

# PFL-2021-G4\_04

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## Members

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## Functions

### **fibRec**

Calculate n-th fibonacci number using recursion.

### **fibLista**

Calculate n-th fibonacci number using a finite list comprehension where each number is the sum of the two previous numbers.

### **fibHelper**

Calculate n-th fibonacci number using a list accumulator and recursion where with each recursive call the new fibonacci number is appended to the list until it is sufficiently long, then last value is returned.

### **fibLista'**

Calculate n-th fibonacci number by calling fibHelper with accumulator list starting at [0, 1].

### **fibListaInfinita**

Calculate n-th fibonacci number using an infinite list comprehension where each number is the sum of the two previous numbers.

### **fibRecBN**

Calculate n-th fibonacci number using recursion using BigInteger for argument and return value.

### **fibListaBN**

Calculate n-th fibonacci number using a finite list comprehension where each number is the sum of the two previous numbers using BigInteger for argument and return value.

### **fibListaInfinitaBN**

Calculate n-th fibonacci number using an infinite list comprehension where each number is the sum of the two previous numbers using BigInteger for argument and return value.

### **isNegative**

Checks whether the BigInteger is negative.

## **isPositive**

Checks whether the BigNumber is positive.

## **minusBNHelper**

Calculates the additive inverse of a given BigNumber.

## **removeLeadingZeros**

Removes leading zeros of a BigNumber.

## **minusBN**

Calculates the additive inverse of a given BigNumber after removing leading zeros.

## **firstBN**

Return the leftmost digit of a BigNumber.

## **tailBN**

Return the given BigNumber without its leftmost element.

## **paddBN**

Padds the first argument with leading zeros until it has the same length as the second argument.

## **scannerHelper**

Converts a string into BigNumber, can only handle positive numbers.

## **scanner**

Converts a string into a BigNumber.

## **output**

Converts BigNumber into a String.

## **somaPosBN**

Add two positive BigNumbers using manual addition algorithm.

## **somaBN**

Add any two BigNumbers.

## **orderedSubPosBN**

Subtract two positive BigNumbers as long as the left is greater than the right using manual subtraction algorithm.

## **lengthBN**

Return the length (number of digits) of a BigNumber.

## **greaterEqualLength**

Compare two BigNumbers of equal length and check if the left is greater than the right.

## **greaterBN**

Compare two BigNumbers and check if the left is greater than the right.

## **subBNHelper**

Subtract any two positive BigNumbers.

## **subBN**

Subtract any two BigNumbers.

## **eBN**

Equivalent to  $x * 10^n$ .

## **baseMulBN**

Multiply two BigNumbers in the format  $x * 10^{ex}$  and  $y * 10^{ey}$  if  $x$  and  $y$  are single digit.

## **digitOrdBN**

Get the  $n$ -th digit of a BigNumber, counting from the left and starting at 0.

## **mulListBN**

Return a list of the results of the smaller multiplications that are made in multiplying bigger numbers using a manual multiplication algorithm.

## **mulPosBN**

Multiply two positive BigNumbers by using foldl with somaBN to add the results of mulListBN.

## **mulBN**

Multiply any two BigNumbers.

## **equalsBN**

Check if two BigNumbers are equal.

## **greaterOrEqualsBN**

Compare two BigNumbers to check if left is greater than or equal to the right.

## lesserOrEqualsBN

Compare two BigNumbers to check if left is lesser than or equal to the right.

## naiveDivBN

Divide BigNumbers using trial and error with multiplication.

## divBNHelper

Divide BigNumbers using a manual division algorithm.

## divBN

Whole division of two BigNumbers by calling divBNHelper with accumulators starting at 0.

## bigNumberToInt

Convert a BigNumber (BN xs x) into an Int, by adding x (already an Int) to 10 times the result of bigNumberToInt xs until xs is empty.

## safeDivBN

Check if trying to divide by 0, if so return Nothing, otherwise call divBN.

## Test Cases

```
--fibRec :: (Integral a) => a -> a
fibRec 0
fibRec 1
fibRec 20

--fibLista :: (Integral a) => a -> a
fibLista 0
fibLista 1
fibLista 20

--fibHelper :: Integral a => Int -> Int -> [a] -> a
fibHelper 5 2 [0,1]
fibHelper 8 5 [0,1,1,2,3]
fibHelper 8 3 [0,1,1]

--fibLista' :: Integral a => Int -> a
fibLista' 8
fibLista' 5

--fibListaInfinita :: (Integral a) => a -> a
fibListaInfinita 8
fibListaInfinita 5

--fibRecBN :: BigNumber -> BigNumber
output (fibRecBN (scanner "8"))
```

```

output (fibRecBN (scanner "5"))

--fibListaBN :: BigNumber -> BigNumber
output (fibListaBN (scanner "8"))
output (fibListaBN (scanner "5"))

--fibListaInfinitaBN :: BigNumber -> BigNumber
output (fibListaInfinitaBN (scanner "8"))
output (fibListaInfinitaBN (scanner "5"))

--isNegative :: BigNumber -> Bool
isNegative (scanner "12")
isNegative (scanner "0")
isNegative (scanner "-12")

--isPositive :: BigNumber -> Bool
isPositive (scanner "12")
isPositive (scanner "0")
isPositive (scanner "-12")

--minusBNHelper :: BigNumber -> BigNumber
output (minusBNHelper (scanner "-12"))
output (minusBNHelper (scanner "12"))

--removeLeadingZeros :: BigNumber -> BigNumber
output (removeLeadingZeros (scanner "000012"))
output (removeLeadingZeros (scanner "100002"))
output (removeLeadingZeros (scanner "120000"))

--minusBN :: BigNumber -> BigNumber
output (minusBN (scanner "-000012"))
output (minusBN (scanner "000012"))
output (minusBN (scanner "12"))

--firstBN :: BigNumber -> Int
firstBN (scanner "-12")
firstBN (scanner "12")

--tailBN :: BigNumber -> BigNumber
output (tailBN (scanner "1245"))
output (tailBN (scanner "12"))

--paddBN :: BigNumber -> BigNumber -> BigNumber
output (paddBN (scanner "12") (scanner "123"))
output (paddBN (scanner "12") (scanner "12367"))
output (paddBN (scanner "12") (scanner "1"))

--scannerHelper :: String -> BigNumber
output (scannerHelper "12")

--scanner :: String -> BigNumber
output (scanner "-12")
output (scanner "12")

```

```

--somaPosBN :: BigNumber -> BigNumber -> BigNumber
output (somaPosBN (scanner "1") (scanner "0"))
output (somaPosBN (scanner "1") (scanner "1"))
output (somaPosBN (scanner "0") (scanner "1"))

--somaBN :: BigNumber -> BigNumber -> BigNumber
output (somaBN (scanner "1") (scanner "0"))
output (somaBN (scanner "1") (scanner "1"))
output (somaBN (scanner "0") (scanner "1"))
output (somaBN (scanner "123") (scanner "-123"))
output (somaBN (scanner "-123") (scanner "-123"))
output (somaBN (scanner "-123") (scanner "123"))

--orderedSubPosBN :: BigNumber -> BigNumber -> BigNumber
output (orderedSubPosBN (scanner "1") (scanner "0"))
output (orderedSubPosBN (scanner "1") (scanner "1"))

--lengthBN :: BigNumber -> Int
lengthBN (scanner "1234567890")

--greaterEqualLengthBN :: BigNumber -> BigNumber -> Bool
greaterEqualLengthBN (scanner "9") (scanner "1")
greaterEqualLengthBN (scanner "9") (scanner "9")
greaterEqualLengthBN (scanner "1") (scanner "9")

--greaterBN :: BigNumber -> BigNumber -> Bool
greaterBN (scanner "1") (scanner "10")
greaterBN (scanner "1") (scanner "1")
greaterBN (scanner "10") (scanner "1")

--subBNHelper :: BigNumber -> BigNumber -> BigNumber
output (subBNHelper (scanner "2") (scanner "1"))
output (subBNHelper (scanner "1") (scanner "2"))

--subBN :: BigNumber -> BigNumber -> BigNumber
output (subBN (scanner "1") (scanner "0"))
output (subBN (scanner "1") (scanner "1"))
output (subBN (scanner "0") (scanner "1"))
output (subBN (scanner "123") (scanner "-123"))
output (subBN (scanner "-123") (scanner "-123"))
output (subBN (scanner "-123") (scanner "123"))

--eBN :: BigNumber -> Int -> BigNumber
output ((scanner "1") `eBN` 1)
output ((scanner "-1") `eBN` 2)
output ((scanner "0") `eBN` 100)

--baseMulBN :: BigNumber -> BigNumber -> Int -> Int -> BigNumber
output (baseMulBN (scanner "1") (scanner "3") 0 0)
output (baseMulBN (scanner "9") (scanner "9") 1 0)

--digitOrdBN :: BigNumber -> Int -> BigNumber
digitOrdBN (scanner "123") 0
digitOrdBN (scanner "123") 1

```

```

--mulListBN :: BigNumber -> BigNumber -> [BigNumber]
mulListBN (scanner "20") (scanner "13")

--mulPosBN :: BigNumber -> BigNumber -> BigNumber
output (mulBN (scanner "1") (scanner "1"))
output (mulBN (scanner "1") (scanner "0"))
output (mulBN (scanner "0") (scanner "1"))
output (mulBN (scanner "123") (scanner "123"))

--mulBN :: BigNumber -> BigNumber -> BigNumber
output (mulBN (scanner "1") (scanner "1"))
output (mulBN (scanner "1") (scanner "0"))
output (mulBN (scanner "0") (scanner "1"))
output (mulBN (scanner "123") (scanner "123"))
output (mulBN (scanner "-123") (scanner "123"))
output (mulBN (scanner "123") (scanner "-123"))
output (mulBN (scanner "-123") (scanner "-123"))

--equalsBN :: BigNumber -> BigNumber -> Bool
equalsBN (scanner "1") (scanner "10")
equalsBN (scanner "1") (scanner "1")
equalsBN (scanner "10") (scanner "1")

--greaterOrEqualsBN :: BigNumber -> BigNumber -> Bool
greaterOrEqualsBN (scanner "1") (scanner "10")
greaterOrEqualsBN (scanner "1") (scanner "1")
greaterOrEqualsBN (scanner "10") (scanner "1")

--lessOrEqualsBN :: BigNumber -> BigNumber -> Bool
lessOrEqualsBN (scanner "1") (scanner "10")
lessOrEqualsBN (scanner "1") (scanner "1")
lessOrEqualsBN (scanner "10") (scanner "1")

--naiveDivBN :: BigNumber -> BigNumber -> BigNumber -> (BigNumber, BigNumber)
naiveDivBN (scanner "12") (scanner "3") (scanner "1")
naiveDivBN (scanner "9") (scanner "3") (scanner "1")

--divBN :: BigNumber -> BigNumber -> (BigNumber, BigNumber)
divBN (scanner "10") (scanner "1")
divBN (scanner "10") (scanner "10")
divBN (scanner "10") (scanner "100")
divBN (scanner "77") (scanner "4")

--bigNumberToInt :: BigNumber -> Int
bigNumberToInt (scanner "0")
bigNumberToInt (scanner "10")
bigNumberToInt (scanner "-10")

--safeDivBN :: BigNumber -> BigNumber -> Maybe (BigNumber, BigNumber)
safeDivBN (scanner "1") (scanner "0")
safeDivBN (scanner "1") (scanner "1")

```

# Strategies

## BigNumber

We used a *data* definition instead of a *type* one since it allowed us to recursively define a new type and to use pattern matching. BigNumber is defined like an inversed list where the "head" would be its last digit to facilitate the algorithms we used.

### scanner

Checks and handles whether the argument string represents a positive or negative number. Reverses string to facilitate storing the number in a BigNumber format.

### output

Converts a BigNumber into a string by converting its last digit to its representative character and appending that value to the result of a recursive call to output with the rest of the BigNumber. If a number is negative, a '-' is inserted at the beginning.

### somaBN and subBN

Add or subtract two BigNumbers using a manual addition/subtraction algorithm. Addition starts by adding their last two digits, if value exceeds 9 then a carry happens to the next digit of the first BigNumber (xs). Subtraction subtracts the last digits of the argument BigNumbers, if the value would be negative carry 10 from the last digit of the first BigNumber (xs). Both construct the result with this value and the result of a recursive call. The functions are complimentary and may call each other to handle negative numbers.

### mulBN

Uses a manual multiplication algorithm. Adds the values of a list of simpler multiplications of all permutations of digit pairs between the two BigNumbers with appropriate order of magnitude.

### divBN

Follows manual division algorithm. Use the quotient and remainder as accumulators starting at 0. Take the remainder, multiply it by 10 and add the first digit of the dividend, then naively divide (using multiplication) that value by the quotient and store the results appropriately in the accumulators. Execute recursive calls with the remaining digits of the dividend and the quotient until dividend is Empty, then return a quotient and remainder pair.

## Fibonacci Comparison

We attempted to calculate the first number where overflow would occur for each type.

### Int

We used this code:

```
head (filter (\(x, y) -> x < 0) (zip (map fibListaInfinita [0::Int .. ]) [0 ..]))
```



returning this:

```
(-6246583658587674878,93)
```

Which indicated to us the largest fibonacci number that could be represented as an *Int* was the 92nd, 7540113804746346429.

## Integer and BigNumber

We attempted the same with both *Integer* and *BigNumber*, but this time using a timeout of 1 minute because reaching a result was taking too long. Code:

```
--Integer
timeout (60*10^6) (print (fst (head (filter (\(x, y) -> x < 0) (zip (map
fibListaInfinita [0::Integer ..]) [0 ..] )))))

--BigNumber
lista = zeroBN : [last lista `somaBN` oneBN]

x = head (filter (\(x, y) -> not (x `greaterOrEqualsBN` zeroBN)) (zip (map
fibListaInfinitaBN lista) [0 ..]))

z = timeout (60*10^6) (print (fst x))
```

We arrived at the conclusion that the largest number both of these types can represent is only limited by the available memory in the computer where they are being stored, allowing them to be arbitrarily large.