3.19 MATLAB: Fourier Representations of Signals and LTI Systems

Frequency Response of LTI Systems
A Continuous Function of Frequency

- Discrete-time LTI system

$$H\left(e^{j\Omega}\right) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}$$

- Continuous-time LTI system

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}d\tau$$

- We can evaluate the frequency response only at discrete values of frequency.
- A large number of values are normally used to capture the details in the system's frequency response.

Frequency Response of LTI Systems

Discrete-time LTI System (1)

- Impulse response
  - $h[n] = 0$ for $n < k_h, n > l_h$
    - k_h: first element of h
    - l_h: last element of h
- Finite-duration complex sinusoidal input signal

$$v[n] = e^{j\Omega n}(u[n] - u[n - l_\nu]), 0 \leq n \leq l_\nu - 1$$

- System output

$$y[n] = h[n] * v[n] = h(n) * e^{j\Omega n}$$

$$= \sum_{k=k_h}^{l_h} h[k]e^{j\Omega(n-k)}, k_h \leq n \leq l_h + l_\nu - 1$$

$$= H\left(e^{j\Omega}\right)e^{j\Omega n}$$

Frequency Response of LTI Systems

Discrete-time LTI System (2)

- Frequency response
  - $y[n] = \left|H\left(e^{j\Omega}\right)\right| e^{j\left(\Omega n + \arg\left\{H\left(e^{j\Omega}\right)\right\}\right)}, k_h \le n < l_h + l_v$
  - $|y[n]| = \left|H\left(e^{j\Omega}\right)\right|$; Magnitude Response
  - $\arg\left\{H\left(e^{j\Omega}\right)\right\} = \arg\{y[n]\} - \Omega n$; Phase Response

Ex 3.22 Moving-Average Systems: Frequency Response: Consider the system with impulse response

$$h_2[n] = \frac{1}{2}(\delta[n] - \delta[n-1]).$$

Let us determine the frequency response and 50 values of the steady-state output of this system for input frequencies $\Omega = \frac{\pi}{4}$ and $\frac{3\pi}{4}$.

Frequency Response of LTI Systems

Discrete-time LTI System (3)

Sol)

$k_h = 0$ and $l_h = 1$

We require $l_v \geq 51$ to obtain 50 values of the steady-state output of this system.

```
>> Omega1=pi/4; Omega2=3*pi/4;
>> v1=exp(j*Omega1*[0:50]);
>> v2=exp(j*Omega2*[0:50]);
>> h=[0.5, -0.5];
>> y1=conv(v1,h); y2=conv(v2,h);
>> n=0:length(y1)-1;
```

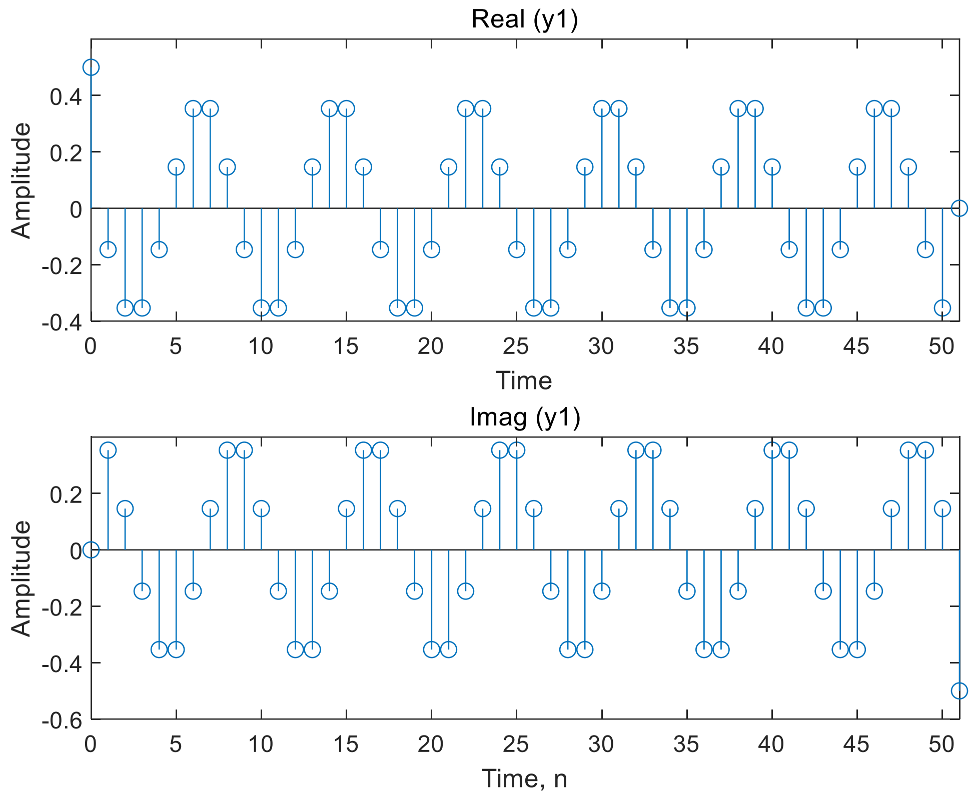
Frequency Response of LTI Systems

Discrete-time LTI System (4)

```
>> subplot(2, 1, 1)
>> stem([0:51], real(y1))
>> xlabel('Time, n'); ylabel('Amplitude');
>> title('Real(y[n]')
>> subplot(2, 1, 2)
>> stem([0:51], imag(y1));
>> xlabel('Time, n'); ylabel('Amplitude');
>> title('Imag(y[n])')
```

Frequency Response of LTI Systems

Discrete-time LTI System (6)



Frequency Response of LTI Systems

Discrete-time LTI System (7)

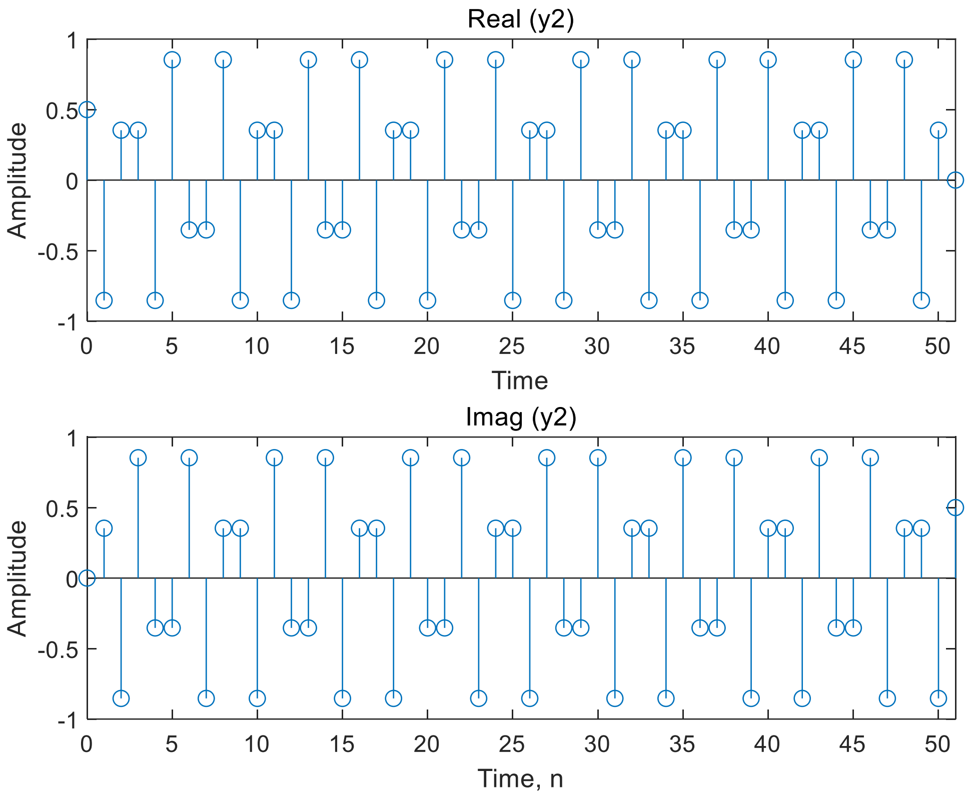


Figure 3.76 Sinusoidal steady-state response computed with the use of MATLAB. The values at times 1 through 50 represent the sinusoidal steady-state response $(h_2(n), \Omega = \frac{\pi}{4}, \frac{3\pi}{4})$

The DTFS (1)

- The only Fourier representation that is discrete valued in both time and frequency
- DTFS

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{j2\pi nk/N}, \qquad n = 0, 1, 2, \cdots, N-1 \qquad (3.10)$$

- DTFS coefficient

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}, \qquad k = 0, 1, 2, \cdots, N-1 \qquad (3.11)$$

The DTFS (2)

- MATLAB commands
  - x: a length-N vector representing one period of an N periodic signal $x[n]$.
  - X: a length-N vector containing the DTFS coefficients $X[k]$.

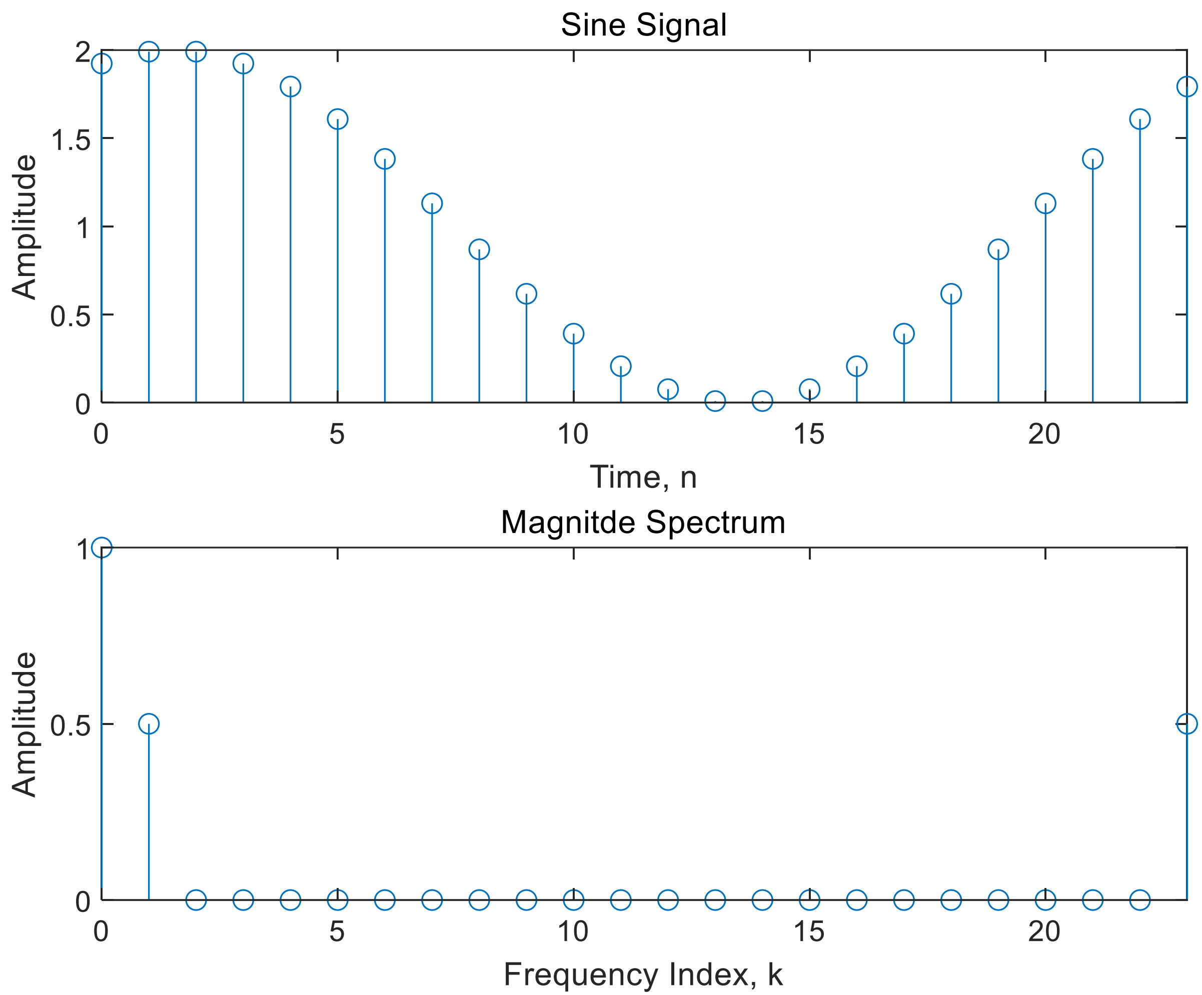
```
>> X=fft(x)/N;    % DFTS coefficients
>> x=ifft(X)*N;   % DTFS
```

The DTFS (3)

Example: $x[n] = 1 + \sin\left(\frac{n\pi}{12} + \frac{3\pi}{8}\right)$

```
% DTFS coefficients
>> n=0:23;
>> x=ones(1,24)+sin(n*pi/12+3*pi/8);
>> X=fft(x)/24;
>> mag_X=abs(X);
>> phase_X=angle(X);
>> Omega=n*2*pi/24;
% DTFS
>> xrecon=ifft(X)*24;
>> xrecon(1:4)
ans = 1.9239    1.9914    1.9914    1.9239
```

The DTFS (4)



Sine Signal
Amplitude
2
1.5
1
0.5
0
0
5
10
15
20
Time, n
Magnitde Spectrum
Amplitude
1
0.5
0
0
5
10
15
20
Frequency Index, k

The FS (1)

Ex 3.14 Square-wave Partial-Sum Approximation

Let the partial-sum approximation to the FS in

$$x(t) = \sum_{k=0}^{\infty} B[k] \cos(k\omega_0 t),$$

be given by

$$\hat{x}_J(t) = \sum_{k=0}^{J} B[k] \cos(k\omega_0 t).$$

This approximation involves the exponential FS coefficients with indices $-J \leq k \leq J$. Consider a square wave with $T = 1$ and $T_0/T = 1/4$. Depict one period of the Jth term in this sum, and find $\hat{x}_J(t)$ for $J = 1, 3, 7, 29$, and 99.

The FS (2)

Sol)

$$B[k] = \begin{cases} 1/2, & k = 0 \\ \big(2/(k\pi)\big)(-1)^{(k-1)/2}, & k \text{ odd} \\ 0, & k \text{ even} \end{cases},$$

20 samples per period

$T_s = T/(20J_{\max})$

Total number of samples in one period: 20 $J_{\max}$

Assuming $J_{\max}$=99 and $T = 1$,

The FS (3)

```
t = [-(20*Jmax-1):10*Jmax]*(1/(20*99));
xJhat(1,:) = B(1)*cos(t*0*2*pi/T);
for k = 2:100
    xJhat(k,:) = xJhat(k-1,:)+B(k)*cos(t*(k-1)*2*pi/T);
end
```

The FS (4)

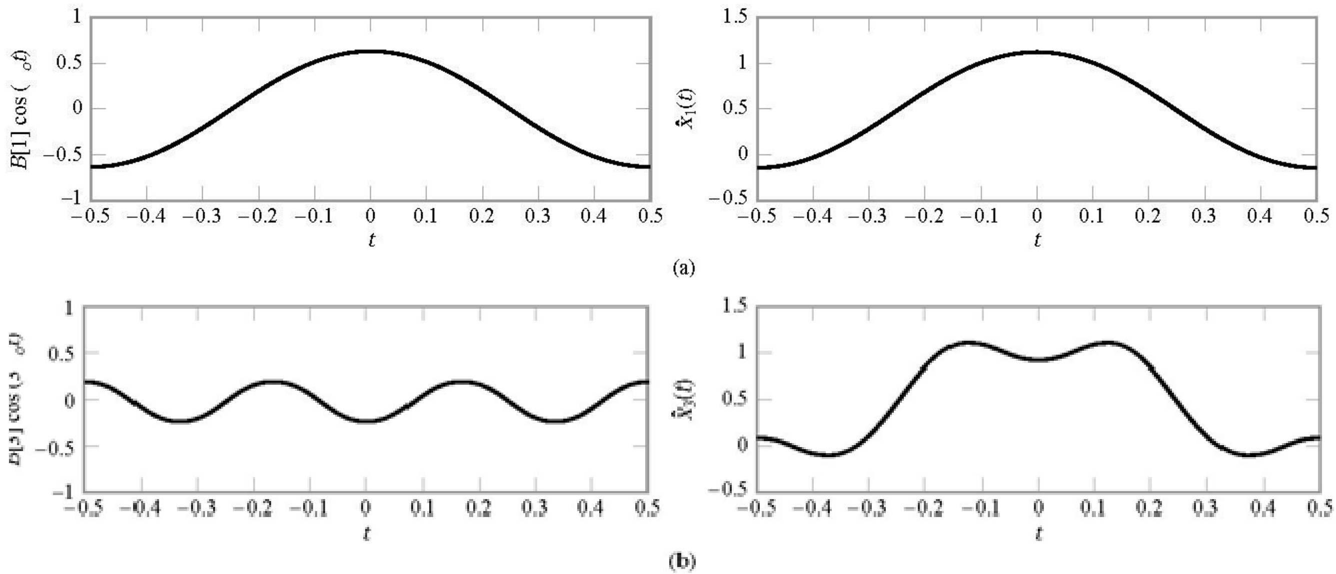
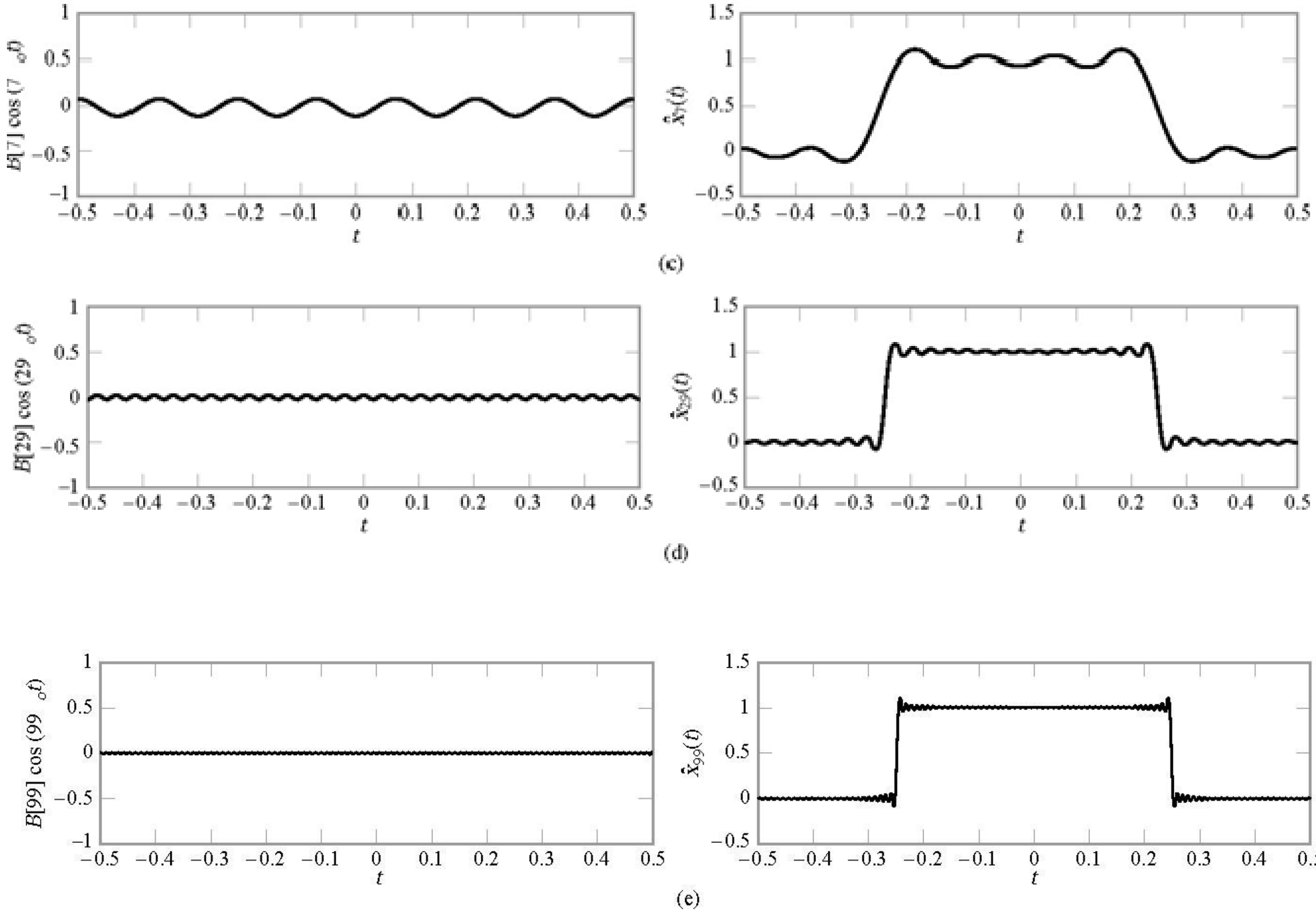


Figure 3.25a (p. 226)

Individual terms (left panel) in the FS expansion of a square wave and the corresponding partial-sum approximations $xj(t)$ (right panel). The square wave has period T = 1 and T_0/T = ¼. The J = 0 term is $x_0(t)$ = ½ and is not shown.

(*a*) J = 1. (*b*) J = 3. (*c*) J = 7. (*d*) J = 29. (*e*) J = 99.

The FS (5)



B[7] cos (7 ot)
$\hat{x}_7(t)$
t
(c)
B[29] cos (29 ot)
$\hat{x}_{29}(t)$
t
(d)
B[99] cos (99 ot)
$\hat{x}_{99}(t)$
t
(e)