» Properties of Matrix Multiplication

★ Matrices are NOT commutative

$$AB \neq BA$$

Let A, B, and C be matrices with dimensions so that the following operations make sense, and let k be a scalar. The following equalities hold:

* $A(BC) = (AB)C$ ← associative
* $A(B + C) = AB + AC$ and $(B + C)A = BA + CA$ ← distributive
* $\underline{k(AB)} = (kA)B = A(kB)$
* $AI = IA = A$

I = identity matrix

associative with scalars

$2(3+4) = 2\cdot3+2\cdot4$

$(3+4)2 = 3\cdot2+4\cdot2$

» Matrix Powers

In the same way that $\boxed{a^0 = 1}$ for a scalar a, We define

$$\boxed{A^0 = I}$$

identity matrix

for a matrix A.

Similarly, for any **positive** integer n, we define

$$A^n = \underbrace{A \cdot A \cdot \cdots \cdot A}_{n \text{ times}}.$$

$3^2 = 3 \cdot 3$

$A^2 = A \cdot A$

$A^3 = A \cdot A \cdot A$

Can we compute A^2 if A is not square? No :(

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 3 \end{bmatrix}$$

(2×3) (2×3)

don't match!

» Important Reminders

* Matrix multiplication is not commutative: $\boxed{AB \neq BA}$
* If we know that $\boxed{AX = BX,}$ we cannot conclude that $A = B$.

Break until 3:05