Assignment-1
CS771 Introduction to Machine Learning

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1 Solution-1

We have,

$$m : \{0, 1\} \rightarrow \{-1, +1\}$$
$$f : \{-1, +1\} \rightarrow \{0, 1\}$$

Let function, $m(x) = 1 - 2x$
if x = 0 then m =1
if x = 1 then m =-1
and
$function,$ f=$\frac{(1-x)}{2}$
if $x = -1$ then $f = 1$
if $x = 1$ then $f = 0$
To Prove,
XOR(b1,b2,b$_3$.....$b_n) = f(\Pi_{i=1}^{n} m(b_i))$ for any set of binary b1,b2,b3...bn for any n $\in$ N
CASE :1
If n is even ,
Case a : If no of 1's are even, hence no of 0's are even

$$L.H.S = 0$$
$$R.H.S = f(m(b_1).m(b_2).m(b_3)...m(b_n))$$
$$= f((1 - 2b_1).(1 - 2b_2).(1 - 2b_3)...(1 - 2b_n)$$
$$= f(-1 * -1... + 1 * +1)$$
$$= f(1)$$
$$= \frac{1 - (1)}{2}$$
$$= 0$$
$$\rightarrow henceL.H.S = R.H.S$$

Case b: If no of 1's are odd, hence no of 0's are odd

$$L.H.S = 1$$

$$R.H.S = f(m(b_1).m(b_2).m(b_3)..m(b_n))$$

$$= f((1 - 2b_1).(1 - 2b_2).(1 - 2b_3)...(1 - 2b_n))$$

$$= f(-1)$$

$$= \frac{1 - (-1)}{2}$$

$$= 1$$

$$\rightarrow hence L.H.S = R.H.S$$

CASE :2
If n is odd ,
Case a: If no of 1's are even, hence no of 0's are odd

$$L.H.S = 0$$

$$R.H.S = f(m(b_1).m(b_2).m(b_3)....m(b_n))$$

$$= f((1 - 2b_1).(1 - 2b_2).(1 - 2b_3)....(1 - 2b_n))$$

$$= f(1)$$

$$= \frac{1 - (1)}{2}$$

$$= 0$$

$$\rightarrow hence L.H.S = R.H.S$$

Case b: If no of 1's are odd, hence no of 0's are even

$$L.H.S = 1$$

$$R.H.S = f(m(b_1).m(b_2).m(b_3)...m(b_n))$$

$$= f((1 - 2b_1).(1 - 2b_2).(1 - 2b_3)...(1 - 2b_n))$$

$$= f(-1)$$

$$= \frac{1 - (-1)}{2}$$

$$= 1$$

$$\rightarrow hence L.H.S = R.H.S$$

2 Solution-2

$$\prod_{i=1}^{n} \text{sign}(\text{r}_i) = sign(\prod_{i=1}^{n} r_i)$$

Taking LHS,

$$\because sign(r_i) = \frac{ri}{|ri|} \text{ if } \forall \text{ri} \notin 0 \text{ and } ri \in R$$
$$\because sign(0) = 0$$
$$\therefore sign(r_i) = 0$$

case-a) when there is atleast one r_i as zero

$LHS :-$

=$\prod sign(r_i)$

=$\frac{r1}{|r1|} * \frac{r2}{|r2|} * \frac{r3}{|r3|} * \frac{r4}{|r4|} * .... \frac{rn}{|rn|}$

if $\exists$i such that sign(r$_i$) $= 0 \rightarrow$r$_i = 0$
then

$\frac{r1}{|r1|} * \frac{r2}{|r2|} * \frac{r3}{|r3|} * \frac{r4}{|r4|} * .... \frac{r9}{|rn|} = 0$

RHS:-

$= sign(\prod r_i)$ if $\exists$i such that $ri = 0 \rightarrow$sign(r$_i$) $= 0$

$\because$ sign(r1*r2*r3*r4......*r$_{i-1} * r9) = sign(0) = 0$

$LHS = RHS$ if $\exists$ i such that $r_i = 0$

case-b) when $\forall \; i \; r_i \neq 0$

$LHS :-$
$sign(r_i) = \frac{r_i}{|r_i|}$

$\because \forall i \; r_i \neq 0$...........(1)

$\prod sign(r_i) = (\frac{r1}{|r1|} * \frac{r2}{|r2|} * \frac{r3}{|r3|} * \frac{r4}{|r4|} * .... \frac{rn}{|rn|})$

$RHS :-$
$= sign(\prod ri)$
$= sign(r1 * r2 * r3 * .... * r9) \forall i \; ri \neq 0$

$using(1)$

$sign(r1 * r2 * r3 * ....rn) = \frac{r1*r2*r3..*rn}{|r1*r2....*rn|}$

$\because |a1 * a2 * a3.....a9| = |a1| * |a2| * |a3| * ... * |an|$

$\therefore \frac{r1}{|r1|} * \frac{r2}{|r2|} * \frac{r3}{|r3|} * \frac{r4}{|r4|} * .... \frac{r9}{|rn|}$

$\therefore LHS = RHS$

3 Solution-3

We have given $\left(\tilde{u}^T\tilde{x}\right)\cdot\left(\tilde{v}^T\tilde{x}\right)\cdot\left(\tilde{w}^T\tilde{x}\right)$. Number of features in each of 'x' feature vectors is 8. To make it simpler we hide the bias the term to get we get $\tilde{x}$ which is 9 dimensional vector. We aim to model the prediction $\left(\tilde{u}^T\tilde{x}\right)\cdot\left(\tilde{v}^T\tilde{x}\right)\cdot\left(\tilde{w}^T\tilde{x}\right)$ quantity into a single linear model with a different dimensionality D. Directly solving for three PUFs:

$$\left(\tilde{u}^T\tilde{x}\right)\cdot\left(\tilde{v}^T\tilde{x}\right)\cdot\left(\tilde{u}^T\tilde{x}\right)=\left(\sum_{i=1}^{9}\tilde{u}_i\tilde{x}_i\right)\cdot\left(\sum_{j=1}^{9}\tilde{v}_j\tilde{x}_j\right)\cdot\left(\sum_{k=1}^{9}\tilde{w}_k\tilde{x}_k\right) \tag{1}$$

$$=\sum_{i=1}^{9}\sum_{j=1}^{9}\sum_{k=1}^{9}\tilde{u}_i\tilde{v}_j\tilde{w}_k\tilde{x}_i\tilde{x}_j\tilde{x}_k \tag{2}$$

$$So,$$

$\left(\tilde{u}^T\tilde{x}\right)\cdot\left(\tilde{v}^T\tilde{x}\right)\cdot\left(\tilde{w}^T\tilde{x}\right)=u_1v_1w_1x_1x_1x_1+u_1v_1w_2x_1x_1x_2+u_1v_1w_3x_1x_1x_3+\ldots+u_9v_9w_9x_0x_9x_9$
There are $9*9*9=729$ terms in our linear equation where $i,j,k\epsilon[1,9]$.
Therefore $9^3=729$ dimensional function that maps $\tilde{x}=(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3)$ to

$$\phi(\tilde{x})=\begin{pmatrix}\tilde{x}_1\tilde{x}_1\tilde{x}_1 & ... & \tilde{x}_1\tilde{x}_1\tilde{x}_9 & ... & \tilde{x}_1\tilde{x}_2\tilde{x}_1 & ... & \tilde{x}_1\tilde{x}_2\tilde{x}_9\\ \tilde{x}_1\tilde{x}_3\tilde{x}_1 & ... & \tilde{x}_1\tilde{x}_3\tilde{x}_9 & ... & \tilde{x}_1\tilde{x}_4\tilde{x}_1 & ... & \tilde{x}_1\tilde{x}_4\tilde{x}_9\\ ... & ... & ... & ... & ... & ... & \\ \tilde{x}_2\tilde{x}_1\tilde{x}_1 & ... & \tilde{x}_2\tilde{x}_1\tilde{x}_9 & ... & \tilde{x}_2\tilde{x}_2\tilde{x}_1 & ... & \tilde{x}_2\tilde{x}_2\tilde{x}_9\\ ... & ... & ... & ... & ... & ... & \\ \tilde{x}_9\tilde{x}_8\tilde{x}_1 & ... & \tilde{x}_9\tilde{x}_8\tilde{x}_9 & ... & \tilde{x}_9\tilde{x}_9\tilde{x}_1 & ... & \tilde{x}_9\tilde{x}_9\tilde{x}_9\end{pmatrix}$$

$$\left(\tilde{u}^T\tilde{x}\right)\cdot\left(\tilde{v}^T\tilde{x}\right)\cdot\left(\tilde{w}^T\tilde{x}\right)=\mathrm{W}^T\phi(\tilde{x})$$

$$W=\begin{pmatrix}\tilde{u}_1\tilde{v}_1\tilde{w}_1 & \tilde{u}_1\tilde{v}_1\tilde{w}_2 & \tilde{u}_1\tilde{v}_1\tilde{w}_3 & .... & \tilde{u}_1\tilde{v}_1\tilde{w}_9\\ \tilde{u}_1\tilde{v}_2\tilde{w}_1 & \tilde{u}_1\tilde{v}_2\tilde{w}_2 & \tilde{u}_1\tilde{v}_2\tilde{w}_3 & .... & \tilde{u}_1\tilde{v}_2\tilde{w}_9\\ ... & ... & ... & ... & ...\\ ... & ... & ... & ... & ...\\ \tilde{u}_9\tilde{v}_9\tilde{w}_1 & \tilde{u}_9\tilde{v}_9\tilde{w}_2 & \tilde{u}_9\tilde{v}_9\tilde{w}_3 & .... & \tilde{u}_9\tilde{v}_9\tilde{w}_9\end{pmatrix}$$

$Actually$, $\phi(\tilde{x})$ and W is 1-Dimensional

Dimentionality $(D)=(8+1)^3=729$, which depends on number of PUFs i.e. 3 and dimentionality of $\tilde{x}=8+1=9$
Hence proved.

4 Solution-4

Solved in submit.py file

5 Solution-5

In the method used, there are two hyperparameters

1 $\rightarrow$ step length
2 $\rightarrow$C

The optimal values of the hyperparameters are obtained by random testing ,we tested a certain number of submission that we selected randomly.

The value of the hyperparameter C is highly affecting the missclassification error.

On decreasing the value of C , the average error increases. We found the optimal value that is C=7 , by slowly increasing the value and found that there comes a point,at which on further increasing the value of C , the average error again starts increasing.

The value of step length η affects the value of hinge loss.We found the optimal value of step length is increased from the found optimal,the hinge loss value increases

The optimal values :- $\eta = 9.4$e$^{-}6$, C = 7

The optimal values provide a trade off between misclassification error and hinge loss.

6 Solution-6