# 2.14 MATLAB: Time-Domain Representations of LTI Systems

# Convolution

## conv

```
>> y=conv(x,h)
```

- Range of index for x, h, y
  - $x[n]: k_x \leq n \leq l_x$
  - $h[n]: k_h \leq n \leq l_h$
  - $y[n]: k_y \leq n \leq l_y, k_y = k_x + k_h$ and $l_y = l_x + l_h$
- The length of $x[n], h[n], y[n]$
  - $L_x = l_x - k_x + 1$
  - $L_h = l_h - k_h + 1$
  - $L_y = l_y - k_y + 1 = L_x + L_h - 1$

# Convolution

## Ex 2.1 Multipath Communication Channel (1)

$$x[n] = \begin{cases} 2, & n = 0 \\ 4, & n = 1 \\ -2, & n = 2 \\ 0, & \text{o.w.} \end{cases}, \qquad h[n] = \begin{cases} 1, & n = 0 \\ 1/2, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

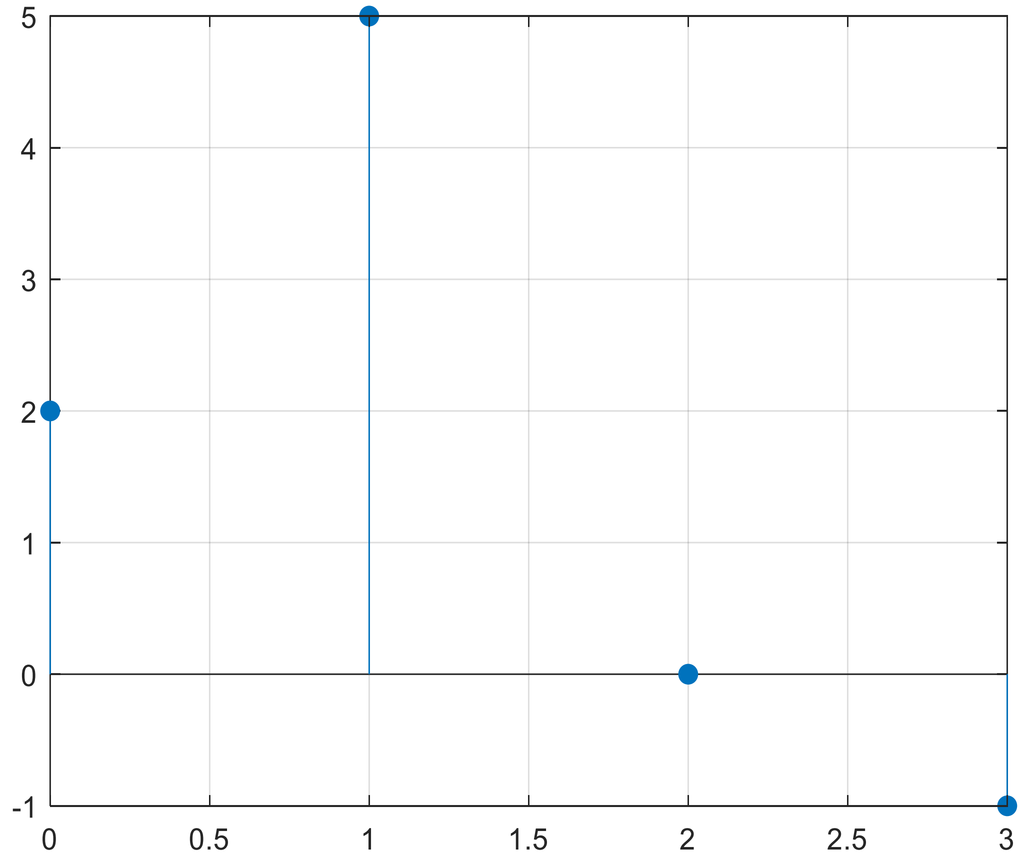
Sol)

- $k_x = k_h = 0 \rightarrow k_y = k_x + k_h = 0$
- $l_x = 2, l_h = 1 \rightarrow l_y = l_x + l_h = 3$
- $L_y = l_y - k_y + 1 = 4$

# Convolution

## Ex 2.1 Multipath Communication Channel (2)

```
>> h=[1,0.5];
>> x=[2,4,-2];
>> y=conv(x,h)
y=
      2  5  0 -1
>> n=0:3;
>> stem(n,y);
```



# Convolution

## Ex 2.3 Moving Average System (1)

$h[n] = (1/4)(u[n] - u[n-4])$

- $k_h = 0, l_h = 3 \rightarrow 0 \leq n \leq 3$

$x[n] = u[n] - u[n-10]$

- $k_x = 0, l_x = 9 \rightarrow 0 \leq n \leq 9$

$y[n] = x[n] * h[n]$

- $k_y = 0, l_y = 12, L_y = 13 \rightarrow 0 \leq n \leq 12$

```
>> h=0.25*ones(1,4);
>> x=ones(1,10);
>> y=conv(x,h);
>> n=0:12;
>> stem(n,y); xlabel('n'); ylabel('y[n]')
```

# Convolution

## Ex 2.3 Moving Average System (2)

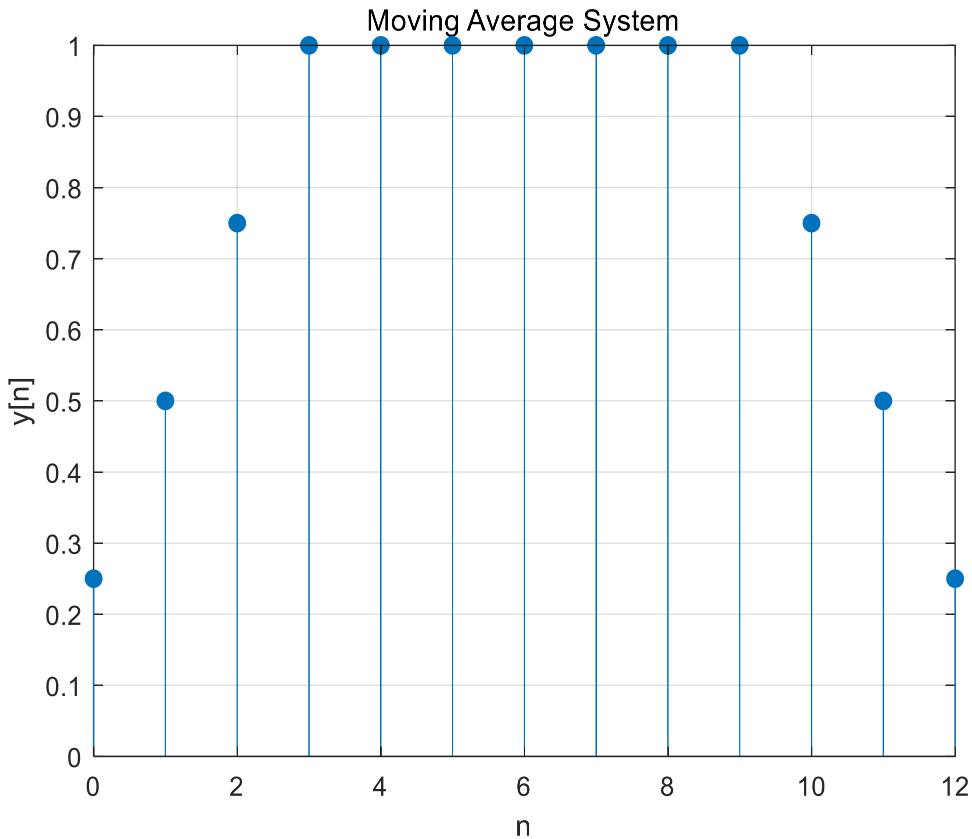



Figure 2.45 (p. 177) Convolution sum computed using MATLAB.

# Simulating Difference Equations (1)

- Linear constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Coefficients of difference equation

– $\boldsymbol{a} = [a_0, a_1, \cdots, a_N]$ and $\boldsymbol{b} = [b_0, b_1, \cdots, b_M]$

- Output of the system

```
>> y=filter(b,a,x)
```

  - The number of output values in y corresponds to the number of input values in x.

```
>> y=filter(b,a,x,zi)
```

  - zi: the initial conditions required by filter

# Simulating Difference Equations (2)

```
>> zi=filtic(b,a,yi)
```

- yi is a vector containing the initial conditions in the order $[y[-1], y[-2], \cdots, y[-N]]$.

## Ex. 2.16 Evaluation of a difference equation by means of a computer

A system is described by the difference equation

$$y[n] - 1.143y[n-1] + 0.4128y[n-2]$$
$$= 0.0675x[n] + 0.1349x[n-1] + 0.675x[n-2]$$

Write a recursive formula that computes the present output from the past outputs and the current inputs. Use a computer to determine the system output when the input is zero and the initial conditions are $y[-1] = 1$ and $y[-2] = 2$.

# Simulating Difference Equations (3)

```
>> a=[1, -1.143, 0.4128]; b=[0.0675, 0.1349, 0.675];
>> x=zeros(1, 50);
>> zi=filtic(b, a, [1 2]);
>> y=filter(b, a, x, zi);
```

# Simulating Difference Equations (4)