Contents

1 Signal & Systems (Fundamental)

1.1 Signal Definition

Definition 1.1.1 (Signal)**.** Any "physical" quantity that varies with time or space (or other independetn variables).

Example 1.1 (Ambulance Siren:)**.**

$$s(t) = (1+t)sin(2\pi[1000t + 10t^2 + 300sin(2\pi t/2)]) \quad (1)$$

- $(1+t)$: amplitude term represents incresing loudness as ambulance approaches
- $1000t$: represents 1kHz siren oscillatione
- $10t^2$: increasing pitch due to the *Doppler effect* as the ambulance approaches.
- $300sim(2\pi 2t)$: the eeh-ooh-eeh-ohh periodic variation in pitch.

Definition 1.1.2 (Systems)**.** A physical "devicec" that performs an operation on a signal.

Definition 1.1.3 (Signal Processing)**.** Take some input signals and produce some related output signals.

Example 1.2 (audio amplifier)**.**

$$S_{out}(t) = aS_{in}(t) \quad (2)$$

Emphasize on continuous-time of analog signals.

1.2 Classification of Signals

1.2.1 Dimensionality

- By the domain of the function, i.e. how many arguments a function has.
- By the dimension of the range of the function, i.e. the space of values the funciton can take.

1.2.2 Time characteristics

Definition 1.2.1 (Continuous-time Signal)**.** A function defined for all times $t \in (-\infty, \infty)$, or at least some interval (a, b).

Classify signals by time characteristics

- Continuous-11:10 signals or analog signals
- Discrete-time signals

1.2.3 Value Characteristics

Definition 1.2.2 (continuous-valued signal or continuous-amplitude signal)**.** Can take any value in some continuous interval.

Definition 1.2.3 (discrete-valued signal or discrete-amplitude signal)**.** Only takes values from a discrete set of possible values.

1.2.4 Determinstic vs Random Signals

Definition 1.2.4 (Determinstic signals)**.** Can be described by an explicit mathematical representation.

Definition 1.2.5 (Random signals)**.** Evolve over time in an unpredictable manner.

1.3 Transformation of CT Signals

1.3.1 Transformations

- Time transformations
  - Folding/reflecting/time-reversal
    $$y(t) = x(-t) \tag{3}$$
  - Time-scaling
    $$y(t) = x(at) \tag{4}$$
  - Time-shifting
    $$y(t) = x(t - t_0) \tag{5}$$

- General time transformations
  Involves all three of the above time transformations.

$$y(t) = x(at - b) = x(\frac{t - t_0}{w}) \tag{6}$$

  where $t_0 = b/a$, $w = 1/a$

- Amplitude transformations
  - reverse

$$y(t) = -x(t) \tag{7}$$

  - scaling

$$y(t) = ax(t) \tag{8}$$

  - shifting

$$y(t) = x(t) + b \tag{9}$$

- Differentiator

$$y(t) = \frac{d}{dt}x(t) \tag{10}$$

  Example 1.3.

$$y(t) = -RC\frac{d}{dt}x(t) \tag{11}$$

- Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau \tag{12}$$

  Example 1.4.

$$y(t) = -\frac{1}{RC}\int_{-\infty}^{t} x(\tau)d\tau \tag{13}$$

- Operation with two signals
  Sum or product at any point

1.4 Signal Characteristic

1.4.1 Periodic signals

$$x(t + T) = x(t)\forall t \tag{14}$$

if no T exists, called aperiodic.

Definition 1.4.1 (Fundamental Period)**.** Smallest T_0 of T.

Theorem 1.4.1. With period $T > 0$,

$$x(t + nT) = x(t) \tag{15}$$

Sum of two periodic signals
Suppose a value $T > 0$ satisfies $T = n_1 T_1$ and $T = n_2 T_2$
then, x(t) is periodic with period T.

Theorem 1.4.2. A sum of two periodic signals is period iff the ratio od their periods is rational.

1.4.2 Even and Odd Symmetry

Definition 1.4.2 (Even Symmetry)**.** iff $x(-t) = x(t) \forall t$

Definition 1.4.3 (Odd Symmetry)**.** iff $x(-t) = x(t) \forall t$

Even and Odd Components
We can decompose any signal into even and odd components:

$$x(t) = x_e(t) + x_o(t) \tag{16}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)] \tag{17}$$

1.4.3 Average value and energy

Definition 1.4.4 (Average Value)**.**

$$A = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt \tag{18}$$

Definition 1.4.5 (Energy)**.**

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \tag{19}$$

Definition 1.4.6 (Average Power)**.**

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \tag{20}$$

Definition 1.4.7 (Energy Signal)**.** If E is finite, then $x(t)$ is called energy signal and $P = 0$.

Definition 1.4.8 (Power Signal)**.** If E is infinite and P is finite and nonzero, $x(t)$ is called power signal.

1.5 Exponential signals

To be done

1.6 Sigularity functions

1.6.1 Transformed rect functions

$rect(\frac{t-t_0}{T})$ centered ar t_0 with width T.

1.6.2 Unit impulse function

$\delta(t)$, area is 1 and width is 0
relationships: $\delta(t) = \frac{d}{dt}u(t),\ u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau$
scaling property: $\delta(at+b) = \frac{1}{|a|}\delta(t+b/a)$

1.6.3 Practical impulse function

$$\delta_\Delta(t) = \begin{cases} \dfrac{1}{\Delta}, & 0 < t < \Delta \\ 0, & otherwise \end{cases} \tag{21}$$

width approaches zero as $\Delta \to 0$; height approaches infinity as $\Delta \to 0$

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t) \tag{22}$$

1.7 Continuous-time system

Definition 1.7.1 (Continuous-time(CT) System)**.** a device that transforms input CT signal into another output CT signal.

$$y(\cdot) = \mathcal{T}(x(\cdot)) \tag{23}$$

Definition 1.7.2 (Input-output Relationship)**.** Precisely defines how the output signal is related to the input signal.

- Series connection

$$x(t) \to \boxed{\mathcal{T}_1} \to \boxed{\mathcal{T}_2} \to y(t) \tag{24}$$

$$y(t) = \mathcal{T}_2[\mathcal{T}_1[x(t)]] \tag{25}$$

- Parallel connection

$$y(t) = \mathcal{T}_1[x(t)] + \mathcal{T}_2[x(t)] \tag{26}$$

1.7.1 Classification of CT Systems

- Amplitude properties
  - A-1 linearity

$$\mathcal{T}[a_1x_1(t) + a_2x_2(t)] = a_1\mathcal{T}[x_1(t)] + a_2\mathcal{T}[x_2(t)] \tag{27}$$

Property

$$\mathcal{T}[ax(t)] = a\mathcal{T}[x(t)] \tag{28}$$

superpostion property

$$\mathcal{T}[\sum_{k=1}^{K} \mathcal{T}[x_k(t)]] \tag{29}$$

$$\mathcal{T}[\int x(t;v)dv] = \int \mathcal{T}[x(t;v)]dv \tag{30}$$

- A-2 stability
  Satisfy BIBO

  Definition 1.7.3 (Bounded-input Bounded-output (BIBO) stable)**.** Every bounded input produces a bounded output

  Triangle Inquality

$$|\sum_{n} a_n| \leq \sum_{n} |a_n| \tag{31}$$

- invertibility

  Definition 1.7.4 (invertible)**.** each output signal is the response to only one input signal.

  Property

$$\mathcal{T}^{-1}[\mathcal{T}[x(t)]] = x(t) \tag{32}$$

- Time properties
  - T-1 causality

    Definition 1.7.5 (Casual System)**.** The output $y(t)$ at time t only depends on the present and (possibly) past inputs, not on future inputs.

    Noncasual systems arise often when t is other variables than time, such as space.

  - T-2 memory

    Definition 1.7.6 (Static system or Memoryless System)**.** The output $y(t)$ at time t only depends on the current input $x(t)$.
    Otherwise is a dynamic system.

  - T-3 time-invariance Systems whose input-output behavior does not change with time

    Definition 1.7.7 (Time Invariant)**.**

$$x(t) \xrightarrow{\mathcal{T}} y(t) \quad \textit{implies that} \quad x(t-t_0) \xrightarrow{\mathcal{T}} y(t-t_0) \tag{33}$$

2 CT LTI Systems

2.1 Introduction

Primary forcus: **CT linear-time-variant (LTI)** systems. Overview:

$$x(t) \rightarrow \boxed{LTI\ with\ imulse\ response\ h(t)} \rightarrow y(t) = x(t) * h(t) \tag{34}$$

where $\delta(t) \xrightarrow{\mathcal{T}} h(t)$
Input-output relationships (given by convolution intergral):

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \tag{35}$$

- Any system whose input and output can be discribed by the above form is LTI system
- An LTI system is completely described by its impulse response $h(t)$
- We can determine the response $y(t)$ due to any input signal with impulse response.

2.2 LTI system properties

2.2.1 Properties of Convolution and Impulse Functions

- commutative property

$$x(t) * h(t) = h(t) * x(t) \tag{36}$$

- associative property

$$[x(t) * h_1(t)] * h_2(t) = x(t) * [h_1(t) * h_2(t)] \tag{37}$$

- distributive property
-

$$x(t) * \delta(t) = x(t) \tag{38}$$

- delay property

$$x(t) * \delta(t - t_0) = x(t - t_0) \tag{39}$$

- If $y(t) = x(t) * h(t)$, then $x(t-t_0) * h(t-t_1) = y(t-t_0-t_1)$

there are four remaining properties in terms of $h(t)$.

- T-1 casusality

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\ &= \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau + \int_{-\infty}^{0^-} h(\tau)x(t-\tau)d\tau \end{aligned}$$

Definition 2.2.1 (LTI causal)**.** An LTI system is causal iff its impulse response $h(t) = 0$ for all $t < 0$.

then using $\tau' = t - \tau$

$$y(t) = \int_0^\infty h(\tau)x(t-\tau)d\tau = \int_{-\infty}^t x(\tau')h(t-\tau')d\tau' \tag{40}$$

- T-2 memory

  Definition 2.2.2 (LTI memoyless)**.** Iff its impulse response is $h(t) = a\delta(t)$. Otherwise is dynamic.

  In this case, the response is $y(t) = ax(t)$
  There are two classes of dynamic systems:

  –

    Definition 2.2.3 (finite impulse response (FIR))**.** has$h(t)$ tha is nonzero only within some finite interval $t_1 < t < t_2$.

  –

    Definition 2.2.4 (infinite impulse response (IIR))**.** has $h(t)$ that persists indefinitely.

- A-2 stability Suppose $x(t)$ is a bounded input signal $|x(t)| \leq M_x < \infty \ \ \forall t$.

$$\begin{aligned}
|y(t)| &= |\int_{-\infty}^\infty h(\tau)x(t-\tau)d\tau| \\
&\leq \int_{-\infty}^\infty |h(\tau)x(t-\tau)|d\tau \quad (trangle\ inequality) \\
&= \int_{-\infty}^\infty |h(\tau)||x(t-\tau)|d\tau \\
&\leq M_x \int_{-\infty}^\infty |h(\tau)|d\tau
\end{aligned}$$

  So, sufficient condition:

$$\int_{-\infty}^\infty |h(\tau)|d\tau < \infty \tag{41}$$

  Definition 2.2.5 (LTI BIBO stable)**.** Iff its imppluse response is absolutely integrable, i.e. $\int_{-\infty}^\infty |h(t)|dt < \infty$

- A-3 invertibility Fact: if a system is LTI, then if it is also invertible, the inverse system is also LTI.

$$x(t) \to \boxed{LTI\ h(t)} \to y(t) \to \boxed{LTIh_i(t)} \to z(t) = x(t) \tag{42}$$

  The cascade of two LTI system is also LTI.

Definition 2.2.6 (LTI invertible)**.** If the system is invertible, then

$$h(t) * h_i(t) = \delta(t) \tag{43}$$

2.3 Step response

A way to find the impluse response $h(t)$ of an LTI system in practice.

3 Fourier Series

3.1 LTI system response for complex-exponential input signals

What happens when pass e^{st} through an LTI system

$$x(t) = e^{st} \rightarrow \boxed{LTI\ h(t)} \rightarrow y(t) \tag{44}$$

By convolution integral:

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau \\ &= e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau \end{aligned}$$

so,

$$x(t) = e^{st} \rightarrow \boxed{LTI\ h(t)} \rightarrow y(t) = e^{st}H(s) \tag{45}$$

where,

$$\boxed{H(s) := \int_{-\infty}^{\infty} h(t)e^{-st}dt} \tag{46}$$

is the system **transfer function**. Also, $H(s)$ is the Laplace transform of $h(t)$. Remark: So the exponential signal passed through an LTI system produces the same exponential signal, but scaled by $H(s)$.

An important case: $s = j\omega$

$$x(t) = e^{j\omega t} \rightarrow \boxed{LTI\ h(t)} \rightarrow y(t) \tag{47}$$

$$y(t) = H(j\omega)e^{j\omega t} = |H(j\omega)|e^{j\angle H(j\omega)}e^{j\omega t} = |H(j\omega)|e^{j[\omega t + \angle H(j\omega)]} \tag{48}$$

- The quantity $H(j\omega)$ is called **frequency response** of the system, and is often just written $H(\omega)$
- In general $H(\omega)$ is complex, so both the magnitude and phase of the complex-exponential signals are affected.

3.1.1 Eigenfunction and eigenvalues

Definition 3.1.1 (eigenfucntion & eigenvalues)**.** When a signal has the property that when passing through a system, it yields the same signal scaled by a (perhaps complex) constant, the signal is called an eigenfucntion and the scaling factor is called the eigenvalues.

- eigenfucntion:e^{st}
- eigenvalues: $H(s)$

3.2 Preview

3.2.1 Euler's identity

$$e^{j\theta} = cos\theta + jsin\theta \tag{49}$$

$$cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta}), \quad sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) \tag{50}$$

3.2.2 Multiplication in frequency domain

$$e^{j\omega t} \rightarrow \boxed{LTI\ h(t)} \rightarrow H(j\omega)e^{j\omega t} \tag{51}$$

Thus, by linearity:

$$x(t) = \sum_{n=1}^{\infty} c_k e^{j\omega_0 kt} \rightarrow \boxed{LTI\ h(t)} \rightarrow y(t) = \sum_{k=1}^{\infty} c_k H(j\omega_0 k) e^{j\omega_0 kt} \tag{52}$$

Convolution in time domain becomes multiplication in frequency domain.

3.2.3 Periodic signal

When an periodic signal is passed through an LTI system

- the output is also periodic
- the Fourier series of the output has coefficients $c_k H(j\omega_0 k)$, where the c_k are the Fourier series coefficients of the input signal.

3.3 Fourier Series

3.4 Synthesis

Definition 3.4.1 (Synthesis)**.** A periodic signal $x(t)$ with fundamental peroid T_0 has the following **Fourier Series** representation:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \tag{53}$$

called synthesis equation.

- T_0: fundamental period
- ω_0: fundamental frequency. $\omega_0 = 2\pi/T_0$
- c_k: Fourier coefficients
- $k\omega_0$: the kth harmonic

3.4.1 Analysis equation

Compute the Fourier coefficients by the following formula,

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt, k = 0, \pm 1 \tag{54}$$

$\int_{T_0}$ denotes the integration over one period.
Note that for k = 0, we get the average value or DC value of the signal:

$$c_0 = \frac{1}{T_0}\int_{T_0} x(t)dt \tag{55}$$

Definition 3.4.2 (Fourier series of periodic CT signal)**.**

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},\ synthesisequatio \tag{56}$$

$$c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt, k = 0, \pm 1,\ analysisequation \tag{57}$$

The above is called the exponential form of Fourier series. And can be applied to complex-value signals.

3.4.2 Hermitian symmetry

There two other FS forms that are useful for real signals. To derive these forms, we first need the following fact:

Definition 3.4.3 (Hermitian symmetry)**.** If $x(t)$ is real, then $c_{-k} = c_k^*$

3.4.3 Trigonometric forms of FS

Although exponential form is useful, often we want a real representation.

Definition 3.4.4 (Combined trigonometric form)**.**

$$x(t) = c_0 + \sum_{k=1}^{\infty} 2|c_k|cos(k\omega_0 t + \theta_k) \tag{58}$$

where

$$c_k = |c_k|e^{j\theta_k} \tag{59}$$

How to derive this:

- if $x(t)$ is real, $c_{-k} = c_k^*$
- thus if $c_k = |c_k|e^{j\theta_k}$ then $c_{-k} = |c_k|e^{-j\theta_k}$, where $\theta_k = \angle c_k$

Definition 3.4.5 (Trigonometric form)**.**

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} A_k cos(k\omega_0 t) - B_k sin(k\omega_0 t) \tag{60}$$

where $A_k = Real(c_k)$, $B_k = Imag(c_k)$.

3.5 Convergence of Fourier Series

In practice we use finite series approximation:

$$x_N(t) = \sum_{k=-N}^{N} c_k e^{jk\omega_0 t} \tag{61}$$

The *error signal*:

$$e_N(t) := x(t) - x_N(t) \tag{62}$$

Definition 3.5.1 (Dirichlet conditions)**.** Following conditions:

- $x(t)$ bounded, or absolutely integrable:

$$\int_{T_0} |x(t)| dt < \infty \tag{63}$$

- has a finite number of max and min in each period
- has at most a finite number of finite discontinuities over one period.

With this condition

$$e_N(t) \to 0 \; as \; N \to \infty \tag{64}$$

3.6 Properties of CT Fourier series

3.6.1 One-signal properties(Fourier series transformations)

From signal $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \tag{65}$$

transform(by a time or amplitude transformation) to form $y(t)$, when $y(t)$ is also periodic, can also express $y(t)$ by a FS:

$$y(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_1 t} \tag{66}$$

where ω_1 is the fundamental frequency od $y(t)$, and may not equal to ω_0
While we don't want to recompute everything, so we need to know the properties.

Amplitude transformations

$$d_k = \begin{cases} b + ac_0, & k = 0 \\ ac_k, & k \neq 0 \end{cases} \tag{67}$$

General time transformations

$$y(t) = ax(t) = b \tag{68}$$

$$y(t) = x(at + b) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_1 t} \tag{69}$$

$$d_k = c_k e^{jk\omega_0 b} \tag{70}$$

where $\omega_1 = a\omega_0$

Time reversal

$$y(t) = x(-t) \tag{71}$$

$$d_k = c_{-k} \tag{72}$$

Time shifting

$$y(t) = x(t = t_0) \tag{73}$$

$$d_k = c_k e^{-jk\omega_0 t_0} \tag{74}$$

Conjugation

$$y(t) = [x(t)]^* \tag{75}$$

$$d_k = c_{-k}^* \tag{76}$$

Complex modulation(frequency shift)

$$y(t) = x(t)e^{j\omega_0 tN} \tag{77}$$

$$d_k = c_{k-N} \tag{78}$$

meaning that the coefficients are all shifted by N.

Differentiation

$$y(t) = \frac{d}{dt}x(t) \tag{79}$$

$$d_k = jk\omega_0 c_k,\ k \neq 0 \tag{80}$$

3.6.2 Two-signal properties

Multiplication

$$c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l} \tag{81}$$

this is called discrete convolution.

Circular convolution

$$y(t) = \frac{1}{T_0}\int_{T_0} x_1(t-\tau)x_2(\tau)d\tau \tag{82}$$

$$d_k = a_k b_k \tag{83}$$

3.6.3 Parsecal's Relation for CT Periodic Signals

$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |c_k|^2 \tag{84}$$

3.7 Power density spectrum

A plot that shows how much power the signal has in each frequency component $k\omega_0$. It is a plot of component power $|c_k|^2$ vs frequency $k\omega_0$

3.7.1 Rectangular Pulse train

For $x(t) = \sum_{n=-\infty}^{\infty} rect(\frac{t-nT_0}{w})$, its FS:

$$c_k = \frac{w}{T_0} sinc(kw\frac{\omega_0}{2\pi}) \tag{85}$$

where the sine cardinal function is:

$$sinc(x) := \begin{cases} \dfrac{sin\pi x}{\pi x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \tag{86}$$

3.8 Fourier Series and LTI Systems

3.8.1 Complex exponential signals

$$x(t) = e^{j\omega t} \rightarrow \boxed{LTI\ h(t)} \rightarrow = H(j\omega)e^{j\omega t} = |H(j\omega)|e^{j(\omega t + \angle H(j\omega))} \tag{87}$$

Mathematical review
Polar form:

$$z = |z|e^{j\theta}, \quad |z| = \sqrt{x^2+y^2},\ \theta \tag{88}$$

4 Fourier Transform

For an aperiodic signal $f(t)$, derive the following relationships:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega,\ F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{89}$$

The functions $f(t)$ and $F(t)$ are called Fourier transform pairs and we write

$$f(t) \overset{\mathcal{F}}{\leftrightarrow} F(t) \tag{90}$$

4.1 FT form FS

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \overset{\mathcal{F}}{\leftrightarrow} X(\omega) = \sum_{k=-\infty}^{\infty} c_k 2\pi\delta(\omega - k\omega_0) \tag{91}$$

4.2 Properties of CT FT

4.2.1 Time-transformations

$$f(at+b) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{|a|}e^{j\omega b/a}F(\omega/a) \tag{92}$$

4.2.2 Time shift

$$f(t-t_0) \overset{\mathcal{F}}{\leftrightarrow} e^{-j\omega t_0}F(\omega) \tag{93}$$

4.2.3 Time scale

$$f(at) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{|a|}F(\omega/a) \tag{94}$$

4.3 Conjugation

$$f^*(t) \overset{\mathcal{F}}{\leftrightarrow} F^*(-\omega) \tag{95}$$

4.3.1 Hermitian symmetric

if $f(t)$ is real,

$$F(\omega) = F^*(-\omega) \tag{96}$$

4.3.2 Real and even

- If $f(t)$ is real and even, $F(t)$ is also real and even
- If $f(t)$ is real and odd, $F(t)$ is purely imaginary and odd.

4.3.3 Duality

FT pairs has the following dual relationship.

$$x(t) = F(t) \overset{\mathcal{F}}{\leftrightarrow} X(\omega) = 2\pi f(-\omega) \tag{97}$$

4.3.4 Time differentiation

$$\frac{d^k}{dt^k} f(t) \overset{\mathcal{F}}{\leftrightarrow} (j\omega)^k F(\omega) \tag{98}$$

4.3.5 Frequancy differentiation

$$(-jt)^n f(t) \overset{\mathcal{F}}{\leftrightarrow} \frac{d^n}{d\omega^n} F(\omega) \tag{99}$$

4.4 Convolution property

$$y(t) = h(t) * x(t) \overset{\mathcal{F}}{\leftrightarrow} Y(\omega) = H(\omega)X() \tag{100}$$