

## Week 3

August 18, 2021

## Matrices - overview

- Rectangular array of numbers written between square brackets.
- 2D array.
- Named as capital letters (A,B,X,Y).
- It allows you to organize, index, and access a large amount of data.
- Dimension of a matrix are [Rows x Columns].
- $A_{(i,j)}$  = entry in  $i^{th}$  row and  $j^{th}$  column.

The diagram shows a matrix  $A$  with 4 rows and 2 columns. The elements are arranged as follows:

1402	191
1371	821
949	1437
147	1448

Handwritten annotations include:

- A red arrow pointing to the first column and a blue arrow pointing to the first row, with the element 1402 circled in blue.
- A red arrow pointing to the second column and a pink arrow pointing to the first row, with the element 191 circled in red.
- A pink arrow pointing to the third row and a pink arrow pointing to the second column, with the element 1437 circled in pink.
- A blue arrow pointing to the fourth row and a blue arrow pointing to the first column, with the element 147 circled in blue.

Handwritten equations to the right of the matrix show the values of specific elements:

- $A_{11} = 1402$  (blue)
- $A_{12} = 191$  (red)
- $A_{32} = 1437$  (pink)
- $A_{41} = 147$  (blue)

## Vectors - overview

- n by 1 matrix.
- Usually referred to as a lower case letter
- $v_i$  is an  $i^{th}$  element.
- It allows you to organize, index, and access a large amount of data.
- Dimension of a matrix are [Rows x Columns].
- $A_{(i,j)}$  = entry in  $i^{th}$  row and  $j^{th}$  column.

$$y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

## Matrix manipulation

### Addition

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}$$

### Multiplication by scalar

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}$$

### Multiplication by vector

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 \\ 5 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

$1 \times 1 + 3 \times 5 = 16$   
 $4 \times 1 + 0 \times 5 = 4$   
 $2 \times 1 + 1 \times 5 = 7$

## Multiplication by matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$$

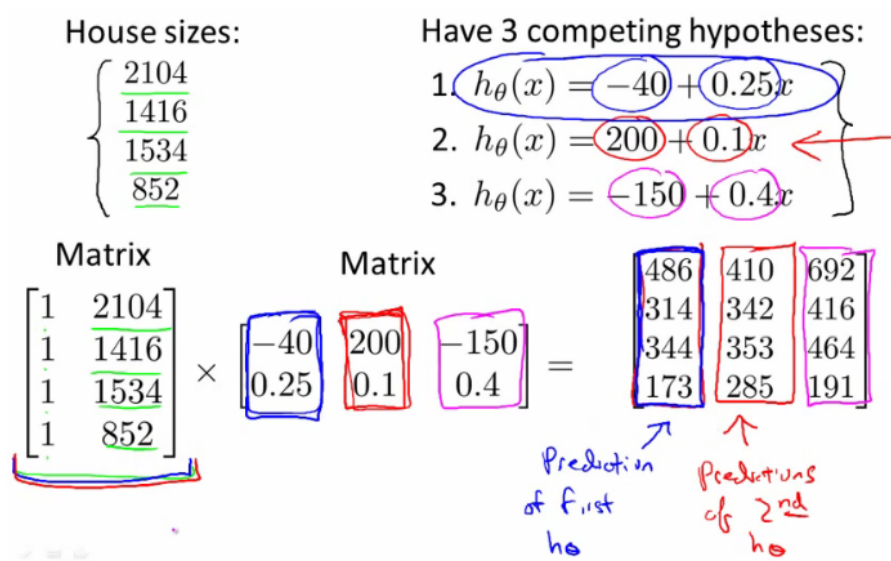
- Take matrix A and multiply by the first column vector from B.
- Take the matrix A and multiply by the second column vector from B.

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

## Implementation/use

- House prices. However, we now have three hypotheses and the same data set.
- We can use matrix-matrix multiplication to efficiently apply all three hypotheses to all data.
- For example, suppose we have four houses and we wish to guess their prices. There are three opposing hypotheses. Because our hypothesis is only one variable, we convert our data (home sizes) vector into a 4x2 matrix by adding an extra column of 1s.



## Matrix multiplication properties

### Lack of Commutativity

When working with raw numbers/scalars multiplication is commutative:

$$3 \cdot 5 == 5 \cdot 3$$

This is not always true for matrix:

$$A \times B \neq B \times A$$

### Associativity

When working with raw numbers/scalars multiplication is associative:

$$3 \cdot 5 \cdot 2 == 3 \cdot 10 == 15 \cdot 2$$

This also holds true for matrix:

$$A \times (B \times C) == (A \times B) \times C$$

## Identity matrix

When working with raw numbers/scalars 1 is always the identity element:

$$z \cdot 1 = z$$

In matrices we have an identity matrix called I:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If multiplication between matrices A and I is possible:

$$A \times I = A$$

## Matrix inverse

When working with raw numbers/scalars multiplication we can usually take their inverse:

$$x \cdot \frac{1}{x} = 1$$

In the space of real numbers not everything has an inverse. E.g. 0 does not have an inverse!

The only matrices that have an inverse are square matrices.

$$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_A \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A^{-1}A} = I_{2 \times 2}$$

There are numerical methods used for finding the inverses of matrices.

## Matrix transpose

If A is an  $m \times n$  matrix B is a transpose of A. Then B is an  $n \times m$  matrix  $A_{(i,j)} = B_{(j,i)}$ .

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix} \quad \underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$