

Why MSD?

- **Quantum Computing (QC)** need **Quantum Error Correction (QEC)** for noise robustness.
- **Clifford gates** are fault-tolerant in common QEC codes.
- **Non-Clifford** resource is necessary for **quantum advantage**.
- MSD is needed to prepare high-fidelity **non-Clifford** resource.

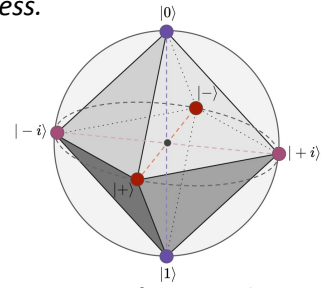
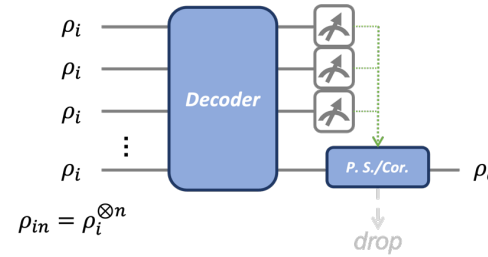


Figure from PennyLane

How to describe MSD? – Stabilizer Reduction

- Almost every MSD protocol can be described using **stabilizer codes**.
- **Stabilizer codes**: QEC codes described by stabilizers S_i s.t.
 $S_i|\psi\rangle = |\psi\rangle$ for all stabilizers and “good” states $|\psi\rangle$
- **Stabilizer Reduction**: 1. Take the input state ρ_{in} . 2. Measure all given stabilizer generators. 3. Post-select on given measurement patterns. 4. Decode the post-measurement state.
- **Technically, just apply the decoder operation and measure every ancilla!**

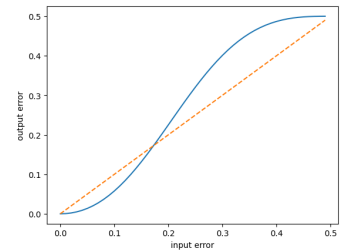


Benchmark MSD

- **Target state**: which state are we distilling? $|T\rangle$ for T gates? Or something else?
- **Distillation efficiency**: How fast can we improve the fidelity asymptotically? **Order of error suppression** and **prefactor**.
- **Distillation threshold**: How good the input states should be, in order for better output.

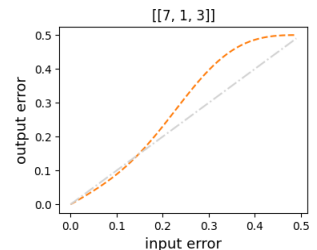
[[5, 1, 3]] protocol

- Stabilizer generators: $XZZXI, IXZZX, XIXZZ, ZXIXZ$
- Logical operators: $X_L = XXXXX, Z_L = ZZZZZ$
- Target state: $|F\rangle\langle F| = (I + (X + Y + Z)/\sqrt{3})/2$



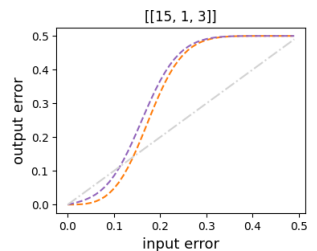
[[7, 1, 3]] protocol

- Stabilizer generators: $XXXXIII, XXIIXXI, XIXIXIX, ZZZZIII, ZZIIZZI, ZIZIZIZ$
- Logical operators: $X_L = XXXXXXX, Z_L = ZZZZZZZ$
- Target state: $|T\rangle = (|0\rangle + e^{i\pi/4}|1\rangle)/\sqrt{2}$
- Linear efficiency: $\epsilon' = \frac{7}{9}\epsilon$
- Tight threshold: 14.148%



[[15, 1, 3]] protocol

- Smallest quantum Reed-Muller code; Smallest QEC codes with transversal T gate
- Cubic efficiency: $\epsilon' = 35\epsilon^3$
- Hard to simulate with matrix method, but easily simulable using dynamic systems!



Visualization with flow diagram

- Map MSD protocols to dynamic systems. See ref arxiv: xxxx:xxxxx
- Single-qubit state: $\rho_i(x, y, z) = (I + xX + yY + zZ)/2$
- Assume homogenous input state: $\rho_{in}(x, y, z) = \rho_i^n = \rho_i$

