Math 107 Lecture 2

Vectors and Matrix Operations

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» Announcements and Objectives

Announcements

* Pre-Notes due before start of next lecture
* Assignments Due Friday (8/30):
  * HW0: Syllabus Quiz
  * HW0: Campuswire Intro (Part A)
  * HW0: Campuswire Intro (Part B)
  * HW0: MATLAB Practice Problem
  * HW0: MATLAB File Upload

Objectives

* Create equally-spaced vectors in MATLAB and perform element-by-element operations
* Access elements of matrices and vectors
* Interpret an image as an RGB matrix

» Warm-up

Question: Suppose someone asked you to list numbers starting at 1, ending at 10, and with a spacing of 3 between each number. How would you think about doing that?

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Question: Suppose someone asked you to list numbers starting at 1, ending at 10, and with a spacing of 3 between each number. How would you think about doing that?

Question: Now suppose someone asked you to list 5 equally-spaced numbers starting at 2 and ending at 10. How would you think about doing that?

Equally-spaced vectors

» Colon operator

MATLAB Syntax:

```
start:end
```

Creates vector with given start and end values and increments by 1

MATLAB Syntax:

```
start:increment:end
```

Creates vector with given start and end values and increments by the middle value.

Remark: Use the colon operator when you know the **spacing** (increment) between elements.

» Question

Poll

Question: Which of the following will create a vector starting at 3, ending at 11, and containing only the odd numbers?

(a) `3:odd:11`
(b) `3:11:2`
(c) `11:-2:3`
(d) `3:2:11`

» linspace()

MATLAB Syntax:

```
linspace(start,end,number_of_elements)
```

Creates vector with given start and end values and the given number of elements.

Remark: Use `linspace` when you know the **number of elements** you want in a vector.

» Question

Poll

Question: Which of the following will create a vector starting at 3, ending at 11, and containing exactly 100 elements?

(a) `linspace(3,100,11)`
(b) `linspace(100,3,11)`
(c) `linspace(3,11,100)`
(d) `3:100:11`

» Length function

MATLAB Syntax: `length(vector)`

Returns an integer telling you how many elements are in the vector.

Example: `length([1, 3, 2, 5])` returns 4

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MATLAB Syntax: `size(matrix)`

Returns two integers telling you how many rows and columns are in the matrix.

Example: `size([1, 3, 2, 5])` returns an array containing the values 1 and 4

Matrices in MATLAB

» Matrices in MATLAB

MATLAB Syntax: The rows of a matrix are created using the same syntax as usual vectors. Separate rows in a matrix by a semicolon (;).

Matrices in MATLAB can be created in a few ways:

* Typing elements individually:
  Example:
  ```
  A = [0, -2, 13; 5, -pi, 6.7]
  ```
* "Stacking" vectors:
  Example:
  ```
  B = [4:8; 19:-2:11; linspace(pi,2*pi,5)]
  ```
* Using special built-in matrix commands:
  Example:
  ```
  C = magic(6)
  ```

Accessing Vector and Matrix and Vector Elements

» Accessing Vector Elements

Example: `v = [-3, 2, 0, 7]`

elements:	-3	2	0	7
indices:	**1**	**2**	**3**	**4**

MATLAB Syntax: Given a vector `v`, you can access the kth element using the syntax `v(k)`.

Example: `v(4)` from above will return 7

» Accessing Matrix Elements

Example: $A = \begin{bmatrix} -3 & 2 & 0 \\ 7 & 1 & 5 \end{bmatrix}$

		column 1	column 2	column 3
row	1	-3	2	0
	2	7	1	5

MATLAB Syntax: Given a matrix A, you can access the element in the *i*th row and *j*th column using the syntax
`A(i,j)`

Example: `A(2,3)` from above will return 5.

» Using Vectors to Index

MATLAB Syntax: You can also use vector notation to access multiple elements of *a* matrix or vector.

```
A([row indices], [column indices])
v(start:end)
```

» Question

Chat blast

Question: Define `C = magic(6)`. What is the syntax for creating a new matrix B consisting of the submatrix from the 2nd row to the 5th row *and* the 3rd column to the 6th column. (Hint: Use the colon operator)

» Question

Chat blast

Suppose you have the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

defined in MATLAB.

Question: How could you use matrix indexing to flip the columns of the matrix to obtain $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$?

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Question: How could you use matrix indexing to flip the columns of the matrix to obtain $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$?

Question: How about flipping the rows of A to obtain $\begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$?

Linear Operations

» Matrix Addition and Scalar Multiplication

Matrix addition: Two $m \times n$ matrices A and B can be added $A + B$ by adding their corresponding components.

Scalar multiplication: The word **scalar** is another word for a number. For an $m \times n$ matrix A and a scalar k, scalar multiplication kA is defined by multiplying each element of A by k.

Matrix Multiplication

» Dot product

Row vector times column vector

Definition: Given a row vector

$$\vec{v} = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

and a column vector

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

the **dot product** $\vec{v} \cdot \vec{w}$ is defined to be

$$\vec{v} \cdot \vec{w} = \sum_{i=1}^{n} v_i w_i = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n.$$

» Dot product

Example

Let

$$\vec{v} = \begin{bmatrix} 3 & -1 & 2 \end{bmatrix} \quad \text{and} \quad \vec{w} = \begin{bmatrix} 0 \\ 4 \\ 7 \end{bmatrix}.$$

Then

$$\vec{v} \cdot \vec{w} = (3)(0) + (-1)(4) + (2)(7) = 10$$

» Dot product

Question

Example: Let

$$\vec{v} = \begin{bmatrix} 2 & 0 & 1 & -1 \end{bmatrix} \quad \text{and} \quad \vec{w} = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 0 \end{bmatrix}.$$

What is $\vec{v} \cdot \vec{w}$?

» Matrix Multiplication

Definition: Let A be an $m \times r$ matrix, and let B be an $r \times n$ matrix. The **matrix product** of A and B, denoted AB, is the $m \times n$ matrix M whose entry in the ith row and jth column is the product of the ith row of A and the jth column of B.

» Matrix Multiplication

Example

Example: Let

$$A = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$

We will compute

$$M = AB.$$

Question: What is the size of M?

» Matrix Multiplication

Example

Example: We have

$$M = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$

We want to find

$$\begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \end{bmatrix}$$

Question: What is m_{11}?

» Matrix Multiplication

Example

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Question: What is m_{11}?

(row 1 of A) $\cdot$ (col 1 of B) $= \begin{bmatrix} -3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = (-3)(0) + (1)(2) = 2$

» Matrix Multiplication

Example

Example: We have

$$M = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$

We want to find

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \end{bmatrix}$$

Question: What is m_{12}?

Special Matrices

» Identity Matrix and Zero Matrix

Definition: The $n \times n$ matrix with 1's in the diagonal and zeros elsewhere is the $n \times n$ **identity matrix**, denoted I_n.

MATLAB Syntax: `eye(n)`

Definition: The $m \times n$ matrix of all zeros, denoted $\mathbf{0}_{m\times n}$, is the **zero matrix**.

MATLAB Syntax: `zeros(m,n)`

Remarks:

* When the dimensions of the zero or identity matrix are clear from the context, the subscript is generally omitted.
* The zero matrix can be any size ($m \times n$, rectangular), but the identity matrix is always a square matrix ($n \times n$, same number of rows as columns).

» Identity Matrix and Zero Matrix

Examples

Example: Some identity matrices include

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example: Some zero matrices include

$$\mathbf{0}_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathbf{0}_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{0}_{4\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrix Multiplication Properties

» Properties of Matrix Multiplication

Let A, B, and C be matrices with dimensions so that the following operations make sense, and let k be a scalar. The following equalities hold:

* $A(BC) = (AB)C$
* $A(B + C) = AB + AC$ and $(B + C)A = BA + CA$
* $k(AB) = (kA)B = A(kB)$
* $AI = IA = A$

» Matrix Powers

In the same way that $a^0 = 1$ for a scalar a, We define

$$A^0 = I$$

for a matrix A.

Similarly, for any **positive** integer n, we define

$$A^n = \underbrace{A \cdot A \cdot \cdots \cdot A}_{n \text{ times}}.$$

» Important Reminders

* Matrix multiplication is not commutative: $AB \neq BA$
* If we know that $AX = BX$, we cannot conclude that $A = B$.