Task 1, list 10

Dawid Żywczak, 18.05.2020

We have to find estimator of parameter p in geometric distribution using MLE method.

Let $X_1, X_2, X_3, \ldots, X_n$ be random samples from geometric distribution, then our likelihood function is given by $L(p) = (1-p)^{(x_1-1)}p(1-p)^{(x_2-1)}p \ldots (1-p)^{(x_n-1)}$. Logarithm of this function is $lnL(p) = nlnp + (\sum_{i=1}^{n}(x_i-1))ln(1-p)$. Now we have to find maximum of this function so we take the derivative and equate it to 0.

$$\frac{dlnL(p)}{dp} = \frac{n}{p} - \frac{\sum_{i=1}^{n}(x_i-1)}{(1-p)} = 0$$

$$\frac{1-p}{p} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\frac{1}{p} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$p = \frac{1}{\bar{x}_n}$$

Let's also check the second derivative to be sure it's maximum.

$$\frac{d^2 lnL(p)}{dp^2} = -\frac{n}{p^2} - \frac{\sum_{i=1}^{n}(x_i-1)}{(1-p)^2}$$

Which is negative for all p, including our calculated above, so now we can be sure it's maximum of function $lnL(p)$.