

## Why MSD?

- **Non-Clifford** resource is necessary for **quantum advantage**.
- MSD is needed to prepare high-fidelity **non-Clifford** resource.

## How to describe MSD? – Stabilizer Reduction

- Almost every MSD protocol can be described using **stabilizer codes**.
- **Stabilizer Reduction**: 1. Take the input state  $\rho_{in}$ . 2. Measure all given stabilizer generators. 3. Post-select on given measurement patterns. 4. Decode the post-measurement state.
- **Technically, just apply the decoder operation and measure every ancilla!**

## Benchmark MSD

- **Target state**: which state are we distilling?  $|T\rangle$  for T gates? Or something else?
- **Distillation efficiency**: How fast can we improve the fidelity asymptotically? **Order of error suppression** and **prefactor**.
- **Distillation threshold**: How good the input states should be, in order for better output.

## [[5, 1, 3]] protocol

- Stabilizer generators:  $XZZXI, IXZZX, XIXZZ, ZXIXZ$
- Logical operators:  $X_L = XXXXX, Z_L = ZZZZZ$
- Target state:  $|F\rangle\langle F| = (I + (X + Y + Z)/\sqrt{3})/2$

## [[7, 1, 3]] protocol

- Stabilizer generators:  $XXXXIII, XXIIXXI, XIXIXIX, ZZZZIII, ZZZIIZI, ZIZIZIZ$
- Logical operators:  $X_L = XXXXXXX, Z_L = ZZZZZZZ$
- Target state:  $|T\rangle = (|0\rangle + e^{i\pi/4}|1\rangle)/\sqrt{2}$
- Linear efficiency:  $\epsilon' = \frac{7}{9}\epsilon$
- Tight threshold: 14.148%

## [[15, 1, 3]] protocol

- Smallest quantum Reed-Muller code; Smallest QEC codes with transversal T gate
- Cubic efficiency:  $\epsilon' = 35\epsilon^3$

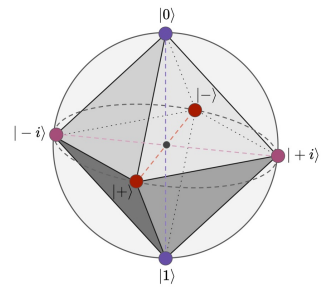
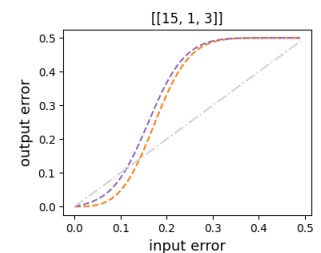
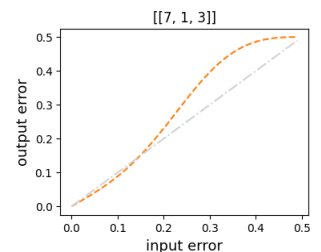
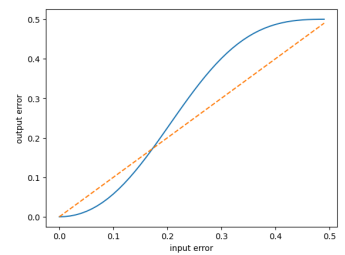
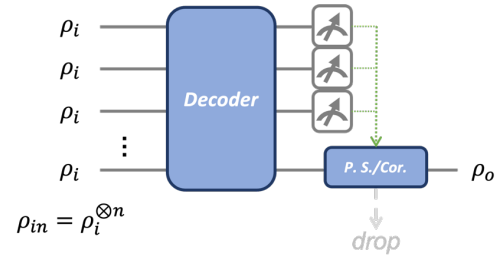


Figure from PennyLane



## Visualization with flow diagram

- Single-qubit state:  $\rho_i(x, y, z) = (I + xX + yY + zZ)/2$
- Assume homogenous input state:  $\rho_{in}(x, y, z) = \rho_i^n = \rho_i$
- Map MSD protocols to dynamic systems. See ref arxiv: xxxx:xxxxx

