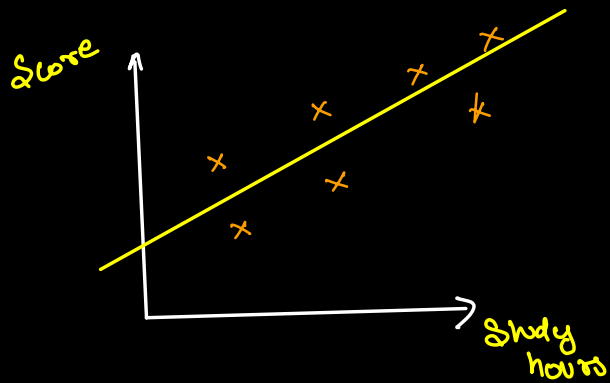


## Simple Linear regression

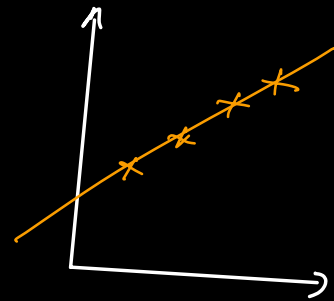
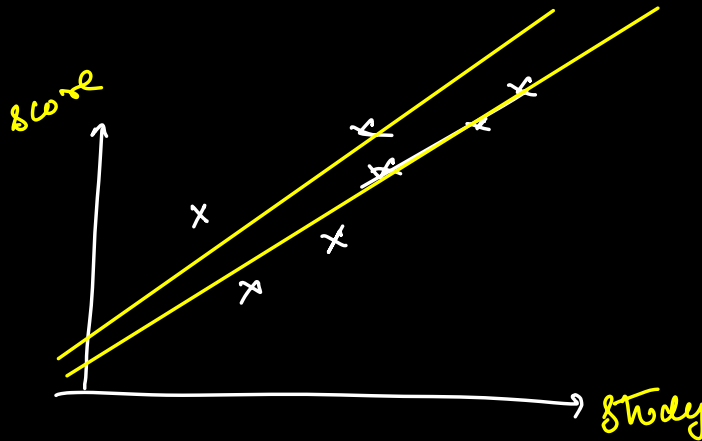


Iris  $\rightarrow$  4D im

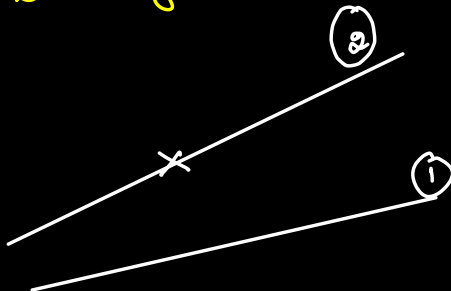
$$y = mx + c$$

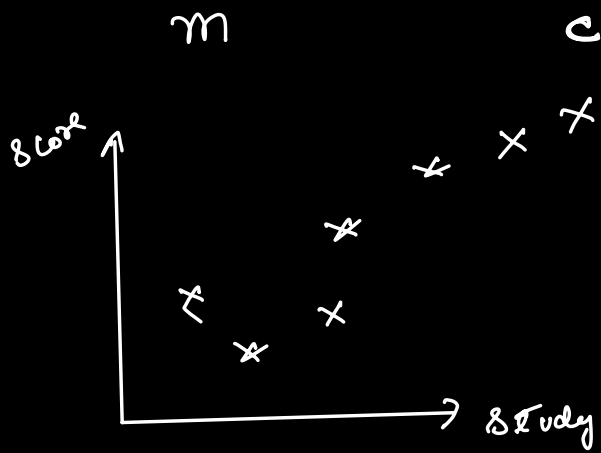
$\Downarrow$

$$h_{\theta}(x) = \theta_1 x_1 + \theta_0$$



Best fit line  $\rightarrow$  min<sup>m</sup> error  
close to all points





$$\text{Score} = \underline{m} \times \text{Study} + c$$

↙  
 maths 100  
 eng 50

$$\text{If study} = 0$$

$$\text{Score} = \underline{c}$$

We have to find the best fit line

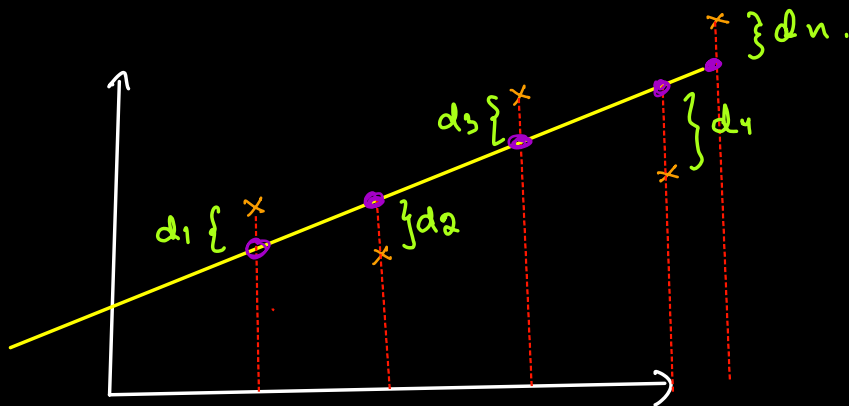
1. Closed form

2] Non-closed form

$$m, =$$
$$c =$$

Gradient  
Descent

OLS

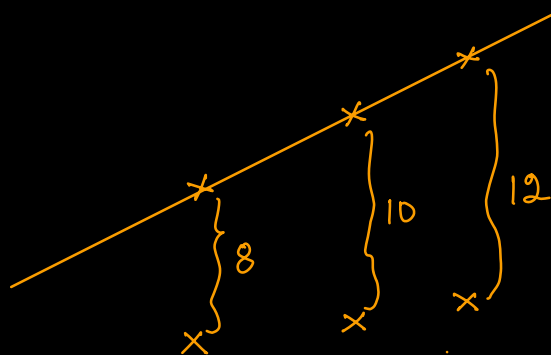
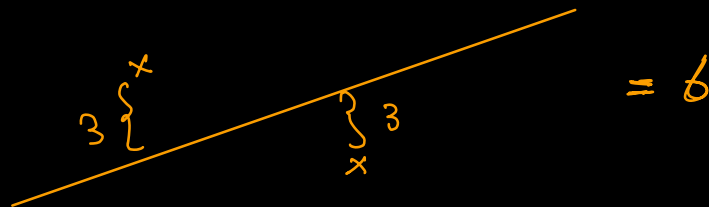


$$\text{error} = d_1 + d_2 + d_3 \dots d_n$$

$$3 \begin{Bmatrix} x \\ x \end{Bmatrix} = 0$$

$$3 + (-3) = 0$$

$$\text{error} = |d_1| + |d_2| + |d_3| + \dots + |d_n|$$

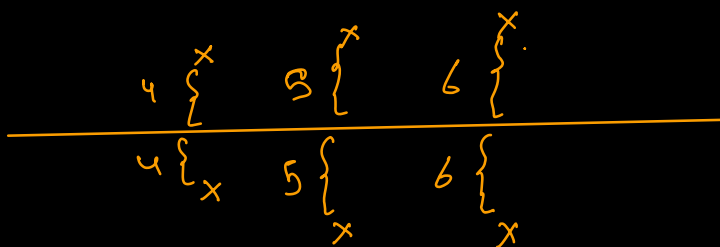


$$\Rightarrow 30$$

$$= 8 + 10 + 12$$

$$2^2 + 10^2 + 12^2 = 308$$

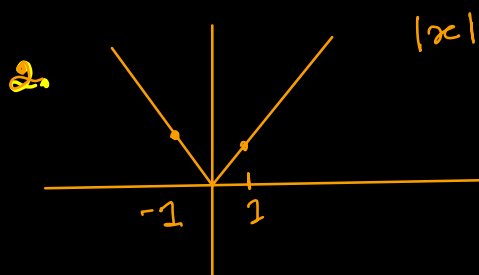
||



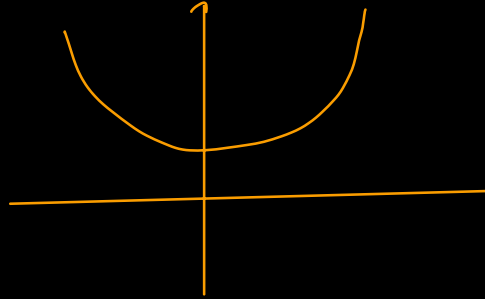
$$= 30$$

$$= 8 + 10 + 12$$

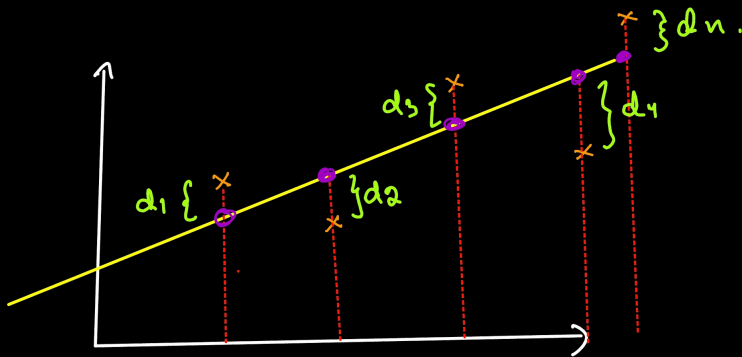
$$2(4^2 + 5^2 + 6^2) = \boxed{154}$$



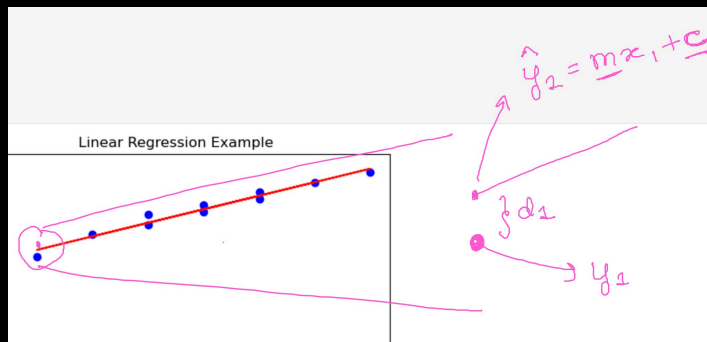
3.  $d^2$ :



$$\varepsilon = \underline{d_1^2} + d_2^2 + d_3^2 + \dots + d_n^2$$



$$d_1 = (y_1 - \hat{y}_1)^2$$



$$\begin{aligned}
 E &= d_1^2 + d_2^2 + d_3^2 \dots d_n^2 \\
 &= (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 \dots (y_n - \hat{y}_n)^2 \\
 &= \sum (y_i - \hat{y}_i)^2 \\
 &\quad \quad \quad (mx_i + b)
 \end{aligned}$$

If my error is reduced, the line will be the best fit line

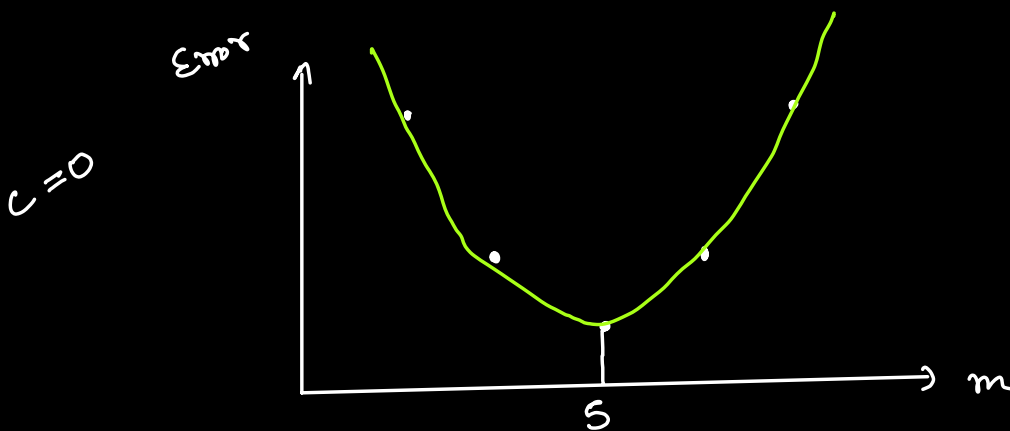
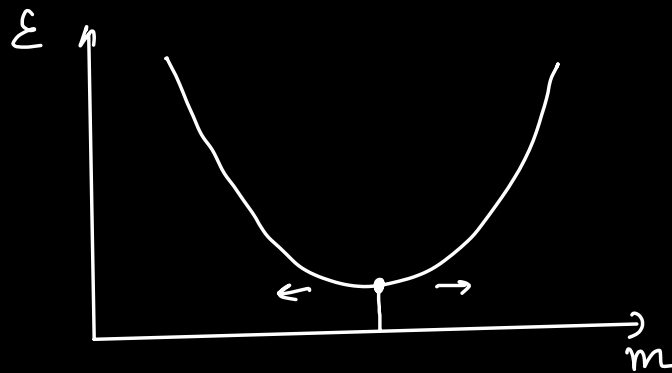
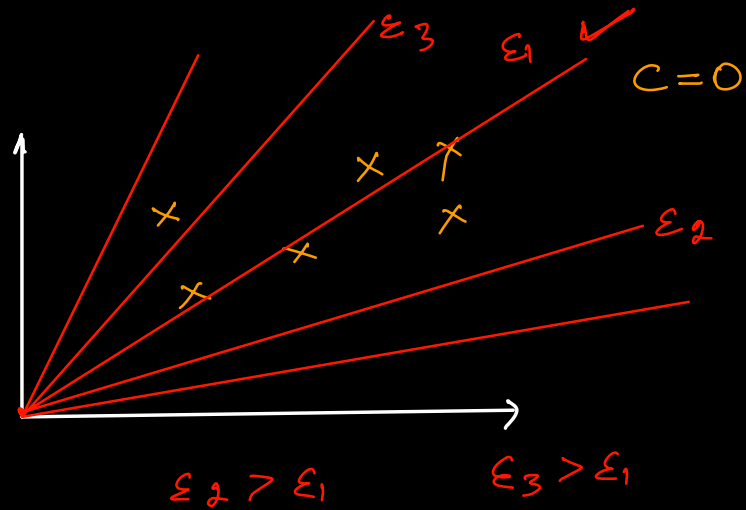
$$E = (y_i - (mx_i + b))^2$$

$$E(m, b) = (\underline{y_i} - (m\underline{x_i} + b))^2$$

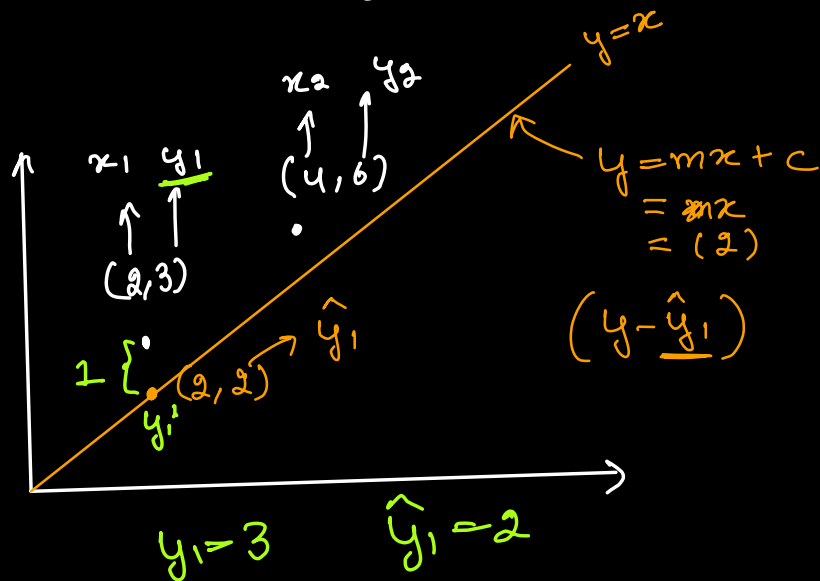
$$y = f(\underline{x})$$

Error/loss

$$E(m, b) = \sum_{i=1}^3 (y_i - (mx_i + b))^2$$



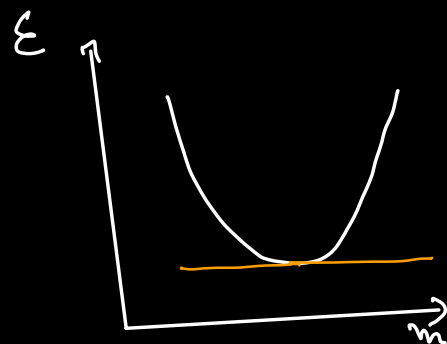
$$E(m, c) = \sum [y_i - [mx_i + c]]^2$$



$$E(m, c) = (y_i - (mx_i + c))^2$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial E}{\partial c} = 0$$



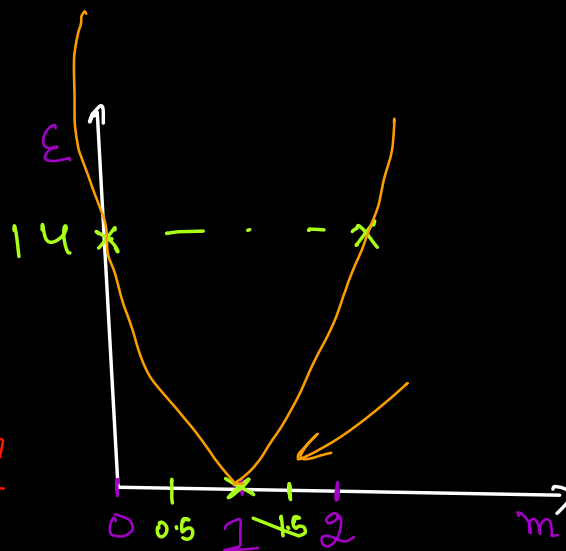
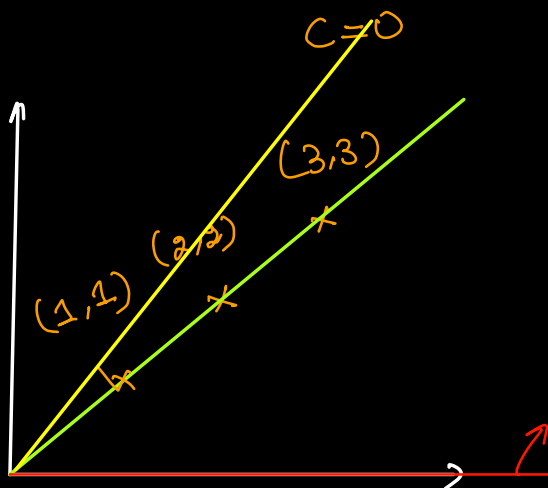


$$\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$$

$$\begin{aligned} x &= 4 \\ y &= 2 \end{aligned} \quad \Leftarrow$$

$x + y = 6$

$$\begin{aligned} x, y &= 3, 3 \\ x, y &= 4, 2 \\ x, y &= 5, 1 \\ x, y &= 6, 0 \end{aligned}$$



$$y = x$$

$$x_1 = 1 \quad \hat{y}_1 = 1$$

$$y = 2x$$

$$\begin{matrix} 1 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 \\ 4 \end{matrix}$$

$$\begin{matrix} 3 \\ 6 \end{matrix}$$

$$y = 0$$

$$\begin{matrix} 1 \\ 0 \end{matrix}$$

$$\begin{matrix} 2 \\ 0 \end{matrix}$$

$$\begin{matrix} 3 \\ 0 \end{matrix}$$

$$E = \sum (y_i - \hat{y}_i)^2$$

$$= \frac{1}{2} \sum (y_i - (mx_i + b))^2$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial E}{\partial b} = 0$$

$m, b$

$\varepsilon \downarrow$

$\frac{\varepsilon}{2} \downarrow$

200

100

$$\frac{\partial E}{\partial b} = 0$$

$$\frac{\partial}{\partial b} \left[ \frac{y_i - mx_i - b}{t} \right]^2 = 0$$

$$\frac{\partial}{\partial b} t^2 = 0$$

$$2t \frac{\partial t}{\partial b} = 0$$

$$\sum_{i=1}^n \frac{\partial}{\partial b} (y_i - mx_i - b) \frac{\partial (y_i - mx_i - b)}{\partial b} = 0$$

-1

$$\sum_{i=1}^n (y_i - mx_i - b) = 0$$

$$\frac{\sum y_i}{n} - m \frac{\sum x_i}{n} - \frac{\sum b}{n} = 0$$

$$\frac{y_1 + y_2 + \dots + y_n}{n} - m \frac{(x_1 + x_2 + \dots + x_n)}{n} - \frac{b + b + \dots + b}{n} = 0$$

$$\sum_{i=1}^n b$$

$$\bar{y} - m\bar{x} - \frac{\sum y}{n} = 0$$

$$b = \bar{y} - \bar{m}\bar{x}$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial}{\partial m} \sum (y_i - m x_i - b)^2 = 0$$

$$\frac{\partial}{\partial m} \left[ y_i - m x_i - (\bar{y} - m \bar{x}) \right]^2 = 0$$

$$\sum_{i=1}^n \frac{\partial}{\partial m} \left[ y_i - m x_i - \bar{y} + m \bar{x} \right] \frac{\partial}{\partial m} \left[ \cancel{y_i - m x_i} - \cancel{\bar{y} + m \bar{x}} \right] = 0$$

$\underline{-x_i + \bar{x} = 0}$

$$\sum \left[ (y_i - \bar{y}) - m \frac{(x_i - \bar{x})}{t} \right] \frac{(x_i - \bar{x})}{t} = 0$$

$\underline{-x_i + \bar{x} = (x_i - \bar{x}) = 0}$

$$\sum (y_i - \bar{y}) t - \sum m t^2 = 0$$

$$\sum (y_i - \bar{y})(x_i - \bar{x}) = m \sum (x_i - \bar{x})^2$$

$$m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$