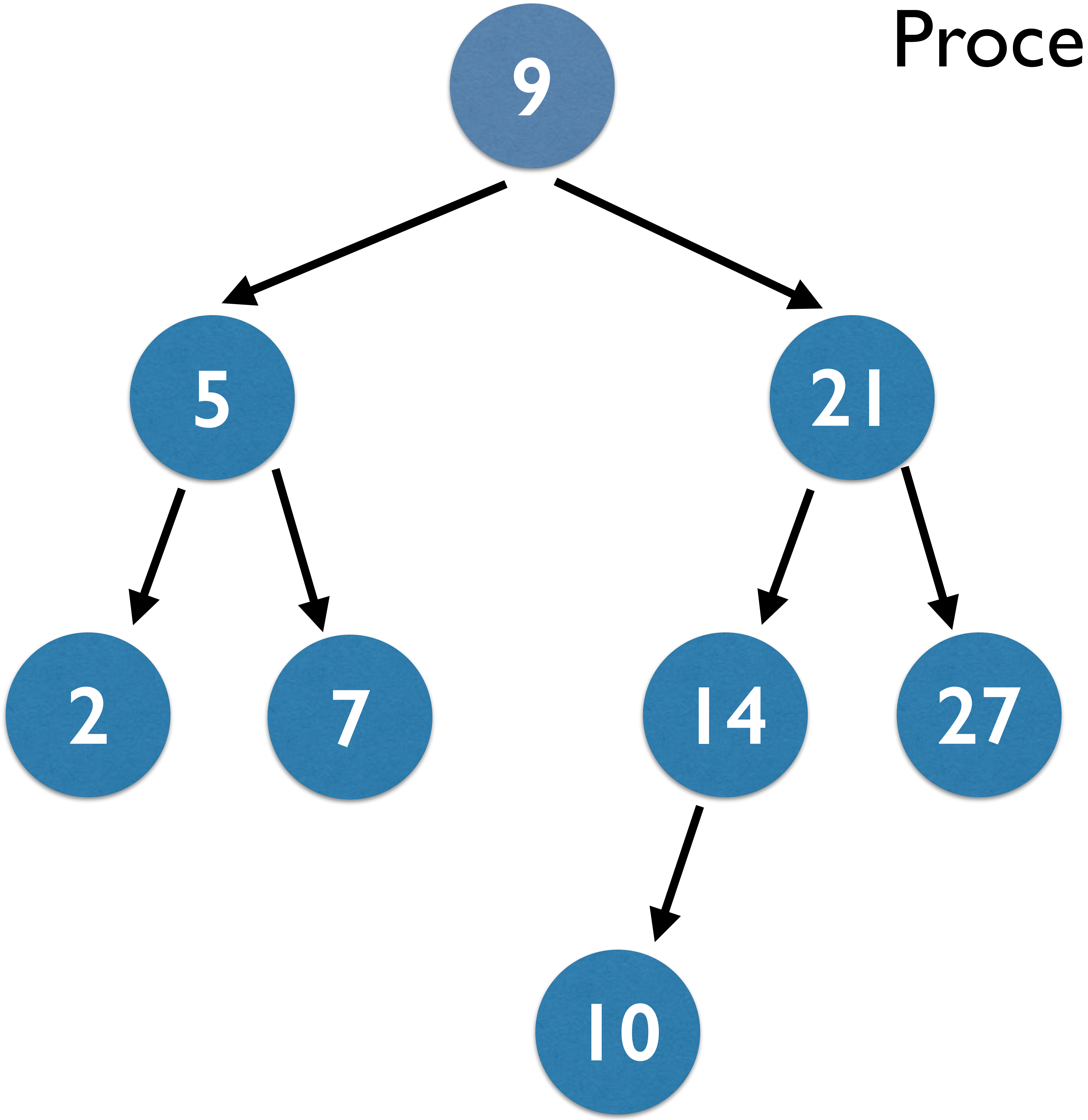
2, 5, 7, 9, 10, 14, 21, 27

- In-order because respects ordering of nodes — nodes are processed smallest to largest value (leftmost to rightmost).
- The most generally useful DFS strategy for BSTs.

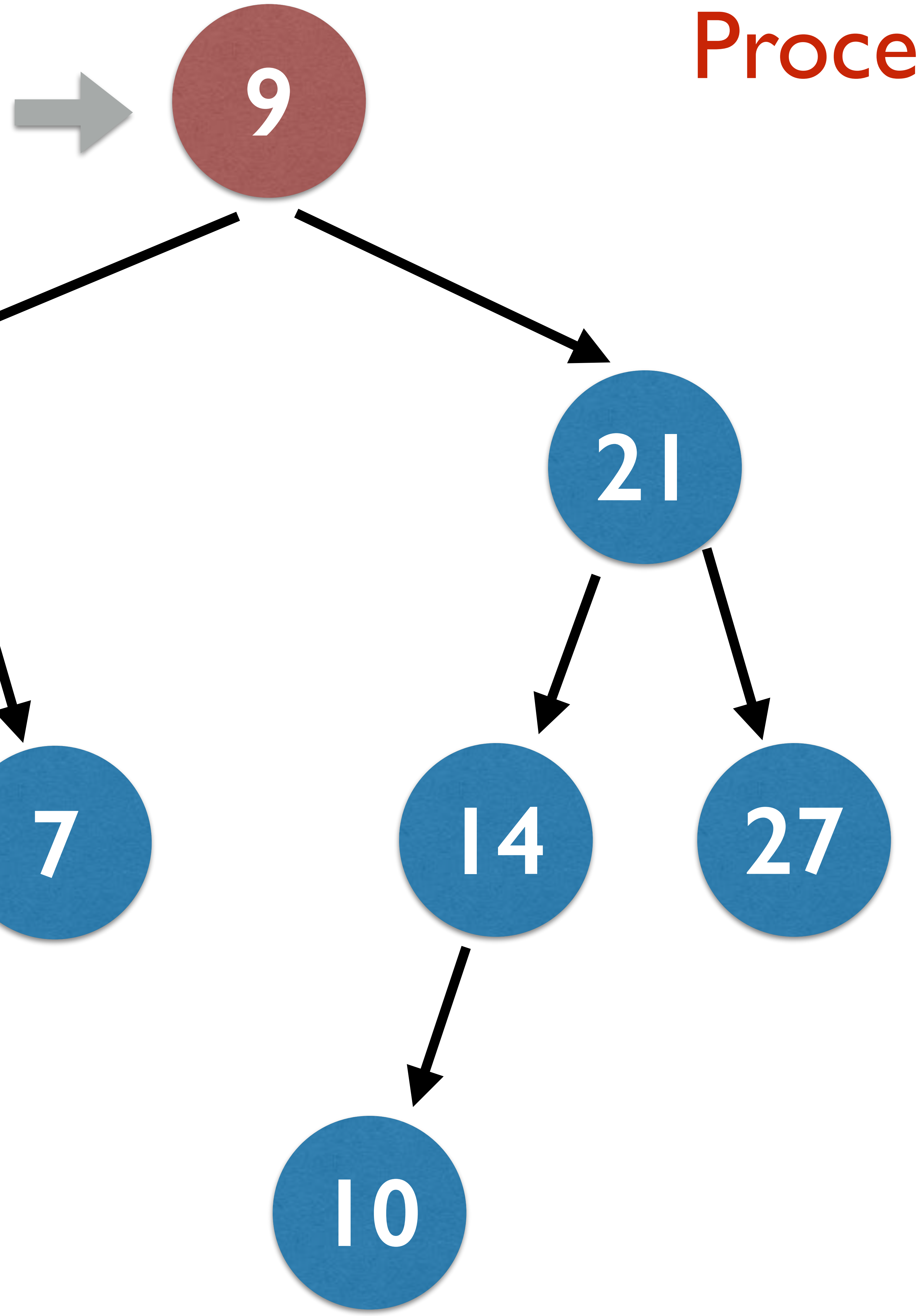
Depth First: Pre-Order

DFS: PRE

Process root · Process left · Process right



DFS: PRE

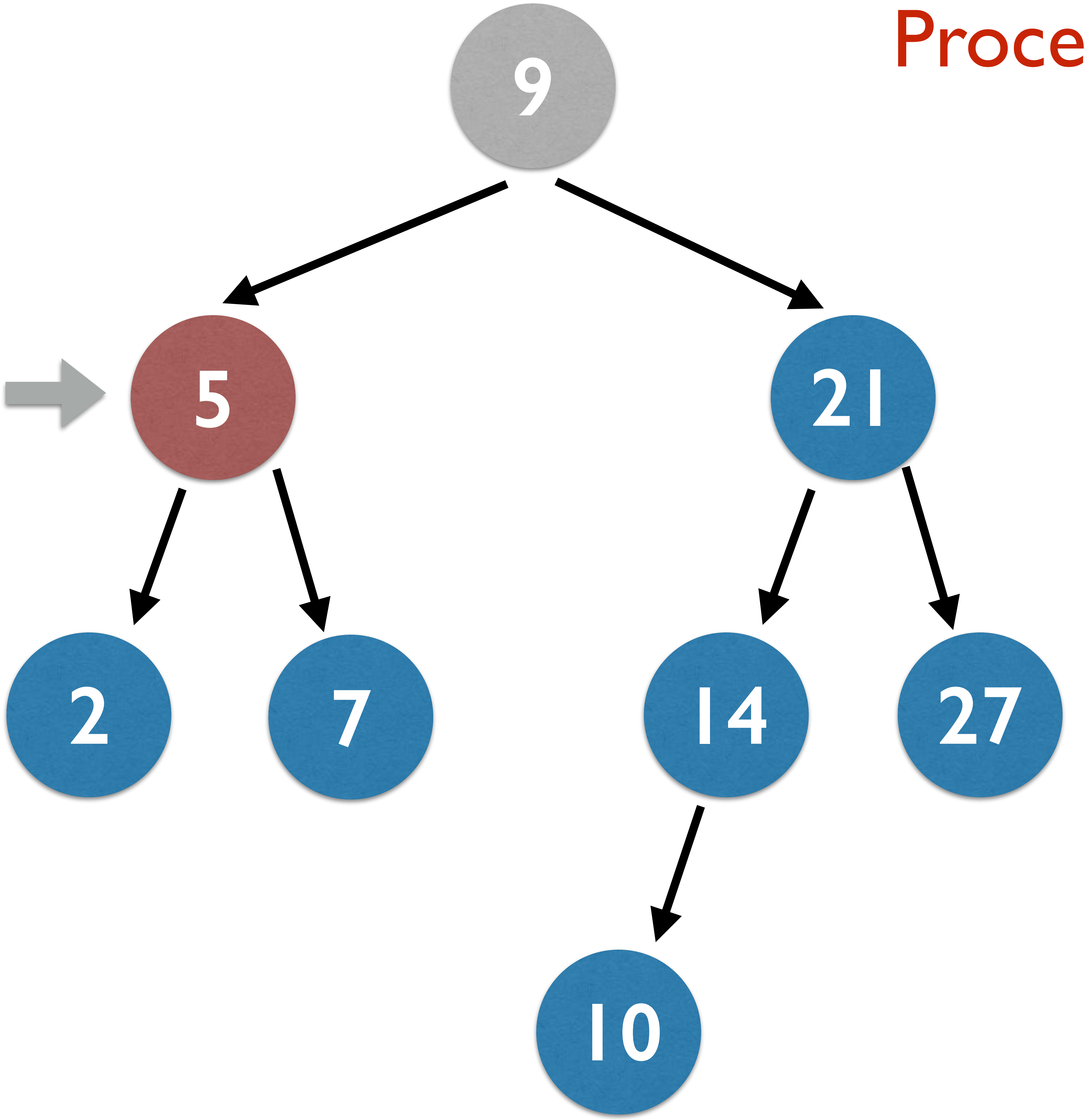


Process root · Process left · Process right

9

DFS: PRE

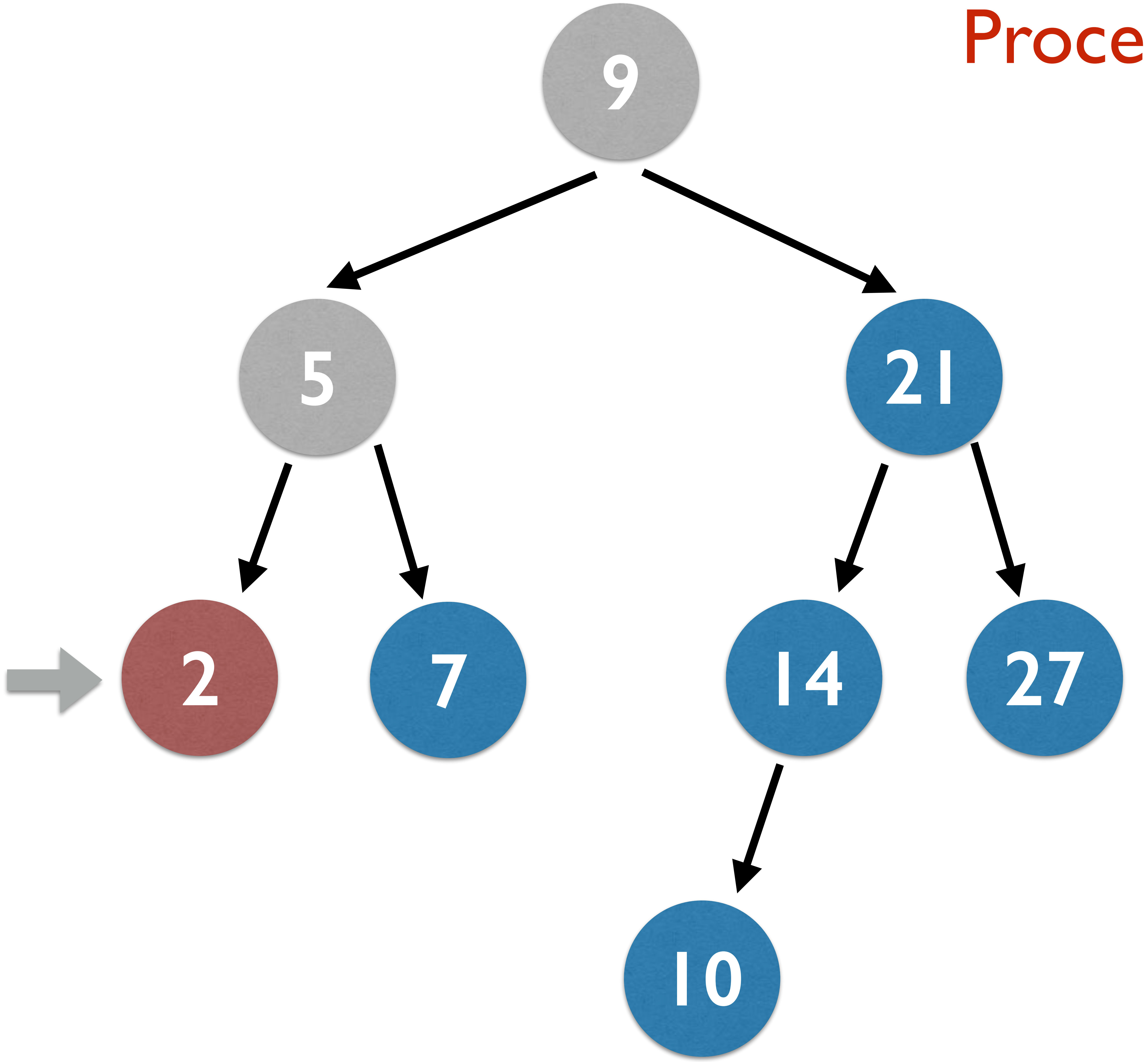
Process root · Process left · Process right



9, 5

DFS: PRE

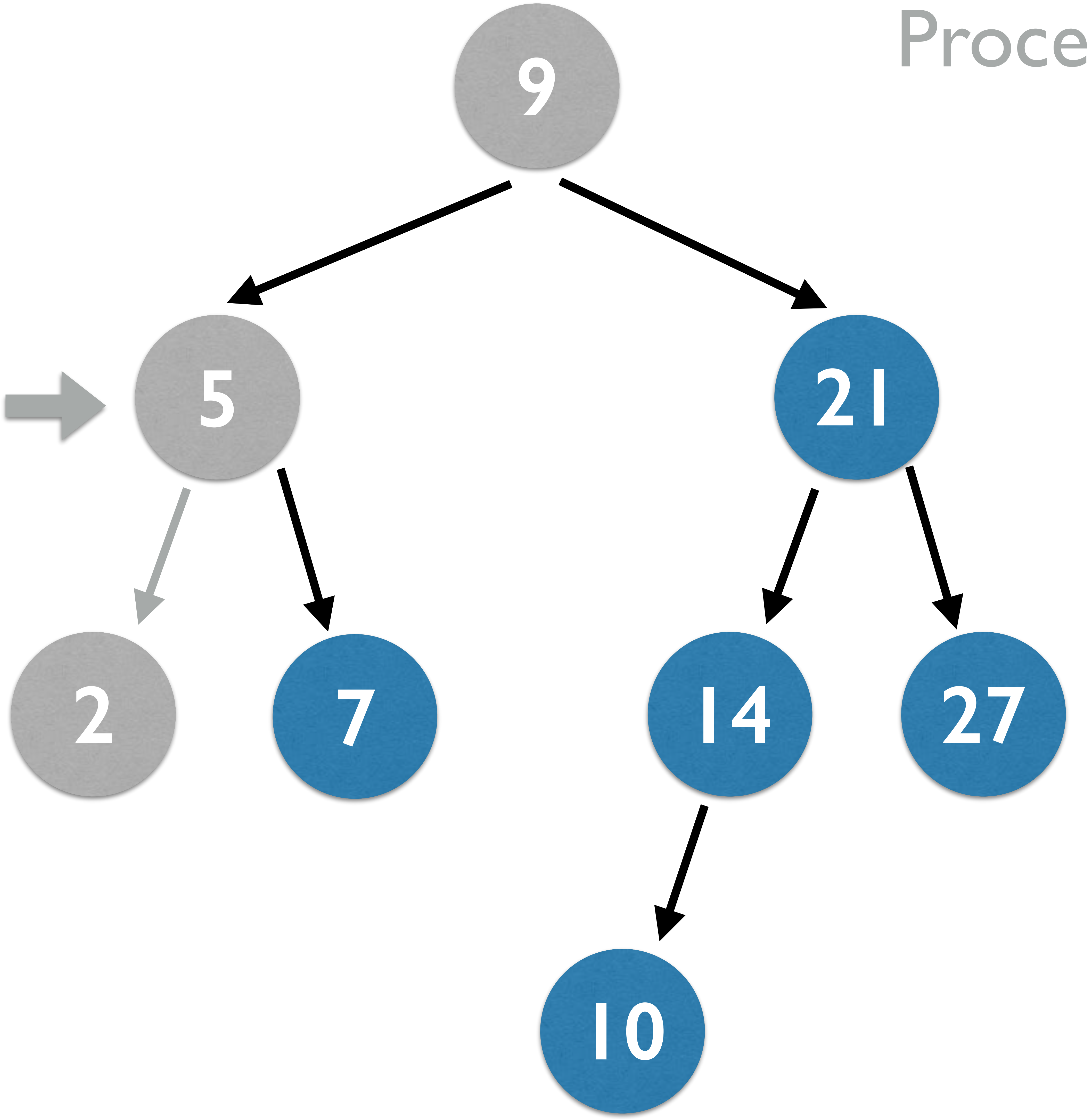
Process root · Process left · Process right



9, 5, 2

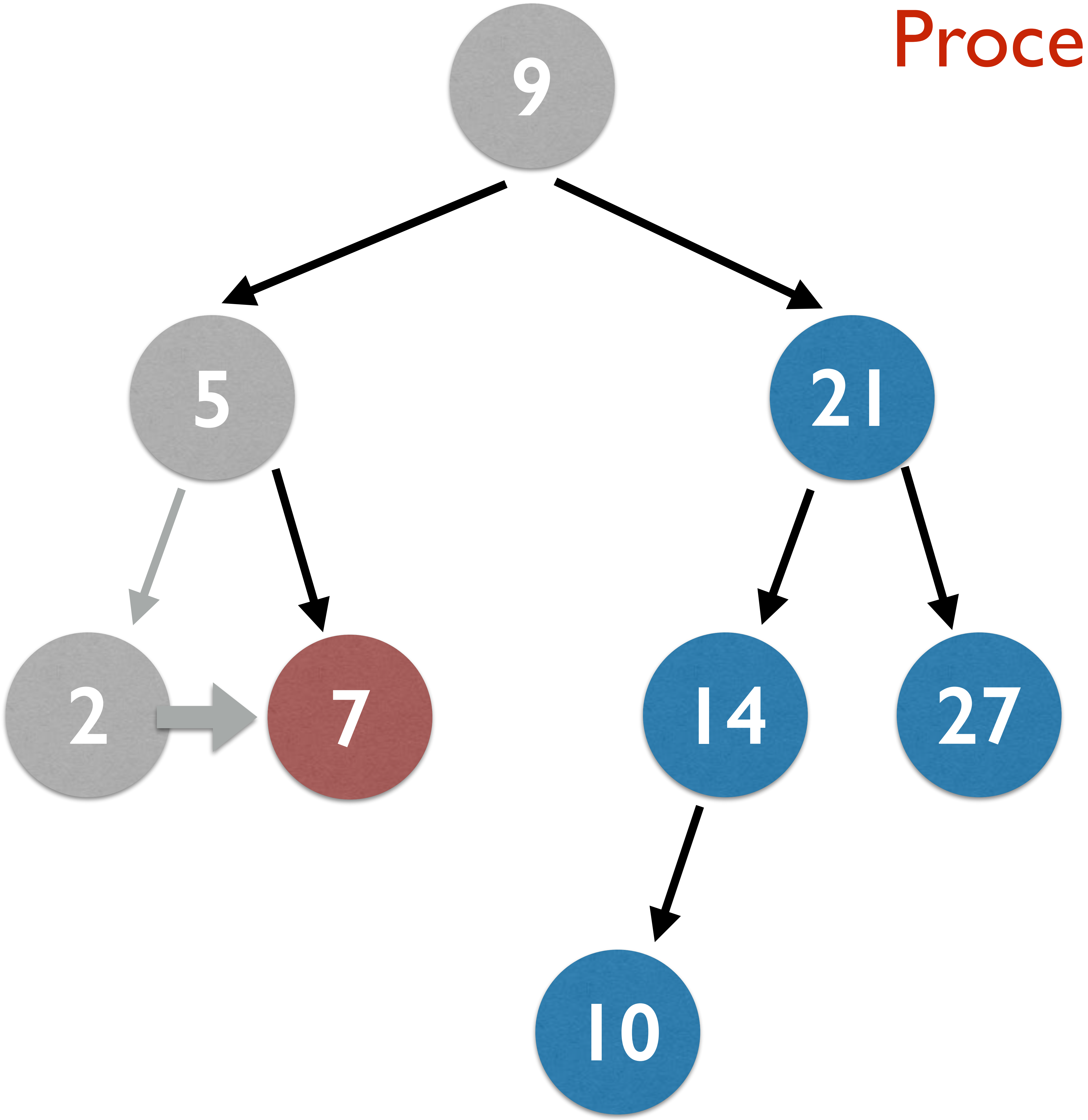
DFS: PRE

Process root · Process left · Process right



9, 5, 2

DFS: PRE

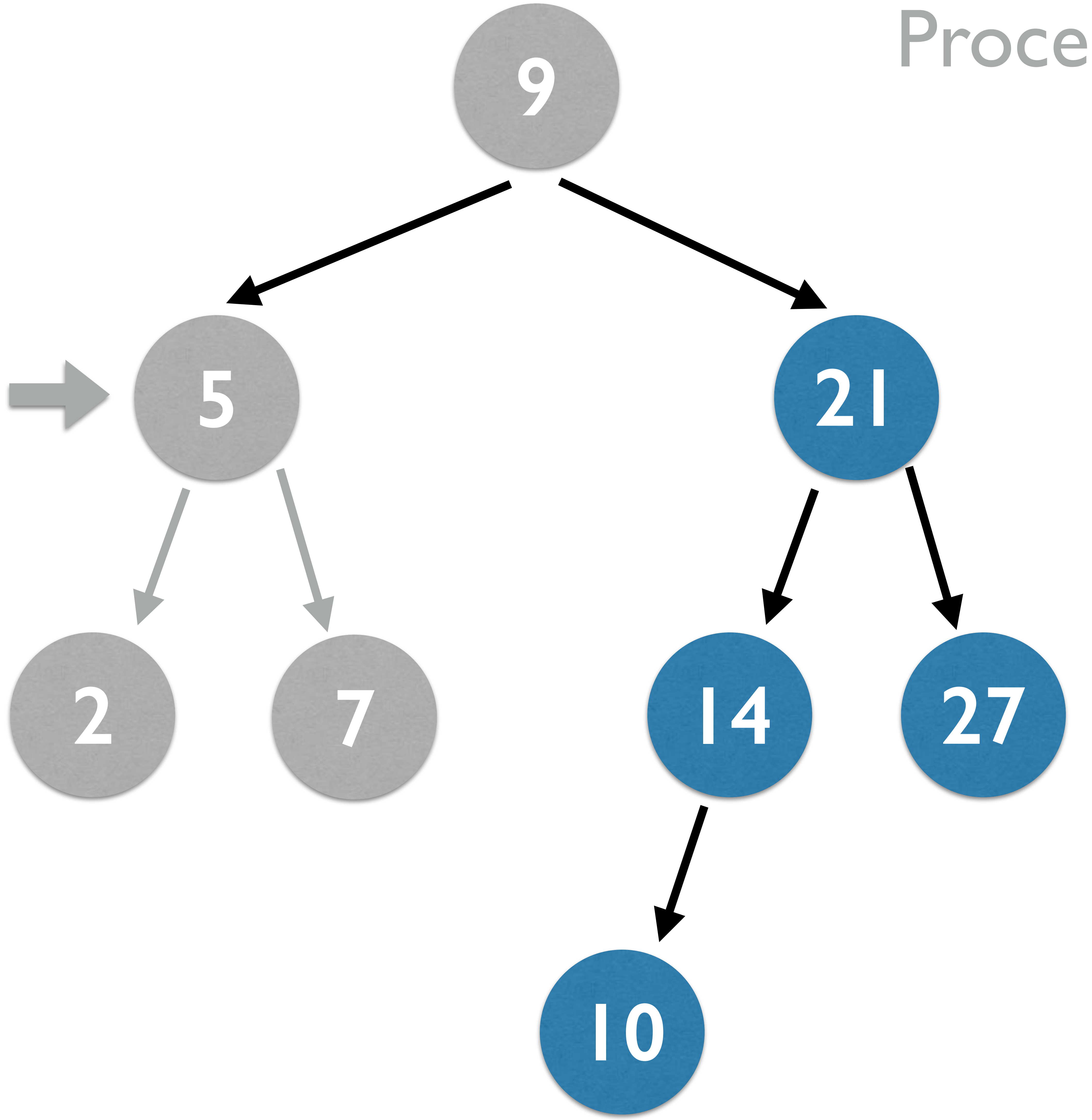


Process root · Process left · Process right

9, 5, 2, 7

DFS: PRE

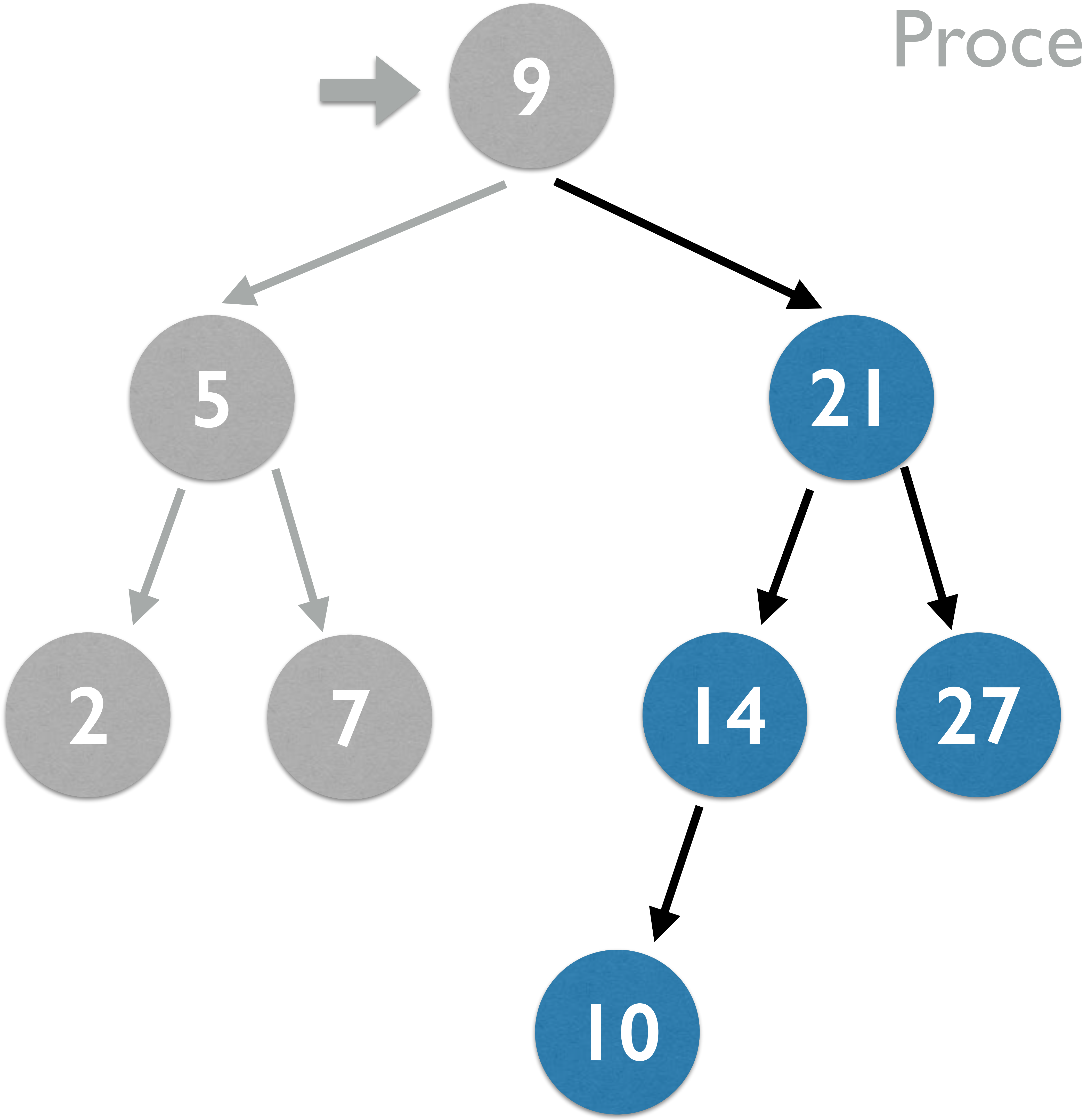
Process root · Process left · Process right



9, 5, 2, 7

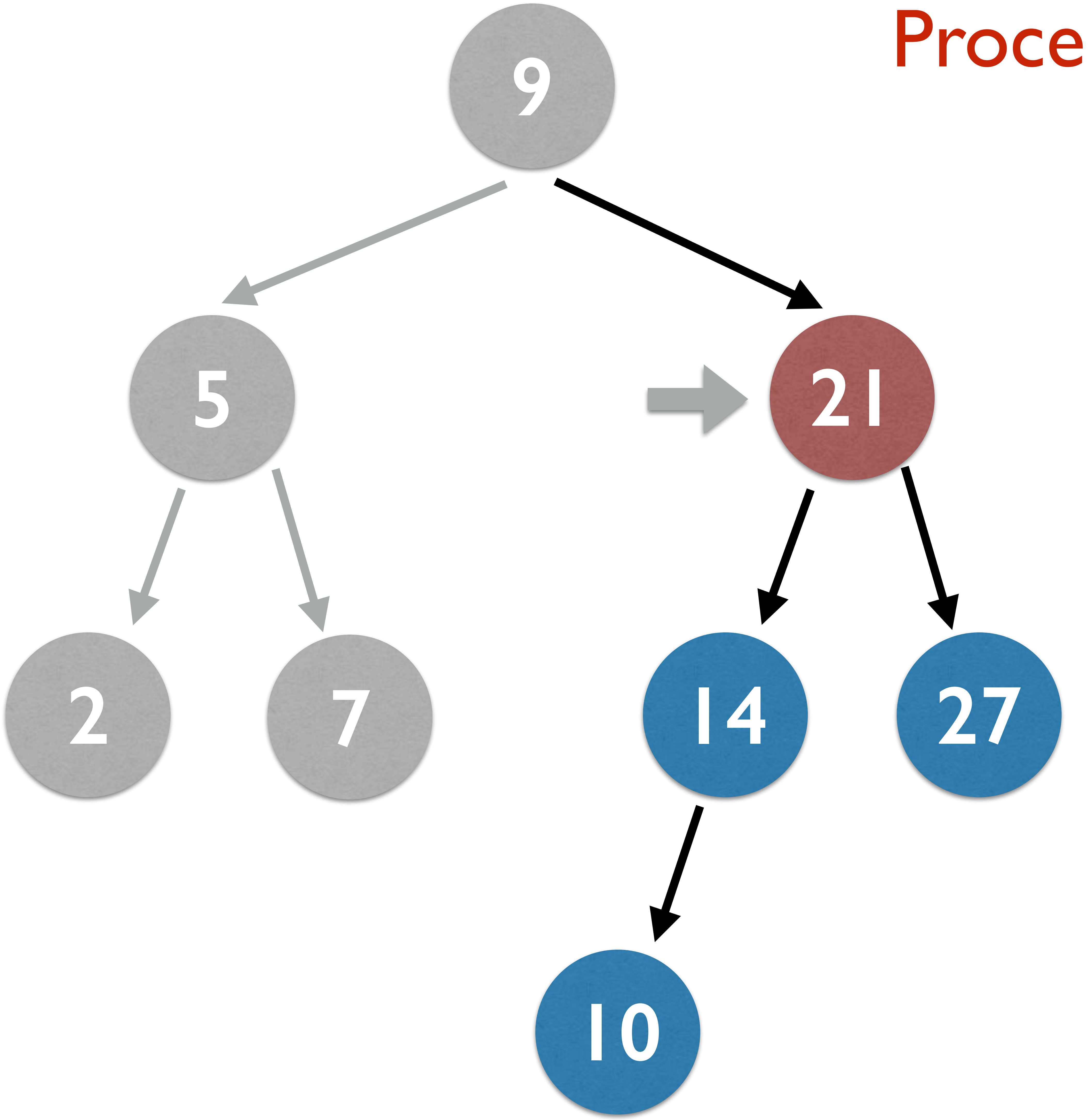
DFS: PRE

Process root · Process left · **Process right**



9, 5, 2, 7

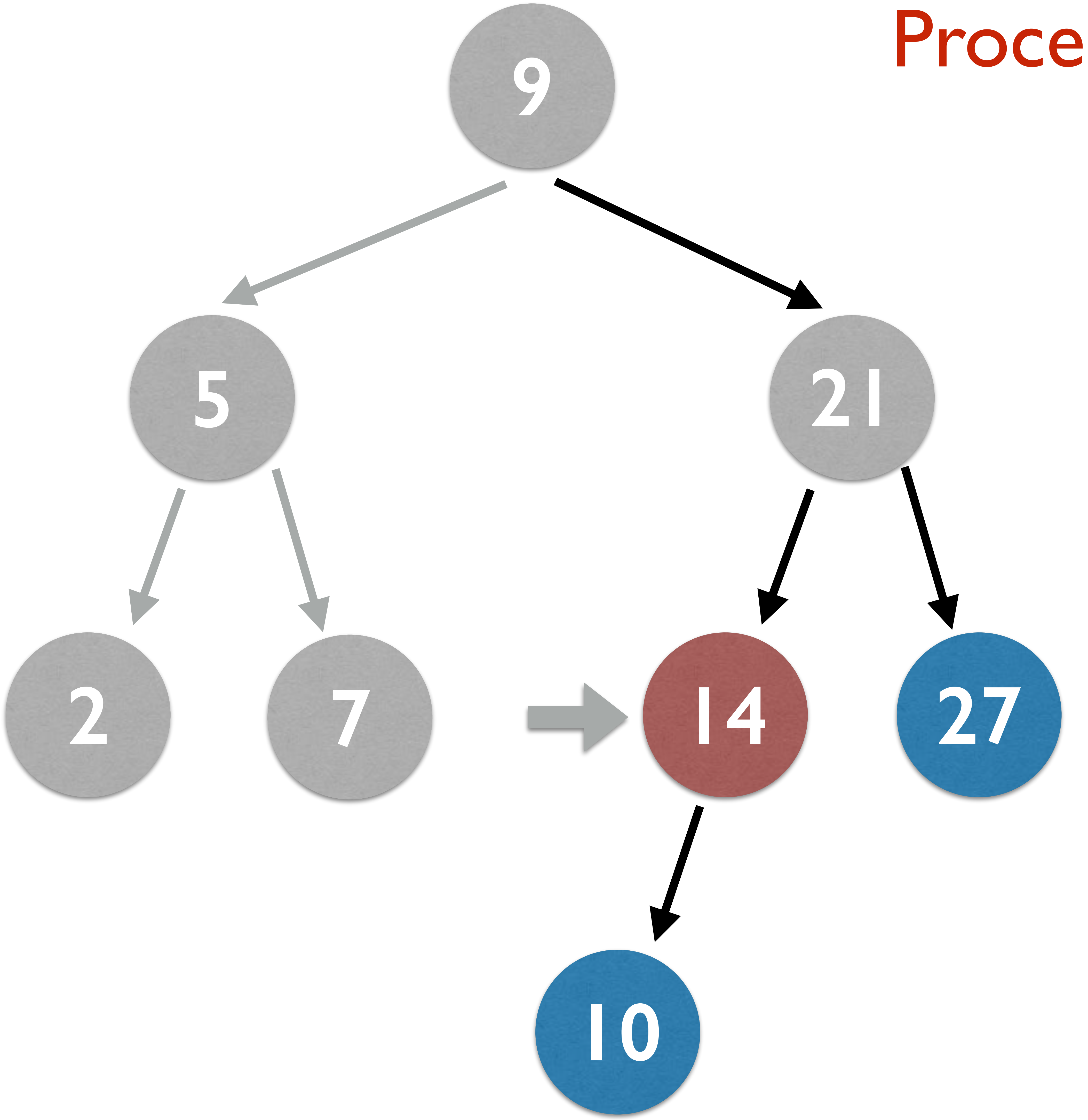
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21

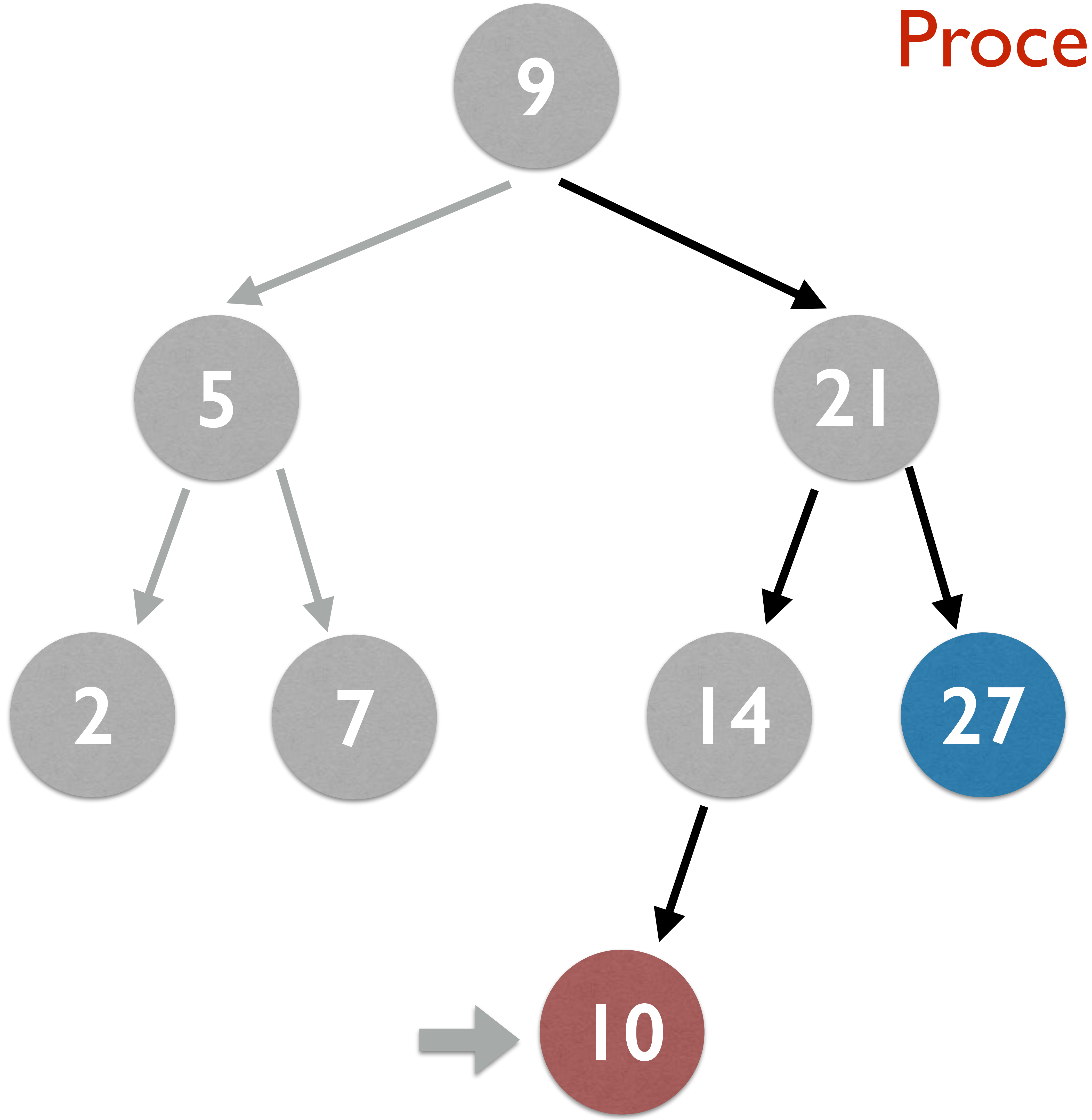
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14

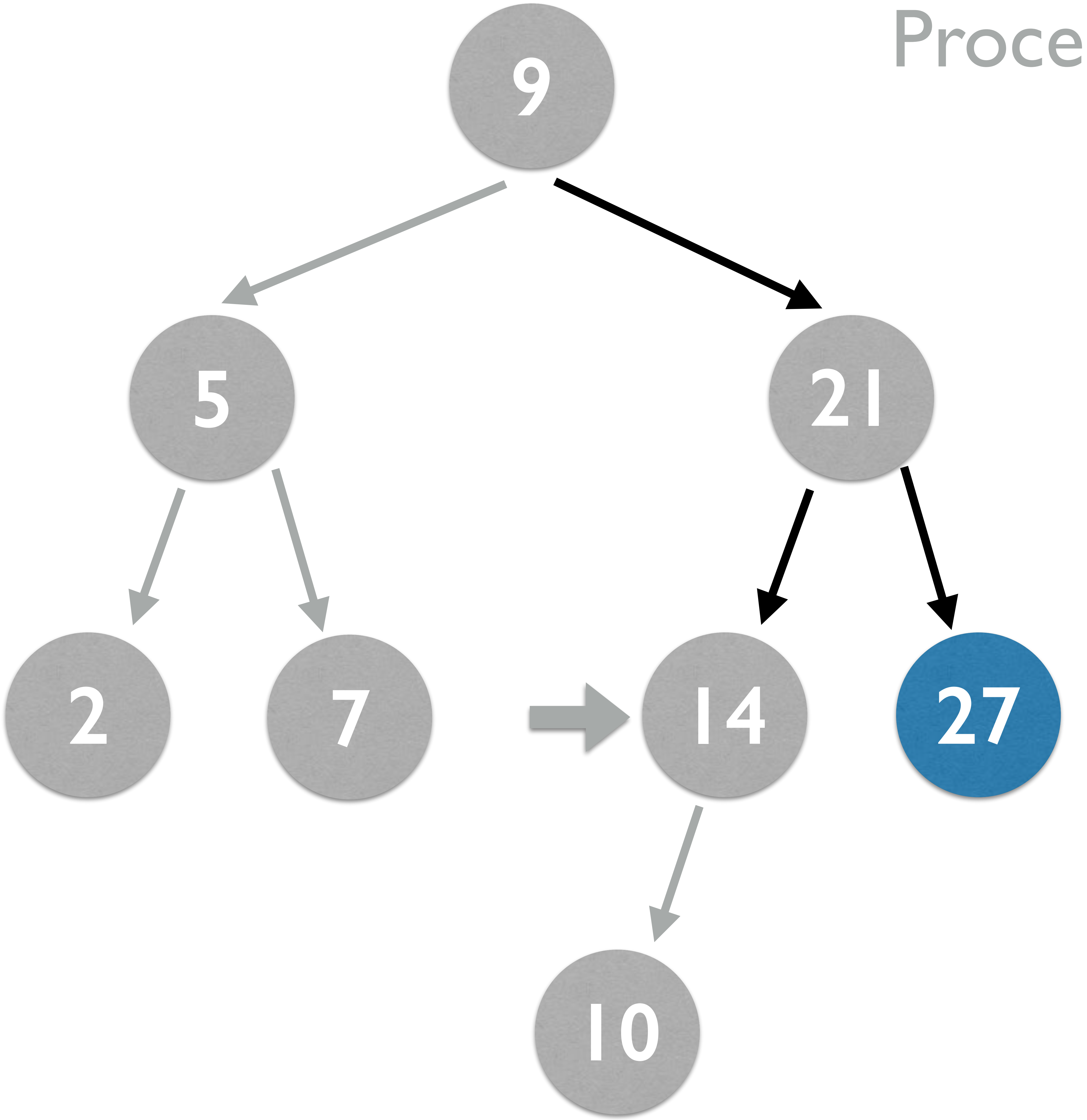
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10

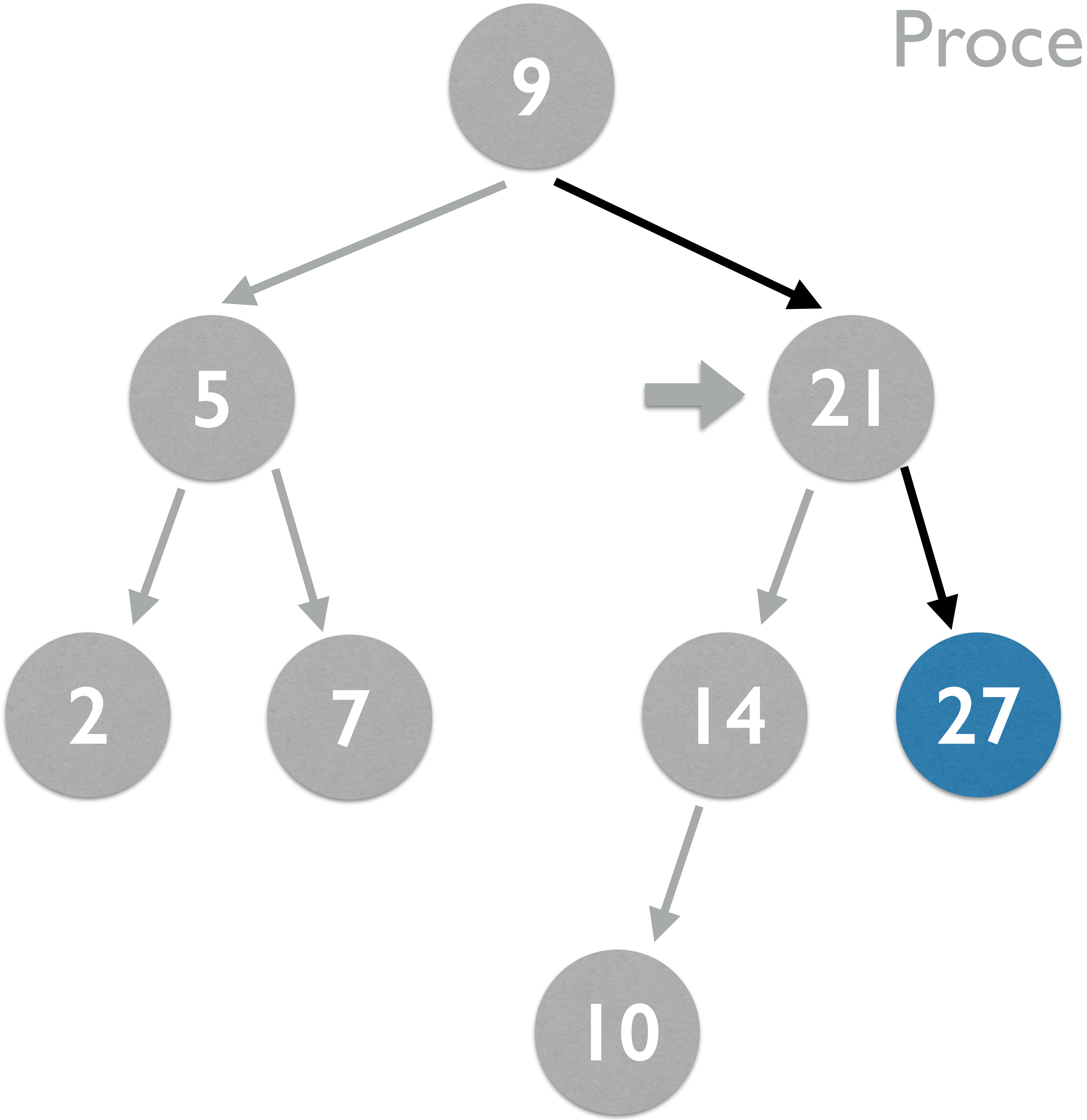
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10

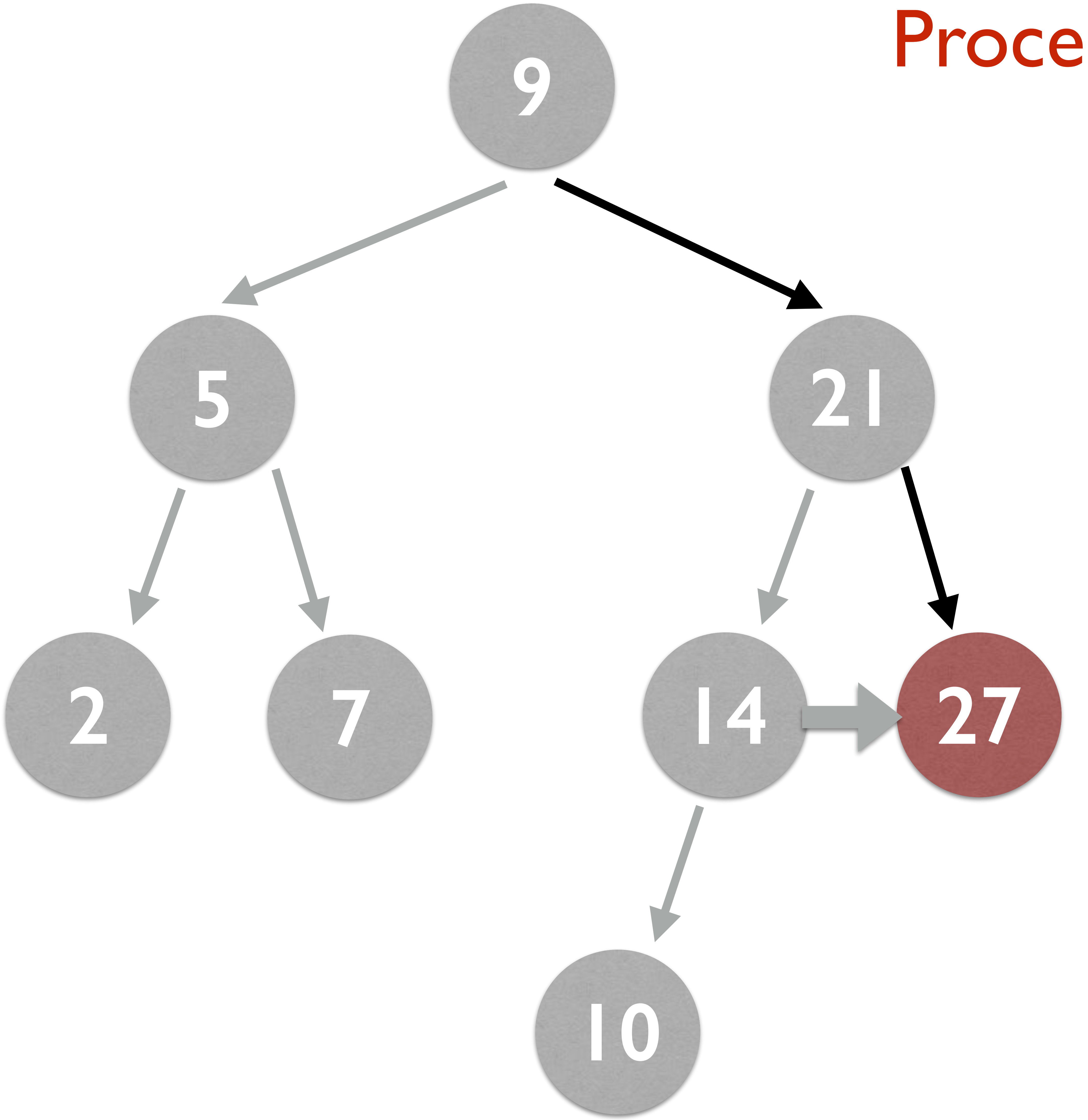
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10

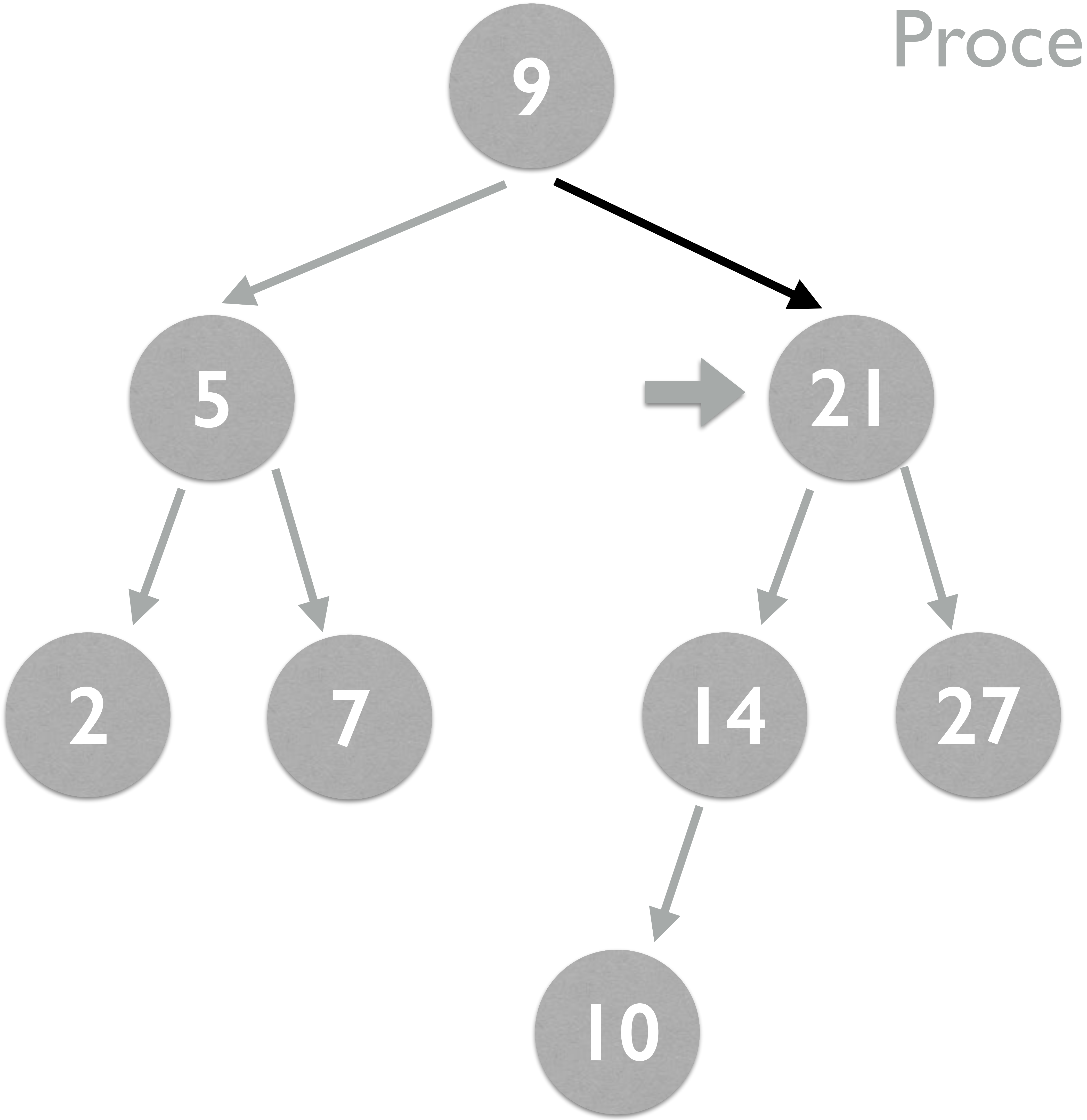
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

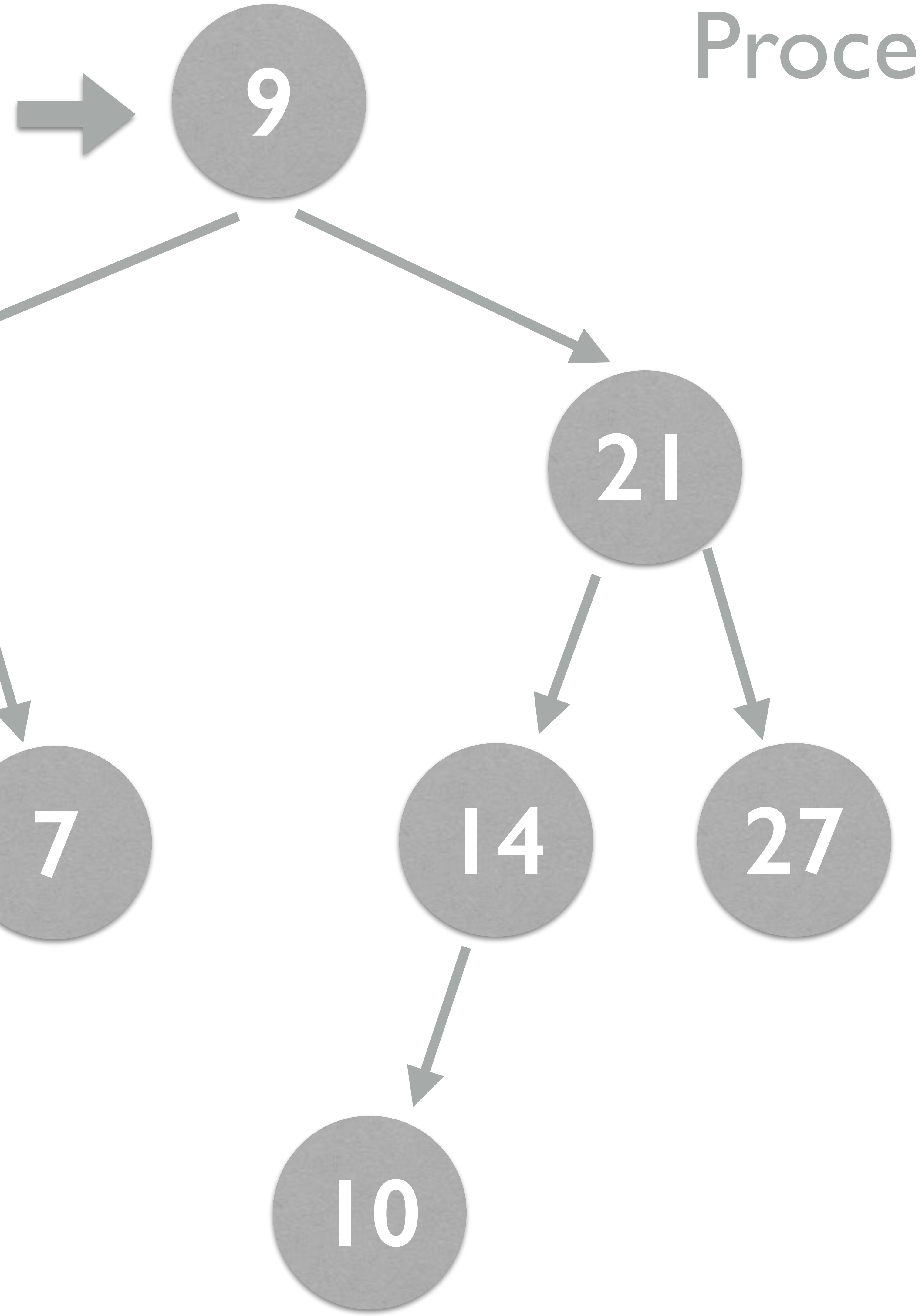
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

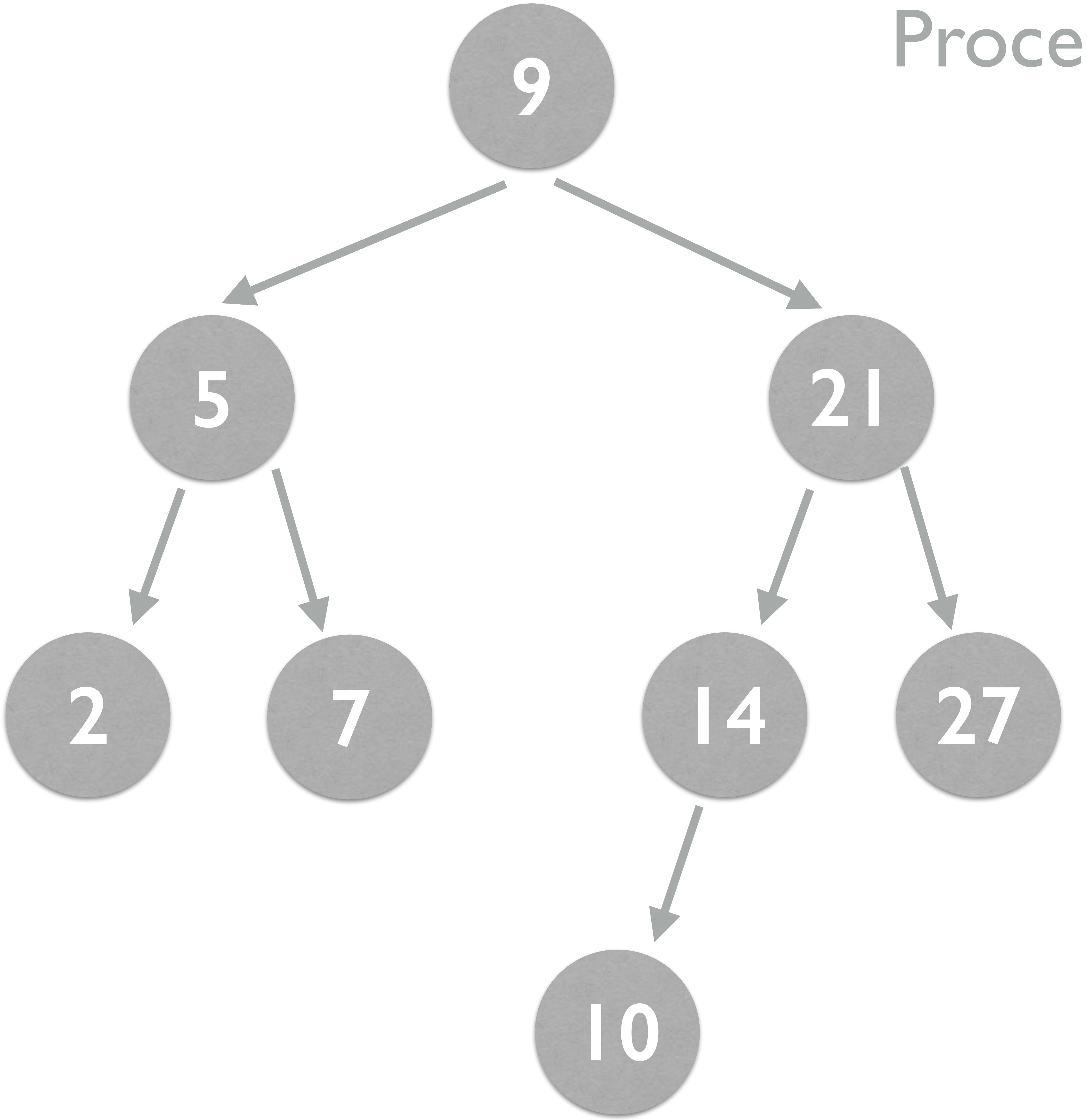
DFS: PRE



Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

DFS: PRE



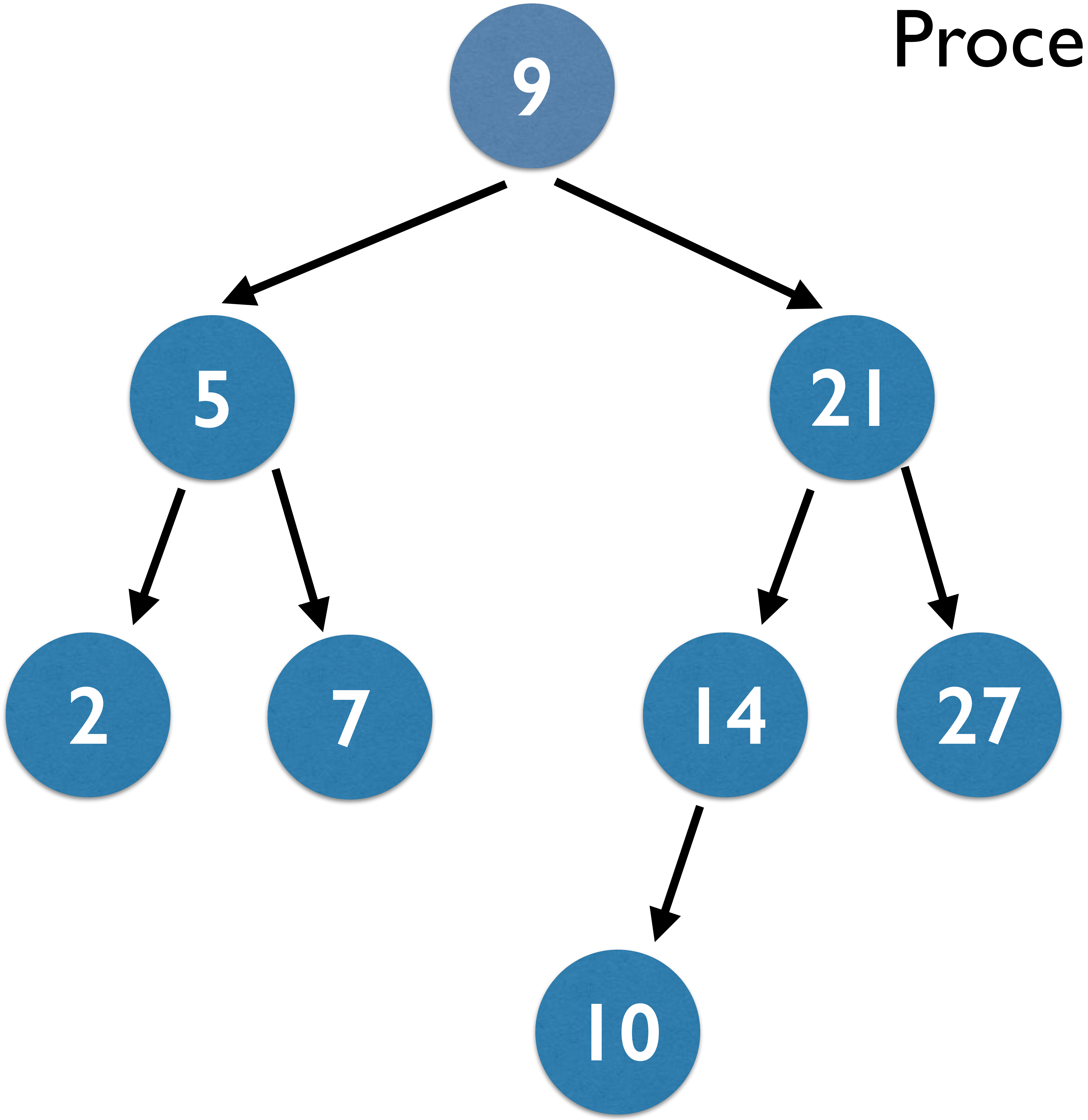
Process root · Process left · Process right

9, 5, 2, 7, 21, 14, 10, 27

- **Output seems random, but actually this has one notable use case. If you create a BST by inserting these values in this order, you get a copy of the original tree. However, the same is true for breadth-first.**

Depth First: Post-Order

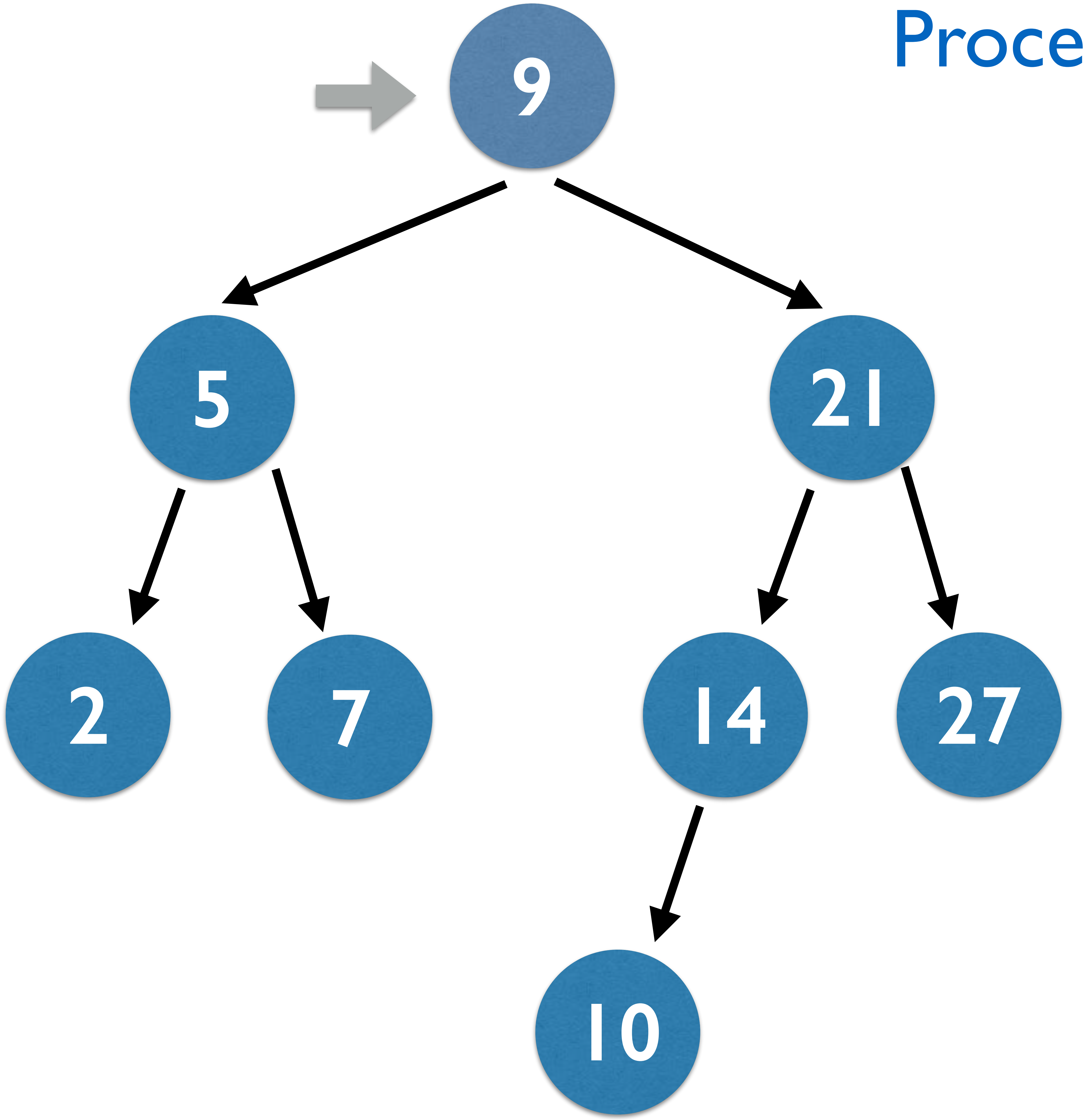
DFS: POST



Process left · Process right · Process root

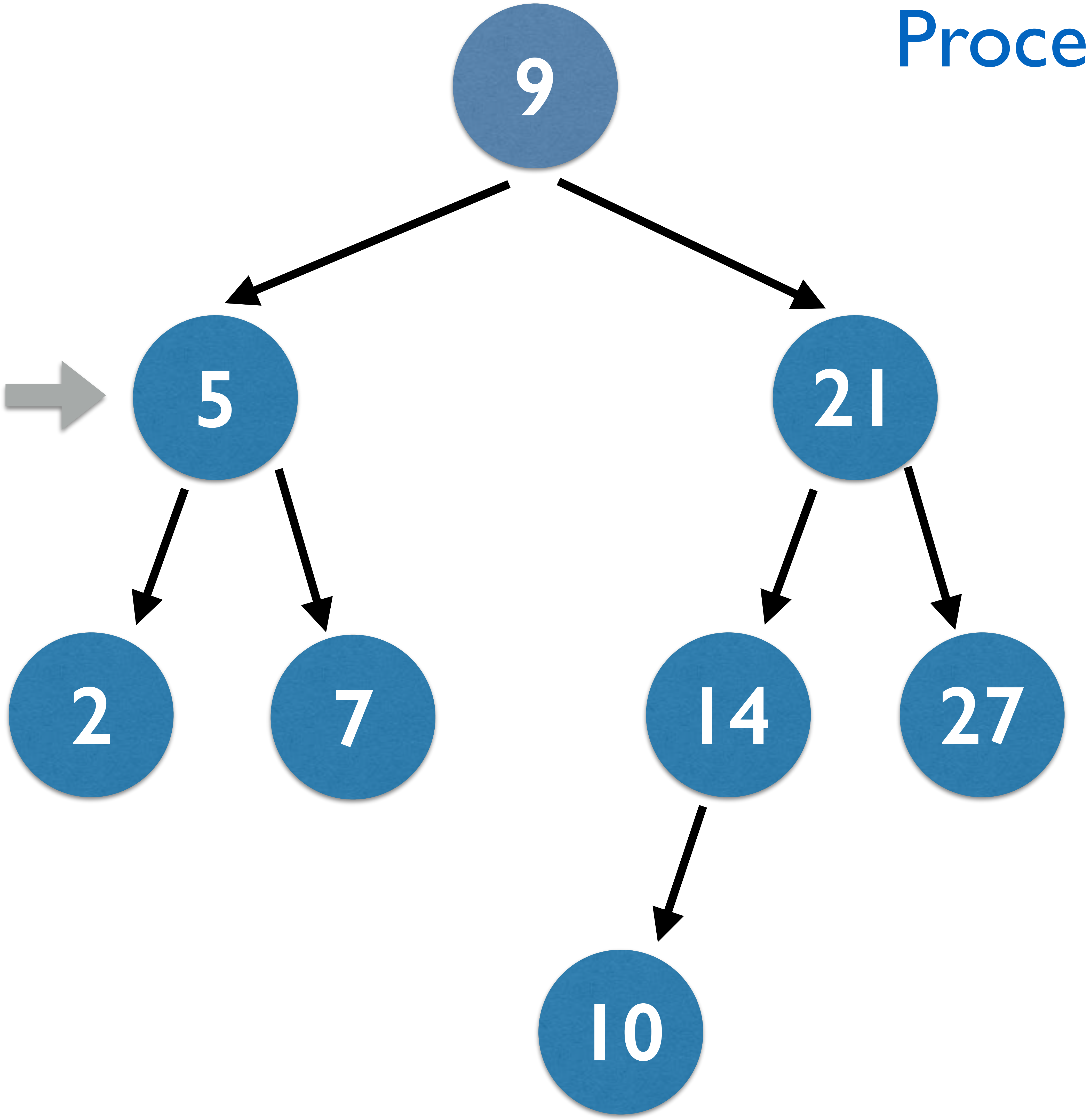
DFS: POST

Process left · Process right · Process root



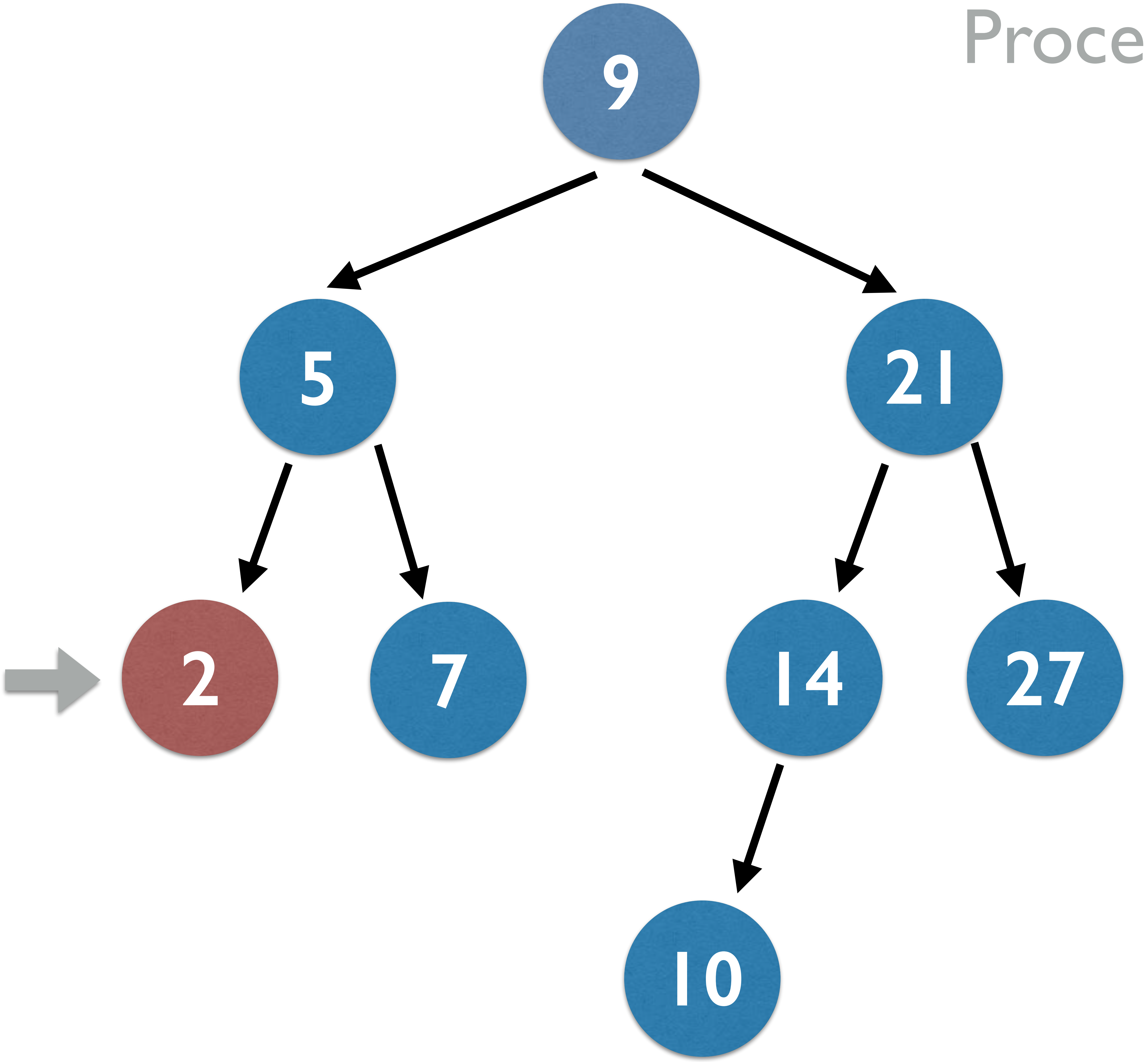
DFS: POST

Process left · Process right · Process root



DFS: POST

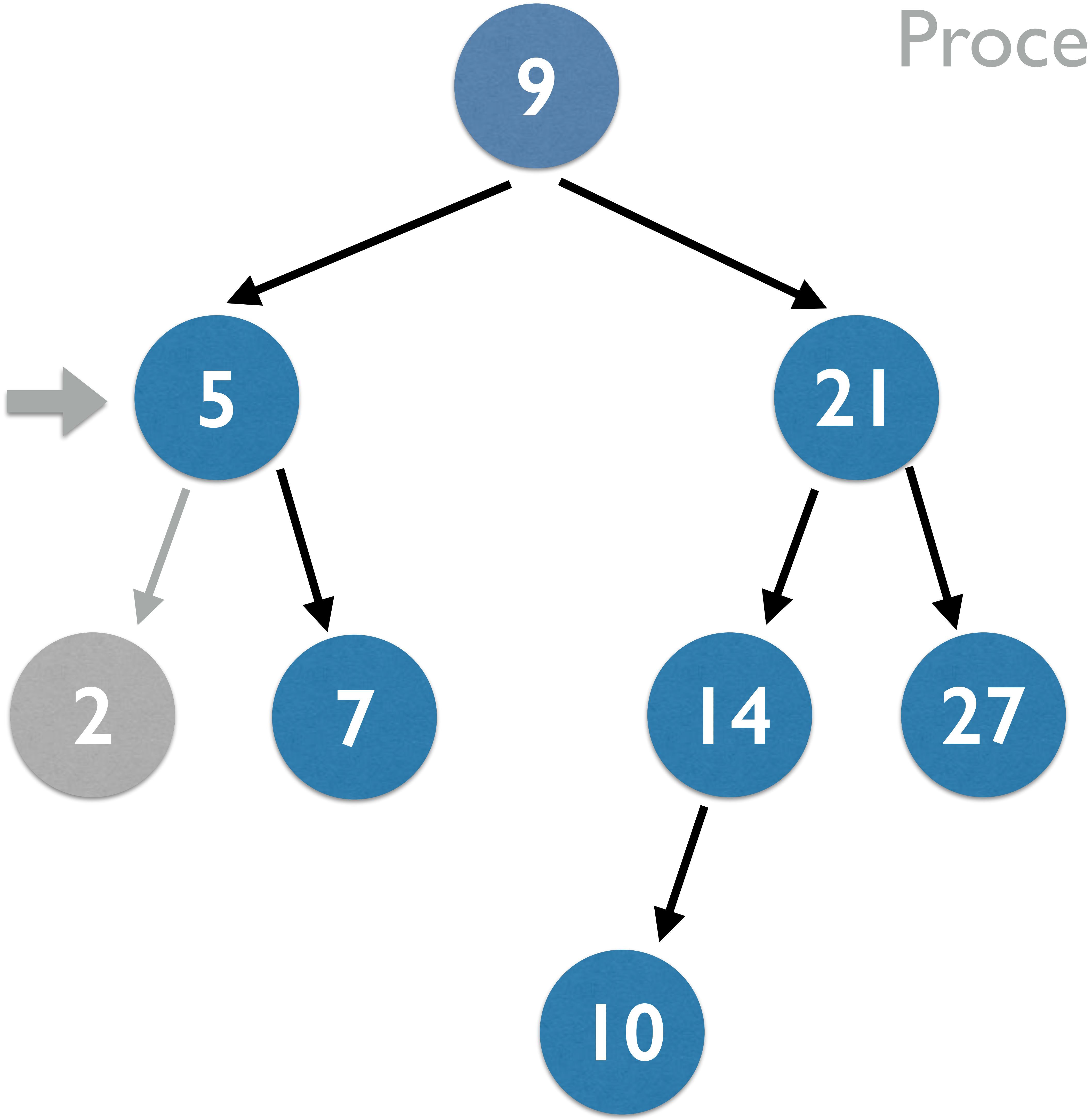
Process left · Process right · Process root



2

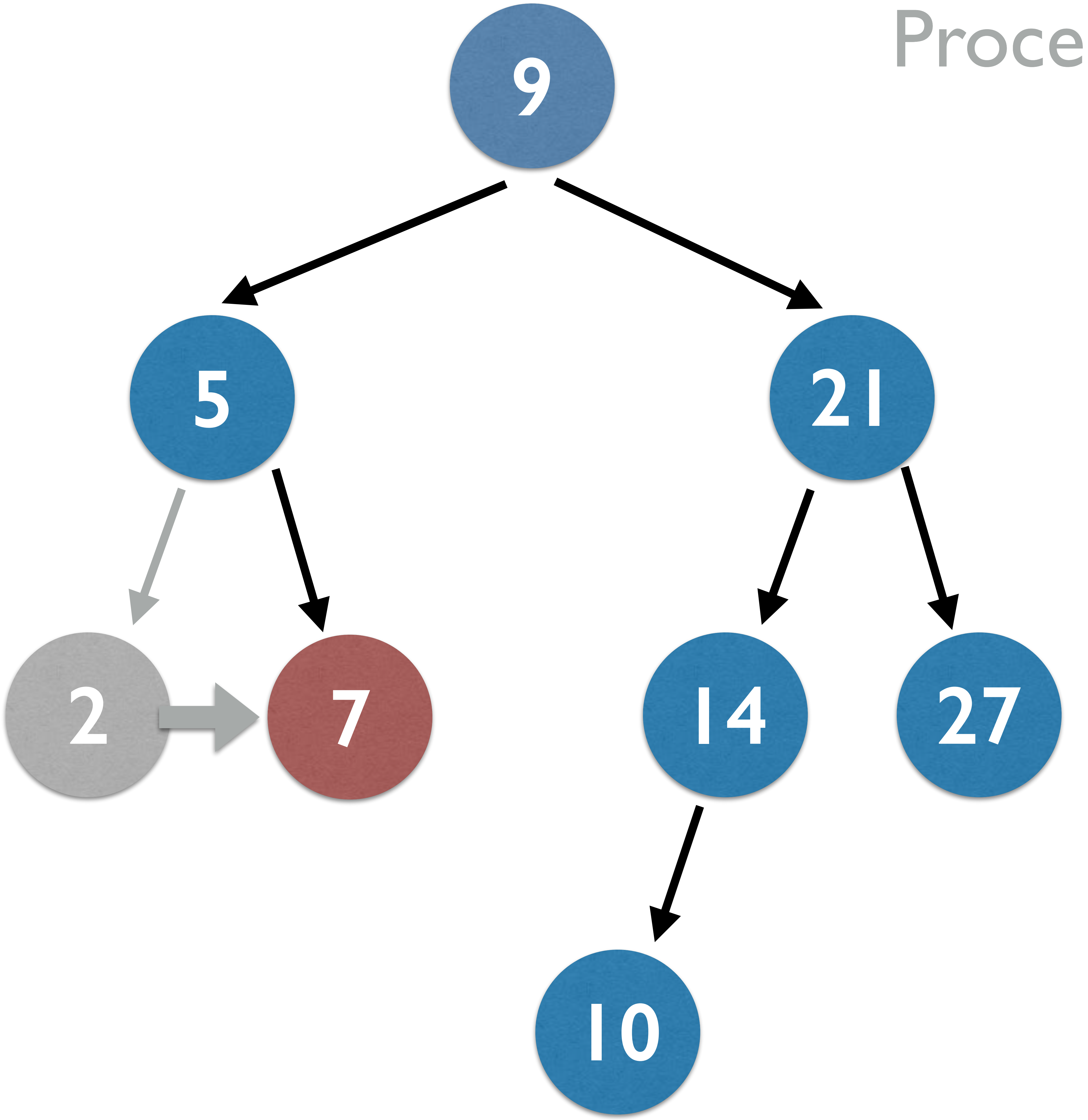
DFS: POST

Process left · Process right · Process root



2

DFS: POST

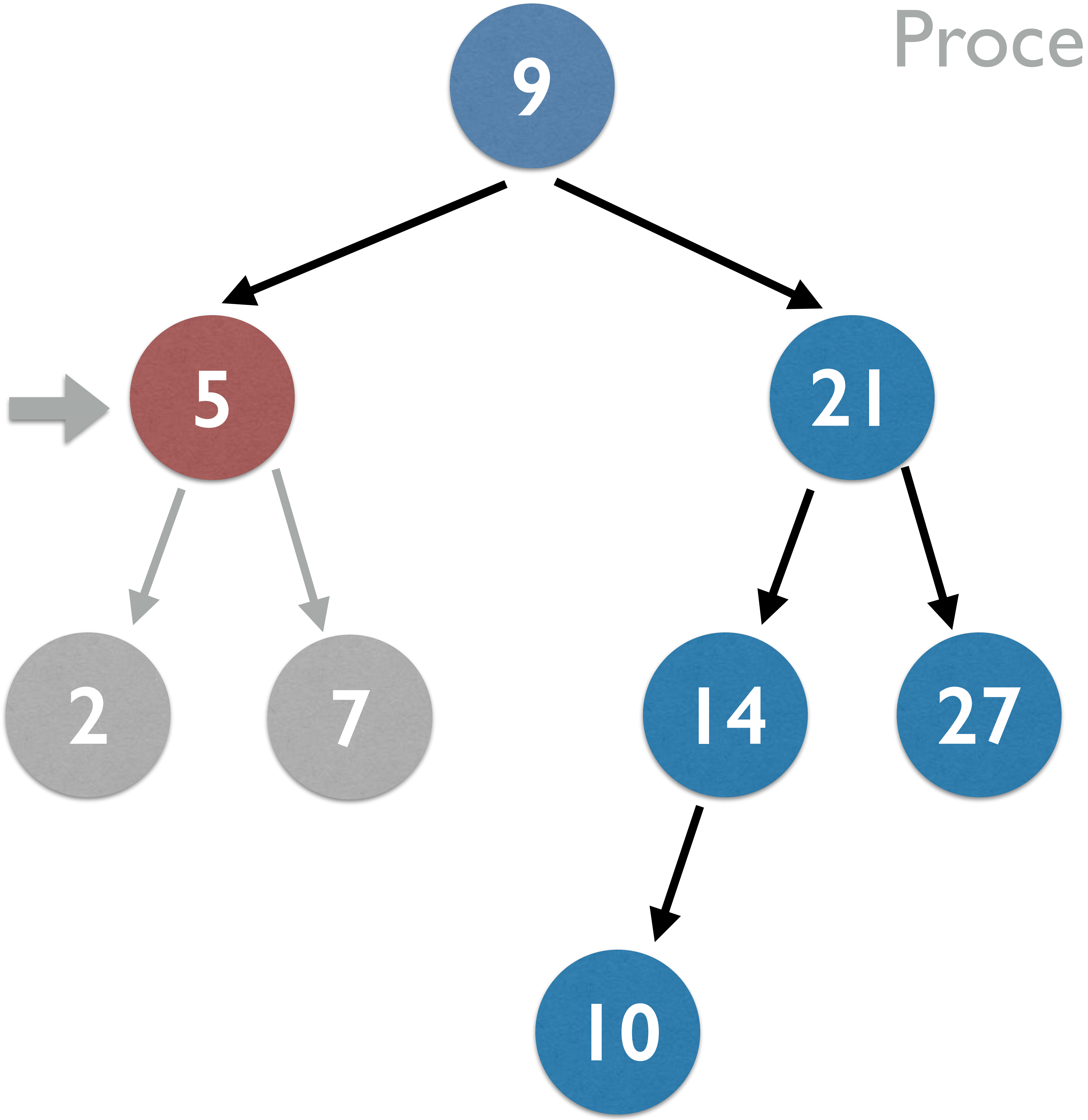


Process left · Process right · Process root

2, 7

DFS: POST

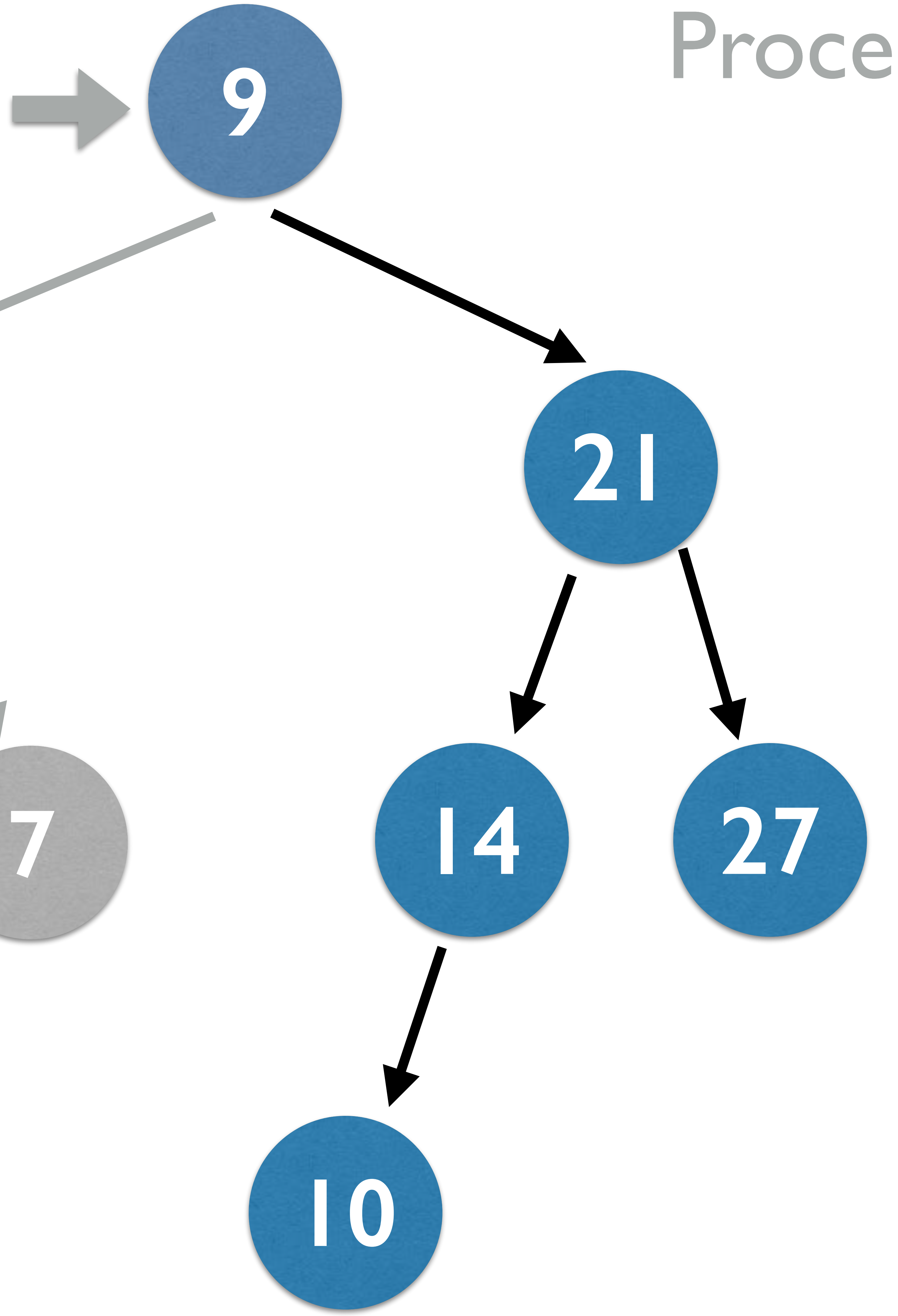
Process left · Process right · Process root



2, 7, 5

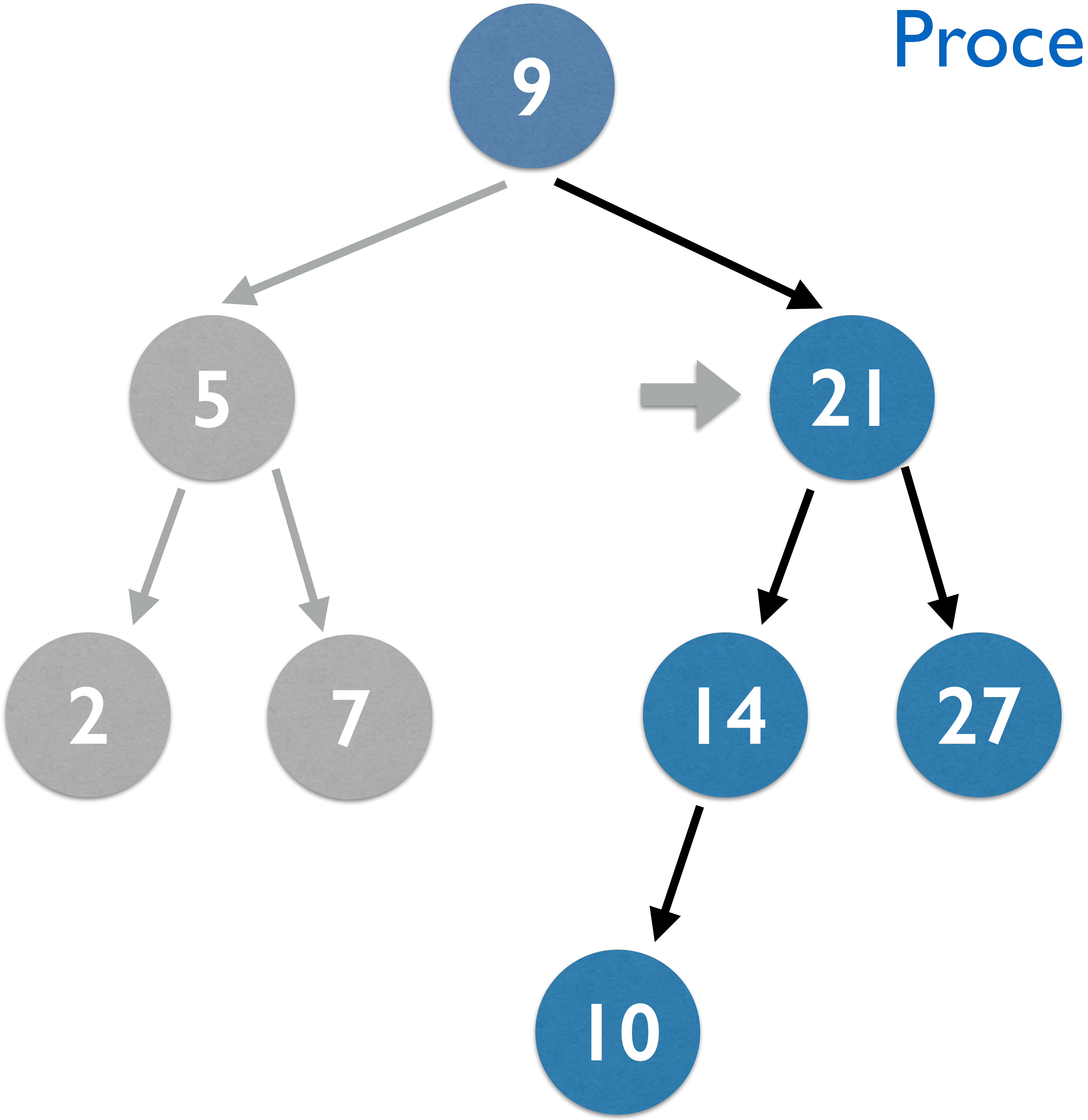
DFS: POST

Process left · Process right · Process root



2, 7, 5

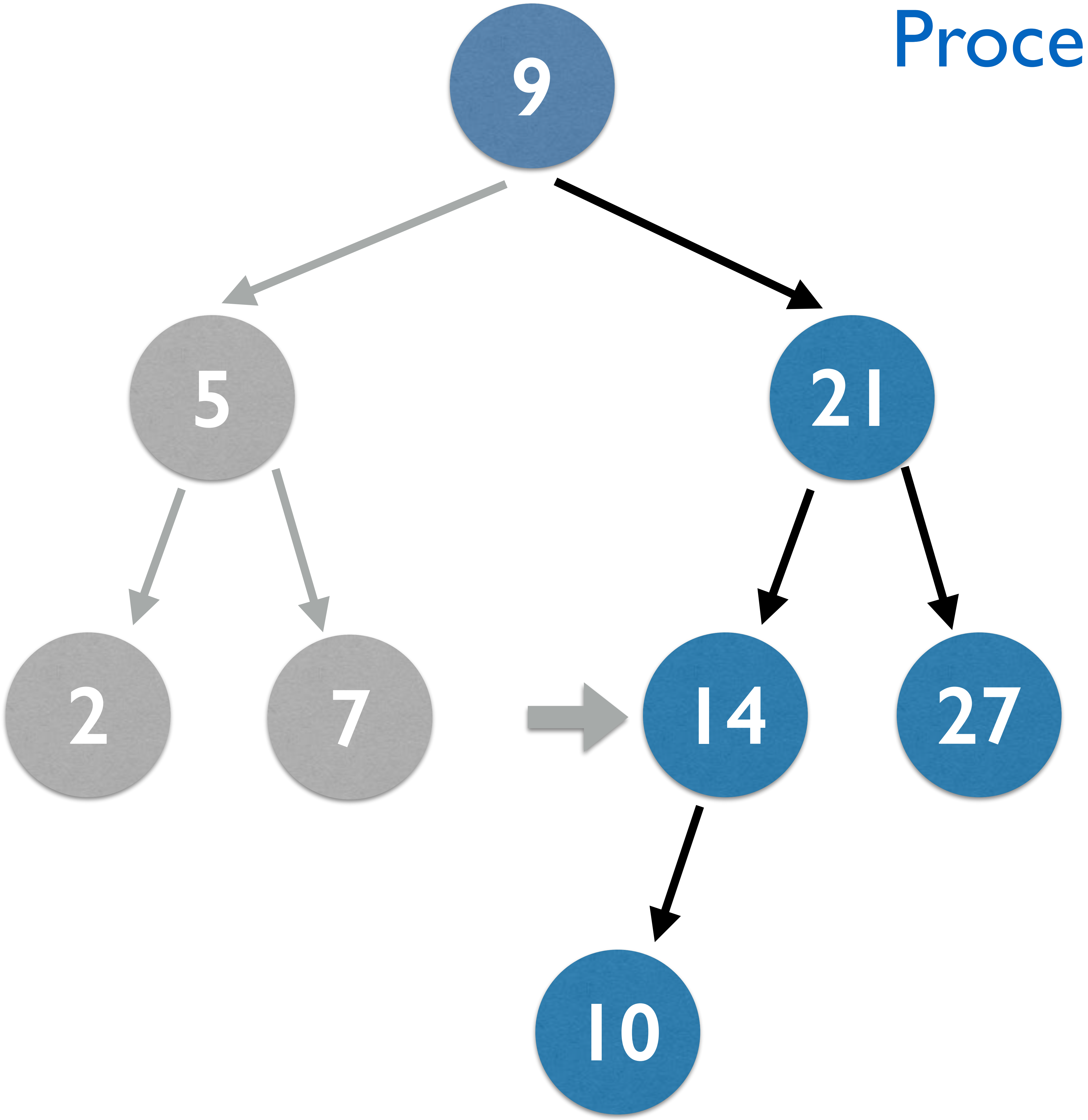
DFS: POST



Process left · Process right · Process root

2, 7, 5

DFS: POST

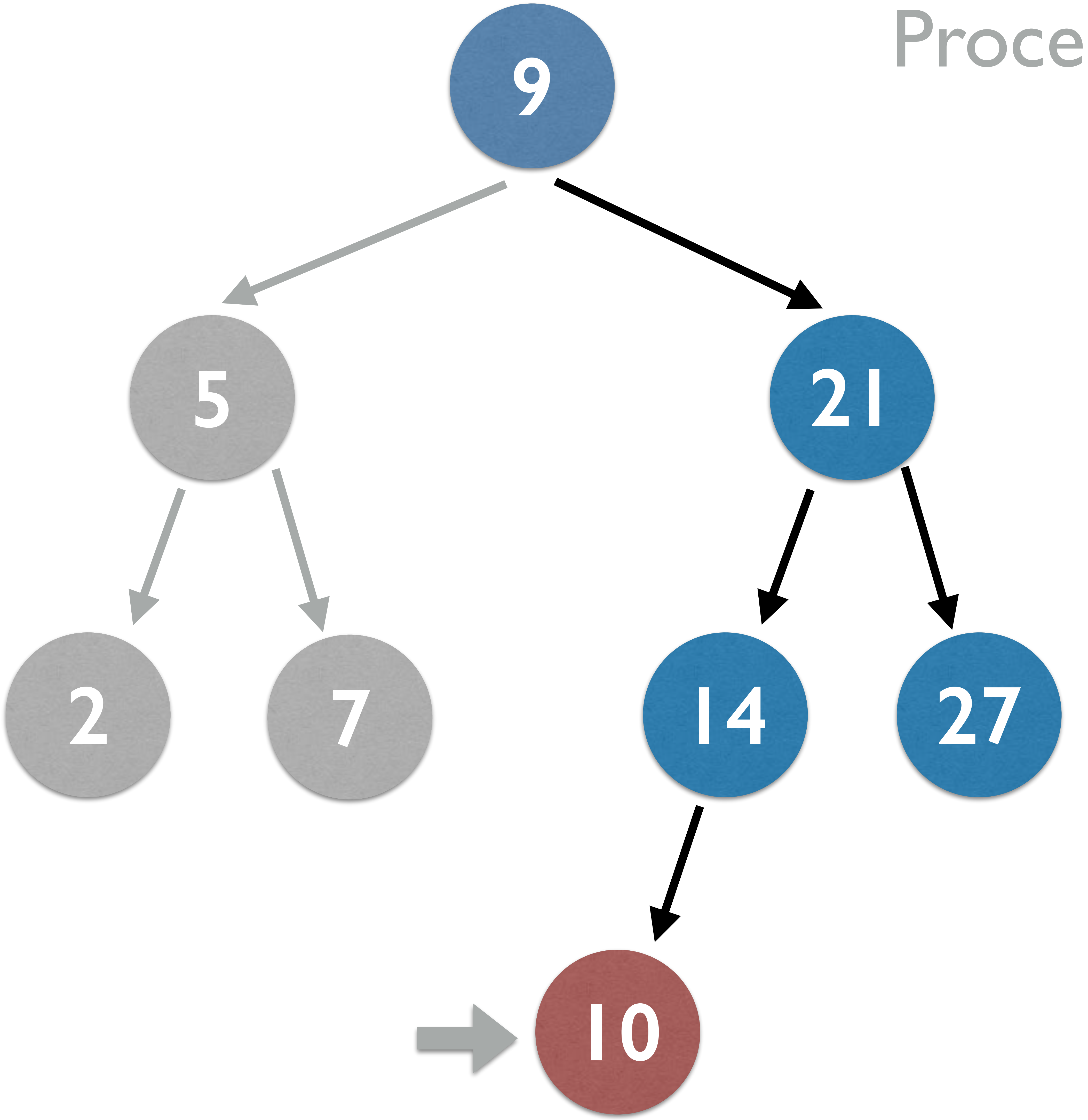


Process left · Process right · Process root

2, 7, 5

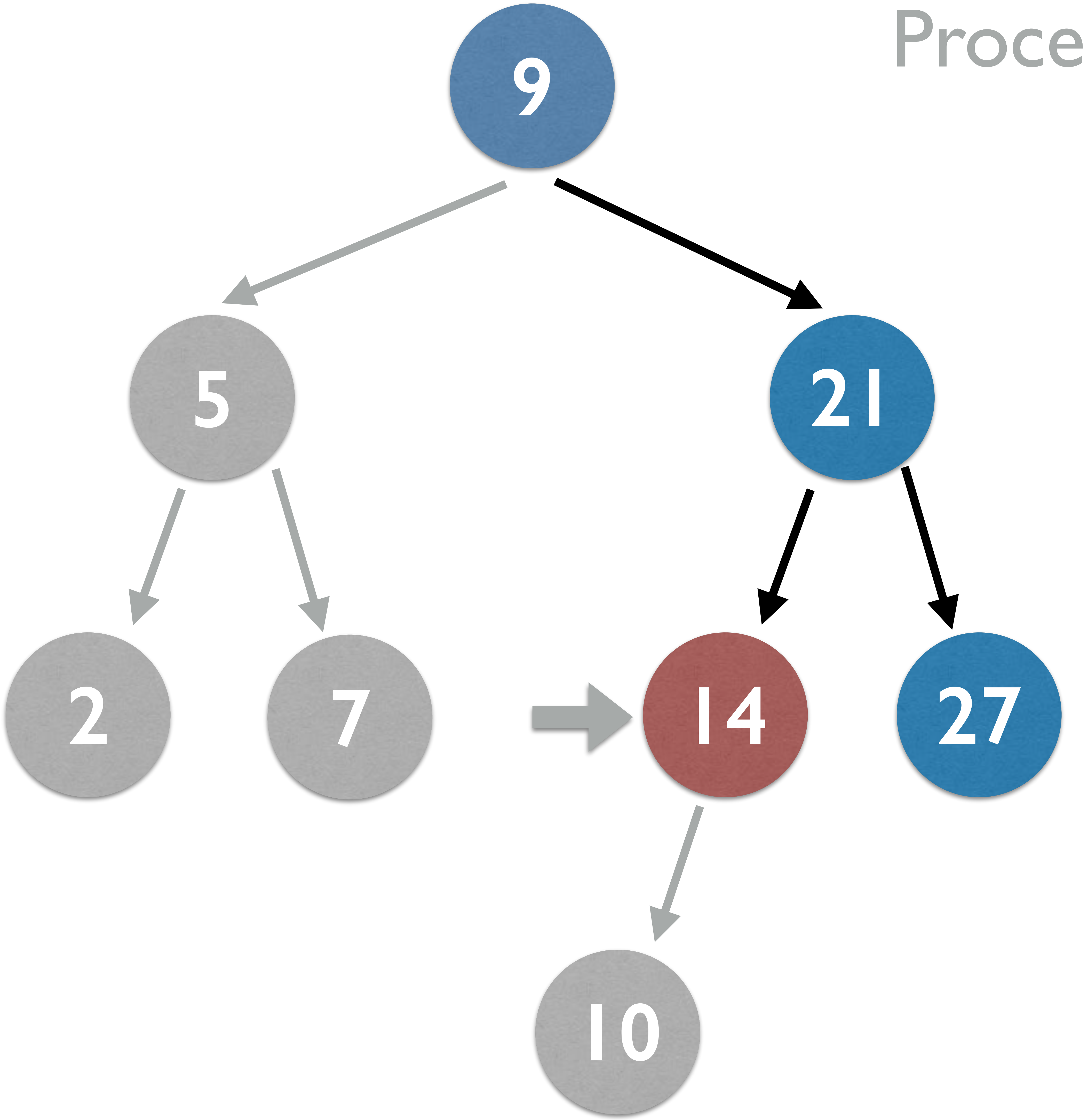
DFS: POST

Process left · Process right · Process root



2, 7, 5, 10

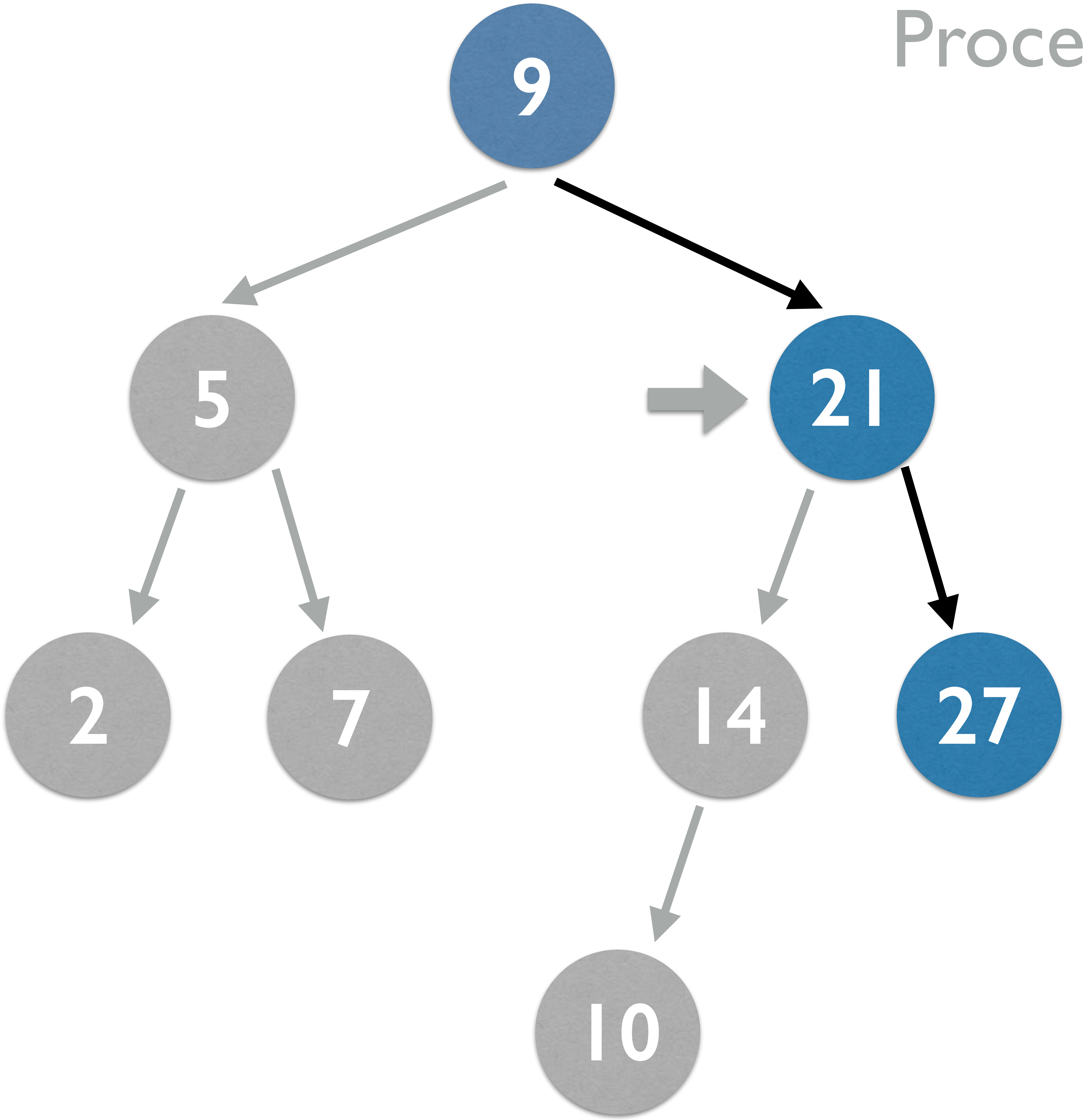
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14

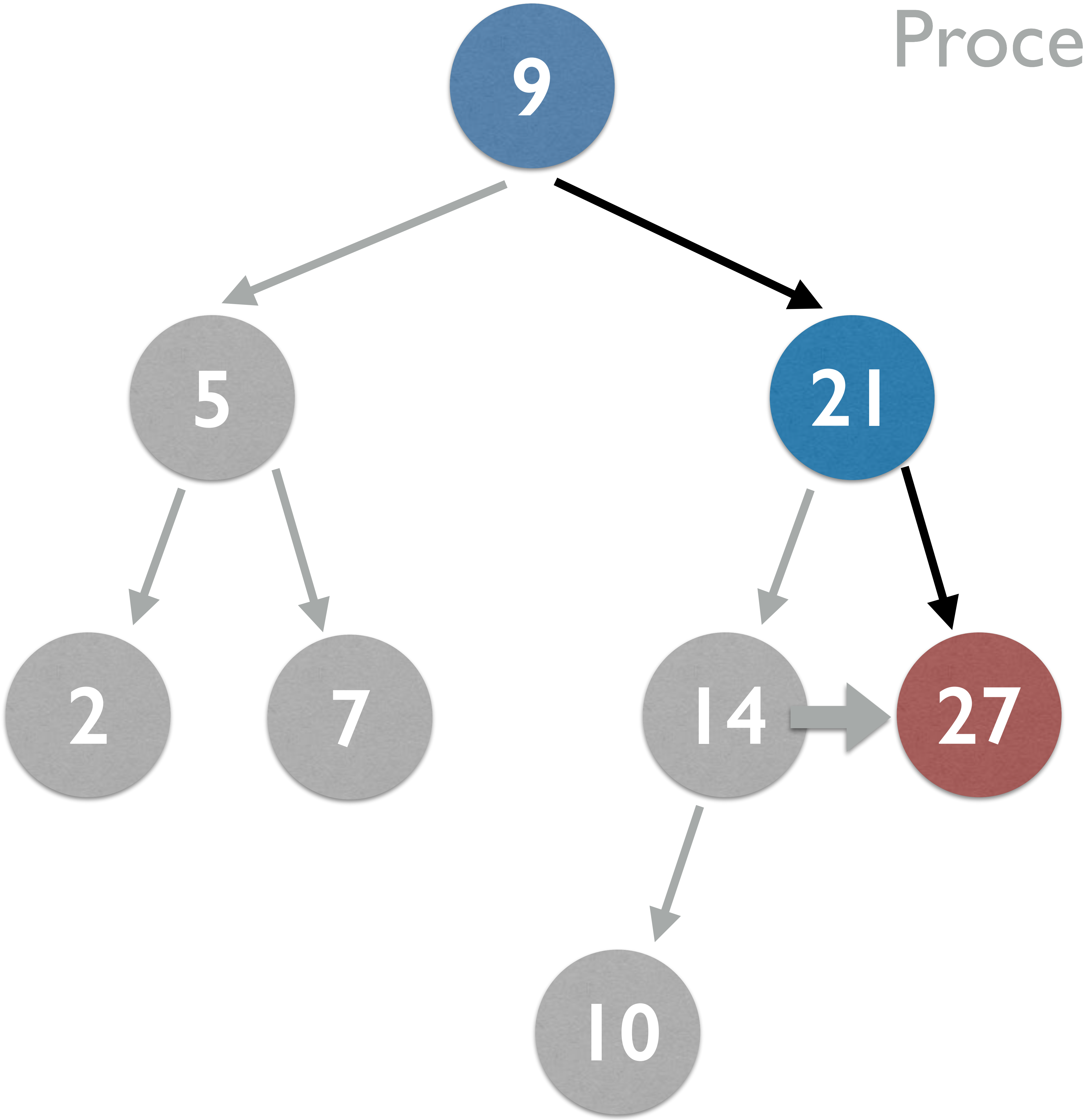
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14

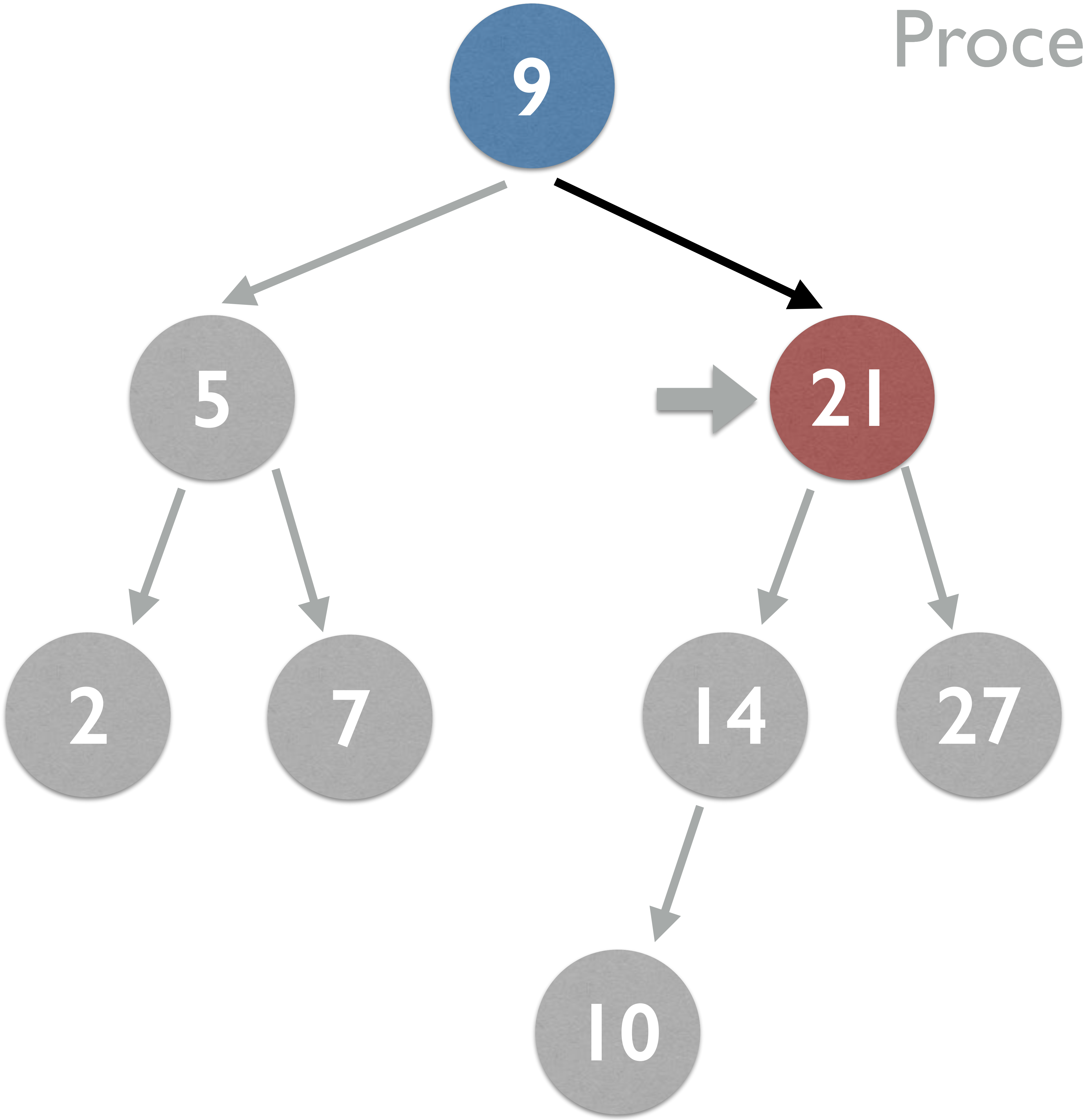
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14, 27

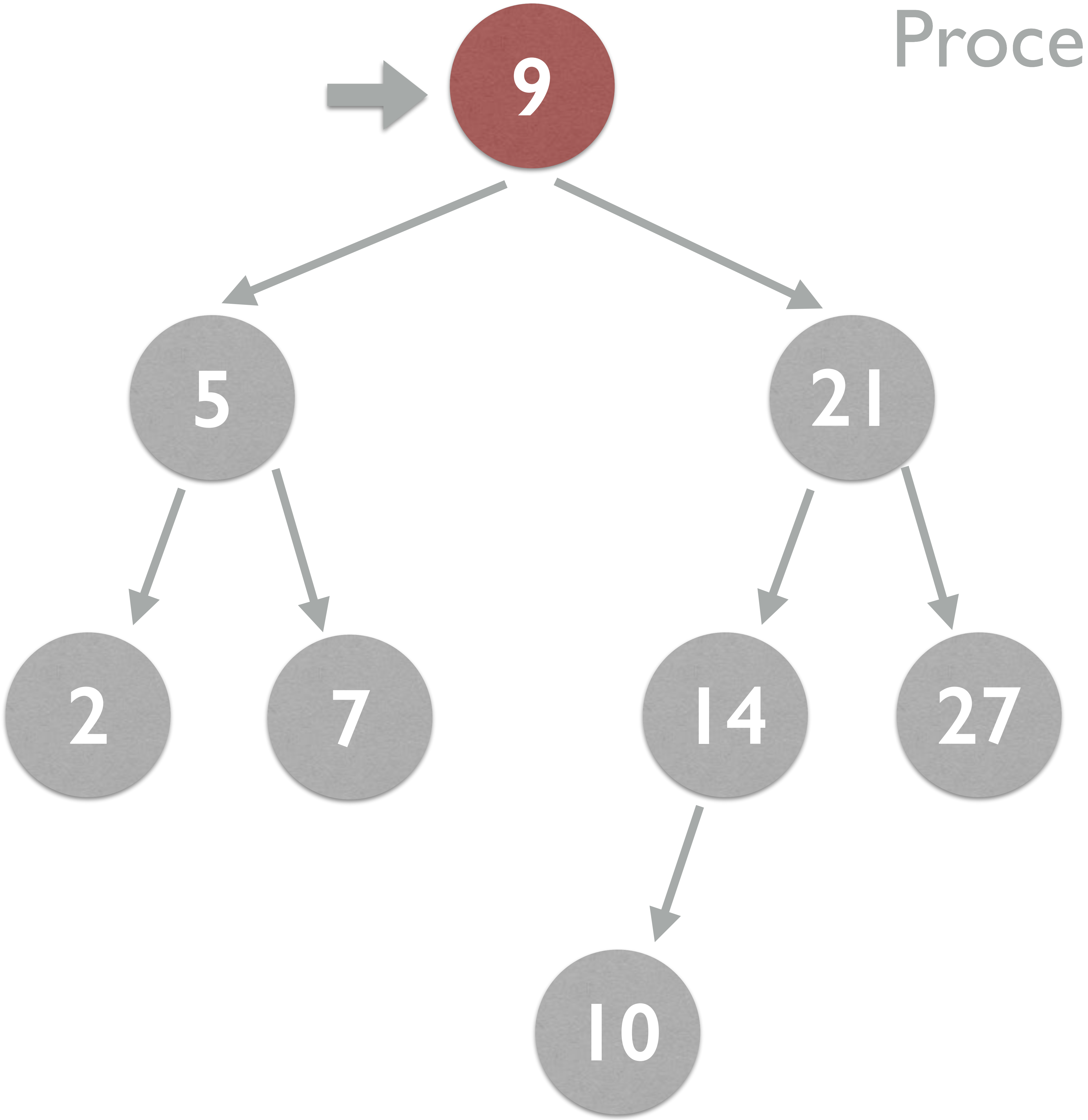
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14, 27, 21

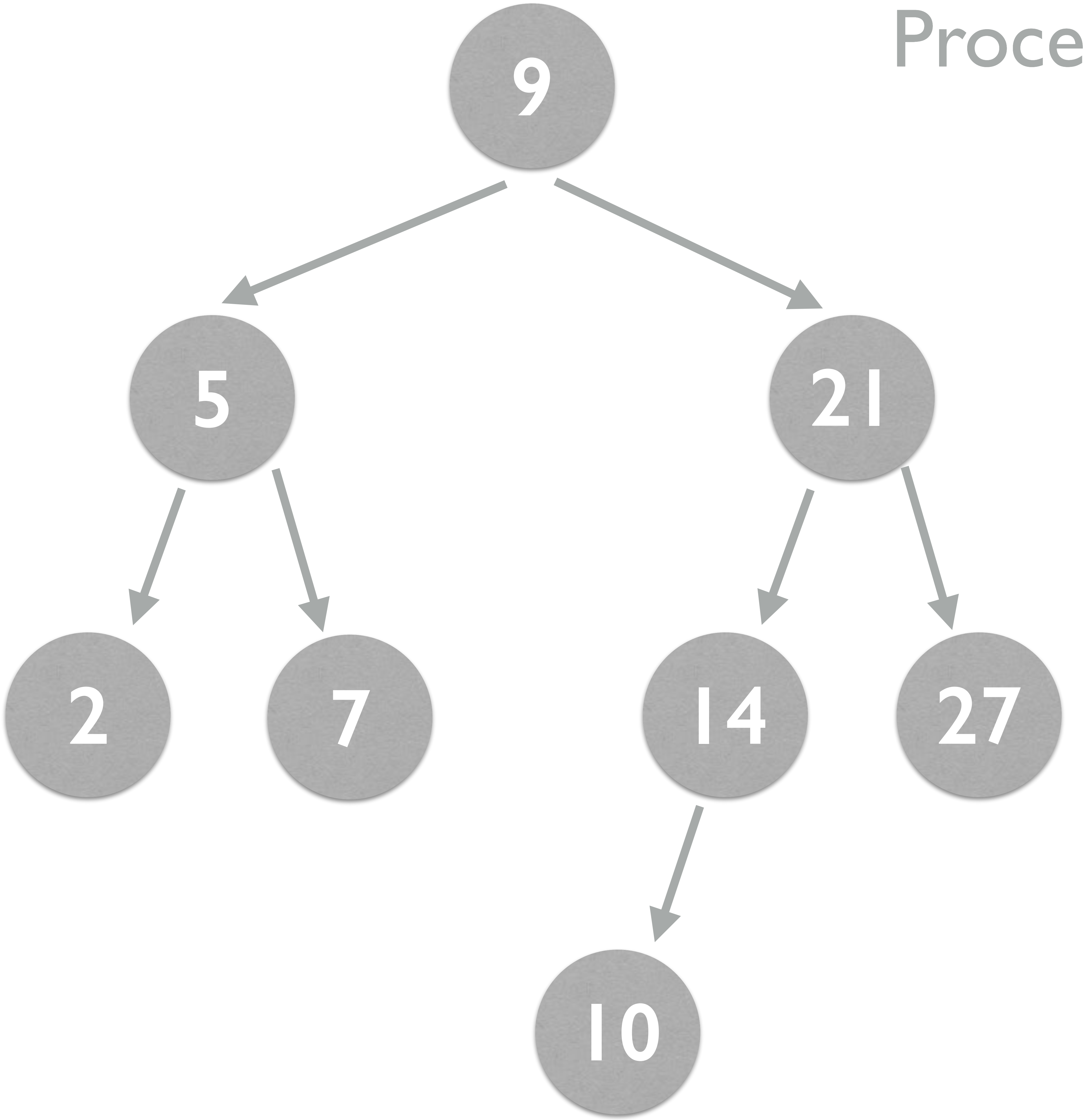
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14, 27, 21, 9

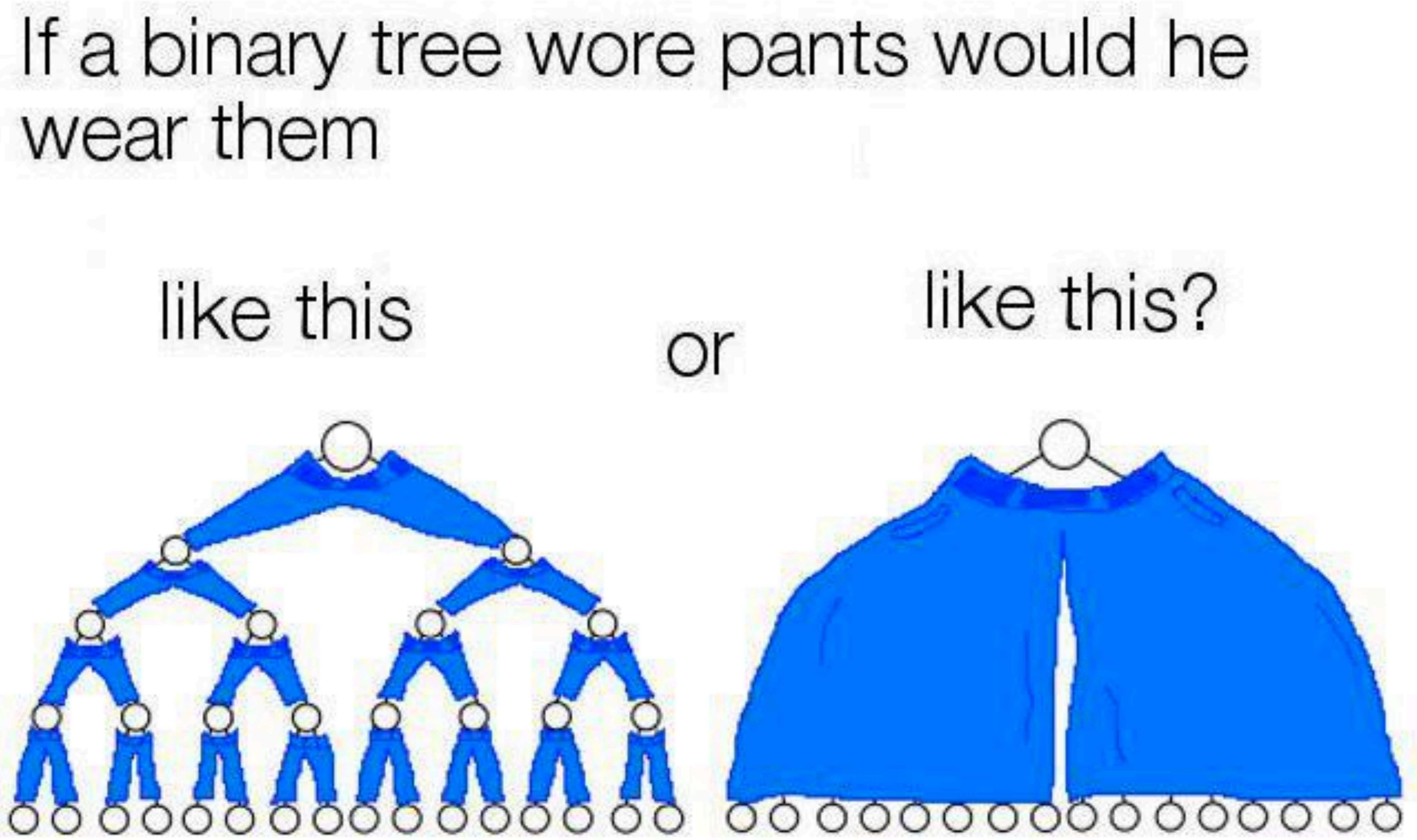
DFS: POST



Process left · Process right · Process root

2, 7, 5, 10, 14, 27, 21, 9

- **Main use case is to safely delete a tree leaf by leaf, in lower-level languages (e.g. C) with no automatic garbage collection. Nodes are only processed once all their descendants have been processed.**


If a binary tree wore pants would he wear them
like this
or
like this?

Recursive Problem Solving

Recursive Problem Solving

- Any problem that can be solved iteratively can be solved recursively, and vice versa
- The catch: this doesn't mean it's equally easy to solve a particular problem both ways
- Binary Search Trees are recursive data structures - it's much easier to write them if you employ recursive problem solving

1. Identify simplest possible input

```
function factorial (n) {

}
```

2. Solve just for the simplest input

```
function factorial (n) {
  // We know that 1! and 0! are 1 – no calculation needed.
  // Therefore, we can simply return 1 in those cases
  if (n === 1 || n === 0) {
    return 1
  }
}
```

3. Solve the problem for the second simplest possible input

```
function factorial (n) {
  if (n === 1 || n === 0) {
    return 1
  } else {
    // we know for sure that we must do two things:
    // a. invoke the func again with the
    //    base case (factorial(1) or factorial(0))
    // b. get our input to factorial by "shrinking" the value of n
    //    (for example, by subtracting 1)

    return 2 * factorial(n - 1)
  }
}
```

4. Generalize in terms of input

```
function factorial (n) {
  if (n === 1 || n === 0) {
    return 1
  } else {
    return n * factorial(n - 1)
  }
}
```


WORKSHOP