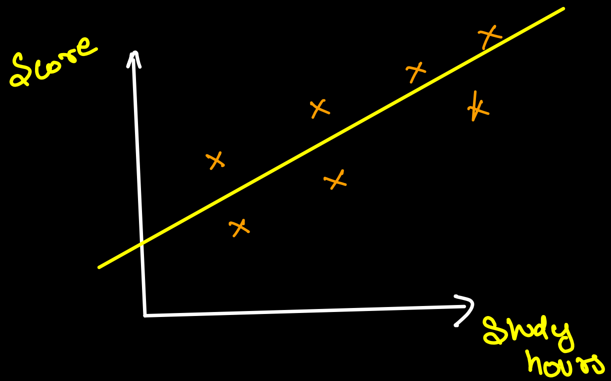
Simple Linear regression

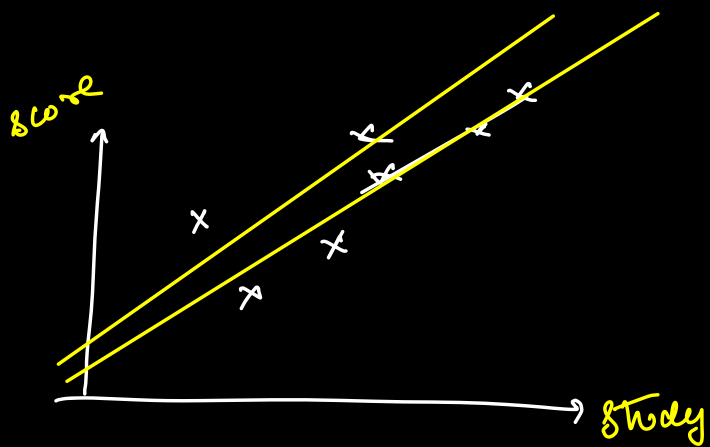


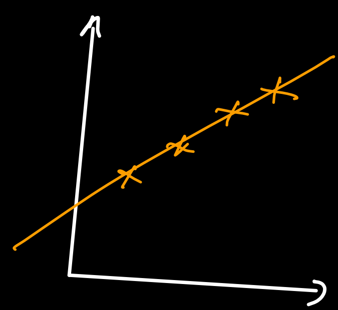
Iris → 4 Dimⁿ

$$y = mx + c$$

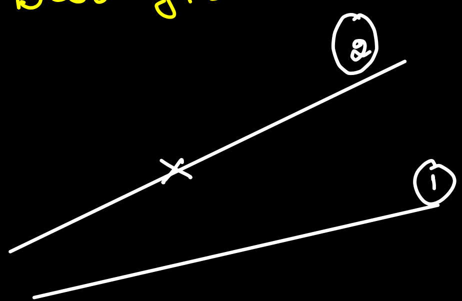
$$\Downarrow$$

$$h_\theta(x) = \theta_1 x_1 + \theta_0$$

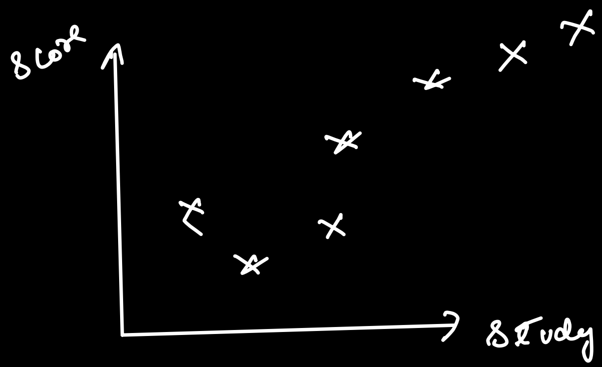




Best fit line → min^m error
close to all points



m c



$$\text{Score} = \underline{m} * \text{Study} + C$$

marks 100

eng 50

If study = 0

$\text{score} = \underline{c}$

We have to find the best fit line

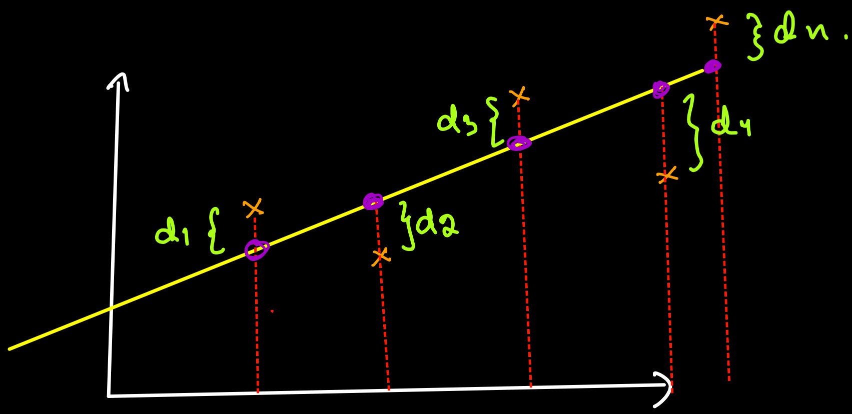
1. Closed form

2] Non-closed form

$m, =$
$c =$

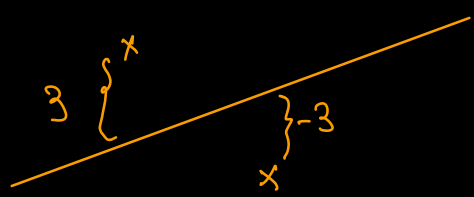
$\boxed{\text{Gradient Descent}}$

OLS



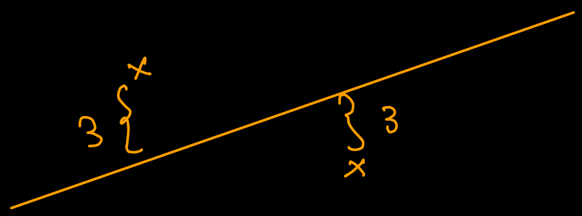
$$\text{error} = d_1 + d_2 + d_3 \ldots d_n$$

$= 0$

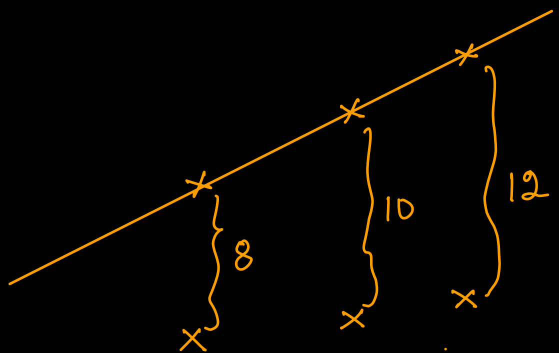


$$3 + (-3) = 0$$

$$\text{error} = |d_1| + |d_2| + |d_3| + \dots |d_n|$$



$= 6$

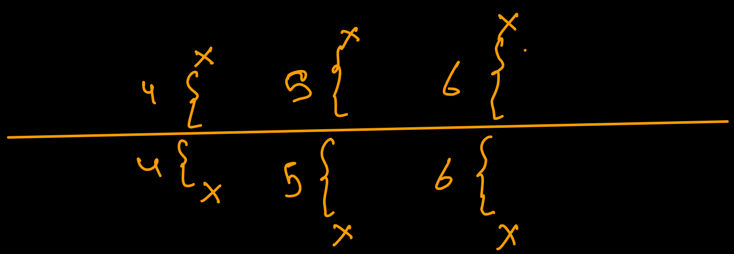


$\Rightarrow 30$

$= 8 + 10 + 12$

$8^2 + 10^2 + 12^2 = 308$

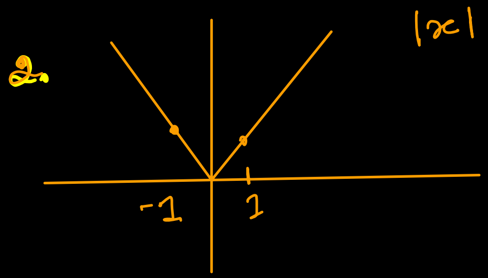
$\parallel$



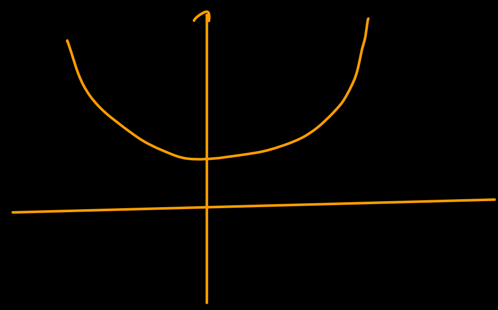
$= 30$

$= 8 + 10 + 12$

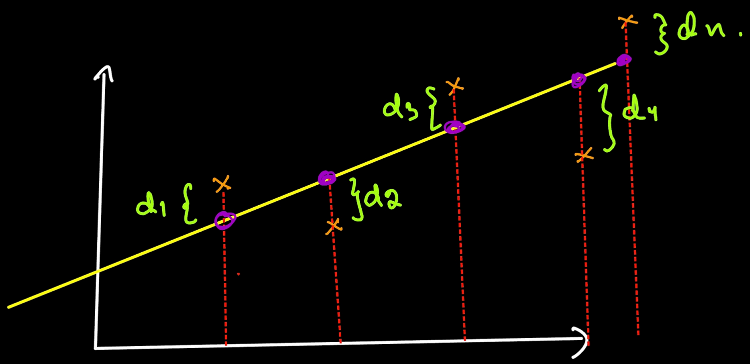
$2(4^2 + 5^2 + 6^2) = \boxed{154}$



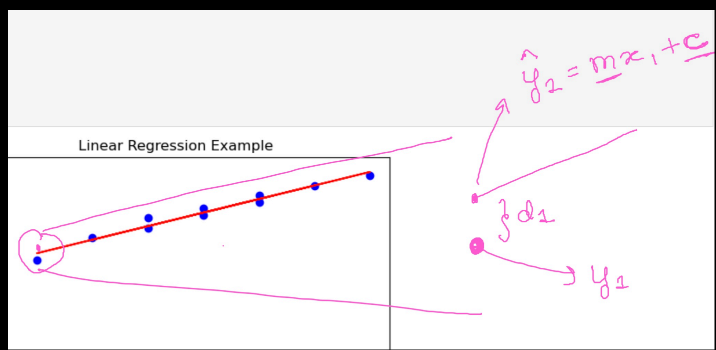
3. d^2



$$\mathcal{E} = d_1^2 + d_2^2 + d_3^2 \cdots d_n^2$$



$$d_1 = (y_1 - \hat{y}_1)^2$$



$$E = d_1^2 + d_2^2 + d_3^2 \ldots d_n^2$$

$$= (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 \ldots (y_n - \hat{y}_n)^2$$

$$= \sum (y_i - \underbrace{\hat{y}_i}_{(mx_i + b)})^2$$

If my error is reduced, the line will be the best fit line

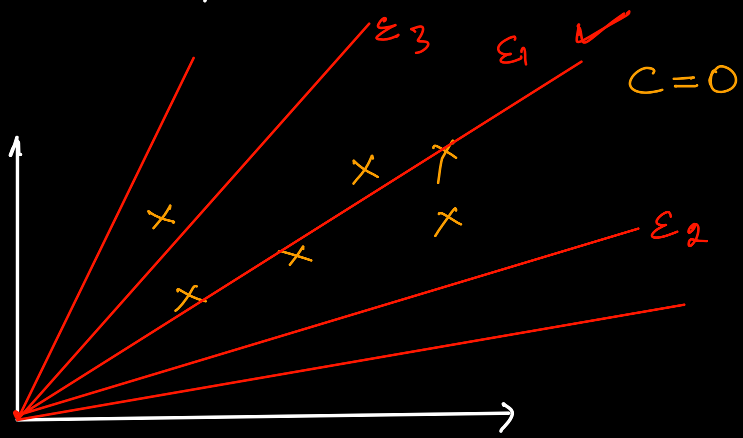
$$E = (y_i - (mx_i + b))^2$$

$$E(m, b) = (\underline{y_i} - (m\underline{x_i} + b))^2$$

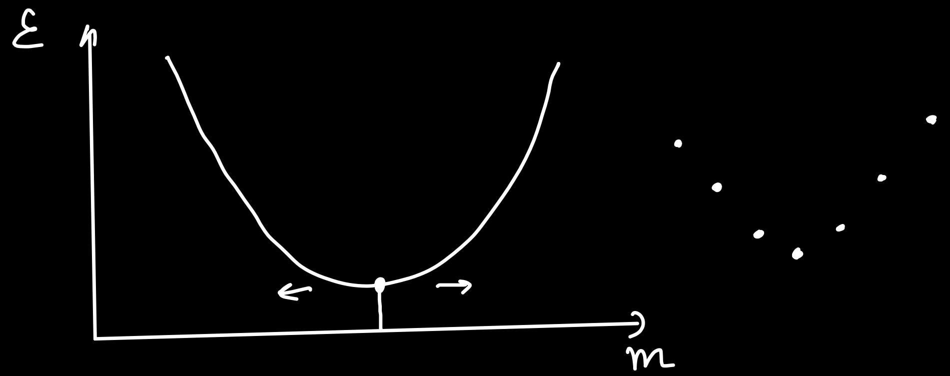
$$y = f(\underline{x})$$

Error/Cost

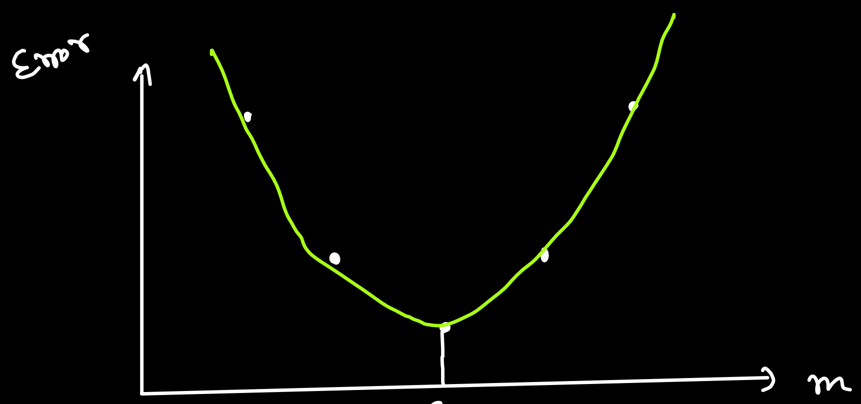
$$\mathcal{E}(m,b) = \sum_{i=1}^{n} \left(y_i - (mx_i + b)\right)^2$$



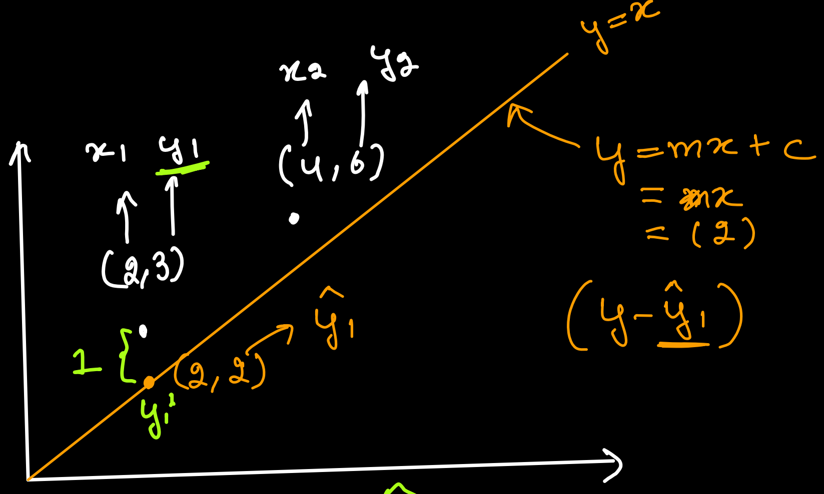
$\mathcal{E}_2 > \mathcal{E}_1$ $\qquad$ $\mathcal{E}_3 > \mathcal{E}_1$



$C=0$



$$E(m,c) = \sum \left[y_i - \left[(mx_i + b)\right]\right]^2$$

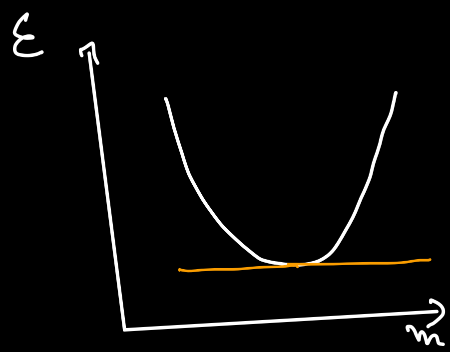


$y_1 = 3$ $\hat{y}_1 = 2$

$$E(m,c) = (y_i - (mx_i + c))^2$$

$$\boxed{\frac{\partial E}{\partial m} = 0}$$

$$\boxed{\frac{\partial E}{\partial b} = 0}$$

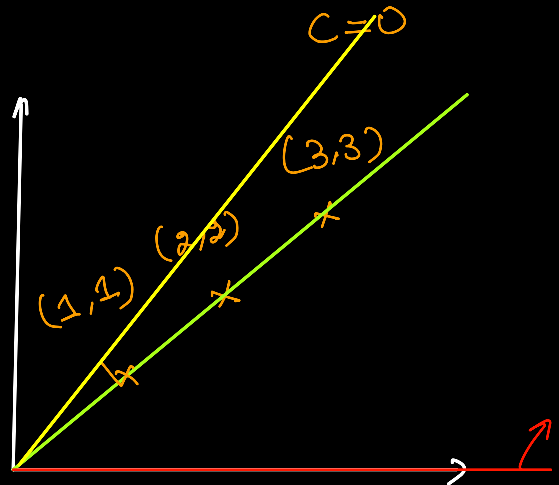


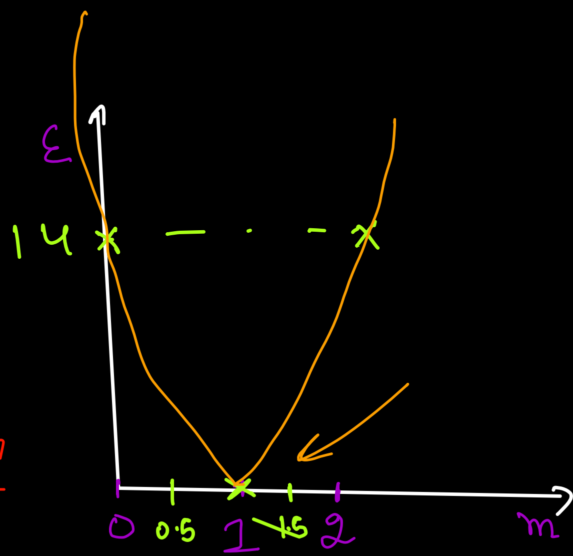
$$\begin{cases} x+y=6 \\ x-y=2 \end{cases} \qquad \Leftarrow$$

$x=4$
$y=2$

$$\boxed{x+y=6}$$

$x\ y = 3,3$
$x,y = 4,2$
$x,y = 5,1$
$x,y = 0,6$





$y=x$

$x_1=1 \qquad \hat{y}_1=1$

$y=2x$

1	2	3
2	4	6

$y=0$

1	2	3
0	0	0

$$E = \sum (y_i - \hat{y}_i)^2$$

$$= \frac{1}{2} \sum \left(y_i - (mx_i + b)\right)^2$$

$$\frac{\partial E}{\partial m} = 0 \qquad \frac{\partial E}{\partial b} = 0$$

m, b

$E \downarrow$ $\qquad$ $\frac{E}{2} \downarrow$

200 $\qquad$ 100

$$\frac{\partial E}{\partial b} = 0$$

$$\frac{\partial}{\partial b}\Big[\underbrace{y_i - mx_i - b}_{t}\Big]^2 = 0$$

$$\frac{\partial}{\partial b} t^2 = 0$$

$$2t \frac{\partial t}{\partial b} = 0$$

$$\sum_{i=1}^{n} 2 (y_i - mx_i - b) \frac{\partial (\cancel{y_i}^{0} - \cancel{mx_i}^{0} - b)}{\partial b} = 0$$

-1

$$\sum_{i=1}^{n} (y_i - mx_i - b)(\cancel{-1}) = 0$$

$*(-1)$ $*(-1)$

$$\frac{\sum y_i}{n} - m\frac{\sum x_i}{n} - \frac{\sum b}{n} = \frac{0}{n}$$

$$\frac{y_1 + y_2 \cdots y_n}{n} - m\left(\frac{x_1 + x_2 + x_3 \cdots x_n}{n}\right) - \frac{b+b+\cdots n}{n} = \frac{0}{n}$$

$$\sum_{i=1}^{n}$$

$$\bar{y} - m\bar{x} - \frac{\cancel{n}b}{\cancel{n}} = 0$$

$$\boxed{b = \bar{y} - m\bar{x}}$$

$$\frac{\partial E}{\partial m} = 0$$

$$\frac{\partial}{\partial m}\left\{ \sum (y_i - mx_i - b)^2 = 0 \right.$$

$$\frac{\partial}{\partial m}\left[y_i - mx_i - (\bar{y} - m\bar{x}) \right]^2 = 0$$

$$\sum_{i=1}^{n} 2\left[y_i - mx_i - \bar{y} + m\bar{x}\right] \frac{\partial}{\partial m}\left[\cancel{y_i} - mx_i - \cancel{\bar{y}} + m\bar{x}\right] = 0$$

$$-x_i \quad + \bar{x}$$

$$-x_i + \bar{x}$$

$$(x_i - \bar{x}) \quad \cancel{= 0}$$

$$\sum \left[(y_i - \bar{y}) - m\underbrace{(x_i - \bar{x})}_{t} \right] \underbrace{(x_i - \bar{x})}_{t} = 0$$

$$\sum (y_i - \bar{y})t - \sum m t^2 = 0$$

$$\sum (y_i - \bar{y})(x_i - \bar{x}) = m \sum (x_i - \bar{x})^2$$

$$\boxed{m = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}}$$

X_train[0] - X_train.mean

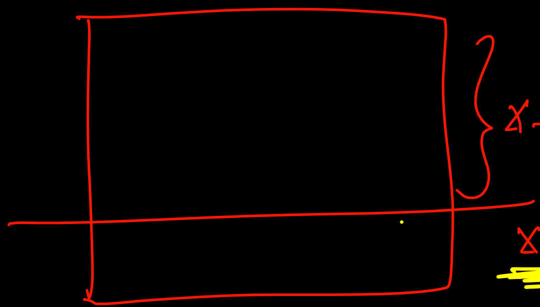
y_tr[0] - y_train.mean()

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$\bar{x}$ = mean of x

$\bar{y}$ = mean of y

$$b = \bar{y} - m\bar{x}$$



X_train, y_train

X_test, y_test

$\Downarrow$

y_pred $\longleftrightarrow$ y_test

Simple linear regression

OLS $\Longrightarrow$ gradient closed.

1 dep   1 independent

Increased no. of predicted

4-D

Petal length   Petal width   sepal len   sepal width

$$y = b + m_1 x_1 \quad m_2 x_2 \quad m_3 x_3 \quad m_4 x_4$$

$$\hat{Y} = mX + b$$

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} \quad \begin{bmatrix} \; & \; & \; & \; \end{bmatrix} + \begin{bmatrix} b \\ b \\ b \end{bmatrix}$$

$$E = (y - \hat{y})$$

$\frac{\partial E}{\partial m_1}$ $\quad \frac{\partial E}{\partial m_2}$ $\quad \frac{\partial E}{\partial m_3}$ $\quad \frac{\partial E}{\partial m_4}$ $\quad \frac{\partial E}{\partial b}$

$= 0$

study	sleep	test	score

$$X \rightarrow \begin{bmatrix} 3 & 1 & 5 \\ 4 & 2 & 7 \\ 5 & 3 & 8 \end{bmatrix}_{3\times 3}$$

$$B = \underline{(X^T X)}^{-1} X^T Y$$

$$\cancel{(100000)^{-1}}$$

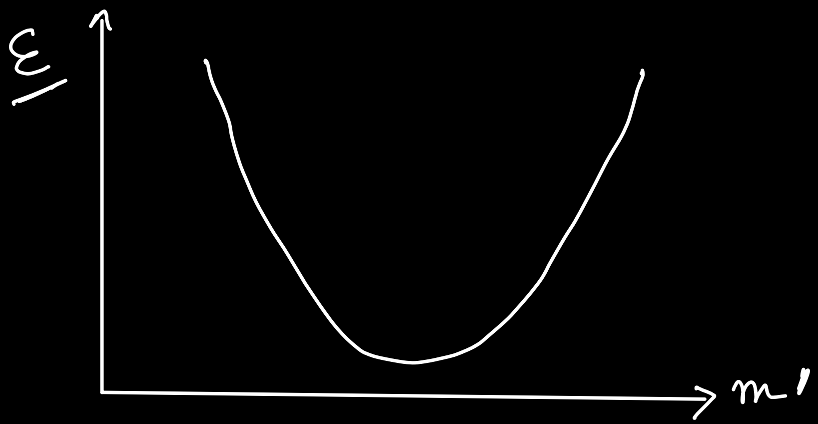
$$(100000)^3$$

$$E(m,c) = \sum \left(y_i - (m x_i + \cancel{b})\right)^2$$

0

$$E = \sum (y_i - m x_i)^2$$

$b = 0$



$$E = \sum \left[y - (m x_i)\right]^2$$

When we go to multiple dim^n,

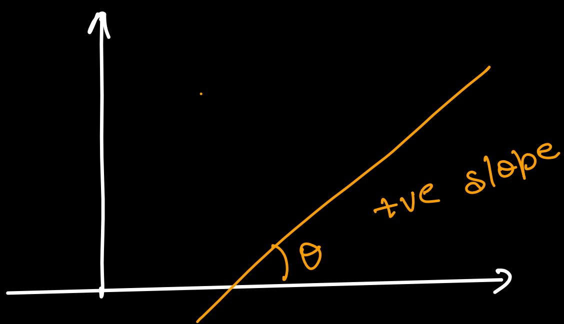
math is very heavy

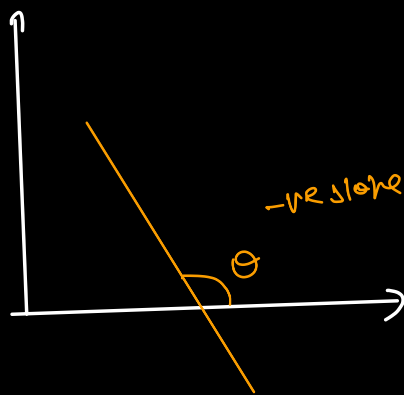
Our error f^n it is a

differentiable f^n

$$E = \sum (y_i - \hat{y}_i)^2$$
$$= \sum (y_i - (mx_i + b))^2$$

It has one global minima



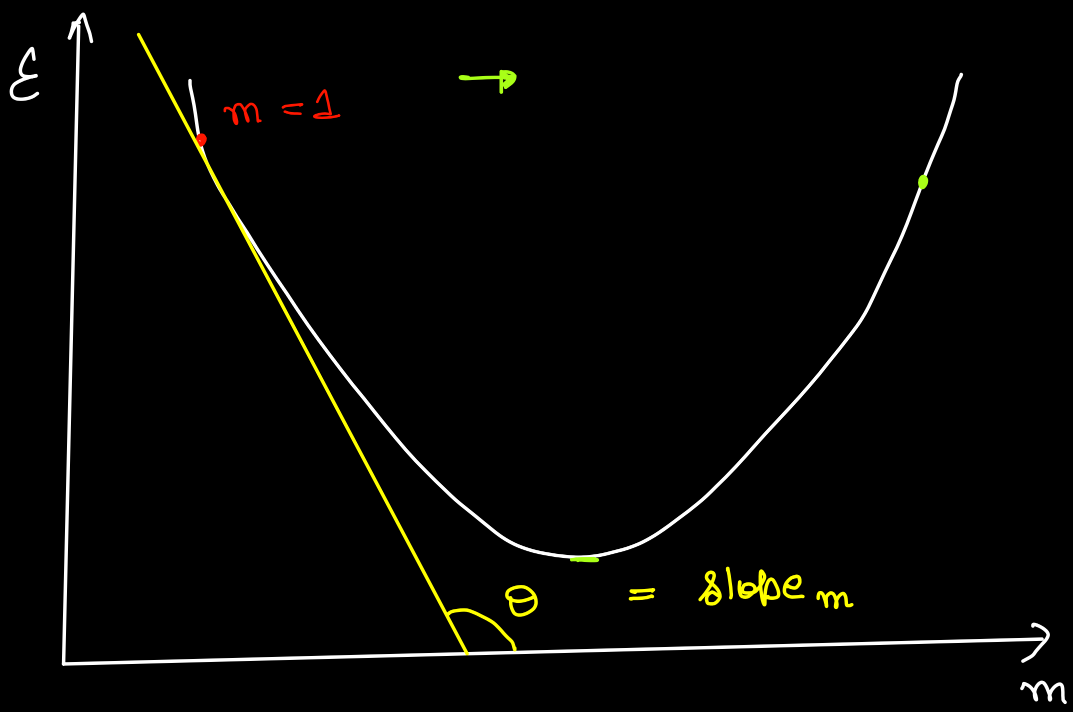


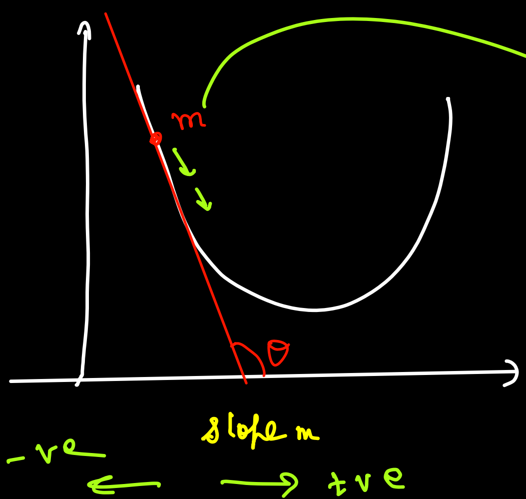
$\Downarrow$ $\tan(\theta)$

$\tan 45° = 1$

$\tan 90° = \infty$

$\tan -\cancel{180} = -$

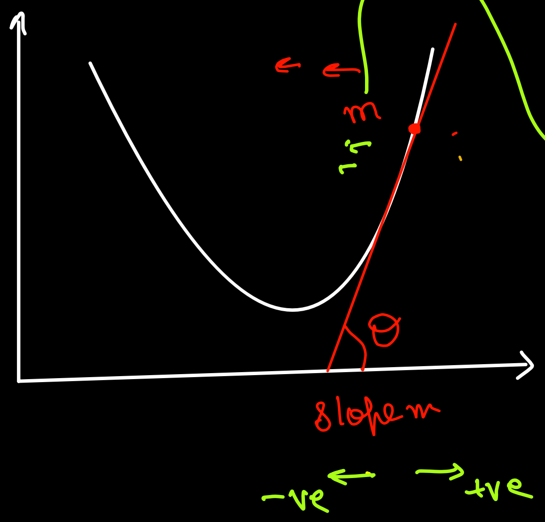




$$m_{new} = m_{old} - \underset{(-ve)}{(slope)}$$

$$= \underline{m_{old}} + (\quad)$$

$$+ (\quad)$$



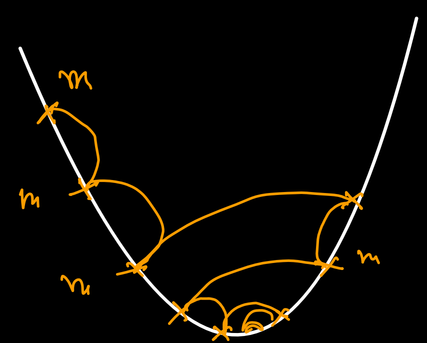
$$m_{new} = m_{old} - \underset{+ve}{\underline{slope}}$$

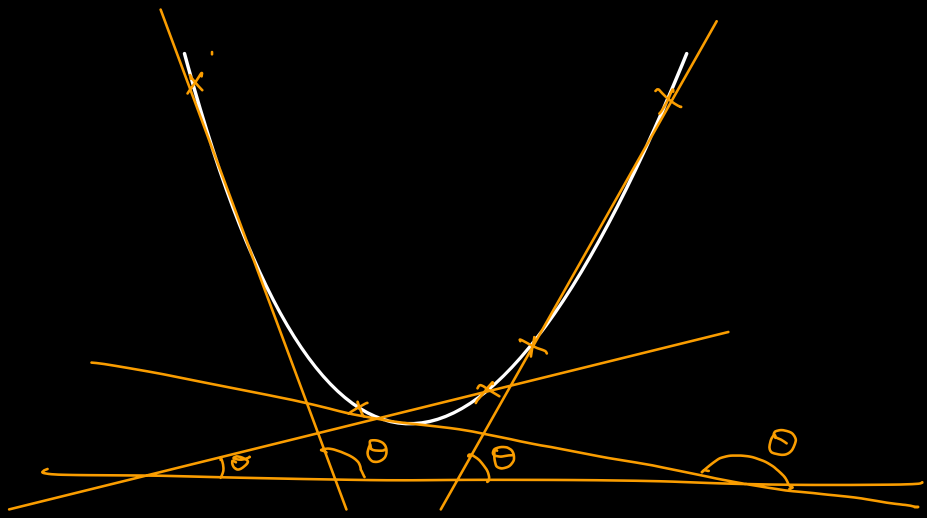
$$= m_{old} - (+ve)$$

$$=$$

for i in range (no. of times)

$$m = m - slope$$




θ
θ
θ
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