Exercise number 2

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Is it possible to find C such that function

$$f_{XY}(x,y) = Cxy + x + y,\ where\ 0 \leq x \leq 3, 1 \leq y \leq 2,$$

would be density of 2-dimensional random variable?

Let's find out! We have to check equation below:

$$\int_0^3 \int_1^2 Cxy + x + y\, dydx = 1$$

$$\int_0^3 \int_1^2 Cxy + x + y\, dxdy = \int_0^3 (\int_1^2 Cxy\, dy + \int_1^2 x\, dy + \int_1^2 y\, dy)dx =$$

$$\int_0^3 (Cx\frac{3}{2} + x + \frac{3}{2})dx = C\frac{3}{2} \cdot \frac{9}{2} + \frac{9}{2} + \frac{9}{2} = \frac{C27}{4} + 9$$

Now when we have integral value we just have to compare it to 1.

$$\frac{C27}{4} + 9 = 1$$
$$C \cdot 27 = -32$$
$$C = \frac{-32}{27}$$

But there is one more condition for f to be density function and that is:

$$f_{XY} \geq 0$$

Now we can find pair (x, y) such that:

$$f_{XY}(3,2) = \frac{-32}{27} * 3 * 2 + 3 + 2 = \frac{-19}{9}$$

So there is no C such that f is density function of 2-dimensional random variable.