Cheat Sheet for **Magic State Distillation (MSD)**

APHY 526 Project
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Why MSD?

- ***Quantum Computing (QC)*** *need* ***Quantum Error Correction (QEC)*** *for noise robustness.*
- ***Clifford gates*** *are fault-tolerant in common QEC codes.*
- ***Non-Clifford*** resource is necessary for ***quantum advantage.***
- *MSD is needed to prepare high-fidelity* ***non-Clifford*** resource.

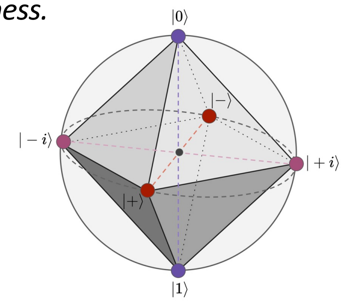
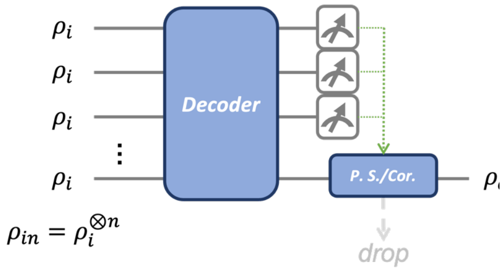


Figure from Pennylane

How to describe MSD? – Stabilizer Reduction

- Almost every MSD protocol can be described using **stabilizer codes**.
- **Stabilizer codes:** QEC codes described by stabilizers S_i s.t.
  $S_i|\psi\rangle = |\psi\rangle$ for all stabilizers and "good" states $|\psi\rangle$
- ***Stabilizer Reduction***: 1. Take the input state ρ_{in}. 2. Measure all given stabilizer generators. 3. Post-select on given measurement patterns. 4. Decode the post-measurement state.
- ***Technically, just apply the decoder operation and measure every ancilla!***

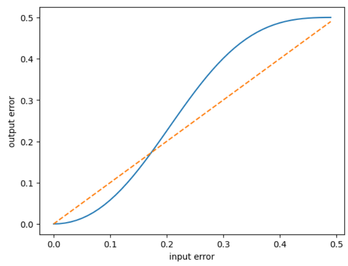


Benchmark MSD

- ***Target state:*** which state are we distilling? $|\mathrm{T}\rangle$ for T gates? Or something else?
- ***Distillation efficiency:*** How fast can we improve the fidelity asymptotically? **Order of error suppression** and **prefactor**.
- ***Distillation threshold:*** How good the input states should be, in order for better output.

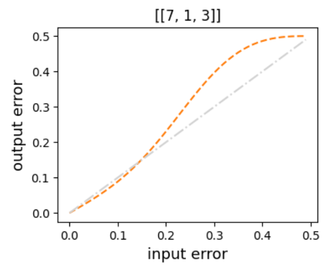
[[5, 1, 3]] protocol

- Stabilizer generators: $XZZXI, IXZZX, XIXZZ, ZXIXZ$
- Logical operators: $X_L = XXXXX, Z_L = ZZZZZ$
- Target state: $|F\rangle\langle F| = \left(I + (X+Y+Z)/\sqrt{3}\right)/2$



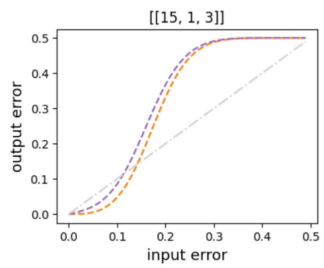
[[7, 1, 3]] protocol

- Stabilizer generators: $XXXXIII, XXIIXXI, XIXIXIX, ZZZZIII, ZZIIZZI, ZIZIZIZ$
- Logical operators: $X_L = XXXXXXX, Z_L = ZZZZZZZ$
- Target state: $|T\rangle = (|0\rangle + e^{i\pi/4}|1\rangle)/\sqrt{2}$
- Linear efficiency: $\epsilon' = \frac{7}{9}\epsilon$
- Tight threshold: 14.148%



[[15, 1, 3]] protocol

- Smallest quantum Reed-Muller code; Smallest QEC codes with transversal T gate
- Cubic efficiency: $\epsilon' = 35\epsilon^3$
- Hard to simulate with matrix method, but easily simulable using dynamic systems!



Visualization with flow diagram

- Map MSD protocols to dynamic systems. See ref arxiv: 2412.04402
- Single-qubit state: $\rho_i(x,y,z) = (I + xX + yY + zZ)/2$
- Assume homogenous input state: $\rho_{in}(x,y,z) = \rho_i^n = \rho_i$