PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 27: Kinematics of rotation | Newton's 2nd law about rotation

CALIFORNIA STATE UNIVERSITY FULLERTON

Chapter 10: Rotation

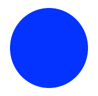
- Learning objectives
  - Kinematics of rotation
  - Dynamics of rotation

Learning goals for today

- Kinematics of rotation
- Moment of inertia
- Torque

Point particle vs. object with a finite volume

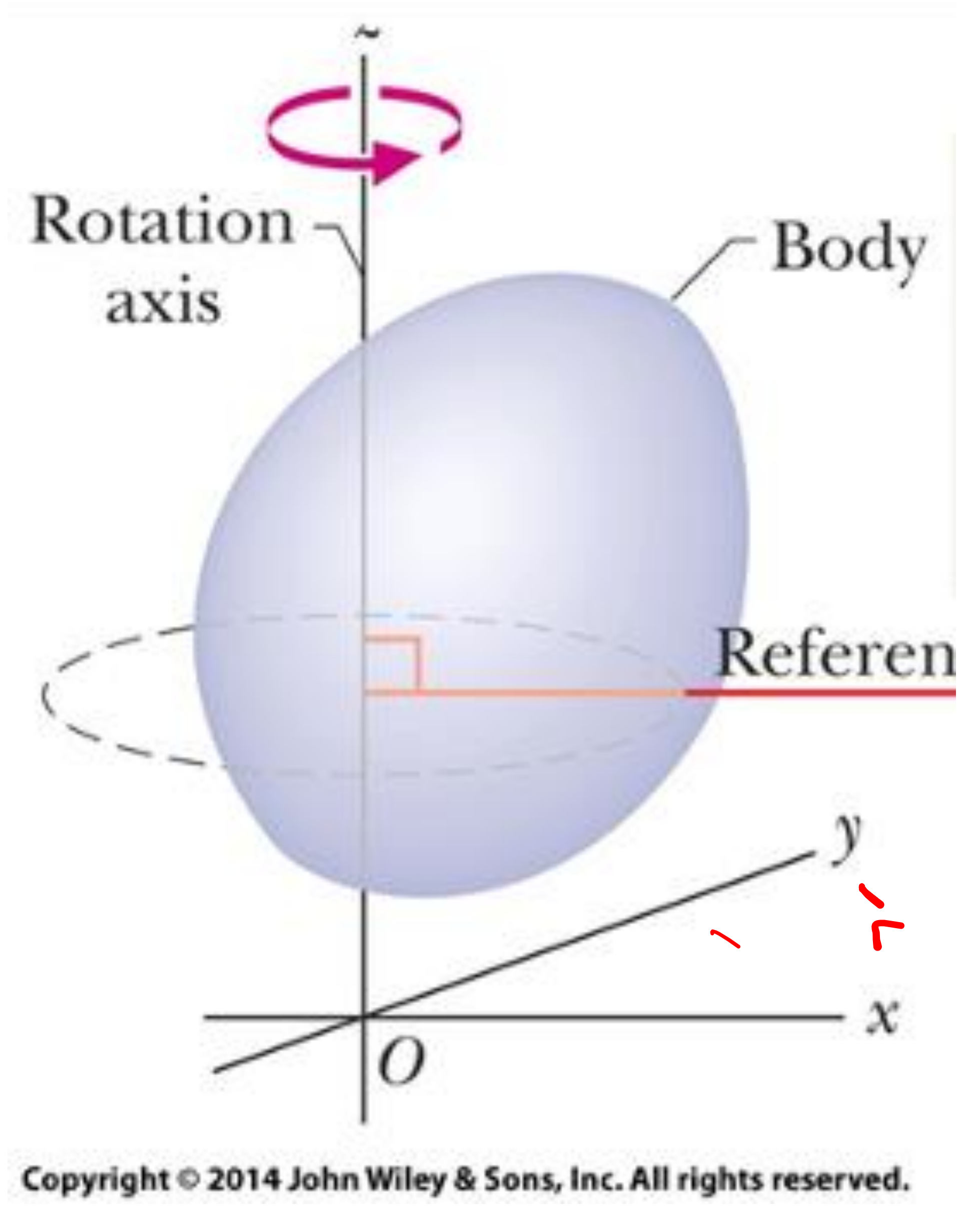
Previously



A point particle

This chapter

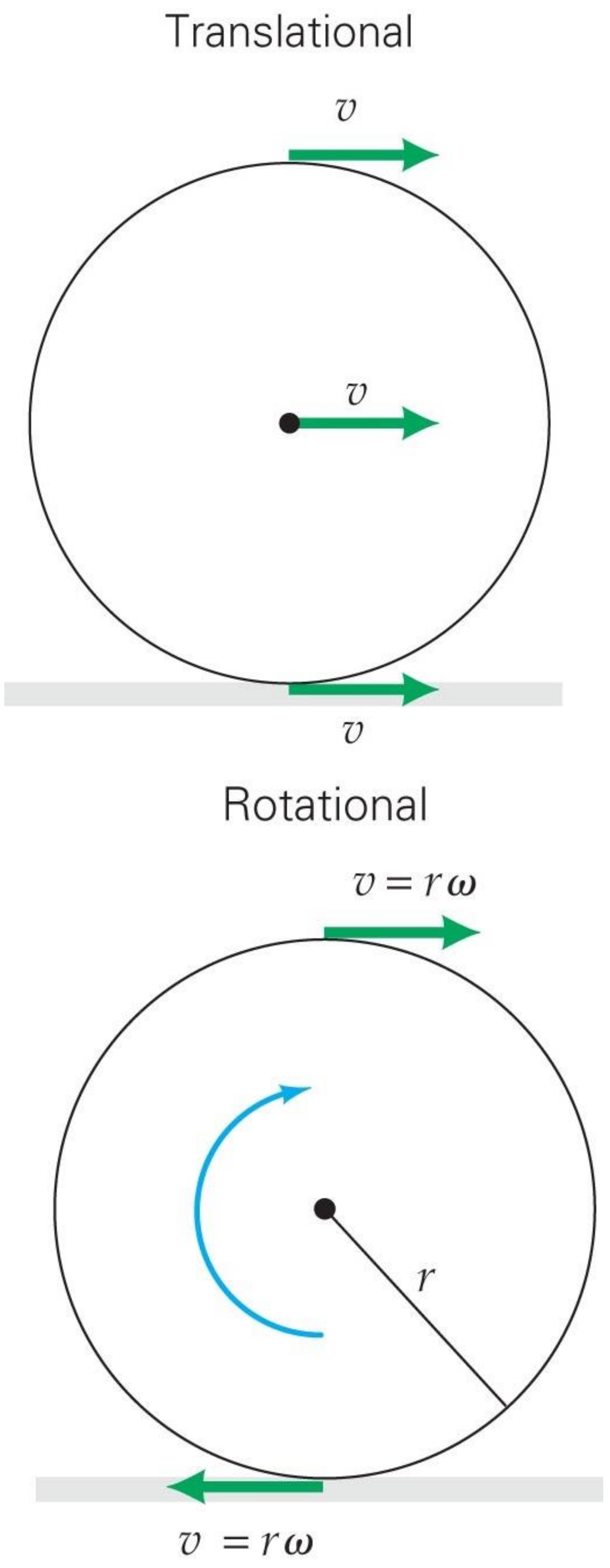
Rotation



Object with a finite volume

Translation & rotation

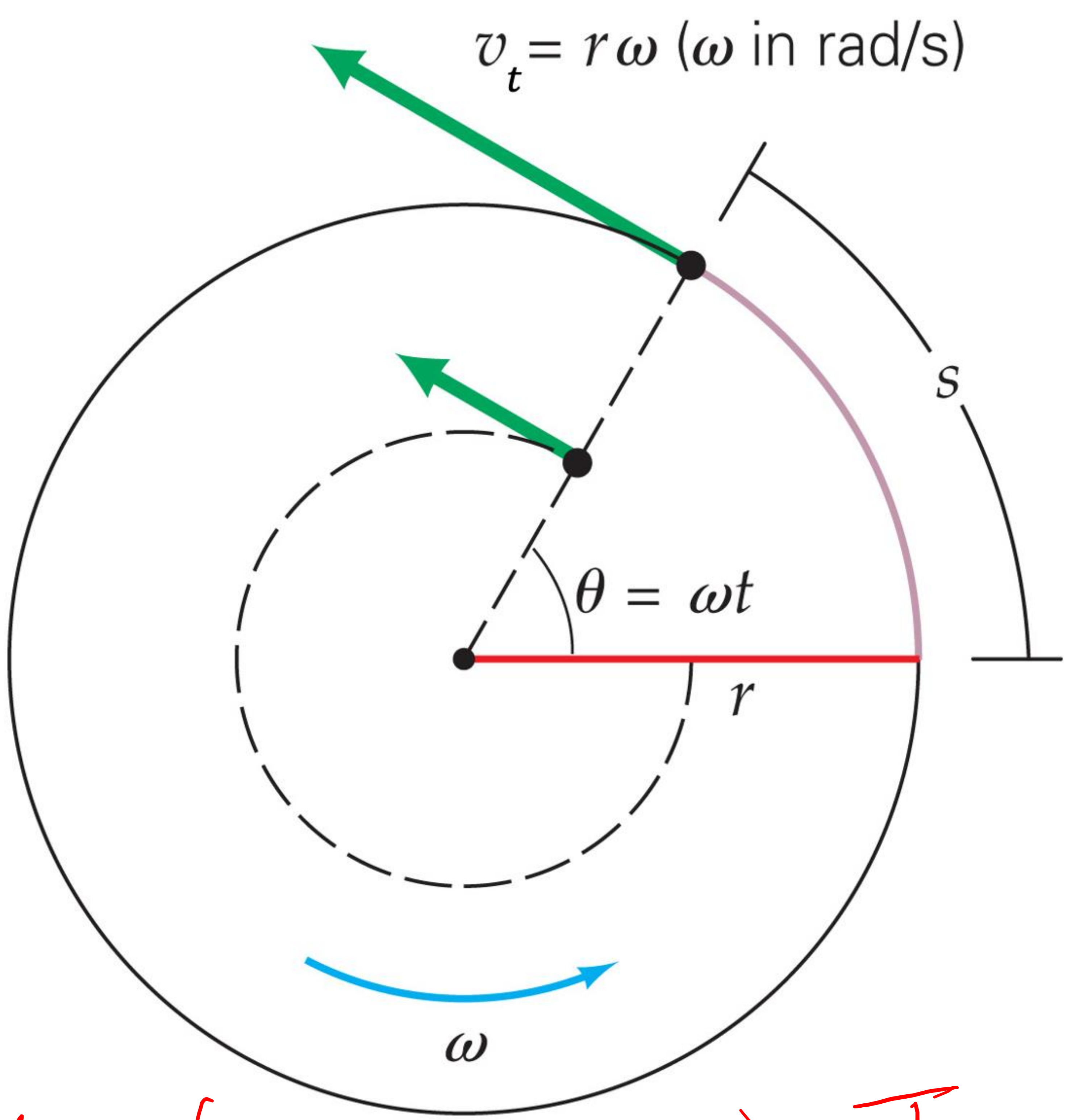
- Kinds of motion
  - **Translational**
    - ❖ All points have equal linear velocities, $\vec{v}$
  - **Rotational**
    - ❖ All points have equal angular velocity, $\vec{\omega}$, about a fixed axis of rotation



1. Angular velocity

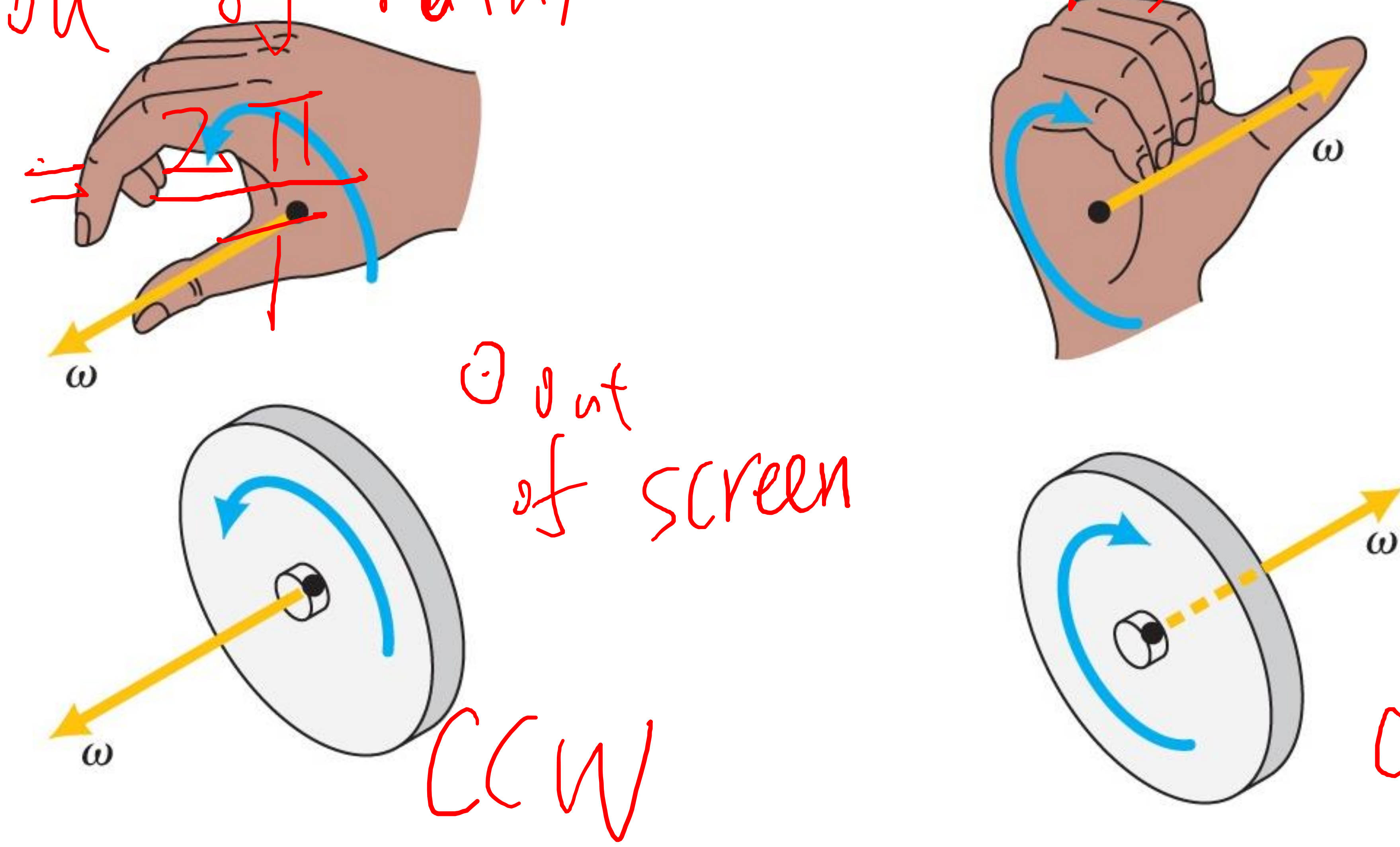
- Angular velocity: $\vec{\omega}$
  - **Magnitude** (angular speed): angle
    - Angle per unit time: $|\omega| = \left|\frac{d\theta}{dt}\right|$
    - Unit: rad s^{-1} or s^{-1} or rpm: $1\,\frac{rev}{min} = \frac{2\pi}{60}\,$s^{-1}

  - **Direction**: right-hand rule (RHR)
    - Points along "axle" of rotation



Right hand rule

If Period of rotation is T, then $|\vec{\omega}| = \frac{2\pi}{T}$



⊙ Out of screen

⊗ Into Screen

CCW

CW

Clicker question 1

RHR

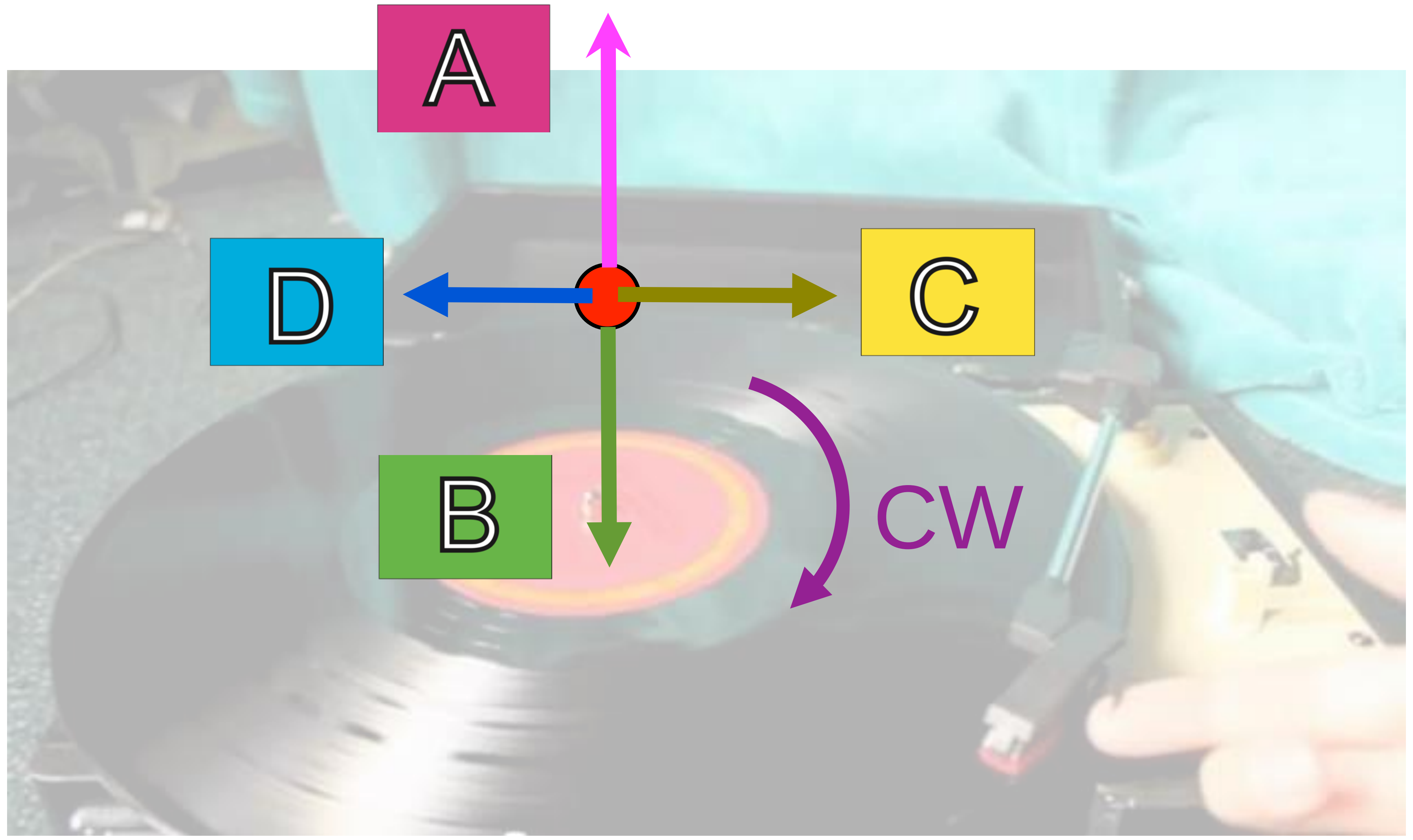
The record rotates **clockwise**. What is the direction of the **angular velocity ($\vec{\omega}$)** at the red dot?

A Up along the axle

B Down along the axle

C To the right

D To the left



2. Angular acceleration, $\vec{\alpha}$

- **Accelerated rotation**: When angular velocity, $\vec{\omega}$, changes over time

Accelerated rotation

Decelerated rotation

- **Angular acceleration:** Rate of change of angular velocity: $\vec{\boldsymbol{\alpha}} = \frac{d\vec{\boldsymbol{\omega}}}{dt}$

Angular acceleration, $\vec{\alpha}$

- Definition: $\vec{\alpha}$ is the rate of change of angular velocity: $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

- Magnitude (Unit: rad s^{-2} or s^{-2}):
  - <u>Average</u> angular acceleration: $|\overline{\vec{\alpha}}| = |\frac{\Delta\vec{\omega}}{\Delta t}|$
  - <u>Instantaneous</u> angular acceleration: $|\vec{\alpha}| = |\frac{d\vec{\omega}}{dt}|$

- Direction: if spinning faster, same as $\vec{\omega}$

  OR if spinning slower, opposite to $\vec{\omega}$

Clicker question 2

If spinning faster then same as $\vec{\omega}$

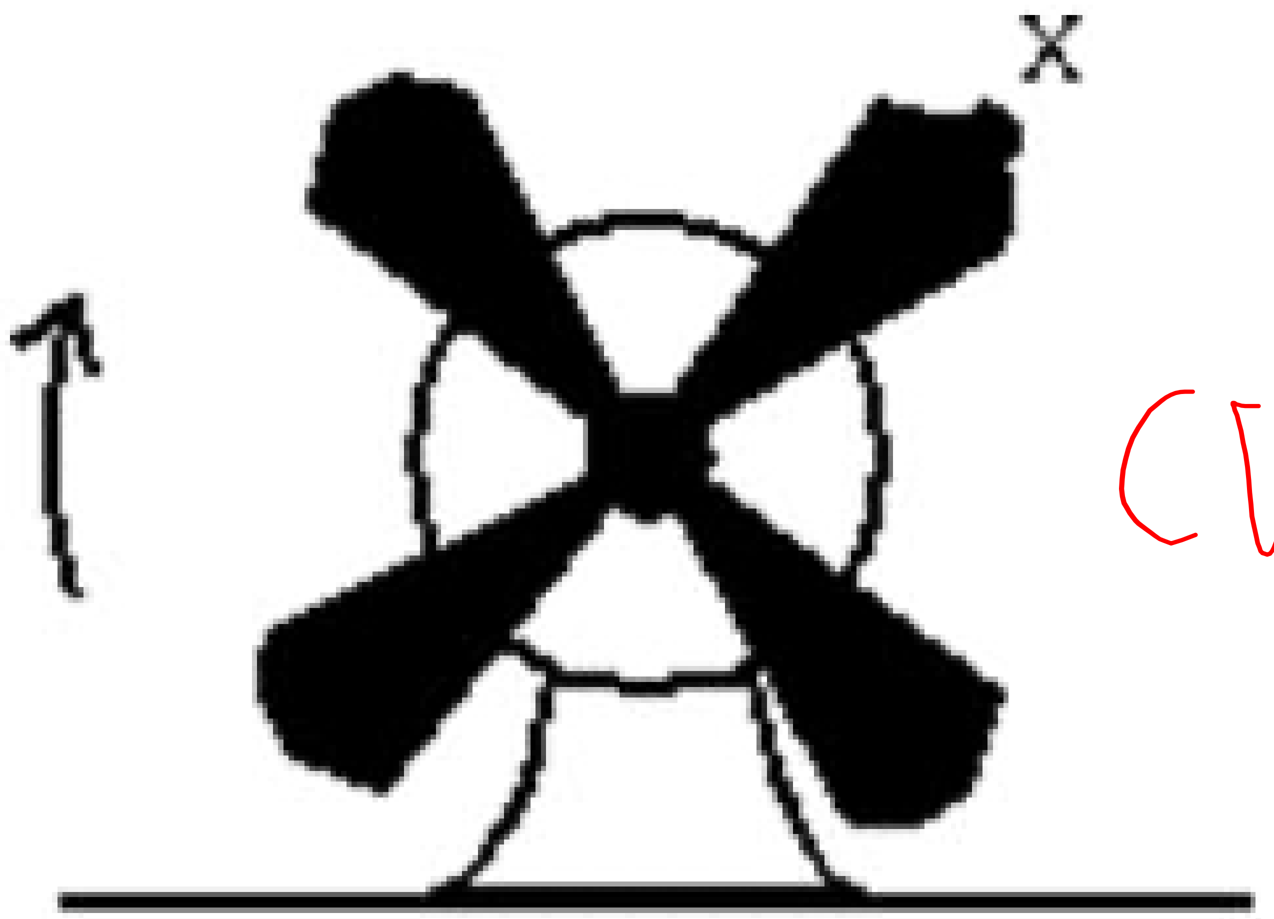
If ~ slower, then oppo. to $\vec{\omega}$

- The fan is turned on and is speeding up to rotate clockwise. What is the direction of angular acceleration, $\vec{\alpha}$?

A Point into the screen

B Point out of the screen

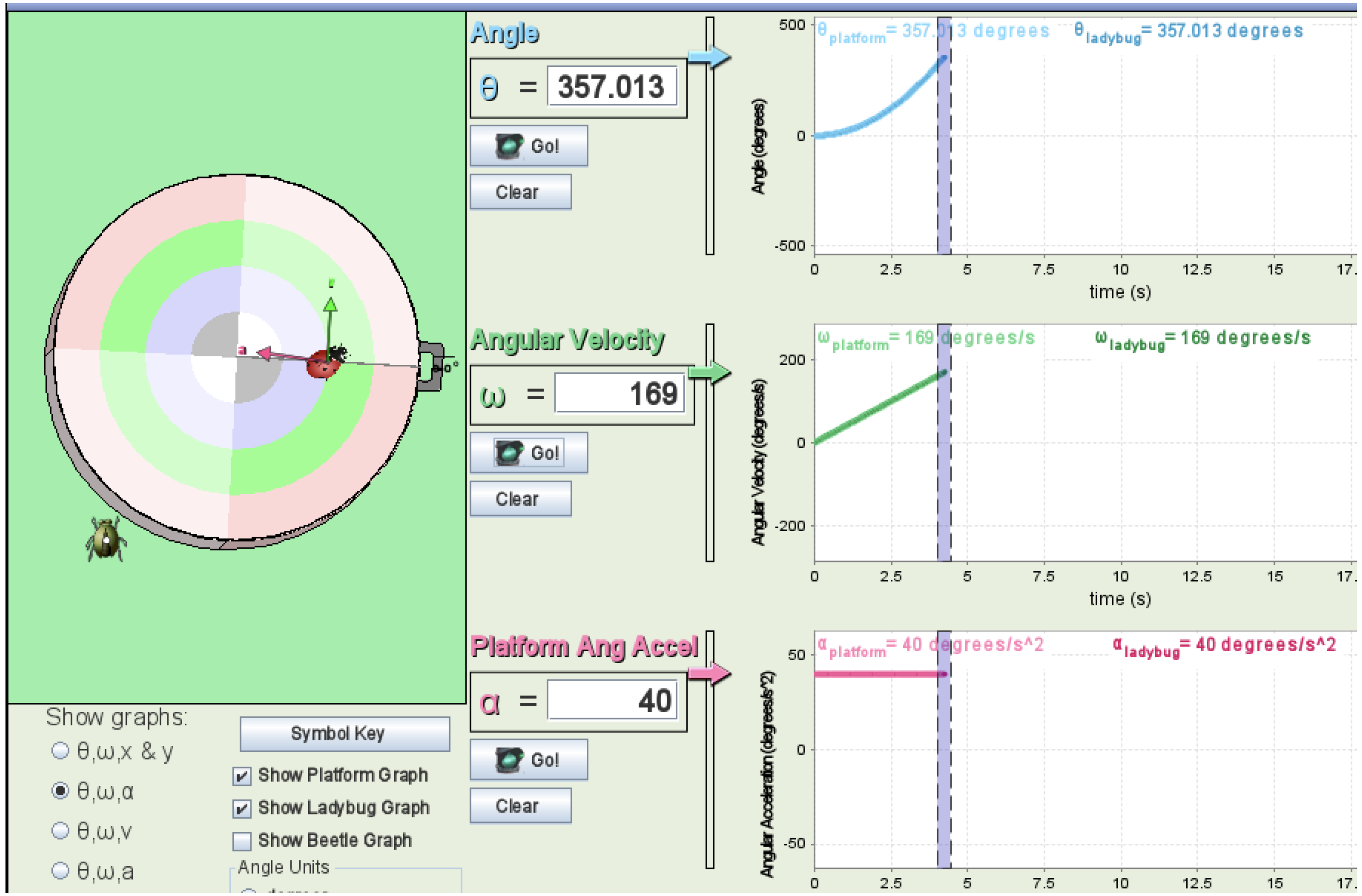
C Zero



CW

RHR

Demo: A constant angular acceleration, $\vec{\alpha}$



https://phet.colorado.edu/sims/cheerpj/rotation/latest/rotation.html?simulation=rotation

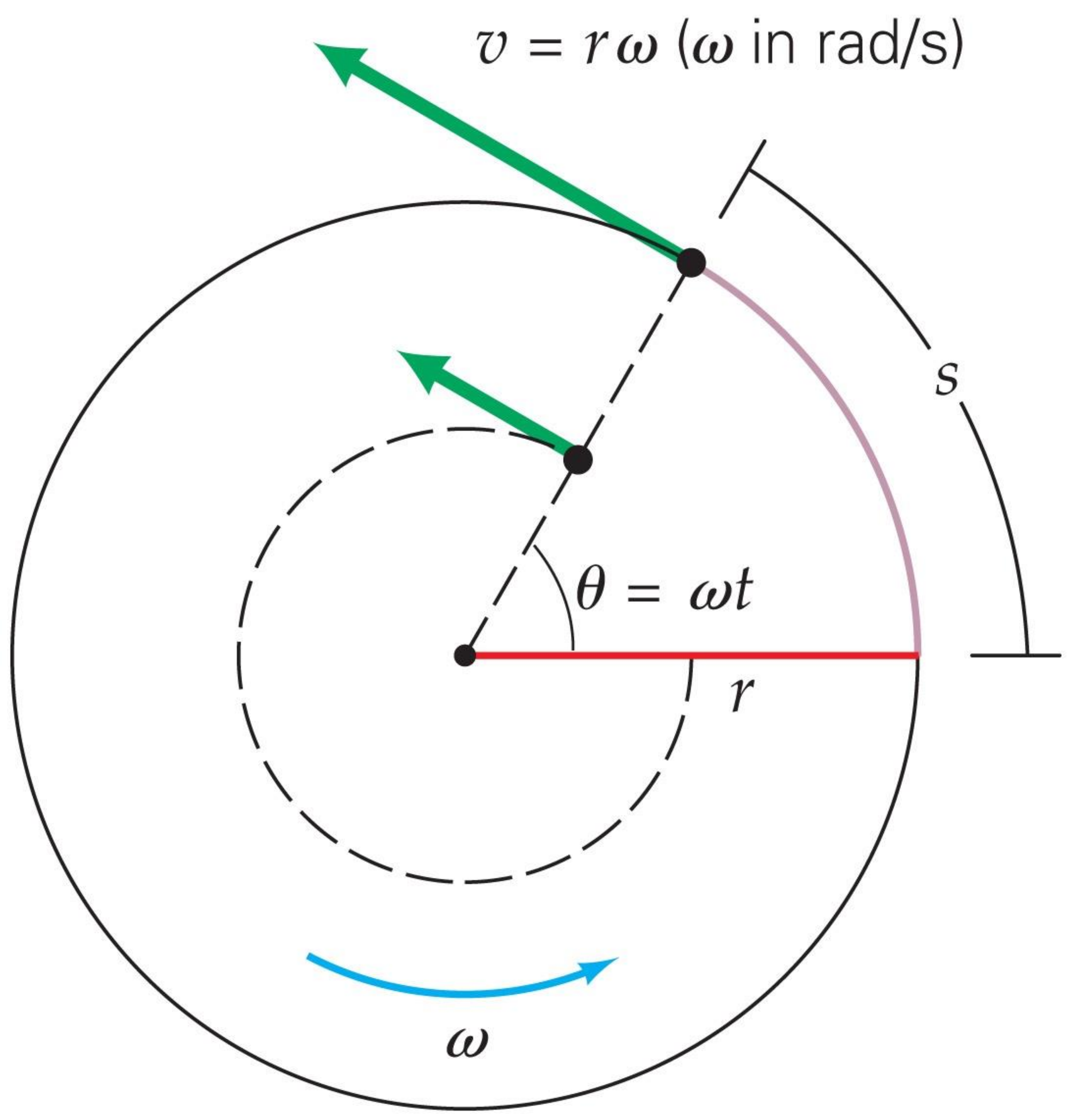
3. Kinematics of rotation with a constant angular acceleration

- If the angular acceleration, $\vec{\alpha}$, is a constant, and $t_0 = 0s$:
  - Angular velocity: $\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t$

    Initial angular velocity Time duration

  - Angle: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

    Initial angle



Analogy between 1D kinematics of linear motion and angular kinematics

Linear	Angular
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$x = x_o + v_o t + \frac{1}{2}at^2$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

Clicker question 3: microwave

Given: ω_0, $\vec{\alpha}$, t

Goal: ω

- A plate rotates in a microwave. The plate accelerates from rest at 0.87 rad/s^2 for 0.50 s. What is the final angular speed?

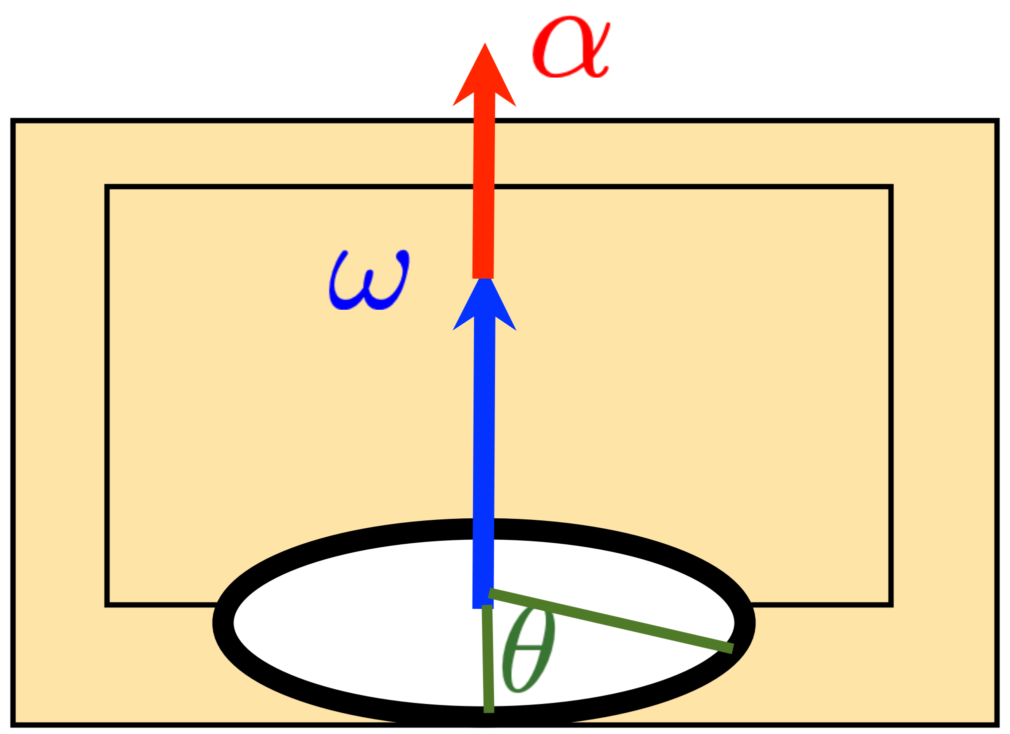
Which principle to use?

Given:

Goal:

A $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

B $\omega = \omega_0 + \alpha t$



http://www.youtube.com/watch?v=ExSKW1bwBq8

Clicker question 4: microwave

Goal: θ

- A plate rotates in a microwave. The plate accelerates from rest at an angular acceleration of 0.87 rad/s^2 for 0.50 s. The initial angle θ_0 =0. How many revolutions does it make?

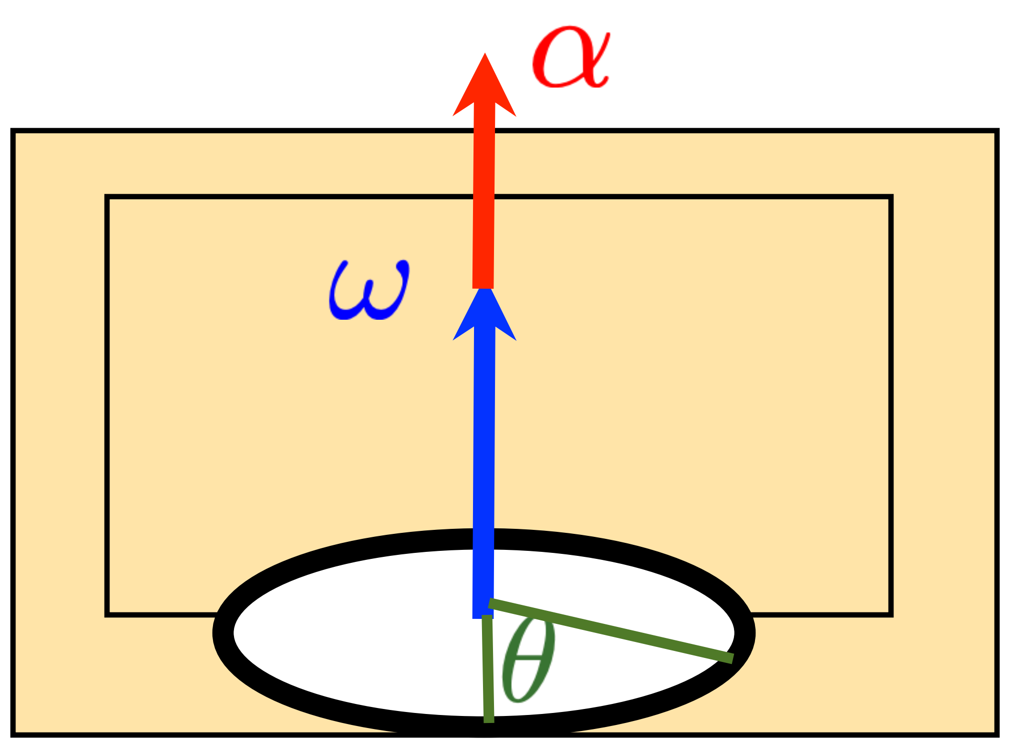
Which principle to use?

Given:

Goal:

A $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

B $\theta = \omega_0 t$



http://www.youtube.com/watch?v=ExSKW1bwBq8

Analogy: Translational/linear motion and rotational motion

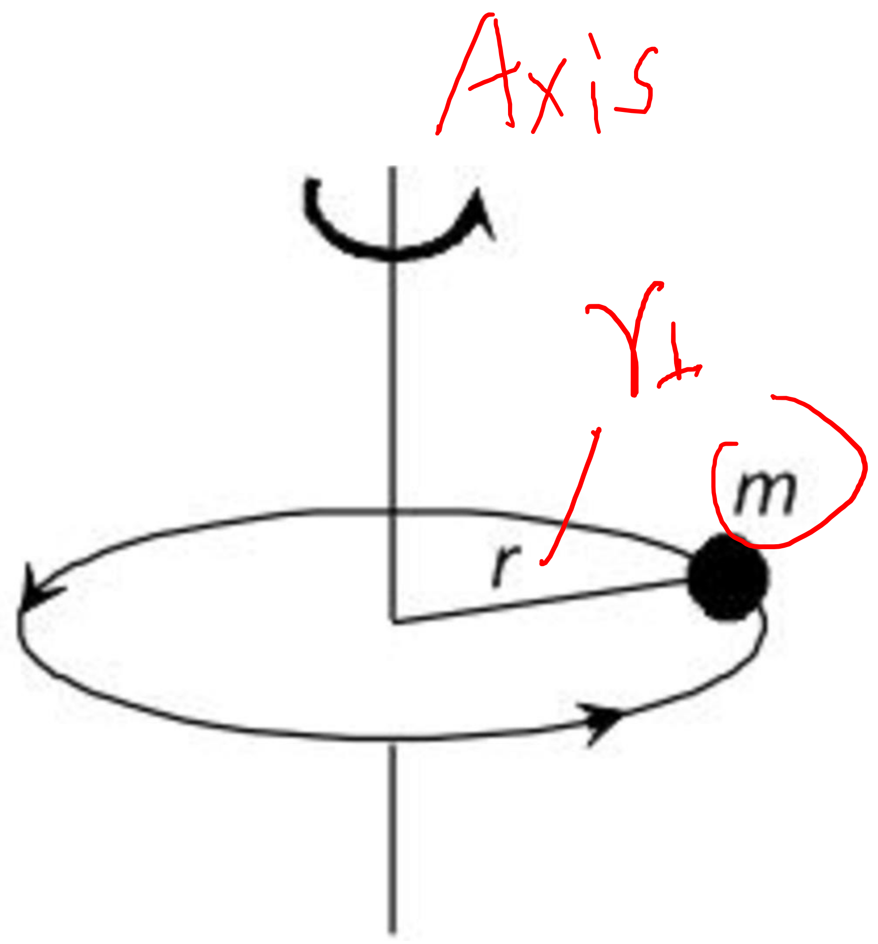
Linear motion	Rotational motion
Velocity, $\vec{v}$	Angular velocity, $\vec{\omega}$
Acceleration, $\vec{a}$	Angular acceleration, $\vec{\alpha}$
Mass, m	Moment of inertia, I
Force, $\vec{F}$	Torque, $\vec{\tau}$
Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$	Newton's 2nd law for rotation: ?

4. Moment of inertia: Rotational analogy of mass

- Moment of inertia (Unit: kg m^2): Resistance for rotation
  - For a point mass:

$$I = m r_\perp^2$$

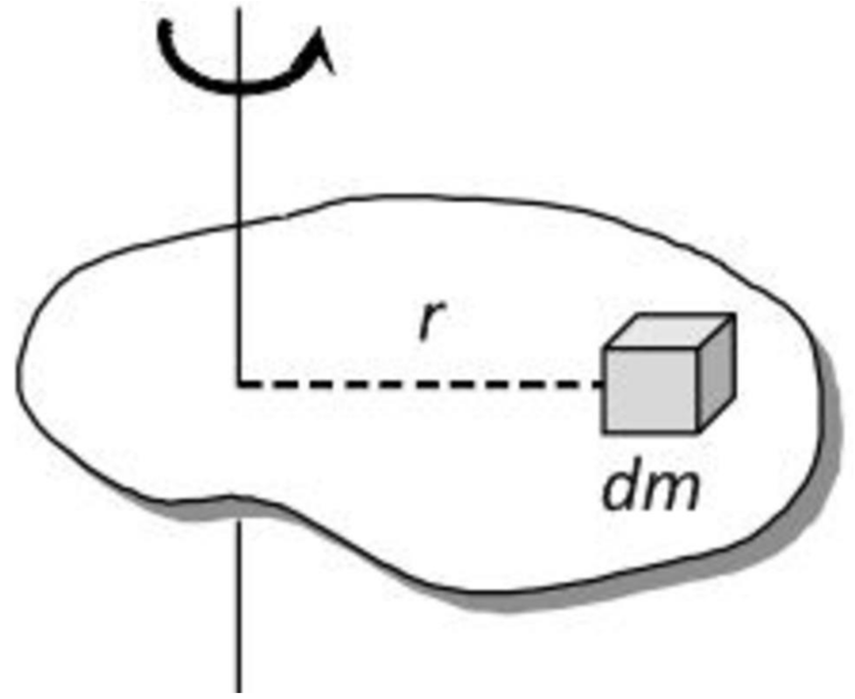
Distance from the point to the rotation axis



  - For a continuous object:

$$I = \int r_\perp^2 dm = \int \rho r_\perp^2 dV$$

Mass density



Clicker question 5.1

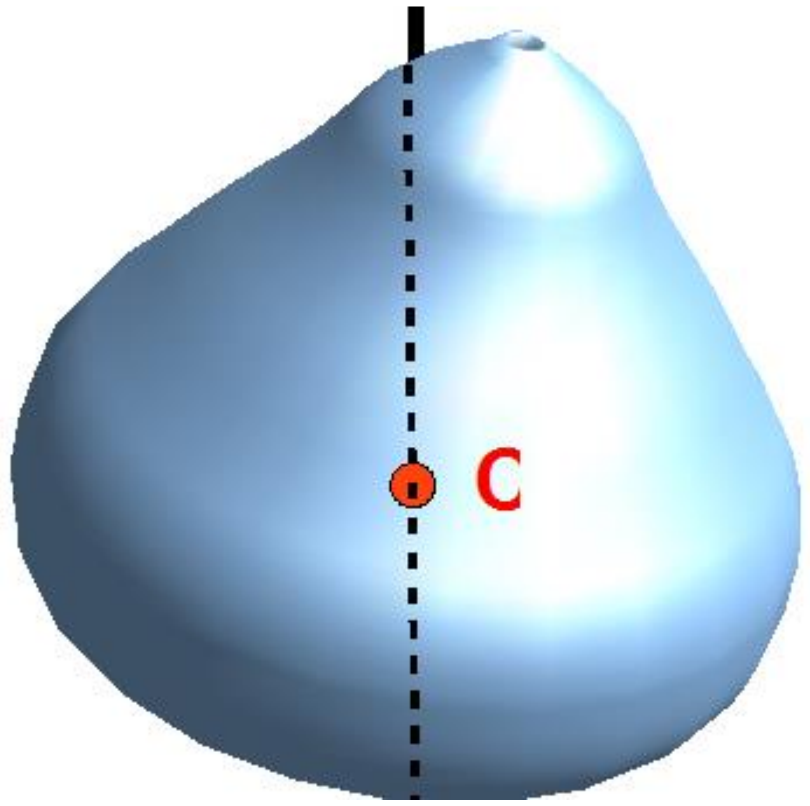
- The moment of inertia of an object does ***NOT*** depend on:

A The mass of the object

B The spatial distribution of the mass

C The location and orientation of the rotation axis

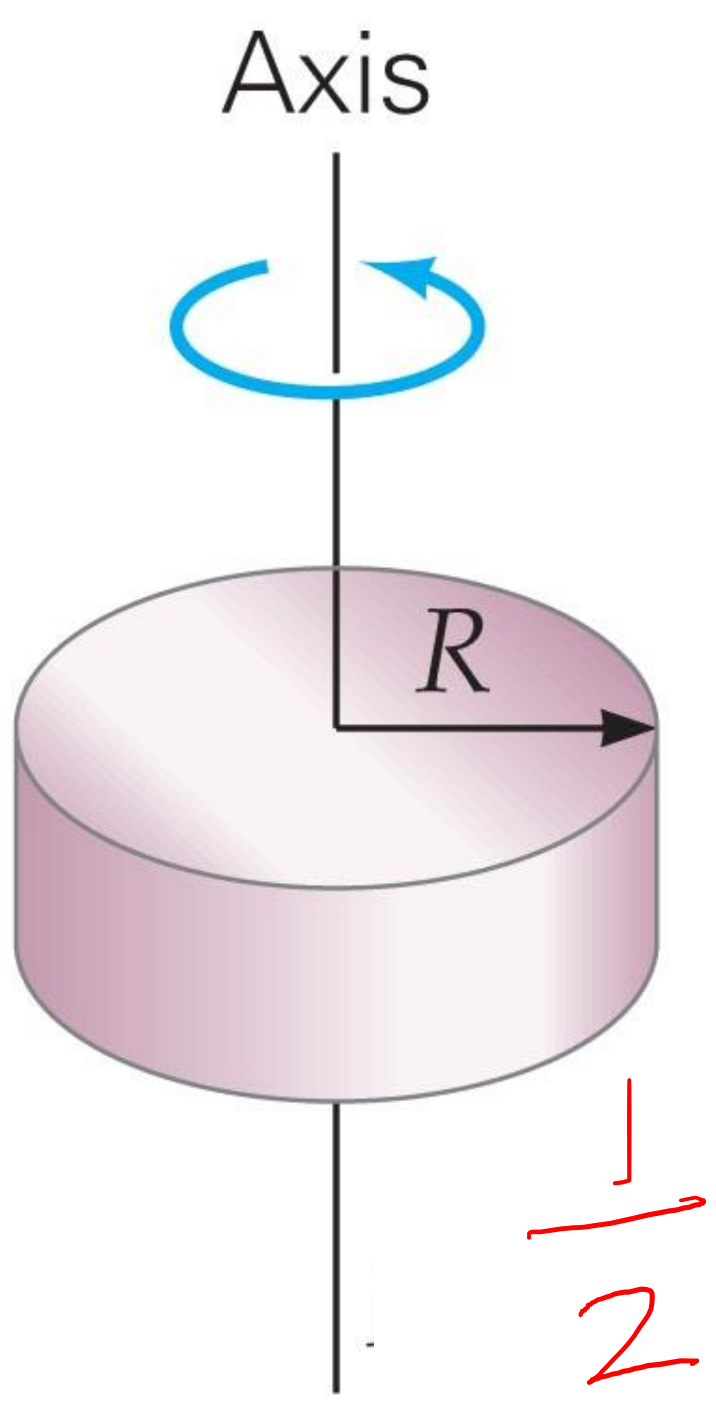
D The angular speed



$$I = \int \rho r_{\perp}^{2} dV$$

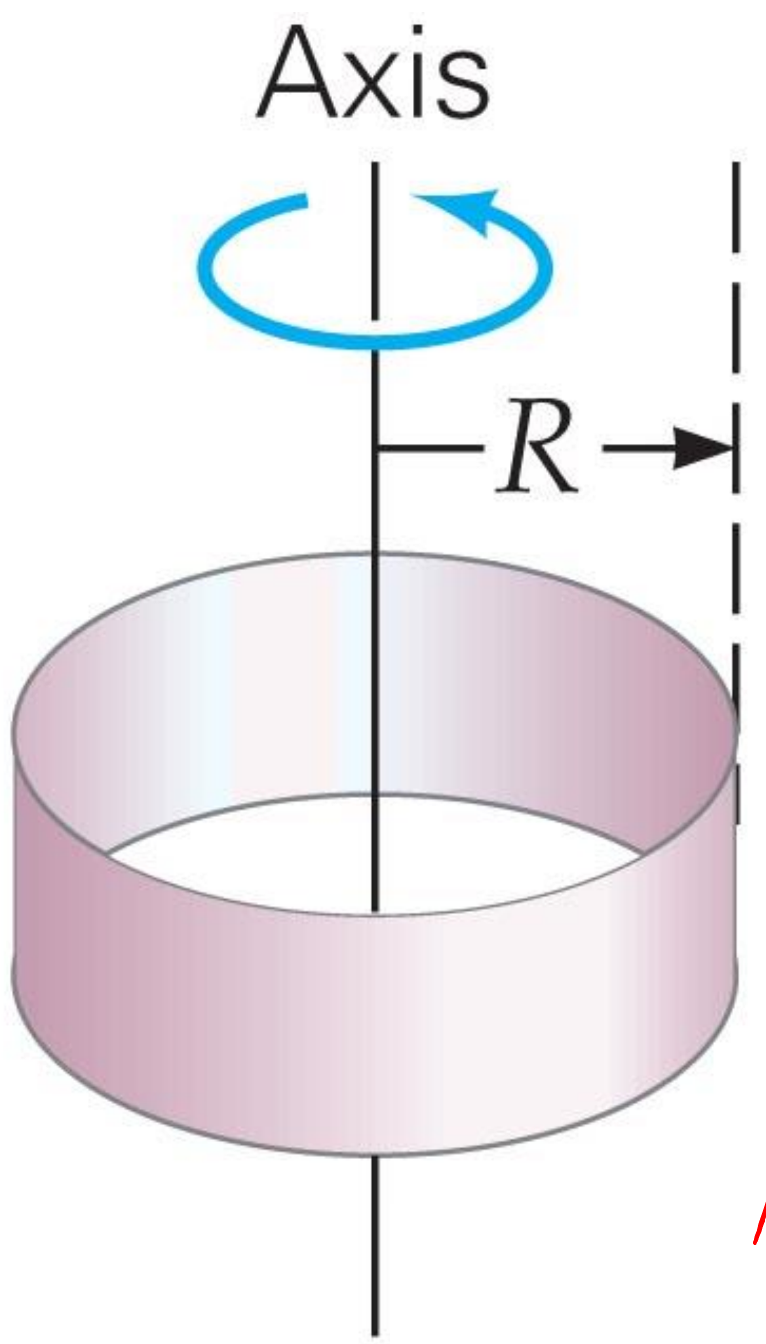
Clicker question 5.2

- Which of the following has a larger moment of inertia w.r.t. the axis specified? Both have same mass M.



$\frac{1}{2}MR^2$

A Solid cylinder or disk

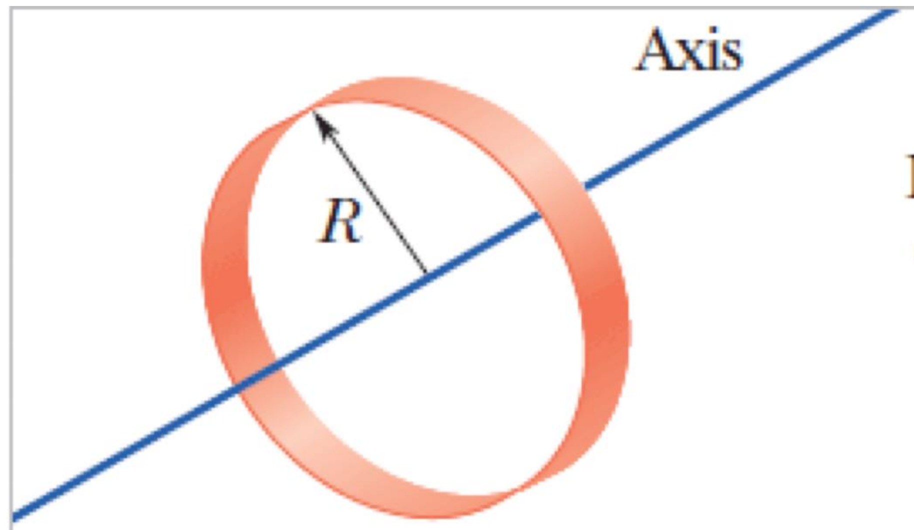


$$I = \int r^2 dm$$

MR^2

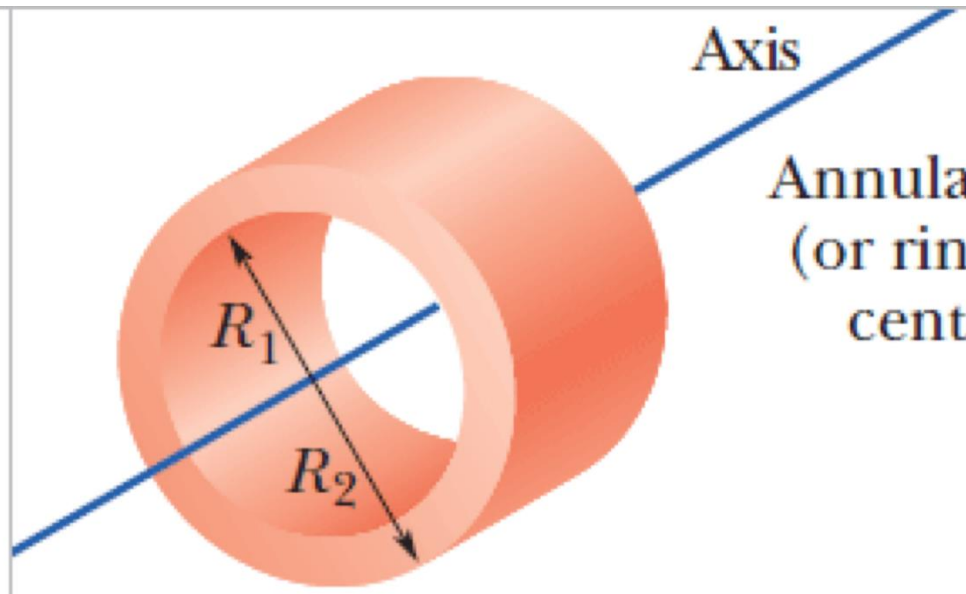
B Thin cylindrical shell, hoop, or ring

Table of moment of inertia for axes through center of mass (com)



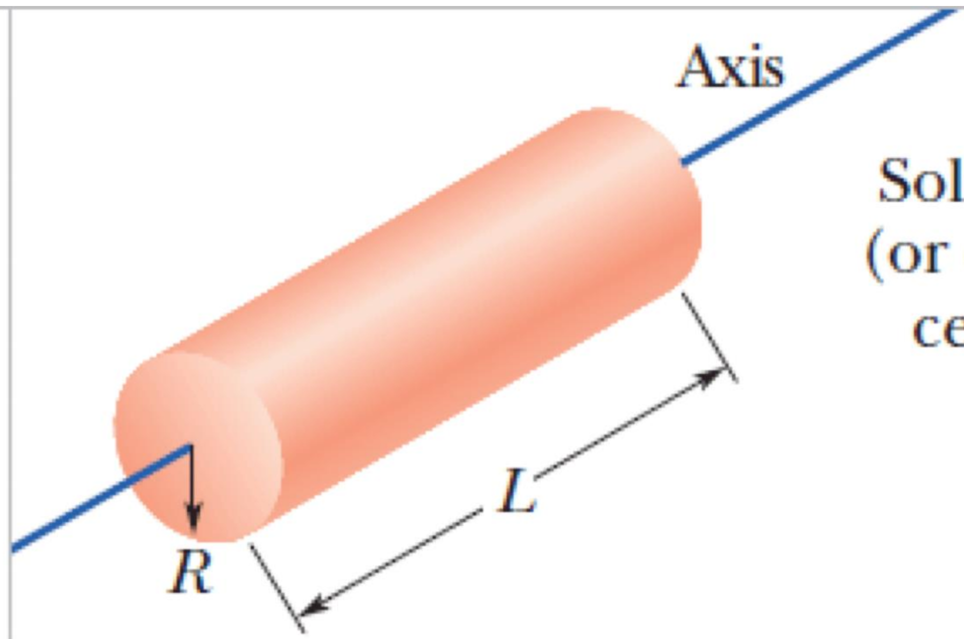
Hoop about central axis

$I = MR^2$ (a)



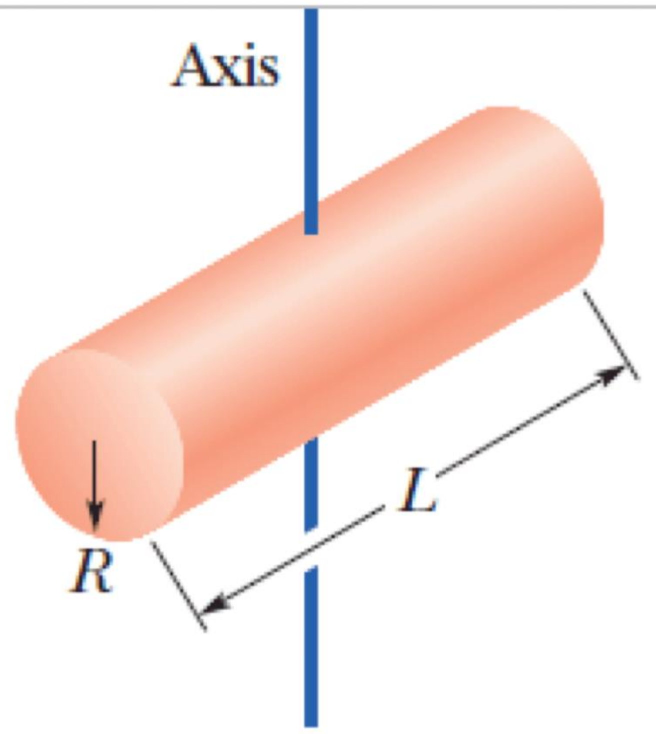
Annular cylinder (or ring) about central axis

$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)



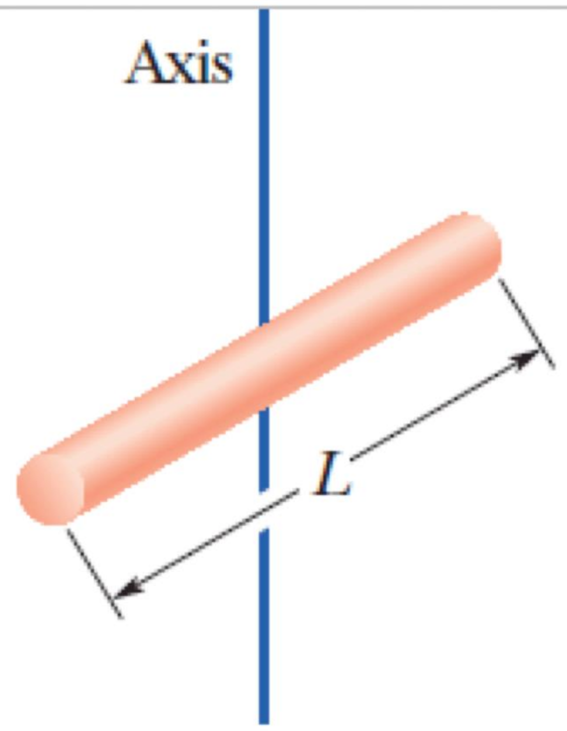
Solid cylinder (or disk) about central axis

$I = \frac{1}{2}MR^2$ (c)



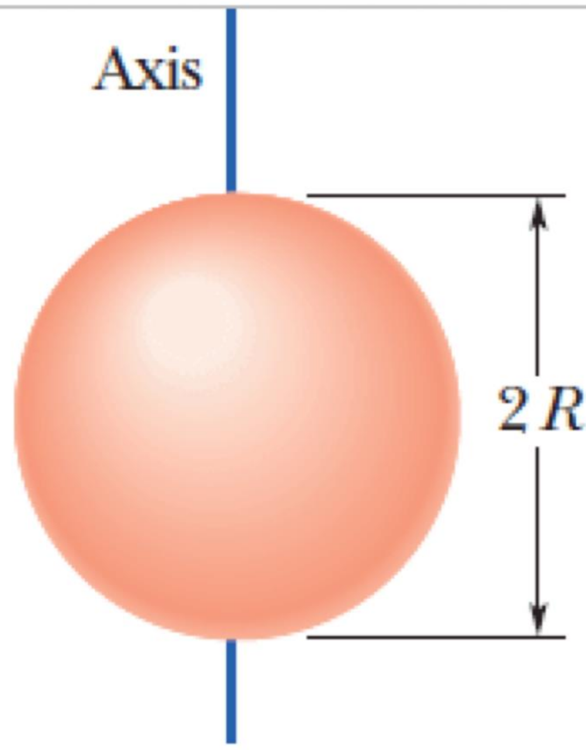
Solid cylinder (or disk) about central diameter

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)



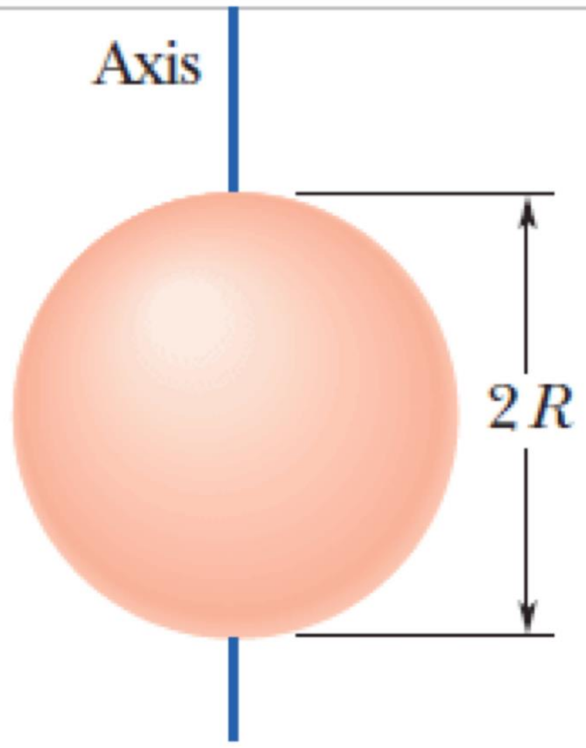
Thin rod about axis through center perpendicular to length

$I = \frac{1}{12}ML^2$ (e)



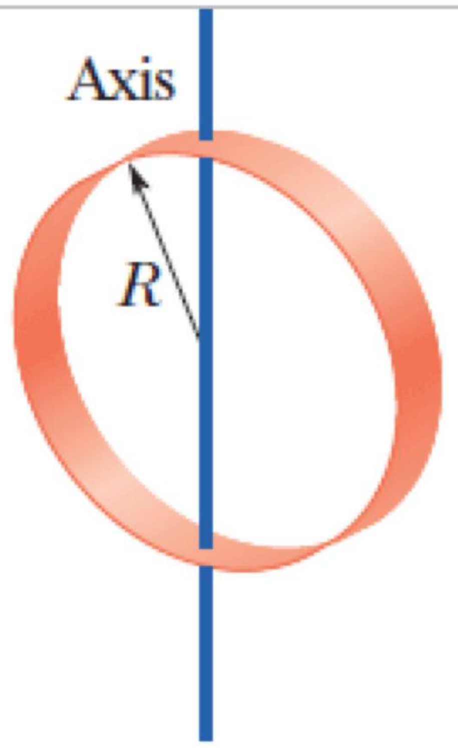
Solid sphere about any diameter

$I = \frac{2}{5}MR^2$ (f)



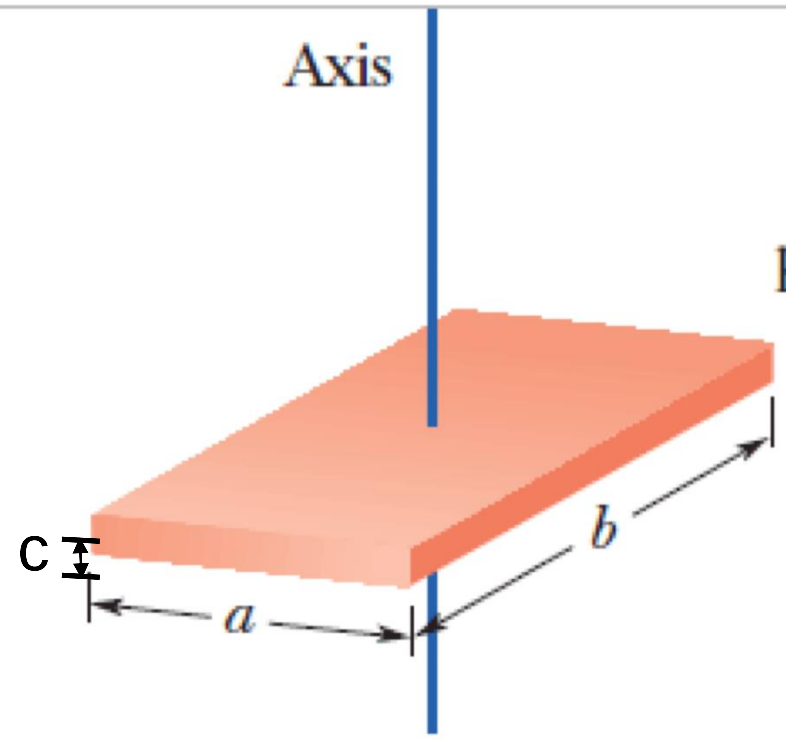
Thin spherical shell about any diameter

$I = \frac{2}{3}MR^2$ (g)



Hoop about any diameter

$I = \frac{1}{2}MR^2$ (h)



Slab about perpendicular axis through center

$I = \frac{1}{12}M(a^2 + b^2)$ (i)

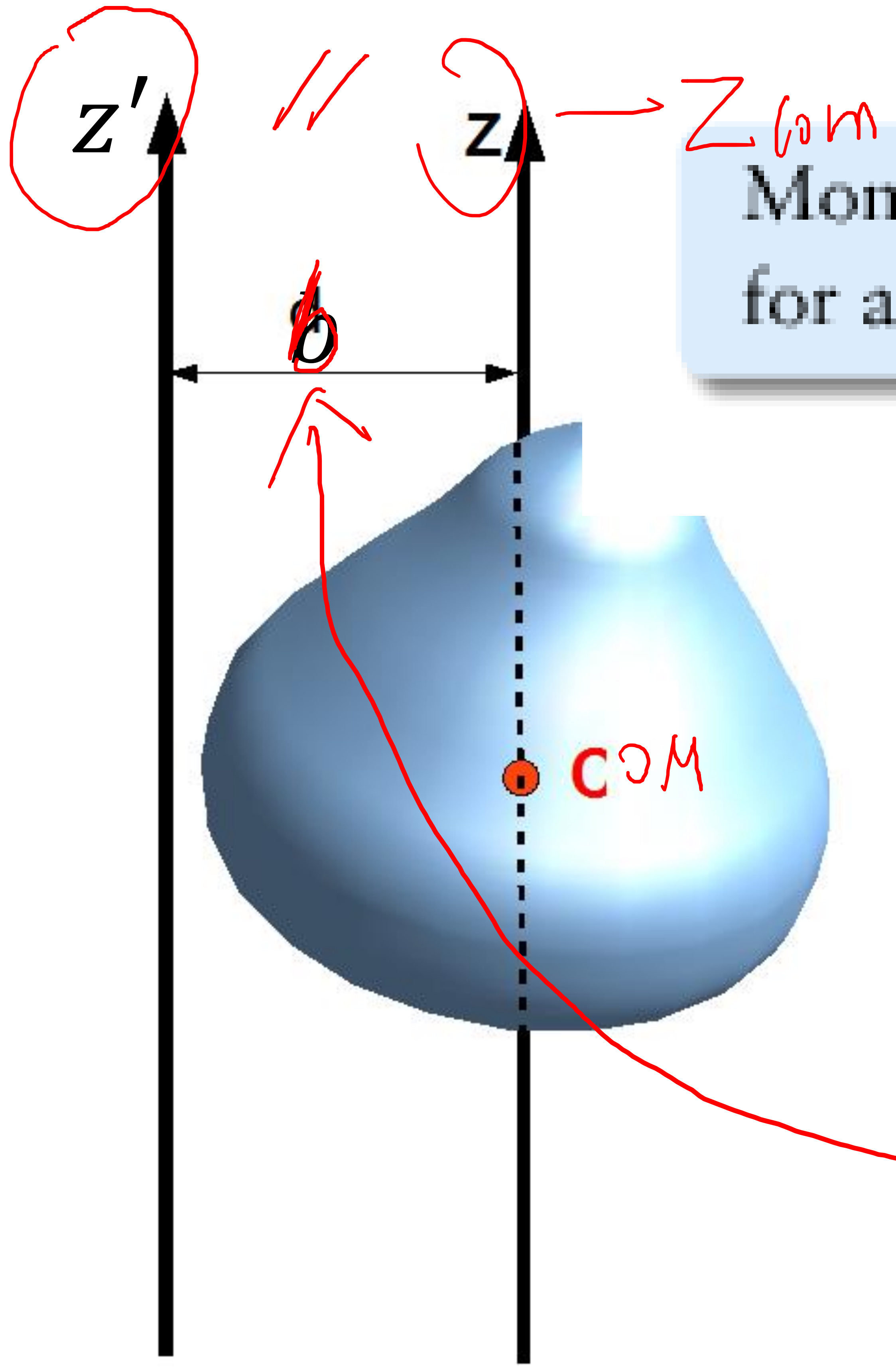
I com

?

I

Parallel axis theorem

- If you know the moment of inertia about an axis, ***z***, passing through the center of mass, then the moment of inertia about some **parallel axis, *z'*,** is



Moment of inertia of an object for a certain rotation axis z'

Moment of inertia of the same object for a second, parallel axis through its center of mass, z

$$I = I_{com} + Mb^2$$

Total mass of the object

Distance between the two parallel axes

Example 1

Given: M, L, z', $I_{com} = \frac{1}{12}ML^2$, $z \parallel z'$

Goal: I

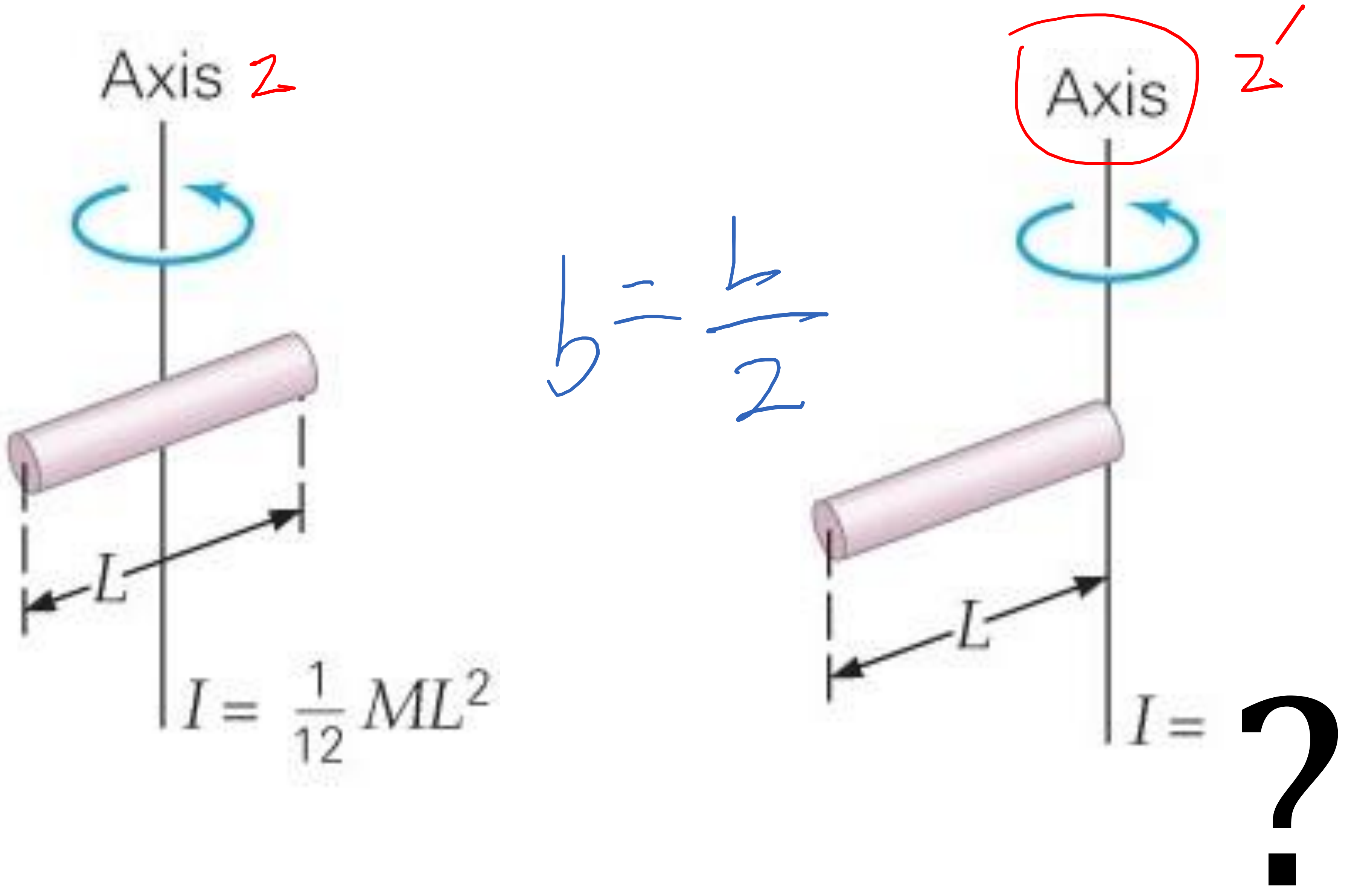
- The rod of mass M =1.0 kg and length of L =1.0 m rotates with respect to the axis perpendicular to the rod at the end of the rod. Please find the moment of inertia of the rod w.r.t. the axis. ***(Hint***: Moment of inertia of a uniform thin rod of length L with respect to its center of mass is $\frac{1}{12}ML^2$)

Step 1: Parallel axis theorem:

$$I = I_{com} + Mb^2$$
$$= I_{com} + M\left(\frac{L}{2}\right)^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$
$$= \frac{1}{3}ML^2$$

Step 2:

$$= \frac{1}{3} \times 1.00\,kg \times (1.0\,m)^2 \approx 0.33\,kg\,m^2$$



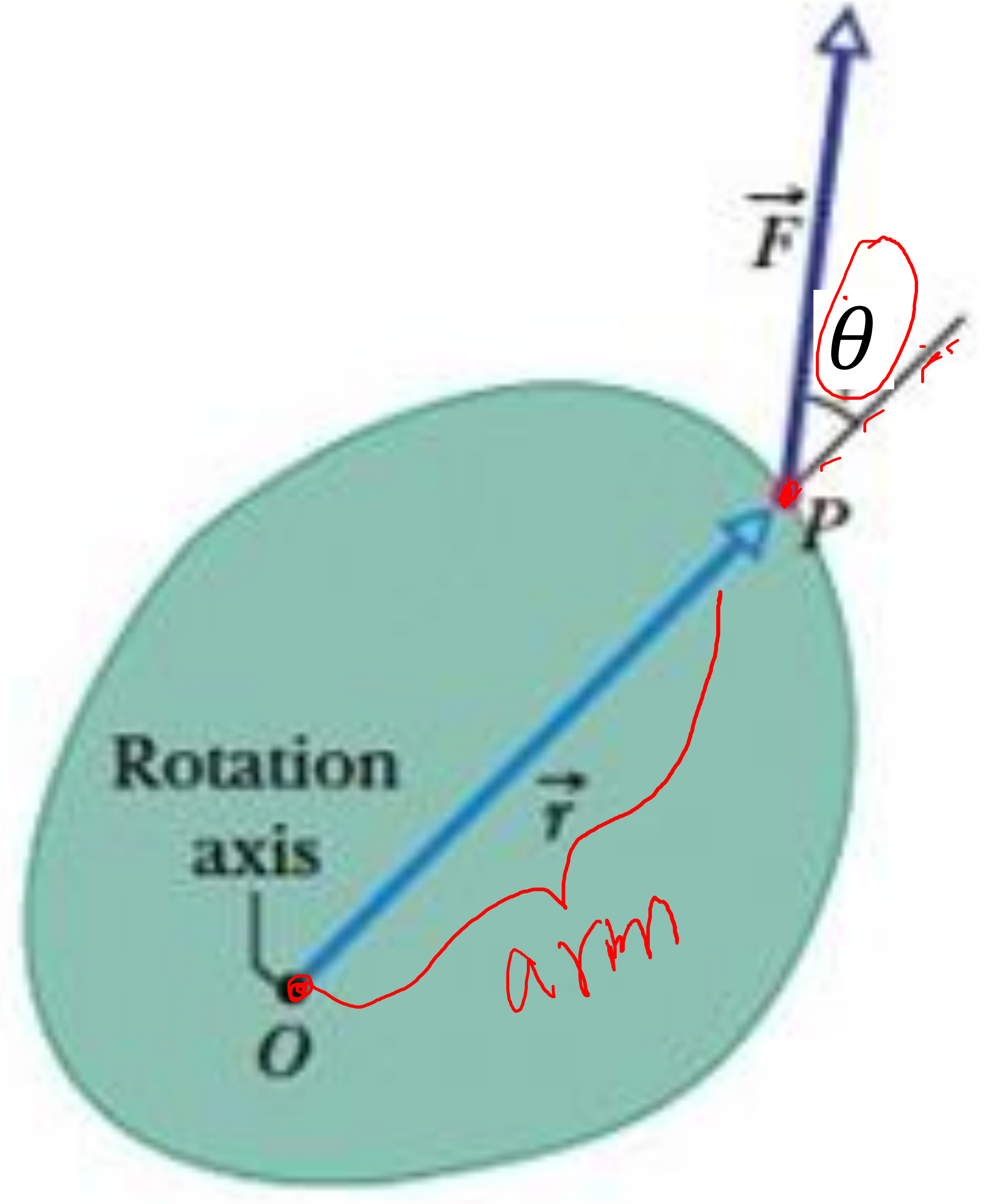
5. Torque

- Torque: $\vec{\tau}$, rotational analog of force
- Torque is a vector: $\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$

  Displacement from the rotation axis to the point

  Angle between $\vec{r}$ and $\vec{F}$

  - Magnitude: $|\vec{\tau}| = |\vec{r}||\vec{F}||\sin\theta|$
  - Direction: right-hand rule
  - Units: m•N
    [NOT J; torque is not energy!]

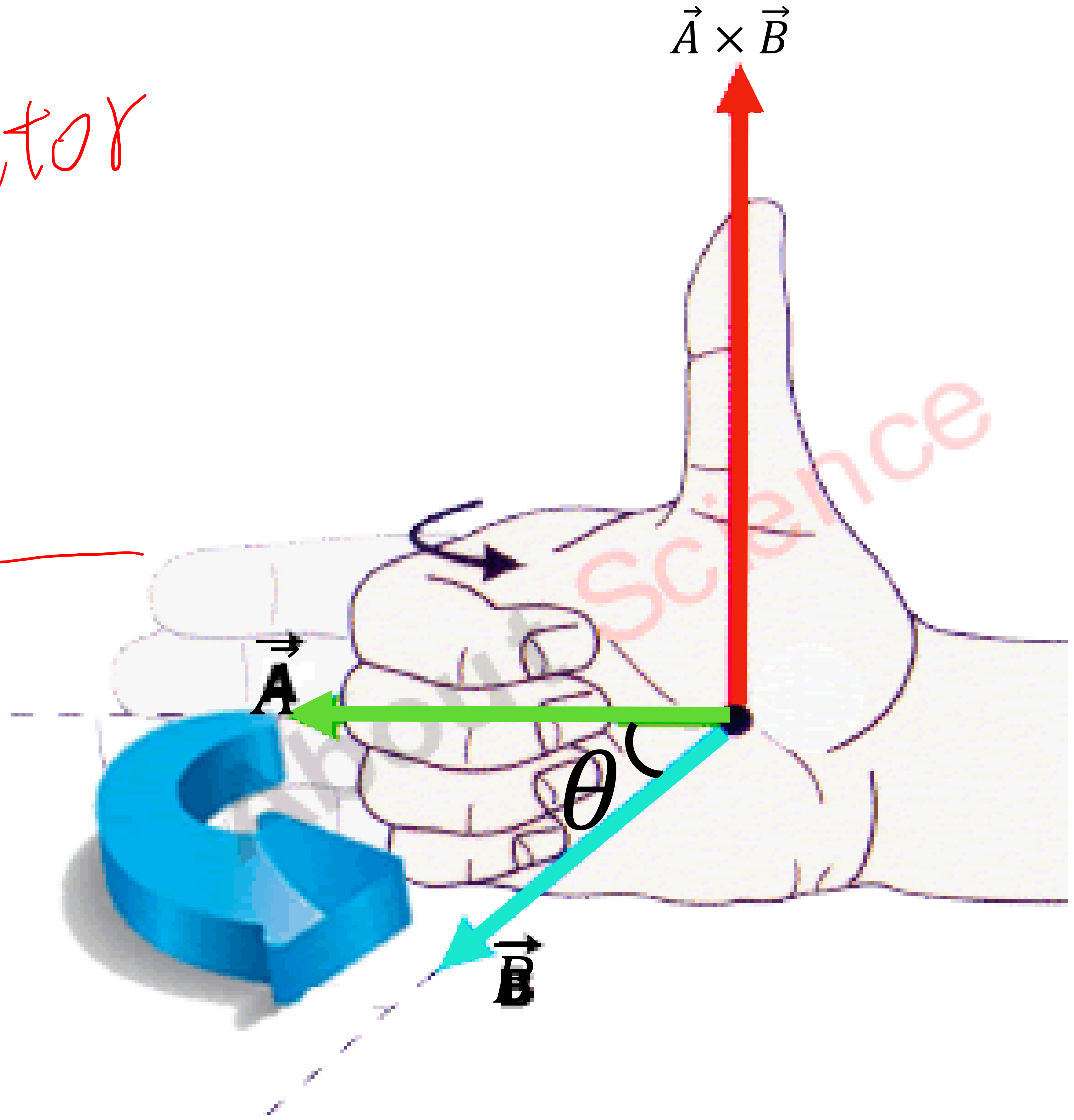


Recap: Cross product

- Cross product (geometric understanding):

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin|\theta|\ \hat{u}$$ unit vector

The direction of $\vec{A} \times \vec{B}$,$\hat{u}$,is determined by the **right-hand rule**



Torque examples

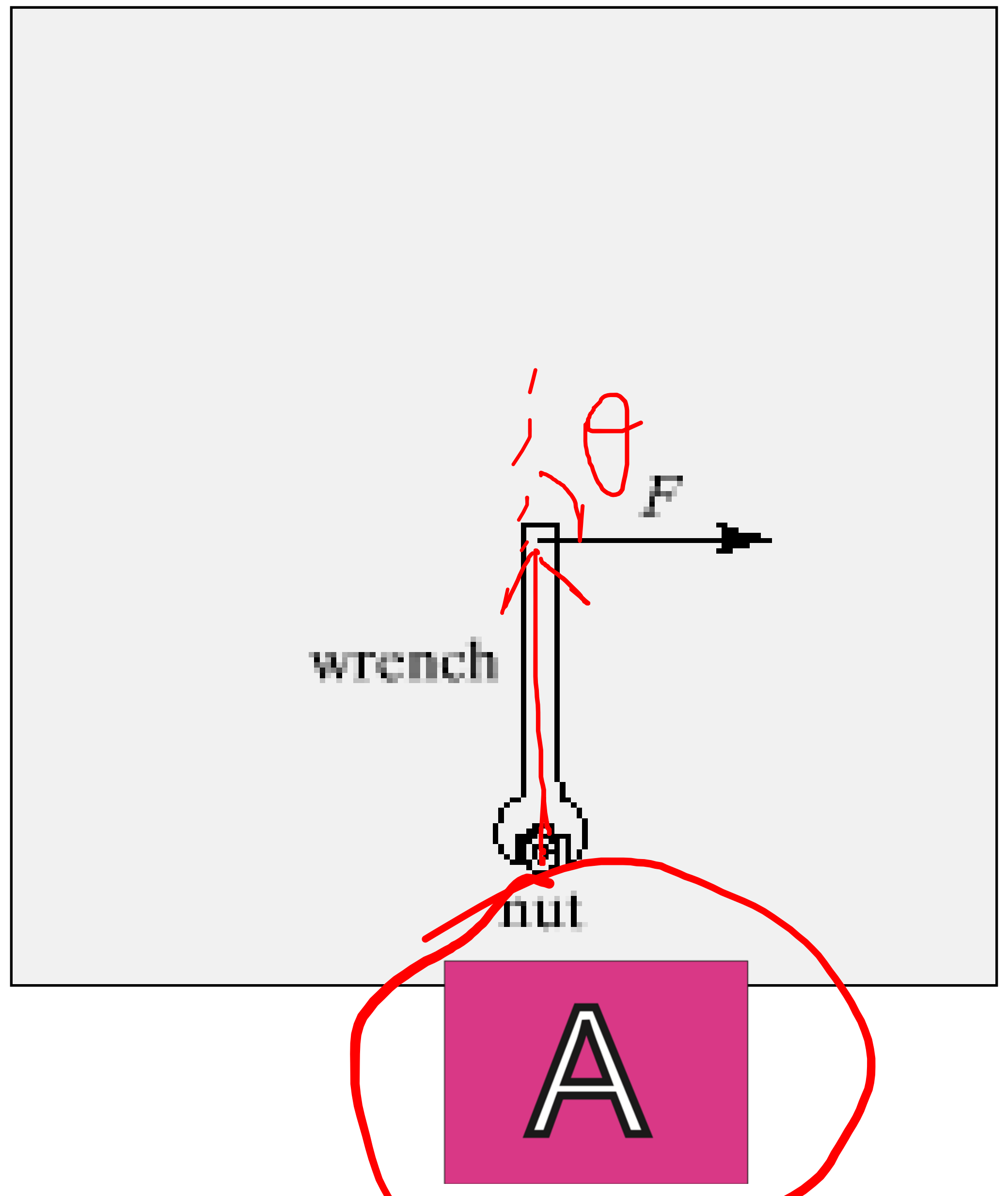
- Tools: screwdrivers and wrenches
- Knobs & handles

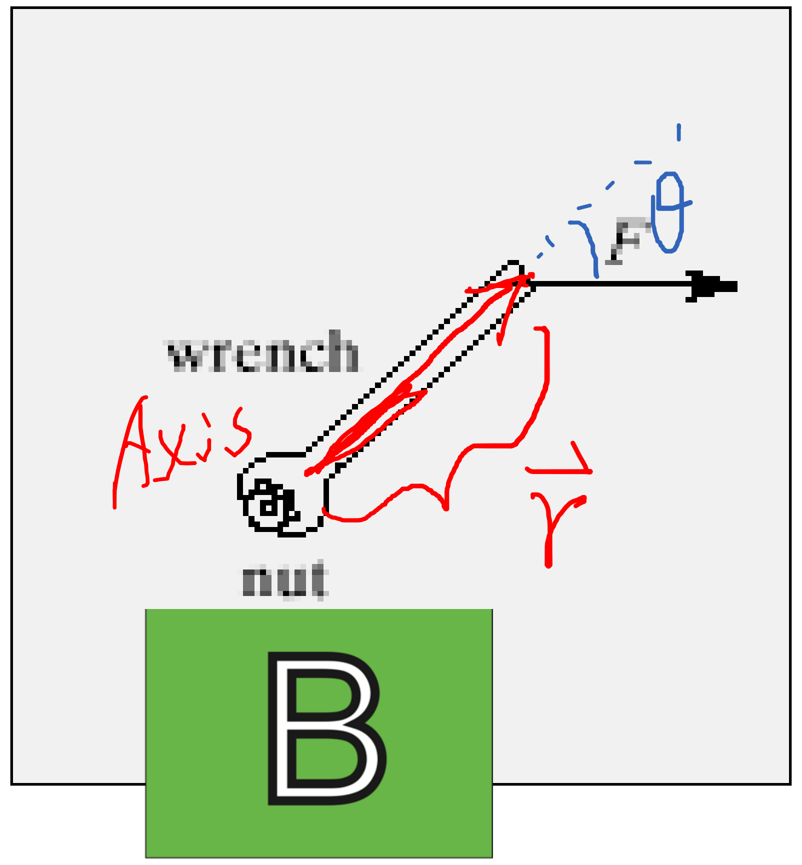
Clicker question 6

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- You are using the same wrench to loosen a rusty nut. Which of the following arrangement will be more effective in loosening the nut?

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\theta_{90^\circ}$$
$$= |\vec{r}||\vec{F}|$$





$0 < \theta_2 < 90^\circ$ ∴ $\sin\theta_2 < 1$

$$|\vec{\tau}_2| = |\vec{r}||\vec{F}|\sin\theta_2$$
$$< |\vec{r}||\vec{F}|$$

6. Newton’s 2nd law for rotation

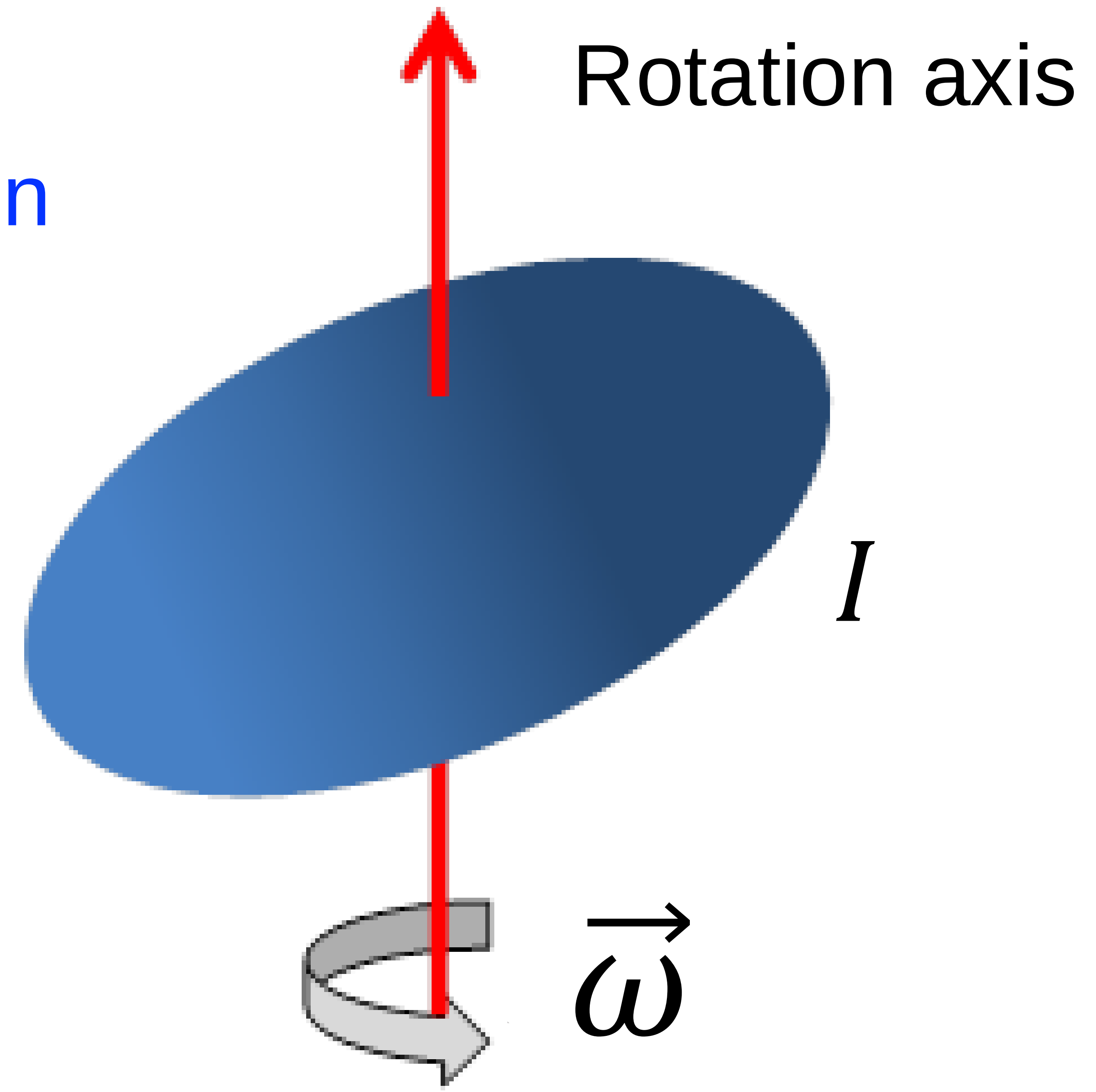
- Newton’s 2nd law for **rotation**:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

net Torque

Angular acceleration

Moment of inertia w.r.t. the axis



Example 2

Given: r, m, $\vec{F}_1$, $\vec{F}_2$

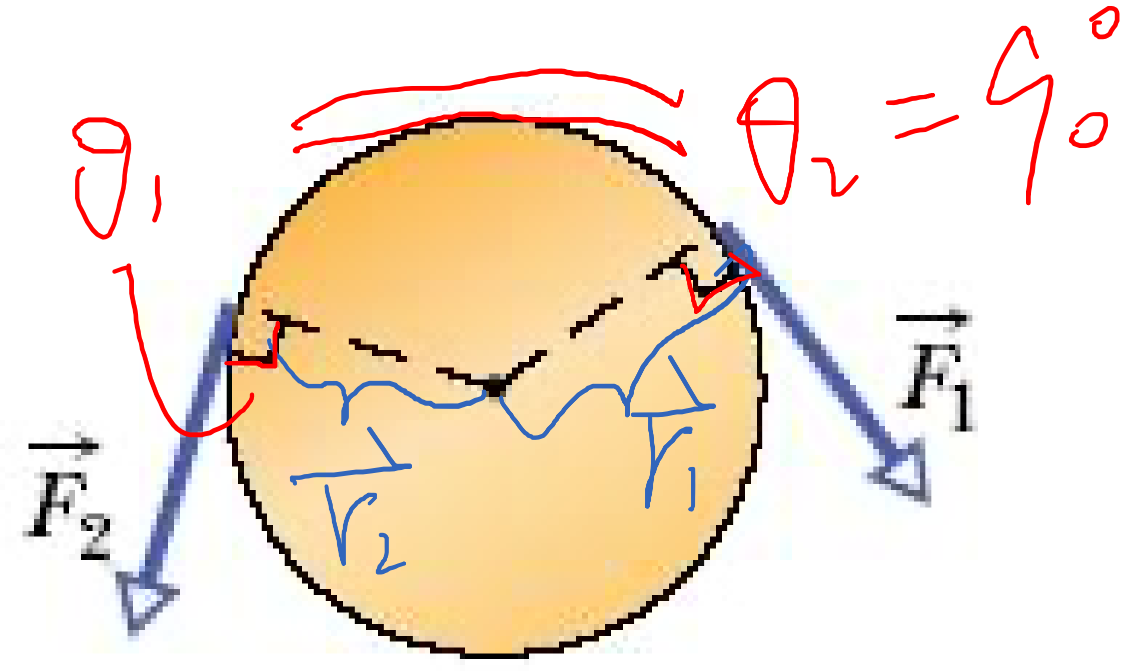
Goal: $\vec{\alpha}$

+z: out of screen

- The figure shows a uniform disk that can rotate around its center. The disk has a radius of r =2.0 cm and a mass of m =20 grams and is initially at rest. Starting at time t = 0, two forces are to be applied tangentially to the rim as indicated. Force $\vec{F}_1$ has a magnitude of 0.10 N, and magnitude $\vec{F}_2$ is 0.20N. What is the angular acceleration of the disk? ***(Hint***: The moment of inertia of the disk w.r.t. the central axis is $I = \frac{1}{2}mr^2$)

Step 1: Newton's 2nd law for Rot.

$$\vec{\tau}_{net} = \vec{I}\,\vec{\alpha}$$



Step 2:

$$\vec{\tau}_1 + \vec{\tau}_2 = \frac{1}{2}mr^2\vec{\alpha}$$

$$\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \frac{1}{2}mr^2\vec{\alpha}$$

$$-r|F_1|\hat{k} + r|F_2|\hat{k} = \frac{1}{2}mr^2\vec{\alpha}$$

Step 3:

$$\vec{\alpha} = \frac{2(|F_2| - F_1)}{mr}\hat{k} = \frac{2(0.2N - 0.1N)}{0.02kg \cdot 0.02m}\hat{k} = 500\,s^{-2}\,\hat{k}$$