EGCP 180: -01/02 – Digital Logic and Computer Structures

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Lecture 1: Number Systems

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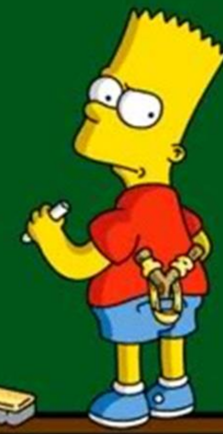
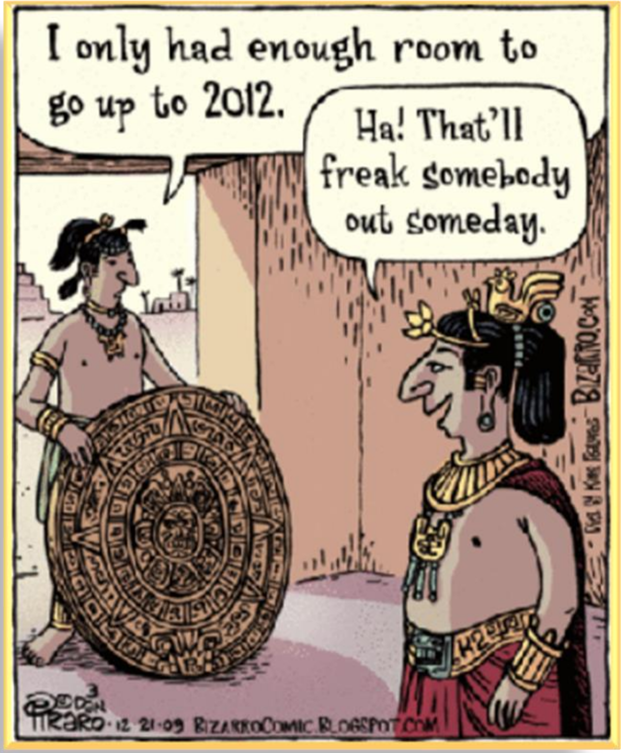
Binary Number Conversions



Binary Number Conversions

- The primary number systems used in digital systems are decimal (base-10), binary (base-2), octal (base-8) and hexadecimal (base-16)
- Octal and Hexadecimal number systems are used to represent binary numbers in a more compact form



Review of Number Systems

Binary

- **Consists of two possible values (coefficients):**
  0 and 1
- **Coefficients are multiplied by powers of 2**

Octal

- **Consists of eight possible values (coefficients):**
  0, 1, 2, 3, 4, 5, 6, and 7
- **Coefficients are multiplied by powers of 8**
- **Octal system is of base 8 or radix 8**

Hexadecimal

- **Consists of sixteen possible values (coefficients):**

0,1, 2, 3, 4, 5, 6, 7, 8, 9, A [$(10)_{10}$], B [$(11)_{10}$], C [$(12)_{10}$], D [$(13)_{10}$], E [$(14)_{10}$], F [$(15)_{10}$]

- **Coefficients are multiplied by powers of 16**
- **Hexadecimal system is of base 16 or radix 16**

Binary, Octal and Hexadecimal

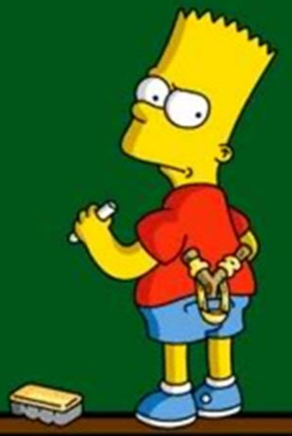
NOTE: binary, octal and hex numbers are all powers of 2

- $2^3 = 8$: each octal digit corresponds to 3 binary digits; with 3 binary digits we can represent 8 combinations; 000 to 111, 0 to 7 decimal

- $2^4 = 16$: each hexadecimal digit corresponds to 4 binary digits; with 4 binary digits we can represent 16 combinations; 0000 to 1111, 0 to F (15) decimal

Octal and Hex are easier to read and more compact to display than Binary

Number Base Conversions



Decimal Number to Base "r"

We will use an Algorithm: efficient procedure (sequence of steps), recipe

Decimal to any base: $(\text{number})_{10} \Rightarrow (\text{number})_r$

1. Separate number into an integer part and a fraction part
2. Integer portion is repeatedly divided by the base to which we are converting. NOTE: remainders are the appropriate coefficients
3. Fractional part is repeatedly multiplied by the base to which we are converting. NOTE: integer portions of resulting numbers are the appropriate coefficients
4. Combine the resulting answers from 2 and 3

EXAMPLE: Convert $(25.375)_{10}$ to base 2 (binary)

Integer = 25   Fraction = 0.375

Integer part:

25/2 = 12 + 1 (least significant bit - lsb)
12/2 = 6 + 0
6/2 = 3 + 0
3/2 = 1 + 1
1/2 = 0 + 1 (most significant bit - msb)

Read back in this direction

$(25)_{10} = (11001)_2$

EXAMPLE: Convert $(25.375)_{10}$ to base 2 (binary)

Integer = 25 Fraction = 0.375

Fraction part:

.375 * 2 = .75 + 0 (most significant bit - msb)
.75 * 2 = .5 + 1
.5 * 2 = 0 + 1 (least significant bit - lsb)

Read back in this direction

$(0.375)_{10} = (011)_2$

Result: $(25.375)_{10} = (11001.011)_2$

EXAMPLE: Convert 53_{10} to binary.

(Practice for Quiz #1)

$2\,\overline{)53}$	
$2\,\overline{)26}$	rem. = 1
$2\,\overline{)13}$	rem. = 0
$2\,\overline{)6}$	rem. = 1
$2\,\overline{)3}$	rem. = 0
$2\,\overline{)1}$	rem. = 1
0	rem. = 1

Read back in this direction

$$53_{10} = 110101_2$$

EX: Convert $.625_{10}$ to binary.

(Practice for Quiz #1)

$$\begin{array}{rr} F_0 = & .625 \\ \times & 2 \\ \hline & 1.250 \end{array} \qquad \begin{array}{rr} F_1 = & .250 \\ \times & 2 \\ \hline & 0.500 \end{array} \qquad \begin{array}{rr} F_2 = & .500 \\ \times & 2 \\ \hline & 1.000 \end{array}$$

$$.625_{10} = .101_2$$

EXAMPLE: Convert 0.7_{10} to binary.

(Practice for Quiz #1)

$$
\begin{array}{r}
.7 \\
\underline{\quad 2} \\
(1).4 \\
\underline{\quad 2} \\
(0).8 \\
\underline{\quad 2} \\
(1).6 \\
\underline{\quad 2} \\
(1).2 \\
\underline{\quad 2} \\
(0).4 \\
\underline{\quad 2} \\
(0).8
\end{array}
$$

$\longleftarrow$ process starts repeating here because 0.4 was previously obtained

$0.7_{10} = 0.1\ \underline{0110}\ \underline{0110}\ \underline{0110}\ \ldots_2$

EXAMPLE: Convert $(720.75)_{10}$ to base 8 (octal)

Integer = 720 **Fraction = 0.75**

Integer part:

720/8 = 90 + 0 (least significant bit - lsd)
90/8 = 11 + 2
11/8 = 1 + 3
1/8 = 0 + 1 (most significant bit - msd)

Read back in this direction

$(720)_{10} = (1320)_8$

EXAMPLE: Convert $(720.75)_{10}$ to base 8 (octal)

Integer = 720   Fraction = 0.75

Fraction part:

.75 * 8 = 6.0 6 (most significant digit - msd) and (least significant digit - lsd)

$(0.75)_{10} = (0.6)_8$

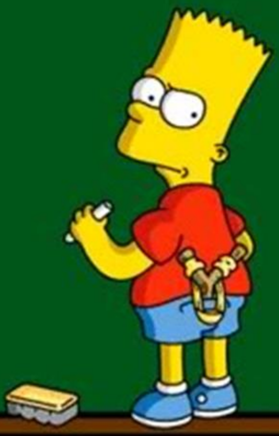
Result: $(720.75)_{10} = (1320.6)_8$

EXAMPLE: Convert 231.3_4 to base 7.

$$231.3_4 = 2 \times 16 + 3 \times 4 + 1 + \tfrac{3}{4} = 45.75_{10}$$

$$\begin{array}{lll}
7\,\underline{/45} & & \\
7\,\underline{/\;\;6} & \text{rem. } 3 & \\
\;\;\;\;\;\;0 & \text{rem. } 6 &
\end{array}
\qquad
\begin{array}{r}
.75 \\
\underline{7} \\
(5)\ .25 \\
\underline{7} \\
(1)\ .75 \\
\underline{7} \\
(5)\ .25 \\
\underline{7} \\
(1)\ .75
\end{array}
\qquad
45.75_{10} = 63.5151\ldots_7$$

Binary, Octal and Hexadecimal Conversions



How to Convert Between Binary, Octal and Hexadecimal

Conversion of base is done by partitioning

- If needed add zeros to extreme left when converting the integer portion
- Add zeros to extreme right when converting fractional part
- Binary to Octal
  - **Partition binary number into groups of three digits starting from binary point**
- Octal to Binary
  - **Each octal digit is converted to its 3-digit binary equivalent**
- Binary to Hexadecimal
  - **Partition binary number into groups of four digits starting from binary point**
- Hexadecimal to Binary
  - **Each hex digit is converted to its 4-digit binary equivalent**

How to Convert Between Binary, Octal and Hexadecimal

Binary to Octal

- **Partition binary number into groups of three digits starting from binary point**

$$(101100.01101)_2$$

$$\underbrace{\underbrace{101}_{5}\quad\underbrace{100}_{4}\quad.\quad\underbrace{011}_{3}\quad\underbrace{010}_{2}}_{(54.32)_8}$$

Octal to Binary

- **Each octal digit is converted to its 3-digit binary equivalent**

$$\overbrace{\overbrace{3}^{011}\quad.\quad\overbrace{1}^{001}}^{(3.1)_8}$$

$$011\quad.\quad001$$

$$(011.001)_2$$

How to Convert Between Binary, Octal and Hexadecimal

Binary to Hexadecimal

- **Partition binary number into groups of four digits starting from binary point**

$$(111010.011)_2$$

$$\underbrace{\underbrace{0011}_{3}\quad\underbrace{1010}_{A}\quad.\quad\underbrace{0110}_{6}}_{(3A.6)_{16}}$$

Hexadecimal to Binary

- **Each hex digit is converted to its 4-digit binary equivalent**

$$\overbrace{\overbrace{0110}^{6}\quad.\quad\overbrace{1101}^{D}}^{(6.D)_{16}}$$

$$(0110.1101)_2$$

Binary ⇔ Hexadecimal Conversion

$$(1001101.010111)_2$$

$$\underbrace{\underbrace{0100}_{4}\;\underbrace{1101}_{D}\;.\;\underbrace{0101}_{5}\;\underbrace{1100}_{C}}_{(4D.5C)_{16}}$$

Conversion from binary to hexadecimal (and conversely) can be done by inspection because each hexadecimal digit corresponds to exactly four binary digits (bits).

How about conversion between Octal and Hexadecimal and vice versa?

EXAMPLE: Convert $(31.7)_8$ to Hex.

(Practice for Quiz #1)

$$(31.7)_8$$

$$\underbrace{011}_{3}\qquad \underbrace{001}_{1}\quad . \quad \underbrace{111}_{7}\qquad \text{Binary}$$

$$\underbrace{0001}_{1}\qquad \underbrace{1001}_{9}\quad . \quad \underbrace{1110}_{E}\qquad \text{Group by 4}$$

$$\underbrace{1\qquad 9\quad . \quad E}_{(19.E)_{16}}$$

EXAMPLE: Convert $(AB.6)_{16}$ to octal.

(Practice for Quiz #1)

$$(AB.6)_{16}$$

$$\underbrace{1010}_{A} \quad \underbrace{1011}_{B} \quad . \quad \underbrace{0110}_{6} \qquad \text{Binary}$$

$$\underbrace{\underbrace{010}_{2} \quad \underbrace{101}_{5} \quad \underbrace{011}_{3} \quad . \quad \underbrace{011}_{3}}_{(253.3)_8} \qquad \text{Group by 3}$$

Q&A