# PHYS 225
# Fundamentals of Physics: Mechanics

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**Fall 2024**

**Lecture 25: Newton's 2nd law for a system | Linear momentum | Impulse**

CALIFORNIA STATE UNIVERSITY
FULLERTON

# Learning goals for today

- Newton's 2nd law for a system of particles
- Linear momentum
- Impulse
- Conservation of linear momentum

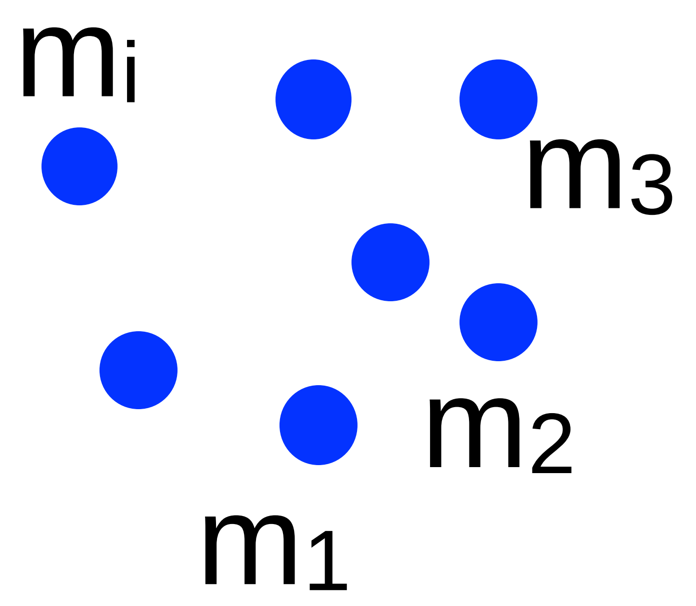
# 1 & 2: Center of mass (COM) position, velocity and acceleration

- Center of mass (position): Weighted average of positions of the particles in the sys.

$$\vec{r}_{com} = \frac{\sum_i m_i \vec{r}_i}{M_{tot}}$$

- Center of mass velocity:

$$\vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{\sum_i m_i \vec{v}_i}{M_{tot}}$$ Total mass



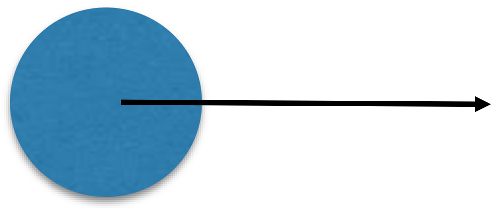

- Center of mass acceleration:

$$\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_i m_i \vec{a}_i}{M_{tot}}$$

# Newton’s 2nd Law for a system of particles

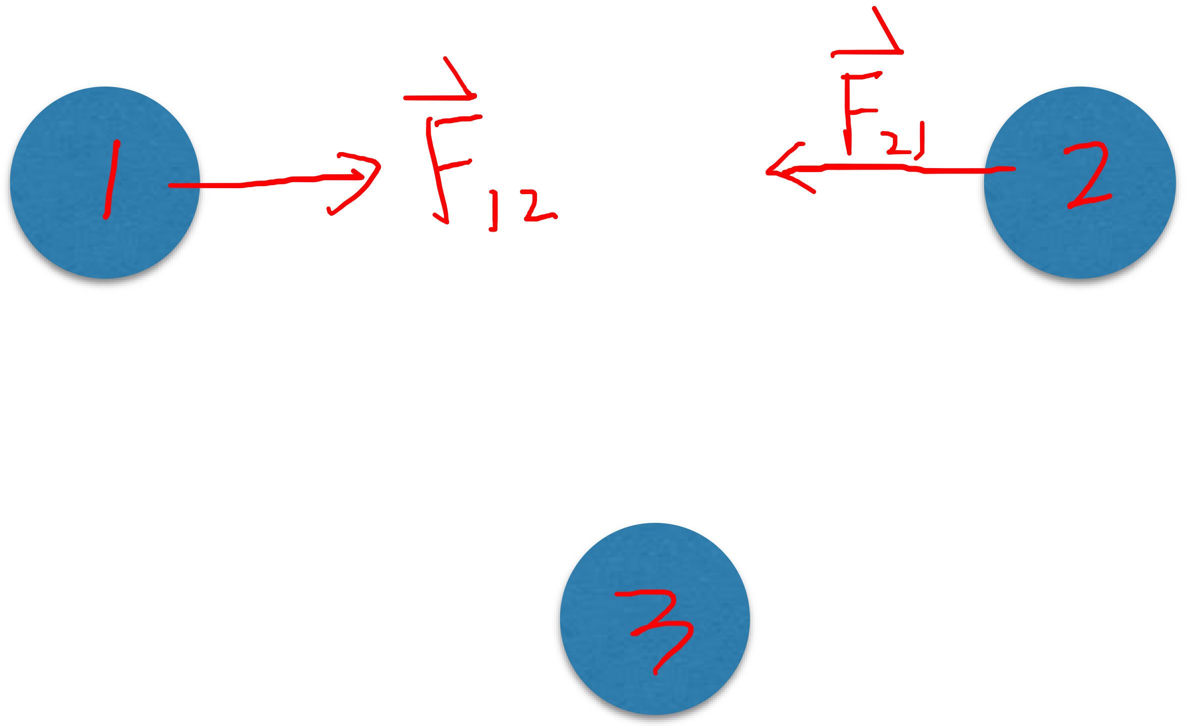
- We already saw, for a single particle:

Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$



- What about the Newton’s 2nd law for many particles?

$$\vec{F}_{tot} = \vec{F}_{external,net} = m_{tot}\vec{a}_{com}$$




3rd law: $\vec{F}_{12} = -\vec{F}_{21}$

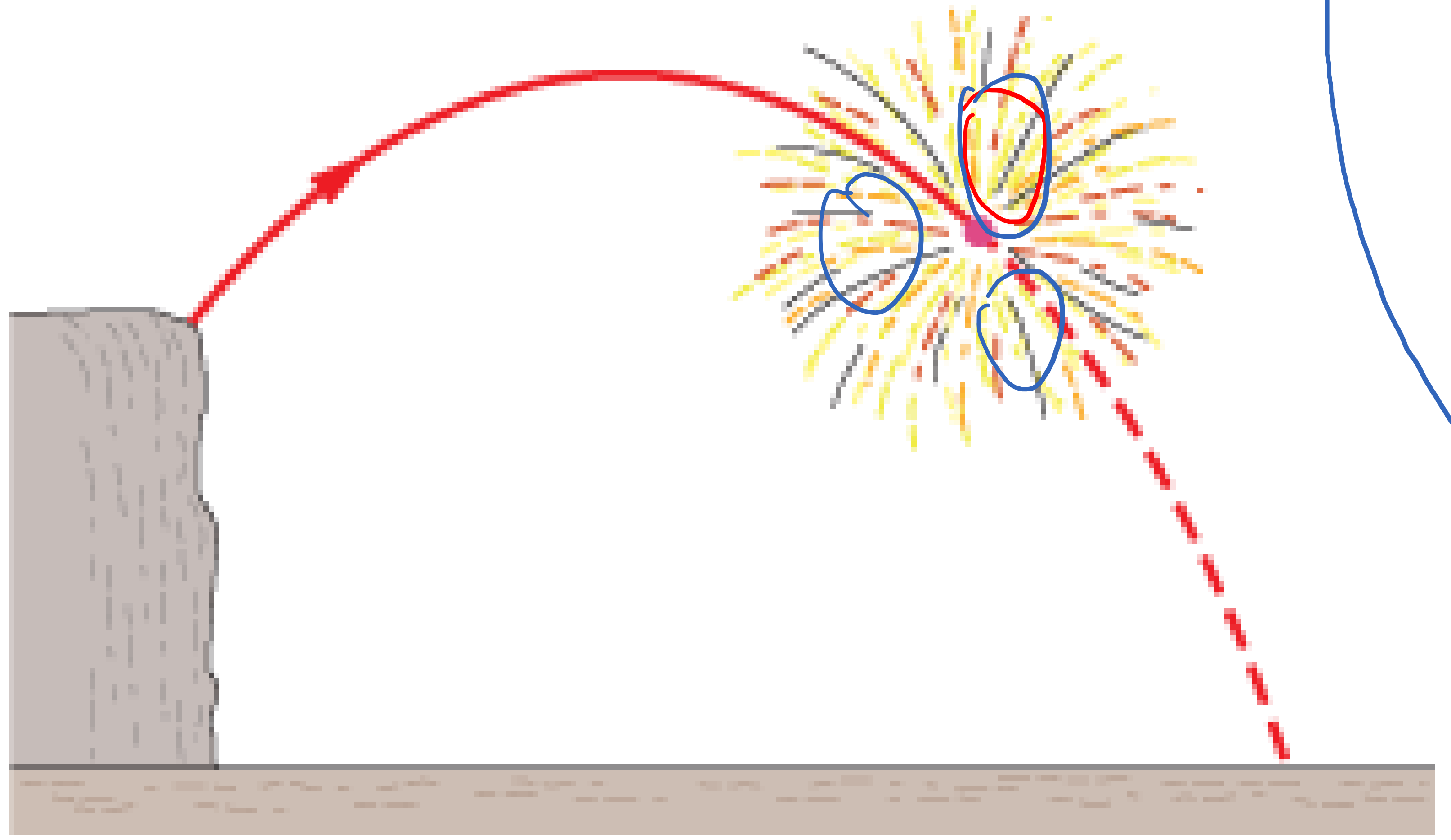
# Clicker Question 3

Near the earth surface

Weight: $\vec{W} = -m_{tot} g \hat{j}$

↑ +y

- The firework explodes, what is the acceleration of the COM of the firework?



$\vec{F}_{ext,net} = m_{tot} \vec{a}_{com}$

| | |
|---|---|
| A | $a_{COM} = 0$ |
| B | $a_{COM} = g$ ↓ |
| C | $a_{COM} = mg$ ↓ |
| D | $a_{COM}$ points ↘ |

# 3. Linear momentum

- Definition of linear momentum:
  - Linear momentum of a single particle: $\vec{P} = \underbrace{m}_{\text{mass}}\underbrace{\vec{v}}_{\text{velocity}}$ (Unit: kg m/s)

    mass Vel.

  - Can be exchanged between objects

# Linear momentum of a system of objects

- Linear momentum of a system of objects: $\vec{P}_{tot} = \sum_i \vec{P}_i = \underbrace{\sum_i m_i \vec{v}_i}$

$$= m_{tot} \Bigg( \underbrace{\frac{\sum_i m_i \vec{v}_i}{m_{tot}}}_{\vec{v}_{com}} \Bigg) = m_{tot} \vec{v}_{com}$$

- Center of mass velocity and linear momentum: $\vec{P}_{tot} = M_{tot} \vec{v}_{com}$

# Clicker question 4

Given: $m_A = 10\,kg$, $m_B = 5\,kg$, $\vec{v}_A = -3\,m s^{-1}\hat{i}$, $\vec{v}_B = 5\,m s^{-1}\hat{i}$

+x

Goal: $\vec{P}_{tot}$

$$\vec{P}_{tot} = m_A \vec{v}_A + m_B \vec{v}_B = 10\,kg \times (-3\,m s^{-1}\hat{i}) + 5\,kg \times (5\,m s^{-1}\hat{i})$$

$$= -5\,kg\,m s^{-1}\hat{i}$$

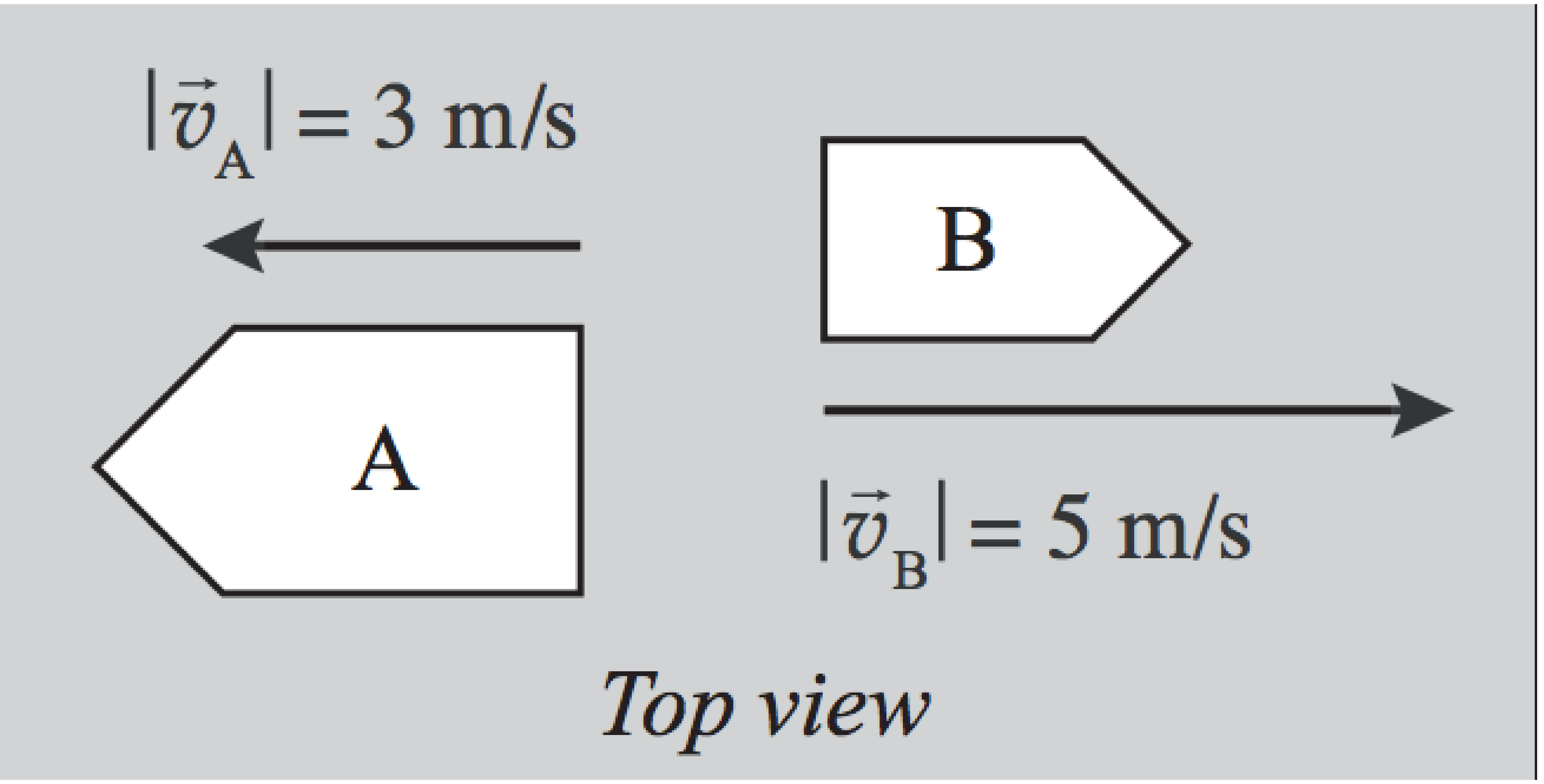
- Boat A has mass 10 kg, and boat B has mass 5 kg. They are seen moving as shown. Which way does the total momentum of the system of the two boats point at?

A To the left

B To the right

C The total momentum is zero.

D Not enough information to know




+x

$$\vec{P}_{tot} = \sum_i \vec{P}_i = \sum_i m_i \vec{v}_i$$

# 4. Newton's 2nd law in terms of linear momentum, $\vec{P}$

- Newton's 2nd law: $\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{P}}{dt}$

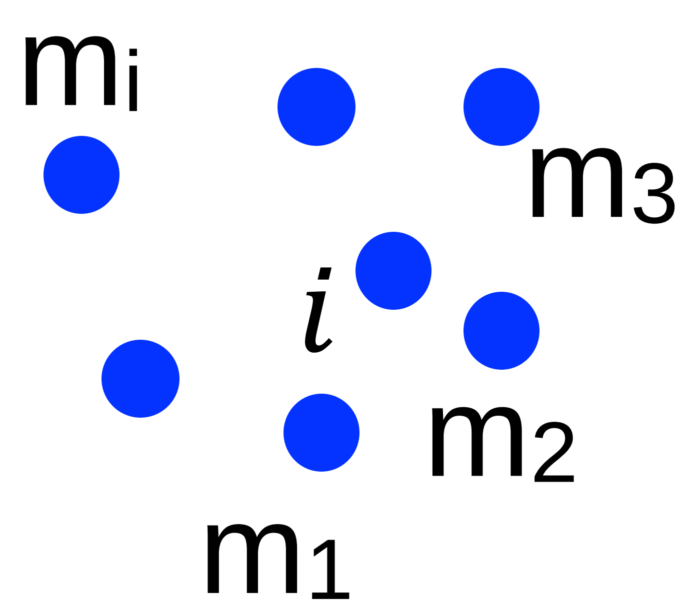
- Newton's 2nd law in terms of linear momentum for a single particle:

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

**The net force is equal to the rate of change of linear momentum.**

- Newton's 2nd law in terms of linear momentum for a system of particles:

$$\boxed{\vec{F}_{ext,tot} = \frac{d\vec{P}_{tot}}{dt}}$$



# Clicker Question 5

$+x$

$$\boxed{\vec{F}_{external,tot} = m_{tot}\vec{a}_{com}} = \frac{d\vec{P}_{tot}}{dt}$$


What is the rate of change of the total linear momentum, $\frac{d\vec{P}_{tot}}{dt}$?

A $\frac{d\vec{P}_{tot}}{dt} = 5\ N\ \hat{\imath}$

B $\frac{d\vec{P}_{tot}}{dt} = 25 N\hat{\imath}$

# 5. Impulse & change of momentum

Unit: N·s

- **Definition of impulse:** Integral of force over time

$$\vec{I} = \int \vec{F} dt \approx \vec{F}_{avg}\underbrace{\Delta t}_{\text{Time duration}}$$

- Recall Newton's 2nd law in terms of momentum:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \vec{F}_{net}\, dt = \vec{I}_{net}$$

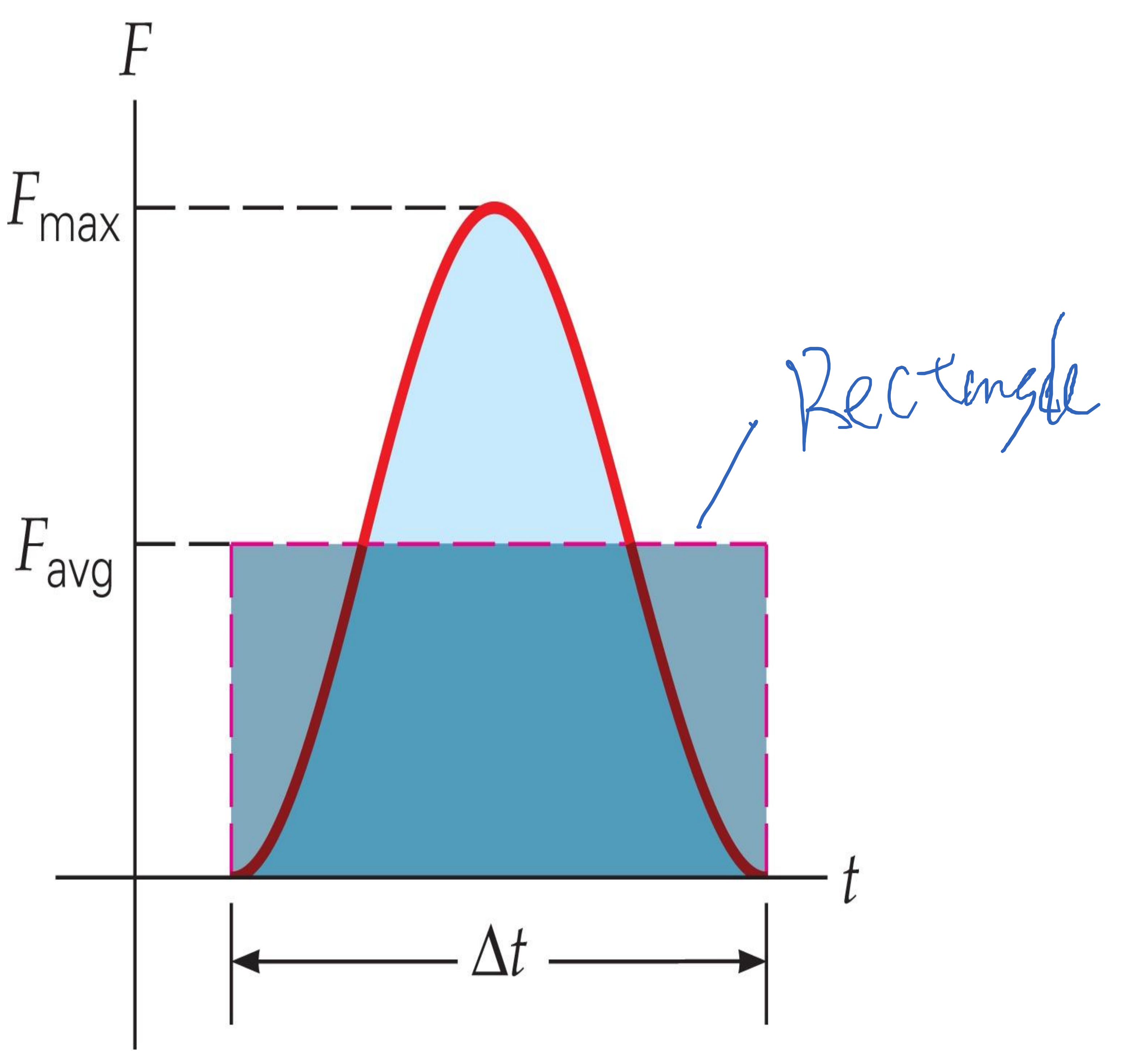
$$\approx \vec{F}_{Ave}\Delta t$$

$$\rightsquigarrow \vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \vec{F}_{net} dt$$

- Therefore, the net impulse is the change of momentum over the same time period:

$$\vec{P}_f - \vec{P}_i = \vec{I}_{net}$$

*Impulse = area under force vs.* ***time*** *curve*



# Clicker question 6

- Impulse has the same unit as that of

| | |
|---|---|
| A | Energy |
| B | Work |
| C | Linear momentum |
| D | Power |

$$\vec{P}_f - \vec{P}_i = \vec{I}_{net}$$

# Impulse: Applications

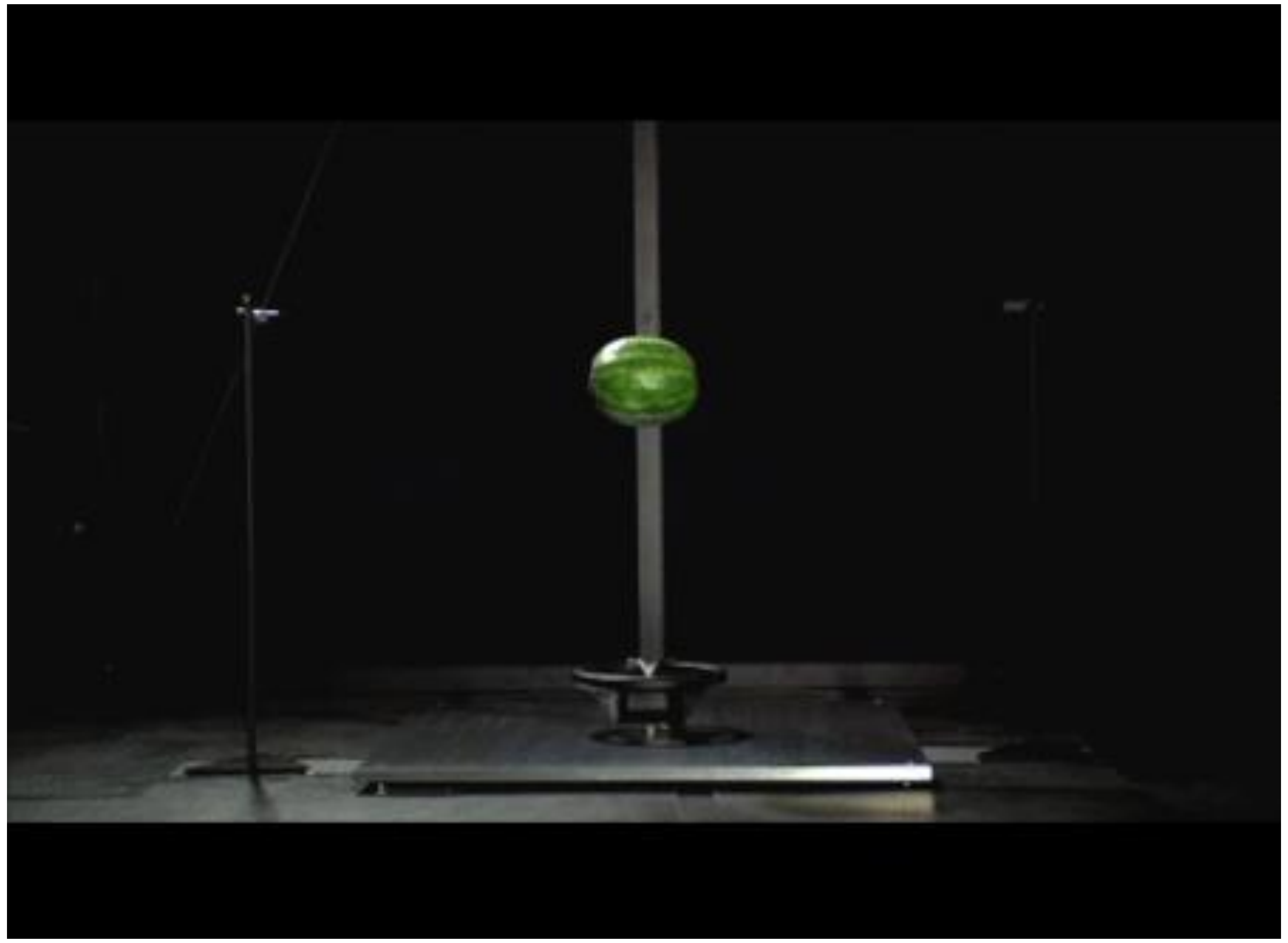
This is how airbags work—they **slow down** collisions considerably—and why cars are built with crumple zones.

$$\vec{I}_{net} = \vec{F}_{avg}\Delta t = \underbrace{\vec{P}_f - \vec{P}_i}$$

For the same $\Delta\vec{P}$

For given $\vec{P}_f$ and $\vec{P}_i$, then $\Delta t$ is increased, $|\vec{F}_{avg}|$ can be decreased.

Longer $\Delta t$ $\rightarrow$ smaller $\vec{F}_{avg}$.



reddit.com

Video: crash test

http://www.youtube.com/watch?v=d7iYZPp2zYY&feature=related

# Clicker question 7

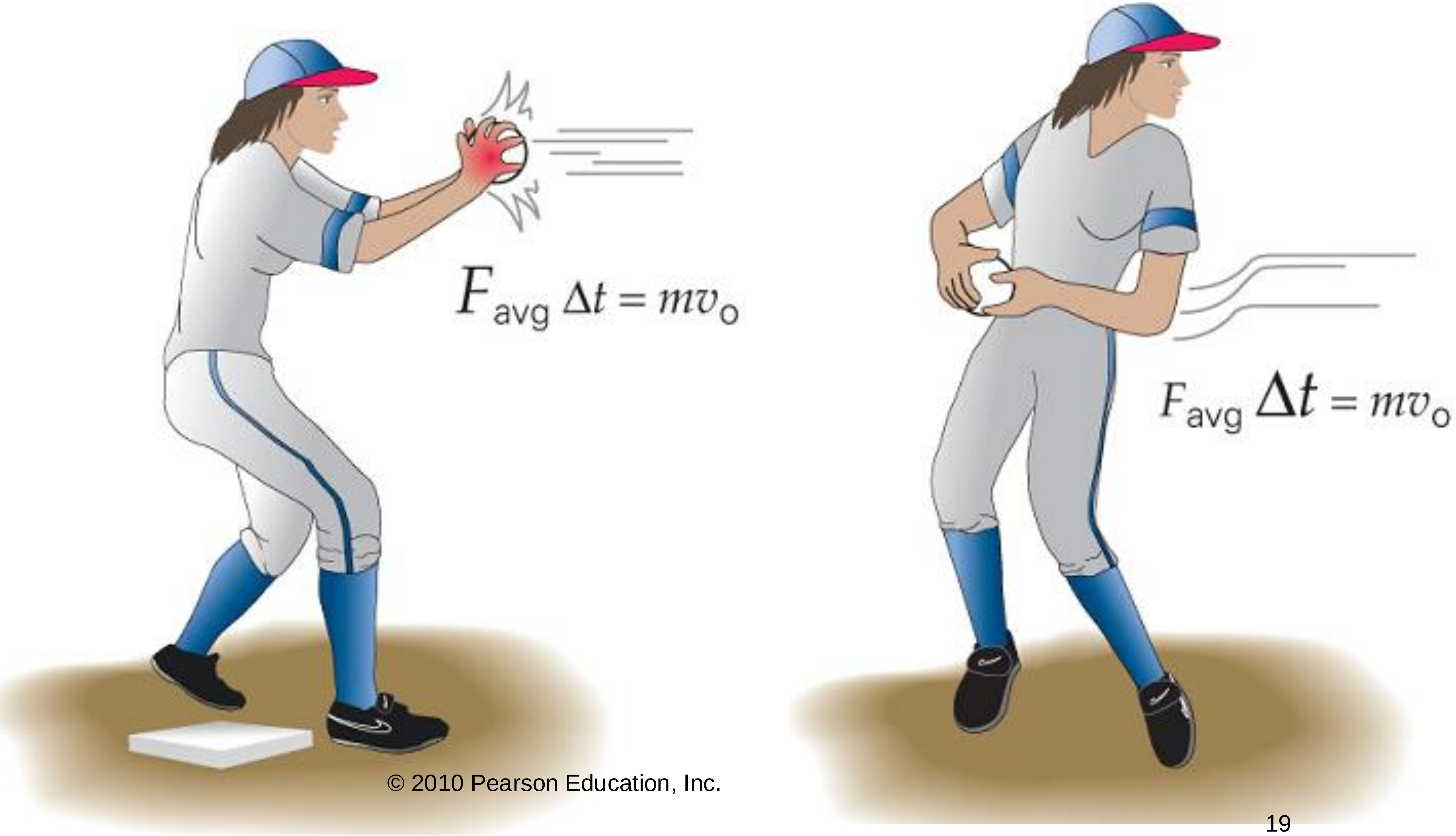
Neglect $\vec{W}$

$F_{ave}\Delta t = \vec{I}_{net} = \vec{P}_f - \vec{P}_i$

$0 \qquad m\vec{v}_0 \quad \rightarrow$

A baseball player wants to catch a ball of mass $m$ and initially moving at velocity $v_0$. To reduce the force between the hand and the ball, is it preferred to catch the ball hardly (fixed hands, short time), or softly (backward moving hands, long time)?

http://www.youtube.com/watch?v=yUpiV2I_IRI




A — Hard catch, small $\Delta t$

B — Soft catch, large $\Delta t$

# Clicker question 8

- The National Transportation Safety Board is doing crashing test on a new car model. The vehicle of mass $m$, moving at an initial velocity $\vec{v}_i$ ,collides with a stationary wall, which stops it at time $t$. What principle to use to calculate the magnitude of the average net force that acts on the car during the impact?

A Conservation of mechanical energy

B $\vec{P}_f - \vec{P}_i = \vec{I}_{net} = \vec{F}_{avg}\Delta t$

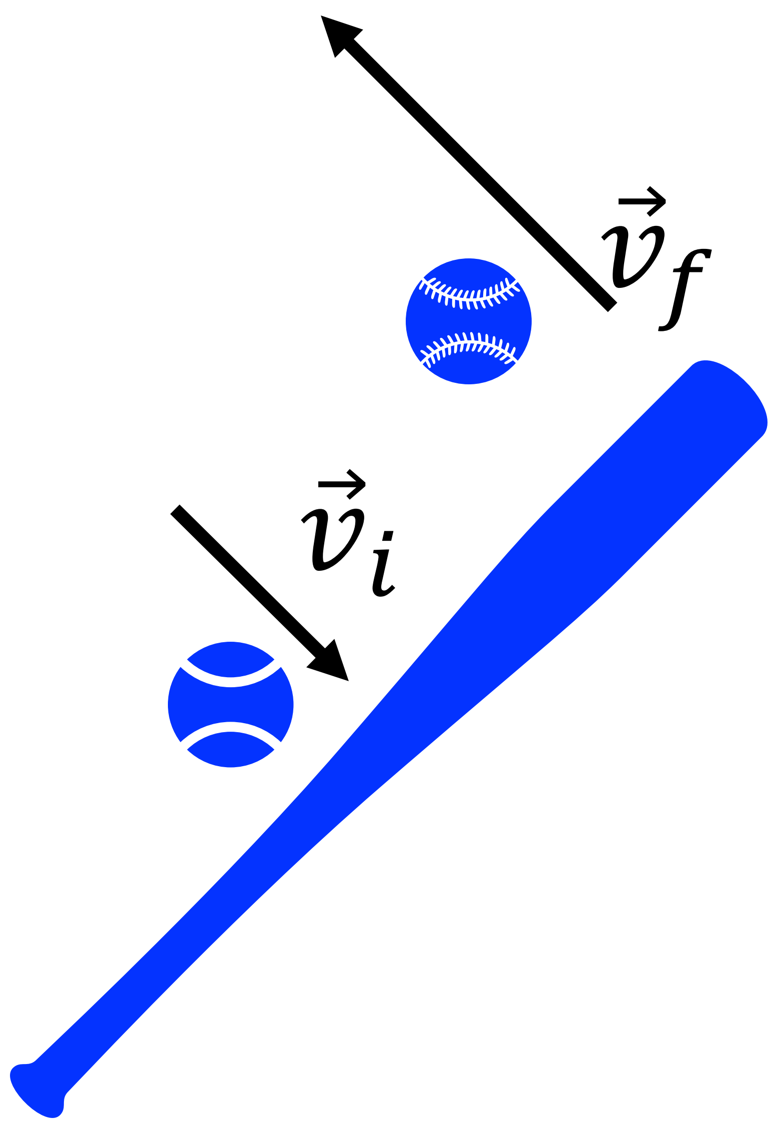
C Conservation of linear momentum

# Example 1

Given: $m, \vec{v}_0, \vec{v}_i, \Delta t$

Goal: $|\vec{F}_{ave}|$

$+x$




- A 150 g baseball pitched at a speed of 44 m/s is hit straight back to the opposite direction at a speed of 57 m/s. What is the magnitude of the average force on the ball if the bat is in contact with the ball for 3.0 ms?

Step 1: $\vec{I}_{net} \approx \vec{F}_{ave} \Delta t = \vec{P}_f - \vec{P}_i$

Step 2: $\vec{F}_{ave} \Delta t = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$

Step 3: $\vec{F}_{ave} = \dfrac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = \dfrac{0.15\,kg(-57\,ms^{-1}\,\hat{i} - 44\,ms^{-1}\,\hat{i})}{0.003\,s}$

$= -5050\,N\,\hat{i}$

Step 4: $|\vec{F}_{ave}| = 5050\,N$

# 6. Conservation of linear momentum

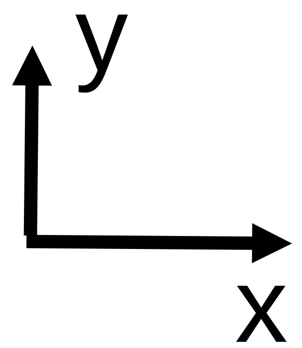
Conditional

- **Conservation of linear momentum:** If the total external force or if the net impulse is 0, then total linear momentum is conserved:

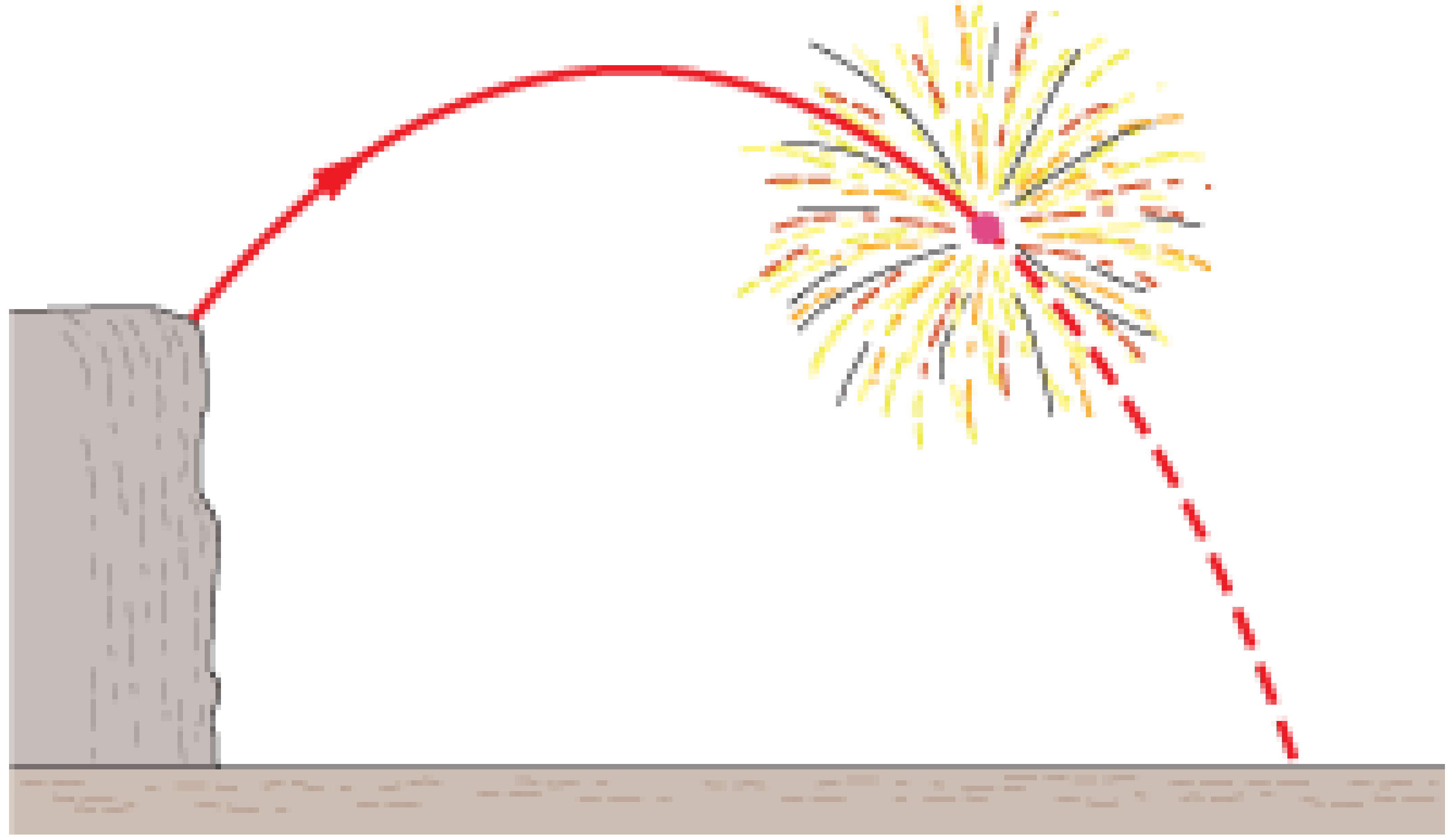
Condition

If $\vec{F}_{ext,tot} = 0$ or $\vec{I}_{net} = 0$, Then $\vec{P}_{tot,f} = \vec{P}_{tot,i}$.

# Clicker Question 9




The firework of total mass ***M*** moves at a velocity $\vec{v}$, and explodes into three pieces, ***M₁, M₂ and M₃.*** ***$M_1$ > $M_2$ >$M_3$***,. Assuming explosion occurs so quickly ($t_f \approx t_i$ (that the impulse by the external force is negligible during explosion, i.e., $\vec{I}_{net} \approx 0$ .What is the total momentum of the three pieces right after the explosion?



$$\Delta\vec{P} = \vec{I}_{net}$$

Here $\vec{I}_{net} = 0$

$$\vec{P}_i = M\vec{v} = \vec{P}_f$$

A $\vec{P}_{after} = 0$

B $\vec{P}_{after} = M\vec{v}$

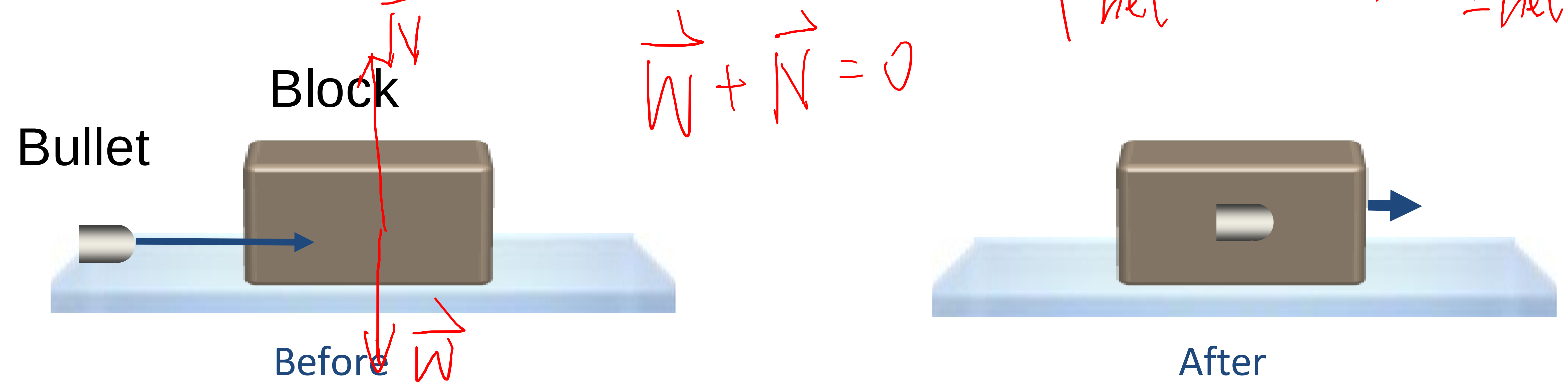
C $\vec{P}_{after} = M_1\vec{v}$

# Clicker Question 10

Bullet hits and lodges into a stationary block on a ***Frictionless horizontal table***. Is the momentum for the ***system containing both the block and the bullet*** conserved?

$\vec{F}_{net} = 0 \rightarrow \vec{I}_{net}$

$\vec{W} + \vec{N} = 0$



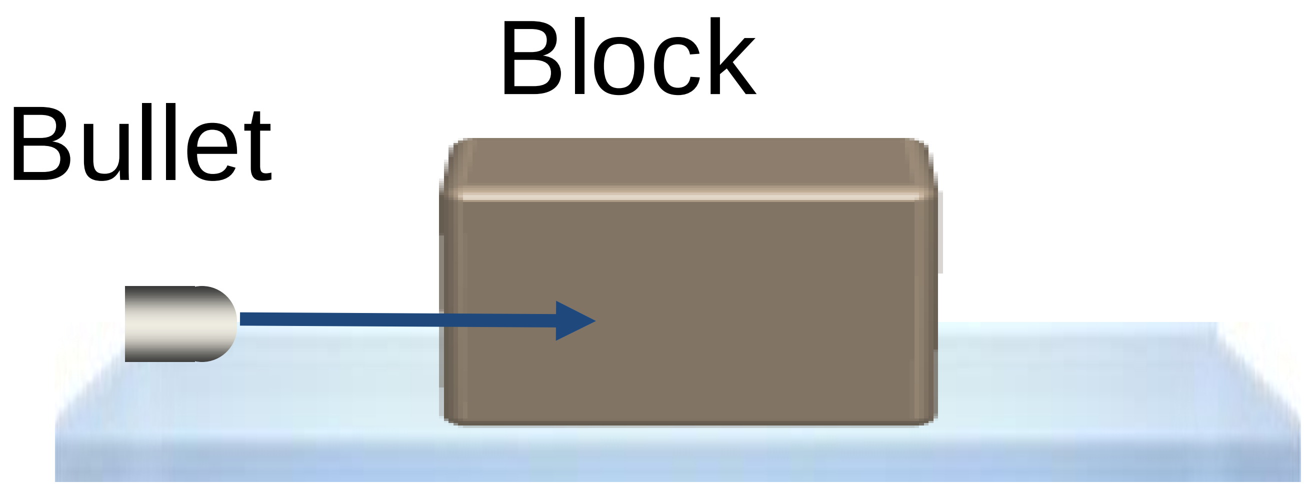

A Yes, total momentum is always conserved regardless of the condition

B Yes, there is no net force acting on the system

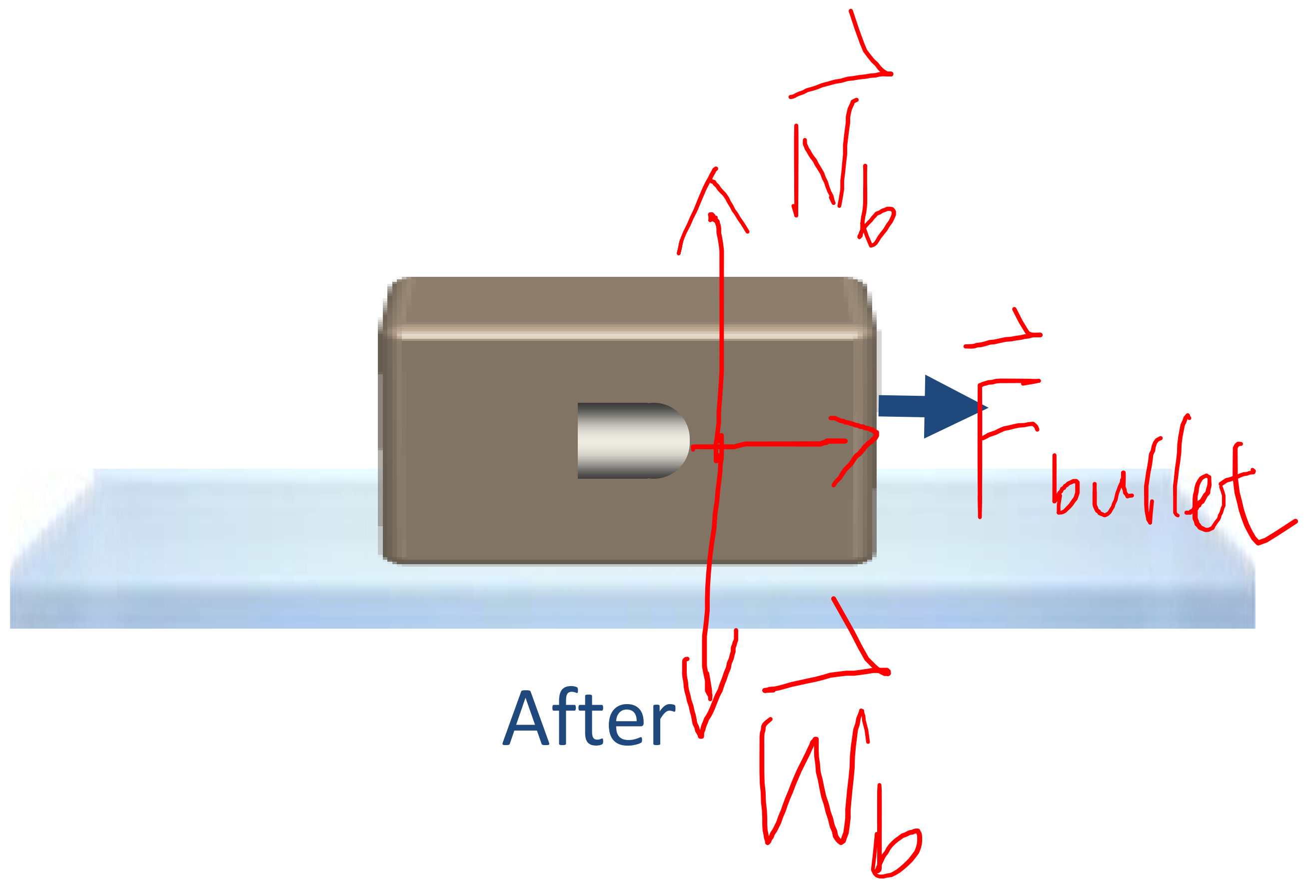
C No, the bullet and block exert a force on each other during the collision

# *Clicker Question 11*

Bullet hits and lodges into a stationary block on a ***Frictionless horizontal table***. Is the momentum for the ***block*** conserved?




Before




After

$\vec{W}_b + \vec{N}_b \approx 0$

$\vec{I}_{net} \neq 0$

A Yes, momentum is always conserved

B Yes, there is no net force acting on the block

C No, the bullet and block exert a force on each other during the collision

# Example 2

Given: $\vec{v}_0$, $m_1 = m$, $\vec{v}_1$, $m_2 = m$, $\vec{v}_2$, $m_3 = 3m$

Goal: $\vec{v}_3$

y

x

A vessel **at rest** at the origin of an *xy* coordinate system explodes into three pieces. Just after the explosion, one piece, of mass ***m***, moves with velocity $\vec{v}_1$ =-30m/s $\vec{\imath}$ and a second piece, also of mass ***m***, moves with velocity $\vec{v}_2$ = -30m/s $\hat{\jmath}$ . The third piece has mass ***3m***. What is the velocity of the third piece, $\vec{v}_3$ , right after explosion?

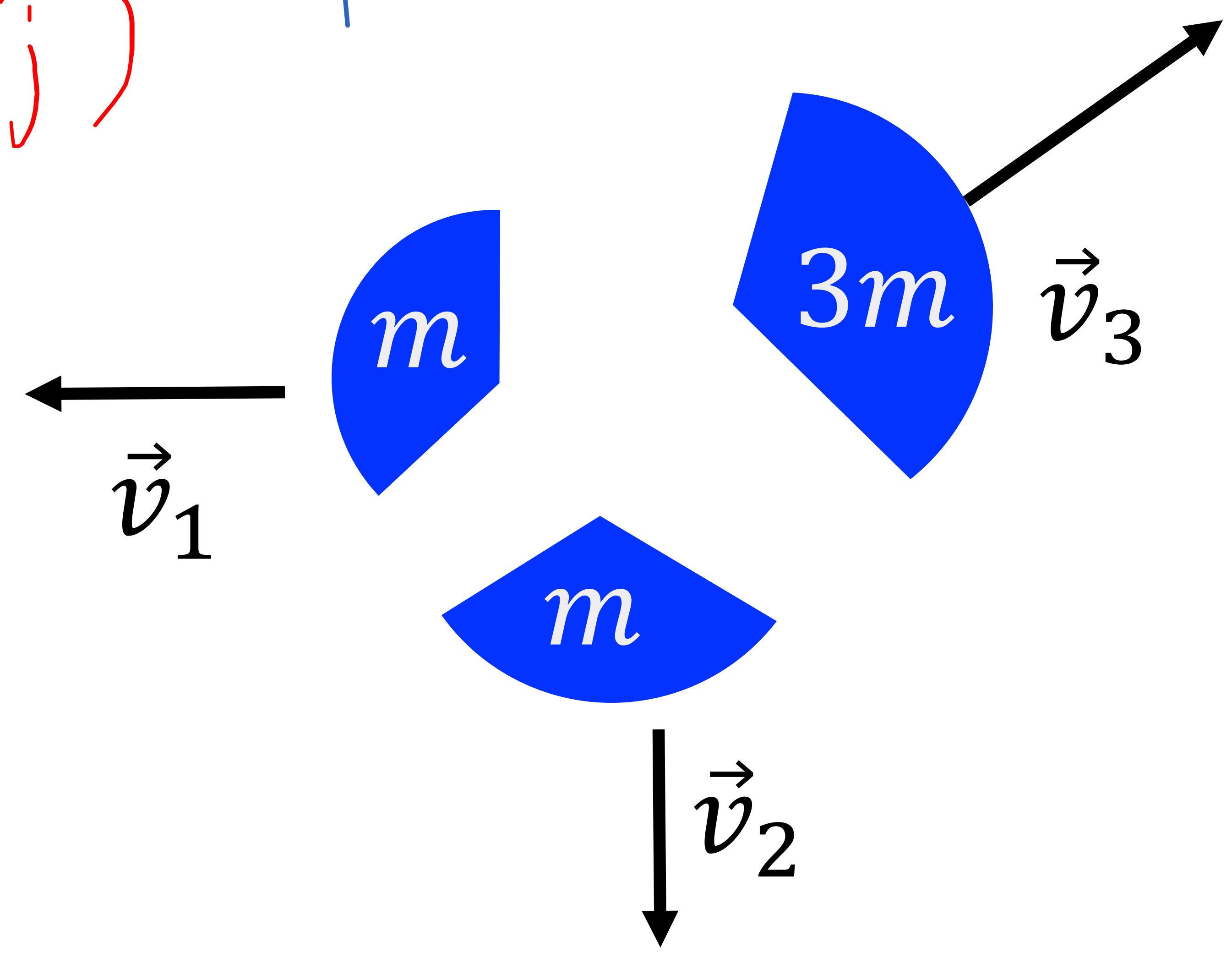
Step 1: $\Delta t \approx 0 \rightarrow \vec{I}_{net} \approx 0$ during explosion:

then: $\vec{P}_f = \vec{P}_i = M_{tot}\vec{v}_0 = 0$

$\vec{P}_f = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = \cancel{m}\vec{v}_1 + \cancel{m}\vec{v}_2 + 3\cancel{m}\vec{v}_3 = 0$

Step2: $\rightarrow \vec{v}_3 = -\frac{1}{3}(\vec{v}_1 + \vec{v}_2) = -\frac{1}{3}(-30ms^{-1}\hat{i} - 30ms^{-1}\hat{j})$

$= 10ms^{-1}\hat{i} + 10ms^{-1}\hat{j}$



# Summary:

- Momentum: $\vec{P} = m\vec{v}$
- Newton's 2nd law for a point particle: $\vec{F}_{net} = \frac{d\vec{P}}{dt}$
- Newton's 2nd law for a system: $\vec{F}_{ext,tot} = \frac{d\vec{P}_{tot}}{dt}$ $= M_{tot}\vec{a}_{com}$
- Impulse: $\vec{I} = \int \vec{F}dt$
- Impulse and change of linear momentum: $\vec{I}_{net} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$
- Conservation of linear momentum: If $\vec{I}_{net} = 0$ ,$\vec{P}_{tot,final} = \vec{P}_{tot,init}$.

# Prelecture 9.1.3

- Before the next class.