

Welcome Back

## Agenda

1. Maths Complete
2. Linear Regression Intuition + Example

Determinant of a matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$|A| = 4 - 2 \cdot 3 = -2$$

## Matrices & Determinants

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a \cdot d - b \cdot c$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = a_1 \begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix} - b_1 \begin{bmatrix} a_2 & c_2 \\ a_3 & c_3 \end{bmatrix} + c_1 \begin{bmatrix} a_2 & b_2 \\ a_3 & b_3 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{1} & \textcircled{2} & \textcircled{3} \\ 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix} \Rightarrow \begin{aligned} &\textcircled{1}[54-56] \\ &\quad -\textcircled{2}[36-40] \\ &\quad \quad +\textcircled{3}[28-30] \end{aligned}$$

$$= 1[-2] - 2[-4] + 3[-2]$$

$$= -2 + 8 - 6$$

$$= 0$$

Inverse of a matrix

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{if } \det(A) \neq 0$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

# Calculus

function  $f(x)$



relationship b/w input & output

Limit on a function

## Calculus

Differentiation

Integration

$$y = f(x) = \boxed{\phantom{000}}$$

Differentiation

Differentiation  $\Rightarrow$  change in  $y$  wrt  $x$

$y$  = dependant variable

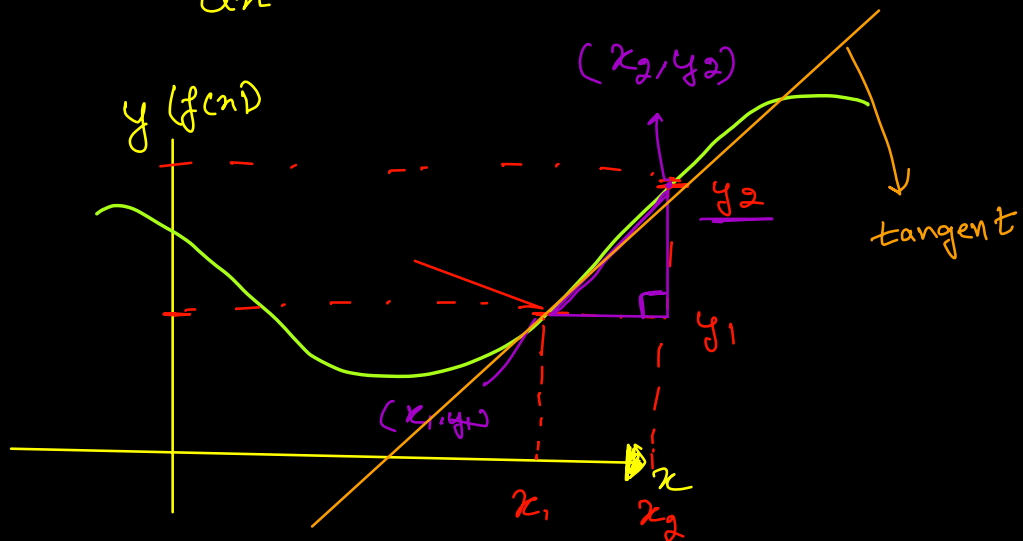
$x$  = independent variable

$$\frac{dy}{dx} = y' = f'(x)$$

\$ student

Study hrs	Scores	Area	Price
→			→
→			→
→			→
→			→
→			→

$$\frac{dy}{dx} = \text{change in } x \text{ wrt } y.$$



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \text{its a change of } y \text{ wrt } x$$

$$= \frac{\Delta y}{\Delta x}$$

$$\boxed{\Delta \text{price}} = 12 - 10 = 2$$

## Partial differentiation

$$\frac{\partial y}{\partial x}$$

$$\begin{array}{ccccc} \text{price of} & y = & 3x_1 & + & 4x_2 \\ \text{house} & & \Downarrow & & \Downarrow \\ & & \text{climate} & & \text{area of house} \end{array}$$

Let us assume that climate & area are somehow dependent on each other.

$$\frac{\partial y}{\partial x_1} = \frac{\partial (3x_1)}{\partial x_1} + \cancel{\frac{\partial (4x_2)}{\partial x_1}} \quad \circ$$

$$\frac{dy}{dx_1} = \frac{d(3x_1)}{dx_1} + \frac{d(4x_2)}{dx_2} \times \left( \frac{dx_2}{dx_1} \right)$$

$$y \rightarrow f(x) \rightarrow g(u)$$

$$\frac{dy}{dx} = \frac{dg(u)}{du} * \frac{du}{dx}$$

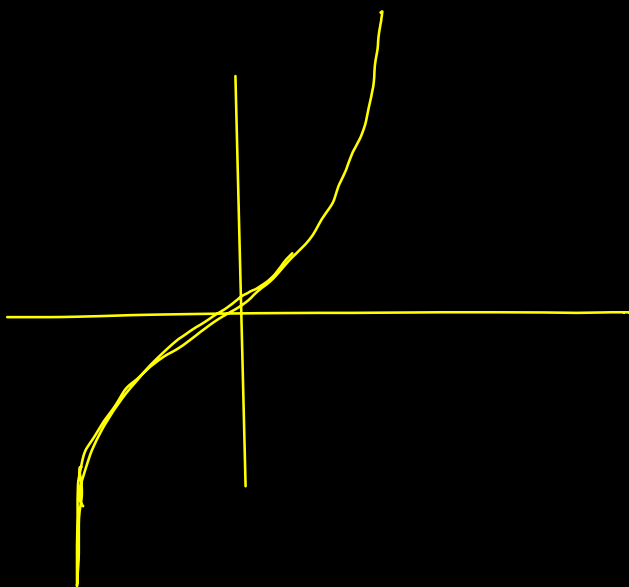
## Rules of Differentiation

Sum rule  $\frac{d(f_1(x) + f_2(x))}{dx} = \frac{df_1(x)}{dx} + \frac{df_2(x)}{dx}$

$$\frac{d(ax_1 + bx_2)}{dz} \Rightarrow \frac{d(ax_1)}{dz} + \frac{d(bx_2)}{dz}$$

## Power Rule

$$\frac{d x^3}{dx} = 3x^2 \quad \frac{dx^n}{dx} = nx^{n-1}$$



## Product rule

$$f(x) = u(x) * v(x)$$

$$\frac{d f(x)}{dx} = u(x) \left( \frac{dv(x)}{dx} \right) + v(x) \left( \frac{du(x)}{dx} \right)$$

## Chain rule

$$y = f(x) = g(u)$$

$$\frac{dy}{dx} = \left( \frac{dy}{du} \right) * \left( \frac{du}{dx} \right)$$

$$y = (x^4 - 1)^{50}$$

$$x^4 - 1 = t$$

$$= t^{50}$$

$$y = \underbrace{(x^4 - 1)^{50}}_{= f(x)} = t^{50} = \underbrace{g(t)}$$

$$y = f(x) = (x^4 - 1)^{50}$$

$$g(x) = (x^4 - 1)^{50}$$

$$n = 50$$

$$t = g(x) = x^4 - 1$$

$$u(t) = t^{50}$$

$$y = f(x) = u(g(x))$$

$$t = x^4 - 1$$

$$u(t) = t^{50}$$

$$u(x^4 - 1) = (x^4 - 1)^{50}$$

$$u(g(x))$$

$$\frac{df}{dx} = \frac{du}{dt} * \frac{dt}{dx}$$

$$= \frac{du}{dt} * \frac{dt}{dx}$$

$$t^{50}$$

$$\downarrow$$

$$= 50t^{49} * 4x^3$$

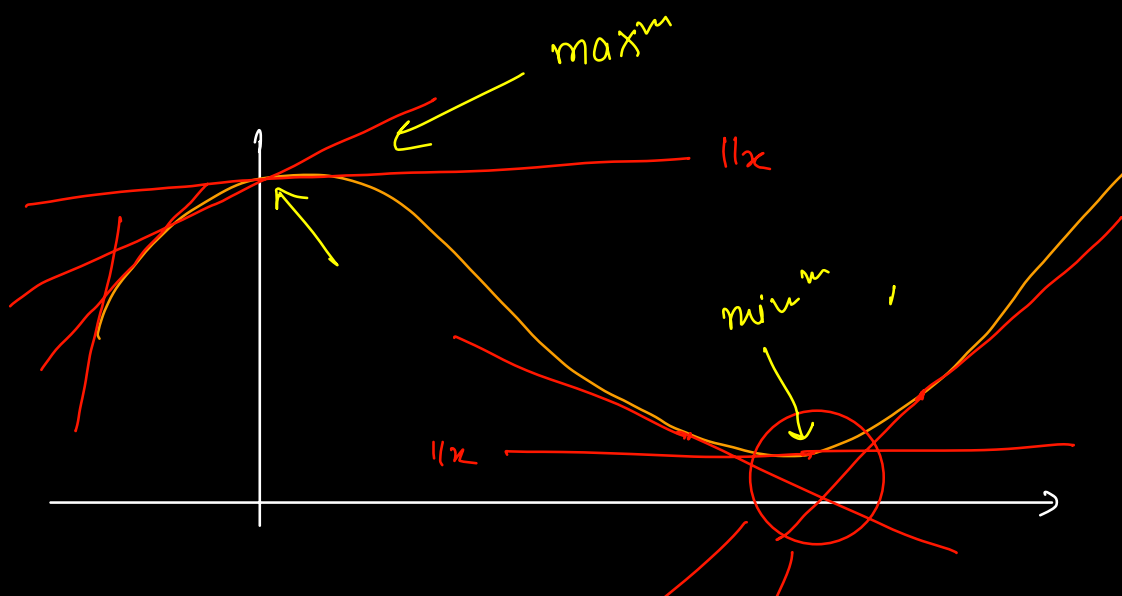
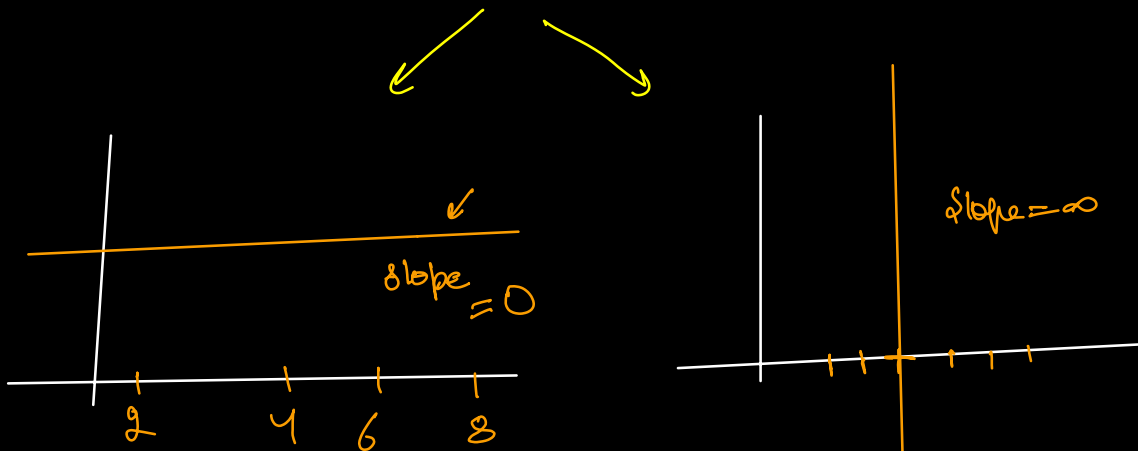
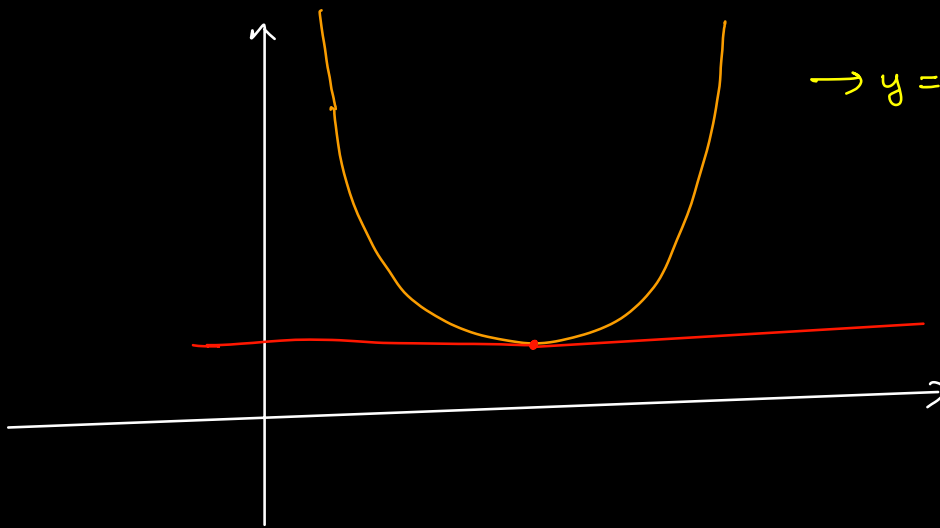
$$= 50(x^4 - 1)^{49} * 4x^3$$

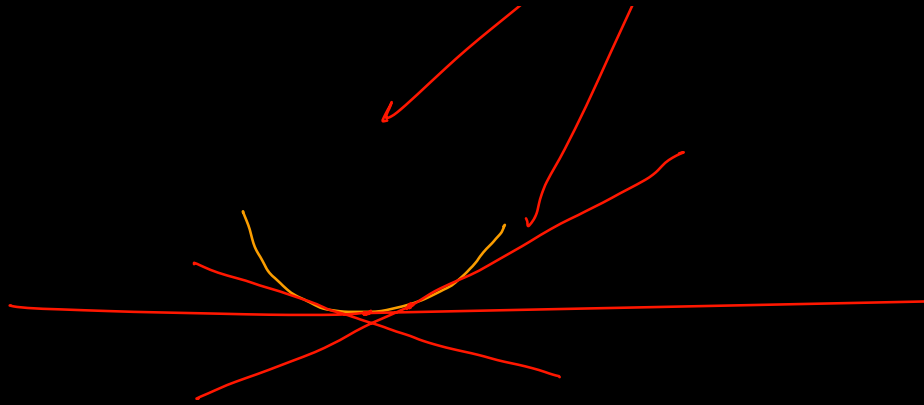
$$t = x^4 - 1$$

$$\frac{dt}{dx} = 4x^3$$



Minima & maxima





When my slope is 0, we get  
the max / min.

$$\frac{dy}{dx} = \text{slope} = 0$$

I am gonna get point  
of maxima / minima.

$$y = 3x^2 + 2$$

$$\frac{dy}{dx} = \frac{d(3x^2)}{dx} + \frac{d(2)}{dx} \quad \begin{matrix} \nearrow 0 \\ \searrow \end{matrix}$$

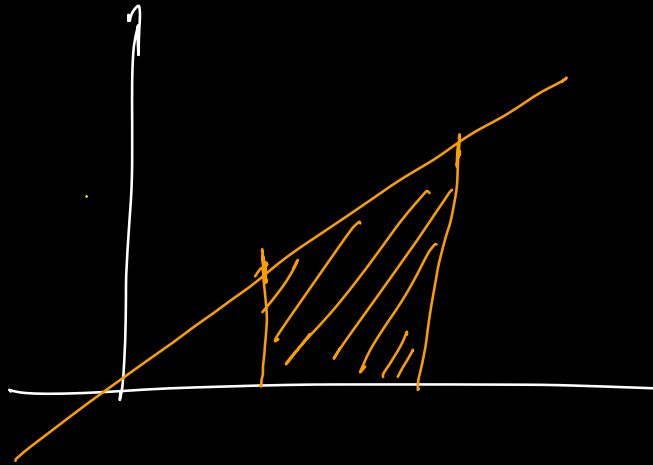
$$= 6x$$

$$3 \left[ \frac{dx^2}{dx} \right]$$

$$\Downarrow \\ 2x^{2-1} = 2x'$$

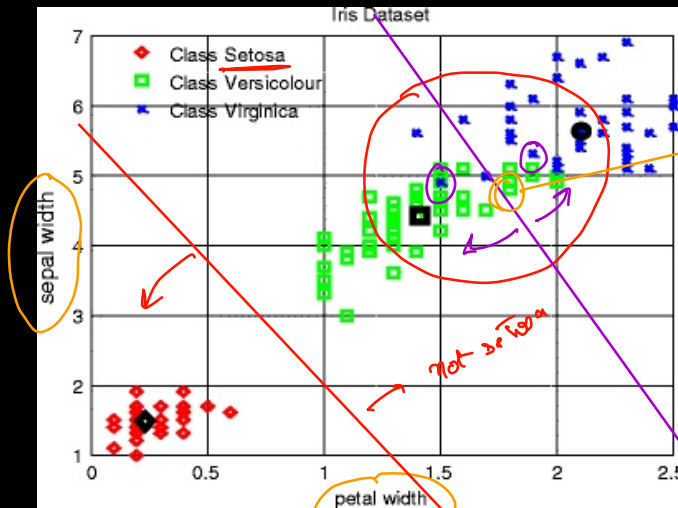
$$\frac{dy}{dx} = 0 \quad \Rightarrow \quad \begin{matrix} 6x = 0 \\ \underline{x = 0} \end{matrix}$$

Integration  $\Rightarrow$



Probability

iris dataset



what is your  
Probability of  
being versicolour  
or  
virginica

$$\text{Probability} = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$$



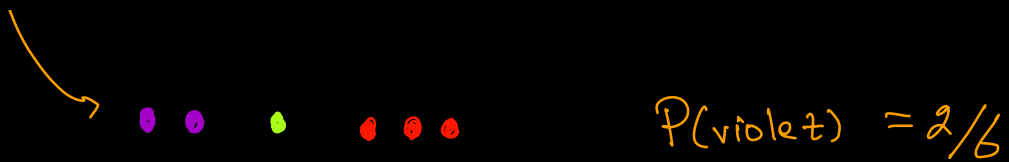
$$P(\text{green}) = \frac{2}{7}$$

condition

$P(\text{violet})$

given that green bowl is

already removed / got out,

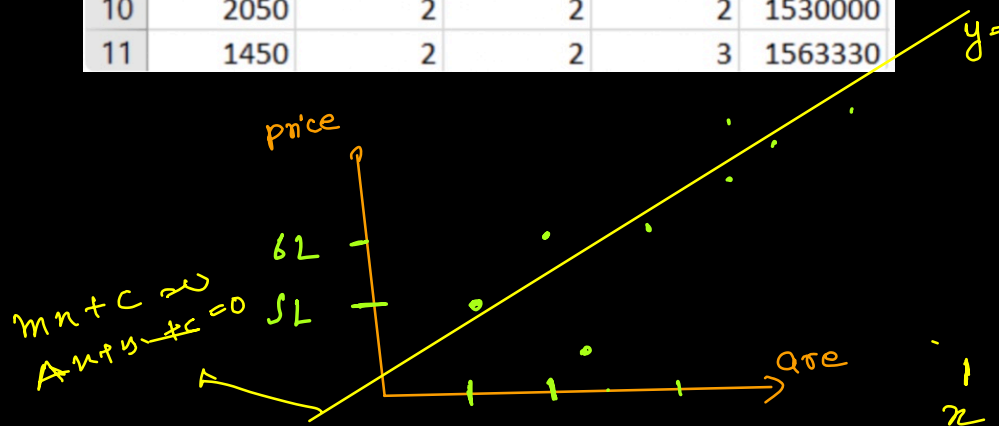


Conditional Probability

Statistics

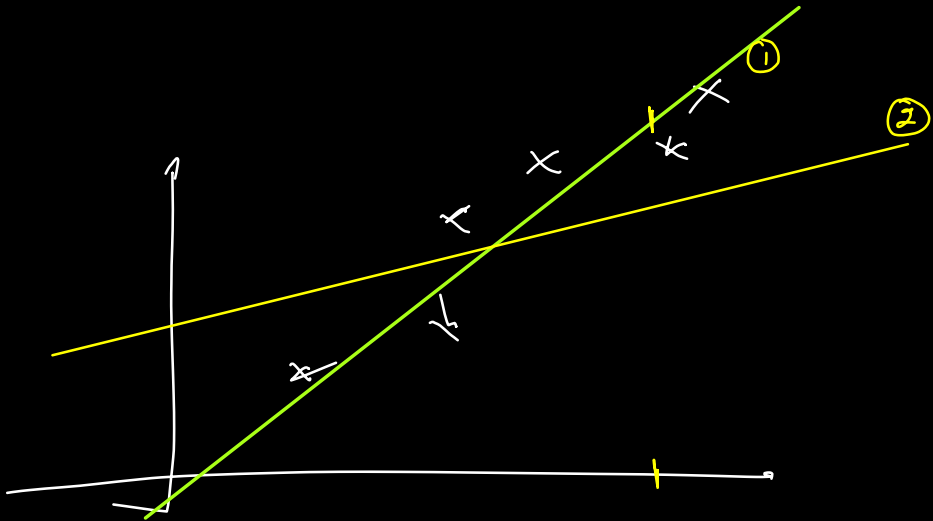
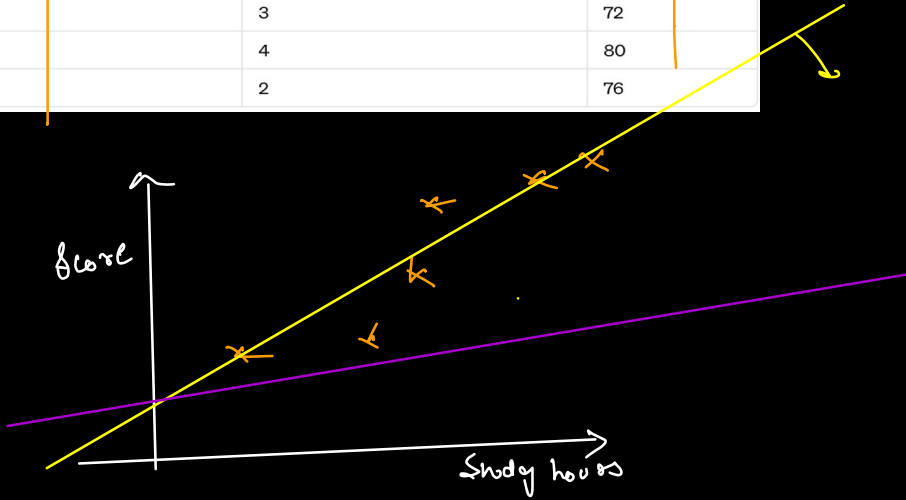
# Linear Regression

	A	B	C	D	E
1	area	bedrooms	balcony	age	price
2	1200	2	0	2	500000
3	2300	3	2	5	620000
4	2500	4	2	1	122500
5	3650	5	3	3	600000
6	1800	3	1	5	2122000
7	3000	3	1	4	120000
8	1222	1	0	2	450000
9	4600	5	3	1	650000
10	2050	2	2	2	1530000
11	1450	2	2	3	1563330





Study Hours	Test Prep Hours	Score
3	2	75
5	1	82
2	3	69
7	2	88
4	4	78
6	1	85
1	2	62
3	3	72
5	4	80
4	2	76



⇒ It is a prediction algorithm

⇒ It is used for regression problem

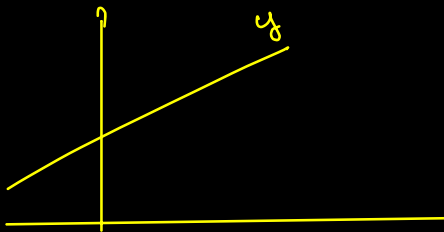
⇓

we have

continuous value as O/P

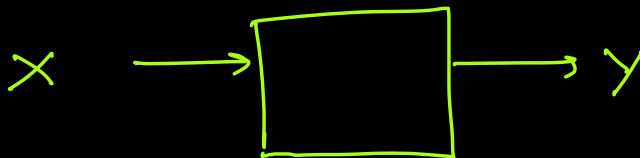
Give training data → learn some  $f^n$

$$\text{Score} = \underline{3}^{\text{rd}} \text{ Study hrs} + \underline{5}$$



$$y = \underline{3x} + 5$$

↑

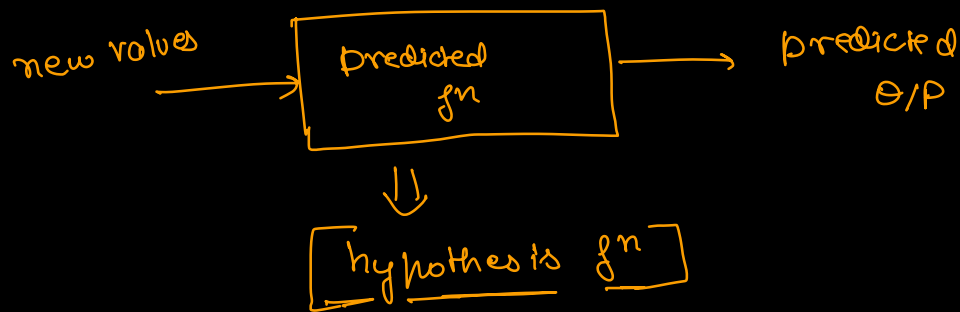


want our algo

Start giving us

some output based

on training.



3 features

$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + c$$

$\Downarrow$   
score

$\Downarrow$   
study hrs

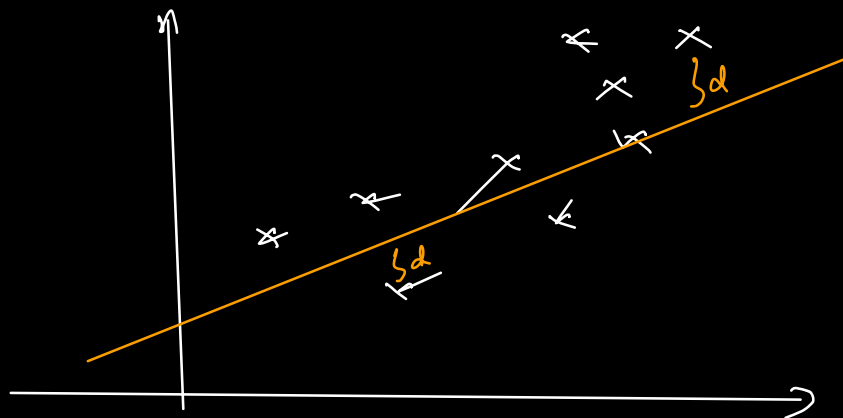
$\Downarrow$   
play hrs

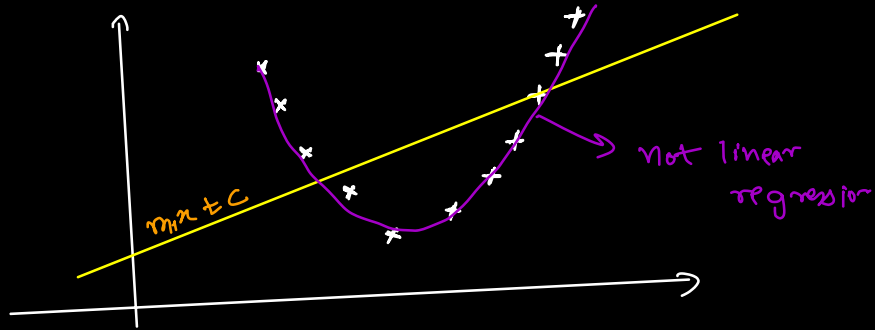
$\Downarrow$   
sleep hrs

$+ m_4 (x_4)$   
 $\Downarrow$   
 phone model

⇒ Our  $f^n$  is dependent on each feature linearly.

⇒ more dependency ⇒ more  $m$





we are still gonna predict a line.

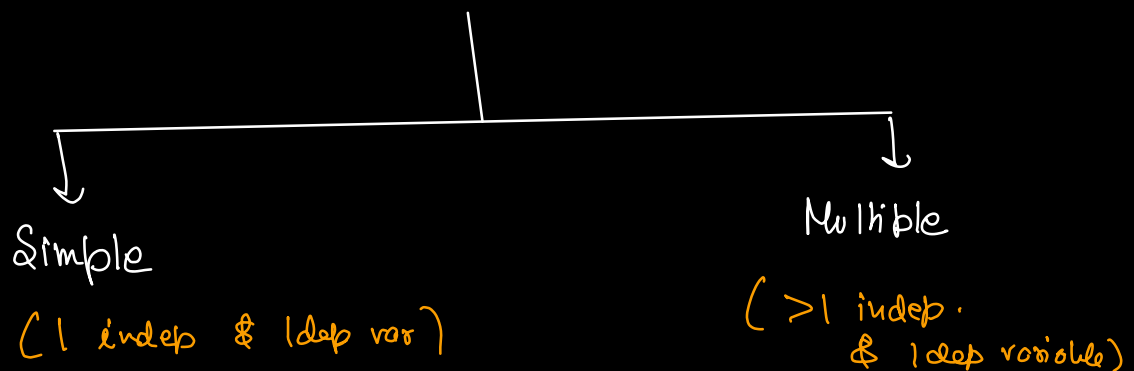
$$y = \underline{m_1 x} + \underline{c} \rightarrow 1 \text{ var}$$

$$y = m_1 x_1 + m_2 x_2 + c \rightarrow 2 \text{ var}$$

$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + c \rightarrow 3 \text{ var}$$

$$\Rightarrow \text{no. of variables} = (n+1)$$

dim<sup>n</sup>



## Simple Linear Regression

1 independent

1 dependent

$$y = \underline{m}x + \underline{c}$$

$$h_{\theta}(x) = \underline{\theta_1}x + \theta_0$$