CALIFORNIA STATE UNIVERSITY FULLERTON

EGEC 180 – Digital Logic and Computer Structures

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Lecture 5: Minterms and Maxterms (2.6)

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Boolean Functions: Minterms and Maxterms

$$A + \overline{A} = B + \overline{B}$$

Why Worry about Minterm and Maxterms

Boolean functions can generally be simplified by using the algebraic techniques. However, two problems arise when algebraic procedures are used:

1. **The procedures are difficult to apply in a systematic way.**
2. **It is difficult to tell when you have arrived at a minimum solution.**

The Karnaugh map method is generally faster and easier to apply than other simplification methods.

Karnaugh Maps

	b'c'	b'c	bc	bc'
a'	m_0	m_1	m_3	m_2
a	m_4	m_5	m_7	m_6

Want to combine the largest number of cells possible to eliminate as many variables as possible, implies that:

Combine 2 cells, eliminate 1 variable
Combine 4 cells, eliminate 2 variables
Combine 2^n, eliminate n variables

a'b'c'+a'b'c
a'b'(c'+c)
a'b'(1)
a'b'

	b'c'	b'c	bc	bc'
a'	m_0	m_1	m_3	m_2
a	m_4	m_5	m_7	m_6

a'bc+a'bc'+abc+abc'
a'(bc+bc')+a(bc+bc')
(a'+a)(bc'+bc)
b (c+c')
b

Minterm and Maxterm Expansions

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

Each of the terms in the above equation is referred to as a minterm. In general, a *minterm* of *n* variables is a product of *n literals* (A, A', B, B', C, and C') in which each variable appears exactly once in either true or complemented form, but not both.

In General a *literal* is a variable or its complement

General Truth Table for Three Variables

Table represents a truth table for a general function of three variables. Each a_i is a constant with a value of 0 or 1.

A B C	F
0 0 0	a_0
0 0 1	a_1
0 1 0	a_2
0 1 1	a_3
1 0 0	a_4
1 0 1	a_5
1 1 0	a_6
1 1 1	a_7

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \cdots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i$$

General Minterm and Maxterm Expansions

We can write the minterm expansion for a general function is as follows; $a_i = 1$ or 0: (Don't forget when $a_i = 0$, a term to dropout since $0.m_i = 0$)

$$F = a_0m_0 + a_1m_1 + a_2m_2 + \cdots + a_7m_7 = \sum_{i=0}^{n} a_i m_i \qquad (4\text{-}12)$$

$$m_0 \Leftrightarrow A'B' or\ A'B'C' or\ A'B'C'D', \quad \cdots \quad , \quad m_n \Leftrightarrow AB or\ ABC or\ ABCD$$

The maxterm expansion for a general function variables is; $a_i = 1$ or 0: (Don't forget when $a_i = 1$, a term to dropout since $1+m_i = 1$ and $1.(\,..) = (\,..\,)$)

$$F = (a_0 + M_0)(a_1 + M_1)(a_2 + M_2) \cdots (a_7 + M_7) = \prod_{i=0}^{n} (a_i + M_i) \qquad (4\text{-}13)$$

$$M_0 \Leftrightarrow A' + B'\ or\ \ A' + B' + C'\ \ or\ \ A' + B' + C' + D', \qquad \cdots$$

$$M_n \Leftrightarrow A + B\ or\ A + B + C\ or\ A + B + C + D$$

Minterms and Maxterms for Three Variables

Row No.	$A\ B\ C$	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Input **Output: Minterms & Maxterms**

minterm Expansions

Minterm expansion for a function is unique. Look at the equation below, it can be rewritten in terms of m-notation as:

A	B	C	F	Term	Coeff	Expansion
0	0	0	0	m_0	a_0=0	0.A'.B'.C' = 0
0	0	1	0	m_1	a_1=0	0.A'.B'.C = 0
0	1	0	0	m_2	a_2=0	0.A'.B.C = 0
0	1	1	1	m_3	a_3=1	1.A'.B.C = A'.B.C
1	0	0	1	m_4	a_4=1	1.A.B'.C' = A.B'.C'
1	0	1	1	m_5	a_5=1	1.A.B'.C = A.B'.C
1	1	0	1	m_6	a_6=1	1.A.B.C' = A.B.C'
1	1	1	1	m_7	a_7=1	1.A.B.C = A.B.C

$$f(A, B, C) = 0 + 0 + 0 + A'BC + AB'C' + AB'C + ABC' + ABC$$

OR

$$f(A, B, C) = m_3 + m_4 + m_5 + m_6 + m_7$$

$$f(A, B, C) = \sum m(3, 4, 5, 6, 7)$$

Minterm Expansions

Find the *minterm* expansion of *F(a,b,c,d) = a'(b' + d) + acd' =* $\underbrace{a'b'}$ + $\underbrace{a'd'}$ + $\underbrace{acd'}$.

These are not *minterms* since a *minterm* should have 4 literals

A	B	C	D	*F(a, b, c, d)*	Terms
0	0	0	0	1+1	m_0
0	0	0	1	1	m_1
0	0	1	0	1+1	m_2
0	0	1	1	1	m_3
0	1	0	0	1	m_4
0	1	0	1	0	M_5
0	1	1	0	1	m_6
0	1	1	1	0	M_7
1	0	0	0	0	M_8
1	0	0	1	0	M_9
1	0	1	0	1	m_{10}
1	0	1	1	0	M_{11}
1	1	0	0	0	M_{12}
1	1	0	1	0	M_{13}
1	1	1	0	1	m_{14}
1	1	1	1	0	M_{15}

$$F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 6, 10, 11)$$

F(A, B, C, D) = A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + A'BC'D' + A'BCD' + AB'CD' + ABCD'

Maxterm Expansions

Maxterm expansion for a function is unique. Look at the equation below, it can be rewritten in terms of m-notation as:

A	B	C	F	Term	Coeff	Expansion
0	0	0	0	M_0	$a_0=0$	0+(A+B+C) = A+B+C
0	0	1	0	M_1	$a_1=0$	0+(A+B+C’) = A+B+C’
0	1	0	0	M_2	$a_2=0$	0+(A+B’+C) = A+B’+C
0	1	1	1	M_3	$a_3=1$	1+(A+B’+C’) = 1
1	0	0	1	M_4	$a_4=1$	1+(A’+B+C) = 1
1	0	1	1	M_5	$a_5=1$	1+(A’+B+C’) = 1
1	1	0	1	M_6	$a_6=1$	1+(A’+B’+C) = 1
1	1	1	1	M_7	$a_7=1$	1+(A’+B’+C’) = 1

$$F(A, B, C) = (A+B+C)(A+B+C')(A+B'+C)(1)(1)(1)(1)(1)$$
$$= (A+B+C)(A+B+C')(A+B'+C)$$

OR

$$F(A, B, C) = M_0 M_1 M_2 \quad F(A, B, C) = \prod M(0, 1, 2)$$

Minterm Expansions

Find the *minterm* expansion of *F(a,b,c,d) = a'(b' + d') + acd' =* $\underbrace{a'b'}$ + $\underbrace{a'd'}$ + $\underbrace{acd'}$.

A	B	C	D	*F(a, b, c, d)*	Terms
0	0	0	0	1+1	m_0
0	0	0	1	1	m_1
0	0	1	0	1+1	m_2
0	0	1	1	1	m_3
0	1	0	0	1	m_4
0	1	0	1	0	M_5
0	1	1	0	1	m_6
0	1	1	1	0	M_7
1	0	0	0	0	M_8
1	0	0	1	0	M_9
1	0	1	0	1	m_{10}
1	0	1	1	0	M_{11}
1	1	0	0	0	M_{12}
1	1	0	1	0	M_{13}
1	1	1	0	1	m_{14}
1	1	1	1	0	M_{15}

These are not *maxterms* since a *maxterm* should have 4 literals and be product of sums

$$F(A, B, C, D) = \prod M(5, 7, 8, 9, 11, 12, 13, 15)$$

$$F(A, B, C, D) = (A+B'+C+D')(A+B'+C'+D')(A'+B+C+D)(A'+B+C+D')(A'+B+C'+D')(A'+B'+C+D)(A'+B'+C+D')(A'+B'+C'+D')$$

Note: Since we know

$$F(A, B, C, D) = \sum m(0, 1, 2, 3, 4, 6, 10, 11)$$

We can write,

$$F(A, B, C, D) = \prod M(5, 7, 8, 9, 11, 12, 13, 15)$$

Moving Between

Definition: Any Boolean function that is expressed as a sum of minterms or as a product of maxterms is said to be in its **canonical form**.

To convert from one canonical form to its other **equivalent** form, interchange the symbols Σ and Π, and list the index numbers that were excluded from the original form.

To convert from one canonical form to its **dual**, interchange the symbols Σ and Π, and list the index numbers from the original form, or use De Morgan's Law or the duality principle.

Ex. $F = m_3 + m_5 + m_6 + m_7 = \Sigma(3, 5, 6, 7)$
$= x' y z + x y' z + x y z' + x y z$

$= M_0 \bullet M_1 \bullet M_2 \bullet M_4 = \Pi(0, 1, 2, 4)$
$= (x+y+z) \bullet (x+y+z') \bullet (x+y'+z) \bullet (x'+y+z)$

$F' = m_0 + m_1 + m_2 + m_4 = \Sigma(0, 1, 2, 4)$
$= x' y' z' + x' y' z + x' y z' + x y' z'$

$= M_3 \bullet M_5 \bullet M_6 \bullet M_7 = \Pi(3, 5, 6, 7)$
$= (x+y'+z') \bullet (x'+y+z') \bullet (x'+y'+z) \bullet (x'+y'+z')$

Σ 1-minterms
equivalent
Π 0-maxterms
inverse
duals
Σ 0-minterms
equivalent
Π 1-maxterms

http://www.cs.ucr.edu/~ehwang/courses/cs120a/minterms.pdf

Conversion of Forms

Summarizes the procedures for conversion between minterm and maxterm expansions of F and F′

GIVEN FORM		DESIRED FORM: Minterm Expansion of F	DESIRED FORM: Maxterm Expansion of F	DESIRED FORM: Minterm Expansion of F'	DESIRED FORM: Maxterm Expansion of F'
GIVEN FORM	Minterm Expansion of F	————	maxterm nos. are those nos. not on the minterm list for F	list minterms not present in F	maxterm nos. are the same as minterm nos. of F
GIVEN FORM	Maxterm Expansion of F	minterm nos. are those nos. not on the maxterm list for F	————	minterm nos. are the same as maxterm nos. of F	list maxterms not present in F

Example

		DESIRED FORM		
	Minterm Expansion of f	Maxterm Expansion of f	Minterm Expansion of f'	Maxterm Expansion of f'
GIVEN FORM $f = \Sigma m(3, 4, 5, 6, 7)$	______________	$\Pi M(0, 1, 2)$	$\Sigma m(0, 1, 2)$	$\Pi M(3, 4, 5, 6, 7)$
GIVEN FORM $f = \Pi M(0, 1, 2)$	$\Sigma m(3, 4, 5, 6, 7)$	______________	$\Sigma m(0, 1, 2)$	$\Pi M(3, 4, 5, 6, 7)$

Minterm Order

A	C	B	D	Terms
0	0	0	0	m_0
0	0	0	1	m_1
0	**0**	**1**	0	m_4
0	**0**	**1**	1	m_5
0	**1**	**0**	0	m_2
0	1	0	1	m_3
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{12}
1	0	1	1	m_{13}
1	1	0	0	m_{10}
1	1	0	1	m_{11}
1	1	1	0	m_{14}
1	1	1	1	m_{15}

A	B	C	D
0	0	0	0
0	0	0	1
0	**1**	**0**	0
0	**1**	**0**	1
0	**0**	**1**	0
0	0	1	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

EGCP180 convention:

Minterms are assigned based on the A, B, C, D column order only

If the equation is given as F(A, C, B, D) it is easier to lay the table out as you see on the left,

However, the minterms are assigned based on the table on the right.

Q&A