Week 3

July 10, 2021

Matrices - overview

- Rectangular array of numbers written between square brackets.
- 2D array.
- Named as capital letters (A,B,X,Y).
- It allows you to organize, index, and access a large amount of data.
- Dimension of a matrix are [Rows x Columns].
- $A_{(i,j)}$ = entry in i^{th} row and j^{th} column.

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

Vectors - overview

- n by 1 matrix.
- Usually referred to as a lower case letter
- v_i is an i^{th} element.
- It allows you to organize, index, and access a large amount of data.
- Dimension of a matrix are [Rows x Columns].
- $A_{(i,j)}$ = entry in i^{th} row and j^{th} column.

$$y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Matrix manipulation

Addition

$$\begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0.5 \\ 2 & 5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0.5 \\ 4 & 10 \\ 3 & 2 \end{bmatrix}$$

Multiplication by scalar

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}$$

Multiplication by vector

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}_{2\times 1} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

$$1\times 1 + 3\times 5 = 16$$

$$4\times 1 + 0\times 5 = 4$$

$$2\times 1 + 1\times 5 = 7$$

Multiplication by matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}$$

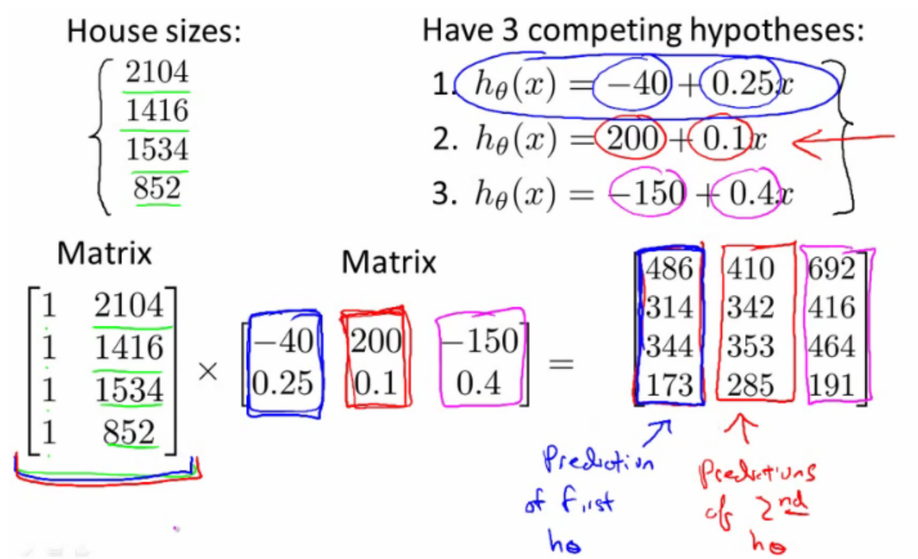
- Take matrix A and multiply by the first column vector from B.
- Take the matrix A and multiply by the second column vector from B.

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

Implementation/use

- House prices. However, we now have three hypotheses and the same data set.
- We can use matrix-matrix multiplication to efficiently apply all three hypotheses to all data.
- For example, suppose we have four houses and we wish to guess their prices. There are three opposing hypotheses. Because our hypothesis is only one variable, we convert our data (home sizes) vector into a 4x2 matrix by adding an extra column of 1s.



Matrix multiplication properties

Lack of Commutativity

When working with raw numbers/scalars multiplication is commutative:

$$3 \cdot 5 == 5 \cdot 3$$

This is not always true for matrix:

$$A \times B \neq B \times A$$

Associativity

When working with raw numbers/scalars multiplication is associative:

$$3 \cdot 5 \cdot 2 == 3 \cdot 10 == 15 \cdot 2$$

This also holds true for matrix:

$$A \times (B \times C) == (A \times B) \times C$$

Identity matrix

When working with raw numbers/scalars 1 is always the identity element:

$$z \cdot 1 = z$$

In matrices we have an identity matrix called I:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If multiplication between matrices A and I is possible:

$$A \times I = A$$

Matrix inverse

When working with raw numbers/scalars multiplication we can usually take their inverse:

$$x \cdot \frac{1}{x} = 1$$

In the space of real numbers not everything has an inverse. E.g. 0 does not have an inverse!

The only matrices that have an inverse are square matrices.

$$\underbrace{\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}}_{\substack{2\times 2 \\ A}} \underbrace{\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A^{-1}A} = I_{2\times 2}$$

There are numerical methods used for finding the inverses of matrices.

Matrix transpose

If A is an $m \times n$ matrix B is a transpose of A. Then B is an $n \times m$ matrix $A_{(i,j)} = B_{(j,i)}$.

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix} \quad \underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$