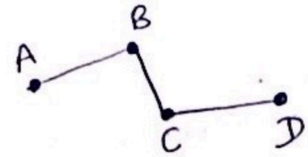
LECTURE-1
(Graph Theory)

Basic Definitions

Graph: A graph is a pair (V, E), consisting of a finite set $V \neq \phi$ and a set E of 2-elt subsets of V.
The elements of V are called vertices & the elements of E are called edges.

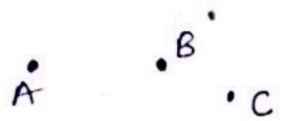
Example:



Vertices = {A, B, C, D}
Edges = {AB, BC, CD}

* Why $V \neq \phi$ and E can be ϕ or $\neq \phi$?

Think: 1) Can an edge be without vertices?
2) Vertices are possible without edges?



G_1

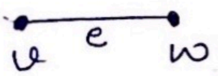
G_2

Graph Notation: $\boxed{G = (V, E)}$ i.e. Graph with vertex set V & edge set E.

Edge Notation: If 'e' is an edge, then $\boxed{e = \{v, w\}}$ where 'v' & 'w' are different elts of V called the end vertices or the ends of 'e'

We usually denote edge 'e' as vw, which is same as 'wv'.

SOME TERMINOLOGY:



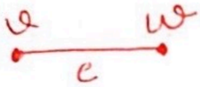
* Its important to learn the language of maths.

× e is next to v / e is joined to v

✓ e is incident with v

Adjacency and Incidence: (To study the relationship b/w edges & vertices)

(I) Incidence: The vertices v & w are incident with the edge e.



The edge e is incident with the vertices v & w.

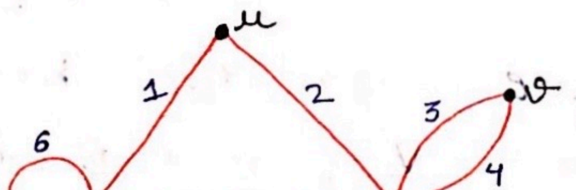
(II) Adjacency:

(a) Adjacent Vertices: 2 vertices are adjacent if they are the end vertices of an edge. OR

The vertices v & w of a graph are adjacent vertices if they are joined by an edge e.

(b) Adjacent Edges: 2 edges are adjacent if they have a vertex in common

Example1:

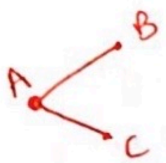


* Edge 6 is incident with ?
* Vertex w is incident with ?
* Are u & x adjacent vertices ?

Degree of a vertex : The number of edges incident with a vertex v is called the degree of 'v', denoted as deg v

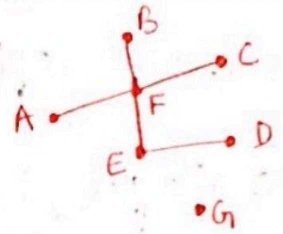
Note: * If deg v is an even no. then v is called an even vertex.

* If deg v is an odd no., vertex v is odd
* A vertex of zero degree is called an isolated vertex.



deg A = ?
deg B = ?
deg C = ?

Figure 2



deg F = 4
deg A = 1
deg G = 0 ∴ G is isolated vertex.

Fig One

Remark : To find whether the given vertex is odd or even, First find degree of that vertex

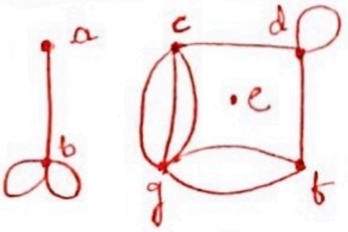
Eg 2)

In figure 1, A is an odd vertex
F is an even vertex

In figure 2, B ?
A ?

Degree Sequence of a Graph : It is the sequence obtained by listing the vertex degree of G in increasing order, with repeats as necessary.

Eg :



Step1: Find degree of each vertex in the graph

Step2: Write in ascending order

deg a = 1
deg b = 5
deg c = 4
deg d = 4
deg e = 0
deg f = 3
deg g = 5

∴ deg seq = (0, 1, 3, 4, 4, 5, 5)

Assignment

Q1 (a) Draw the graph having the following vertex set and edge set :

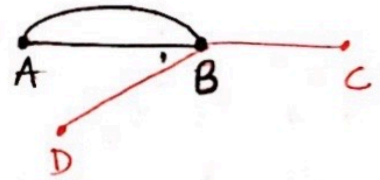
$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E = \{v_1v_4, v_1v_6, v_2v_5, v_4v_5, v_5v_6\}$

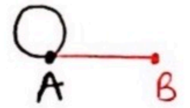
(b) After drawing the graph, spot the isolated vertice, if any.

Some Other Definitions :

① **Multiple Edges** : In a graph, 2 or more edges joining the same pair of vertices are called multiple edges.

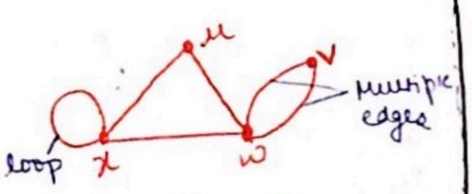


② **Loop** : An edge joining a vertex to itself is called a loop.

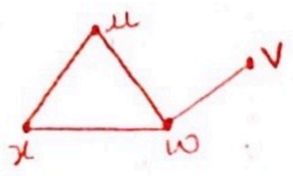


③ **Simple Graph** : A graph with no multiple edges or loops.

④ **Pseudograph** : A graph that contains loops &/or multiple edges.



~~Mult~~ Pseudograph

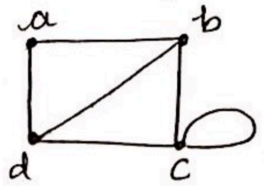


Simple graph

Note : while counting the degree of a vertex, a loop is counted twice

Assignment

Q-2) Write down the vertices and edges of the given graph. Also check if the graph is simple or not ?



Q-3) Draw the graph whose vertices & edges are :

Vertices = $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Edges = $\{12, 22, 23, 34, 35, 67, 68, 78\}$

Euler's Theorem / Handshaking Lemma

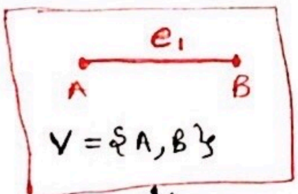
In any graph, the sum of the degrees of all the vertices is equal to twice the no. of edges.

In symbols, If $G(V,E)$ is any graph then

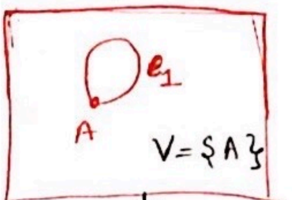
$$\sum_{v \in V} \deg v = 2|E|$$

Proof: In any graph, there are 2 types of edges: one which have different end vertices & one which have same end vertex (loop).

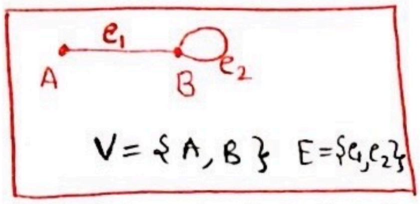
Now, edges having different end vertices contribute 1 to the degree of each of its end vertices & a loop contribute 2 to the degree of the vertex incident with it.



$$\sum_{v \in V} \deg v = \deg A + \deg B = 2$$



$\deg A = 2$



$$\sum_{v \in V} \deg v = \deg A + \deg B = 1 + 3 = 4 = 2 \cdot 2$$

LECTURE-2
(Graph Theory)

Corollary of Euler's Theorem: In any graph, the no. of vertices of odd degree is even.

Proof: Let V denote the vertex set of the graph. Then by Euler's Thm,

$$\sum_{v \in V} \deg v = 2|E| \quad \text{where } E \text{ denotes the no. of edges.}$$

$$\Rightarrow \sum_{\substack{v \in V \\ \deg v \text{ is even}}} \deg v + \sum_{\substack{v \in V \\ \deg v \text{ is odd}}} \deg v = 2|E|$$

$$\Rightarrow \sum_{\substack{v \in V \\ \deg v \text{ is odd}}} \deg v = 2|E| - \sum_{\substack{v \in V \\ \deg v \text{ is even}}} \deg v$$

Now, RHS is clearly even (∵ difference of even no. is even)

∴ LHS should be even i.e.

$$\sum_{\substack{v \in V \\ \deg v \text{ is odd}}} \deg v = \text{even}$$

Hence Proved!

SUBGRAPHS

In mathematics, we often study complicated objects by looking at simpler objects of the same type contained in them

Assignment 1 (MFDS-II)

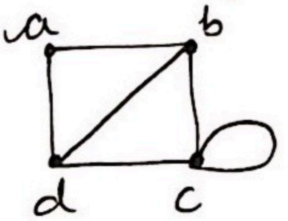
Q-1) (a) Draw the graph having the following vertex set and edge set : (1 Mark)

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E = \{v_1v_4, v_1v_6, v_2v_5, v_4v_5, v_5v_6\}$

(b) Spot the isolated vertices in above graph formed, if any.

Q-2) Write down the vertices & edges of the graph G. Also check if the graph is simple or not ? (1 Mark)



Q-3) Draw the graph whose vertices and edges are: (1 Mark)

Vertices = $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Edges = $\{12, 22, 23, 34, 35, 67, 68, 78\}$

Q-4) Let G be a graph with degree sequence $(1, 2, 3, 4)$. Write down the no. of edges & vertices of G and construct such a graph. Are there any simple graphs with degree sequence $(1, 2, 3, 4)$? (2 Marks)