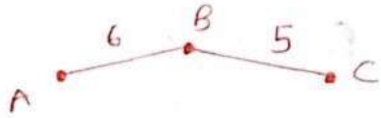
Lecture - 10

(Graph Theory)

Shortest Path Algorithm



Some definitions :

(1) Weighted Graph : A weighted graph is a graph G in which each edge, e, is assigned a non-negative real number denoted as w(e), called the weight of e.

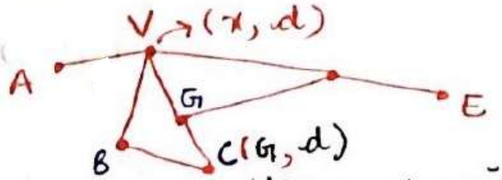
(*) The weight of a graph is the sum of the weights of the edges (all) of the graph.

(2) Shortest Path : A shortest path b/w 2 vertices in a weighted graph is a path of least weight.

Note : In an unweighted graph, a shortest path means the one with the least number of edges.

Shortest Path Algorithm

The algorithm is to find the shortest path from a specified vertex A to another specified vertex E. It procede by progressively assigning to each vertex V in the graph an ordered pair (x, d) where d is the shortest distance from A to V & xv is the last edge on the shortest path.



Here x will be A.

<u>Note</u> : If the vertex E never gets labelled, there is no path from A to E, the graph is thus not connected.

<u>Procedure</u> : To find the shortest path from vertex A to vertex E in a weighted graph, carry out the following procedure.

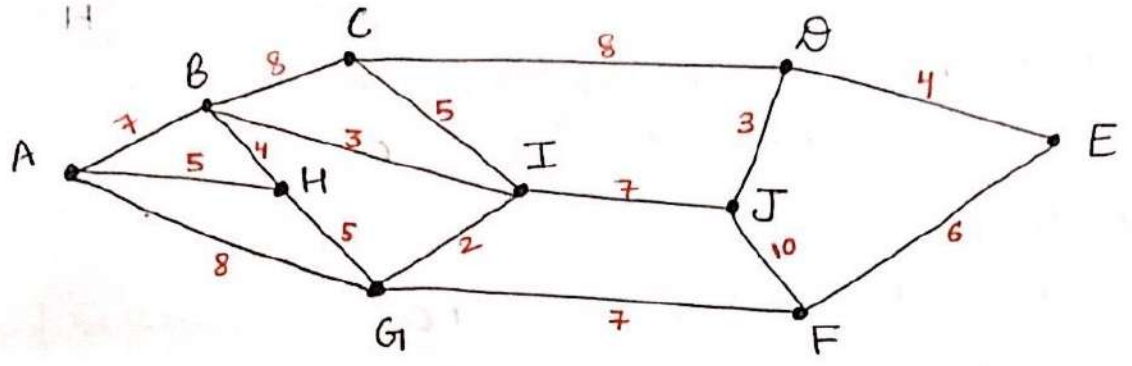
<u>Step 1</u> : Assign the vertex A the label $(-, 0)$

<u>Step 2</u> : Until E is labelled or no further labels can be assigned, do the following

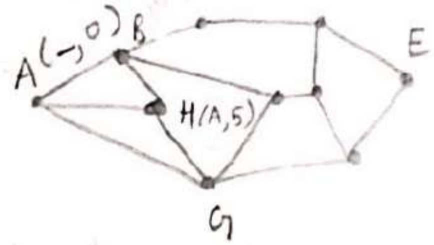
(a) For each labelled vertex $u(x, d)$ & for each unlabelled vertex V adjacent to u, compute $d + w(e)$, $e = uV$

(b) For each labelled vertex u and adjacent unlabelled vertex v giving minimum $d' = d + w(e)$, assign to v the label (u, d'). If a vertex can be labelled (x, d') for various x, make any choice.

Q-1) Find the shortest path from A to E in the given weighted graph.



Solution:



① Assign the label $(-, 0)$ to A

② There are 3 vertices adjacent to A i.e. B, H & G.

Calculate $d + w(e)$ for all three:

For B, $d + w(e) = 0 + 7 = 7$

For H, $d + w(e) = 0 + 5 = 5$ ✓

For G, $d + w(e) = 0 + 8 = 8$

∵ H has the smallest value of $d + w(e)$

∴ H acquires the label $(A, 5)$

③ Now, the unlabelled vertices adjacent to labelled vertex A are: B & G

∴ numbers $d + w(e)$ are: $\underbrace{0 + 7 = 7}_{\text{for B}}$ & $\underbrace{0 + 8 = 8}_{\text{for G}}$

Also, unlabelled vertices adjacent to labelled vertex H are: B & G

∴ The numbers $d+w(e)$ are : $5+4=9$ & $5+5=10$

(∵ now $d=5$)

B G

The smallest $d+w(e)=7$

i.e. corresponding to labelled vertex A & unlabelled vertex B

∴ label vertex B as $(A, 7)$

(4) Now there are 3 labelled vertices A, H & B.

* (a) Adjacent to A, there is one unlabelled vertex i.e. G with $d+w(e)=0+8=8$
* Adjacent to H, there is one unlabelled vertex with $d+w(e)=5+5=10$ [vertex G]
* Adjacent to B, there are 2 unlabelled vertices.

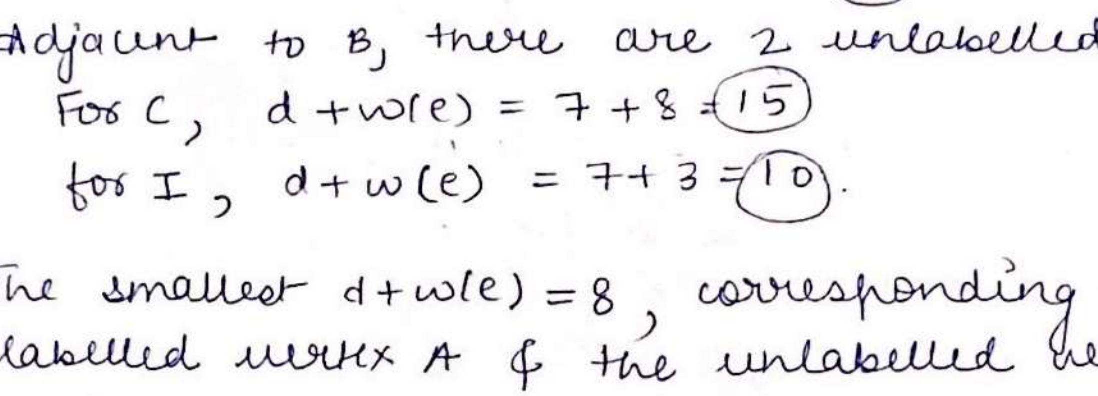
For C, $d+w(e)=7+8=15$

for I, $d+w(e)=7+3=10$.

The smallest $d+w(e)=8$, corresponding to the labelled vertex A & the unlabelled vertex G

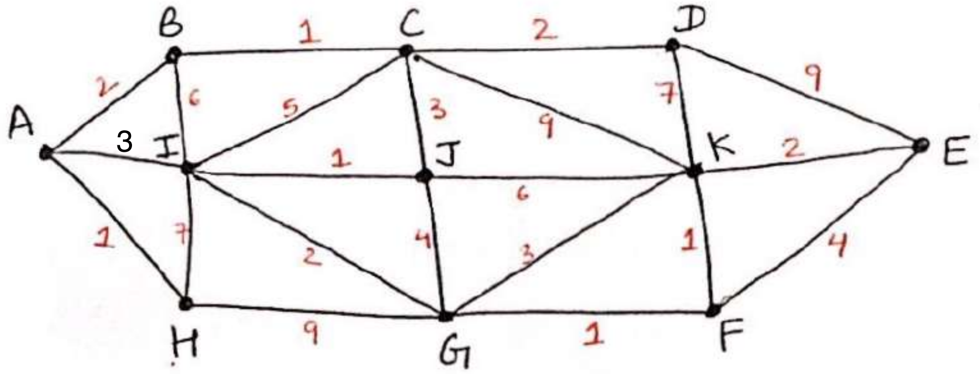
∴ G gets the label $(A, 8)$.

(5) Continuing in this manner, the vertices acquire the following labels.



Thus the shortest route from A to E has weight 21 (Route: AGFE)

Practice Question : Find the shortest path from A to E in the following figure.



Assignment - 8

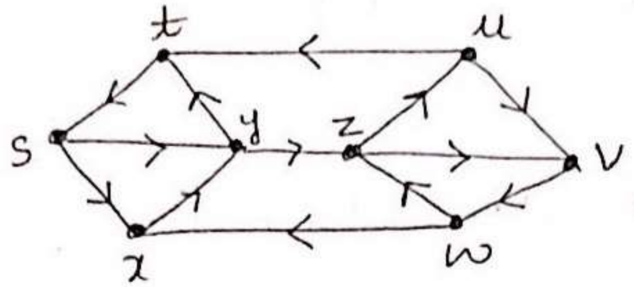
Q-1) For the digraph, write down:

(a) All the paths from t to w

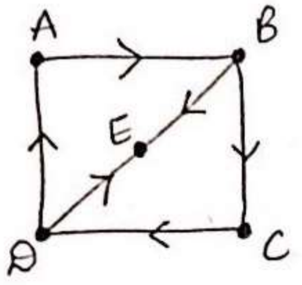
(b) All the paths from w to t

(c) A closed trail of length 8 containing t & z

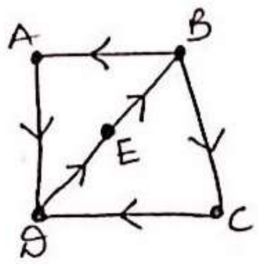
(d) All the cycles containing both t & w.



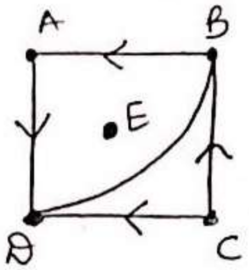
Q-2) Classify each of the following digraphs as disconnected, connected but not strongly connected or strongly connected.



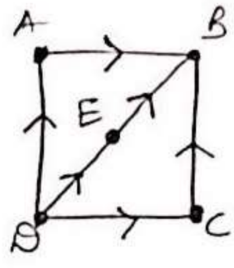
(a)



(b)



(c)



(d)

Q-3) Check whether given digraph is Eulerian and/or Hamiltonian.