MEC85 Fall 2020 Notes

Drake Lin

November 11, 2020

Contents

Chapter 2: Vectors

Scalars

Scalars are positive/negative values specified by magnitude (length, time)

Vectors

Vectors have a magnitude + direction (force, position, moment)
3D vectors: $||A|| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
Position vector: $\vec{AB} = (x_B - x_A)i + (y_B - y_A)i + (z_B - z_A)i$
Dot product: $A * B = ||\vec{A}||||\vec{B}||cos\theta = A_xB_x + A_yB_y + A_zB_z$

Chapter 3: Moment

Moment

The tendency of body to rotate about point (torque)

$$\vec{M_o} = \vec{r}x\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

moment of a force $\vec{F}$ about position O

$\vec{r}$ = position vector directed from O to any point on line of action of F

Sum of many moments = $M_{Ro} = \sum \vec{r}x\vec{F}$

Moment of force about an axis = $\vec{M_a} = \vec{u_a} * rxF = M_a * \vec{u_a} = \begin{vmatrix} \vec{u_{ax}} & \vec{u_{ay}} & \vec{u_{az}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

Cross Product

$$\vec{C} = \vec{A}x\vec{B} = (ABsin\theta)\vec{u}_c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$c = ABsin\theta$ (magnitude)

$\vec{u}_c$ = direction = right hand rule

Chapter 4: Rigid Body

Equilibrium of a rigid body: $\vec{F_x} = 0, \vec{M_R} = 0$
Two force members: Forces applied only at 2 points of member. For the member to be in equilibrium, the forces must be the same magnitude, in the opposite direction of each other, but be in the same line of action.

Equilibrium in 2D

Free Body Diagram: understand all the known/unknown forces + moments on the body
Support Reaction: prevents translation of body by exerting force, prevents rotation by providing moment
Common support reactions:

- roller:
- pin:
- fixed:

Other forces:

- Springs: $F = kx, x = \Delta l$
- Friction: $F_s = u_s N$

Equations of Equilibrium: $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases} \quad \begin{cases} \sum M = 0 \end{cases}$

Equilibrium in 3D

Common support reactions:

- roller:
- ball and socket:
- fixed:

Equations of Equilibrium: $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$

Chapter 5: Structural Analysis

Designing structures. Goal = maximize bending stiffness: change material or structure
Bending stiffness = $R = \rho/\delta$

Trusses

Trusses are structures designed to suport loads such as roofs or buildings. They are slender members and joined at endpoints with each other. Its weight is minimized to maximize its strength to weight ratio. When analyzing trusses, we make 4 key assumptions:

- all loadings applied at joints
- weight of bar ignored
- joined together by smooth points
- consists of two force members

For truss analysis:

1. determine support reactions
2. determine zero force members (members that carry zero force)
3. determine forces supported by individual members of truss

Method of Joints

With the method of joints, we evaluate individual joints or pin connections. We assume tension in members and solve by summing forces in the x-y direction.

Method of Sections

With the method of sections, we evaluate a section including multiple joints by cutting through structural members. This section must be in equilibrium so $\sum F = 0$ and $\sum M = 0$. We cut a section to reduce the number of unknown variables.

Chapter 6: Center and Moment

The centroid of an area is the geometric center, denoted as $(\tilde{x}, \tilde{y})$

Center of Mass

$\overline{x} = \frac{\int_A \tilde{x} dA}{\int_A dA}, \overline{y} = \frac{\int_A \tilde{y} dA}{\int_A dA}$

Case 1: Vertical	Case 2: Horizontal
$\tilde{x} = x$	$\tilde{x} = \frac{x}{2}$
$\tilde{y} = \frac{y}{2}$	$\tilde{y} = y$
$dA = ydx$	$dA = xdy$

Composite bodies: connected, simpler parts can be added together with the following formula to find the center of mass of a more complex shape. $\overline{x} = \frac{\sum \tilde{x} A}{\sum A}, \overline{y} = \frac{\sum \tilde{y} A}{\sum A}$

Moment of Inertia

Moment of inertia is the quantity that determines the torque needed for a desired angular acceleration about a rotational axis (just like how mass relates force and acceleration)
Equations:

- $I_x = \int_A y^2 dA$
- $I_y = \int_A x^2 dA$
- $J_o = \int_A r^2 dA = I_x + I_y$ (polar moment of inertia)

The Parallel Axis Theorem lets us solve for the moment of inertia, then shift it to some point.

- $I_x = \tilde{I}_x + Ady^2$
- $I_y = \tilde{I}_y + Adx^2$
- $J_o = \tilde{J}_c + Ad^2$

Chapter 7: Stress and Strain

Normal Force (N): force $\perp$ to area

Shear Force (v): force that lies in plane of area

Bending Moment (m): bend of body across axis, moment caused by external loads

Factor of safety = $\frac{F_{fail}}{F_{allowable}}$

Deformation: When force is applied to a body, it tends to change the body's shape and size

Stress

$\sigma = \frac{N}{A}$; Average Force = Internal resultant normal force / Area

$T_{avg} = \frac{V}{A}$; Shear force of section = Shear force / Area of section

Strain

$\sigma = \frac{L - L_o}{L_o} = \frac{\delta}{L_o}$; strain = change of length / initial length

$\gamma = \frac{\pi}{2} - \theta$; Shear strain of section

Small strain analysis

Small strain analysis is used when the $\Delta\theta$ is small

- $sin\Delta\theta \approx \Delta\theta$
- $cos\Delta\theta \approx 1$
- $tan\Delta\theta \approx \Delta\theta$

Chapter 8: Mechanical Loads