Units and Measurement

Errors

Difference between measured and actual values

- Absolute error (ΔE)
- Relative error $\left(\dfrac{\Delta E}{E}\right)$
- Percentage error $\left(\dfrac{\Delta E}{E}\times 100\right)$

Physical Quantity (P.Q.)

- **Fundamental Quantities :**

  Set of independent P.Q. used to express all physical quantities.

- **Derived Quantity :**

  Derived from fundamental quantities

Dimensions

Powers to which fundamental P.Q. raised to express derived quantity

Applications:

- Conversion of units ($n_1 \mu_1 = n_2 \mu_2$)
- Principle of homogeneity
- Derivation of empirical formula

Measurement

- **Significant Figures :**

  Digits that have significance

- **Accuracy :**

  Closeness of measured value to actual value

- **Precision :**

  Upto what limit, measurement can be done. Depends on least count of instrument.

Propagation of Errors

- If $A \pm B = C$

  then, $\Delta C = \Delta A + \Delta B$

- If $C = \dfrac{A}{B}$ or $C = AB$

  then, $\dfrac{\Delta C}{C}\times 100 = \dfrac{\Delta A}{A}\times 100 + \dfrac{\Delta B}{B}\times 100$

- If $C = \dfrac{A^n B^m}{D^o}$

  then,

  $$\frac{\Delta C}{C}\times 100 = n\left(\frac{\Delta A}{A}\times 100\right) + m\left(\frac{\Delta B}{B}\times 100\right) + o\left(\frac{\Delta D}{D}\times 100\right)$$

- If $\dfrac{1}{C} = \dfrac{1}{A} + \dfrac{1}{B}$

  then, $\dfrac{\Delta C}{C^2} = \dfrac{\Delta A}{A^2} + \dfrac{\Delta B}{B^2}$

Measuring Devices

- Vernier Scale

  $$L.C. = \frac{1\,\text{MSD}}{\text{No. of Vernier Division}}$$

  Correct Reading = MSR + VSR – Error

- Screw Gauge

  $$L.C. = \frac{\text{Pitch}}{\text{Circular Scale Division}}$$

  Correct Reading = MSR + CSR – Error

Motion in a Straight Line

Instantaneous Speed and Velocity

$$v = \frac{dx}{dt}$$

$$\underbrace{\Delta x}_{\text{Displacement}} = \int v\,dt$$

Equations of Motion (Constant acceleration)

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s_n = u + \frac{a}{2}(2n-1)$$

Acceleration

Average Acceleration

$$\langle a \rangle = \frac{v-u}{t}$$

Instantaneous Acceleration

$$a = \frac{dv}{dt}$$

Relative Motion

$$\vec{r}_{B,A} = \vec{r}_B - \vec{r}_A$$

$$\vec{v}_{B,A} = \vec{v}_B - \vec{v}_A$$

$$\vec{a}_{B,A} = \vec{a}_B - \vec{a}_A$$

Graphs

Graphs	x - t	v - t	a - t
Slope	Velocity	Acceleration	
Area under curve		Displacement	Change in velocity

Distance and Displacement

- **Distance :**

  Total path length travelled by object

- **Displacement ($\Delta\vec{x}$) :**

  Vector joining initial to final position

  $\Delta\vec{x} = x_f - x_i$

Motion under Gravity

Projected up with velocity u

Time of flight $t = \frac{2u}{g}$

Max. height $h = \frac{u^2}{2g}$

Final velocity $-u$

Dropped ($u = 0$) from height h

Time of flight $t = \sqrt{\frac{2h}{g}}$

Final velocity

$v = \sqrt{2gh}$

Non-uniform Acceleration (Variable Acceleration)

$$a = \frac{dv}{dt}$$

$$\downarrow$$

$$v - u = \int_{t_1}^{t_2} a\,dt$$

When acceleration is function of t

$$a = \frac{v\,dv}{dx}$$

$$\int_u^v v\,dv = \int_{x_1}^{x_2} a\,dx$$

When a is function of position x

Average Speed and Velocity

Average Speed $\langle v \rangle = \frac{\text{Total distance}}{\text{Time}}$

If particle travels with speed v_1 and v_2 for

* Equal time interval $\langle v \rangle = \frac{v_1 + v_2}{2}$

* Equal distance $\langle v \rangle = \frac{2v_1v_2}{v_1 + v_2}$

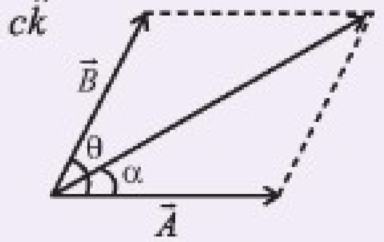
Average velocity $= \frac{\text{Total displacement}}{\text{Time}}$

Motion in a Plane

Vectors

→ Vector in plane - $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

* Addition of Vectors
* Parallelogram Method -



$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + \cos\theta}}$$

$$\boxed{\begin{aligned} R_{max} &= A + B \\ R_{min} &= |A - B| \end{aligned}}$$

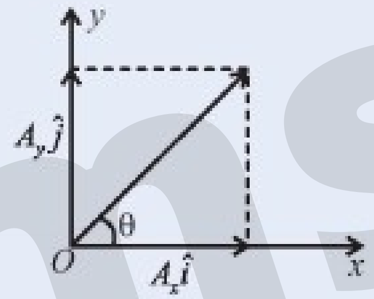
Resolution of Vectors

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}}$$

$$\boxed{A_x = A\cos\theta} \quad \boxed{A_y = A\sin\theta}$$

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

$$\boxed{\tan\theta = \frac{A_y}{A_x}}$$



When $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Dot Product of Vectors

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ $\quad \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\boxed{\vec{a}\cdot\vec{b} = a_1a_2 + b_1b_2 + c_1c_2}$$

$$\boxed{\hat{i}\cdot\hat{i} = 1,\ \hat{j}\cdot\hat{j} = 1,\ \hat{k}\cdot\hat{k} = 1} \quad \boxed{\hat{i}\cdot\hat{j} = 0,\ \hat{j}\cdot\hat{k} = 0,\ \hat{k}\cdot\hat{i} = 0}$$

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\theta} \quad \boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{ab}}$$

→ Projection of a vector on another vector

$$\boxed{A\cos\theta = A\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]}$$

Cross Product

$\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k},\ \vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= (b_1c_2 - b_2c_1)\hat{i} - (a_1c_2 - a_2c_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

$$\boxed{\vec{a}\times\vec{b} = ab\sin\theta\ \hat{n}}$$

→ Direction of $\hat{n}$ can be found using right hand rule

→ Circle Rule -

$$\boxed{\begin{aligned} \hat{i}\times\hat{j} &= \hat{k} \\ \hat{j}\times\hat{k} &= \hat{i} \\ \hat{k}\times\hat{i} &= \hat{j} \end{aligned}} \quad \boxed{\begin{aligned} \hat{i}\times\hat{i} &= 0 \\ \hat{j}\times\hat{j} &= 0 \\ \hat{k}\times\hat{k} &= 0 \end{aligned}}$$

Kinematics

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}}$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}}$$

→ Equations of Motion

1. $\vec{v} = \vec{u} + \vec{a}t$
2. $\Delta\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
3. $|\vec{v}|^2 = |\vec{u}|^2 + 2\vec{a}\cdot\Delta\vec{x}$

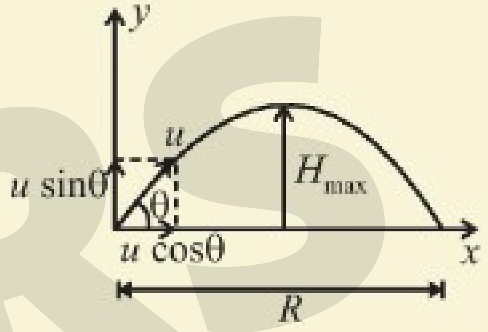
$$\boxed{\begin{aligned} \vec{a} &= a_x\hat{i} + a_y\hat{j} \\ \Delta\vec{r} &= x\hat{i} + y\hat{j} \\ \vec{v} &= u_x\hat{i} + u_y\hat{i} \end{aligned}} \quad \boxed{\begin{aligned} v_x &= u_x + a_xt \\ v_y &= u_y + a_yt \\ \Delta x &= u_xt + \frac{1}{2}a_xt^2 \\ \Delta y &= u_yt + \frac{1}{2}a_yt^2 \end{aligned}}$$

Projectile Motion

$$T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g}$$

$$H = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2\theta}{2g}$$

$$R = \frac{2u_xu_y}{a_y} = \frac{u^2\sin 2\theta}{g}$$



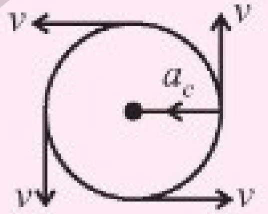
→ Equation of Trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

* If (a) $u = 0$ → linear path ($a \neq 0$)
  (b) $u \neq 0$ and $u \parallel a$ → linear path
  (c) $u \neq 0$ and u not $\parallel a$ → parabolic

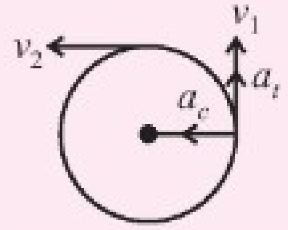
Circular Motion

• Uniform Circular Motion -
→ Speed = Constant,
Velocity ≠ Constant
→ Acceleration ≠ Constant



$$\boxed{|\vec{a_c}| = \frac{v^2}{r}} \quad \boxed{T = \frac{2\pi r}{v}}$$

• Non-uniform Circular Motion -
→ Speed ≠ Constant,
Velocity ≠ Constant
→ Acceleration ≠ Constant



$$\boxed{|\vec{a_c}| = \frac{|\vec{v}|^2}{r}} \quad \boxed{|\vec{a_t}| = \frac{d|\vec{v}|}{dt}} \quad \boxed{\vec{a}_{net} = \vec{a_c} + \vec{a_t}}$$

• Relative Motion - $\boxed{\vec{V_{AB}} = \vec{V_A} - \vec{V_B}}$
→ River-Boat
* For minimum time - $\boxed{t = \frac{d}{v_m}}$
* For minimum distance - $\boxed{t = \frac{d}{\sqrt{v_m^2 - v_r^2}}}$

v_m = velocity of man w.r.t. river
v_r = velocity of river w.r.t. ground

Laws of Motion

Force and Newton's Laws

Force - a push or pull which tries to change the state or shape of an object.

- Gravitational Force
- Electromagnetic Force
  - Normal
  - Tension
  - Spring Force
  - Friction
- Weak Nuclear Force
- Strong Nuclear Force

Newton's Law of Motion -

→ First Law - Law of Inertia

→ Second Law - $\boxed{F \propto \dfrac{dp}{dt} \Rightarrow F = ma}$

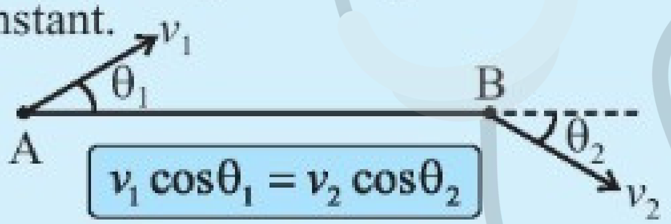
→ Third Law - Action-Reaction Law

Solving Problem - System → Identify the force → Free Body Diagram → Use Newton's Laws

String Constraint and Circular Motion

String Constraint -

→ Velocity along the string should remain constant.



$$\boxed{v_1 \cos\theta_1 = v_2 \cos\theta_2}$$

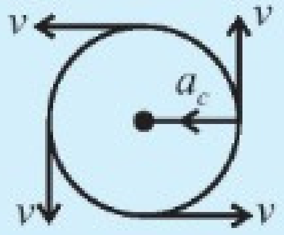
→ Virtual - Work Method -

$$\boxed{\vec{T_1} \cdot \vec{v_1} + \vec{T_2} \cdot \vec{v_2} + \ldots = 0}$$

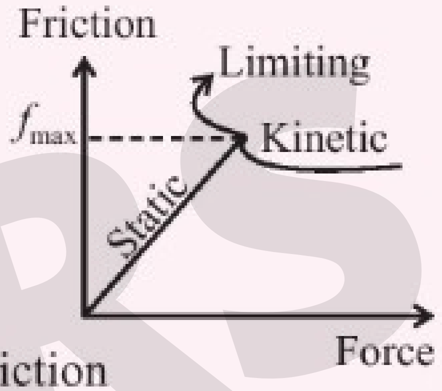
Uniform Circular Motion -

$$F_{net} = ma_c$$

$$F_{net} = \frac{mv^2}{r}$$



Friction



- **Static Friction -**

  $f_{static} = f_{applied}$

- **Limiting Friction -**

  $f_{Limiting} = \mu_s N$

  μ_s = coefficient of static friction

- **Kinetic Friction -**

  $f_{Kinetic} = \mu_k N$

  μ_k = coefficient of kinetic friction

- **Conservation of Momentum -**

  → When external force = 0

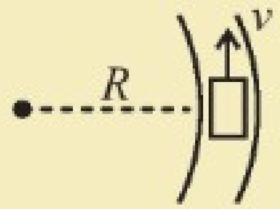
  → Momentum of the system remains conserved.

  Recoil velocity of the gun $= -\dfrac{m\vec{v}}{M}$

Banking on Roads

→ Without Banking -

$$v_{max} = \sqrt{\mu R g}$$

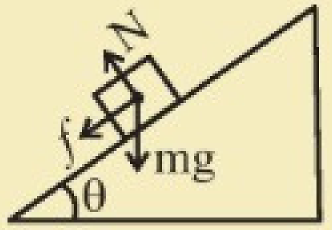


→ With Banking -

$$v_{max} = \sqrt{gR\tan(\phi + \theta)}$$

$\phi = \tan^{-1}(\mu)$

θ = angle of banking



Some Other Forces

→ Thrust Force - $F = -\dfrac{vdm}{dt}$

→ Spring Force - $F = Kx$

→ Pseudo Force - $\boxed{\vec{F} = -m\vec{a}}$ → negative sign shows that the direction of force is opposite to the direction of acceleration of frame

Here, m = mass of body,
a = acceleration of frame

Work, Energy and Power

Work

* When force is constant -

$\boxed{W = F \cdot d \cos\theta}$

- If $\theta = 0°$, $W = +ve$
- If $\theta = 90°$, $W = 0$
- If $\theta = 180°$, $W = -ve$

1 erg	10^{-7} J
1 eV	1.6×10^{-19} J
1 cal	4.186 J
1 kWh	3.6×10^{6} J

* Word done by a variable force -

$$W = \int \vec{F} \cdot d\vec{r}$$

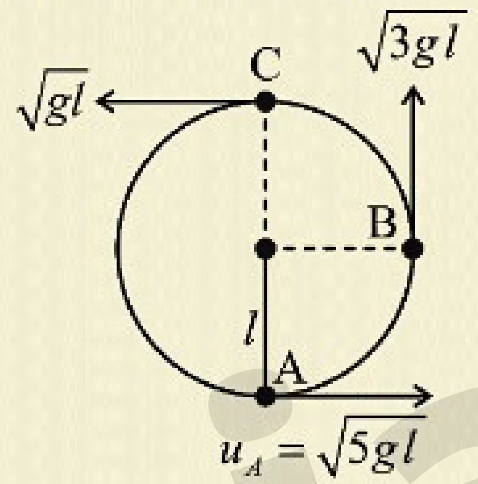
$$W = \int_{x_1}^{x_2} F_x \cdot dx + \int_{y_1}^{y_2} F_y \cdot dy + \int_{z_1}^{z_2} F_z \cdot dz$$

* Work Energy Theorem -

Work done by all forces $= \Delta K$

ΔK = change in K.E.

Vertical Circular Motion



→ Minimum velocity required at each point to complete circle

Conservation of Energy and Power

* Conservation of Mechanical Energy -

→ In the absence of non-conservative forces -

$$K_i + U_i = K_f + U_f$$

→ Mechanical Energy $= K + U$

* Power - rate of doing work

$$P_{avg} = \frac{W}{t} \qquad P_{inst} = \vec{F} \cdot \vec{v}$$

* Relation between Potential Energy and Force

$$W_c = \int \vec{F}_c \cdot d\vec{r} = -\Delta U$$

$$\vec{F} = -\frac{\delta U}{\delta x}\hat{i} - \frac{\delta U}{\delta y}\hat{j} - \frac{\delta U}{\delta z}\hat{k}$$

Kinetic Energy and Potential Energy

* Kinetic Energy - $\boxed{K = \frac{1}{2}mv^2}$

→ Conservative Forces -
Work done does not depend on the path
Example - Gravitational Force, Electrostatic Force

→ Non-Conservative Forces -
Work done depends on the path. *Example* - Friction

* Potential Energy - Only defined for conservative forces

→ Gravitational potential energy $= mgh$

→ Spring potential energy $= \frac{1}{2}kx^2$

→ Work done by spring force -

$$W_s = -\Delta U = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Collisions

→ Elastic Collisions - $(e = 1)$

Kinetic energy before collisions = Kinetic energy after collisions

→ Inelastic Collisions - $(0 < e < 1)$

Kinetic energy before collisions ≠ Kinetic energy after collisions

$$e = \frac{\text{Velocity of seperation}}{\text{Velocity of approach}}$$

→ For all types of collisions

Initial Momentum = Final Momentum

→ For perfectly inelastic collision - $(e = 0)$

Both bodies stick to each other

System of Particles and Rotational Motion

Centre of Mass

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{m_1 + m_2 + m_3}$$

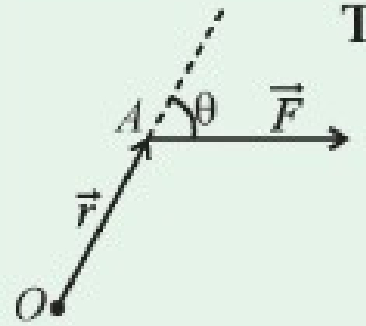
$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3}{m_1 + m_2 + m_3}$$

→ $\boxed{\vec{F}_{ext} = m\vec{a}_{cm}}$

→ When there is no external force - $\boxed{\vec{a}_{cm} = 0}$

$\boxed{\text{If } \vec{v}_{cm} = 0 \Rightarrow \Delta\vec{x}_{cm} = 0,\ \Delta\vec{y}_{cm} = 0}$

Torque



$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

$\boxed{|\vec{\tau}| = rF\sin\theta = r_\perp F = rF_\perp}$

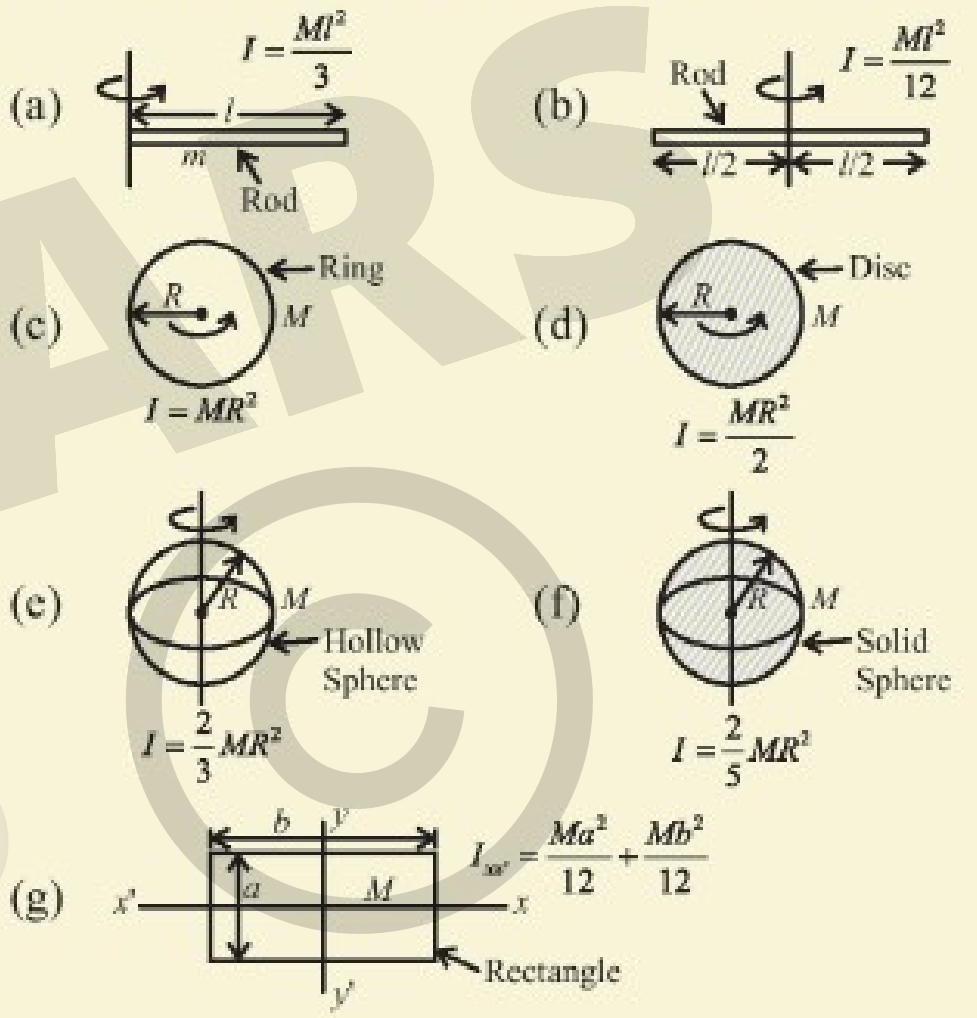
Equilibrium -

Translational Equilibrium - $\sum \vec{F} = 0$

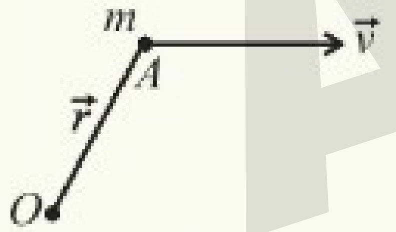
Rotational Equilibrium - $\sum \vec{\tau} = 0$

→ If $\vec{F}_{net} = 0$ on a rigid body, then $\vec{\tau}_{net}$ is same about every point.

Moment of Inertia of Different Bodies



Angular Momentum



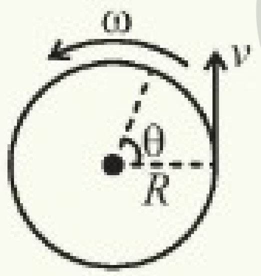
$\boxed{\vec{L} = \vec{r} \times \vec{p}}$

$\boxed{\vec{L} = \vec{r} \times m\vec{v}}$

* Angular Variable

$\theta = \frac{arc}{R} \Rightarrow arc = R\theta$

$|\vec{v}| = R\omega$



$\boxed{\omega = \frac{d\theta}{dt}}$ $\boxed{\alpha = \frac{d\omega}{dt}}$ $\boxed{\omega = \frac{2\pi n}{60}}$ n = revolution per minute (rpm)

→ If α = Constant : -

(i) $\omega = \omega_0 + \alpha t$

(ii) $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

→ Relation between $\vec{\tau}$ and $\vec{L}$:-

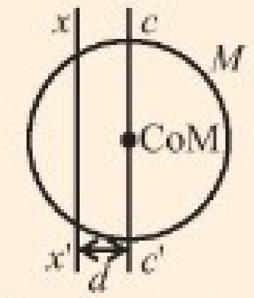
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Moment of Inertia

$\boxed{I = \int r^2 dm}$

→ Parallel Axis Theorem :-

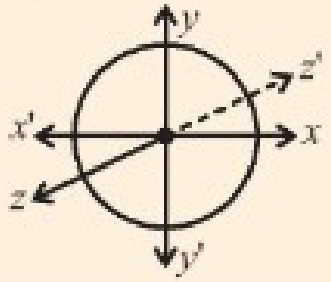
$\boxed{I_{xx'} = I_{cc'} + Md^2}$



→ Perpendicular Axis Theorem :-

$\boxed{I_{zz'} = I_{xx'} + I_{yy'}}$

Only for planar bodies



Rolling Motion

→ $\boxed{\sum \vec{F}_{ext} = M\vec{a}_{cm}}$ → $\boxed{\sum \vec{\tau}_{ext} = I\vec{\alpha}}$

→ $\boxed{v = R\omega}$ $\boxed{a = R\alpha}$ → Only for contact point

→ $\boxed{\text{Total K.E.} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2}$

→ $\boxed{\vec{L}_{system} = I_{cm}\vec{\omega} + m\vec{r}_{cm} \times \vec{v}_{cm}}$

* Work done by Torque

$\boxed{W = \int \tau\ d\theta}$ Power, $P = \vec{\tau} \cdot \vec{\omega}$

* Conservation of Angular Momentum -

When, $\vec{\tau}_{ext} = 0 \Rightarrow \vec{L} =$ constant

Gravitation

Kepler's Laws and Newton's Law of Gravitation

* **Kepler's 1st Law -** All the planets revolve around the sun in elliptical orbits and the sun lies at one of the focii of the orbit.
* **Kepler's 2nd Law -** Planets cover equal area in equal time intervals.

  Areal velocity $\Rightarrow \dfrac{d\vec{A}}{dt} = \dfrac{\vec{L}}{2m_p}$
* **Kepler's 3rd Law -** $\boxed{T^2 \propto r^3}$
* **Newton's Law of Gravitation -**

$$\boxed{F_g = G\frac{m_1 m_2}{r^2}} \qquad \boxed{G = 6.67\times10^{-11}\ \text{Nm}^2/\text{Kg}^2}$$

Gravitational Potential

- Due to a point mass : $V = \dfrac{-GM}{r}$
- Due to a ring : $V_{\text{centre}} = \dfrac{-GM}{R}$, $V_{\text{axis}} = \dfrac{-GM}{\sqrt{R^2+x^2}}$
- Due to a shell : $V_{\text{inside}} = \dfrac{-GM}{R}$, $V_{\text{outside}} = \dfrac{-GM}{r}$
- Due to a solid sphere : $V_{\text{inside}} = \dfrac{-GM}{2R^3}\left(3R^2 - r^2\right)$

  $V_{\text{outside}} = \dfrac{-GM}{r}$, $V_{\text{surface}} = \dfrac{-GM}{R}$
- Relation between V and $\vec{I}$: $V(\vec{r}) = -\int \vec{I}\cdot d\vec{r}$

  $\rightarrow \vec{I} = -\dfrac{\delta V}{\delta x}\hat{i} - \dfrac{\delta V}{\delta y}\hat{j} - \dfrac{\delta V}{\delta z}\hat{k}$

Gravitational Field (I)

- Due to a point mass - $\boxed{\left|\vec{I}_g\right| = \dfrac{\vec{F}}{m_0} = \dfrac{GM}{r^2}}$
- Due to a ring -

  $\boxed{I_{\text{centre}} = 0}$ $\boxed{I_{\text{axis}} = \dfrac{Gmx}{\left(a^2+x^2\right)^{3/2}}}$
- Due to a sold sphere -

  $\boxed{I_{\text{centre}} = 0}$ $\boxed{I_{\text{inside}} = \dfrac{GMr}{R^3}}$ $\boxed{I_{\text{outside}} = \dfrac{GM}{r^2}}$
- Due to a spheric shell -

  $\boxed{I_{\text{inside}} = 0}$ $\boxed{I_{\text{outside}} = \dfrac{GM}{r^2}}$
- Due to a disc - $\boxed{I_{\text{axis}} = \dfrac{GM}{R^2}\left[1 - \dfrac{x}{\sqrt{a^2+x^2}}\right]}$

Acceleration Due to Gravity

- Acceleration due to gravity : $\boxed{g_P = \dfrac{GM_P}{R_P^2}}$

  For Earth, $\boxed{g_e = 9.8\ \text{m/s}^2}$
- Effects on acceleration due gravity :

  (a) Effect of Altitude : $\boxed{g' = \dfrac{GM}{(R+h)^2} = \dfrac{gR^2}{(R+h)^2}}$

  For small heights : $\boxed{g' = g\left[1 - \dfrac{2h}{R}\right]}$

  (b) Effect of Depth : $\boxed{g'' = \dfrac{GM(R-d)}{R^3} = g\left[1 - \dfrac{d}{R}\right]}$

  (c) Effect of Rotation : $\boxed{g_{\text{eff}} = g - \omega^2 R\cos^2\lambda}$

  (At poles :- $g' = g$) (λ = Latitude)

  (At equator :- $g' = g - \omega^2 R$)

Escape Velocity and Satellites

- **Escape Velocity -** $\boxed{v_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{2gR}}$
- **Geostationary Satellite -** appear to be stationary because it moves with same angular velocity as that of earth with time period of 24 hours.
- **Polar Satellite -** orbital plane coincides with the axis of rotation of earth.
- Orbital Velocity, Time Period and Energy of a Satellite

  $\boxed{v_0 = \dfrac{v_e}{\sqrt{2}} = \sqrt{gR}}$ $\boxed{T = 2\pi\sqrt{\dfrac{R}{g}}}$ If $h << R$ $\boxed{K.E. = \dfrac{GMm}{2r}}$ $\boxed{P.E. = \dfrac{-GMm}{r}}$

Gravitational Potential Energy

- Two point mass-system : $\boxed{U = -\dfrac{GM_1M_2}{r}}$
- Self Potential Energy of a Shell : $\boxed{U_{\text{shell}} = \dfrac{-GM^2}{2R}}$
- Self Potential Energy of a Solid Sphere :

  $\boxed{U_{\text{sphere}} = \dfrac{-3GM^2}{5R}}$
- Gravitational Potential Energy of Earth Body System :

  For large height $\boxed{\Delta U = -Gm_1m_2\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right]}$

  When $h << R$ $\boxed{\Delta U = mgh}$

Mechanical Properties of Solids

Stress and Strain

→ **Elasticity :** property of the body due to which it tends to regain its original shape and size.

→ **Plasticity :** property due to which the body does not regain its original shape and size.

- **Stress :** The restoring force per unit area.

$$\sigma = \frac{F}{A}$$

- **Strain :** Change in configuration per unit original configuration.

$$\text{Longitudinal Strain} = -\frac{\Delta L}{L}$$

$$\text{Volumetric Strain} = \frac{\Delta V}{V}$$

$$\text{Shearing Strain} = \frac{\Delta x}{L} = \tan\theta \approx \theta$$

Poisson's Ratio

$$v = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

- **Important Relations :**

$$Y = 3B(1 - 2v)$$
$$Y = 2\eta(1 + v)$$
$$Y = \frac{9B\eta}{\eta + 3B}$$

Stress-Strain Curve

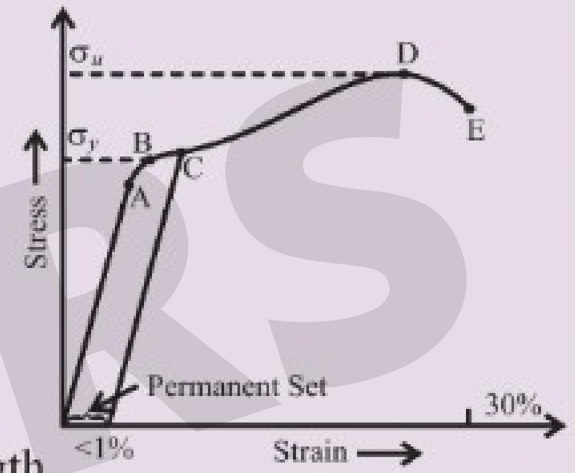
A = Proportional Limit

B = Yield Point

E = Fracture Point

σ_u = Yield Strength

σ_y = Ultimate Tensile Strength



→ If D & E are close :- Brittle Material

→ If D & E are far apart :- Ductile Material

Hooke's Law and Stress-Strain Curve

- **Hooke's Law :**

For small deformations :

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = k \times \text{Strain}$$

k = modulus of elasticity

Energy Stored in a Stretched Wire

$$E = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$$

Elastic Moduli

1. Young's Modulus of Elasticity :

$$Y = \frac{\text{Tensile or compressive stress}}{\text{Longitudinal strain}} = \frac{\sigma}{\varepsilon}$$

$$Y = \frac{FL}{A\Delta L}$$

2. Shear Modulus :

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F}{A\theta}$$

3. Bulk Modulus :

$$B = \frac{\text{Hydraulic stress}}{\text{Hydraulic strain}} = \frac{-P}{\Delta V/V}$$

Mechanical Properties of Fluids

Pressure

→ **Fluid :** that can flow, like - Gases, liquids

- **Pressure :** $P_{av} = \dfrac{F}{A}$

$1\ \text{Pa} = 1\ \text{N/m}^2$
$1\ \text{bar} = 10^5\ \text{Pa}$
$1\ \text{torr} = 1\ \text{mm of Hg}$
$1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$

- **Pascal's Law :** Pressure is same at all points at the same horizontal level for a fluid at rest

→ At depth h, absolute pressure $= P_0 + \rho g h$
$P_0 =$ Atmospheric pressure

→ Gauge pressure = Absolute pressure – Atmospheric pressure

→ Pressure applied at any point is transmitted equally in all directions

Equation of continuity and Bernoulli's Theorem

→ In stream line flow, velocity of particles passing at any point is same at that point.

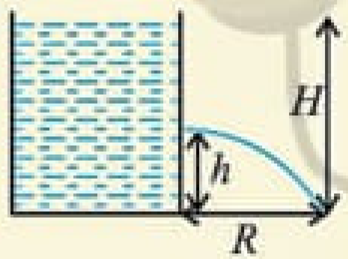
→ Equation of Continuity : (using conservation of mass) $A_1 v_1 = A_2 v_2$ for incompressible fluid

→ Bernoulli's Principle : For an incompressible and non-viscous fluid

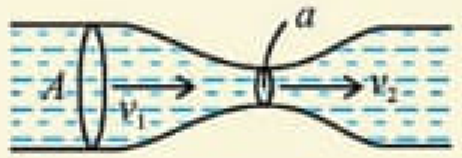
$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$ using conservation of energy

→ Torricelli's Law : (when the tank is open) $v_{\text{efflux}} = \sqrt{2gh}$

→ Range of Fluid : $R = 2\sqrt{h(H-h)}$
when $h = H/2 \Rightarrow R = \max$



→ Venturimeter : $v_1 = \sqrt{\dfrac{2h\rho_m g}{\rho\left(\dfrac{A^2}{a^2} - 1\right)}}$



Archimedes' Principle

Upthrust = weight of liquid displaced by the body

Upthrust $= V\rho g$

ρ = density of liquid

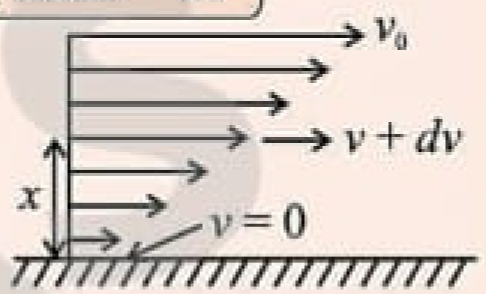
V = volume of immersed part of the body

Viscosity and Stoke's Law

→ Viscosity : a resistance to fluid motion.

→ Coefficient of Viscosity : $\eta = \dfrac{\text{Shearing stress}}{\text{Shearing strain}} = \dfrac{Fl}{vA}$

→ Viscous Force: (between layers) $F = -\eta A \dfrac{dv}{dx}$



* Stoke's Law : Viscous force on a body moving in the fluid is given by $F = 6\pi\eta r v$

* Terminal Velocity : $v_T = \dfrac{2r^2(\sigma - \rho)g}{9\eta}$

* Reynold's Number : $R_N = \dfrac{\rho v D}{\eta}$

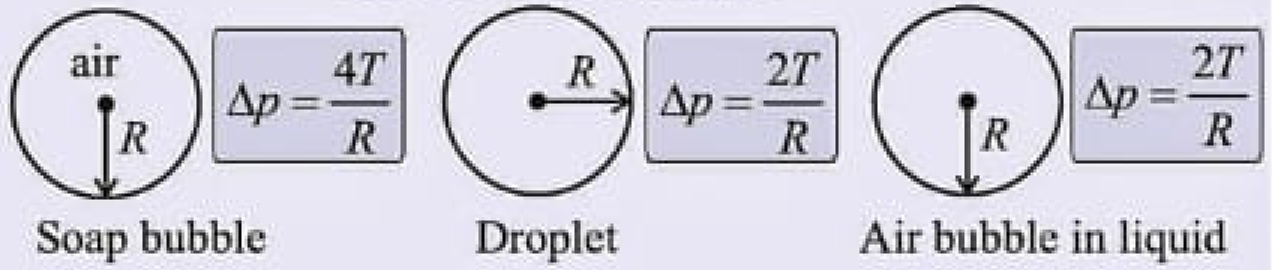
$R_N < 1000 \rightarrow$ streamline flow
$1000 < R_N < 2000 \rightarrow$ transition flow
$R_N > 2000 \rightarrow$ turbulent

Surface Tension and Capillary Rise

* Surface Tension : Force per unit length $T = \dfrac{F}{l}$

→ Surface Energy : amount of work to be done to form a surface $E = S \times A$

→



* Angle of Contact : angle between tangent to the liquid surface at the point of constant and solid surface inside the liquid.

→ when angle of contact is obtuse, the liquid will fall in capillary tube

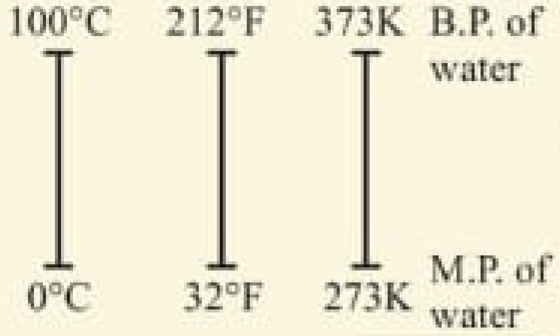
→ when angle of contact is acute, the liquid will rise in capillary tube

* Capillary Rise : $h = \dfrac{2 \cdot T\cos\theta}{r\rho g}$

Thermal Properties of Matter

Temperature and State Change

- **Temperature :** Degree of hotness or coldness



$$\boxed{\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{X-M.P.}{B.P.-M.P.}}$$

X = temperature at general scale

$$\boxed{F=\frac{9}{5}C+32}$$

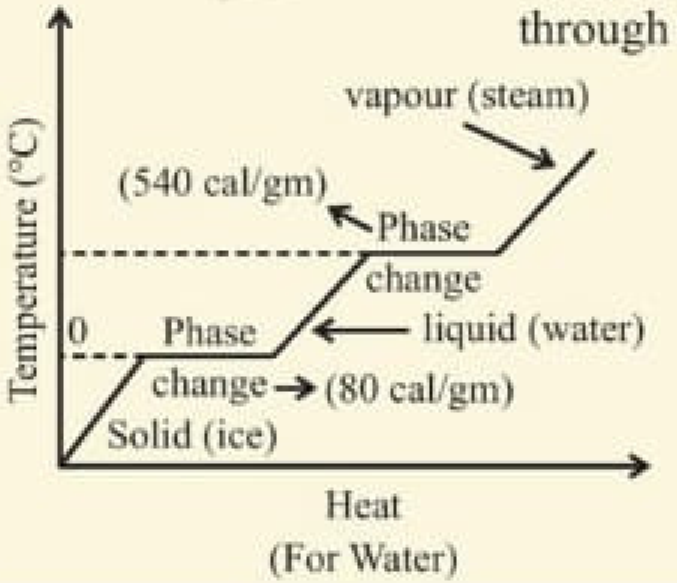
- **Heat Capacity :** $\boxed{\frac{Q}{\Delta T}=S}$
- **Specific Heat Capacity :** $\boxed{s=\frac{S}{m}=\frac{Q}{m\Delta T}}$

s_{water} = 1 cal/gm-°C = 4186 J/kg-K
s_{ice} = 0.5 cal/gm-°C

- **Principle of Calorimetry :** $\boxed{Q_1+Q_2+Q_3+...=0}$

Heat given by one system = Heat taken by other system

- **Water Equivalent :** $\boxed{s_c = Wm_w}$ where W is the water equivalent (in g/kg)
- **Change of State :**
  - * Solid to liquid → melting
  - * Liquid to solid → fusion
  - * Liquid to vapour → vapourisation
  - * Solid to vapour without passing through the liquid state → sublimation



Latent Heat : Heat required/unit mass to change the state

Thermal Expansion

- **Linear Expansion :** $\boxed{\Delta l = l\alpha\Delta T}$

$\boxed{l_f = l(1+\alpha\Delta T)}$ α = coefficient of linear expansion

- **Areal Expansion :** $\boxed{\Delta A = A\beta\Delta T}$

$\boxed{A_f = A(1+\beta\Delta T)}$ β = coefficient of areal expansion

- **Volume Expansion :** $\boxed{\Delta V = V\gamma\Delta T}$

$\boxed{V_f = V(1+\gamma\Delta T)}$ γ = coefficient of volume expansion

$\boxed{\gamma = 3\alpha;\ \beta = 2\alpha}$

- **Thermal Stress :** $\boxed{Y\alpha\Delta T}$

Heat Transfer

- **Conduction :** heat is transferred due to vibrations between the molecules.
- **Rate of flow of heat :** $\boxed{H=\frac{-KA\Delta T}{l}}$ K = coefficient of thermal conductivity
- **Analogy Between Electric Current and Heat Current :**

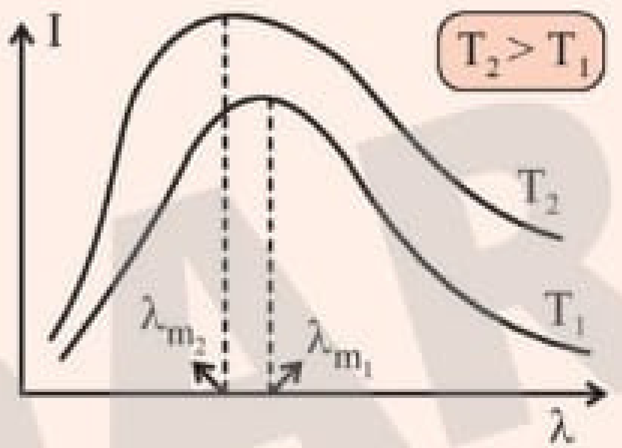
$\boxed{I=\frac{\Delta V}{R}} \equiv \boxed{H=\frac{\Delta T}{R}}$ For heat current: $\boxed{R=\frac{l}{KA}}$

→ **Convection :** Heat is transferred due to the transfer of mass

→ **Radiation :** Heat is transferred due to the electro-magnetic waves

Heat Radiation and Law of Cooling

- **Wien's Displacement Law :** $\boxed{\lambda_m T = b}$



b = Wien's constant $= 2.9\times10^{-3}$ m-K

Total energy carried by all radiation = Total area of I-λ graph

- **Stefan's Boltzman Law :**

Total power radiated by a black body : $\boxed{P=\sigma AT^4}$

For a body of emissivity e, $\boxed{P=e\sigma AT^4}$

If the temperature of surrounding = T_s

Power loss ⇒ $\boxed{P=e\cdot\sigma A\left(T^4-T_s^4\right)}$

- **Newton's Law of Cooling :**

Rate of Cooling, $\boxed{\frac{-dT}{dt}=K\left(T-T_0\right)}$

T = temperature of the body

T_0 = temperature of surrounding

When temperature drops from T_1 to T_2 :

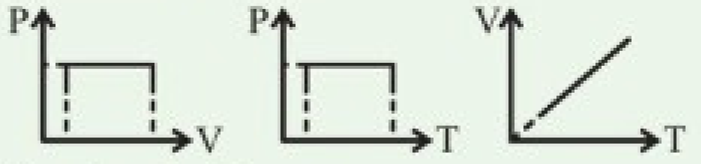
$$\boxed{\frac{-(T_2-T_1)}{t}=K\left[\frac{T_1+T_2}{2}-T_s\right]}$$

Thermodynamics

Zeroth Law and Thermodynamic Process

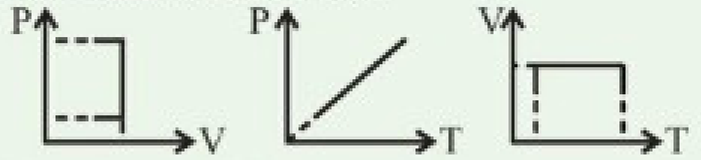
- **Zeroth Law :** If A is in thermal equilibrium of B and B is in thermal equilibrium of C, then A and C are also in thermal equilibrium. (A and C at same temperature)
- **Thermodynamic Process :**

1. Isobaric Process :



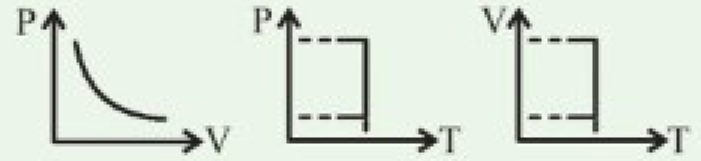
P = Constant
V ∝ T

2. Isochoric Process :



V = Constant
P ∝ T

3. Isothermal Process :



T = Constant
PV = Constant

4. Adiabatic Process :

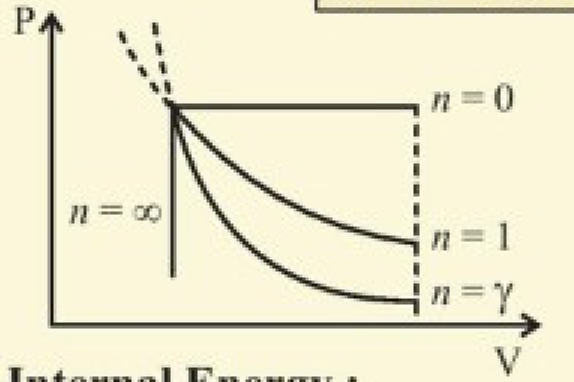
PV^{γ} = Constant or $TV^{\gamma-1}$ = Constant

$$\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$$

Polytropic Process and Internal Energy

- **Polytropic Process :** PV^n = Constant

$n = 0 \rightarrow$ Isobaric,	$n = \gamma \rightarrow$ Adiabatic
$n = 1 \rightarrow$ Isothermal,	$n = \infty \rightarrow$ Isochoric



$$\frac{dP}{dV} = -\frac{nP}{V}$$

- **Internal Energy :**

$f = 3 \rightarrow$ monoatomic
$f = 5 \rightarrow$ di-atomic
} for rigid molecules

$$U = \frac{f}{2}nRT$$

f = degree of freedom

Molar Specific Heat Capacity

$$C = \frac{Q}{n\Delta T}$$

$$C_P = \frac{Q}{n\Delta T} = R\left(\frac{f}{2}+1\right)$$

$$C_V = \frac{Q}{n\Delta T} = \frac{f}{2}R$$

For polytropic process :

$$C_n = C_V + \frac{R}{1-n}$$

$$C_n = \frac{R}{(\gamma-1)} + \frac{R}{(1-n)}$$

Word Done by Gas

$$W = \int_{V_1}^{V_2} PdV$$

1. Isochoric Process : $dV = 0 \Rightarrow W = 0$

2. Isobaric Process : $W = \int_{V_1}^{V_2} PdV = nR(T_2 - T_1)$

3. Isothermal Process :

$$W = nRT\ell n\left[\frac{V_2}{V_1}\right] = 2.303nRT\log_{10}\left[\frac{V_2}{V_1}\right]$$

4. Polytropic Process : $W = \frac{nR(T_2-T_1)}{1-n} = \frac{P_2V_2 - P_1V_1}{1-n}$

for adiabatic process : $n = \gamma$

- Work done = area under P-V curve
- **Work Done for Cyclic Process :**
  Work done → +ve for clockwise process
  Work done → –ve for anti-clockwise process

Laws of Thermodynamics

- **First Law :** $Q = \Delta U + W$

→ for adiabatic process : $Q = 0$

$W = -\Delta U$ & $\gamma = \frac{f+2}{f}$

for monoatomic gas, $\gamma = 5/3$
for di-atomic gas, $\gamma = 7/5$

→ for isothermal process $\Delta T = 0$

$Q = W$ $\Delta U = 0$

→ for isochoric process : $W = 0$

$Q = \Delta U$

- **Heat Engine :** $\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$ η = Efficiency
- **Refrigerator :** $\alpha = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$ α = Coefficient of performance
- **Second Law :**

→ **Statement - 1 (Clausius) :** No process is possible whose sole result is the transfer of heat from a colder object to a hotter object.

→ **Statement - 2 (kelvin-Planck) :** No process is possible whose sole result is the absorption of heat from a reservoir and then complete conversion into work.

Kinetic Theory of Gases

Behaviour of Ideal Gases and Basic Terms

→ No interaction between the molecules.

→ Total interval energy of the gas = kinetic energy of the molecules

→ $\sum \vec{v}_i = 0$

- **Pressure :** Force exerted by the gas per unit area.
- **No. of moles :** measure the amount of gas.

  1 mole = 6.023×10^{23} molecules
- **Boyle's Law :** $\boxed{P \propto \frac{1}{V}}$ at T = Constant
- **Charle's Law :** $\boxed{V \propto T}$ at P = Constant
- **Gay-Lussac's Law :** $\boxed{P \propto T}$ at V = Constant
- **Avogadro's Law :** At the same temperature and pressure, equal volume of all the gases contains equal number of molecules.

Kinetic Energy of Gas

- **Kinetic Energy :** $\boxed{K.E._T = \frac{1}{2} mN < v^2 >}$

  $\boxed{K.E._T = \frac{3}{2} PV = \frac{3}{2} nRT}$
- **Law of Equipartition of Energy :**

  $\boxed{(K.E.)_x = (K.E.)_y = (K.E.)_z = \frac{nRT}{2}}$

  For gases with fixed molecules Total K.E. :

  For mono-atomic gas : $\boxed{K.E. = \frac{3}{2} nRT}$

  For di-atomic gases : $\boxed{K.E. = \frac{5}{2} nRT}$
- **Speeds of Molecules :**

  $\boxed{V_{mp} = \sqrt{\frac{2RT}{M}}}$, $\boxed{V_{avg} = \sqrt{\frac{8RT}{\pi M}}}$, $\boxed{V_{rms} = \sqrt{\frac{3RT}{M}}}$

Ideal Gas Equation

$\boxed{PV = nRT}$ R = Gas constant = 8.31 J/mol-K

- **Pressure of Gas :**

  $$\boxed{P = \frac{mN < v^2 >}{3V} = \frac{1}{3} \rho < v^2 >}$$

  m = mass of one molecule

  N = Total number of molecules

  $< v^2 >$ = mean square speed

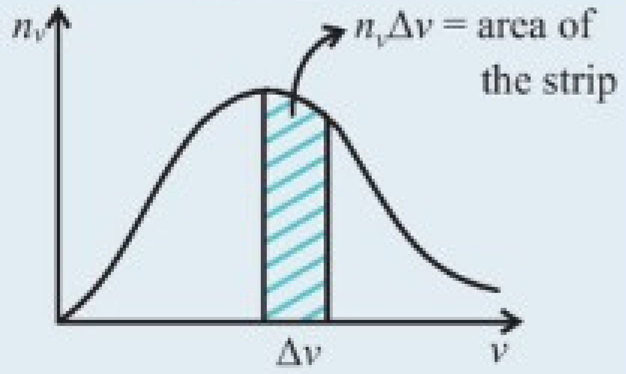
  V = Total volume

  ρ = density of gas

Maxwell-Distribution Function

→ Total area under the curve = total number of molecules

→ Area of strip = number of molecules with velocities lying between v and $v + \Delta v$.



- **Graham's Diffusion Law :**

  $\boxed{\text{Rate of diffusion} \propto v_{rms}}$

  At same temperature : $\boxed{\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}}$ $\boxed{R \propto \frac{1}{\sqrt{M}}}$
- **Mean Free Path :**

  $\boxed{\lambda = \frac{1}{\sqrt{2}\pi d^2 n}}$ d = diameter of molecules

  $n = \frac{\text{Number of molecules}}{\text{Volume}}$

Oscillations

Simple Harmonic Motion

- **Periodic Motion :** a motion that repeats itself after a regular interval of time.
- **Oscillation :** a motion in which a body moves to and fro or back and forth about a fixed point.
- **Simple Harmonic Motion :** acceleration of an oscillating body is directly proportional to the displacement of the body from the mean position.

$$\boxed{a = -ky}$$

- **Displacement in S.H.M. :**

$$\boxed{y = A\sin(\omega t + \phi)}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

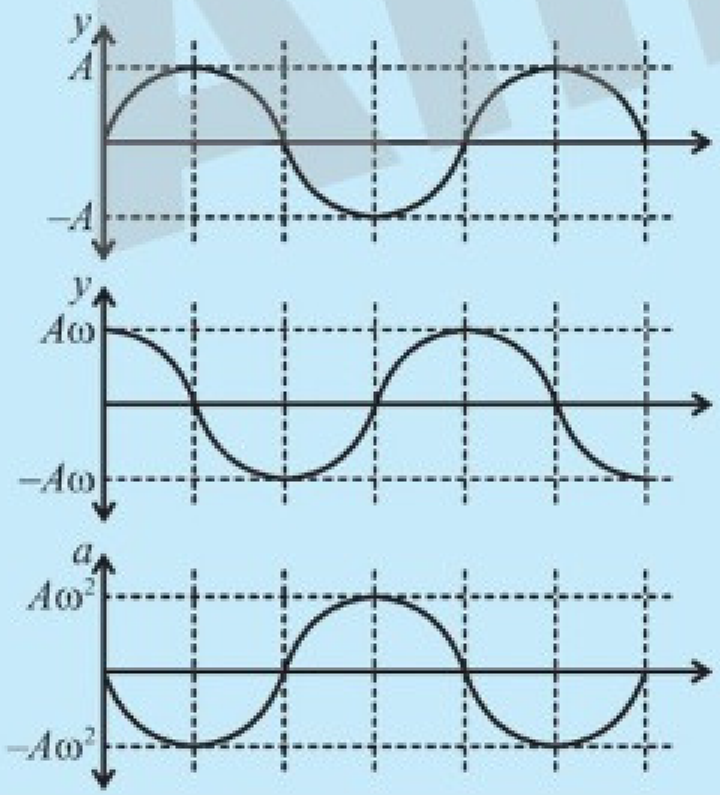
A = amplitude
ω = angular velocity
ϕ = initial phase

- **Velocity of a body in S.H.M. :**

$$\boxed{v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - y^2}}$$

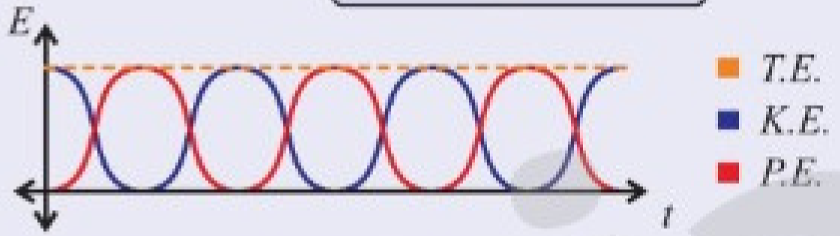
- **Acceleration of a body in S.H.M. :**

$$\boxed{a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 y}$$



Kinetic and Potential Energy in SHM

- **Kinetic Energy :** $\boxed{K.E. = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)}$
- **Potential Energy :** $\boxed{P.E. = \frac{1}{2}m\omega^2 y^2}$
- **Total Energy :** $\boxed{T.E. = \frac{1}{2}m\omega^2 A^2}$



Simple Pendulum and Physical Pendulum

- **Time period of a simple pendulum :** $T = 2\pi\sqrt{\frac{l_{eff}}{g_{eff}}}$

If length is comparable to radius of earth :

$$\boxed{T = 2\pi\sqrt{\frac{\text{Re}}{\left(1+\text{Re}/l\right)g}}}$$

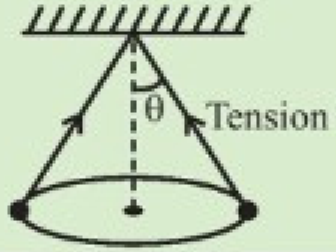
- **Physical Pendulum :**

I_0 = M.O.I. of body about the axis passing through point of suspension
l = distance of C.O.M. from point of suspension

$$\boxed{T = 2\pi\sqrt{\frac{I_0}{mgl}}}$$

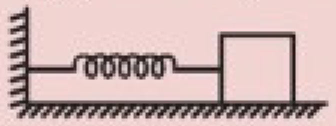
- **Conical Pendulum :**

Time Period $\boxed{T = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$



Other Systems

- **Torsional Pendulum :** $\boxed{T = 2\pi\sqrt{\frac{I}{K}}}$ K = Torsional Constant
- **Spring Block System :** $\boxed{T = 2\pi\sqrt{\frac{m}{K_{eff}}}}$ K = Spring Constant



* **Combination of Springs :**

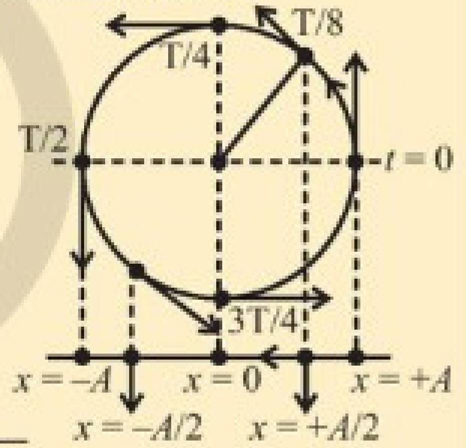
→ In series : $\boxed{\frac{1}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots}$

→ In parallel : $\boxed{K_{eff} = K_1 + K_2 + K_3 + \ldots}$

Circle Diagram and Damped Oscillations

- **Analogy between SHM and Circular Motion :**

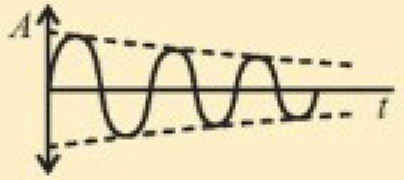
→ The projection of the particle doing uniform circular motion on x-axis or y-axis does S.H.M.



- **Damped Oscillations :**

Damped Frequency, $\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$

Amplitude, $A = A_0 e^{\frac{-bt}{2m}}$



Forced Oscillations and Resonance

(a) For small damping : (driving frequency far from natural frequency)

$$\boxed{A = \frac{F_0}{m\left(\omega^2 - \omega_d^2\right)}}$$

(b) Driving frequency close to Natural frequency (Resonance):

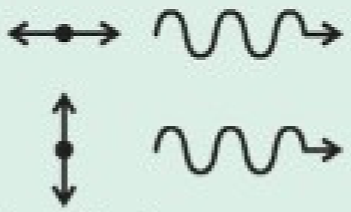
$$\boxed{A = \frac{F_0}{\omega_0 b}}$$

ω = natural frequency
ω_d = driving frequency

Waves

Wave Equation

- **Mechanical Waves :**



Longitudinal waves

Transverse waves

(Oscillating particles)

- **Wave function and wave equation :**

Wave function : $y = f(x \pm vt)$

Wave equation of a wave travelling +ve x direction :

$$\boxed{y = A\sin(\omega t - kx)}$$

Sound

- **Speed of Sound :** $\boxed{v = \sqrt{\dfrac{B}{\rho}}}$ B = Bulk modulus of medium, ρ = Density of medium

→ According to Newton :

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

Laplace Correction : $\boxed{v = \sqrt{\dfrac{\gamma P}{\rho}}}$

- **Loudness of Sound :**

$$\boxed{L = 10\log_{10}\left[\frac{I}{I_0}\right]}$$

I = intensity of sound

I_0 = threshold of hearing $= 10^{-12}\,\dfrac{W}{m^2}$

Standing Waves

- **Superposition of Waves :**

$$\boxed{y = A\sin(\omega t - \phi)}$$

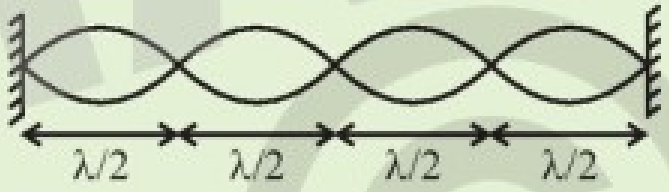
→ If $\phi = 2n\pi$ → Constructive interference

→ If $\phi = (2n-1)\pi$ → Destructive interference

- **On a stretched string :**

$$\boxed{\lambda = \frac{2L}{n}} \Rightarrow \boxed{f = \frac{nv}{2L}}$$

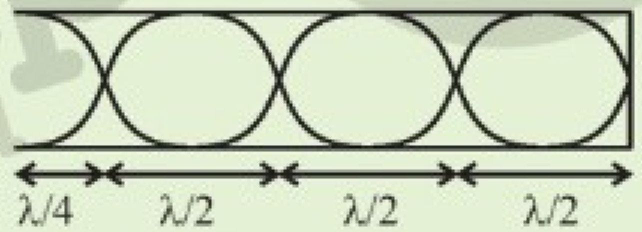
n = nth harmonic = 1, 2, 3,...



- **Closed organ pipe :**

$$\boxed{\lambda = \frac{4L}{(2n+1)}} \Rightarrow \boxed{f = \frac{(2n+1)v}{4L}}$$

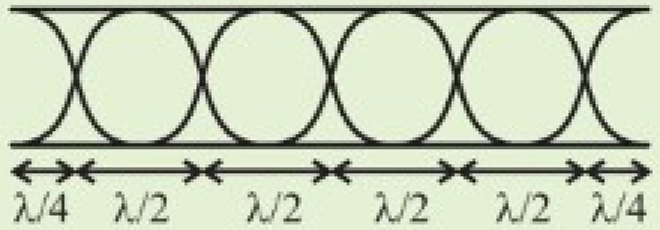
n = nth overtone = 0,1, 2, 3,...



- **Open organ pipe :**

$$\boxed{\lambda = \frac{2L}{n}} \Rightarrow \boxed{f = \frac{nv}{2L}}$$

n = nth harmonic = 1, 2, 3,...



- **Beats :**

$$\boxed{\text{Beat frequency} = f_1 - f_2}$$

Transverse Wave

- **Velocity of a Wave on a String :**

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

T = Tension in string

μ= mass / length

- **Power :** $\boxed{P_{av} = \dfrac{1}{2}\mu\omega^2 A^2 v}$

- **Intensity :** $\boxed{I_{av} = \dfrac{1}{2}\rho\omega^2 A^2 v}$

→ Phase difference $= \dfrac{2\pi}{\lambda} \times$ path difference

Doppler's Effects

- **Source Moving, Observer Stationary :**

$f = f_0\left[1 - \dfrac{v_S}{v}\right]$ O → $v_0 = 0$ S → $v_S \neq 0$

$f = f_0\left[1 + \dfrac{v_S}{v}\right]$ O → $v_0 = 0$ S ← $v_S \neq 0$

- **Observer Moving, Source Stationary :**

$f = f_0\left[1 + \dfrac{v_0}{v}\right]$ O → $v_0 \neq 0$ S → $v_S = 0$

$f = f_0\left[1 - \dfrac{v_0}{v}\right]$ ← O $v_0 \neq 0$ S → $v_S = 0$

- **Both Source and Observer Moving :**

$f = f_0\left[\dfrac{v + v_0}{v - v_S}\right]$ → v_0 ← v_S

$f = f_0\left[\dfrac{v - v_0}{v + v_S}\right]$ ← v_0 → v_S

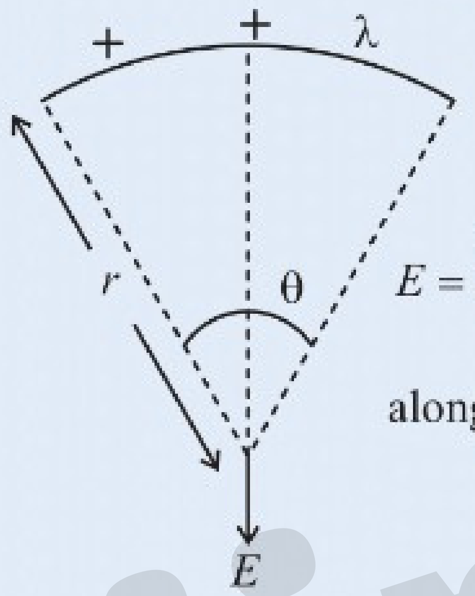
ELECTRIC CHARGES AND FIELDS

Electric Field (E)

Force on a unit positive charge

* Due to point charge, $E = \frac{kq}{r^2}$

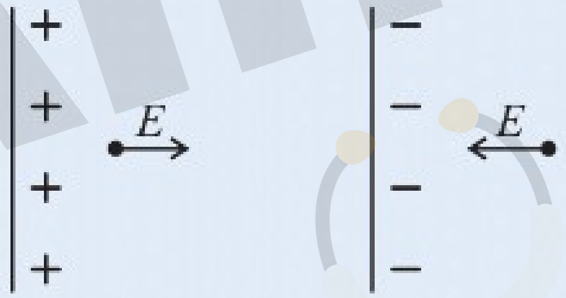
* Circular arc subtending θ angle at center



$$E = \frac{2k\lambda}{r}\sin\left(\frac{\theta}{2}\right)$$

along angular bisector

* Infinite sheet, $E = \frac{\sigma}{2\varepsilon_0}$



* Spherical Conductor/Hollow Sphere

$E = 0 \qquad (r < R)$

$E = \frac{kq}{r^2} \qquad (r \geq R)$

* Solid Non-conducting Sphere

$E = \frac{kqr}{R^3} \qquad (r < R)$

$E = \frac{kq}{r^2} \qquad (r \geq R)$

Charge

* Properties of Charge
  - → Quantization
  - → Conservation
* Ways of Charging
  - → Friction/Rubbing
  - → Induction
  - → Conduction

Coulomb's Law

Force between two point charges

$$F = \frac{kq_1q_2}{r^2}$$

In air/vaccum, $k = \frac{1}{4\pi\varepsilon_0}$

In medium with dielectric constant K,

$$k = \frac{1}{4\pi K\varepsilon_0}$$

Electric Field Lines

* Properties
* Represent direction of force on +ve charge

Electric Flux and Gauss's Law

Electric Flux (ϕ) : Measure of electric field lines crossing an area

$$d\phi = EdA\cos\theta$$

$$\phi = \vec{E}\cdot\vec{A}$$

* Gauss Law :

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{enc}}{\varepsilon_0}$$

Note : $\cdot q_1 \quad \cdot -q_2 \qquad \phi = \frac{q_1}{\varepsilon_0}$

Electric Dipole ($\vec{p}$)

$\vec{p} = q(2d) \quad (\rightarrow)$

* Important for studying polar molecules

* $\vec{E}_{axial} = \frac{2k\vec{p}}{r^3}$

* $\vec{E}_{equatorial} = \frac{-k\vec{p}}{r^3}$

* $E_{general} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Dielectrics and Polarisation

* Dielectric : Non conducting substance

  For dielectric, $\vec{E}_{\text{Net, inside}} \neq 0$

  $$\vec{E}_{\text{induced}} = \frac{-\vec{E}_{ext}}{K}$$

* Polarisation $\vec{P} = \chi \vec{E}$

  (dipole moment per unit volume)

Electric Potential Energy

* Two point charges, $U_{12} = \frac{kq_1q_2}{r}$

  (with sign consideration)

* Multiple charges,

  $U = U_{12} + U_{13} + U_{23} + \ldots$

* Dipole in ext. field $U = -\vec{p} \cdot \vec{E}$
* Energy conservation

Electric Potential (V)

Work done in moving a unit +ve charge from infinity to point

* Point Charge, $V = \frac{kq}{r}$
* Ring
  - At center, $V = \frac{kq}{r}$
  - Along axis, $V = \frac{kq}{\sqrt{r^2 + x^2}}$
* Electric Dipole
  - $V_{\text{axial}} = \frac{kp}{r^2}$
  - $V_{\text{equatorial}} = 0$
* Hollow Sphere/Metallic Conductor
  - $V = \frac{kq}{R}$ $(r \leq R)$
  - $V = \frac{kq}{r}$ $(r \geq R)$

Capacitor (Charge Storage Device)

$C = \frac{Q}{V}$

* Parallel Plate Capacitor
  - Air medium, $C = \frac{A\varepsilon_0}{d}$
  - Dielectric completely filled between plates, $C = \frac{KA\varepsilon_0}{d}$
  - Dielectric filled upto thickness t and same area as plates, $C = \frac{A\varepsilon_0}{d - t\left(1 - \frac{1}{K}\right)}$
* Series/Parallel Combination
* Energy Stored $E = \frac{Q^2}{2C} = \frac{V^2C}{2}$
* RC Circuit

  In charging $\left\}\begin{array}{l}\text{At } t = 0, \text{C as wire} \\ t = \infty, \text{C as open circuit}\end{array}\right.$

  (Initially uncharged capacitor)

Electrostatics of Conductors

* $E_{\text{inside}} = 0$
* $E_{\text{surface}} = \frac{\sigma}{\varepsilon_0}$ (along normal direction)
* Excess charge only on surface
* $V_{\text{inside}} = V_{\text{surface}}$
* Electrostatic Shielding

  (E inside uncharged cavity is always zero)

Potential Difference and Field

* $\Delta V = V_f - V_i$
* $dV = -\vec{E} \cdot \vec{dr}$ or $E = \frac{-dV}{dr}$
* Work done in moving charge q, $W = q\Delta V$
* Equipotential Surfaces

CURRENT ELECTRICITY

Electric Current (I)

- **Current Carriers :-** Charged particles

$\oplus \rightarrow$ / $\longrightarrow$ conventional current / $\leftarrow \ominus$

- **Electric Current:-** $\boxed{I = \dfrac{dq}{dt}}$
  - → Ampere
  - → Slope of q-t graph = I
  - → Area of I-t graph = q
- **Current in Conductors :-**
  - → Drift velocity $\vec{V}_d = \dfrac{-e\vec{E}\tau}{m}$
  - → Mean free path = 10^{-10} m
  - → Relaxation time = 10^{-14} sec
  - → Current $I = neA\vec{V}_d$, Current density, $\vec{J} = ne\vec{V}_d$

Grouping of Resistors

- → In Series - $R_{net} = R_1 + R_2 + R_3 + ...$
- → In Parallel - $\dfrac{1}{R_{net}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + ...$

Grouping of Cells

- → In Series -

$E_{net} = E_1 + E_2 + ...,\quad r_{net} = r_1 + r_2 + ...$

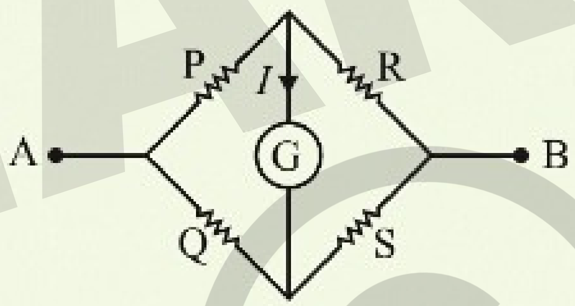
- → In Parallel -

$E_{net} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2},\quad r_{net} = \dfrac{r_1 r_2}{r_1 + r_2}$

Kirchoff's Laws :-

- → (1) Incoming Current = Outgoing Current
- (2) $\sum V = 0$ (for closed loop)

Wheatstone Bridge :-



If $\boxed{\dfrac{P}{Q} = \dfrac{R}{S}}$ → $\boxed{I = 0}$

Meter Bridge :-

$\boxed{\dfrac{P}{l} = \dfrac{Q}{100 - l}}$

Potentiometer :-

- → Comparing emf of two cells : -

$\boxed{\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}}$

- → Internal resistance of a cell : -

$\boxed{r = \dfrac{R(l - l')}{l'}}$

- → Sensitivity : -

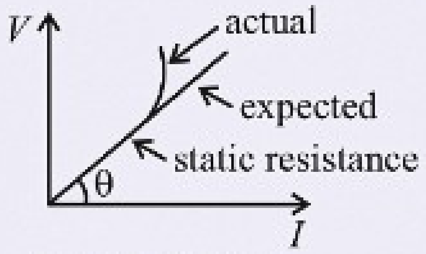
$\boxed{K = \left[\dfrac{E_r}{R + r}\right]\dfrac{1}{l}}$

Ohm's Law $\boxed{I = VR \text{ or } I \propto V}$

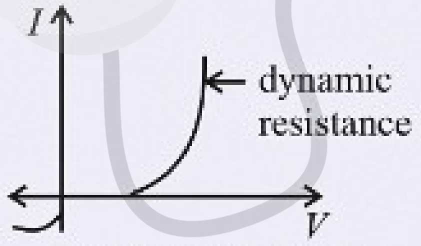
→ $\boxed{R = \dfrac{ml}{ne^2 A\tau} = \dfrac{\rho l}{A}}$ & $\boxed{\rho = \dfrac{m}{ne^2\tau}}$ → $\boxed{\rho = R \text{ If } l = 1 \text{ \& } A = 1}$

Resistance　　Resistivity

Conductivity (s) = $\dfrac{1}{\rho}$ & Conductance (G) = $\dfrac{1}{R}$



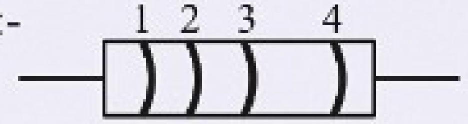
Ohmic Devices



P-N Junction Diode (Non-ohmic Device)

$\boxed{R_t = R_0 (1 + \alpha \Delta t)}$

→ Colour Coding :-



1. 1st digit 2. 2nd digit 3. multiplier 4. tolerence

Heating Effects and Electric Bulb

- → $\boxed{P = VI = I^2R = V^2/R}$ ← in parallel generally; ← in series generally
- → $H = VIt = I^2Rt = V^2t/R$
- → Combination of Appliances :-
  - * In Parallel : - $P_{net} = P_1 + P_2$
  - * In Series : - $\dfrac{1}{P_{net}} = \dfrac{1}{P_1} + \dfrac{1}{P_2}$
- → For Variable Current :-

$\boxed{H = \int_{t_1}^{t_2} I^2 R\, dt}$

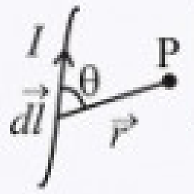
- → For a Fuse :- $\boxed{i^2 \propto r^3}$

Moving Charges and Magnetism

Magnetic Field

→ A moving charge produced varying E.F. which in turn produces constant M.F.

- **Biot-Savart's Law**



$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2} \qquad \mu_0 = 4\pi\times10^{-7}\,TmA^{-1}$$

Or

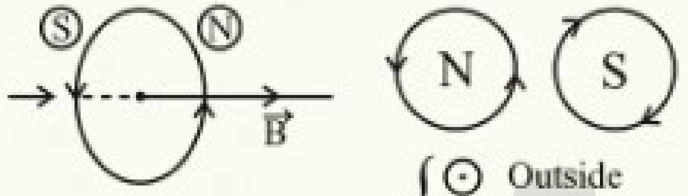
$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\left(\overrightarrow{dl}\times\vec{r}\right)}{r^3}$$

SI unit = T
CGS unit = G
1 Tesla = 10^4 G

- **$\vec{B}$ due to an Arc**

$$B = \frac{\mu_0 I}{4\pi a}(\alpha)$$

- **Direction of $\vec{B}$ due to Coil**



⊙ Outside
⊗ Inside

- **$\vec{B}$ due to a Finite Wire**

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\phi_2 + \sin\phi_1\right]$$

Direction :- Right hand grip rule

- **$\vec{B}$ due to a Semi-infinite Wire**

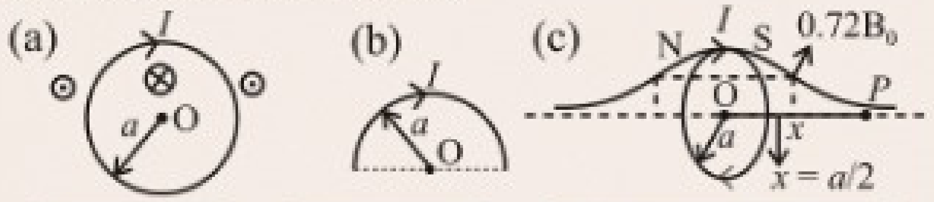
(a) Semi-infinite wire

$$B = \frac{\mu_0 I}{4\pi a}$$

(b) Infinite wire

$$B = \frac{\mu_0 I}{2\pi a}$$

- **$\vec{B}$ due to a Circular Coil**



(a) $B_0 = \frac{\mu_0 I}{2a}$

(b) $B_0 = \frac{\mu_0 I}{4a}$

(c) $B_P = \frac{\mu_0 I a^2}{2\left(a^2+x^2\right)^{3/2}}$

$$\frac{B_0}{B_P} = \left(1+\frac{x^2}{a^2}\right)^{\frac{3}{2}} \qquad B_{x=\frac{a}{2}} = 0.72\ B_{centre}$$

Ampere's Circuital Law

- **Ampere's Circuital Law :**

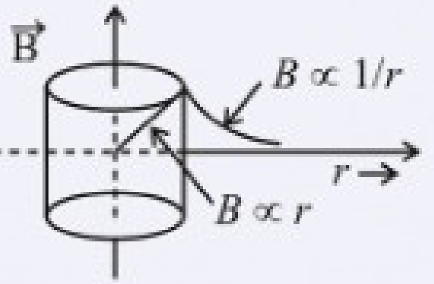
$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0\ \left(I_{enclosed}\right)$$

- **$\vec{B}$ due to a Solid Cylinder**

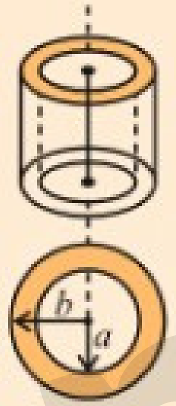
(a) $B_{inside} = \frac{\mu_m I r}{2\pi R^2}$

(b) $B_{surface} = \frac{\mu_0 I}{2\pi R}$

(c) $B_{outside} = \frac{\mu_0 I}{2\pi r}$



- **$\vec{B}$ due to a Hollow Cylinder**



(a) When $r < a$:- B = 0

(b) When $a < r < b$:-

$$B = \frac{\mu_m I\left(r^2-a^2\right)}{2\pi r\left(b^2-a^2\right)}$$

(c) When $r > b$:-

$$B = \frac{\mu_0 I}{2\pi r}$$

- **$\vec{B}$ due to Solenoid and Toroid**

→ Long Solenoid :-

$$B_{solenoid} = \mu_0 ni \quad \text{(inside)}$$

$$B_{solenoid} = \frac{\mu_0 ni}{2} \quad \text{(at edge)}$$

→ Toroid :-

(a) $r < r_1$ $\quad B = 0$

(b) $r_1 < r < r_2$ $\quad B = \frac{\mu_0 NI}{2\pi r}$

(c) $r > r_2$ $\quad B = 0$

- **Magnetic Force on a Charged Particle**

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

→ Force is perpendicular to both v and B

→ Direction of force can be find using Fleming's Left Hand Rule

- **Motion of Charged Particle in $\vec{B}$**

$$r = \frac{mv}{qB},\quad T = \frac{2\pi m}{qB} \qquad f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- **Lorentz Force** $\quad \vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$

- **Force on Wire in $\vec{B}$** $\quad \vec{F} = i(\vec{l}\times\vec{B})$

- **Force between two Wires**

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi r}$$

If $r = 1m,\ i_1 = i_2 = 1A$

$$\frac{F}{l} = 2\times10^{-7}\,N/m$$

- **Force on a loop in $\vec{B}$** $\quad \vec{F} = 0$

- **Magnetic Moment of loop in $\vec{B}$**

$$\vec{M} = NIA\hat{n}$$

- **Torque on a loop in $\vec{B}$**

$$\vec{\tau} = \vec{M}\times\vec{B} = MB\sin\theta\ \hat{n}$$

- **Moving Coil Galvanometer**

→ For Galvanometer :- $i = \frac{K}{nAB}\theta$

→ Sensitivity $= \frac{nAB}{K}$

→ To convert into ammeter :-

$$I_g R_g = \left(I - I_g\right)\times R_S$$

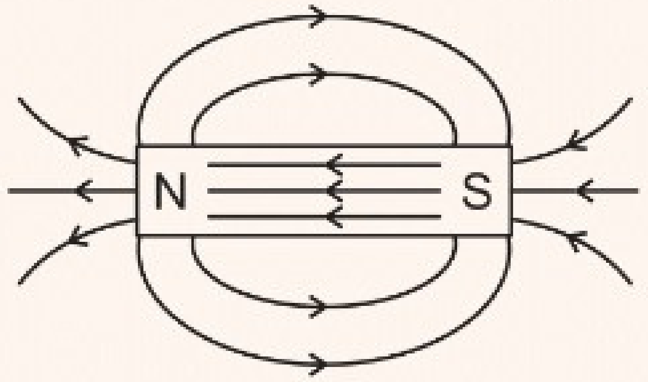
→ To convert in voltmeter :-

$$V = I_g\left(R_S + R_g\right)$$

Magnetism And Matter

Magnetic Field Lines

→ Travel N to S outside the magnet



→ Form closed continuous loops

→ Do not intersect each other

→ No. of field lines ∝ strength of magnetic field

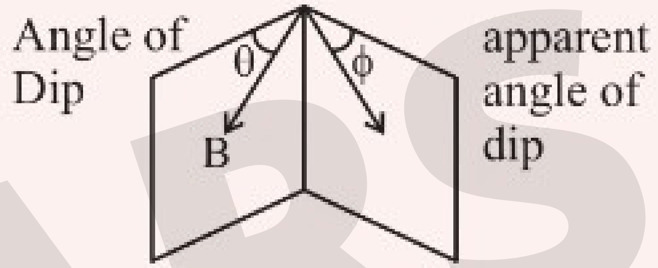
→ Same poles repel each other & opposite poles attract

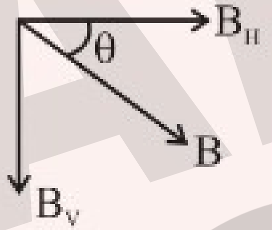
Earth's Magnetism

$B_H = B\cos\theta$

$B_V = B\sin\theta$

$\tan\theta = \dfrac{B_V}{B_H}$





$\theta_{equator} = 0$

$\theta_{poles} = 90^\circ$

$B_H = 0$ (at magnetic pole)

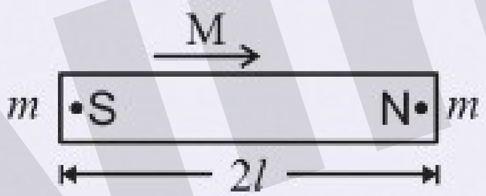
$B_V = 0$ (at magnetic equator)

Gauss's Law

$\phi_B = \oint \vec{B}\cdot\vec{ds} = 0$ → through any closed surface

Short Bar Magnet

→ Magnetic Dipole Moment



Time period

$T = 2\pi\sqrt{\dfrac{I}{MB}}$

$\vec{M} = m(2\vec{l})$

→ Magnetic Field

On axial line - $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{2\vec{M}}{d^3}$

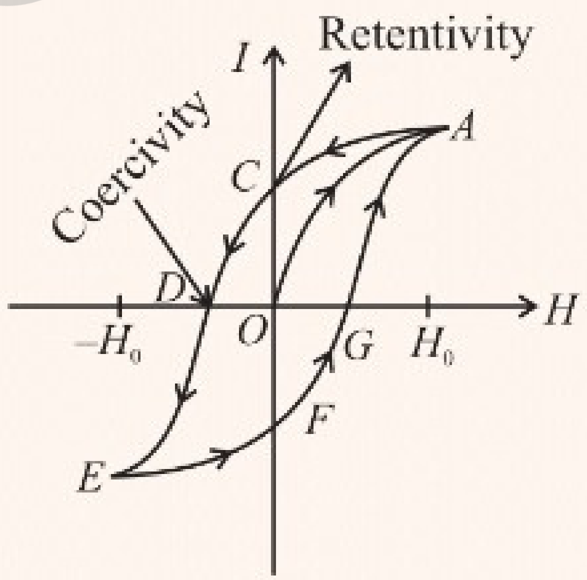
On equitorial line - $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{\vec{M}}{d^3}$

→ Torque - $\tau = MB\sin\theta$, $\vec{\tau} = \vec{M}\times\vec{B}$

→ Potential Energy - $U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\Delta U = W = -MB(\cos\theta_2 - \cos\theta_1)$

Hysteresis Curve



Some Important Terms

→ Relative Magnetic Permeability :-

$\mu_r = \dfrac{\mu}{\mu_0}$ or $\mu_r = \dfrac{B}{B_0}$

→ Magnetic Intensity :- $H = \dfrac{B_0}{\mu_0}$ or $H = \dfrac{B}{\mu}$

→ Magnetisation Intensity (I) :-

$I = \dfrac{\text{Magnetic Moment}}{\text{Volume}} = \dfrac{M}{V}$

→ Magnetic Susceptibility :- $\chi_m = \dfrac{I}{H}$, $\mu_r = 1 + \chi_m$

→ Curie Law :- $\chi_m = \dfrac{C}{T - T_C}$ (For Ferromagnetic)

Substance	χ_m
Diamagnetic	$-1 \le \chi_m < 0$
Paramagnetic	$0 < \chi_m < 1$
Ferromagnetic	$\chi_m >> 1$

Electromagnetic Induction

Faraday's Law and Lenz's Law

Faraday's Law

→ whenever the magnetic flux changes an emf is induced in the circuit.

$$\boxed{e = \frac{-d\phi}{dt}}$$

For coil having N turns

$$\boxed{I = \frac{-N}{R}\frac{d\phi}{dt}} \quad \text{and} \quad \boxed{\Delta q = \frac{N\Delta\phi}{R}}$$

ϕ : Flux through single coil

Δq : Charge flow in time Δt

→ flux can be changed by changing magnetic field, area or angle.

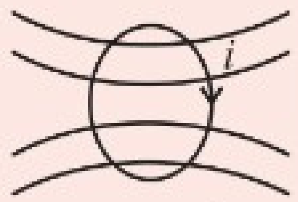
Lenz's Law

→ Induced emf opposes cause of generation

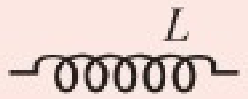
Self Inductance

$\boxed{\phi \propto i}$

$\boxed{\phi = Li}$



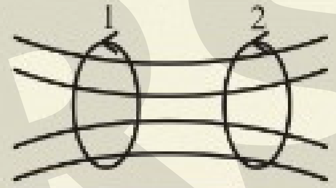
$$\boxed{e = \frac{-d\phi}{dt} = \frac{-Ldi}{dt}} \rightarrow$$



Inductance

→ L for Solenoid :- $\boxed{L = \frac{\mu_0 N^2 A}{L}}$

Mutual Inducatnce



$\boxed{\phi_{21} = M_{21} i_1}$

$\boxed{e_2 = -M_{21} \cdot \frac{di_1}{dt}}$

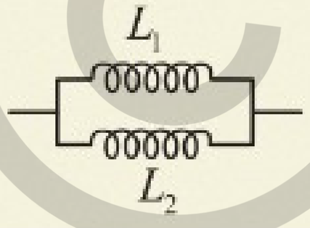
$\boxed{M_{21} = M_{12}}$ Theorem of reciprocity

→ For two coils :-

$\boxed{L_{eq} = L_1 + L_2}$

→ For two solenoids :-

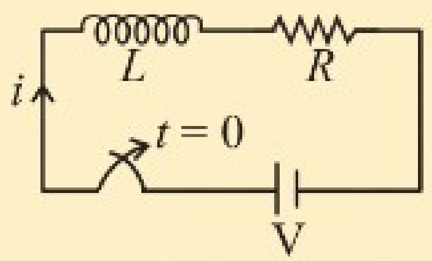
$\boxed{M = \frac{\mu_0 N_1 N_2 A}{L}}$



$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$

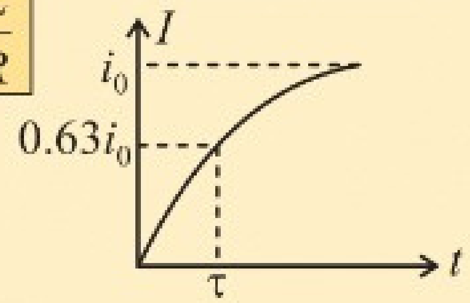
LR Circuit

→ Growth of Current :-

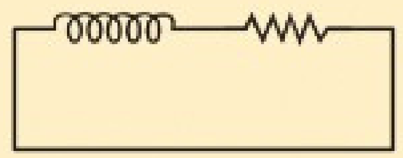


$\boxed{\tau = \frac{L}{R}}$

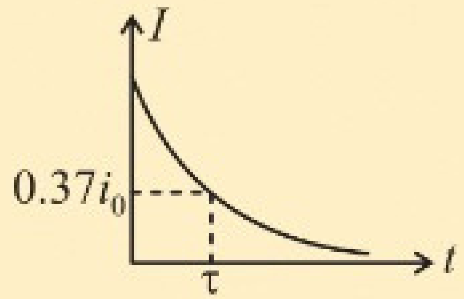
$\boxed{i = i_0\left(1 - e^{-tR/L}\right)}$



→ Decay of Current :-



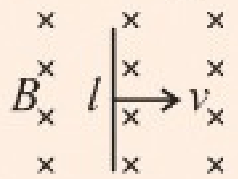
$\boxed{i = i_0 e^{-tR/L}}$



$$\boxed{\text{Energy stored} = \frac{1}{2} Li^2}$$

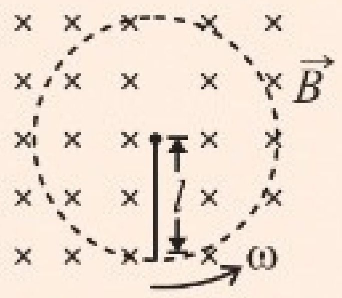
Motional EMF

→ A Moving Wire :-



$\boxed{e = Bvl}$

→ Rotating Metallic Rod :-



$$\boxed{e = \frac{1}{2} Bl^2\omega = \pi f Bl^2}$$

Induced Electric Field

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}}$$

Alternating Current

AC Current

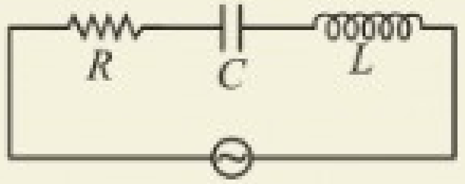
→ Direction of current changes alternatively

$i = i_0 \sin\omega t$, $V = V_0 \sin\omega t$

$i_{rms} = \frac{i_0}{\sqrt{2}}$, $V_{rms} = \frac{V_0}{\sqrt{2}}$

$i_{avg} = \frac{2i_0}{\pi}$, $V_{avg} = \frac{2V_0}{\pi}$

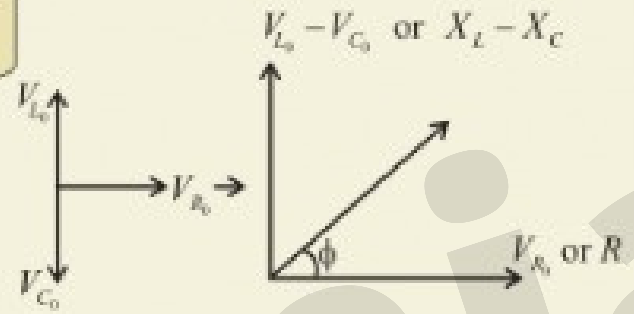
L-C-R Circuit



$V_0 = I_0\sqrt{(X_L - X_C)^2 + R^2}$

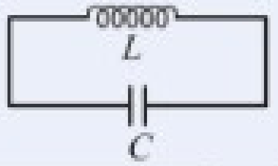
$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$V_{rms} = I_{rms} Z$



$\langle P \rangle = i_{rms} V_{rms} \cos\phi = \frac{i_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} \cos\phi$ & $\cos\phi = \frac{R}{Z}$, $\tan\phi = \frac{X_L - X_C}{R}$

LC Oscillation



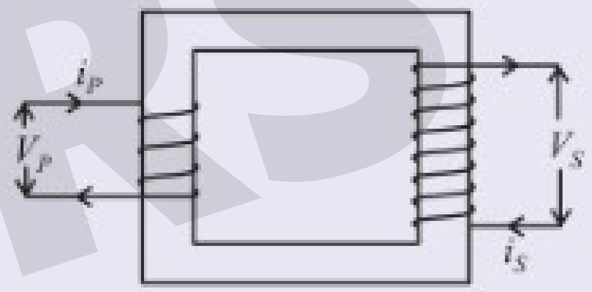
$q = q_0 \cos\omega t$, $I = \frac{-dq}{dt}$

$\omega = \frac{1}{\sqrt{LC}}$, $E = \frac{q^2}{2C} + \frac{1}{2}LI^2$

Transformers

→ used to change an alternating voltage from one to another.

$V_S = -N_S \frac{d\phi}{dt}$



$\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$V_P I_P = V_S I_S$

$\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$

→ $N_S > N_P$ → step up

→ $N_P > N_S$ → step down

→ No loss in flux and energy → Ideal

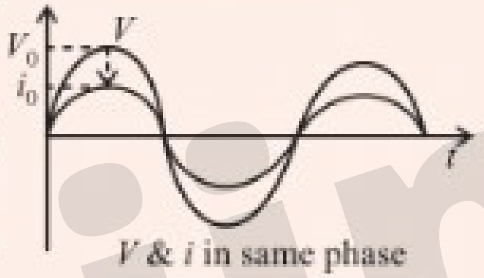
→ $\eta = \frac{P_{out}}{P_{in}} \times 100$

R-Circuit

$i = i_0 \sin\omega t$, $V = V_0 \sin\omega t$

$i_0 = \frac{V_0}{R}$, $i_{rms} = \frac{V_{rms}}{R}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$, $V_{rms} = \frac{V_0}{\sqrt{2}}$



V & i in same phase

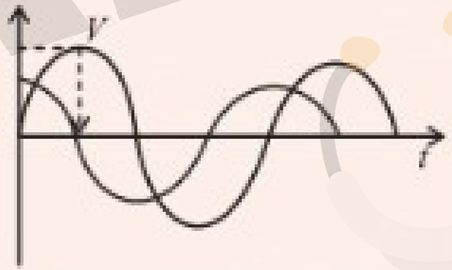
C-Circuit

$V = V_0 \sin\omega t$, $i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

$i_0 = \frac{V_0}{X_C} = \frac{V_0}{\frac{1}{\omega C}} = \omega C V_0$

$\langle P \rangle = 0$

→ Current leads voltage by $\pi/2$



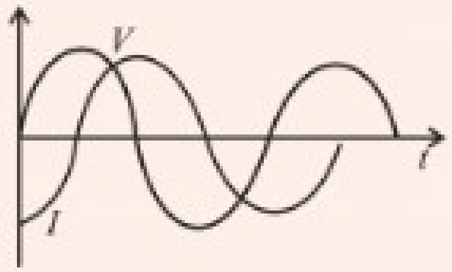
L-Circuit

$V = V_0 \sin\omega t$, $i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

$i_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$

$\langle P \rangle = 0$

→ Voltage leads current by $\pi/2$



Resonance in L-C-R

$V = V_0 \sin(\omega t + \phi)$

$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$

Resonant frequency

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

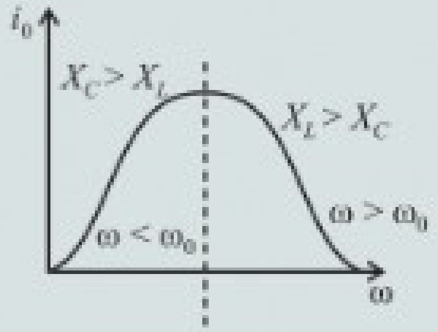
→ At resonance

→ $i_0 = \max$

→ $X_L = X_C$

→ $Z = R$

→ $\cos\phi = 1$, $\omega_0 = \frac{1}{\sqrt{LC}}$



Quality Factor

$Q = \frac{\omega_0}{\text{Bandwidth}}$

$Q = \frac{\omega_0 L}{R}$

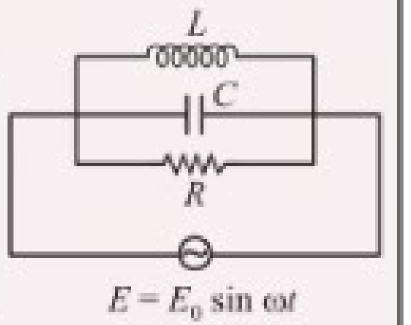
Efficiency of Motor

$\eta = \frac{P_{out}}{P_{in}} = \frac{E}{V}$

Parallel L-C-R

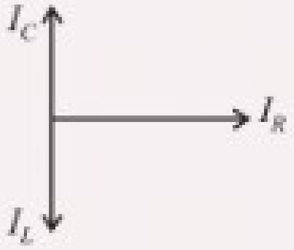
$I = \sqrt{(I_C - I_L)^2 + (I_R)^2}$

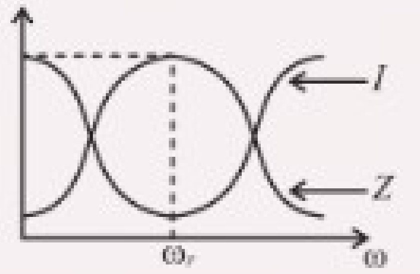
$\frac{1}{Z} = \sqrt{\left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 + \left(\frac{1}{R}\right)^2}$

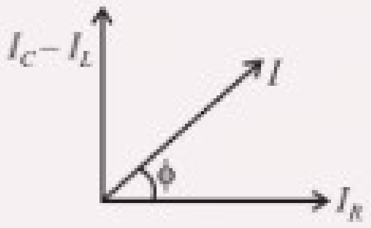


At resonance

$\omega_r = \frac{1}{\sqrt{LC}}$ & $\phi = 0$







Electromagnetic Waves

Maxwell's Equations

(1) $\oint \vec{E}\cdot\vec{dA} = \dfrac{q_{\text{env}}}{\varepsilon_0}$ → Gauss's law for electricity

(2) $\oint \vec{B}\cdot\vec{dA} = 0$ → Gauss's law for magnetism

(3) $\oint \vec{E}\cdot\vec{dl} = \dfrac{-d\phi_B}{dt}$ → Faraday's law

(4) $\oint \vec{B}\cdot\vec{dl} = \mu_0 i_c + \mu_0\varepsilon_0\dfrac{d\phi_E}{dt}$ → Ampere Maxwell law

$$\boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}}, \quad \boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Displacement Current

Ampere Maxwell Law

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0\left(i_c + i_d\right)}$$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i_c + \mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

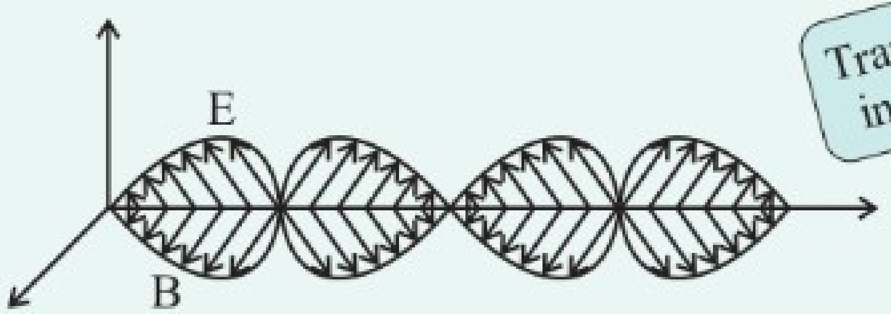
$$\rightarrow \boxed{i_d = \varepsilon_0\frac{d\phi_E}{dt}}$$

Production of EM Waves

The EM Waves are produced by the accelerated or oscillating charge.

$$\boxed{\begin{aligned} E_z &= E_0\sin\left(\omega t + kx\right) \\ B_y &= B_0\sin\left(\omega t + kx\right)\end{aligned}} \quad \boxed{c = \frac{E_0}{B_0}}, \quad \boxed{\omega = \frac{2\pi}{T}}$$

Transverse in nature



→ Average electric energy density

$$\boxed{U_E = \frac{1}{2}\varepsilon_0 E_0^2}$$

→ Average magnetic energy density

$$\boxed{U_B = \frac{1}{2}\frac{B_0^2}{\mu_0}}$$

→ Total energy per unit volume

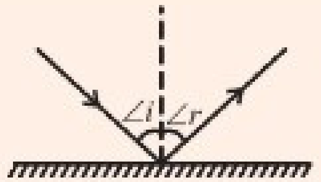
$$\boxed{U_{av} = U_E + U_B}$$

Type	Wavelength	Uses
Radiowave	> 0.1 m	Radio and television communication
Microwave	0.1 m to 1 mm	Microwave oven, Radar system
Infrared	1 mm to 700 nm	Remote Switches and Household electronic devices
Visible rays	700 nm to 400 nm	To see objects
Ultraviolet	400 nm to 1 nm	Eye surgery, Water purifier
X-rays	1 nm to 10^{-3} nm	Medical diagnosis
Gamma	$< 10^{-3}$ nm	Medical treatment

Ray Optics

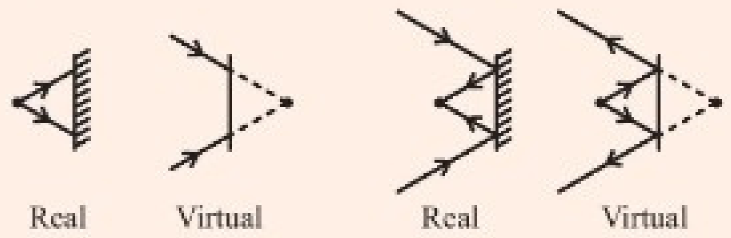
Mirror

Reflection



- $\angle i = \angle r$
- incident ray, reflected ray and normal lie in same plane

→ Object → Image



→ Properties of image by a plane mirror

(i) Real image of virtual object.

(ii) Virtual image of real object.

(iii) Dist. of image = Dist. of object

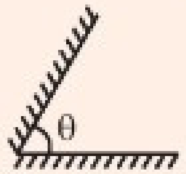
(iv) Size of image = Size of object

(V) Laterally inverted

→ Velocity of image in a plane mirror

$\boxed{(\vec{V}_{I.M})_{\perp} = (\vec{V}_{O.M})_{\perp}}$ & $\boxed{(\vec{V}_{I})_{\parallel} = -(\vec{V}_{O})_{\parallel}}$

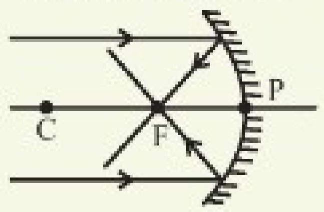
$\boxed{\omega_{I.M} = 2\omega_{O.M}}$

→  $\boxed{N = \dfrac{360}{\theta}}$

- If N = even → no. of images = N – 1
- If N = odd → no. of images = N – 1 (object is on angle bisector)
- If N = odd → no. of images = N (object is not on angle bisector)
- If N ≠ integer → count manually

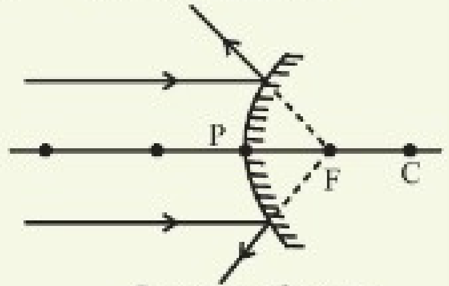
Spherical Mirrors

Concave Mirrors



$R = -\text{ve}, f = -\text{ve}$

Convex Mirrors



$R = +\text{ve}, f = +\text{ve}$

→ Mirror formula

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

→ Magnification

$$\boxed{m = \frac{-v}{u}}$$

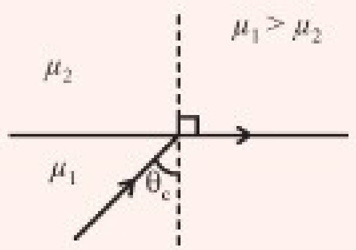
→ $|m| > 1$ → Enlarged

→ $|m| < 1$ → Diminished

→ $m < 0$ → Inverted

→ $m > 0$ → Erect

Total Internal Reflection

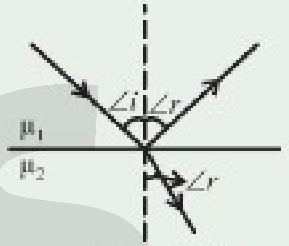


$$\boxed{\sin\theta_c = \frac{\mu_2}{\mu_1}}$$

For $i > \theta_c$ No Refraction

Refraction

$\boxed{\mu_{21} = \dfrac{v_1}{v_2}}$, $\boxed{\mu_{ma} = \dfrac{C}{v}}$ $\boxed{\dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{\mu_2}{\mu_1}}$



→ Snell's Law - $\boxed{\mu_1 \sin i = \mu_2 \sin r}$

→ Apparent depth - $\boxed{h_I = \dfrac{h_0}{\mu}}$, $\boxed{\text{Shift} = h_0\left[1 - \dfrac{1}{\mu}\right]}$

→ Lens Maker's Formula - $\boxed{\dfrac{1}{f} = (\mu_{21} - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]}$

→ For curved surface $\boxed{\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}}$

→ Lens Formula - $\boxed{\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}}$

→ Magnification $\boxed{n = \dfrac{h_1}{h_0} = \dfrac{v}{u}}$

→ Velocity of image - $\boxed{\overrightarrow{V_{IM}} = \overrightarrow{V_{OM}} \times m^2}$

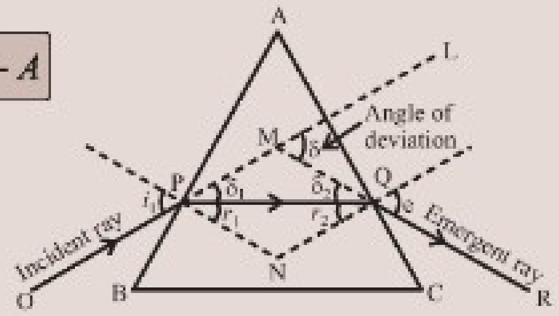
→ $\boxed{\text{Power} = \dfrac{1}{f}}$

→ Lens kept close to each other - $\boxed{\dfrac{1}{f_{eq}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \ldots}$ $\boxed{P_n = P_1 + P_2 + \ldots}$

Prism

$\boxed{A = r_1 + r_2}$, $\boxed{\delta = i + e - A}$

$$\boxed{\mu = \frac{\sin\left[\dfrac{\delta_{min} + A}{2}\right]}{\sin\left(\dfrac{A}{2}\right)}}$$



Thin prism $\delta = (\mu - 1)A$

Concave Mirror

Object	Image			
	Position	Size	Nature	Orientation
at ∞	at F	Point	Real	Inverted
b/w ∞ & C	b/w C & F	Diminished	Real	Inverted
at C	at C	Same	Real	Inverted
b/w C & F	b/w C & ∞	Magnified	Real	Inverted
at F	at ∞	Highly Magnified	Real	Inverted
b/w F & P	other side	Large	Virtual	Erect

Convex Mirror

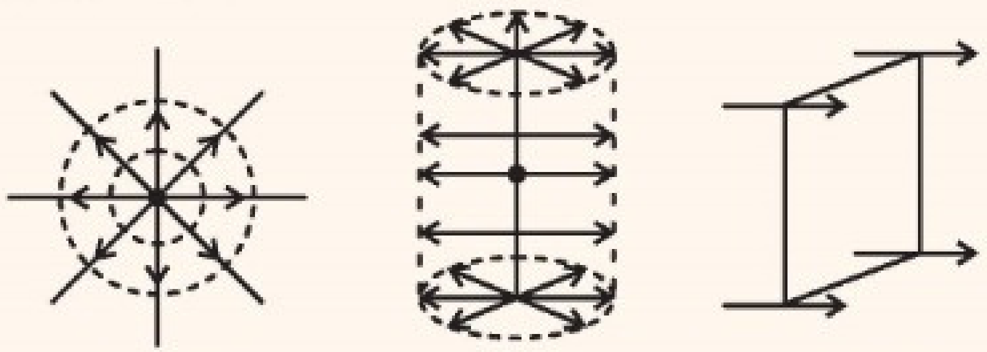
Object	Image			
	Position	Size	Nature	Orientation
at ∞	at F	Point	Virtual	Erect
b/w ∞ & P	b/w P & F	Diminished	Virtual	Erect

Instrument	Image at ∞ (Normal Adjustment)	Image at least Distance (LDDV)
Simple Microscope	Magnification $= D/f$	Magnification $= D/f + 1$
Compound Microscope	Total Magnification $= \dfrac{-L}{f_0}\dfrac{D}{f_e}$	Magnification $= \dfrac{-L}{f_0}\left[1 + \dfrac{D}{f_e}\right]$
Astronomical Telescope	Magnification $= -\dfrac{f_0}{f_e}$	Magnification $= -\dfrac{f_0}{f_e}\left[1 + \dfrac{f_e}{D}\right]$

Wave Optics

Wave Front and Huygens Principle

Wave Front



Spherical Cylindrical Plane

Huygens Principle

→ each point on the wavefront is the source of a secondary disturbance

Interference

→ phase difference does not change with time → coherent sources

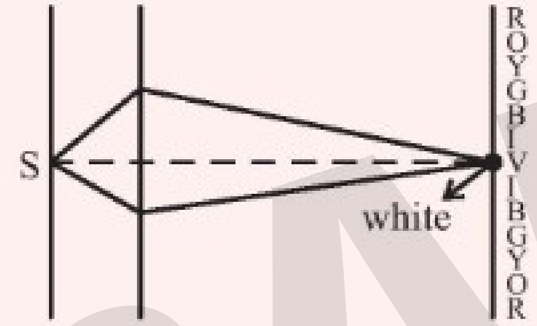
→ $\boxed{A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

→ $\boxed{I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi}$

→ YDSE

- Fringe width, $\beta = \dfrac{\lambda D}{d}$
- Position of nth bright fringe - $y_{bright} = \dfrac{n\lambda D}{d}$
- Position of nth dark fringe - $y_{dark} = \dfrac{(2n-1)\lambda D}{2d}$
- For bright fringe - $\Delta x = n\lambda$
- For dark fringe - $\Delta x = \left(n + \dfrac{1}{2}\right)\lambda$

YDSE with White Light



If $\Delta x = \dfrac{\lambda_{red}}{2}$ → red missing

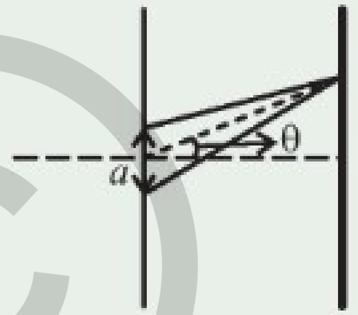
If $\Delta x = \dfrac{\lambda_{violet}}{2}$ → violet missing

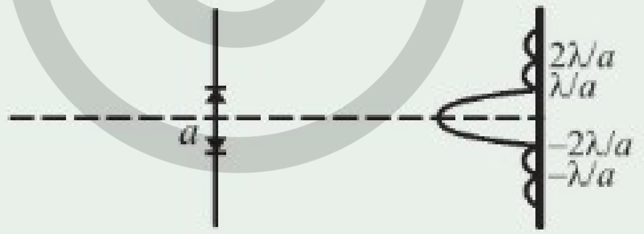
Diffraction

For minima - $\boxed{a\theta = n\lambda}$

For maxima - $\boxed{a\theta = \left[n + \dfrac{1}{2}\right]\lambda}$

Angular width $= 2\theta = \dfrac{2\lambda}{a}$





Polarisation

- **Unpolarised :** → $\vec{E}$ perpendicular to propagation
  → in all directions
- **Polarised :** → $\vec{E}$ perpendicular to propagation
  → confined only in 1-direction
- **Law of Malus :** $\boxed{I_T = I_0\cos^2\theta}$
- **Resolving Power :**

For Telescope - $R.P. = \dfrac{1}{\Delta\theta_{min}} = \dfrac{D}{1.22\lambda}$

For Microscope - $R.P. = \dfrac{1}{d_{min}} = \dfrac{2\mu\sin\theta}{1.22\lambda}$

Dual Nature of Matter and Radiation

Photoelectric Effect

→ When light of sufficient small wavelength is incident on a metal surface, electrons are ejected.

→ Electrons are called photo-electrons.

→ Minimum energy required to bring an electron out of the surface is work function.

Wave Nature

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

Principle of uncertainty - $\boxed{\Delta x \Delta p \geq \frac{h}{4\pi}}$

For a photon -

$$\boxed{p = \frac{hv}{c} \Rightarrow \frac{h}{p} = \frac{c}{v} = \lambda}$$

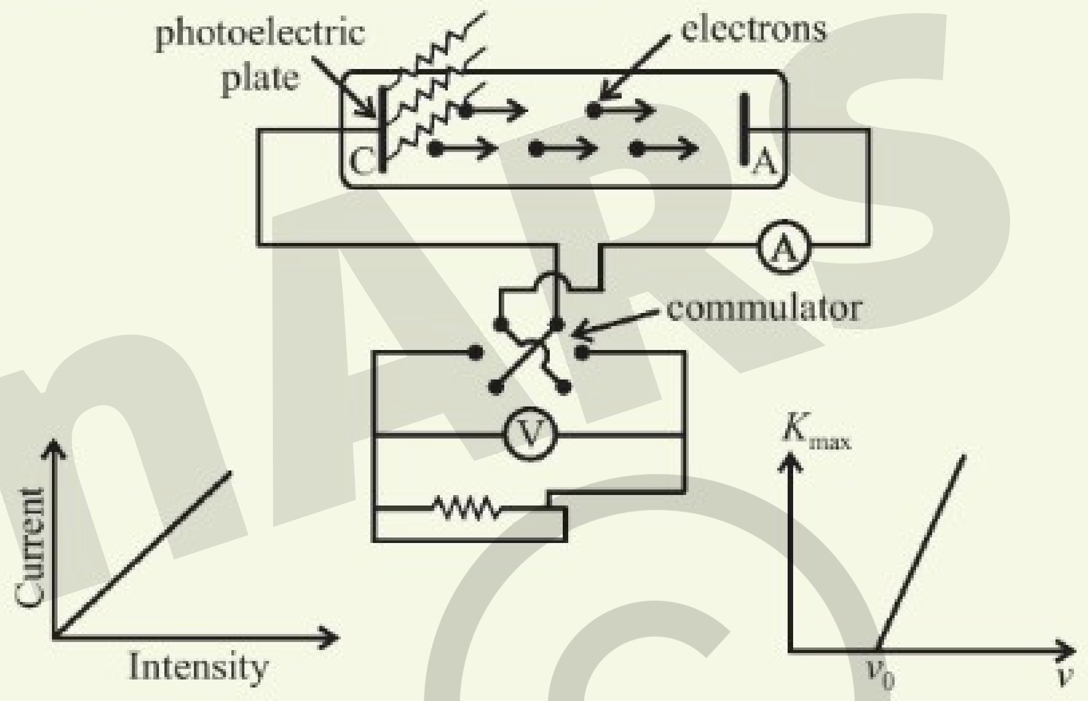
For an electron -

$$\boxed{K = eV = \frac{p^2}{2m}}$$

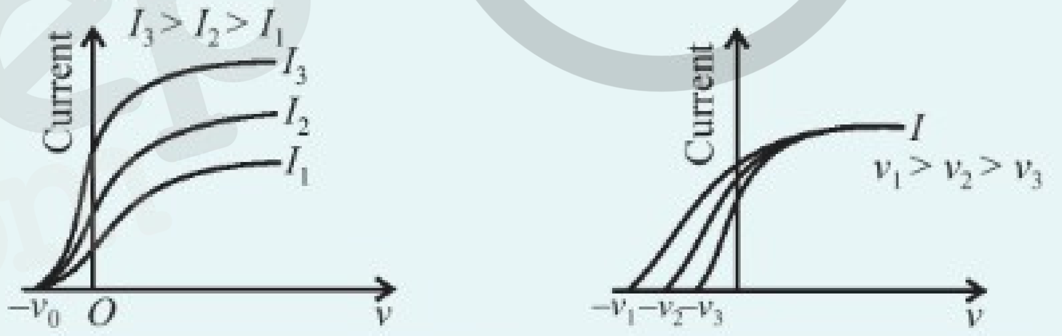
$$\Rightarrow \boxed{p = \sqrt{2mK} = \sqrt{2meV}}$$

$$\Rightarrow \boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}} \quad \boxed{\lambda = \frac{1.227}{\sqrt{V}}} \rightarrow \text{nm}$$

Photoelectric Experiment



Graphs



→ same frequency
→ different intensity

→ different frequency
→ same intensity

Einstein's Explanation

$$\boxed{E = K_{max} + \phi}$$

OR

$$\boxed{K_{max} = E - \phi = eV_0}$$

$$\boxed{K_{max} = hv - hv_0}$$

$$\boxed{\phi = \frac{hc}{\lambda_0}}$$

$$\boxed{\phi = hv_0}$$

→ If $\lambda > \lambda_0$ *i.e.* $E < \phi$ → no electron will come out.

Atoms

Rutherford's Model

Postulates - Atoms have a central, massive, positively charged core around which electrons revolve.

→ Size of nucleus = 1 fermi = 10^{-15} m

Drawbacks - Doesn't explain the stability of atom.

→ doesn't explain the atomic spectra.

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

$$U = \frac{-e^2}{4\pi\varepsilon_0 r}$$

Bohr's Model

Postulates -

→ Electrons revolve around the nucleus in stationary orbits.

→ Angular momentum : - $mv_n r_n = \frac{nh}{2\pi}$

→ An electron can make transition to a lower energy state.

Energy of the photon released -

$$hf = E_i - E_f$$

→ For H-like atoms -

$$v_n = \left[\left(2.18\times10^6 \times \frac{Z}{n}\right)\right] \text{m/s}$$

$$r_n = \left[\left(0.53\times\frac{n^2}{Z}\right)\right] \text{Å}$$

$$K.E. = \frac{KZe^2}{2r}$$

$$P.E. = \frac{-KZe^2}{r}$$

$$E = -13.6\frac{Z^2}{n^2} eV$$

Atomic Spectra

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series (U.V. region) -

$n_1 = 1$ and $n_2 = 2, 3, 4, ...$

For Balmer series (Visible region) -

$n_1 = 2$ and $n_2 = 3, 4, 5, ...$

For Paschan series (Infrared region) -

$n_1 = 3$ and $n_2 = 4, 5, 6, ...$

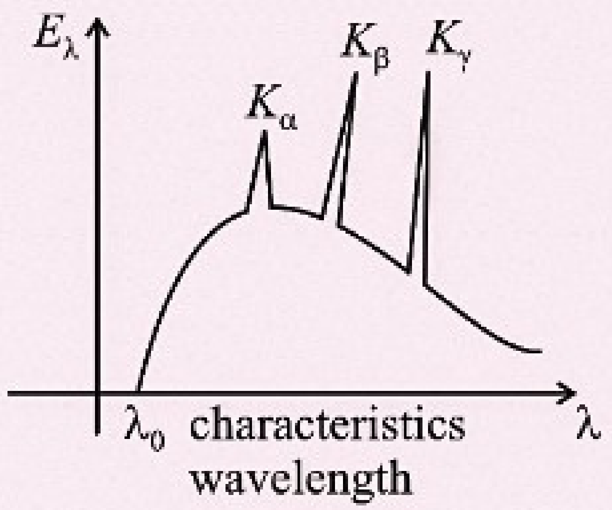
X-Rays

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{12400}{V} \text{Å}$$

$$\sqrt{f} = \sqrt{\frac{3Rc}{4}}(Z-1)$$

$$\sqrt{f} = a(Z-1)$$



Nuclei

Nucleus

→ Mass ∝ Volume of nucleus

→ 1 amu = 1 atomic mass unit

$= \frac{1}{12}$th of mass of C-12

$= 1.660539 \times 10^{-27}$ kg

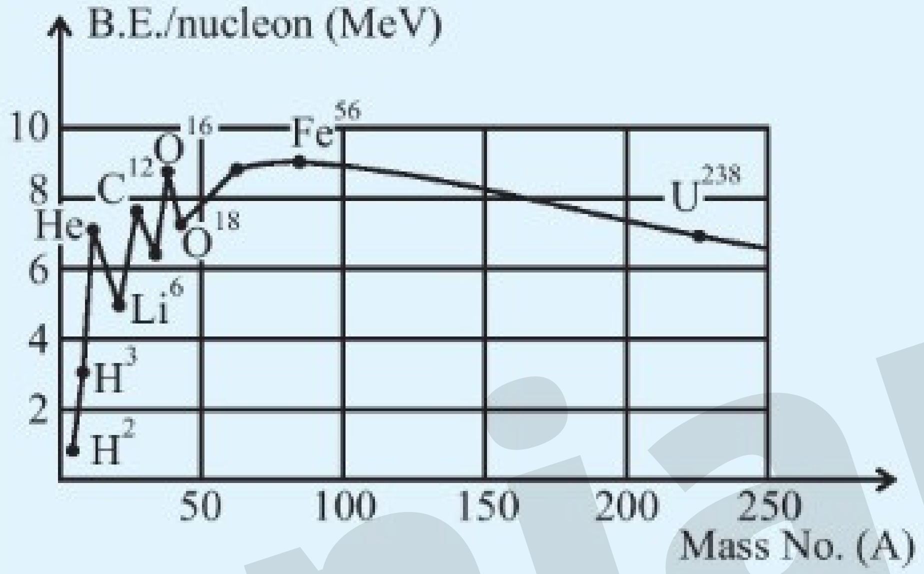
→ Radius of nucleus :-

$R = R_0 A^{1/3}$ $\quad$ $R_0 = 1.2$ fm

→ Density of nucleus :-

$\text{density} = 2.3 \times 10^{17}\ \text{kg/m}^3$

Graph of B.E. per Nucleon



Radioactivity

* Law of Radioactivity :-

$\frac{dN}{dt} = -\lambda N$

$R = R_0 e^{-\lambda t}$

$N = N_0 e^{-\lambda t}$

1 Becquerel = 1 decay/sec

$T_{1/2} = \frac{\ln(2)}{\lambda}$

$T_{1/2} = \frac{0.693}{\lambda}$

1 Curie = 3.7×10^{10} Becquerel

$N = N_0 \left[\frac{1}{2}\right]^n$

$\tau = \frac{1}{\lambda}$

$n = \frac{t}{T_{1/2}}$

Nuclear Binding Energy

* Nuclear Force :- independent of charge, short range.

* Mass-Energy Equivalent:- $E = mc^2$

→ Electron's Rest Mass-Energy -

$E = 511\ KeV$

$1\ eV = 1.6 \times 10^{-19} V$

→ Energy from 1 amu = 931.5 MeV

* $B.E. = \left(\left[Zm_p + (A-Z)m_n\right] - M\right)c^2$

* B.E. per nucleon :- $E_{bn} = \frac{E_b}{A}$

→ E_{bn} → low for $A < 30$ & $A > 170$

Q-Value

$Q = B.E._{\text{products}} - B.E._{\text{reactant}}$

Also,

$Q = (\Delta m)c^2$

$Q = \left(M_{\text{reactants}} - M_{\text{products}}\right)c^2$

Types of Radioactivity

* α-decay -

${}_Z X^A \longrightarrow {}_{Z-2}Y^{A-4} + \alpha\text{-particle}$

* β-decay -

(a) ${}_Z X^A \longrightarrow {}_{Z+1}Y^A + \beta^- + \bar{\nu}$ $\quad$ (β^- decay)

(b) ${}_Z X^A \longrightarrow {}_{Z-1}Y^A + \beta^+ + \nu$ $\quad$ (β^+ decay)

* γ-decay -

${}_Z X^A \longrightarrow {}_Z Y^A + \gamma$ $\quad$ (γ radiation)

Semiconductor Electronics

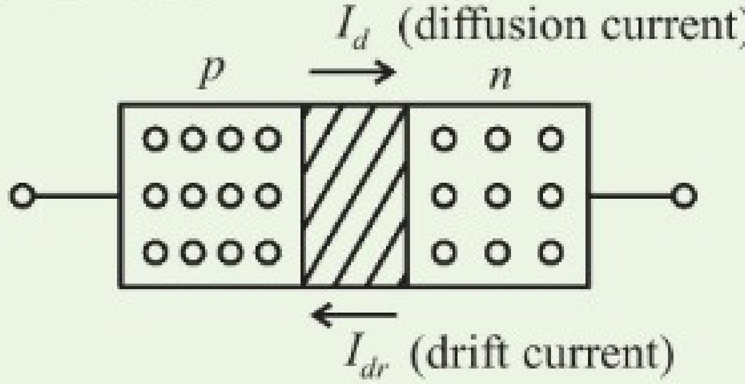
Semiconductor and Types

Materials

	Conductors	Semiconductors	Insulators
ρ	$10^{-8} - 10^{-2}\ \Omega m$	$10^{-6} - 10^{-5}\ \Omega m$	$10^{11} - 10^{19}\ \Omega m$
σ	$10^{2} - 10^{8}\ (\Omega m)^{-1}$	$10^{5} - 10^{6}\ (\Omega m)^{-1}$	$10^{-19} - 10^{-11}\ (\Omega m)^{-1}$

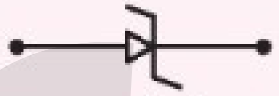
- Energy band gap - gap between top of valence band and bottom of conductor band.
  - → For conductors - $E_g \approx 0$
  - → For insulators - $E_g > 3$ eV
  - → For semiconductors - $E_g < 3$ eV
- Intrinsic - pure semiconductors
- Extrinsic - doped semiconductors

PN Junction

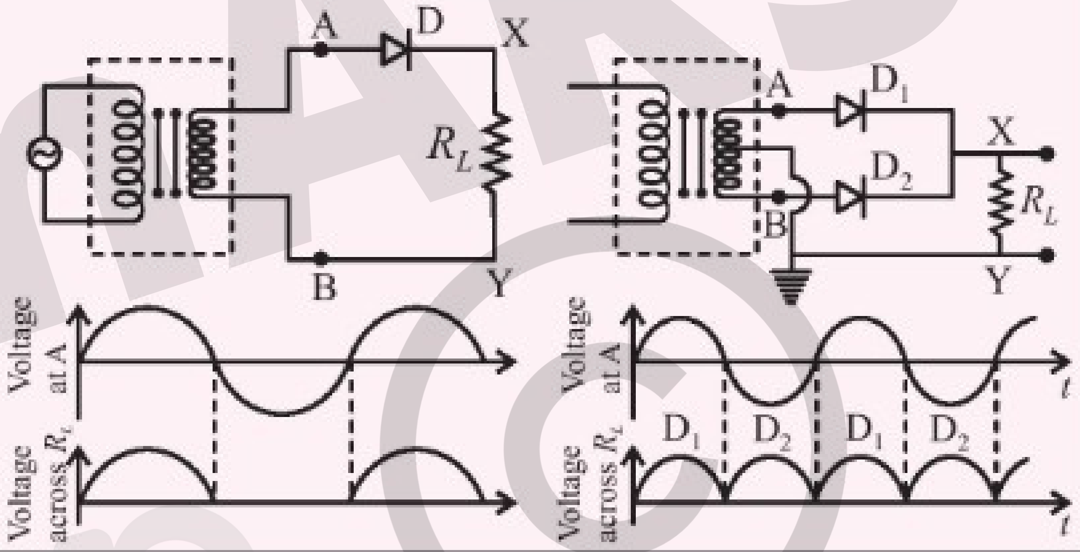


- Biasing of *p-n* junction
  - → when *p* - side is given higher potential → forward biased
  - → when *n* - side is given higher potential → reverse biased

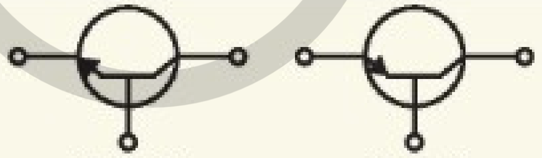
Zener Diode



- → operates under reverse bias in the break-down voltage, used as a voltage regulator.
- → depletion region is very thin.
- Half wave and full wave rectifier



Transistor



$I_E = I_C + I_B$

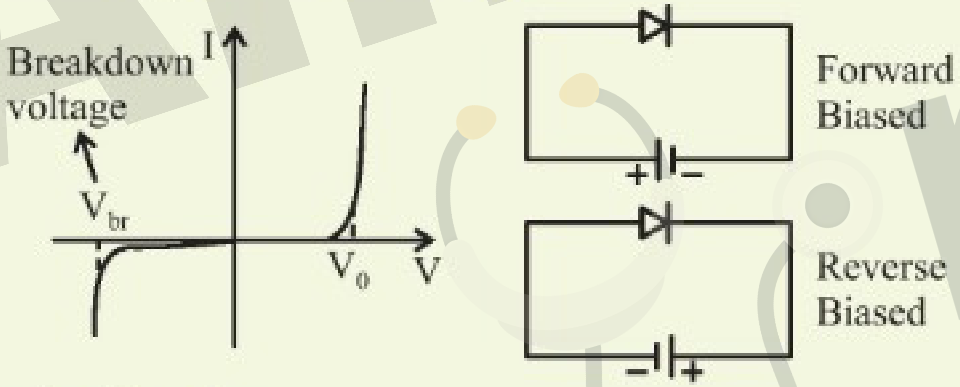
→ As an amplifier

Voltage gain, $A_v = \dfrac{\Delta V_c}{\Delta V_{in}}$ $\quad A_R = R_L / R_i$, $\quad A_v = -A_R \beta_{AC}$

Power gain, $A_p = \beta_{AC} A_v$ $\quad \beta = \dfrac{\Delta I_C}{\Delta I_B}$

Trans-conductance, $g_m = \dfrac{\Delta I_C}{\Delta V_B}$

Biasing of PN Junction



- **Applications**
  - (i) Photodiode - is PN junction whose function is controlled by the light allowed to fall on it.
  - (ii) LED - used in T.V. or electronic gadgets.
    -V-I characteristics are similar to that of a *Si* function diode
  - (iii) Solar cell - generates emf when solar radiation falls on it.

Extrinsic Semiconductors

- *n*-type - pentavalent dopant
  - → donor
  - → $n_e > n_h$
- *p*-type - trivalent dopant
  - → acceptor
  - → $n_e < n_h$

→ For semiconductors - $n_e n_h = n_i^2$

Logic Gates

OR Gate | **AND Gate** | **NOT Gate** | **NAND** | **NOR**

OR Gate: $Y = A + B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

AND Gate: $Y = A \cdot B$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

NOT Gate: $Y = \overline{A}$

A	Y
0	1
1	0

NAND: $Y = \overline{A \cdot B}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR: $Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0