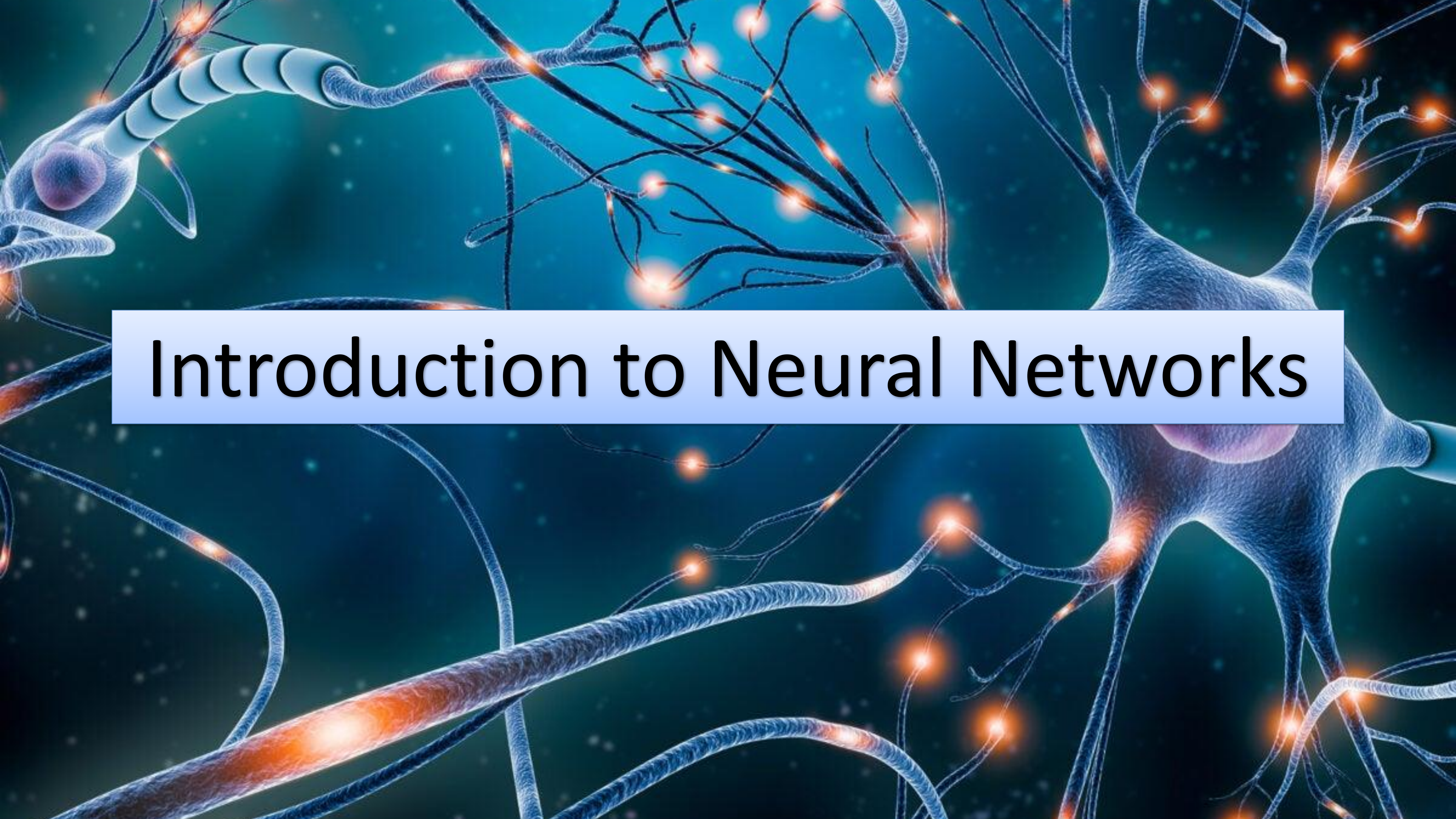
Introduction to Neural Networks



What is Deep Learning?

Artificial Intelligence

Any technique that enables computers to mimic human behavior

Machine Learning

Ability to learn without explicitly being programmed

Deep Learning

Extract patterns from data using neural networks

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Why Deep Learning and Why Now?

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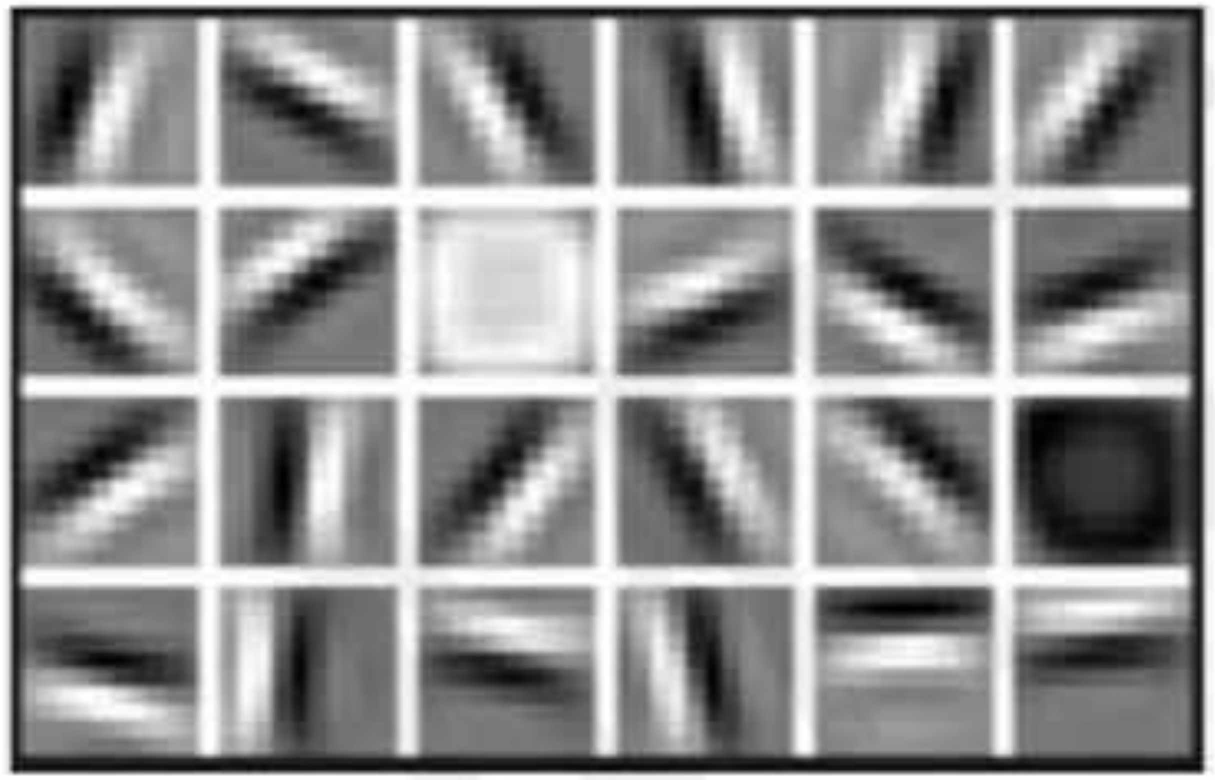
Why Deep Learning?

Hand engineered features are time consuming, brittle, and not scalable in practice

Can we learn the **underlying features** directly from data?

Low Level Features



Lines & Edges

Mid Level Features

Eyes & Nose & Ears

High Level Features

Facial Structure

Why Now?

1952 Stochastic Gradient Descent

1958 Perceptron
- Learnable Weights

⋮

1986 Backpropagation
- Multi-Layer Perceptron

1995 Deep Convolutional NN
- Digit Recognition

⋮

Neural Networks date back decades, so why the resurgence?

1. Big Data
- Larger Datasets
- Easier Collection & Storage

IMAGENET

WIKIPEDIA
The Free Encyclopedia

2. Hardware
- Graphics Processing Units (GPUs)
- Massively Parallelizable

3. Software
- Improved Techniques
- New Models
- Toolboxes

TensorFlow

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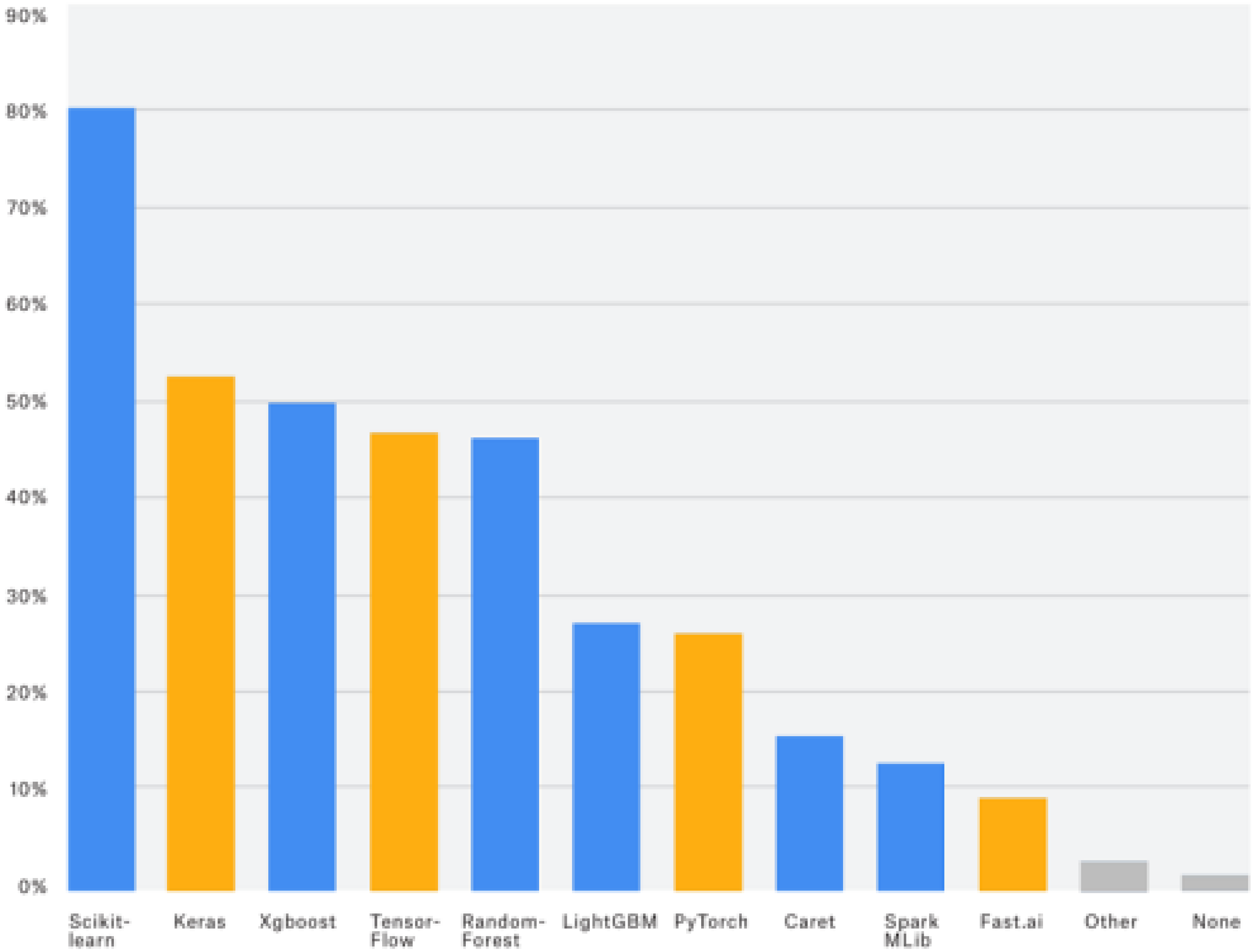

Trends in Deep Learning

The Perceptron

The structural building block of deep learning

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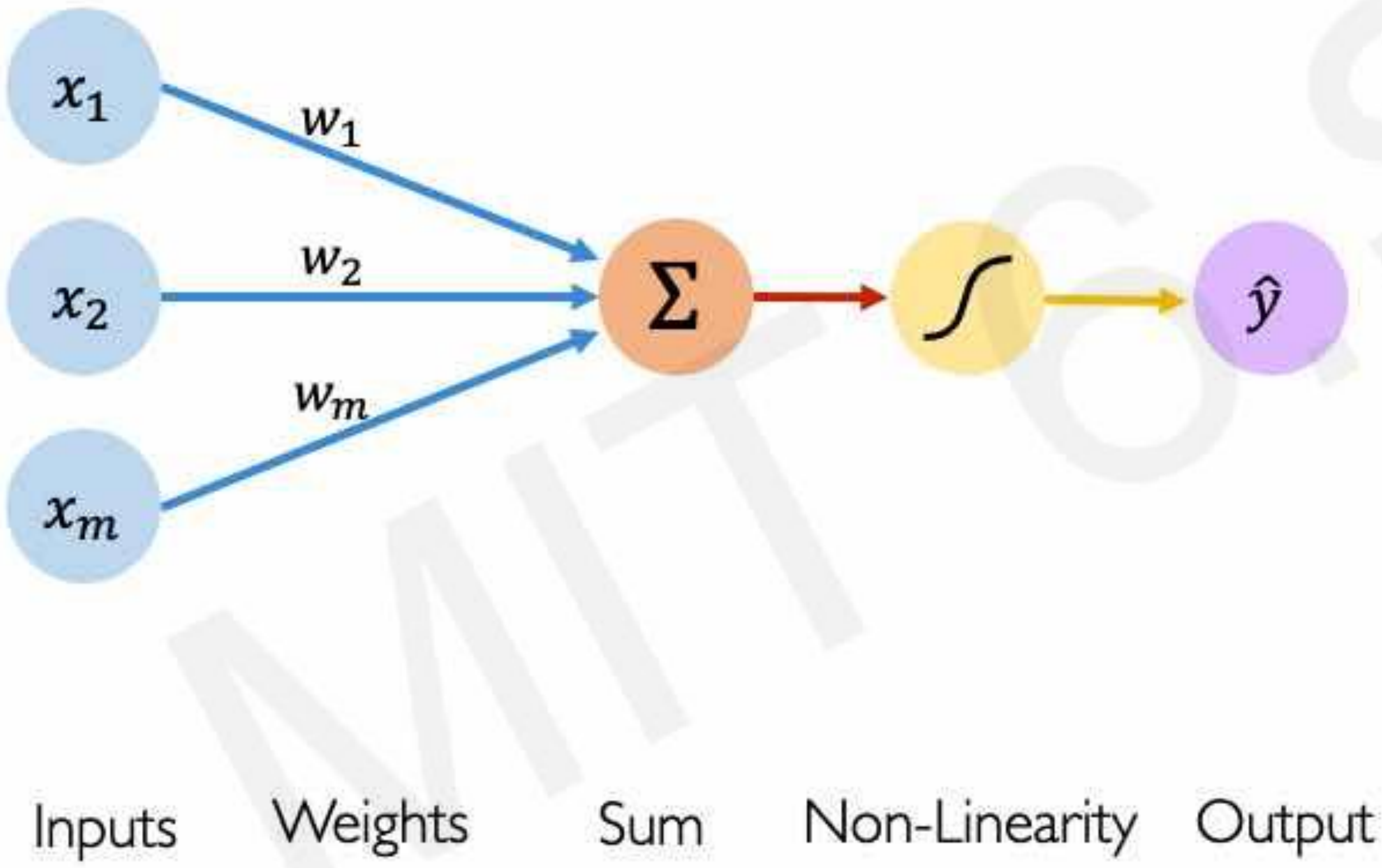
The Perceptron: Forward Propagation



Output

Linear combination of inputs

$$\hat{y} = g\left(\sum_{i=1}^{m} x_i \, w_i\right)$$

Non-linear activation function

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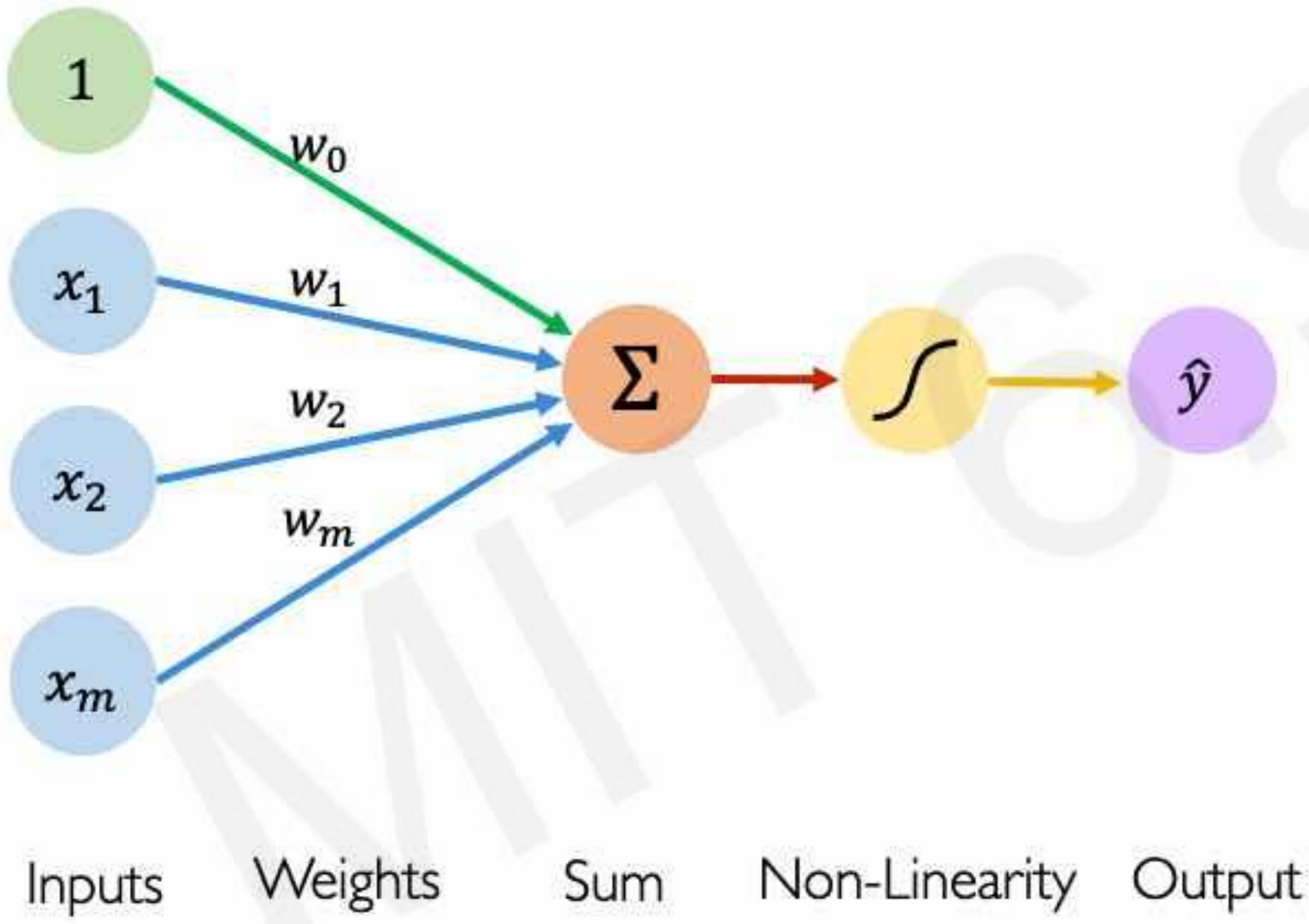
The Perceptron: Forward Propagation



Output

Linear combination of inputs

$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i \, w_i \right)$$

Non-linear activation function

Bias

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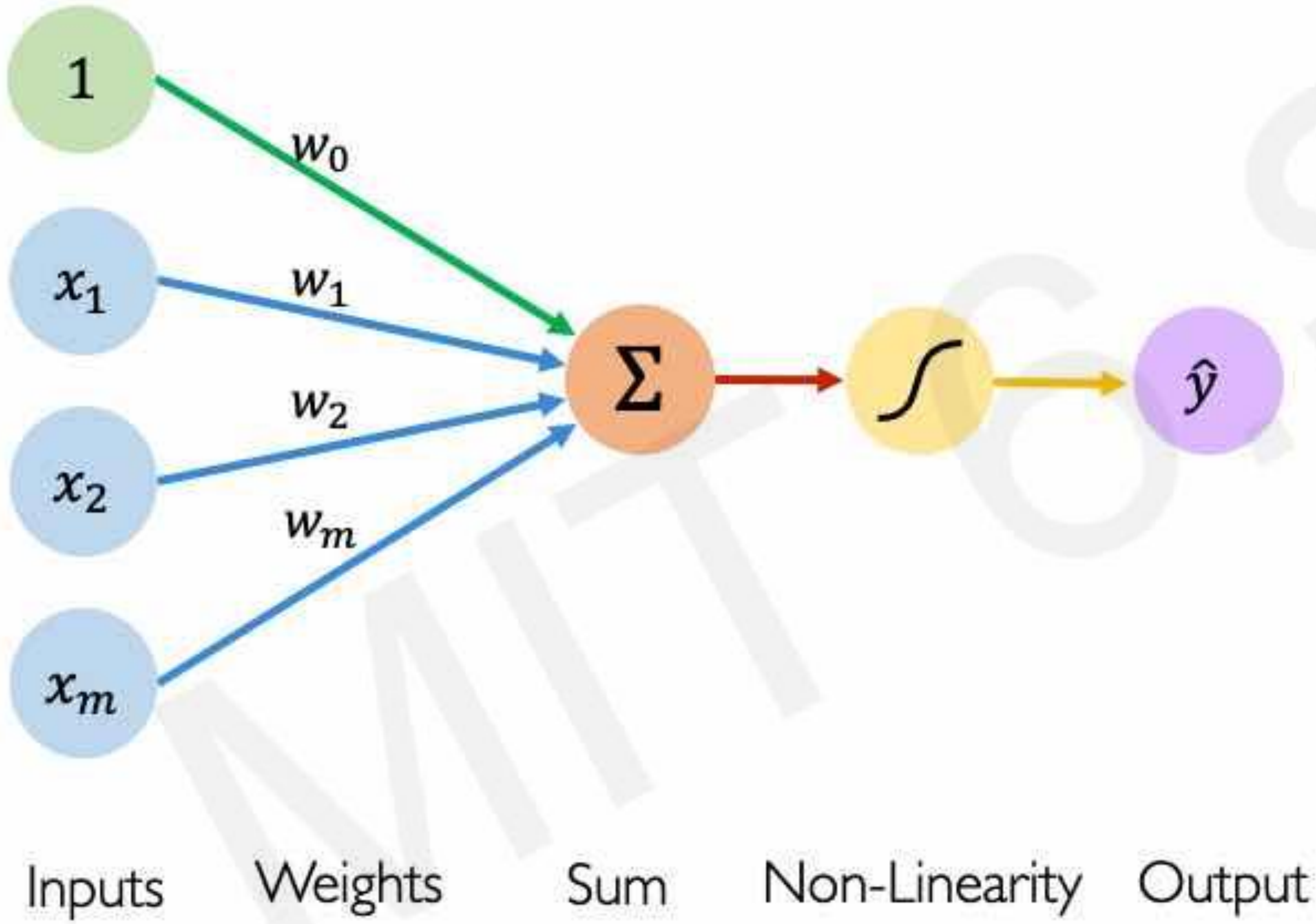
The Perceptron: Forward Propagation



$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i \, w_i \right)$$

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

where: $\boldsymbol{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$ and $\boldsymbol{W} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$

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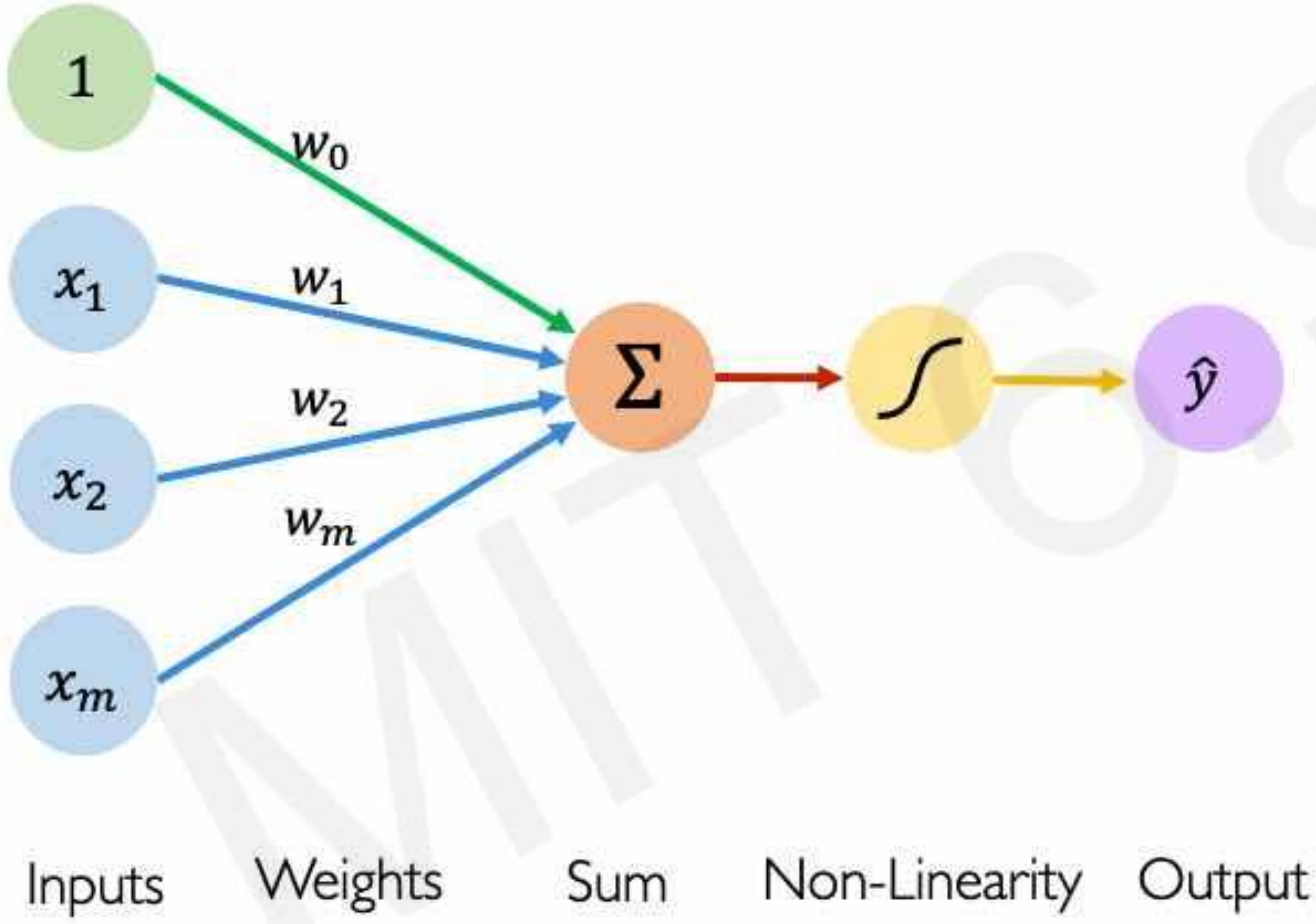
The Perceptron: Forward Propagation

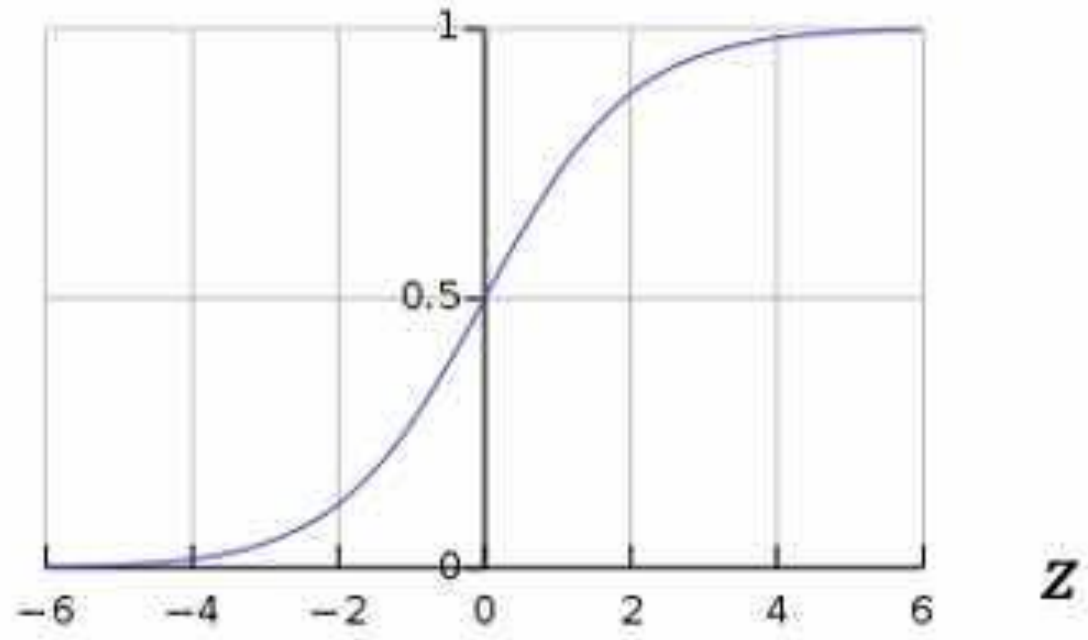


Activation Functions

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

- Example: sigmoid function

$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$



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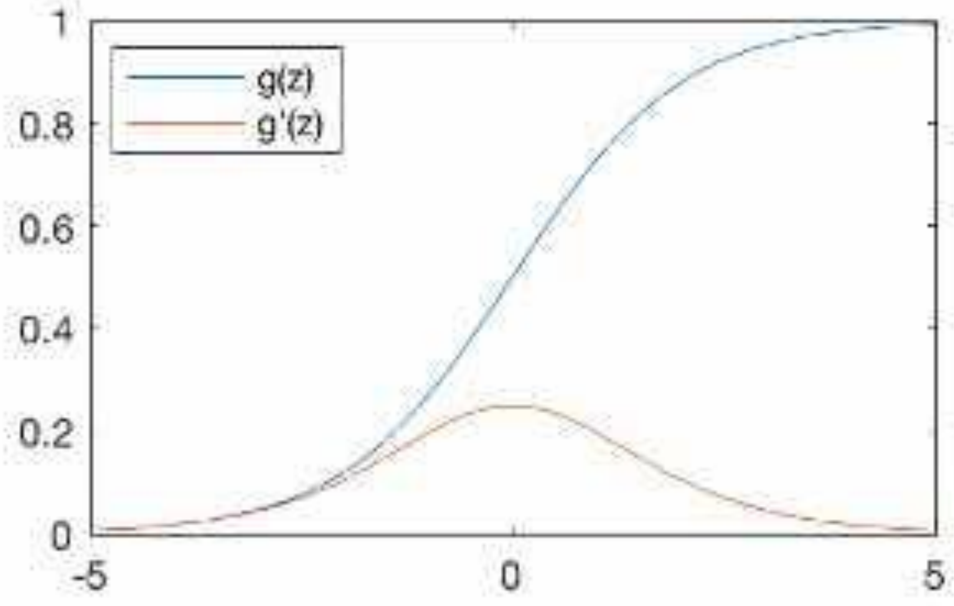
Common Activation Functions

Sigmoid Function

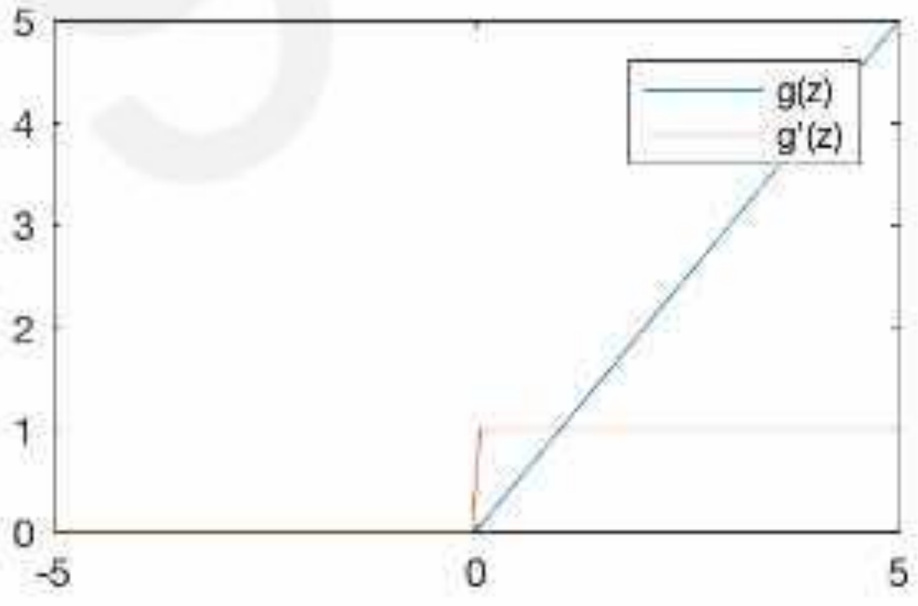


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

```
tf.math.sigmoid(z)
```

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

```
tf.nn.relu(z)
```

TensorFlow code blocks

NOTE: All activation functions are non-linear

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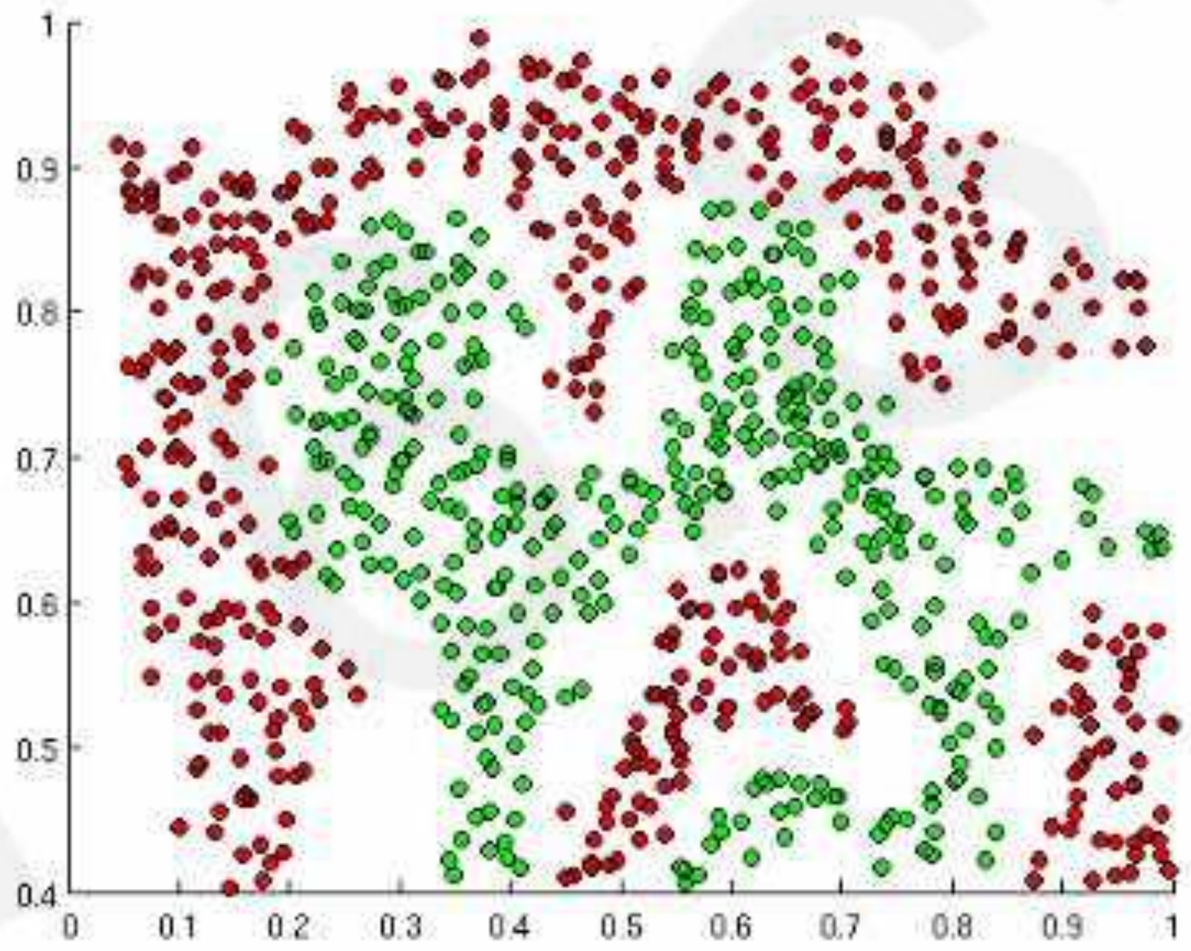
Importance of Activation Functions

*The purpose of activation functions is to **introduce non-linearities** into the network*



What if we wanted to build a neural network to
distinguish green vs red points?

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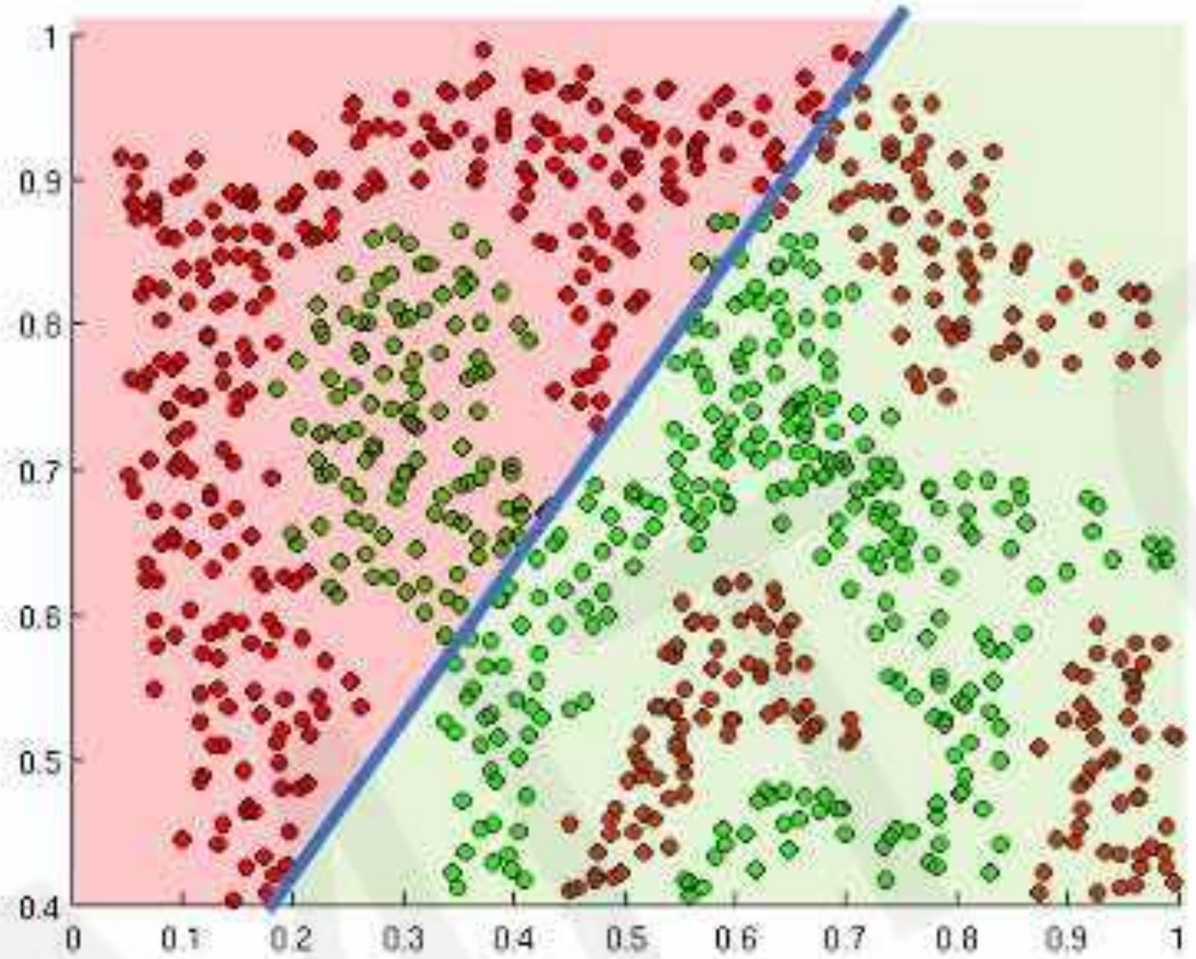
Importance of Activation Functions

The purpose of activation functions is to ***introduce non-linearities*** *into the network*



Linear activation functions produce linear decisions no matter the network size

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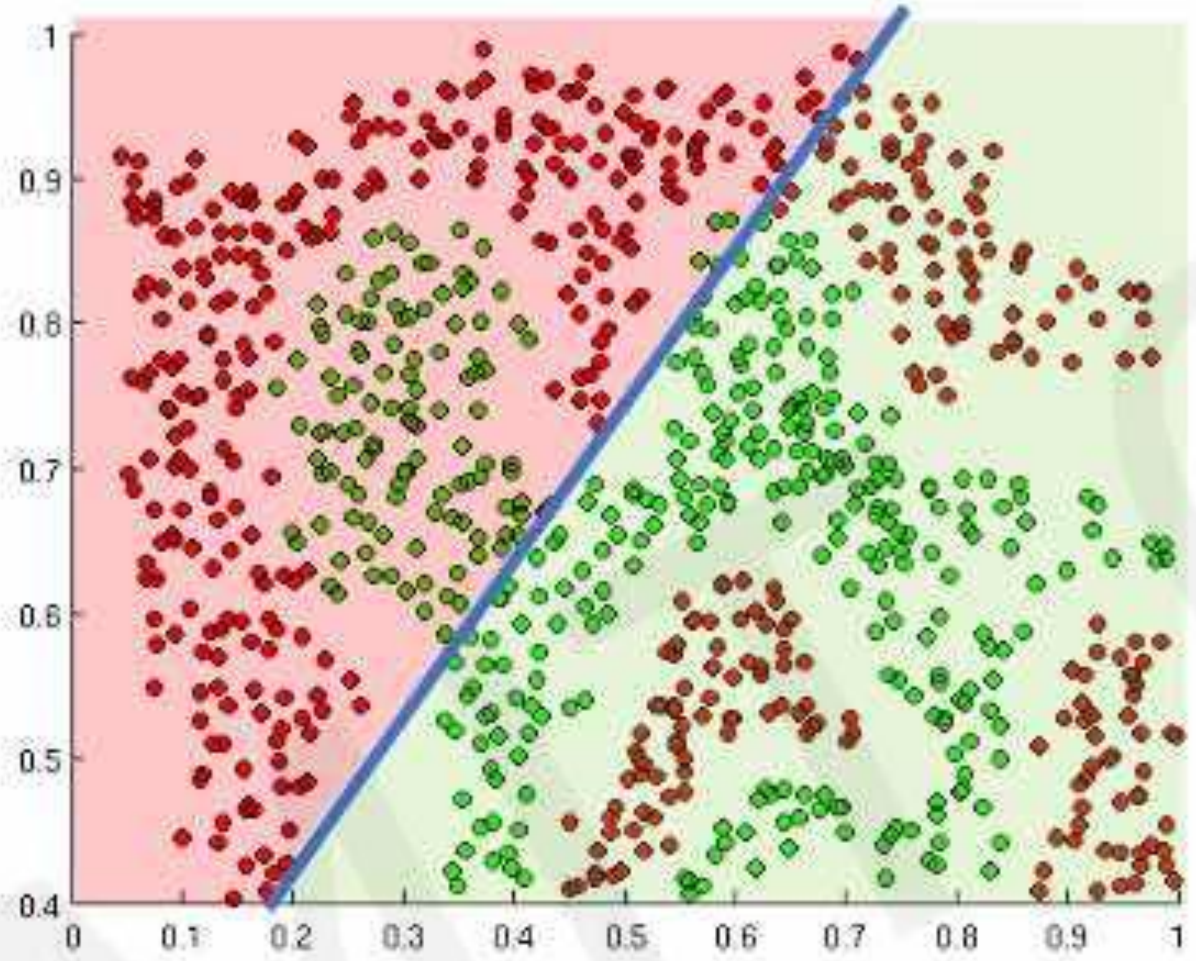
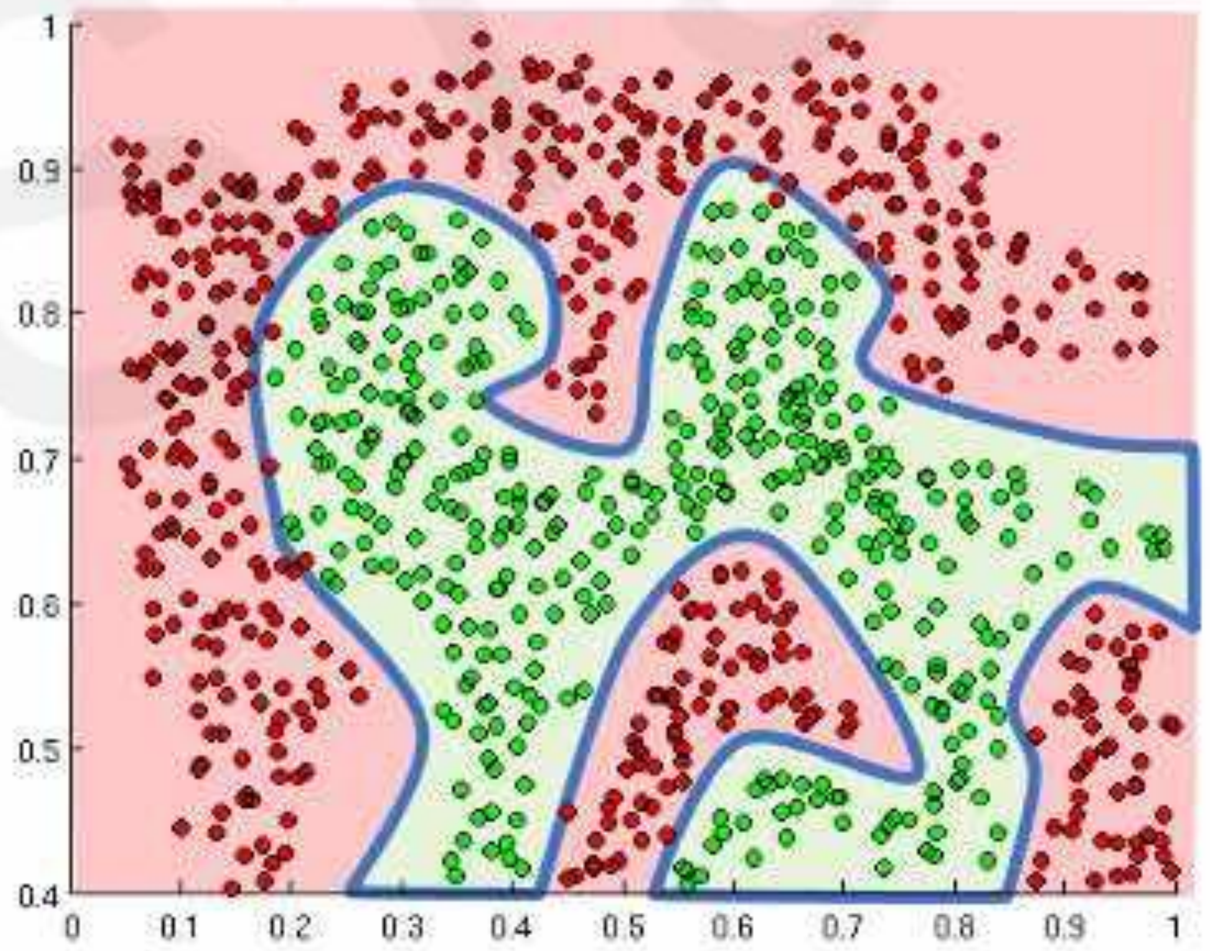
Importance of Activation Functions

*The purpose of activation functions is to **introduce non-linearities** into the network*



Linear activation functions produce linear decisions no matter the network size



Non-linearities allow us to approximate arbitrarily complex functions

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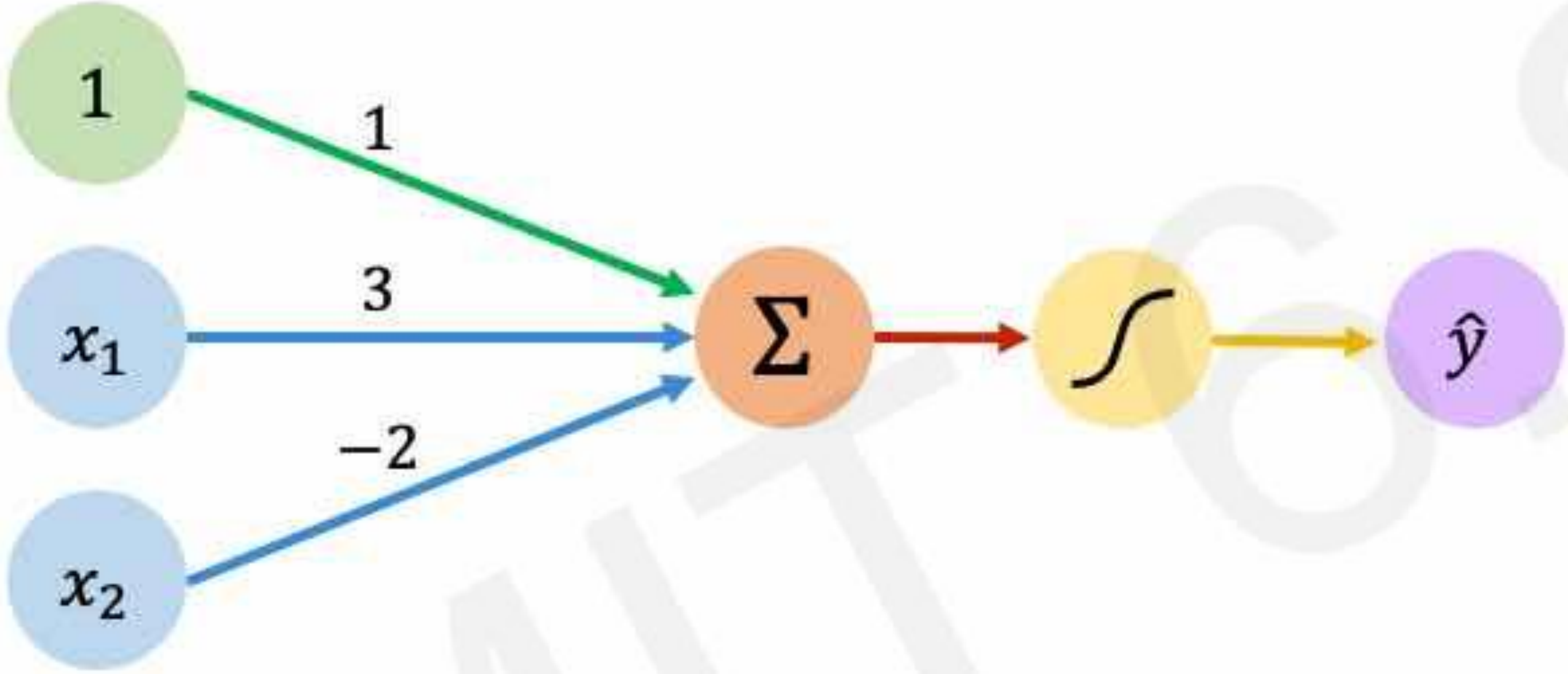
The Perceptron: Example



We have: $w_0 = 1$ and $\boldsymbol{W} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$
$$= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$$
$$\hat{y} = g\underbrace{(1 + 3x_1 - 2x_2)}_{\text{This is just a line in 2D!}}$$

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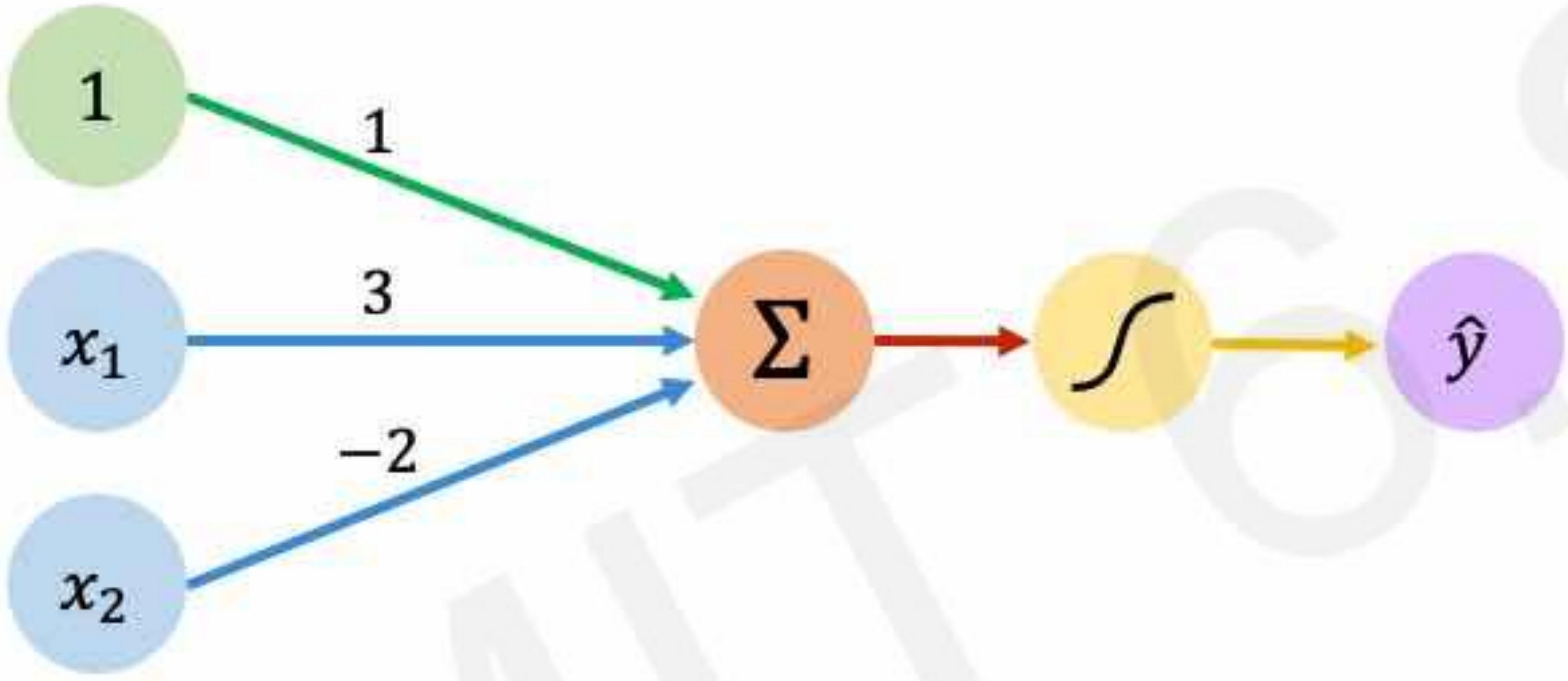
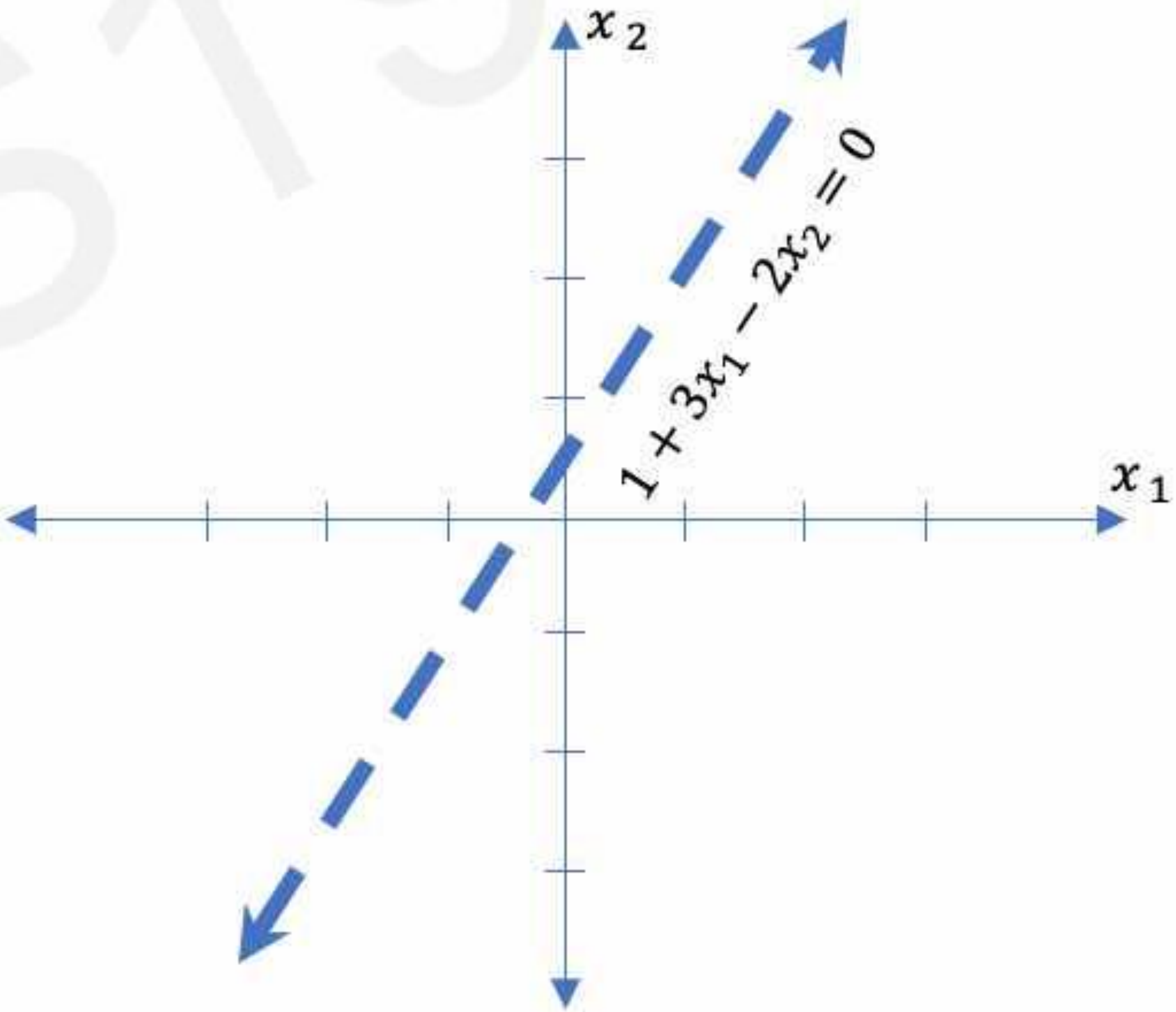
The Perceptron: Example



$$\hat{y} = g(1 + 3x_1 - 2x_2)$$



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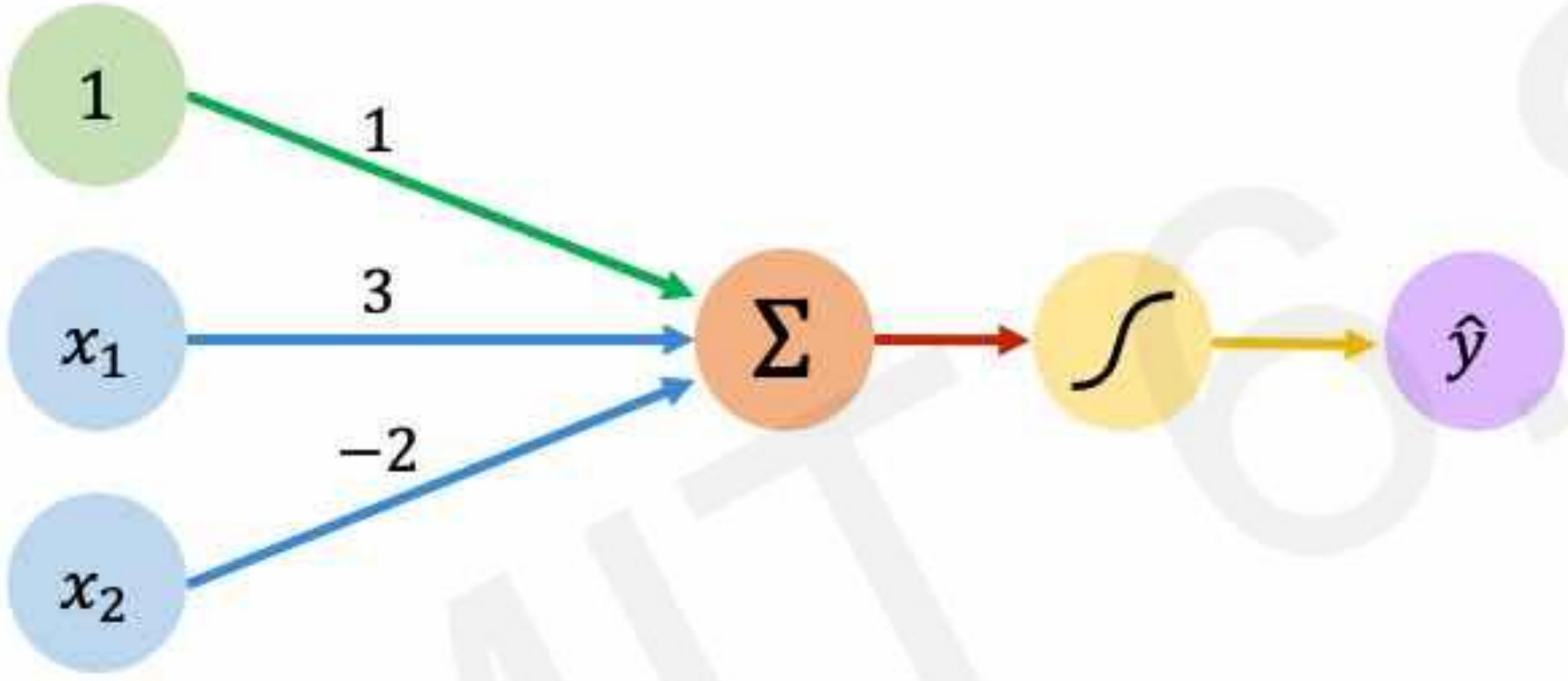
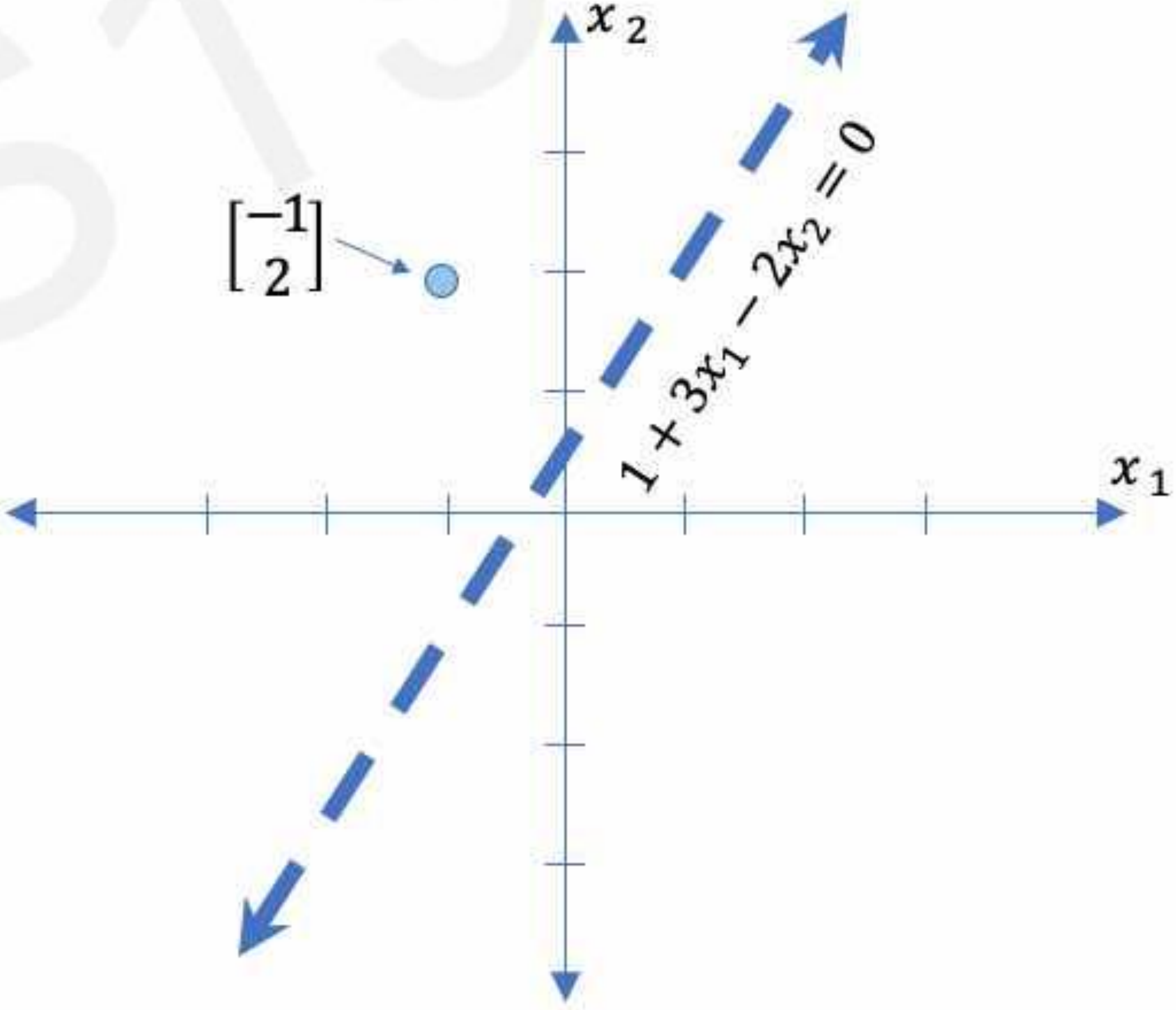
The Perceptron: Example



Assume we have input: $\boldsymbol{X} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\hat{y} = g\big(1 + (3*-1) - (2*2)\big)$$
$$= g(-6) \approx 0.002$$

$$\hat{y} = g(1 + 3x_1 - 2x_2)$$



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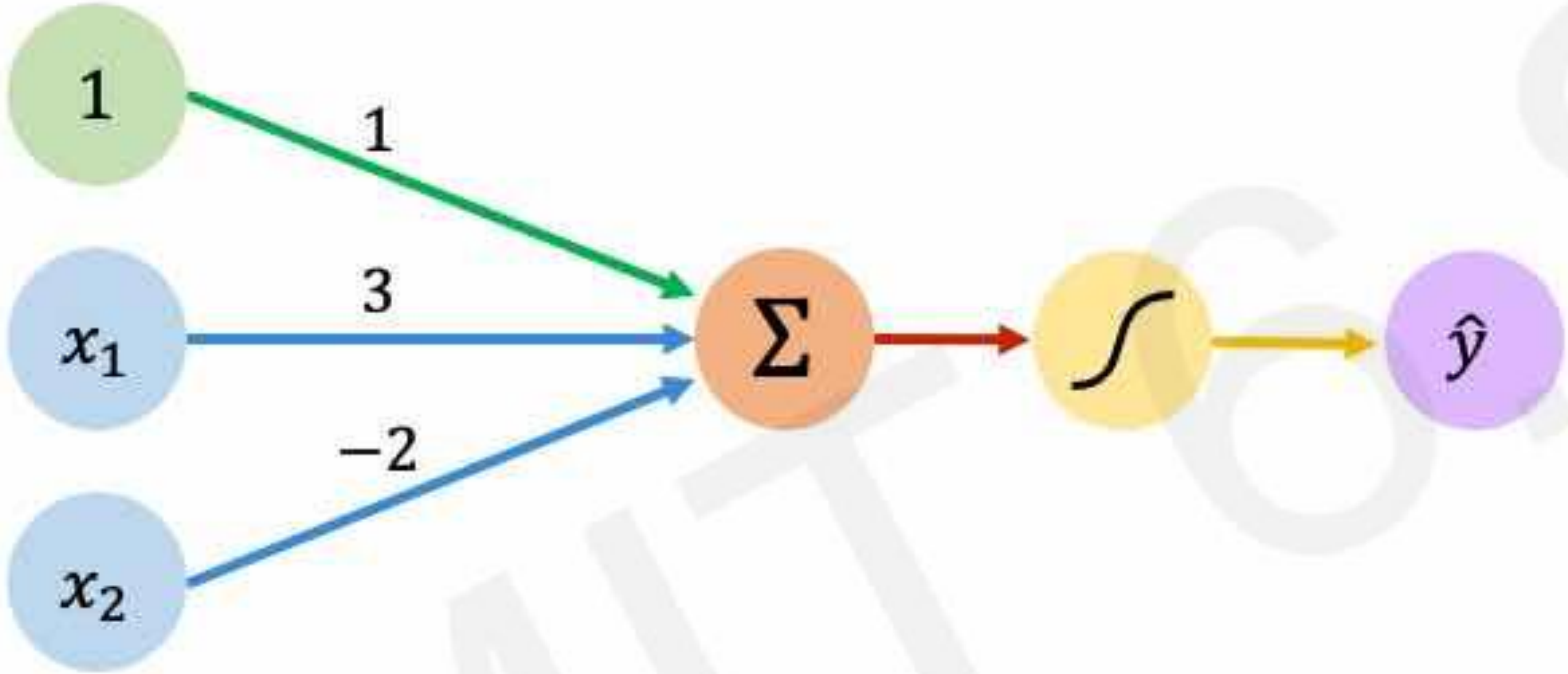
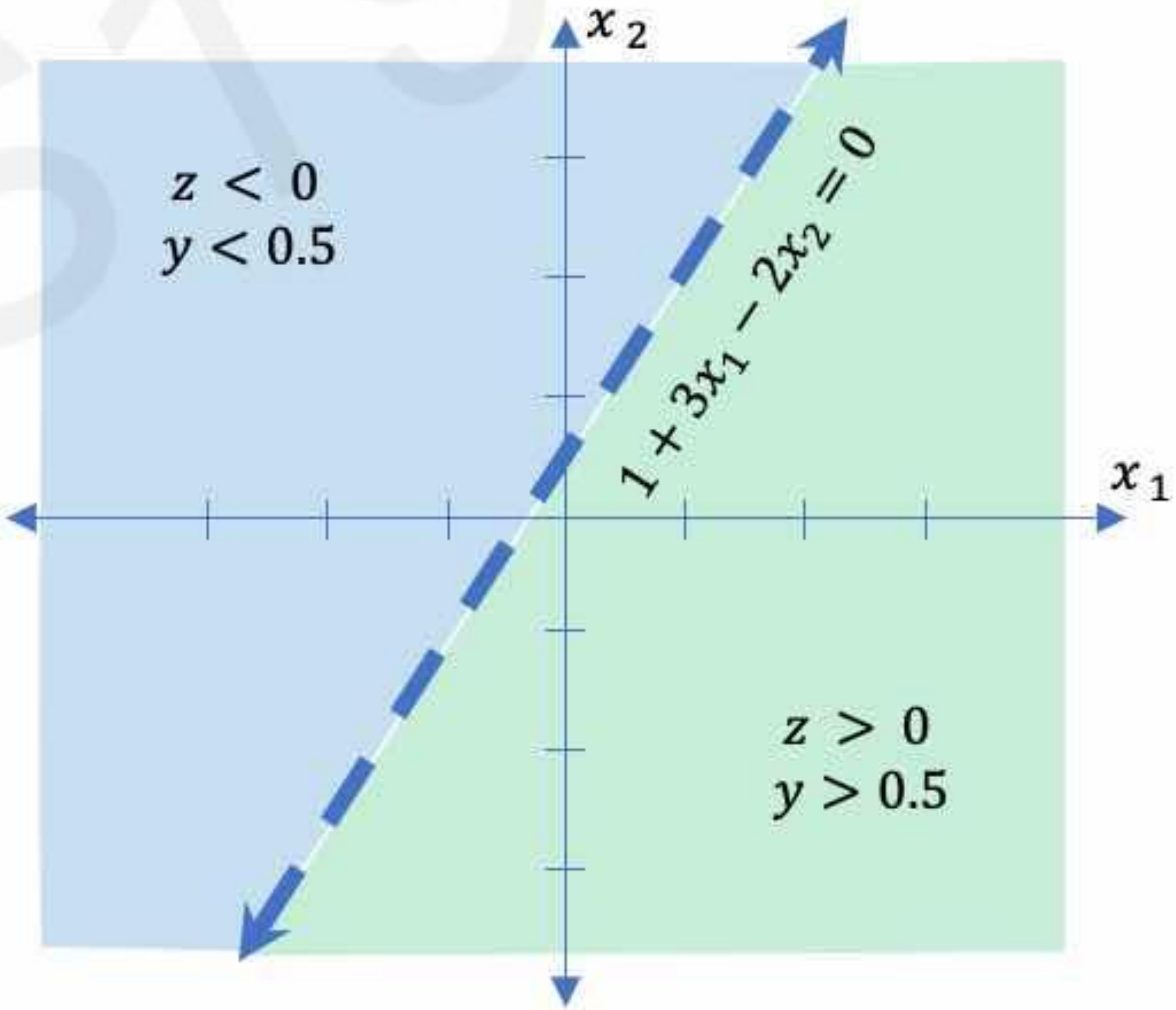
The Perceptron: Example



$\hat{y} = g(1 + 3x_1 - 2x_2)$



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Building Neural Networks with Perceptrons

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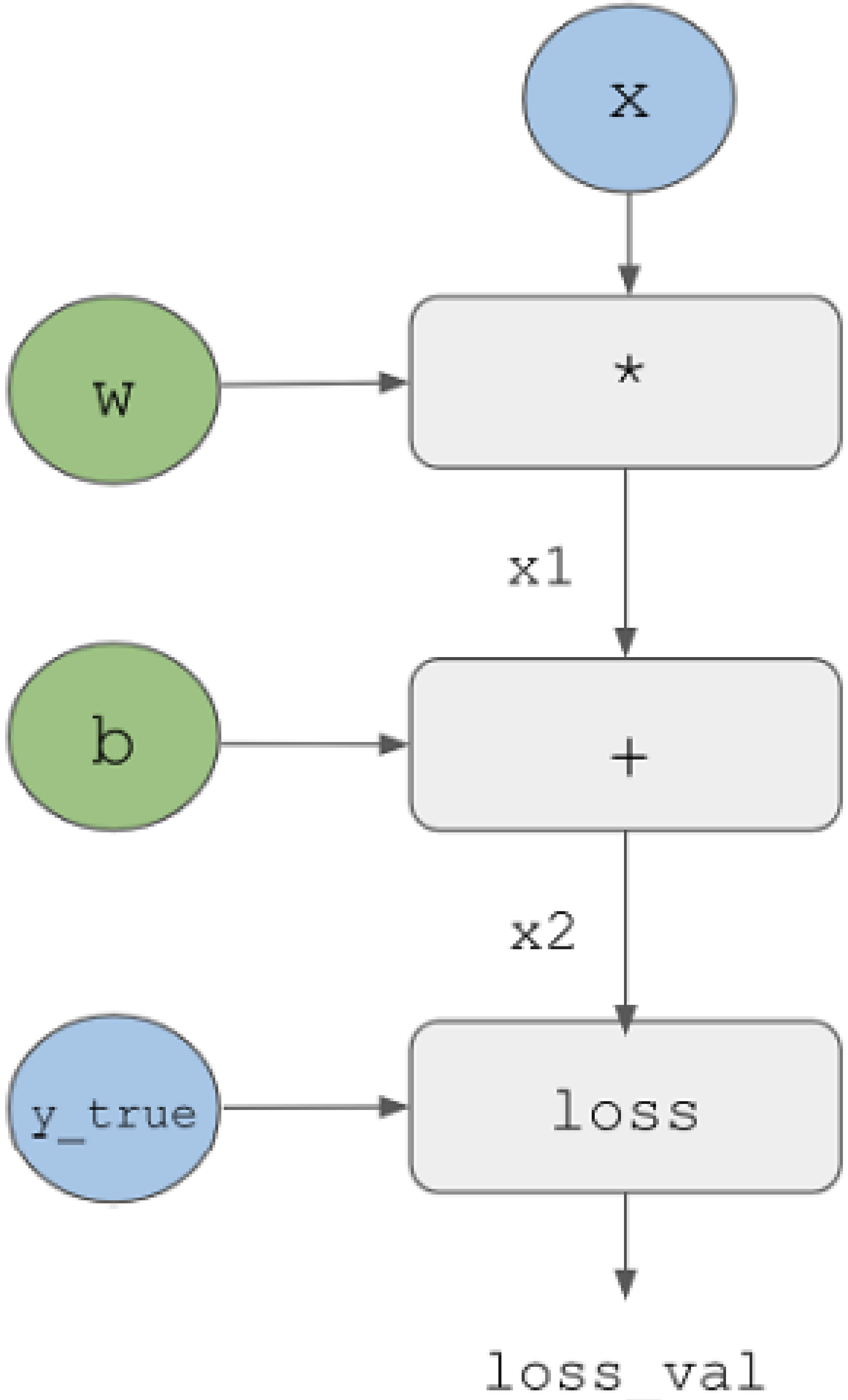


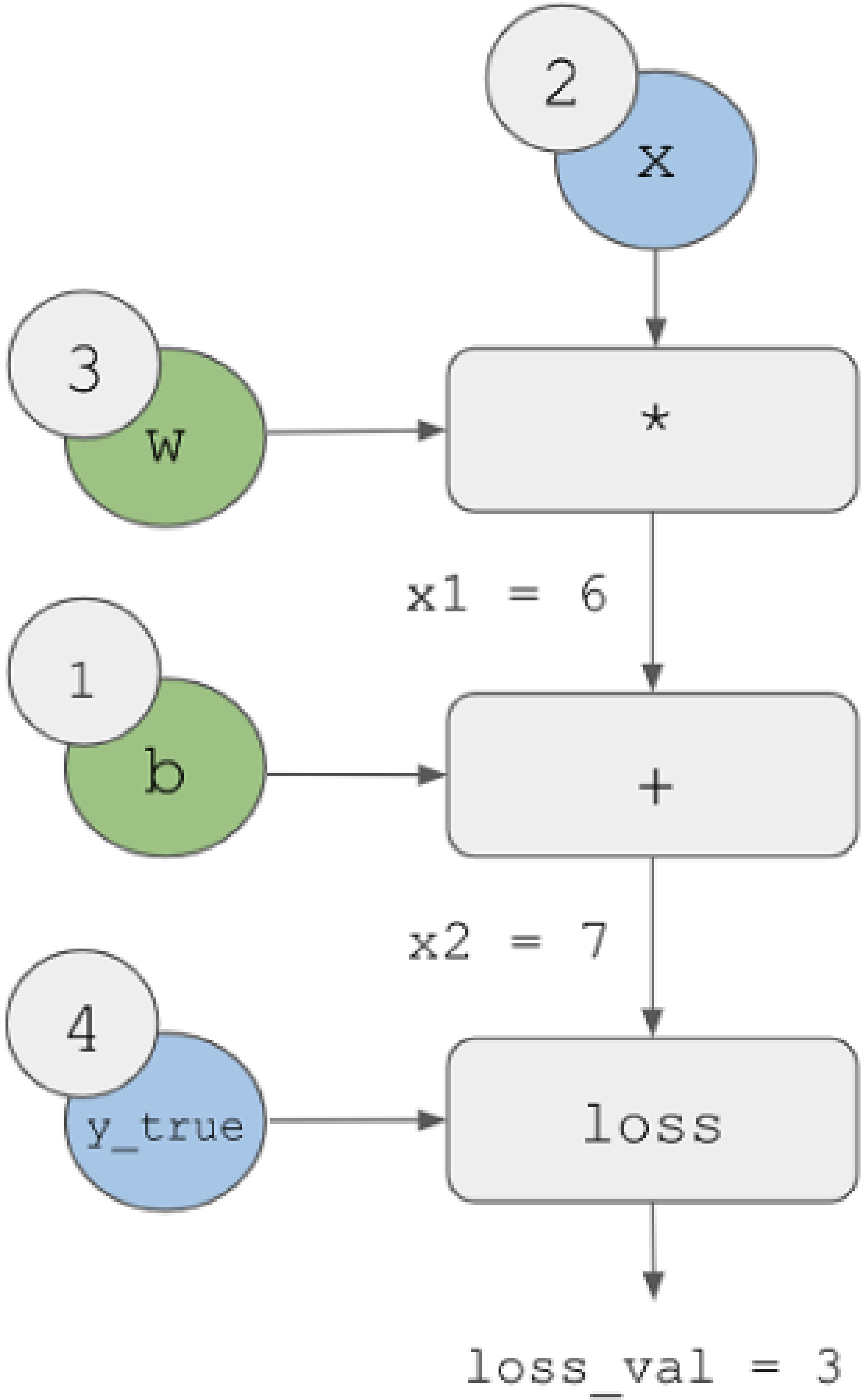


Figure 2.21 Running a forward pass

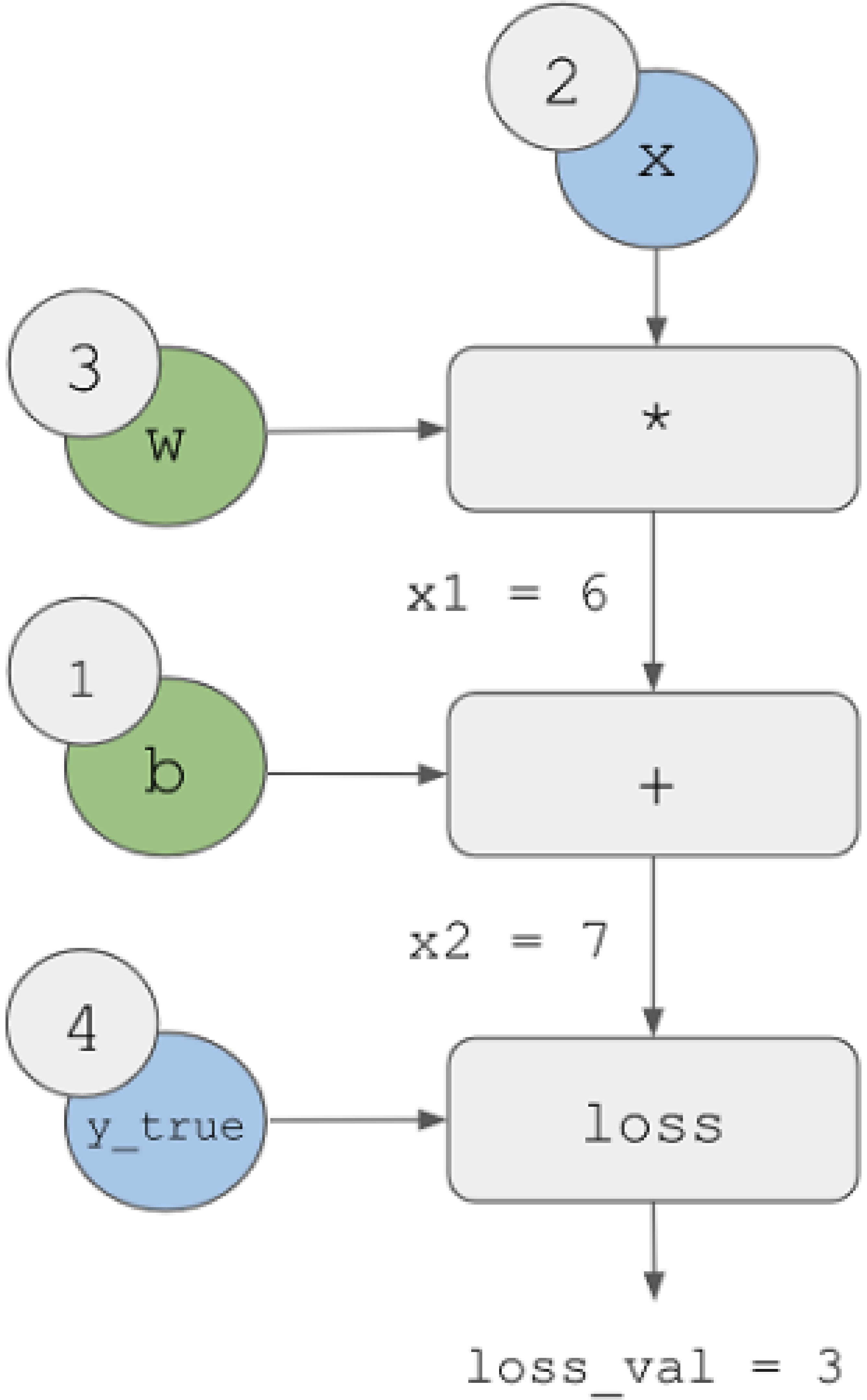


Figure 2.21 Running a forward pass

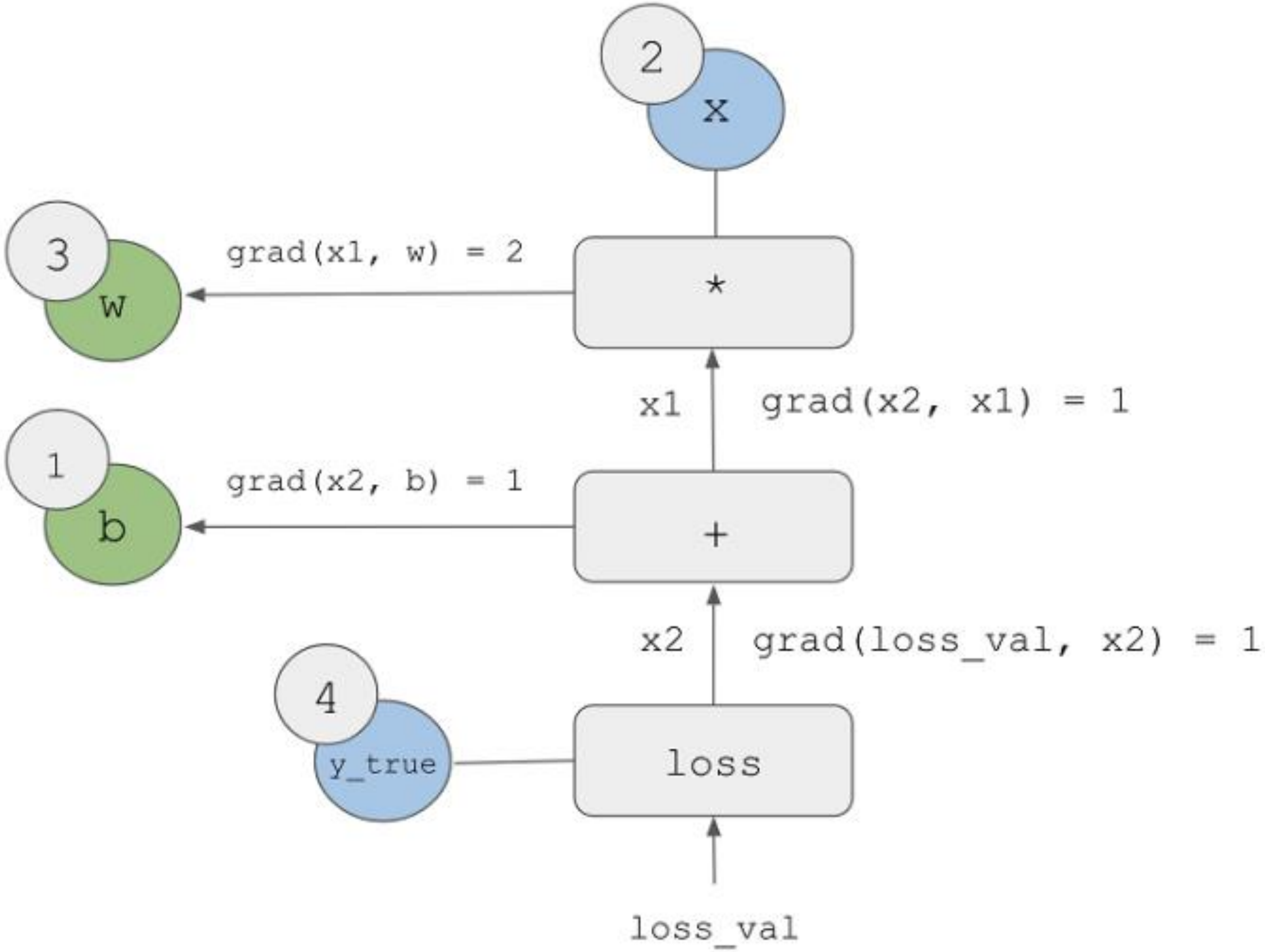
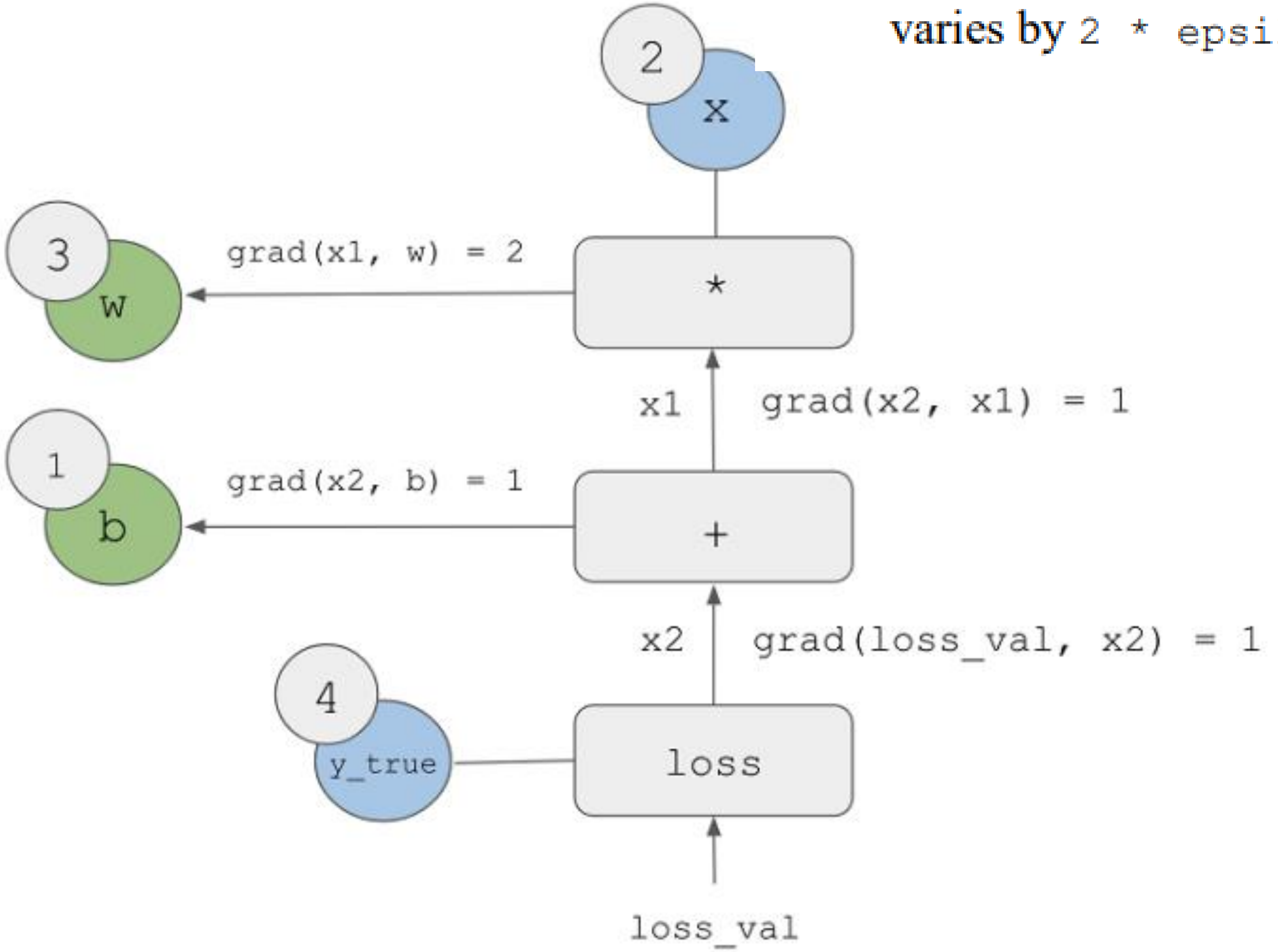


Figure 2.22 Running a backward pass

We have:

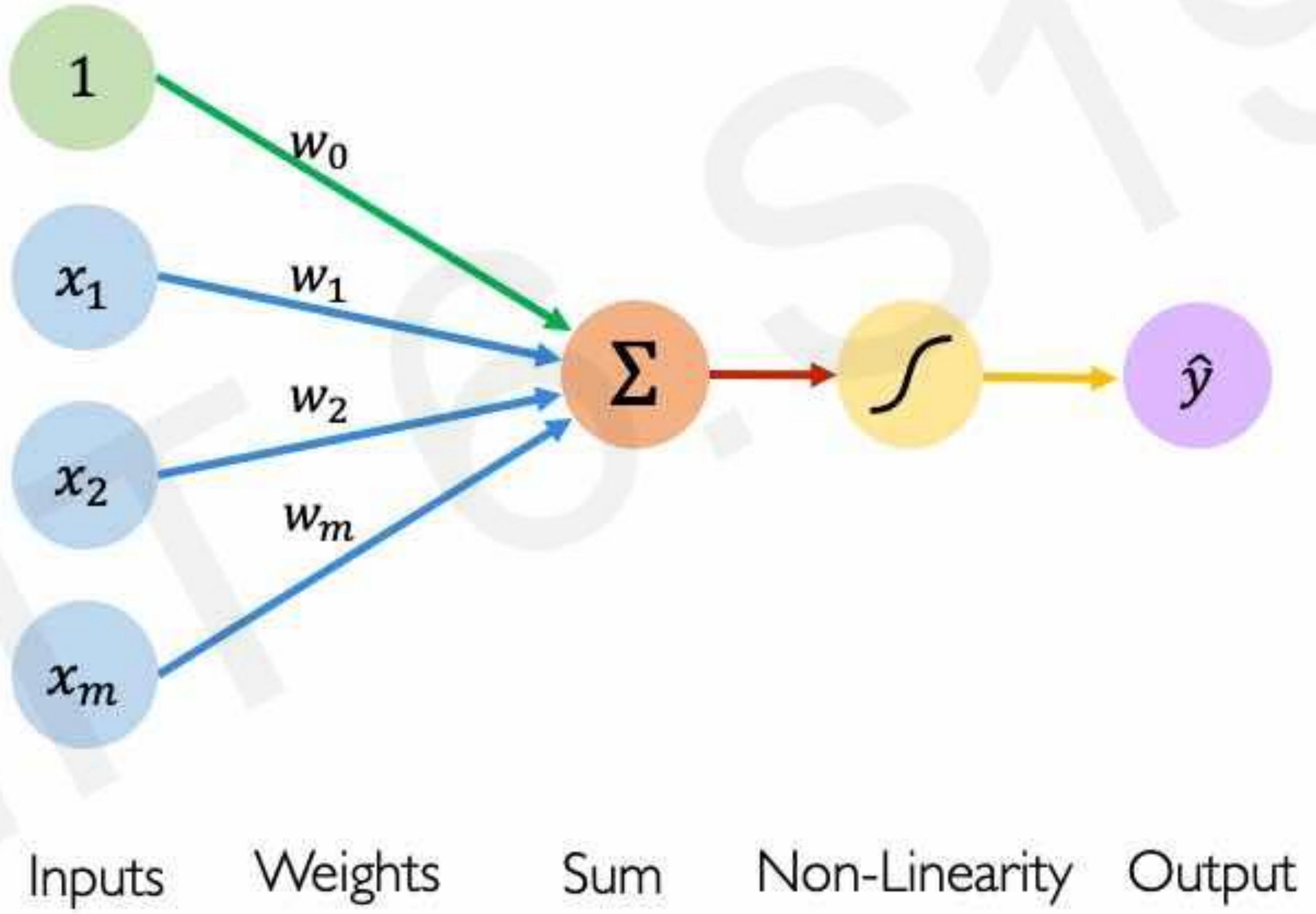
- `grad(loss_val, x2) = 1`, because as `x2` varies by an amount epsilon, `loss_val = abs(4 - x2)` varies by the same amount.
- `grad(x2, x1) = 1`, because as `x1` varies by an amount epsilon, `x2 = x1 + b = x1 + 1` varies by the same amount.
- `grad(x2, b) = 1`, because as `b` varies by an amount epsilon, `x2 = x1 + b = 6 + b` varies by the same amount.
- `grad(x1, w) = 2`, because as `w` varies by an amount epsilon, `x1 = x * w = 2 * w` varies by `2 * epsilon`.



Backward-Pass a.k.a. Backpropagation

The Perceptron: Simplified

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$



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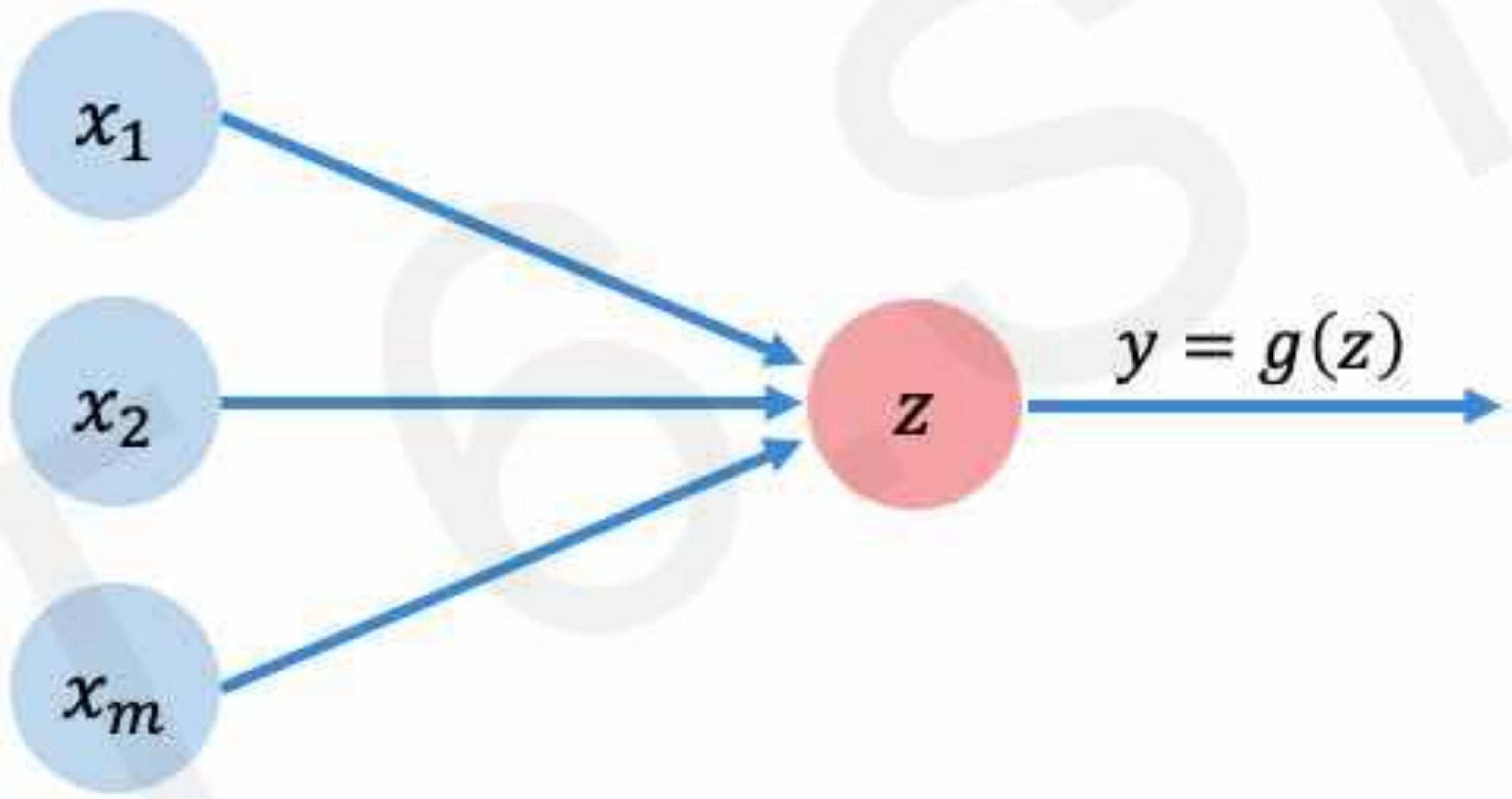
The Perceptron: Simplified



$$z = w_0 + \sum_{j=1}^{m} x_j \, w_j$$

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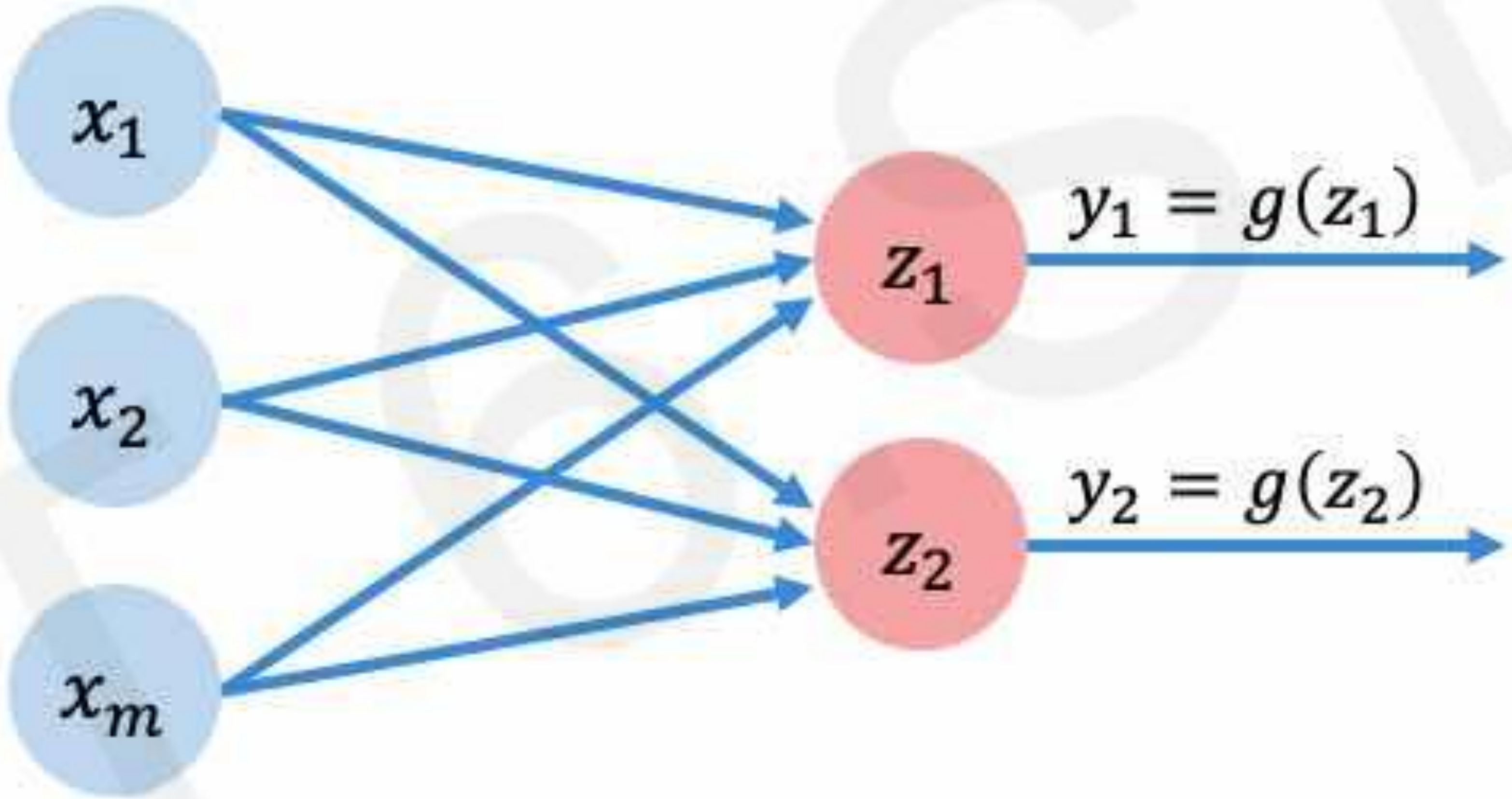
Multi Output Perceptron

Because all inputs are densely connected to all outputs, these layers are called **Dense** layers



$$z_i = w_{0,i} + \sum_{j=1}^{m} x_j \, w_{j,i}$$

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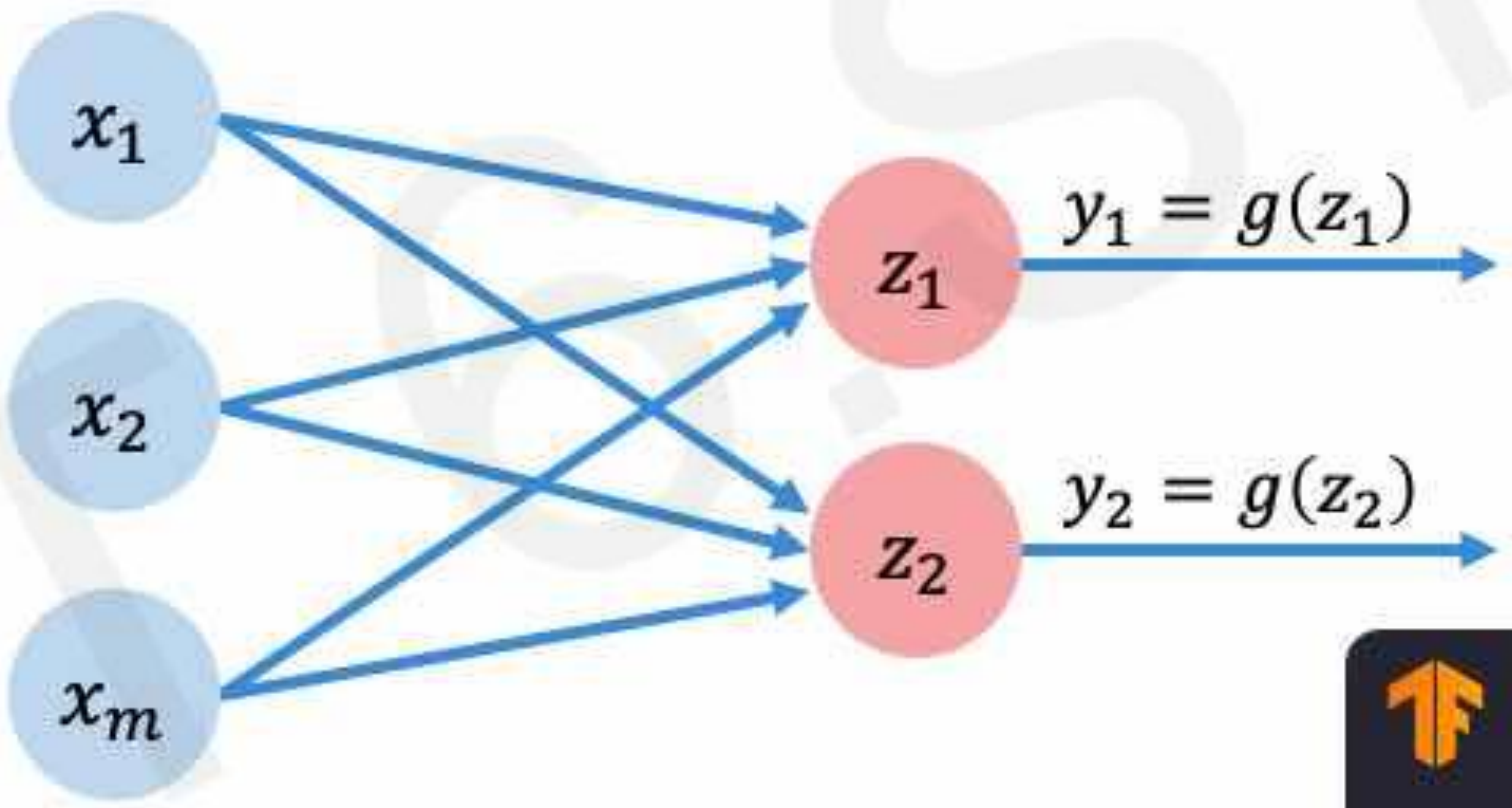
Multi Output Perceptron

Because all inputs are densely connected to all outputs, these layers are called **Dense** layers




```python
import tensorflow as tf

layer = tf.keras.layers.Dense(
    units=2)
```

$$z_i = w_{0,i} + \sum_{j=1}^{m} x_j \, w_{j,i}$$

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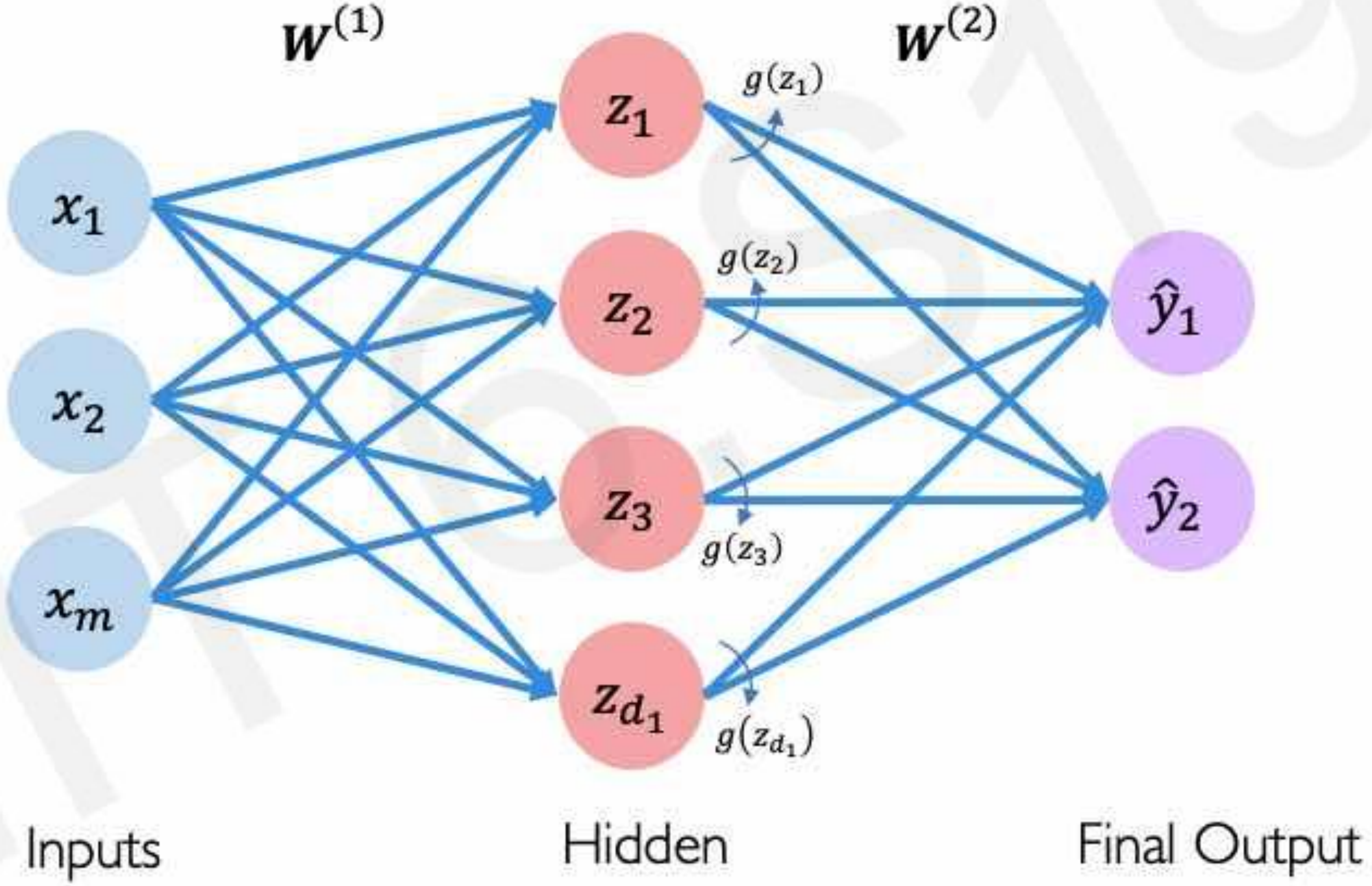
Single Layer Neural Network



$$z_i = w_{0,i}^{(1)} + \sum_{j=1}^{m} x_j \, w_{j,i}^{(1)} \qquad \hat{y}_i = g\left(w_{0,i}^{(2)} + \sum_{j=1}^{d_1} g(z_j) \, w_{j,i}^{(2)}\right)$$

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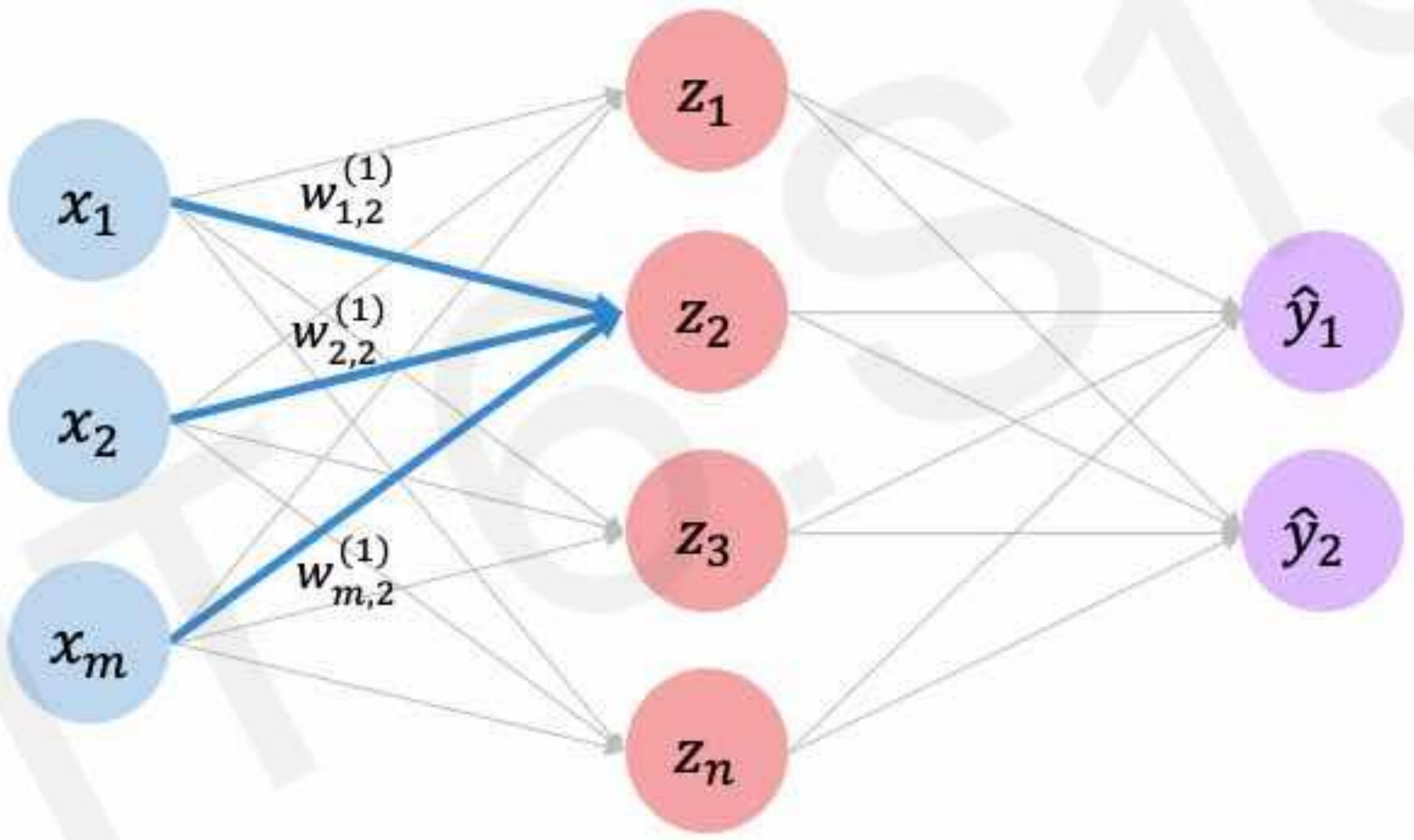
Single Layer Neural Network



$$z_2 = w_{0,2}^{(1)} + \sum_{j=1}^{m} x_j\, w_{j,2}^{(1)}$$

$$= w_{0,2}^{(1)} + x_1\, w_{1,2}^{(1)} + x_2\, w_{2,2}^{(1)} + x_m\, w_{m,2}^{(1)}$$

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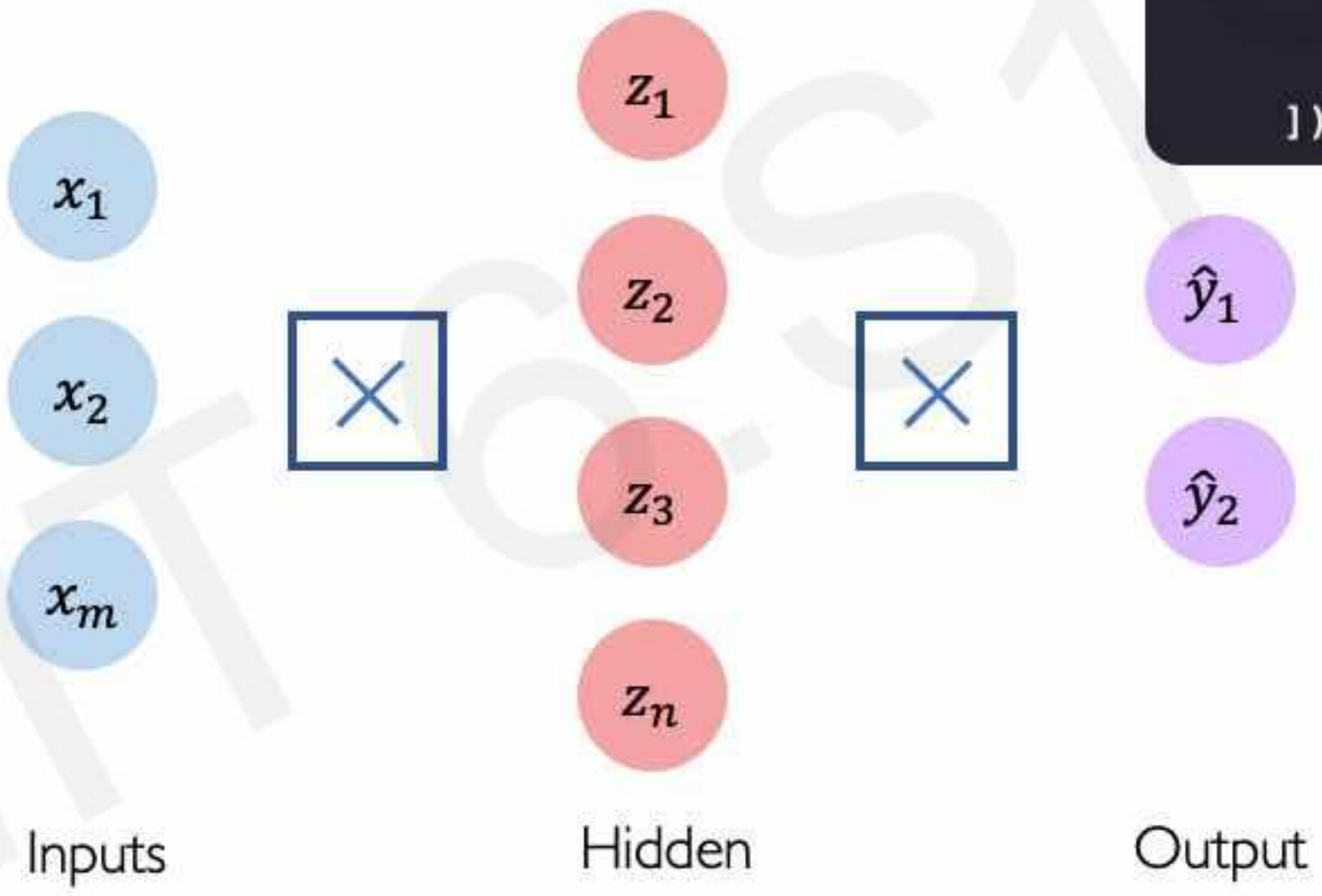
Multi Output Perceptron

```
import tensorflow as tf

model = tf.keras.Sequential([
    tf.keras.layers.Dense(n),
    tf.keras.layers.Dense(2)
])
```



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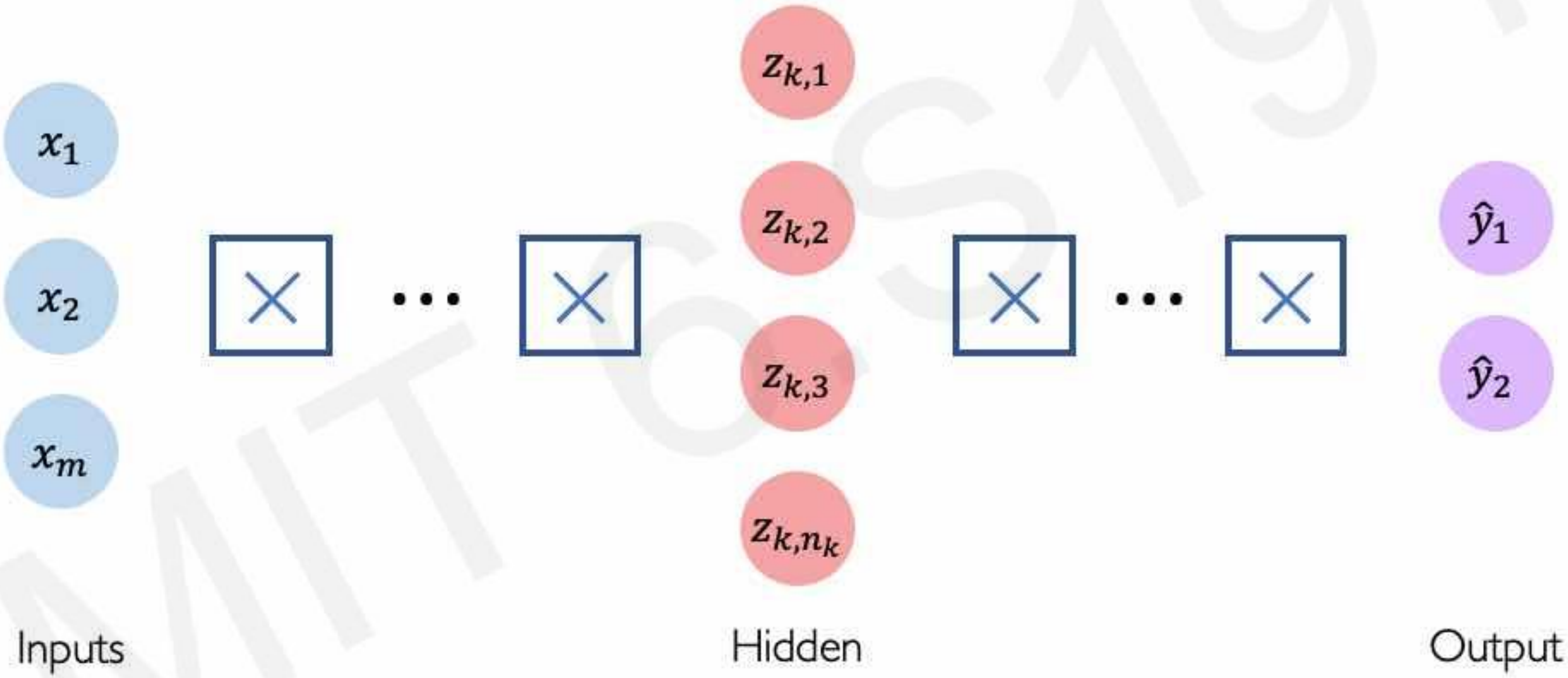
Deep Neural Network



$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$

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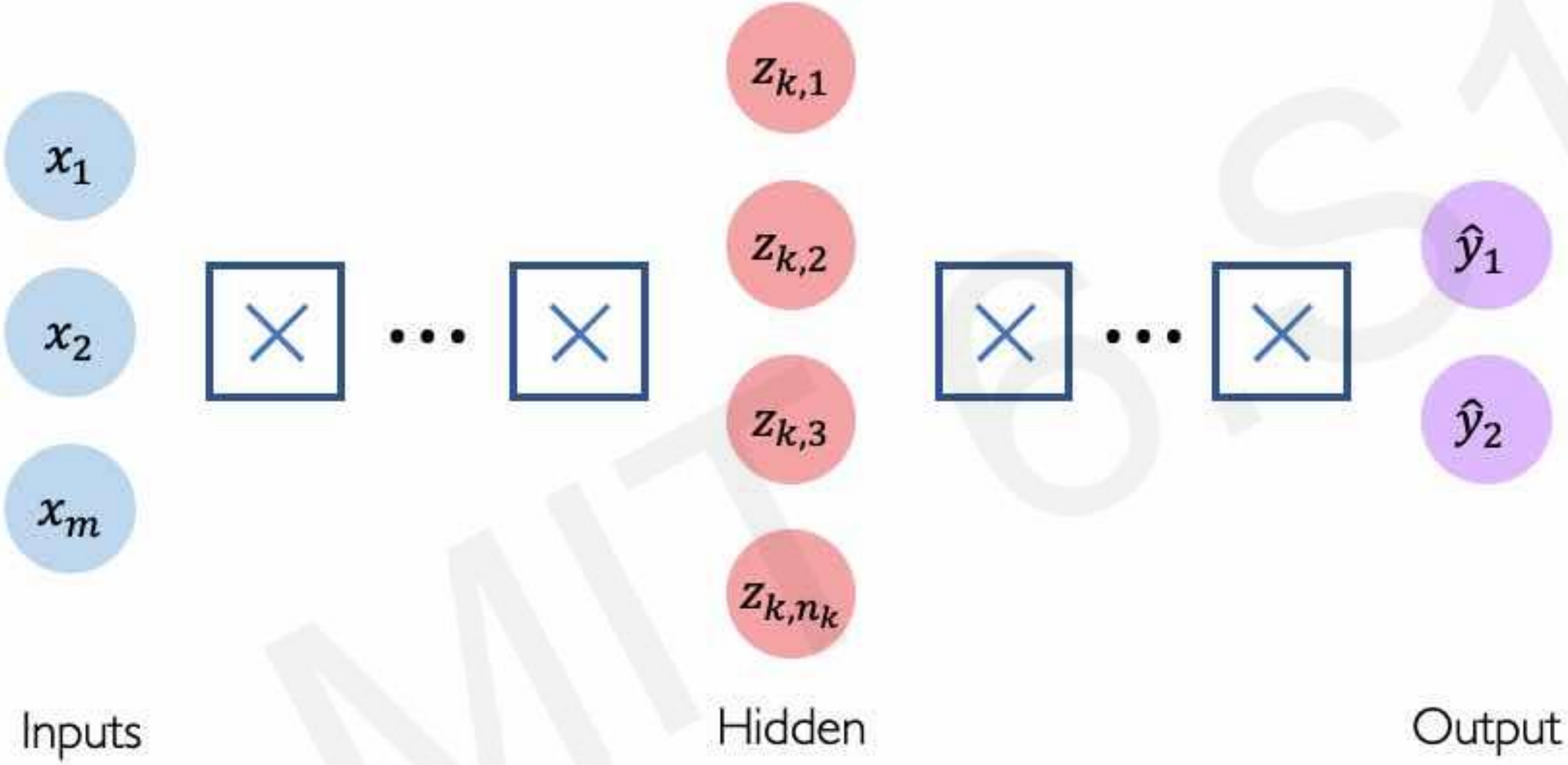
Deep Neural Network




```python
import tensorflow as tf

model = tf.keras.Sequential([
  tf.keras.layers.Dense(n₁),
  tf.keras.layers.Dense(n₂),
  ⋮
  tf.keras.layers.Dense(2)
])
```

$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$

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Applying Neural Networks

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Example Problem

Will I pass this class?

Let's start with a simple two feature model

x_1 = Number of lectures you attend

x_2 = Hours spent on the final project

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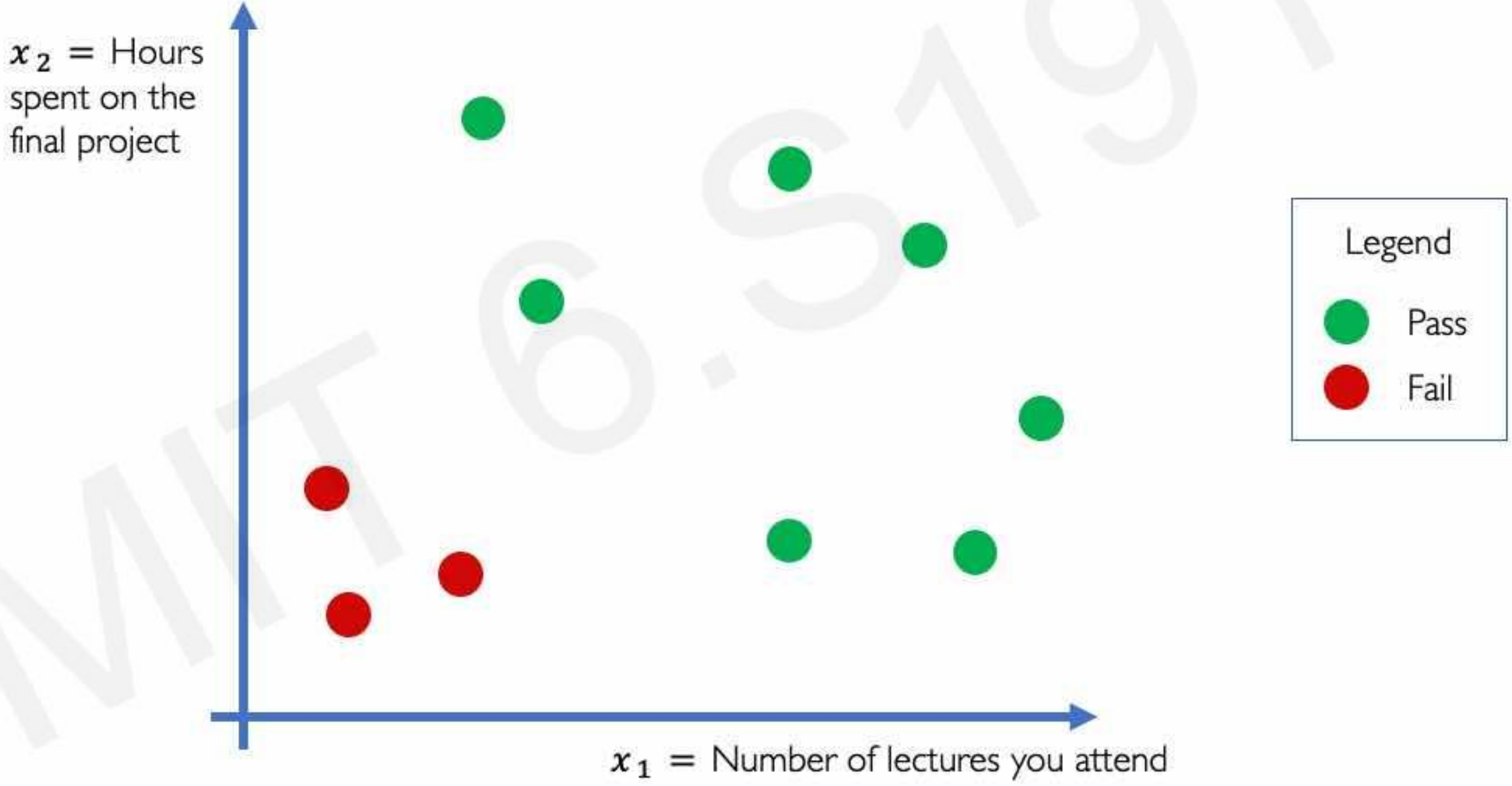
Example Problem: Will I pass this class?



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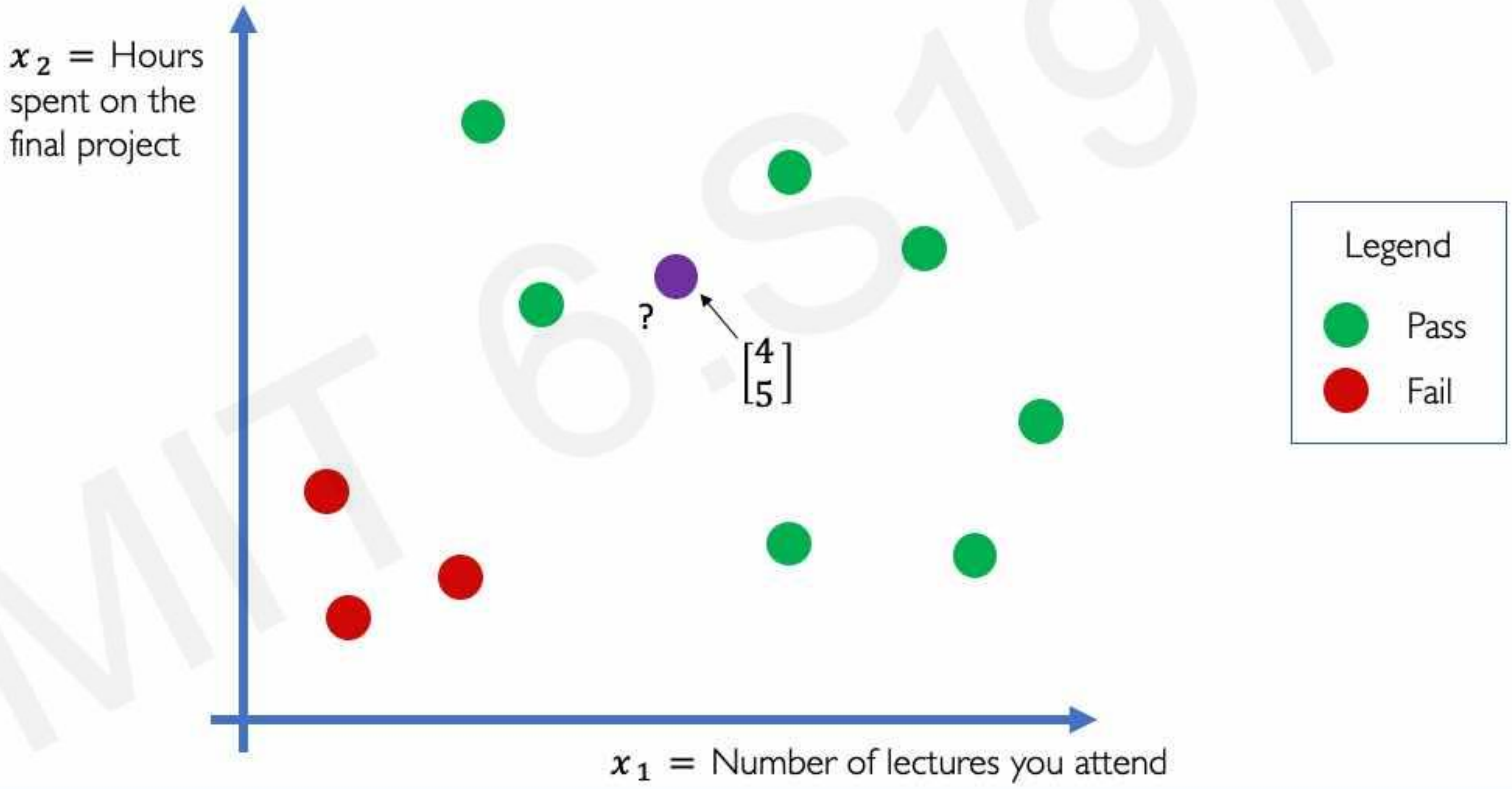
Example Problem: Will I pass this class?



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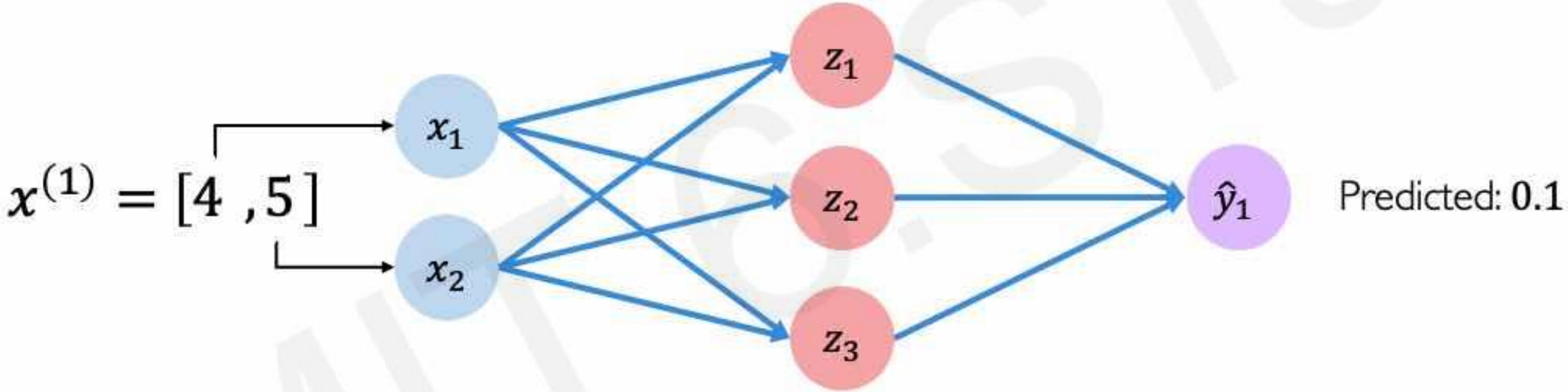

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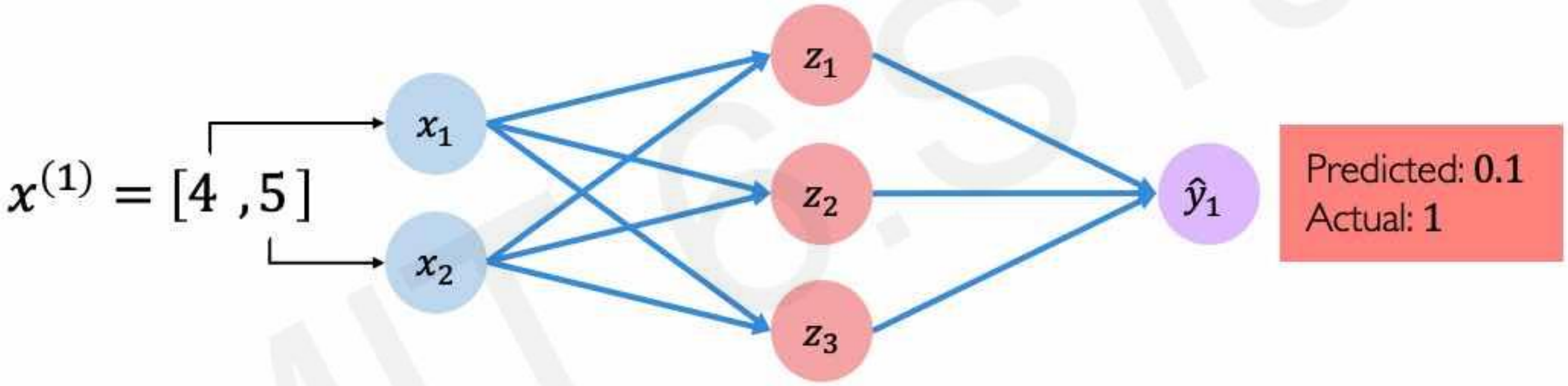
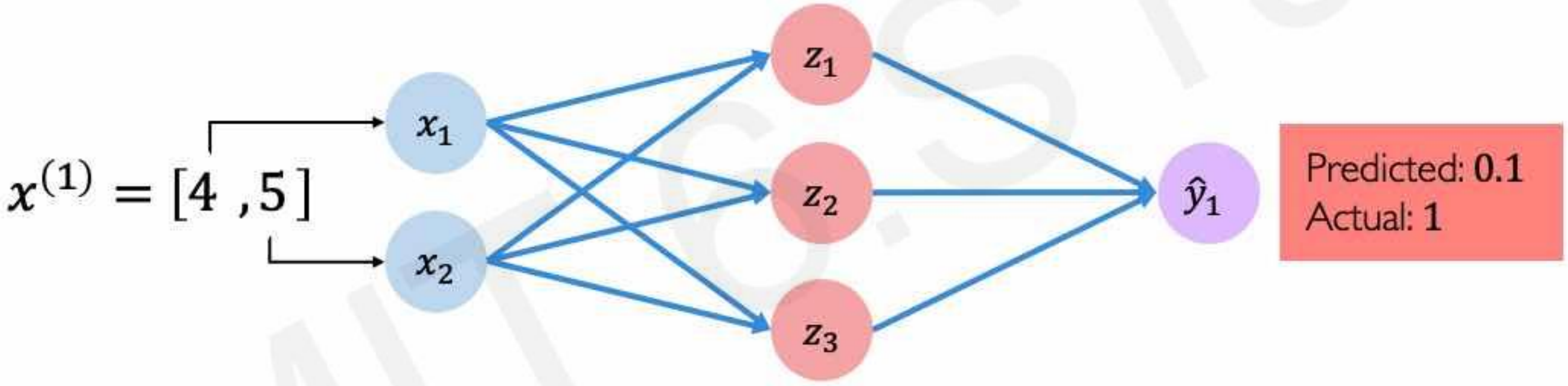


Quantifying Loss

*The **loss** of our network measures the cost incurred from incorrect predictions*



$$\mathcal{L}\Big(\underbrace{f\left(x^{(i)}; \boldsymbol{W}\right)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\Big)$$

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