CALIFORNIA STATE UNIVERSITY FULLERTON™

EGEC 180 – Digital Logic and Computer Structures

Spring 2024

Lecture 6: K-Map(2.7)

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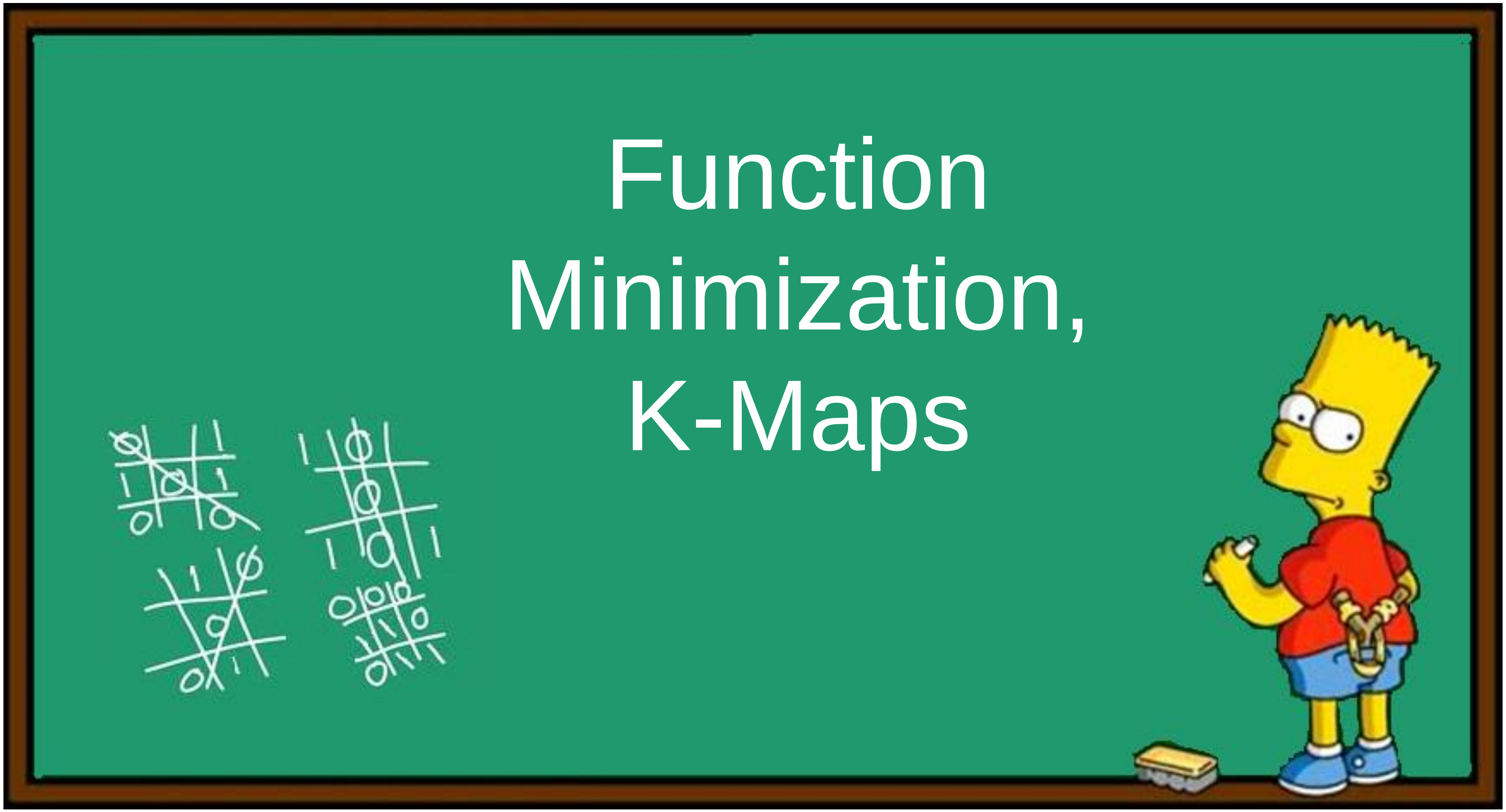
Function Minimization, K-Maps



Karnaugh Maps

Switching/Boolean functions can generally be simplified by using the algebraic techniques. However, two problems arise when algebraic procedures are used:

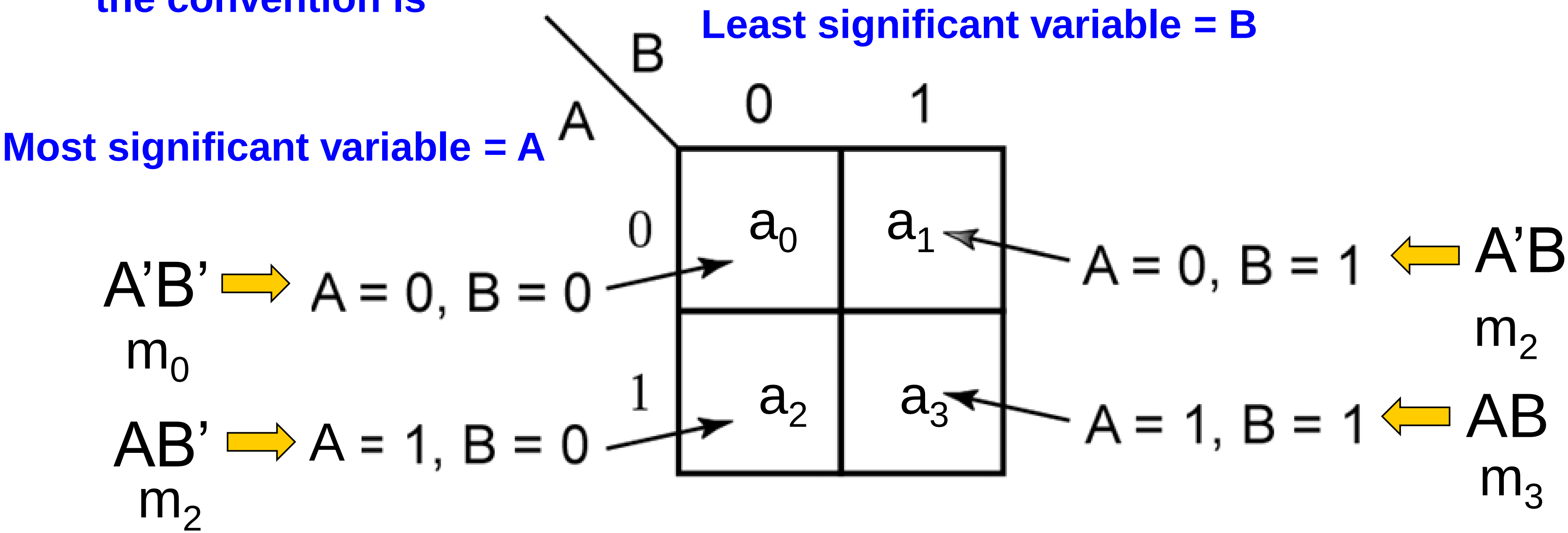
1. The procedures are difficult to apply in a systematic way.
2. It is difficult to tell when you have arrived at a minimum solution.

The Karnaugh map method is generally faster and easier to apply than other simplification methods.

Two- Variable Karnaugh Maps

Just like a truth table, the Karnaugh map of a function specifies the value of the function for every combination of values of the independent variables.

ORDER Does not matter BUT the convention is



Example: Two – Variable K-map

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

A \ B	0	1
0	a_0	a_1
1	a_2	a_3

A \ B	0	1
0	$a_0 = 1$	$a_2 = 0$
1	$a_1 = 1$	$a_3 = 0$

A \ B	0	1
0	1	0
1	1	0

A'B'

AB'

F(A,B) = A'B'+AB'

Why it works

B'(A'+A)

B'(1)

B'

A \ B	0	1
0	1	0
1	1	0

Since A changes and B' does not the answer is B'

Karnaugh Maps
(2 – Variable MINTERM Rules)

A \ B	0	1
0	1	0
1	1	0

F(A,B) = A'B'+AB' = (A'+A)B' = B'

A \ B	0	1
0	1	1
1	0	0

F(A,B) = A'B'+A'B = A'(B'+B) = A'

A \ B	0	1
0	0	1
1	0	1

F(A,B) = A'B+AB = (A'+A)B = B

A \ B	0	1
0	0	0
1	1	1

F(A,B) = AB'+AB = A(B'+B) = A

How to Do 2 Groups

- Now suppose F(A,B) = A’B+A’B’ +AB, the Karnaugh Map looks like

A \ B	0	1
0	1	1
1	0	1

Since A’ does not change in the RED terms but B does then A’ remains and B is eliminated.

Since B does not change in the BLUE terms but A does then B remains and A is eliminated.

$$
\begin{aligned}
F(A,B) &= A'B' + A'B + AB \\
&= A'B' + A'B + A'B + AB \\
&= A'(B' + B) + (A' + A)B \\
&= A'(1) + (1)B \\
&= A' + B
\end{aligned}
$$

So,

F(A,B) = A’B+A’B’ +AB = A’ + B

Rules for Combining Squares (Simplification, Elimination)

1. Can combine 2^k adjacent cells; where $0 \leq k \leq n$ and n is the number of variables; implies that
   - Can combine 1, 2, 4, 8, 16, etc, cells; a power of 2
   - Cannot combine 3, 5, 6, 7, 9, etc cells

2. Want to combine the largest number of cells possible to eliminate as many variables as possible, implies that:
   - Combine 2 cells, eliminate 1 variable
   - Combine 4 cells, eliminate 2 variables
   - Combine 8 cells, eliminate 3 variables
   - Combine 2^n, eliminate n variables

3. Once a minterm (maxterm) is used we can use it again

4. Avoid redundancy!!!

Karnaugh Maps
(2 – Variable MAXTERM Rules)

A \ B	0	1
0	1	0
1	1	0

$$
\begin{aligned}
F(A,B) &= (A+B')(A'+B') \\
&= AA'+AB'+B'A'+B'B' \\
&= 0 + AB' + B'A' + 0 \\
&= AB' + B'A' \\
&= (A + A')B' \\
&= (1)B' \\
&= B'
\end{aligned}
$$

A \ B	0	1
0	1	1
1	0	0

$F(A,B) = A'$

A \ B	0	1
0	0	1
1	0	1

$F(A,B) = B$

A \ B	0	1
0	0	0
1	1	1

$F(A,B) = A$

Any 2 maxterms in adjacent squares that are ANDed will cause removal of the different variable

Looking Back

- Now suppose F(A,B) = A’B+A’B’ +AB, the Karnaugh Map looks like

A \ B	0	1
0	1	1
1	$\boxed{0}$	1

$$F(A,B) = A'B' + A'B + AB$$
$$= A'B' + A'B + A'B + AB$$
$$= A'(B' + B) + (A' + A)B$$
$$= A'(1) + (1)B$$
$$= A' + B$$

So,

$$F(A,B) = M_2 = A' + B$$

Some times it easier to work the maxterms than the minterms and vice versa

Examples: 2 – Variable K-Maps

EX: simplify the function

X \ Y	0	1
0	0	1
1	1	0

F(X,Y) = XY’ + X’ Y ⇨ **XOR gate**

These 2 configurations that result in selecting the 2 minterms by themselves.

These Represent an

X \ Y	0	1
0	1	0
1	0	1

F(X,Y) = X’Y + XY’ ⇨ **XNOR gate**

Practice Problem

- Simplify the function $F(X,Y) = X'Y' + XY' + X'Y$

X \ Y	0	1
0	1	1
1	1	0

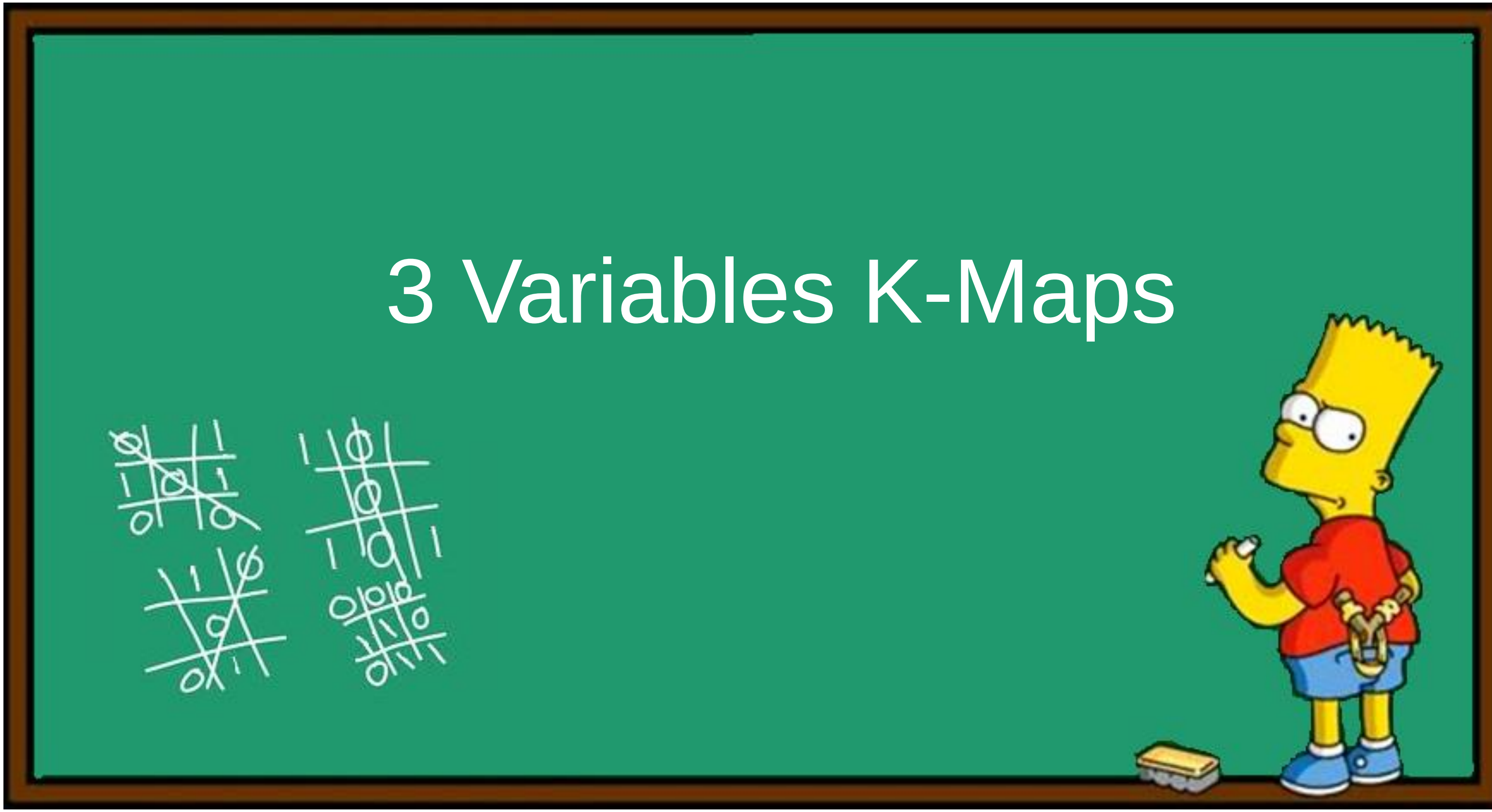
Since X' does not change in the RED terms but Y does then X' remains and Y is eliminated.

Since Y' does not change in the BLUE terms but X does then Y' remains and X is eliminated.

$$
\begin{aligned}
F(X,Y) &= X'Y' + X'Y + XY' \\
&= X'Y' + X'Y + XY' + X'Y' \\
&= X'(Y' + Y) + (X + X')Y' \\
&= X'(1) + (1)Y' \\
&= X' + Y'
\end{aligned}
$$

So,

$$
\begin{aligned}
F(X, Y) &= X'Y' + XY' + X'Y \\
&= X'+Y'
\end{aligned}
$$



3 Variables K-Maps

Three- Variable Karnaugh Maps

A \ BC	00	01	11	10
0	a_0	a_1	a_3	a_2
1	a_4	a_5	a_7	a_6

As with the 2 variable K-MAP the truth table is moved mapped into the table as shown

A	B	C	F	Term	Coeff
0	0	0	0	m_0	a_0
0	0	1	0	m_1	a_1
0	1	0	0	m_2	a_2
0	1	1	1	m_3	a_3
1	0	0	1	m_4	a_4
1	0	1	1	m_5	a_5
1	1	0	1	m_6	a_6
1	1	1	1	m_7	a_7

Gray Code

A	B	C	Terms
0	0	0	m_0
0	0	1	m_1
0	1	1	m_3
0	1	0	m_2
1	1	0	m_6
1	1	1	m_7
1	0	1	m_5
1	0	0	m_4

The table uses a gray code for the columns so that the Absorbtion property holds

Groupings of 2

So a group of 2 variables across to adjacent columns eliminates the variable that changes between columns.

A'B' A'B

A \ BC	00	01	11	10
0	a_0	a_1	a_3	a_2
1	a_4	a_5	a_7	a_6

AB' AB

A'C AC'

A \ BC	00	01	11	10
0	a_0	a_1	a_3	a_2
1	a_4	a_5	a_7	a_6

AC AC'

Q&A