

1 Gauss-Jordan elimination!

Find all solutions to the following linear system of equations.

1.1 Task 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 7 & 2 & -3 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

1.2 Task 2

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \vec{0}.$$

1.3 Task 3

I have 32 bills in my wallet, in the denominations of \$1, \$5, and \$10, worth \$100 in total. How many do I have of each denomination? **Hint: there are 2 solutions!**

2 A classic problem!

Consider the system

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & k & 4 \\ 1 & 2 & k+2 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}, \quad k \in \mathbb{R}.$$

- For which values of the constant k does this system have no solution?
- When are there infinitely many solutions?
- When is there a unique solution?

3 Can we work backwards?

3.1 Task 1

Find a system of linear equations with three unknowns whose solutions are the points on the line through $(1, 1, 1)$ and $(3, 5, 0)$.

3.2 Task 2

Find a system of linear equations with three unknowns x, y, z whose solutions are

$$x = 6 + 5t, \quad y = 4 + 3t, \quad \text{and} \quad z = 2 + t,$$

where t is an arbitrary constant.

PILOT Tip of the Week!

Top Ten List to Stop Procrastination

1. Break it down! Look at your work and break it into smaller steps in order to simplify it.
2. Change your environment – find a new place to study.
3. Create a detailed timeline with specific deadlines.
4. Eliminate your procrastination pit-stops – disable email and social media notifications.
5. Hang out with people who inspire you!
6. Set small goals and reward yourself – study for 45 minutes and then watch 15 minutes of Netflix.
7. Tell others about your goals – it makes you more accountable to achieve them.
8. Seek out someone who has already achieved the outcome you want and ask their advice.
9. Take a much-needed break and give your brain a rest! A little bit of rest time can help you strengthen your focus.
10. Stop over-complicating things – there is never a perfect time to start studying or a perfect pen/notebook to take notes - just take-action and start!