

Environmental Noise and Ross-Macdonald Transmission Models

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Introduction

Climate change is expected to lead to shifts in the risk of mosquito-borne disease outbreaks throughout the world. But it is unclear how increases in the variability of temperature and rainfall might impact these outbreaks. The goal of this project is to investigate how environmental and demographic noise affect key measures of the transmission of mosquito-borne diseases.

Question

How does the strength of environmental noise influence the states of a simple Ross-Macdonald model?

Does the strength of environmental noise of different vector epidemiological traits (parameters) affect the critical states?

Models

Simple Ross-Macdonald ODE

$$\frac{dH}{dt} = \frac{b\tau_{HV}}{N_H}V(N_H - H) - \gamma_H H \quad (1)$$

$$\frac{dV}{dt} = \frac{b\tau_{VH}}{N_H}H(N_V - V) - \mu_V V \quad (2)$$

where,

- H is the number of infected hosts
- V is the number of infected vectors
- N_H is the total population/carrying capacity of hosts
- N_V is the total population/carrying capacity of vectors
- b is the per capita mosquito biting rate
- τ_{ij} is the probability of successful transmission from j to i
- $\frac{1}{\gamma_H}$ is the infectious period of the host
- $\frac{1}{\mu_V}$ is the lifespan/infectious period of vectors

The basic reproductive number of this model is:

$$R_0 = \sqrt{\frac{b^2 \tau_{HV} \tau_{VH} N_V}{N_H \mu_V \gamma_H}} \quad (3)$$

For our chosen set of parameters, $R_0 = 1.054$.

Simple SDE

Following O'Regan et al. 2016, we can convert this system of ODEs to SDEs. We can integrate demographic stochasticity with the following set of time varying equations:

$$dH = \left(\frac{b\tau_{HV}}{N_H} V(N_H - H) - \gamma_H H \right) dt + \sqrt{\frac{b\tau_{HV}}{N_H} V(N_H - H) + \gamma_H H} dW_1 \quad (4)$$

$$dV = \left(\frac{b\tau_{VH}}{N_H} H(N_V - V) - \mu_V V \right) dt + \sqrt{\frac{b\tau_{VH}}{N_H} H(N_V - V) + \mu_V V} dW_2 \quad (5)$$

Here, the dW_i are independent standard Wiener processes.

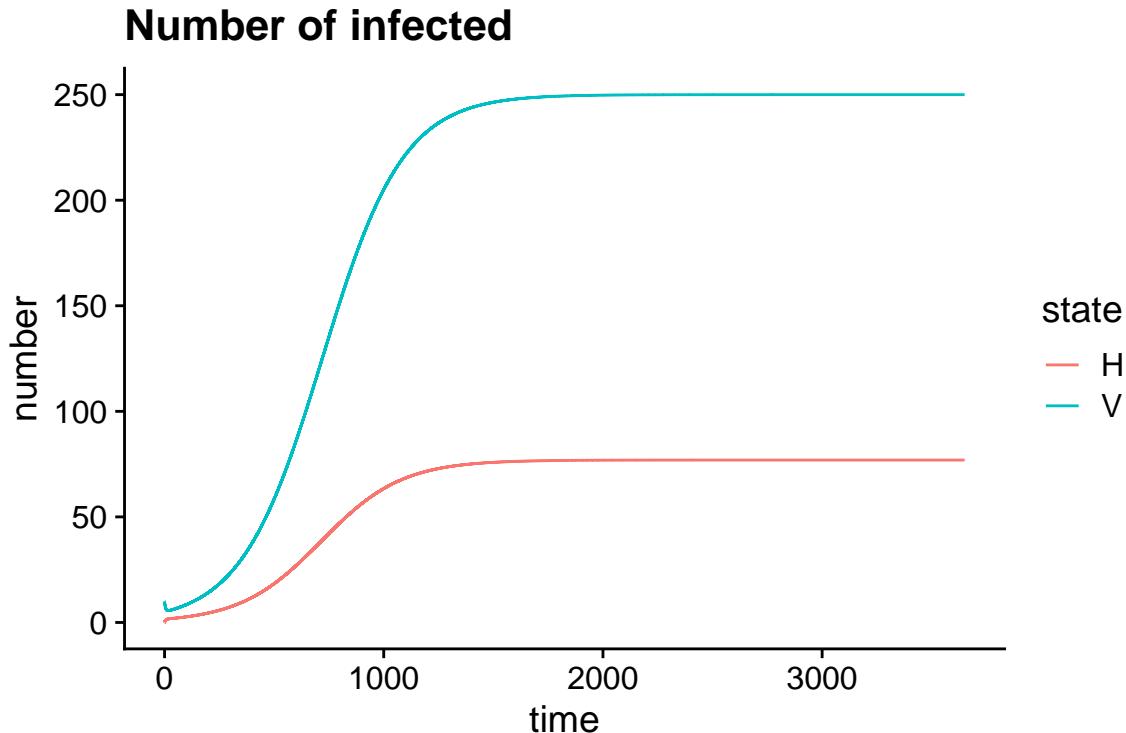


Figure 1: The number infected over time in the ODE model. Both populations are initialized with 10 infected vectors.

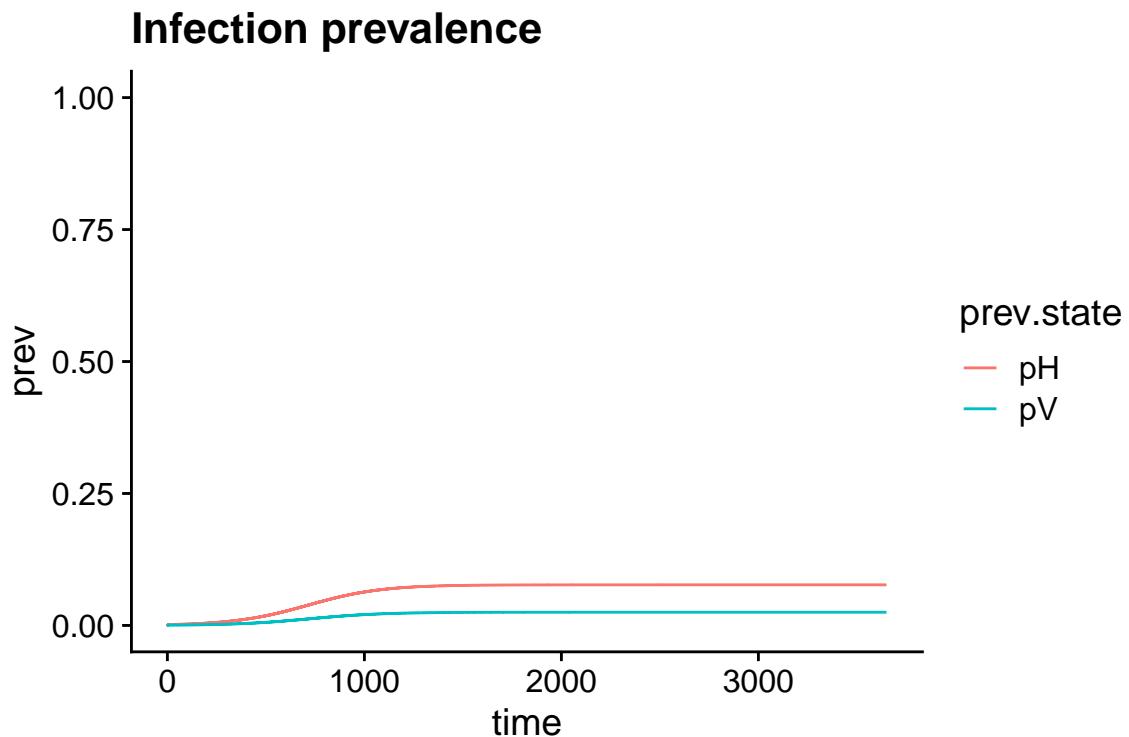


Figure 2: The prevalence of infections over time in the ODE model. Both populations are initialized with 10 infected vectors.

Number of infected (no environmental stochasticity)

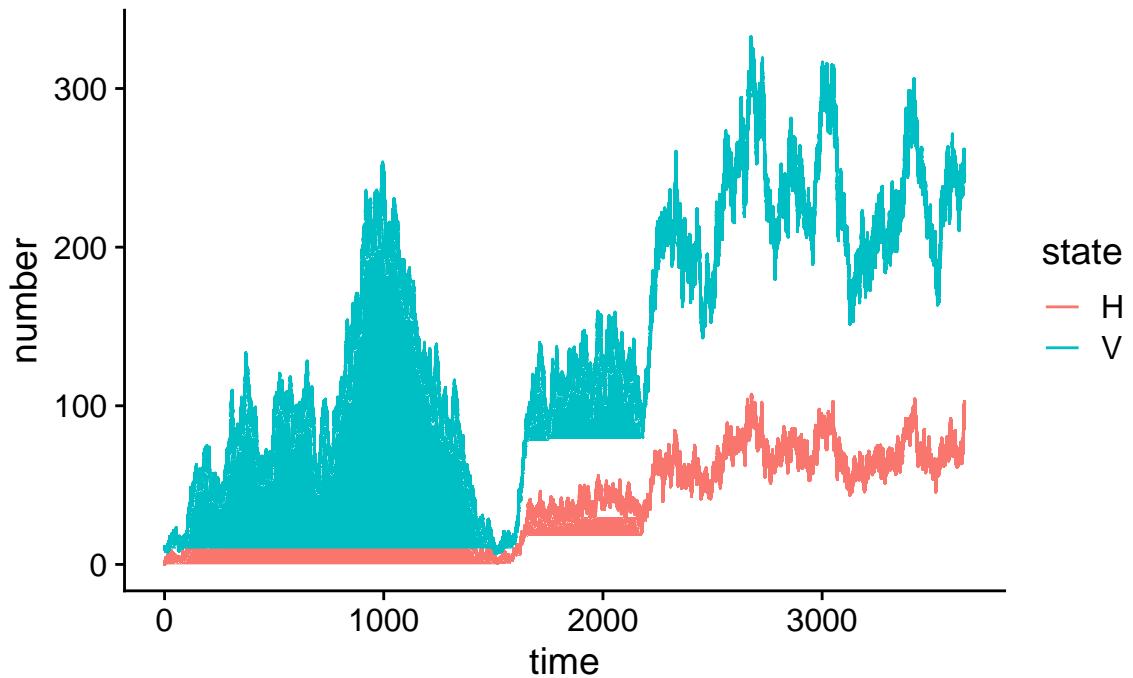


Figure 3: The number of infections over time in the SDE model without demographic stochasticity. Both populations are initialized with 10 infected vectors.

Distribution of the final number of infected vectors

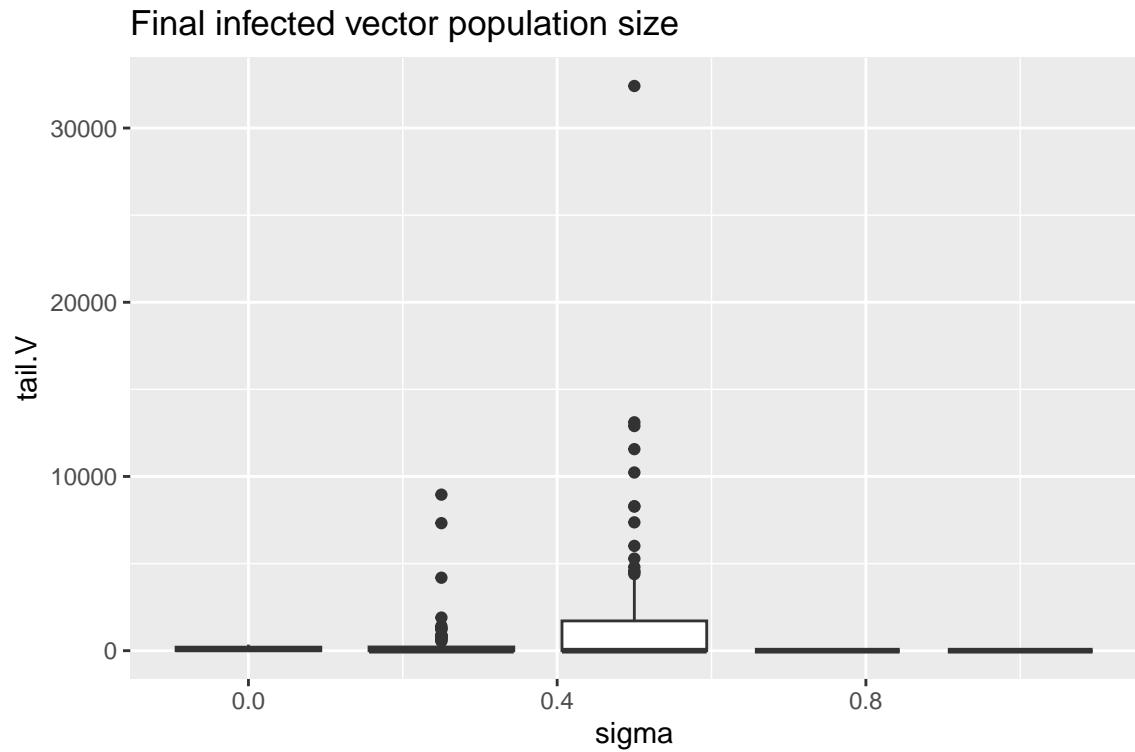


Figure 4: Distribution of the number of infected vectors at the last time point of the SDE model with environmental stochasticity. Both populations are initialized with 10 infected vectors.

Distribution of the final number of infected hosts

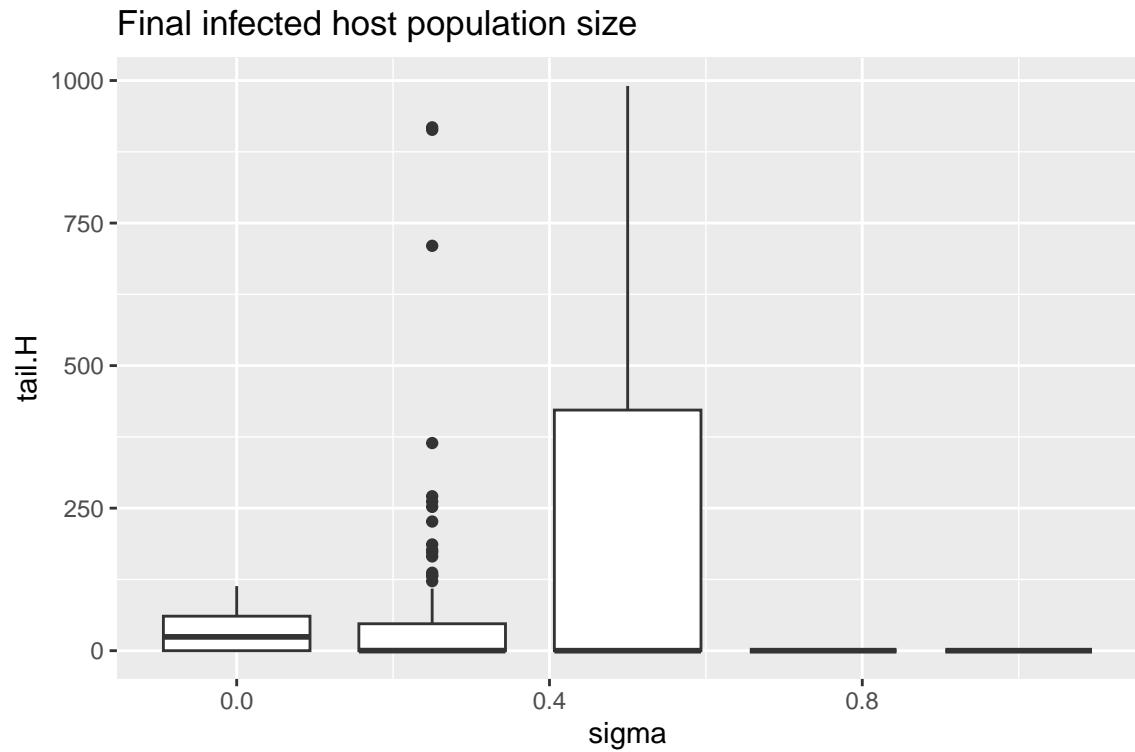


Figure 5: Distribution of the number of infected hosts at the last time point of the SDE model with environmental stochasticity. Both populations are initialized with 10 infected vectors.

Simulation Statistics

Notes: * when $\mathcal{R}_0 = 4.74$, we don't see any changes as σ increases

Stats Test

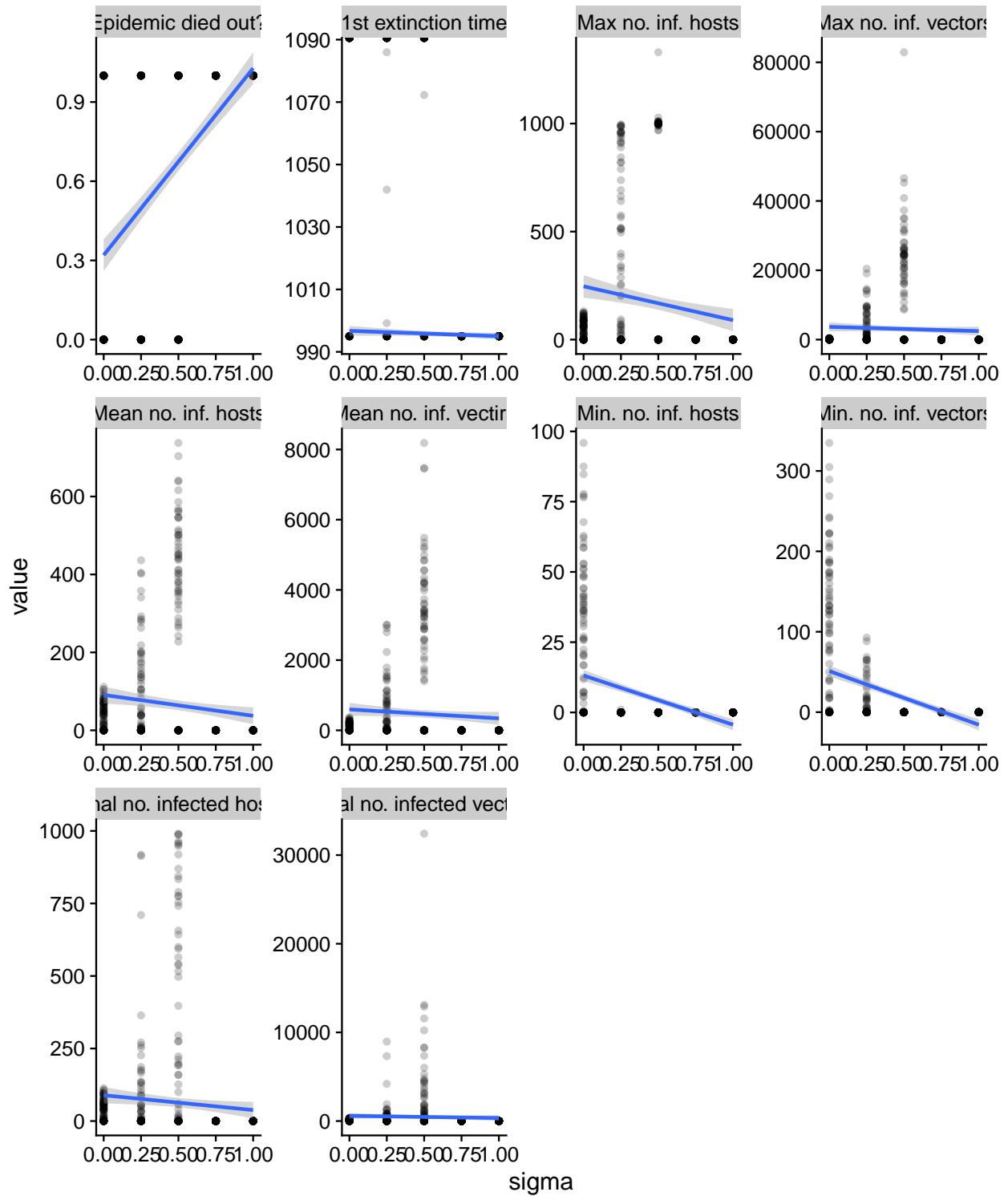


Figure 6: Testing the stats function to make sure it returns correct values.

Stats Test with demographic noise

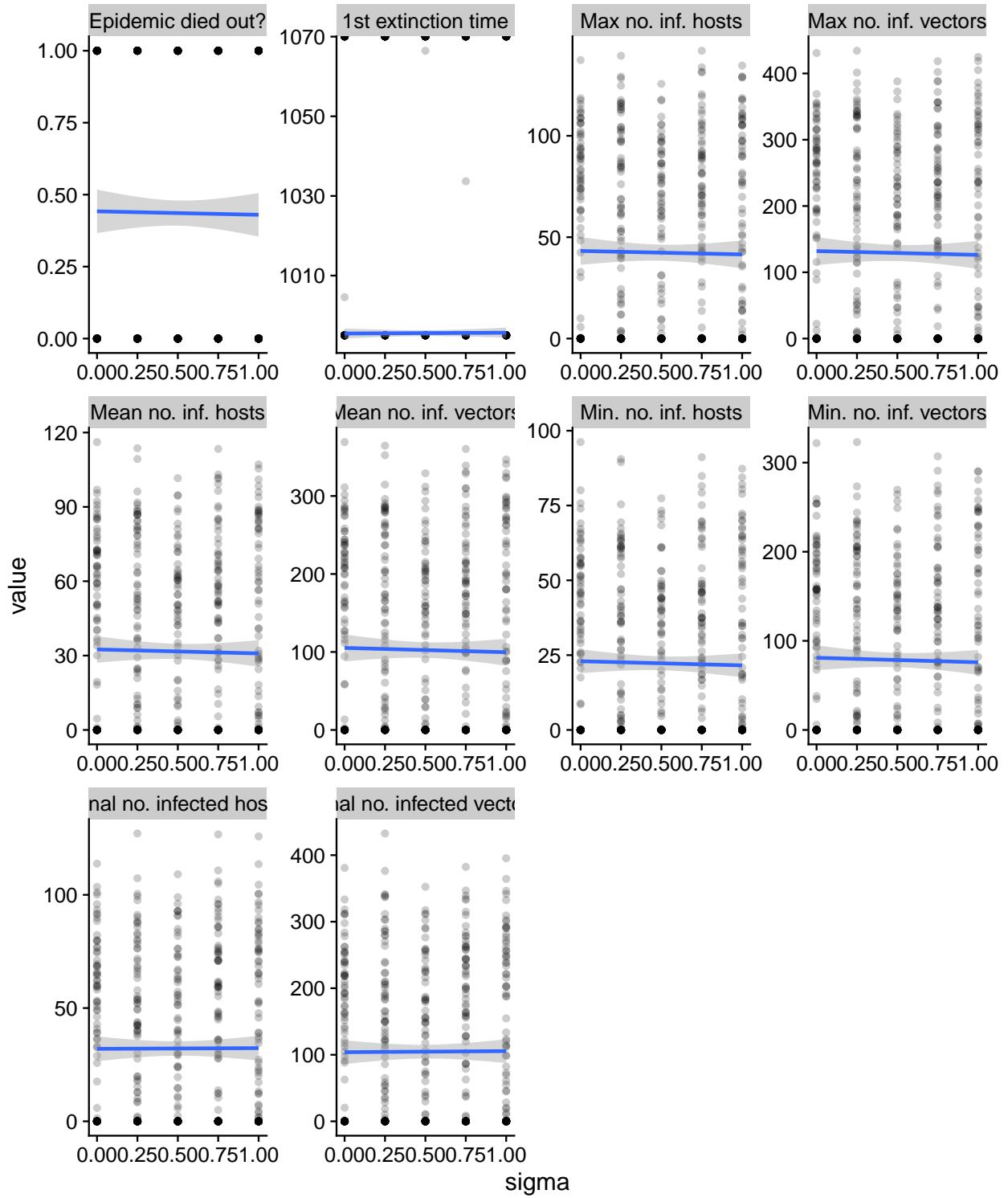


Figure 7: Testing the stats function to make sure it returns correct values.