Good work

SMA 2480 - COMPLEX ANALYSIS REVISION.

Ex.

1. Given $z_1 = 3+4i$ and $z_2 = 5+3i$. Compute:

(i) $z_1 \cdot z_2^{-1}$

$z_2^{-1} = ?$ but $z_2 \cdot z_2^{-1} = 1$ --(i)

where $z_2 = 5+3i$ --(ii)

z_2^{-1} from (i) $= z_2^{-1} = \frac{1}{z_2}$

$z_2^{-1} = \frac{1}{5+3i}$

so $z_1 \cdot z_2^{-1} = \frac{3+4i}{5+3i} \frac{(5-3i)}{(5-3i)}$

$z_1 \cdot z_2^{-1} = \frac{3(5-3i)+4i(5-3i)}{5^2 - -9}$

$z_1 z_2^{-1} = \frac{15-9i+20i--12}{34}$

$z_1 \cdot z_2^{-1} = \frac{27+11i}{34} = \frac{27}{34} + \frac{11}{34} i = \frac{27}{34} + \frac{11}{34} i$

(ii) $z_2 \cdot z_1^{-1}$

given $z_1 \cdot z_1^{-1} = 1$

$z_1^{-1} = \frac{1}{z_1}$ where $z_1 = 3+4i$

$z_1^{-1} = \frac{1}{3+4i}$

so $z_2 \cdot z_1^{-1} = \frac{5+3i}{3+4i}$

$z_2 \cdot z_1^{-1} = \frac{5+3i}{(3+4i)} \frac{(3-4i)}{(3-4i)} = \frac{5(3-4i)+3i(3-4i)}{3^2 - -4^2}$

$= \frac{15-20i+9i-12i}{25} = \frac{27-11i}{25}$

(iii) $z_1 . z_2$

where $z_1 = 3+4i$

so $z_1 . z_1 = (3+4i)^2$

$= 9 + -16$

$= -7$

$(3+4i)(3+4i)$

$3(3+4i) + 4i(3+4i)$

$9 + 12i + 12i - 16$

$-7 + 24i$

2. Show that $z_1 . z_2 = z_2 . z_1$

Solving L.H.S

$z_1 . z_2 = (3+4i)(5+3i)$

$= 3(5+3i) + 4i(5+3i)$

$= 15 + 9i + 20i + -12$

$= 3 + 29i$

Solving R.H.S

$z_2 . z_1 = (5+3i)(3+4i)$

$= 5(3+4i) + 3i(3+4i)$

$= 15 + 20i + 9i + -12$

$= 3 + 29i$

R.H.S = L.H.S = $3 + 29i$ so $z_1 . z_2 = z_2 . z_1$ (associative law

PART 2: DIVISION OF COMPLEX NUMBERS

Exercise

(i) $\frac{3+4i}{1-2i} = \frac{3+4i}{1-2i} \frac{(1+2i)}{(1+2i)} = \frac{3(1+2i) + 4i(1+2i)}{1^2 - -2^2}$

$= \frac{3 + 6i + 4i + -8}{5} = \frac{10i - 5}{5}$

$= 2i - 1$

(ii) $\frac{2i+4}{3i-7} = \frac{2i+4}{3i-7} \frac{(3i+7)}{(3i+7)}$

$\frac{2i(3i+7) + 4(3i+7)}{-9 - 49}$

$= \frac{-6 + 14i + 12i + 28}{-58} = \frac{22 + 26i}{-58} = \frac{-22}{58} \frac{-26}{58} i$