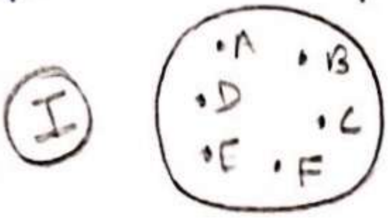
SUBGRAPHS

* In mathematics, we often study complicated objects by looking at simpler objects of the same type contained in them.

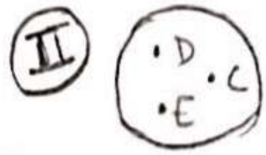
Example:

Group



Every alphabet is friend of other.

subgroup

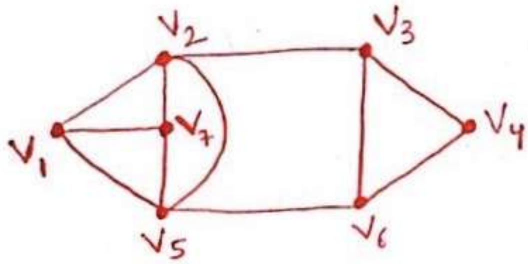


Application: chemical bonding

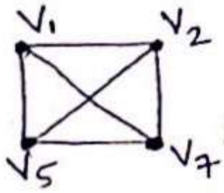
Definition for subgraph: A graph G_1 is called a subgraph of graph G iff the vertex set & the edge set of G_1 are respectively subsets of the vertex & edge sets of G.

* Vertices in G' will be in G
* Edges in G' will be in G.

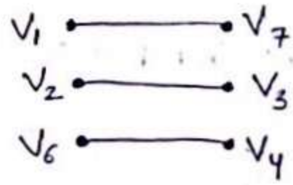
G' = subgraph
G = graph



G



G_1

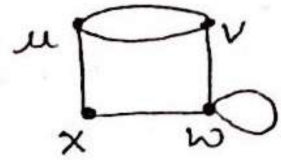


G_2

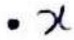
Note: Subgraphs do not have to be drawn in the same way as they appear in the presentation of graph.

Q1 (class) Which of the following are subgraphs of G :

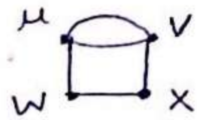
(I)



(a) ✓



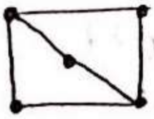
(b) ✓

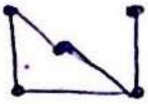


(c) ✗

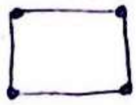
Note : To check whether G' is a subgraph of G,
① vertices in G' should be in G
② Edges b/w vertices in G' should be b/w vertices in G.

(II)

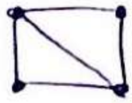




(a)



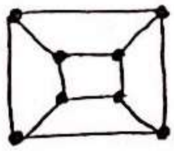
(b)

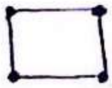


(c)

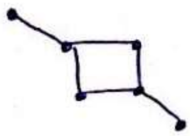
★ What if the graph is unlabelled?

(III)





(a)

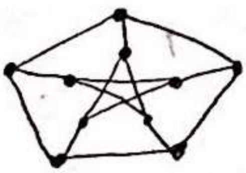


(b)

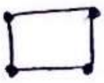
(c)

① Check :
number of vertices in G' will be $\leq$ vertices in G

(IV)



(a)



(b)

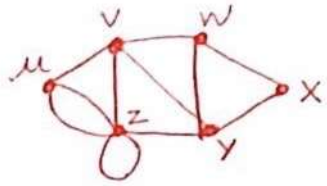
(c)

② Try to find similar diagram in the original graph.

length of walk
uvwxywvu
$\boxed{7}$ - length

n: vertices
(n-1): length

Walk [no restriction]



Trail
[edges cannot repeat]

Path
[① edges cannot repeat
② vertices cannot repeat]

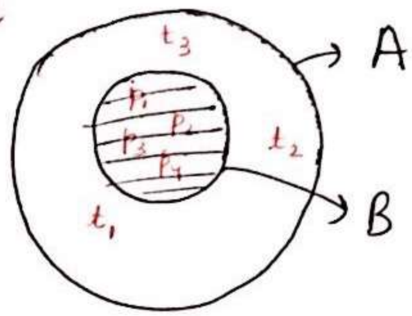
Example: $\boxed{WYVZ}$ is a path [of length 3]

↓

$\boxed{WYVZ}$ is a trail [of length 3]

Result: Every path is a trail
But converse is not true (why?)

Eg: $\boxed{VWYV}$ is a trail
but not a path



$\boxed{B}$ contains all the paths.
$\boxed{A}$ contains all the trails.
p_1 = WYVZ

Review
A statement holds for all x

To prove this statement wrong we need to find atleast 1 x for which the statement fails.

Eg: A student will get 'A' grade when scores above 90% in each subject.

Closed Walk

[① First & last vertex will be same
② No restriction on repetition of edges & vertices]

Closed Trail

① ~~Closed~~ First & last vertex will be same
② Edges cannot be repeated

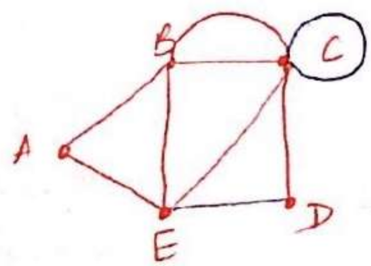
Closed Path

① First & last vertex will be same
② Vertices [except first & last] cannot repeat.
Edges cannot repeat

Closed Path
↓
Closed Trail

[Converse is not true]

Eg:



ABEA is a closed path
" " " trail

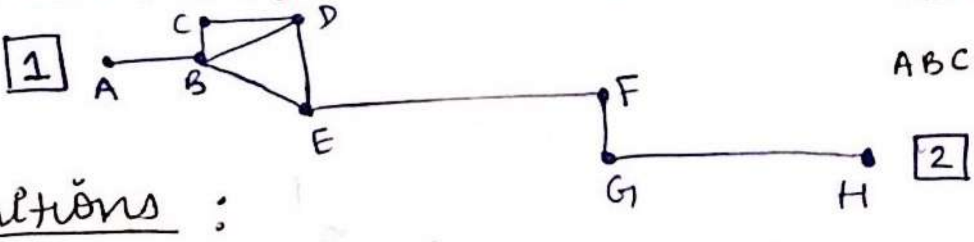
(BUT)

~~ABEBA~~
BECCB is a closed trail but not a closed path

PATHS & CYCLES

* To understand how we move from one vertex to another, we study paths & cycles.

Eg

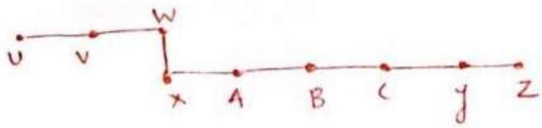


Friend 1 :
ABCDBEFGH

Friend 2 :
ABDEFGH

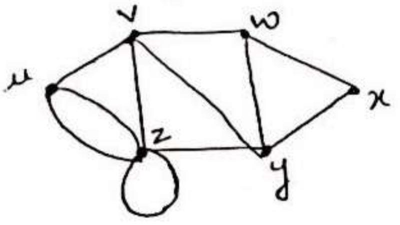
Definitions :

① **Walk** : A walk of length 'k' in a graph is a succession of 'k' edges of the form uv, vw, wx, ... yz. This walk is denoted by uvwx.....yz and is referred to as a walk b/w u & Z.



Note : In a walk, we do not require all the edges or vertices to be different.

Eg :



no. of edges covered
||

In this graph,
uvwxywvzzy is a walk of $\boxed{\text{length}}$ 9 b/w the vertices 'u' & 'y' which includes the edge vw twice & the vertices v, w, y & z twice

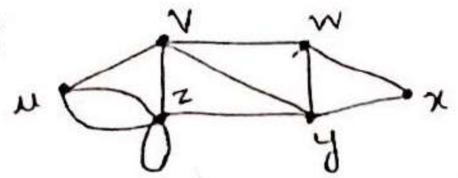
* It is sometimes useful to be able to refer to a walk under more restrictive conditions in which we require all the edges or all the vertices to be different.

② **Trail** : A trail is a walk in which all the edges (but not necessarily all the vertices) are different.

(3) Path : A path is a walk in which all the edges & all the vertices are different.

Note : Every path is a trail but converse is not true.

Example :

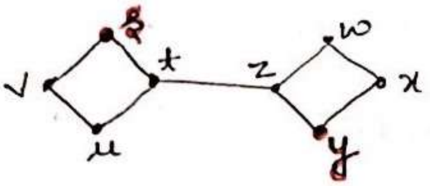


xyzzvy is a trail of length 5 b/w x & y. This trail is not a path as vertices y & z are being repeated.

uvyz is a path of length 3 b/w u & z.

(class)

Q2: Find all the paths b/w s & y in graph



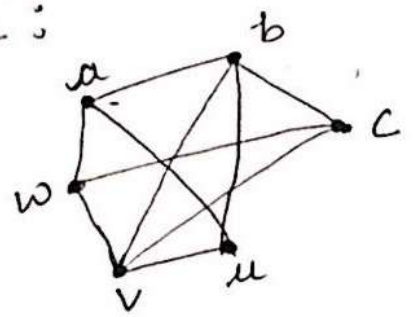
Some more definitions :

(1) Closed Walk : A closed walk in a graph is a succession of edges of the form uv, vw, wx, . . . yz, zu that starts & ends at the same vertex.

(2) Closed Trail : A closed trail is a closed walk in which all edges are different.

(3) Cycle /closed path : A cycle is a closed walk in which all the edges & all the intermediate vertices are different.

Example :



Ex abcvbc is a walk but not a trail.
abcvbu is a trail but not a path.
abcvbua is a closed trail but not a cycle.
abcwvua is a cycle.

(4) Acyclic : A graph is called acyclic if it does not contain a cycle.

Note : (1) A walk or trail is open if it starts and ends at different vertices

(2) A cycle of length 3 is called a triangle.

(3) In describing closed walks, we can allow any vertex to be the starting vertex.

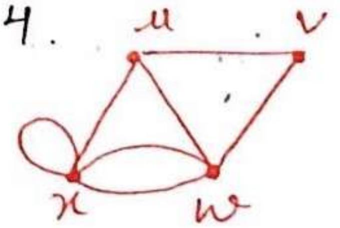
(Assignment)

Q1 For given graph, write:

(a) A closed walk that is not a closed trail. uwvwu

(b) A closed trail that is not a cycle uvwxwu

(c) All cycles of length 1, 2, 3 & 4.



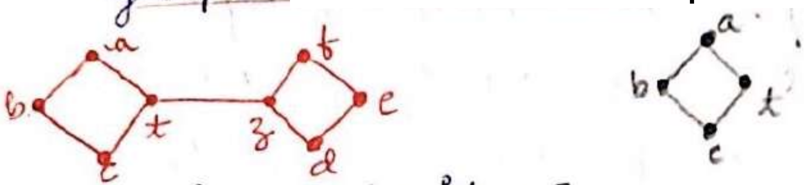
LECTURE-3
(Graph Theory)

CONNECTEDNESS

① <u>Connected Graph</u>: A graph is connected if there is a <u>path</u> between <u>each pair</u> of vertices. Examples ?

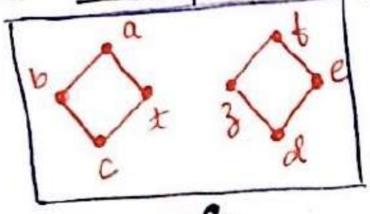
② <u>Disconnected Graph</u>: A graph is disconnected if it is not connected. Examples?

③ <u>Bridge</u>: An edge ~~is~~ in a connected graph is a bridge if <u>its removal leaves the graph</u> into 2 or more components

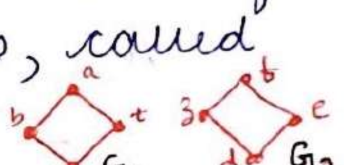


tz is a bridge?

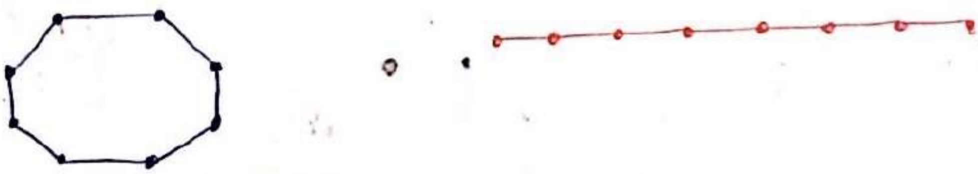
④ <u>Components</u>: Every <u>disconnected graph</u> can be split up into a number of <u>connected subgraphs</u>, called components.



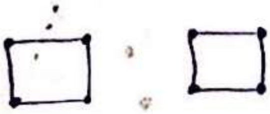
G_1



Q(a) Draw a connected graph with 8 vertices:



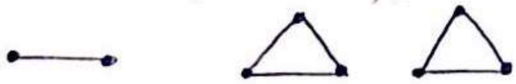
(b) a disconnected graph having 8 vertices & 2 components:



i.e. draw two connected graphs with 4 vertices each.

Note: To draw components, we are actually drawing connected graphs.

ⓒ a disconnected graph with eight vertices & 3 components

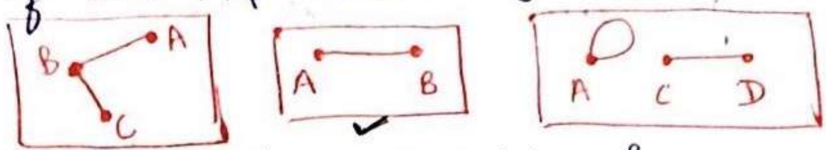


2 Important classes of graphs :

① Regular Graphs

② Bipartite Graphs

① <u>Regular Graph</u> : A graph is called regular if all its vertices have same degree.



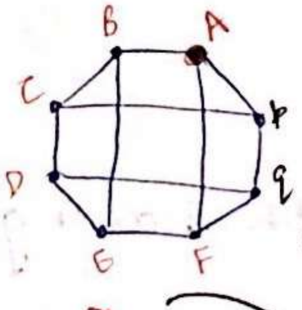
<u>r-regular graph</u> : A regular graph is r-regular or regular of degree r, if the degree of each vertex is r.

<u>Example</u> :

A 2-regular graph

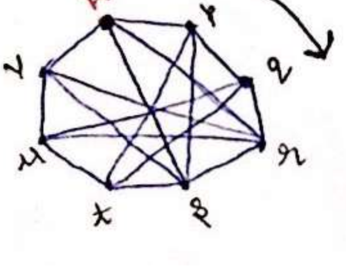
<u>Q1</u> Draw an r-regular graph with 8 vertices when:
(class)

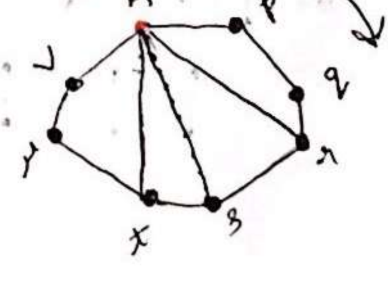
ⓐ $r = 3$



(TRY)
ⓒ $r = 5$?

ⓑ $r = 4$



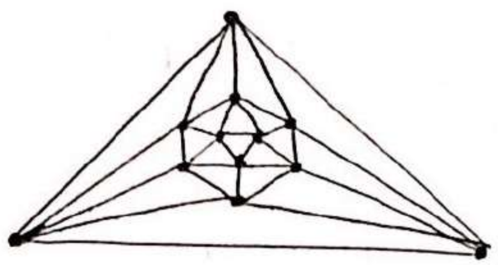


<u>Thm</u>: Let G be an <u>r-regular graph</u> having n vertices. Then G has $\frac{nr}{2}$ edges.

<u>Proof</u>: No. of vertices of $G = n$.
degree of each vertex $= r$
$\therefore$ sum of ~~ead~~ degrees of all vertices $= nr$

$\therefore$ By handshaking lemma, no. of edges $= \frac{nr}{2}$.
(i.e. $\sum_{v \in V} \deg v = 2|E|$)

<u>Q-3)</u> Find the no. of edges in this 5-regular graph.



<u>Q4</u> Prove that there are no 3 regular graphs with 7 vertices.

<u>Soln</u> By above theorem, 3 regular graph with 7 vertices has $\frac{3\times 7}{2} = \frac{21}{2}$ edges,
which is not possible.

(Assignment)
<u>Q5</u> Prove that there are no r-regular graphs having n vertices when n & r both are odd.

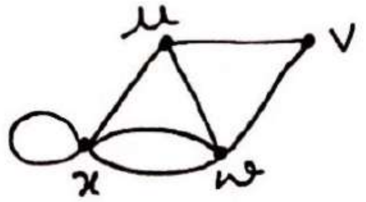
Some <u>Important</u> <u>classes</u> <u>of</u> <u>regular graphs</u>

① <u>Complete Graphs (K_n)</u>: A complete graph is a graph in which each vertex is joined to each of the others by exactly one edge.

Assignment - 2

Q-1) For given graph, write:

1. A closed walk that is not a closed trail
2. A closed trail that is not a cycle
3. All cycles of length 1, 2, 3 & 4.



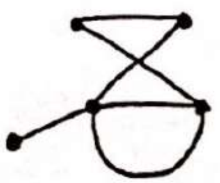
(3 Marks)

Q-2) Prove that there are no r-regular graphs having 'n' vertices when 'n' & 'r' both are odd. (2 Marks)

(Hint: Try to use theorem for r-regular graph)

Q-3) Draw a regular graph with 8 vertices and 12 edges. (2 Marks)

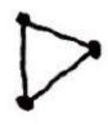
Q-4) Check if the given graphs are subgraphs of G or not.



G

G_1

G_2



G_3

(3 Marks)