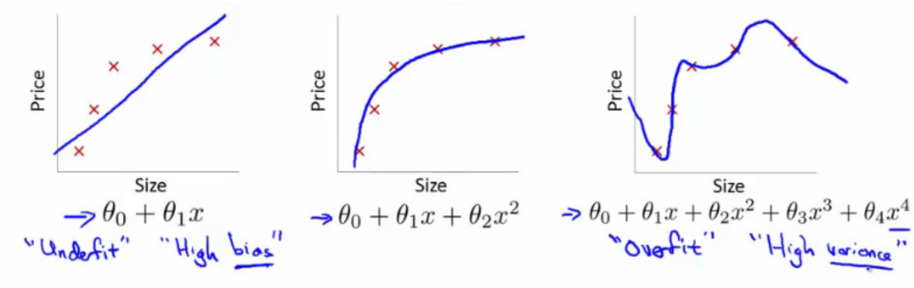
Week 7

July 15, 2021

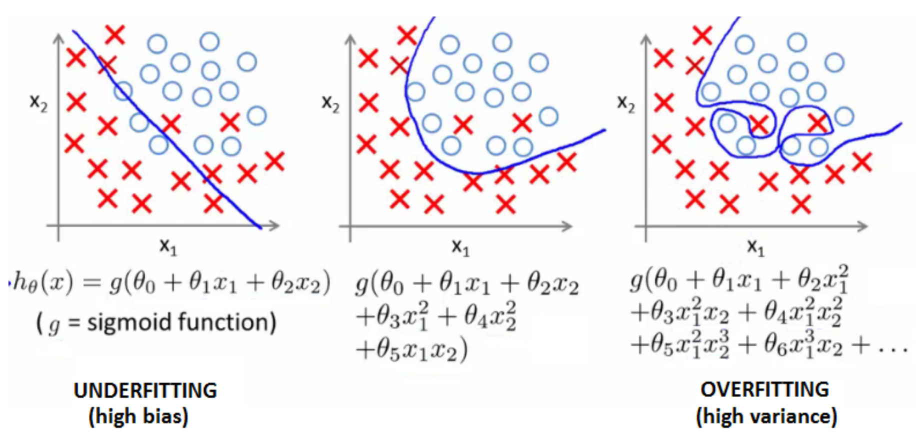
Overfitting with linear regression

- We will use our house pricing example yet another time.
- If we fit a linear function to the data, we have underfitting - also known as high bias.
- If we fit a quadratic function, it will fit better than a straight line, but it will not pass through all of the points.
- If we fit 4th order polynomial, the curve fit's through all five examples. This is overfitting - also known as high variance.



Overfitting with linear regression

- The same thing may happen with logistic regression.
- Sigmoidal function is an underfit.
- A high order polynomial, on the other hand, results in overfitting (high variance hypothesis).

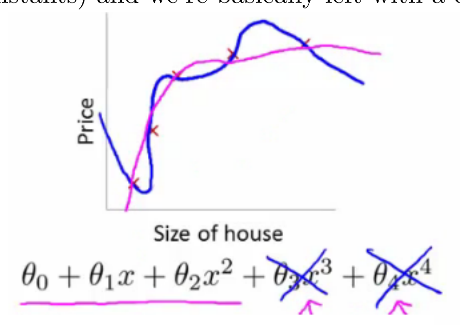


Cost function optimization for regularization

Penalize and make some of the θ parameters really small.

$$min \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)} + y^{(i)})^2 + 1000\theta_3^2 + 1000\theta_4^2$$

So we simply end up with θ_3 and θ_4 being near to zero (because of the huge constants) and we're basically left with a quadratic function.



Regularization
Small parameter values correlate to a simpler hypothesis (you effectively get rid of some of the terms). Overfitting is less likely with a simpler hypothesis.

$$J(\theta) = \frac{1}{2m} [\sum_{i=1}^{m} (h_\theta(x^{(i)} + y^{(i)})^2 + \lambda \sum_{j=1}^{m} \theta_j^2]$$

λ is the regularization parameter that controls a trade off between our two goals:

- Want to fit the training set well.
- Want to keep parameters small.

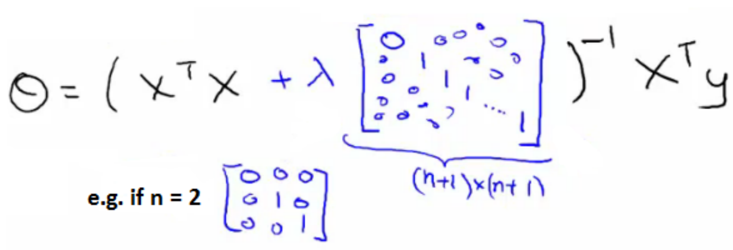
Later in the course, we'll look at various automated methods for selecting λ.

Regularized linear regression

Algorithm 1 Regularized linear regression

while not converged **do**
$\theta_j := \theta_j - \alpha[\frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)} + y^{(i)}) x_j^{(i)} + \frac{\lambda}{m} \theta_j]$
$(for\ j = 1, 2, 3, ..., n)$

Regularization with the normal equation



Regularized logistic regression

Logistic regression cost function is as follows:

$$J(\theta) = -[\frac{1}{m}\sum_{i=1}^{m} y^{(i)} log(h_\theta(x^{(i)} + (1-y^{(i)}) log(1-h_\theta(x^{(i)}))] + \frac{\lambda}{2m}\sum_{j=1}^{m}\theta_j^2$$

Algorithm 2 Regularized logistic regression

while not converged **do**

$\quad \theta_j := \theta_j - \alpha[\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)} + y^{(i)})x_j^{(i)} + \frac{\lambda}{m}\theta_j]$

$\quad (for\ j = 1, 2, 3, ..., n)$

It appears to be the same as linear regression, except that the hypothesis is different.