EGEC 281: Designing with VHDL

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Lecture 4: Boolean Functions- Minterms and Maxterms

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Canonical or Standard Forms

- A binary variable may be in its normal form, X, or on its complement form, X' .
- Minterm or Standard Product (chase the 1s):
  - A product term in which all variables appear exactly once in either true or complemented form
  - Product term is derived from a truth table for which the function is equal to 1
  - Each minterm is obtained from an AND operation of all variables, with each variable complemented if corresponding bit of the binary number is 0 and uncomplemented if it is a 1
  - For n variables, there is 2^n distinct minterms (0 to [2^n -1])
  - Symbol for minterms is m_j; j is the decimal equivalent of the binary number

Canonical or Standard Forms

- A binary variable may be in its normal form, X, or on its complement form, X'.
- Maxterm or Standard Sum (chase the 0s):
  - Each sum term is obtained from an OR operation of all the variables, with each variable being complemented if the corresponding bit is 1 and uncomplemented if 0
  - For n variables, there is 2^n distinct maxterms ($0 – [2^n - 1]$)
  - Symbol for a maxterm is M_j , j is the decimal equivalent of the binary number
  - All variables appear exactly once in either true or complemented form

Canonical or Standard Forms
One variable

A	Function	Minterm	Maxterm
0		m0 – A′	M0 – A
1		m1 – A	M1 – A'

Each minterm is the complement of corresponding maxterm and vice versa

Canonical or Standard Forms
Two variables

A	B	Function	Minterm	Maxterm
0	0		m0 - A'B'	M0 - A+B
0	1		m1 - A'B	M1 - A+B'
1	0		m2 - AB'	M2 - A'+B
1	1		m3 - AB	M3 - A'+B'

Each maxterm is the complement of corresponding minterm and vice versa

Ex: 2 variables A and B

For 2 variables $m_2 = AB'$

$m_2' = (AB')' = A' + B \longrightarrow M_2$

Each maxterm is the complement of corresponding minterm and vice versa

Canonical or Standard Forms
Two variables

A	B	Function	Minterm	Maxterm
0	0	1	m0 - A'B'	M0 - A+B
0	1	0	m1 - A'B	M1 - A+B'
1	0	0	m2 - AB'	M2 - A'+B
1	1	1	m3 - AB	M3 - A'+B'

XNOR

Sum of minterns, denotes ORing:

$F = A'B' + AB = m_0 + m_3 = \sum m(0,3)$

Complement of F:

$F' = A'B + AB' = m_1 + m_2 = \sum m(1,2)$

Canonical or Standard Forms Two variables

A	B	Function	Minterm	Maxterm
0	0	1	m0 - A'B'	M0 - A+B
0	1	0	m1 - A'B	M1 - A+B'
1	0	0	m2 - AB'	M2 - A'+B
1	1	1	m3 - AB	M3 - A'+B'

XNOR

Product of maxterms, denotes ANDing:

F = (A + B’) (A’ + B)= M_1 M_2 = Õ $M(1,2)$

Complement of F:

F’ = (A + B)(A’ + B’) = M_0 M_3 = Õ $M(0,3)$

Canonical or Standard Forms
Two variables

A	B	Function	Minterm	Maxterm
0	0	1	m0 - A'B'	M0 - A+B
0	1	0	m1 - A'B	M1 - A+B'
1	0	0	m2 - AB'	M2 - A'+B
1	1	1	m3 - AB	M3 - A'+B'

XNOR

NOTE: $m_j' = M_j$ and vice versa

Canonical or Standard Forms Three variables

A	B	C	Function	Minterm	Maxterm
0	0	0		m0 - A'B'C'	M0 - A+B+C
0	0	1		m1 - A'B'C	M1 - A+B+C'
0	1	0		m2 - A'BC'	M2 - A+B'+C
0	1	1		m3 - A'BC	M3 - A+B'+C'
1	0	0		m4 - AB'C'	M4 - A'+B+C
1	0	1		m5 - AB'C	M5 - A'+B+C'
1	1	0		m6 - ABC'	M6 - A'+B'+C
1	1	1		m7 - ABC	M7 - A'+B'+C'

Canonical or Standard Forms Four variables

A	B	C	D	Function	Minterm	Maxterm
0	0	0	0		m0 - A'B'C'D'	M0 - A+B+C+D
0	0	0	1		m1 - A'B'C'D	M1 - A+B+C+D'
0	0	1	0		m2 - A'B'CD'	M2 - A+B+C'+D
0	0	1	1		m3 - A'B'CD	M3 - A+B+C'+D'
0	1	0	0		m4 - A'BC'D'	M4 - A+B'+C+D
0	1	0	1		m5 - A'BC'D	M5 - A+B'+C+D'
0	1	1	0		m6 - A'BCD'	M6 - A+B'+C'+D
0	1	1	1		m7 - A'BCD	M7 - A+B'+C'+D'
1	0	0	0		m8 - AB'C'D'	M8 - A'+B+C+D
1	0	0	1		m9 - AB'C'D	M9 - A'+B+C+D'
1	0	1	0		m10 -AB'CD'	M10 - A'+B+C'+D
1	0	1	1		m11 - AB'CD	M11 - A'+B+C'+D'
1	1	0	0		m12 - ABC'D'	M12 - A'+B'+C+D
1	1	0	1		m13 - ABC'D	M13 - A'+B'+C+D'
1	1	1	0		m14 - ABCD'	M14 -A'+B'+C'+D
1	1	1	1		m15 - ABCD	M15 - A'+B'+C'+D'

Canonical or Standard Forms
Five variables

A	B	C	D	E	Function	Minterm	Maxterm
0	0	0	0	0		m0 - A'B'C'D'E'	M0 - A+B+C+D+E
0	0	0	0	1		m1 - A'B'C'D'E	M1 - A+B+C+D+E'
0	0	0	1	0		m2 - A'B'C'DE'	M2 - A+B+C'+D'+E
0	0	0	1	1		m3 - A'B'CD	M3 - A+B+C'+D'
0	0	1	0	0		m4 - A'B'C'D'	M4 - A+B'+C+D
0	0	1	0	1		m5 - A'BC'D	M5 - A+B'+C+D'
0	0	1	1	0		m6 - A'BCD'	M6 - A+B'+C'+D
0	0	1	1	1		m7 - A'BCD	M7 - A+B'+C'+D'
0	1	0	0	0		m8 - AB'C'D'	M8 - A'+B+C+D
0	1	0	0	1		m9 - AB'C'D	M9 - A'+B+C+D'
0	1	0	1	0		m10 - AB'CD'	M10 - A'+B+C'+D
0	1	0	1	1		m11 - AB'CD	M11 - A'+B+C'+D'
0	1	1	0	0		m12 - ABC'D'	M12 - A'+B'+C+D
0	1	1	0	1		m13 - ABC'D	M13 - A'+B'+C+D'
0	1	1	1	0		m14 - ABCD'	M14 - A'+B'+C'+D
0	1	1	1	1		m15 - ABCD	M15 - A'+B'+C'+D'
1	0	0	0	0		m0 - A'B'C'D'	M0 - A+B+C+D
1	0	0	0	1		m1 - A'B'C'D	M1 - A+B+C+D'
1	0	0	1	0		m2 - A'B'CD'	M2 - A+B+C'+D
1	0	0	1	1		m3 - A'B'CD	M3 - A+B+C'+D'
1	0	1	0	0		m4 - A'B'C'D'	M4 - A+B'+C+D
1	0	1	0	1		m5 - A'BC'D	M5 - A+B'+C+D'
1	0	1	1	0		m6 - A'BCD'	M6 - A+B'+C'+D
1	0	1	1	1		m7 - A'BCD	M7 - A+B'+C'+D'
1	1	0	0	0		m8 - AB'C'D'	M8 - A'+B+C+D
1	1	0	0	1		m9 - AB'C'D	M9 - A'+B+C+D'
1	1	0	1	0		m10 - AB'CD'	M10 - A'+B+C'+D
1	1	0	1	1		m11 - AB'CD	M11 - A'+B+C'+D'
1	1	1	0	0		m12 - ABC'D'	M12 - A'+B'+C+D
1	1	1	0	1		m13 - ABC'D	M13 - A'+B'+C+D'
1	1	1	1	0		m14 - ABCD'	M14 - A'+B'+C'+D
1	1	1	1	1		m15 - ABCDE	M15 - A'+B'+C'+D'+E'

Complete/Correct the Truth Table

Canonical or Standard Forms

NOTE: Boolean Functions are said to be in "Canonical Form" if expressed as:

- Sum of Minterms (SOM)
- Product of Maxterms (POM)

NOTE: for SOM or POM each term must contain all the variables

- to implement the function in hardware (logic gates) the Boolean function needs to be minimized (SOP, POS)

EX: F(X,Y,Z) = $\sum m(1,2,5,7) = \prod M(0,3,4,6)$

Boolean Functions

A Boolean Function may be expressed algebraically from a Truth Table by:

- 1) Sum of Minterms:
    - forming a minterm for each combination of the variables that produce a “1” in the function, and taking the OR of all minterms
- 2) Product of Maxterms:
    - forming a maxterm for each combination of all the maxterms variables which produce a “0” in the function, and takes the AND of all maxterms

Minterm and Maxterm Expansions

$f = A'BC + AB'C' + AB'C + ABC' + ABC$

Each of the terms in the above equation is referred to as a minterm. In general, a *minterm* of *n* variables is a product of *n* literals in which each variable appears exactly once in either true or complemented form, but not both.

(A *literal* is a variable or its complement)

Minterms and Maxterms for Three Variables

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

Input **Output: Minterms & Maxterms**

Minterm and Maxterm Expansions

Minterm expansion for a function is unique. Look at the equation bellow, it can be rewritten in terms of m-notation as:

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$f(A, B, C) = m_3 + m_4 + m_5 + m_6 + m_7$$

This can be further abbreviated by listing only the decimal subscripts in the form:

$$f(A, B, C) = \Sigma\, m(3, 4, 5, 6, 7)$$

Minterm Expansion Example

Find the minterm expansion of *f(a,b,c,d) = a'(b' + d) + acd'.*

$$f = a'b' + a'd + acd'$$

$$\begin{aligned} f &= a'b' + a'd + acd' \\ &= a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b') \\ &= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + \cancel{a'b'c'd} + \cancel{a'b'cd} \\ &\qquad + a'bc'd + a'bcd + abcd' + ab'cd' \qquad (4\text{-}9) \end{aligned}$$

$$f = \underset{0\,0\,0\,0}{a'b'c'd'} + \underset{0\,0\,0\,1}{a'b'c'd} + \underset{0\,0\,1\,0}{a'b'cd'} + \underset{0\,0\,1\,1}{a'b'cd} + \underset{0\,1\,0\,1}{a'bc'd} + \underset{0\,1\,1\,1}{a'bcd} + \underset{1\,1\,1\,0}{abcd'} + \underset{1\,0\,1\,0}{ab'cd'}$$

$$f = \Sigma\, m(0, 1, 2, 3, 5, 7, 10, 14) \qquad (4\text{-}10)$$

Maxterm Expansion Example

Find the maxterm expansion of *f(a,b,c,d) = a'(b' + d) + acd'.*

$$
\begin{aligned}
f &= a'(b' + d) + acd' \\
&= (a' + cd')(a + b' + d) = (a' + c)(a' + d')(a + b' + d) \\
&= (a' + bb' + c + dd')(a' + bb' + cc' + d')(a + b' + cc' + d) \\
&= (a' + bb' + c + d)(a' + bb' + c + d')\cancel{(a' + bb' + c + d')} \\
&\qquad (a' + bb' + c' + d')(a + b' + cc' + d) \\
&= \underset{1000}{(a' + b + c + d)}\underset{1100}{(a' + b' + c + d)}\underset{1001}{(a' + b + c + d')}\underset{1101}{(a' + b' + c + d')} \\
&\qquad \underset{1011}{(a' + b + c' + d')}\underset{1111}{(a' + b' + c' + d')}\underset{0100}{(a + b' + c + d)}\underset{0110}{(a + b' + c' + d)} \\
&= \Pi\, M(4, 6, 8, 9, 11, 12, 13, 15) \qquad (4\text{-}11)
\end{aligned}
$$

General Truth Table for Three Variables

Table represents a truth table for a general function of three variables. Each a_i is a constant with a value of 0 or 1.

A B C	F
0 0 0	a_0
0 0 1	a_1
0 1 0	a_2
0 1 1	a_3
1 0 0	a_4
1 0 1	a_5
1 1 0	a_6
1 1 1	a_7

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \cdots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i$$

General Minterm and Maxterm Expansions

We can write the minterm expansion for a general function of three variables as follows; $a_i = 1$:

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \cdots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i \qquad (4\text{-}12)$$

The maxterm expansion for a general function of three variables is; $a_i = 0$:

$$F = (a_0 + M_0)(a_1 + M_1)(a_2 + M_2) \cdots (a_7 + M_7) = \prod_{i=0}^{7} (a_i + M_i) \qquad (4\text{-}13)$$

Conversion of Forms

Summarizes the procedures for conversion between minterm and maxterm expansions of F and F′

GIVEN FORM	DESIRED FORM: Minterm Expansion of *F*	DESIRED FORM: Maxterm Expansion of *F*	DESIRED FORM: Minterm Expansion of *F*′	DESIRED FORM: Maxterm Expansion of *F*′
Minterm Expansion of *F*	————	maxterm nos. are those nos. not on the minterm list for *F*	list minterms not present in *F*	maxterm nos. are the same as minterm nos. of *F*
Maxterm Expansion of *F*	minterm nos. are those nos. not on the maxterm list for *F*	————	minterm nos. are the same as maxterm nos. of *F*	list maxterms not present in *F*

Example

		DESIRED FORM		
	Minterm Expansion of f	Maxterm Expansion of f	Minterm Expansion of f'	Maxterm Expansion of f'
GIVEN FORM: $f = \Sigma\, m(3, 4, 5, 6, 7)$	________	$\Pi\, M(0, 1, 2)$	$\Sigma\, m(0, 1, 2)$	$\Pi\, M(3, 4, 5, 6, 7)$
GIVEN FORM: $f = \Pi\, M(0, 1, 2)$	$\Sigma\, m(3, 4, 5, 6, 7)$	________	$\Sigma\, m(0, 1, 2)$	$\Pi\, M(3, 4, 5, 6, 7)$

Truth Table for EXAMPLE 1-7

(*Decimal*)	X	Y	Z	$F1$
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

Truth Table for EXAMPLE 1.8

(Decimal)	*X*	*Y*	*Z*	*F2*
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

Q&A