PHYS 225
Fundamentals of Physics: Mechanics

Prof. Meng (Stephanie) Shen

Fall 2024

Lecture 6: Vectors

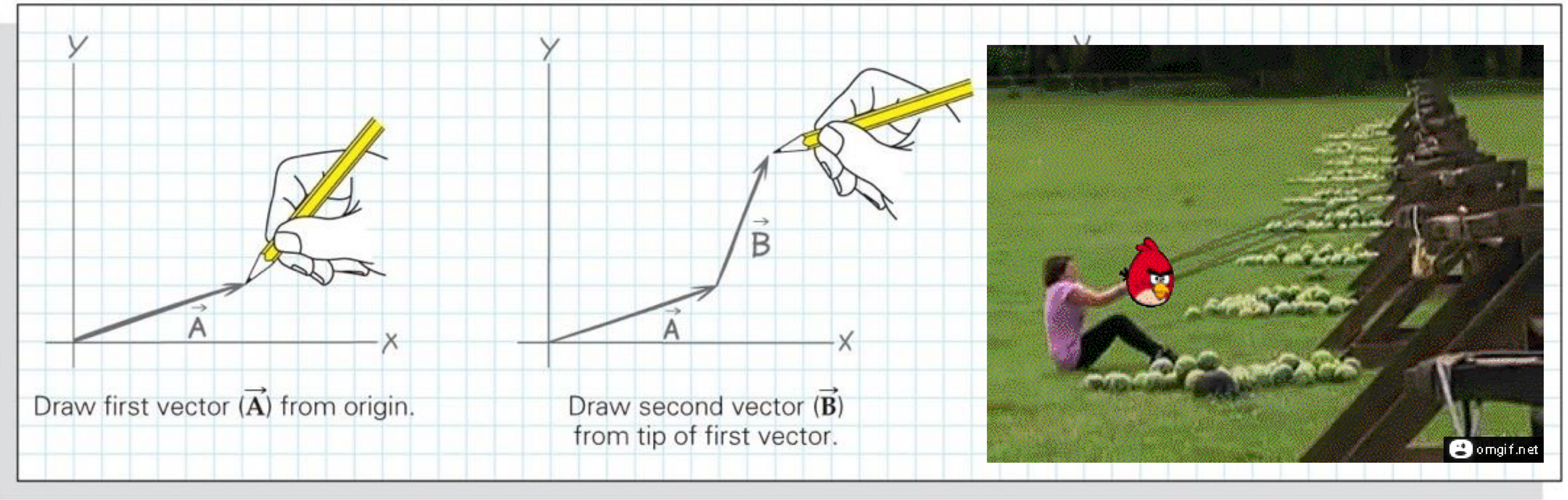
CALIFORNIA STATE UNIVERSITY
FULLERTON

Chapter 3: Vectors

- Learning goals:
  - Apply vector operation in two and three dimensions
    - Vector addition and Vector components
    - Vector multiplication

Learning goals for today

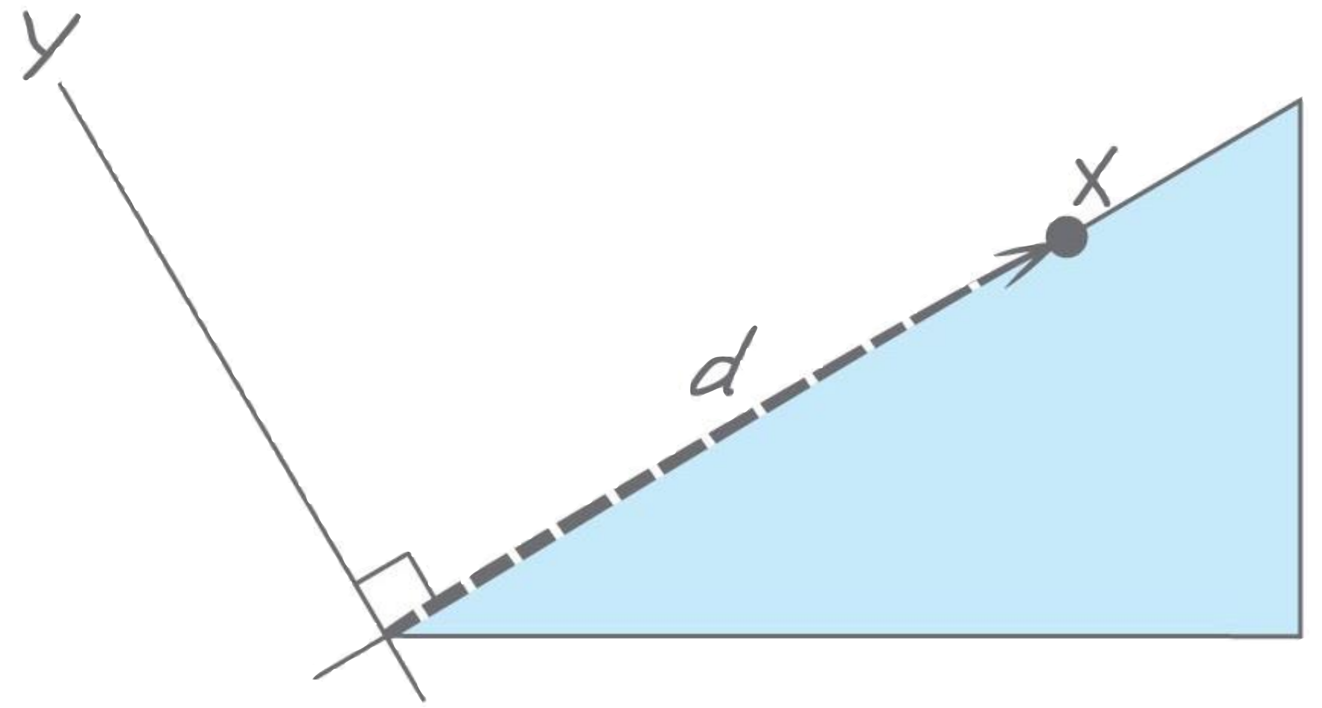
- Concepts about vectors in higher dimensions
- Vector addition
  - Head-tail rule
  - By components
- Vector decomposition



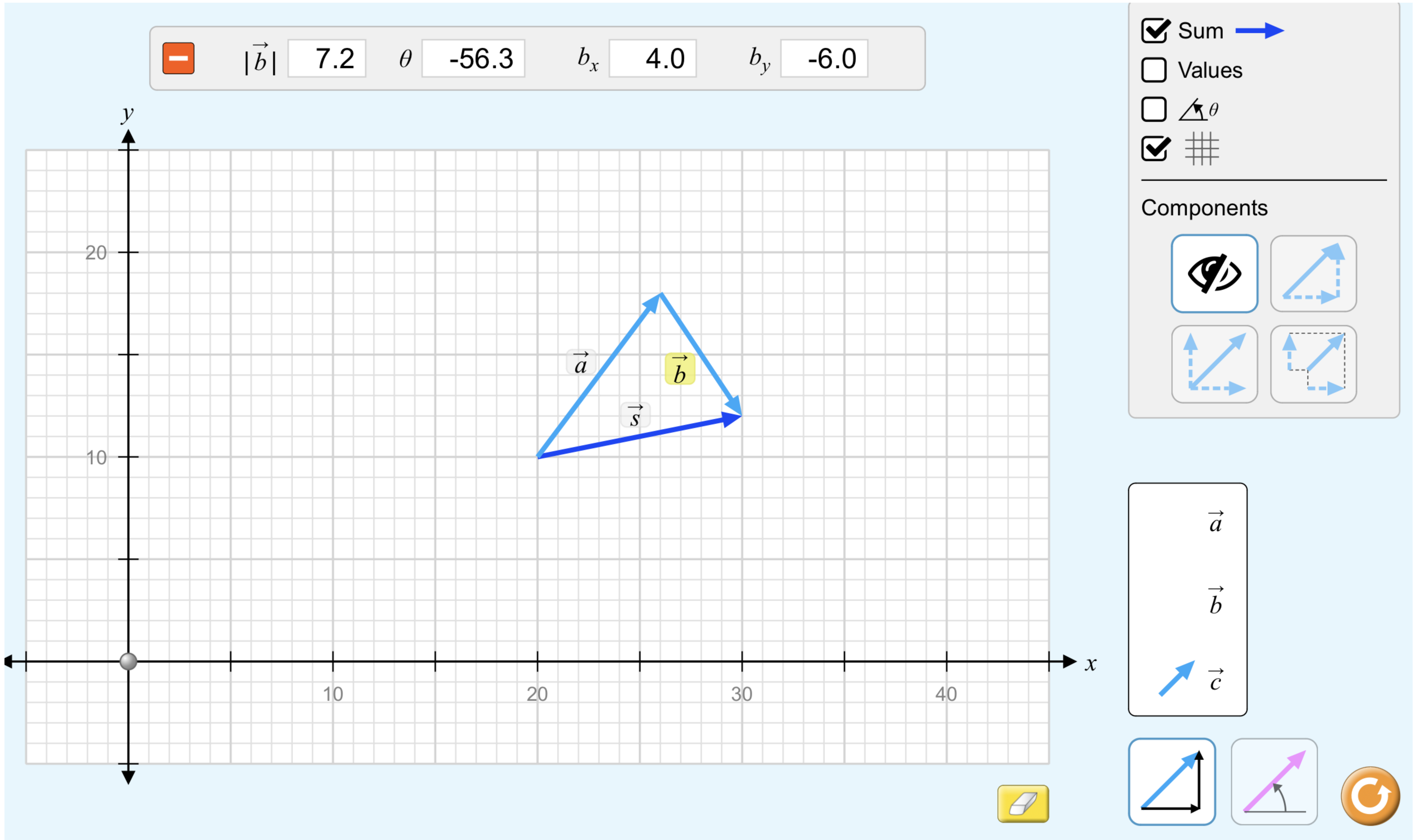
$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

Vectors

- Vectors has magnitude and direction
  - **Magnitude** = Size of the vector
  - **Direction**: from start point to end point
- 1D vectors
  - Sign (+/–) to show direction
- What about 2D? 3D?



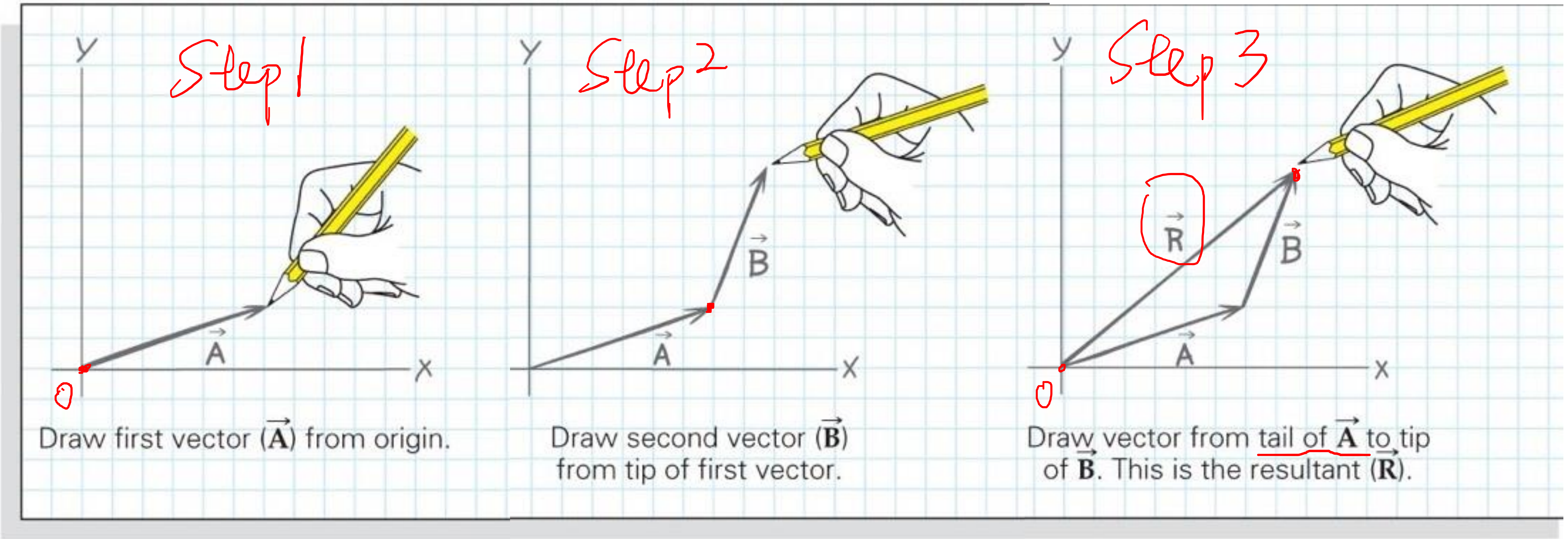
Demo



https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_all.html

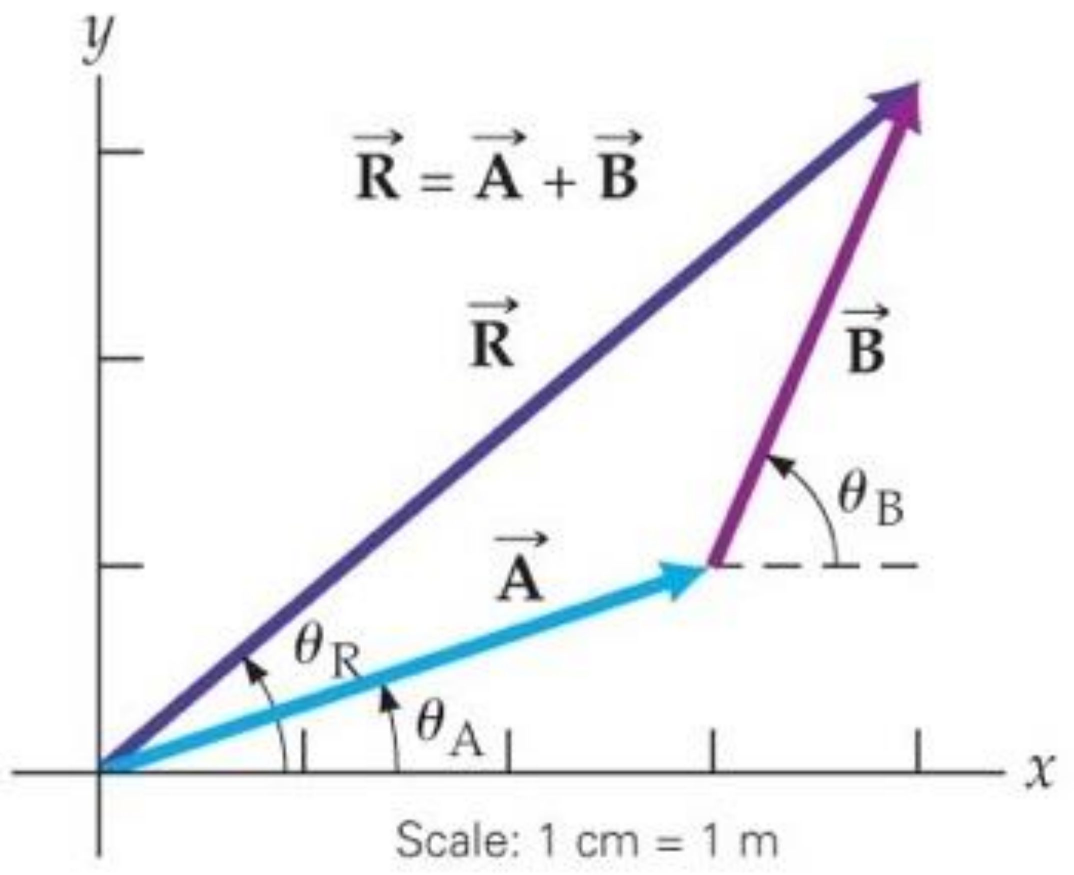
Adding vectors geometrically in higher dimensions

Adding vectors geometrically in higher dimensions

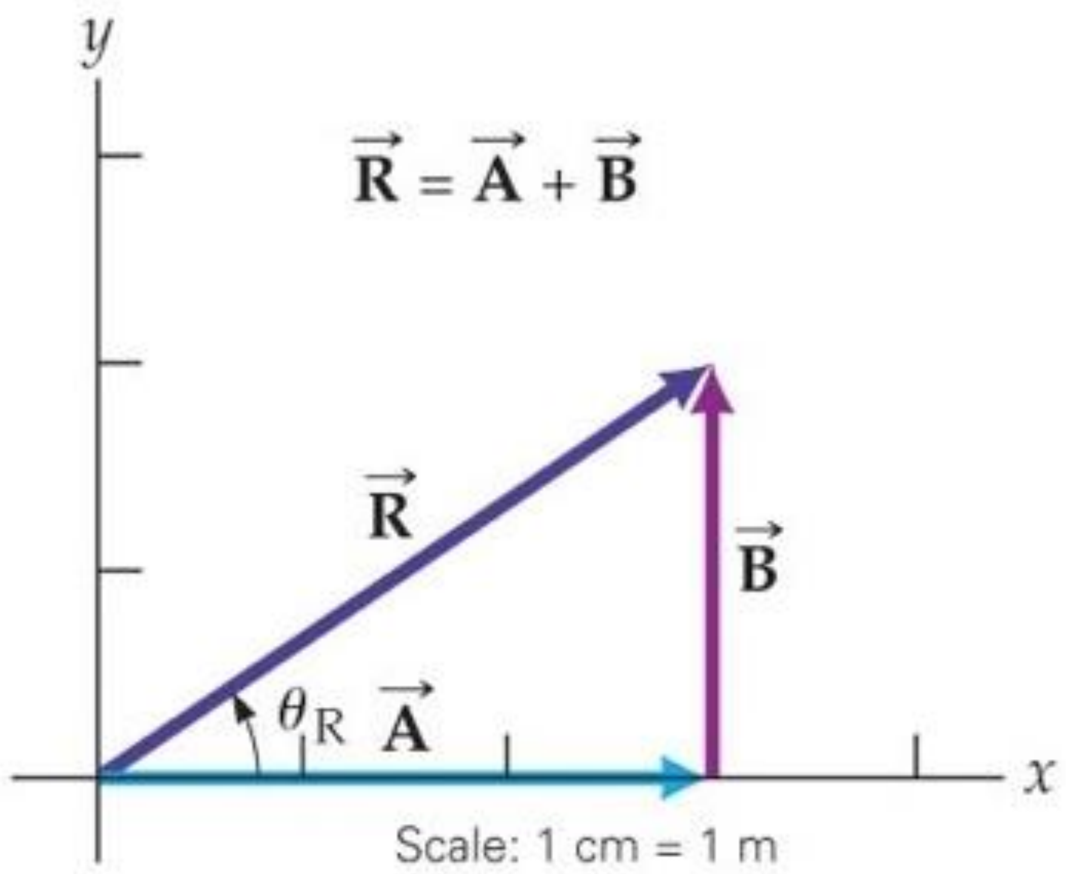


$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

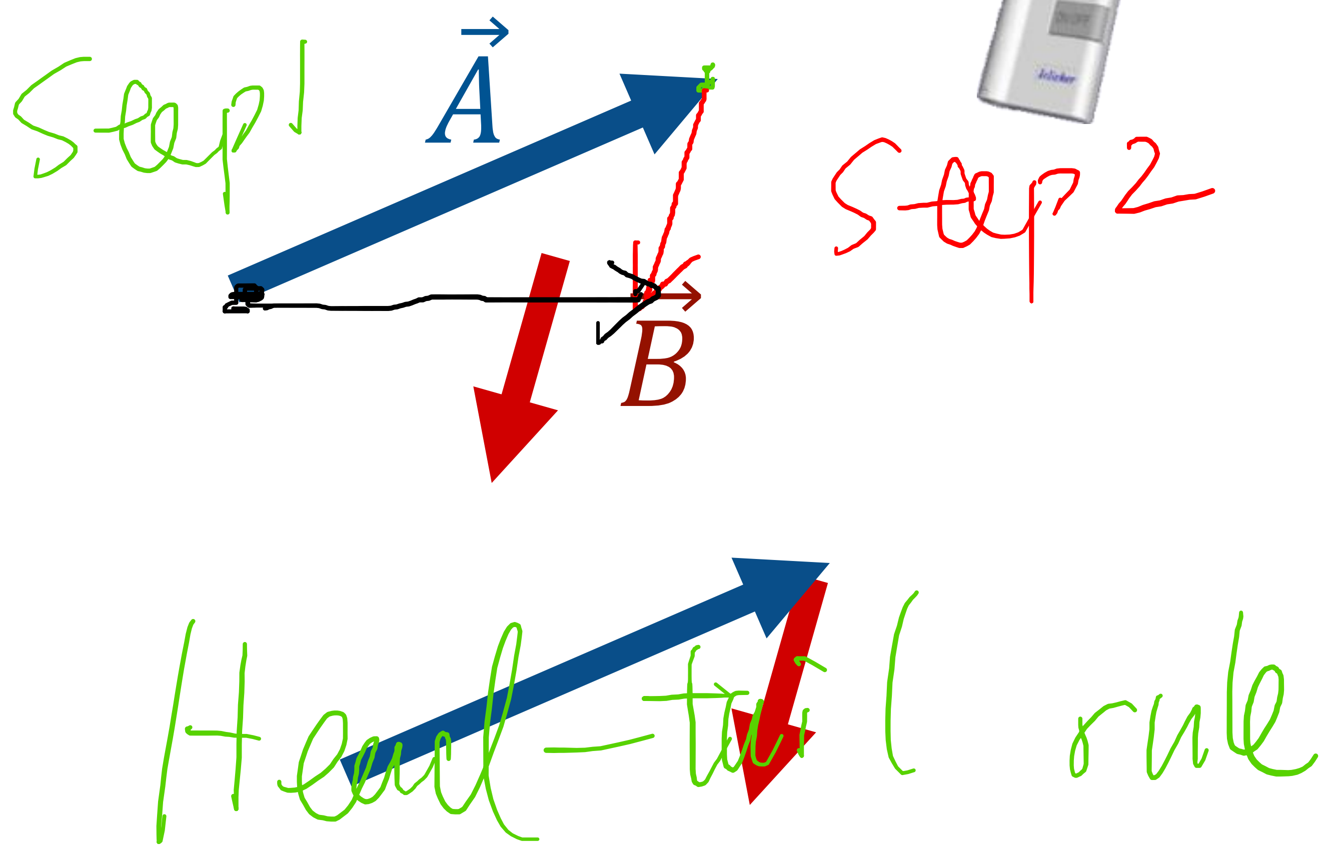
Head-tail rule



Example 1



Example 2



Vectors $\vec{A}$ and $\vec{B}$ are shown to the right.

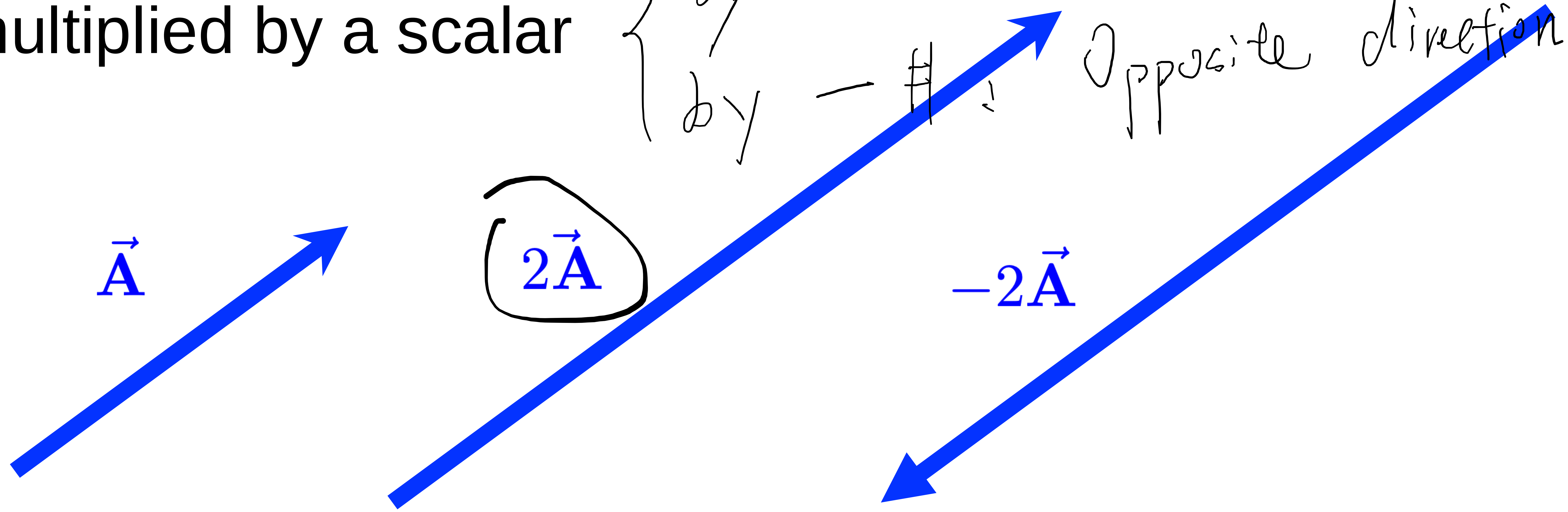
Which of the following best describes $\vec{A} + \vec{B}$?

A B C

Vector scaling

- Vector multiplied by a scalar

by + #: Same direction
by − #: Opposite direction



Multiplying a vector by a scalar changes the length

The direction reverses also, if the scalar is negative

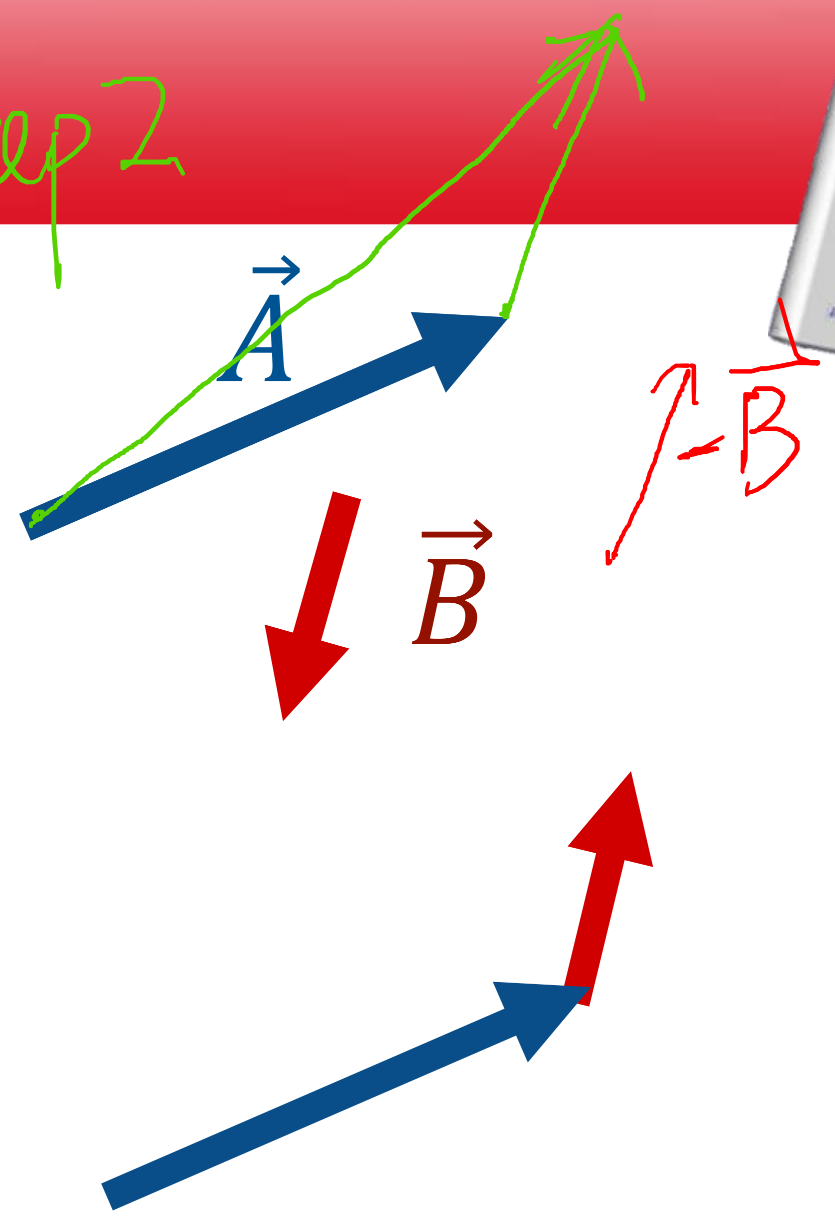
Clicker Question 2

Vectors $\vec{A}$ and $\vec{B}$ are shown on the right.

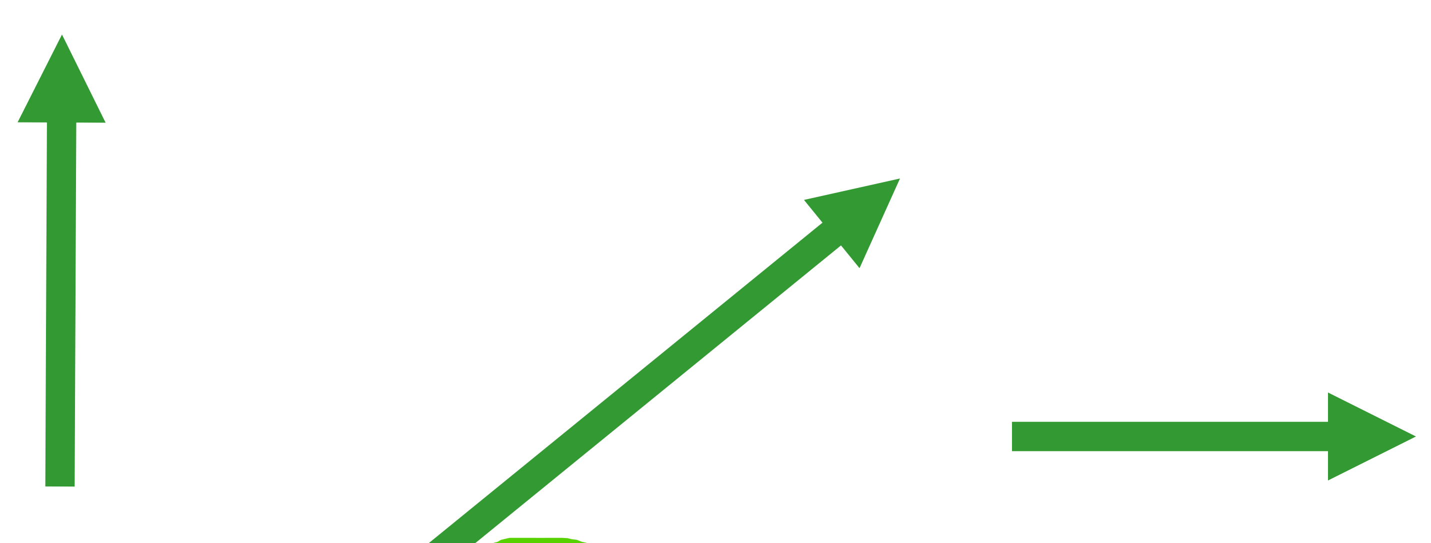
Which of the following best describes $\vec{A} - \vec{B}$?

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Step 2



Step 1: Scaling



A B C D

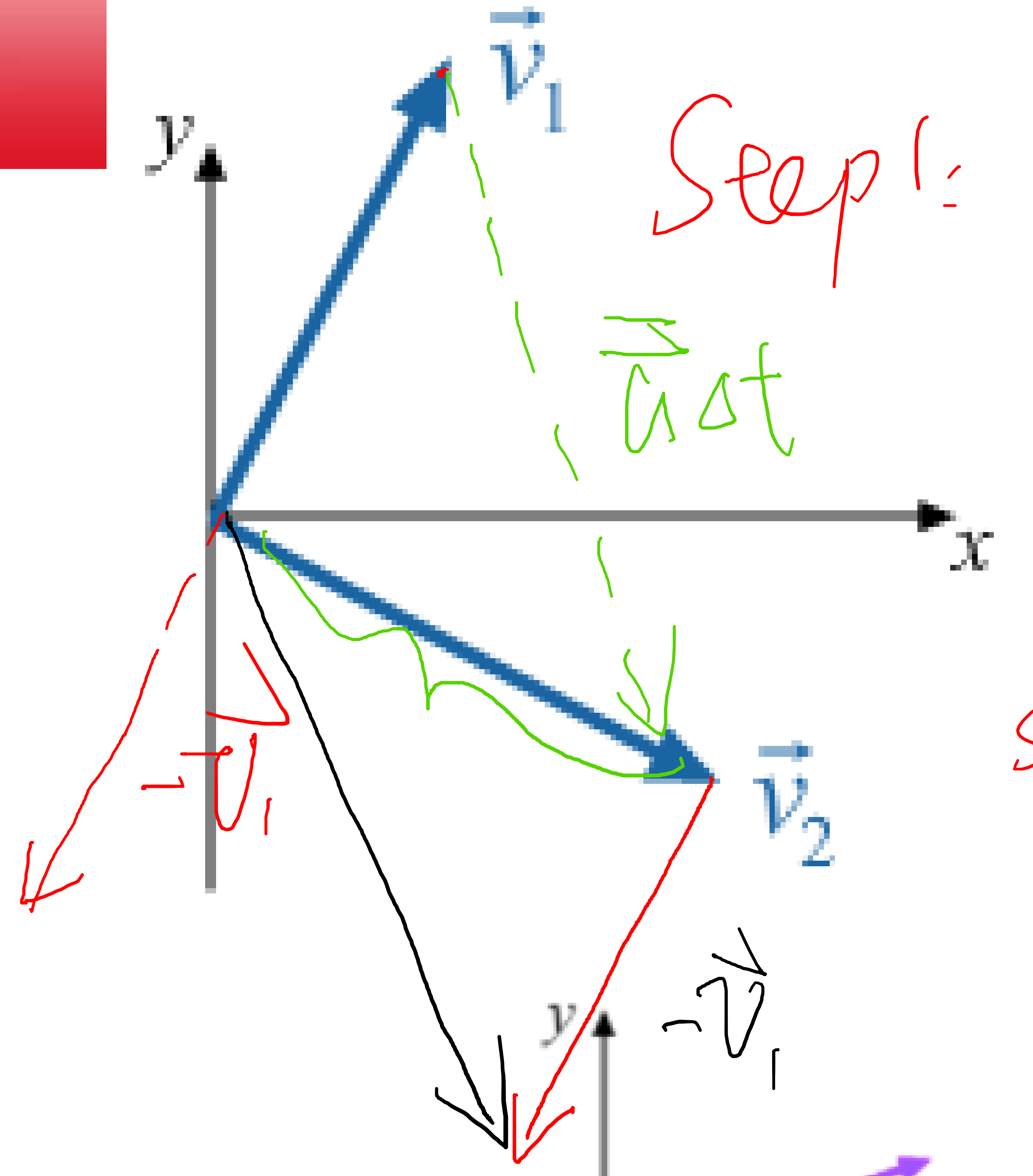
An object has velocity v_1 at time t=0 and v_2 at time t=1 s. Which of the following are the average acceleration?

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$\vec{v}_2 + (-\vec{v}_1)$

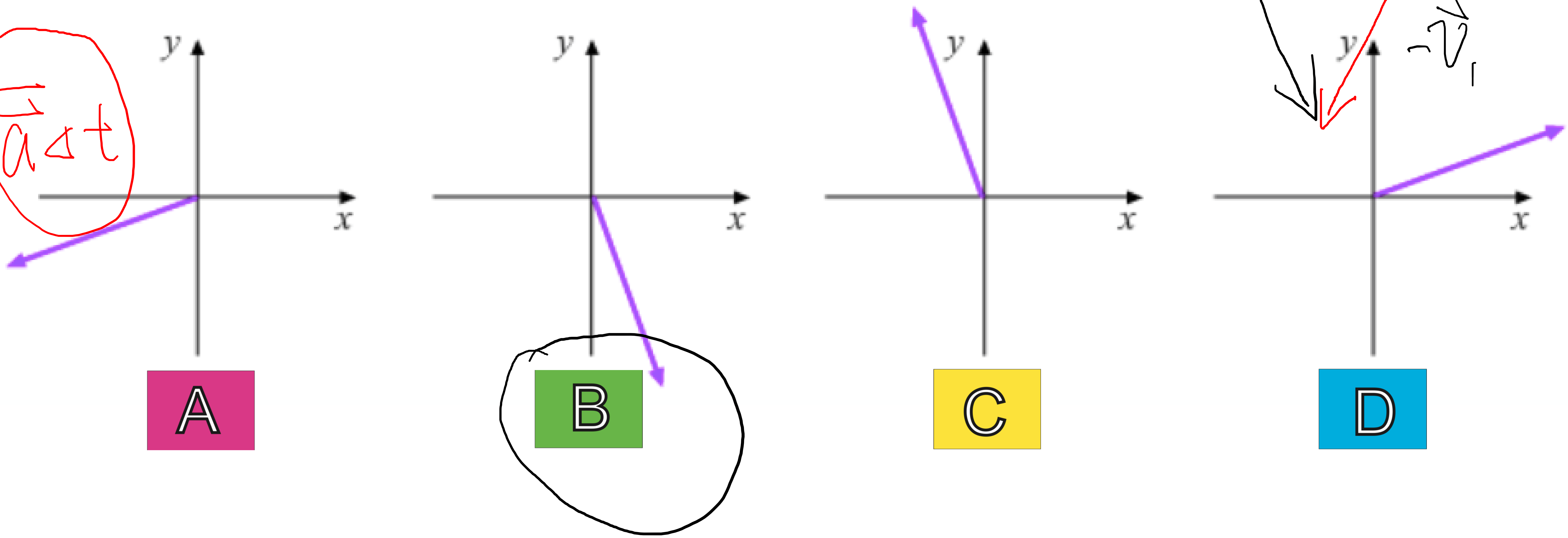
Step 1: Scaling

Step 2: HT



Rewrite

$\vec{v}_2 = \vec{v}_1 + \vec{a}\Delta t$



Vector decomposition

- Vectors in higher dimensions can be decomposed into 1D components
  - Each component points to either the x, y or z direction
- Operation by components makes complex physics clear

Vector decomposition

- Vectors in higher dimensions can be decomposed into 1D components
  - Each component points to either the x, y or z direction
- Operation by components makes complex physics clear
- For example:

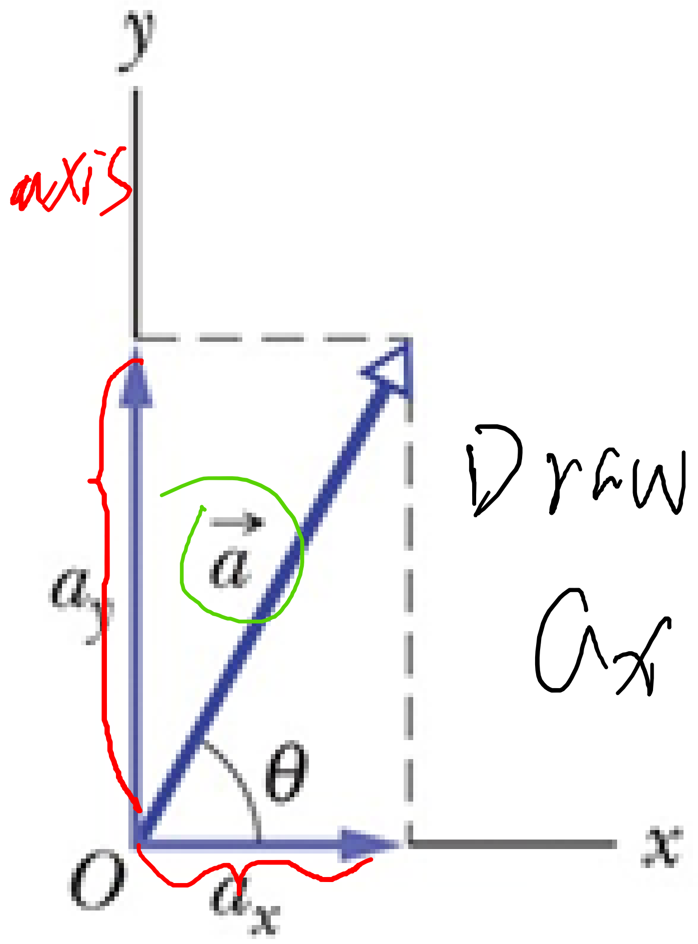
$\vec{A}$: A_x, A_y, A_z

2D vector : x & y

components :

$\vec{a}$: a_x , a_y

Draw a line ⊥ y axis intercept with y is a_y

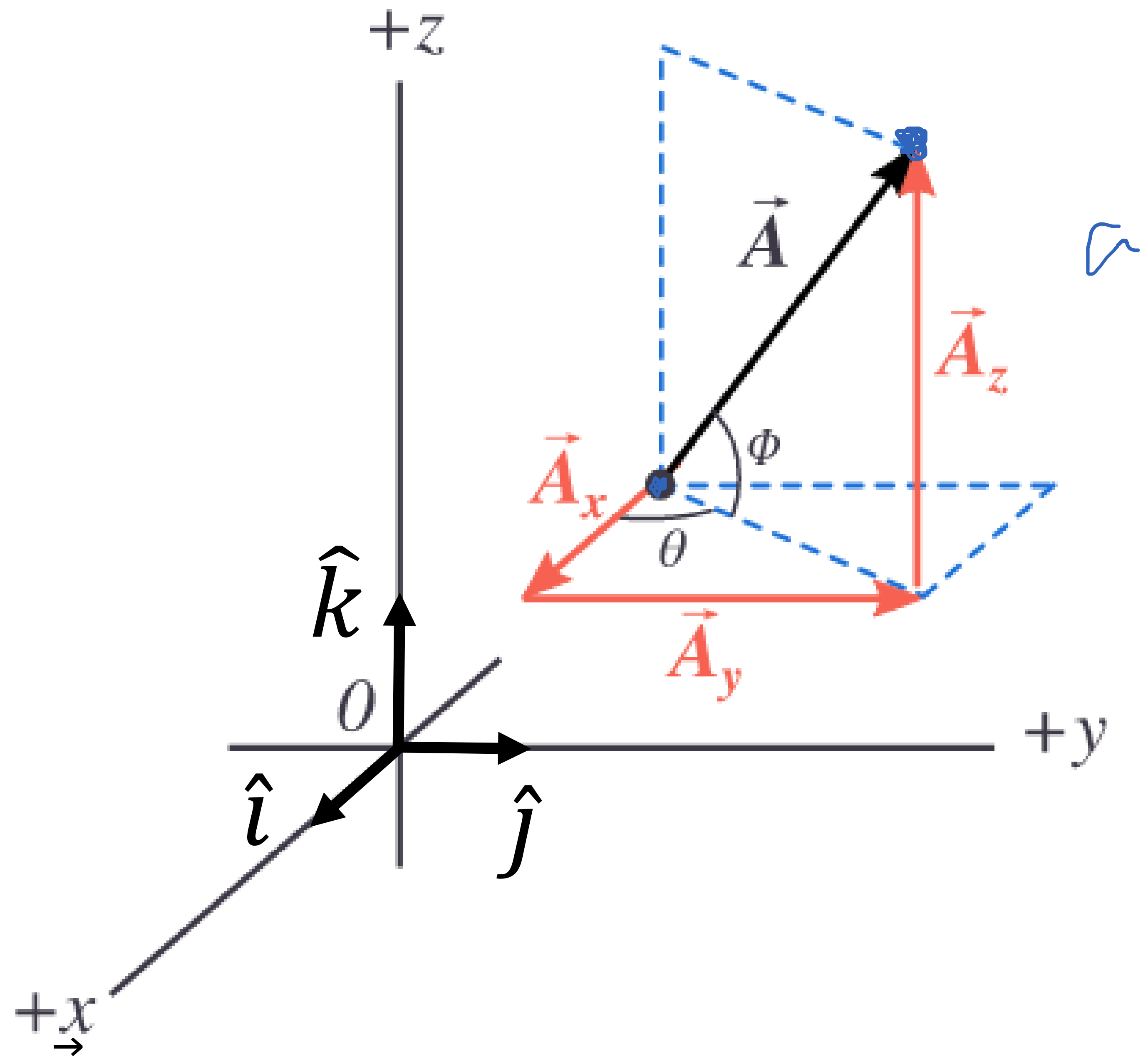


Draw a line ⊥ x axis

a_x is the intercept with x axis

along x, y, z axis

+/−



2D Vector $\vec{a}$ is decomposed into $\vec{a}_x$ and $\vec{a}_y$

$$\vec{a} = \vec{a}_x + \vec{a}_y$$

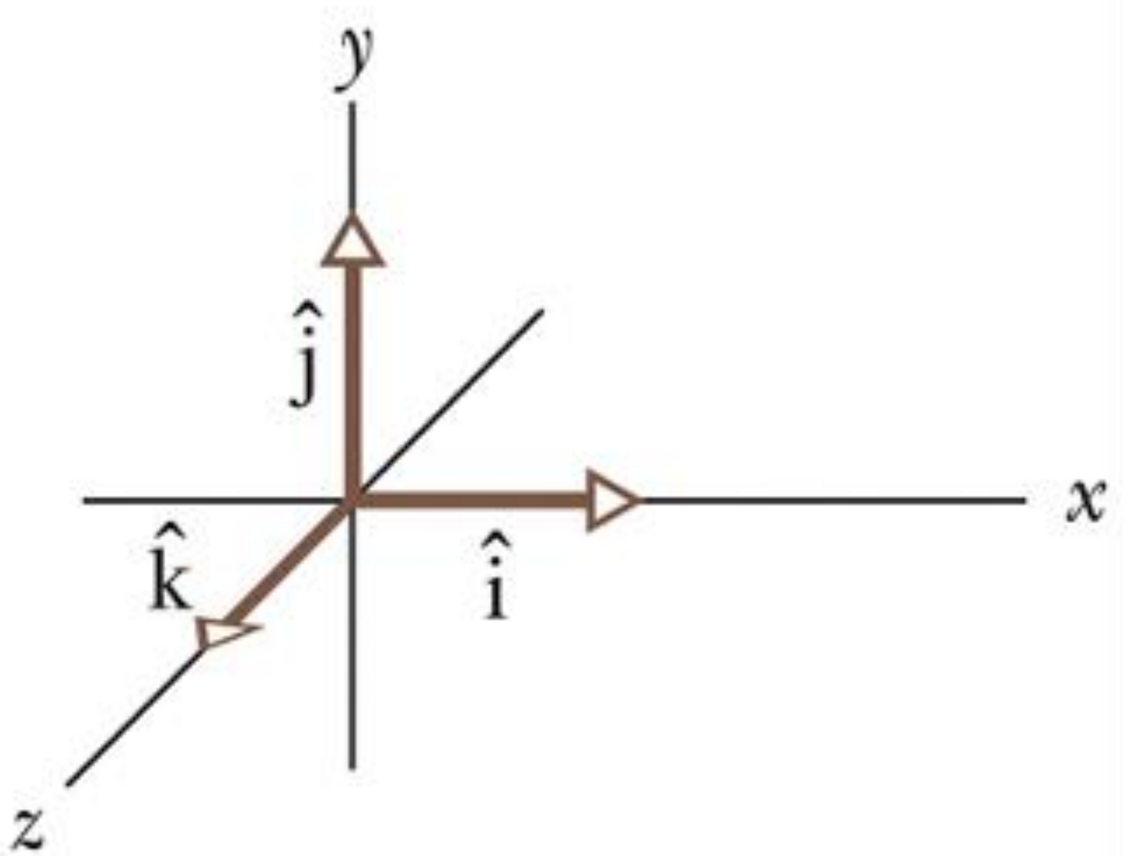
3D Vector A is decomposed into $\vec{A}_x$,$\vec{A}_y$ and $\vec{A}_z$

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

Unit vector notation

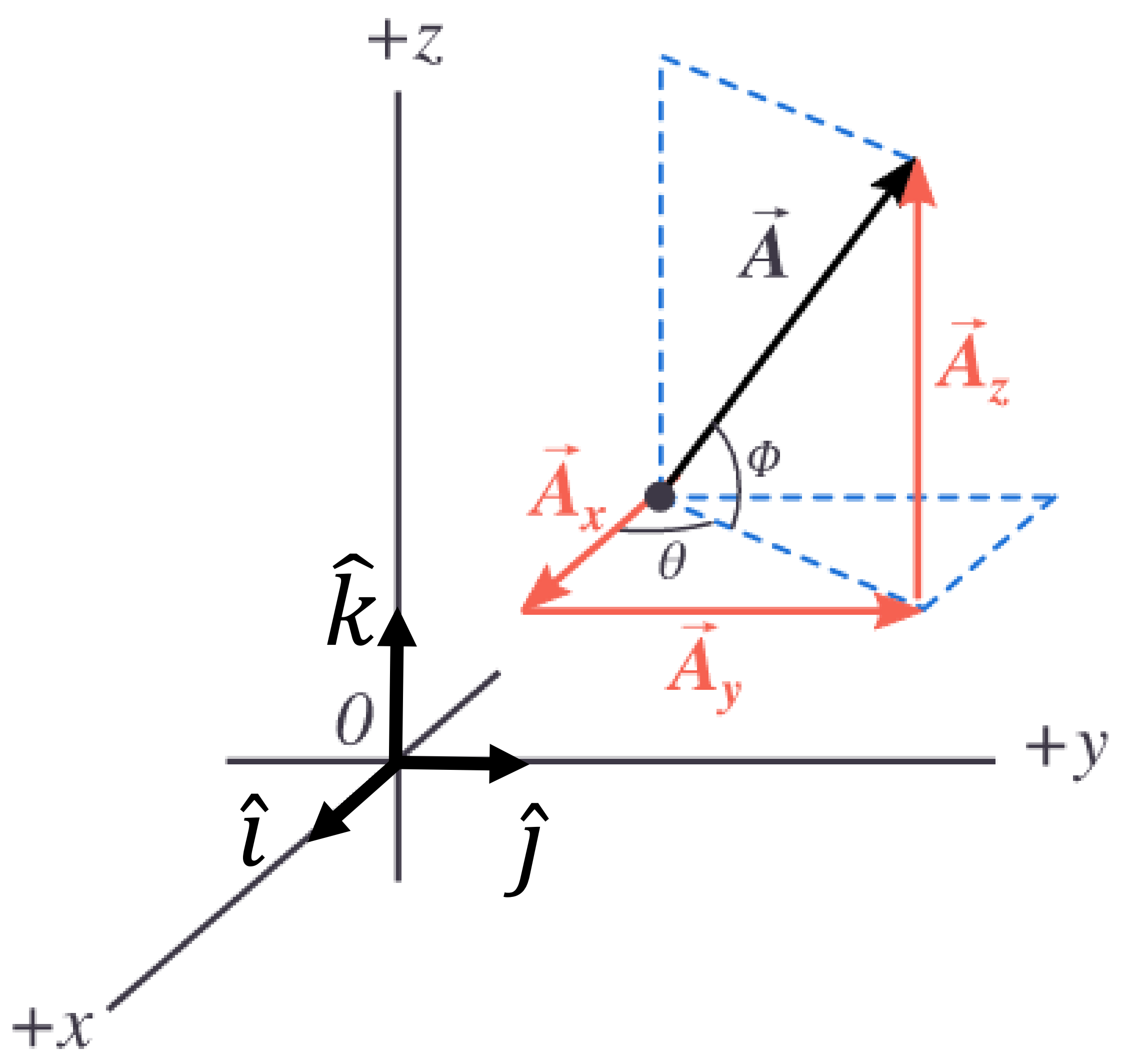
($\hat{x}$ or $\hat{\imath}$: x direction)
($\hat{y}$ or $\hat{\jmath}$: y ~)
($\hat{z}$ or $\hat{k}$: z ~)

The unit vectors point along axes.



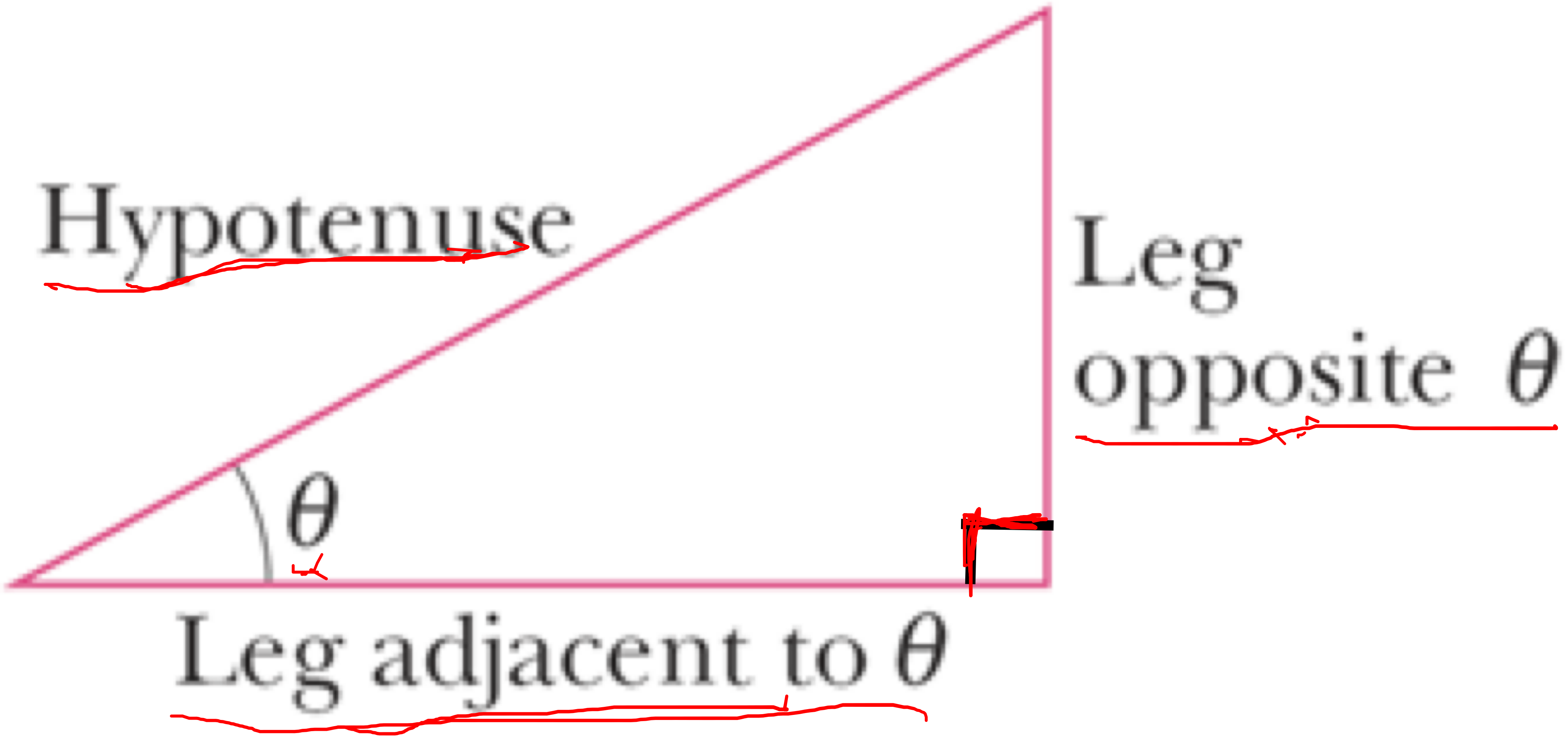
- A **unit vector** has a magnitude of 1
  - Its sole purpose is to point in a direction
  - The **unit vectors** $\hat{\imath}, \hat{\jmath}, \hat{k}$ point along the x, y and z axes, respectively
- Any vector can be written in terms of its components and unit vectors. For example,
  - $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

Unit vector notation



Trig mini-review

- Know these basic trigonometric functions



$$\sin\theta = \frac{\text{leg opposite }\theta}{\text{hypotenuse}}$$

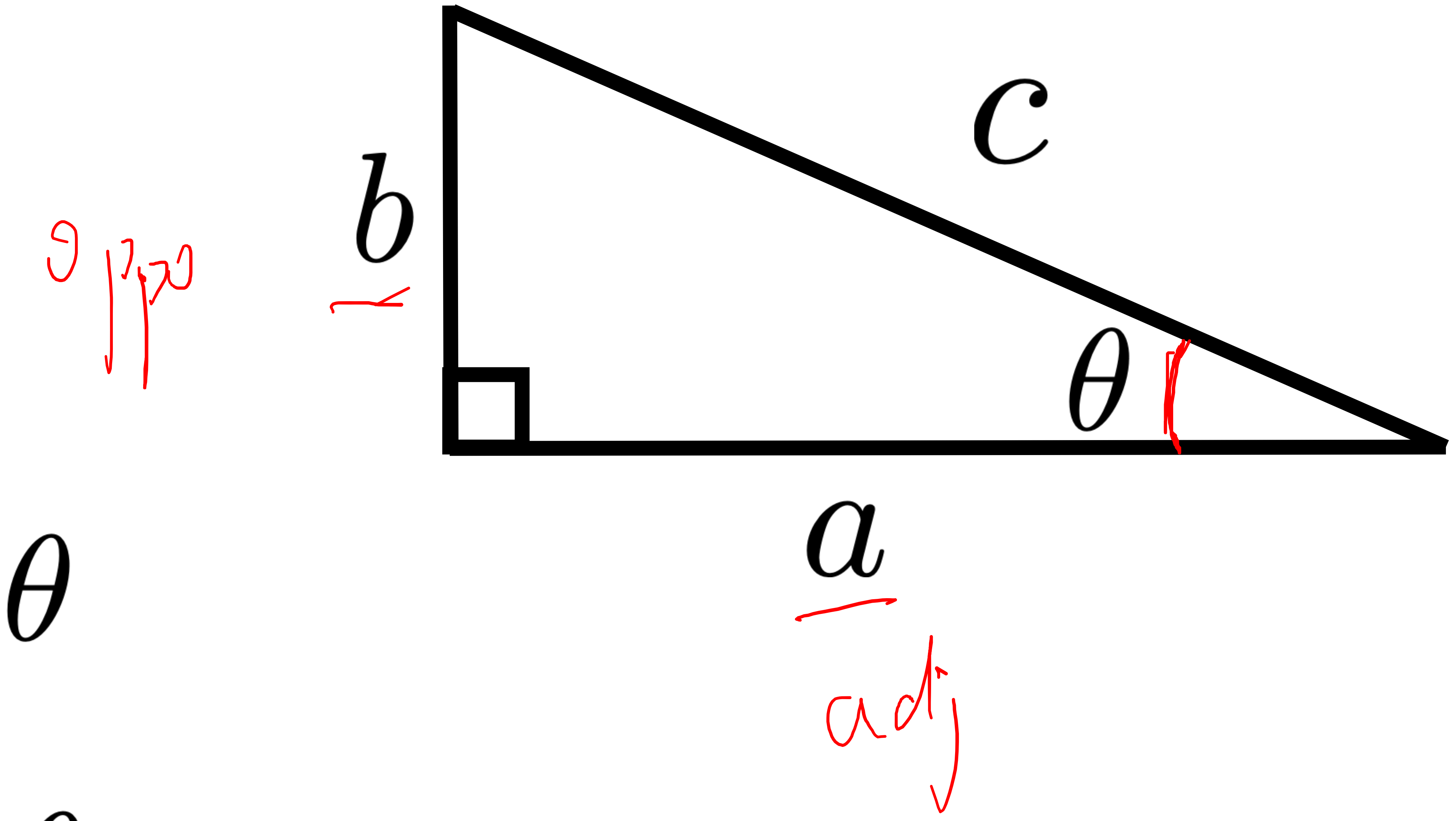
$$\cos\theta = \frac{\text{leg adjacent to }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{leg opposite }\theta}{\text{leg adjacent to }\theta}$$

$\text{Hypotenuse}^2 = \text{adjacent}^2 + \text{opposite}^2$

Clicker question 3

- What is *b* / *a* ?



A $\sin\theta$

B $\cos\theta$

C $\tan\theta$

D θ

$$\frac{\text{oppo}}{\text{adj}} = \text{tg}\,\theta$$

Clicker question 4

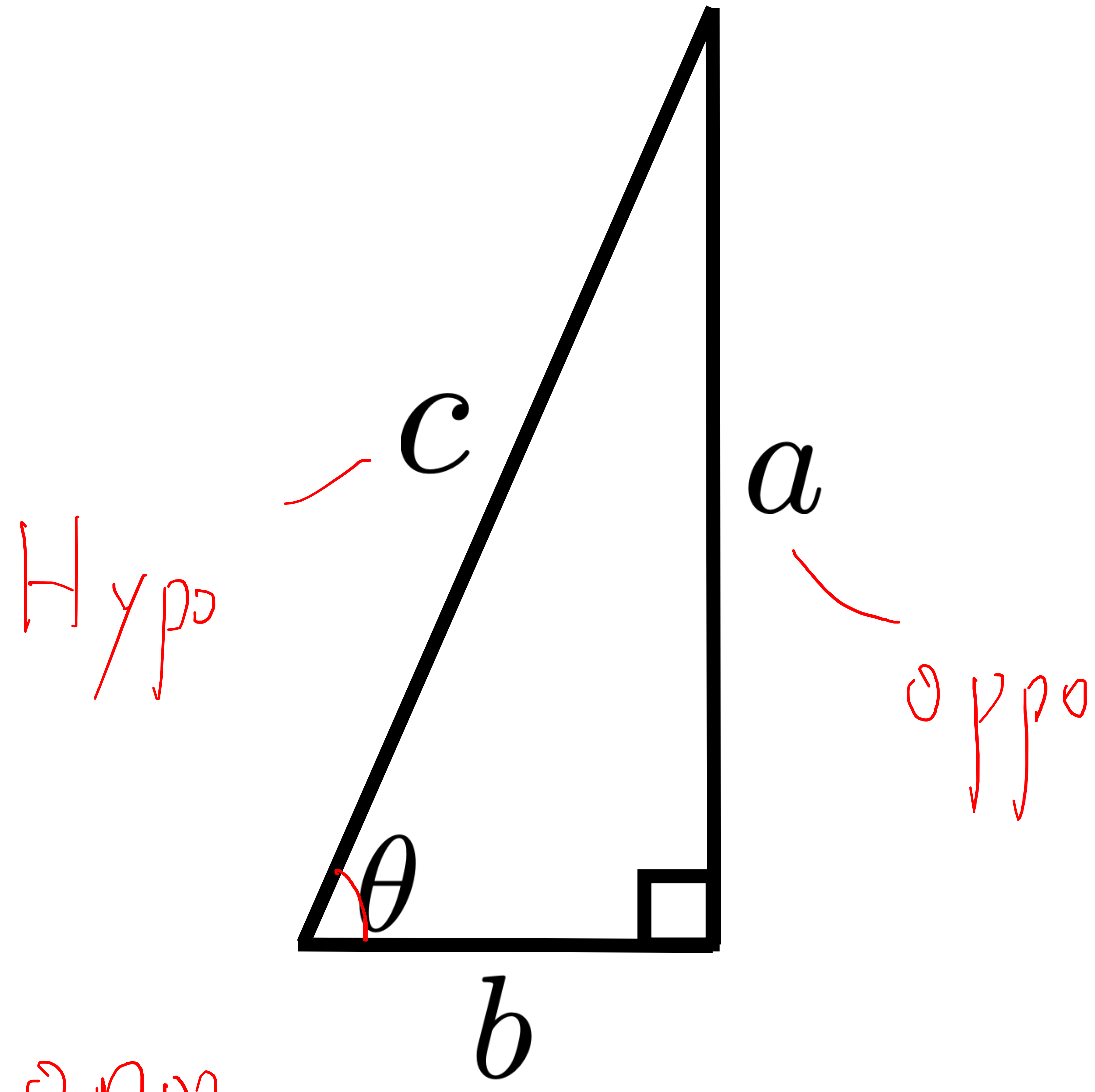
- What is *a* / *c* ?

A $\sin\theta$

B $\cos\theta$

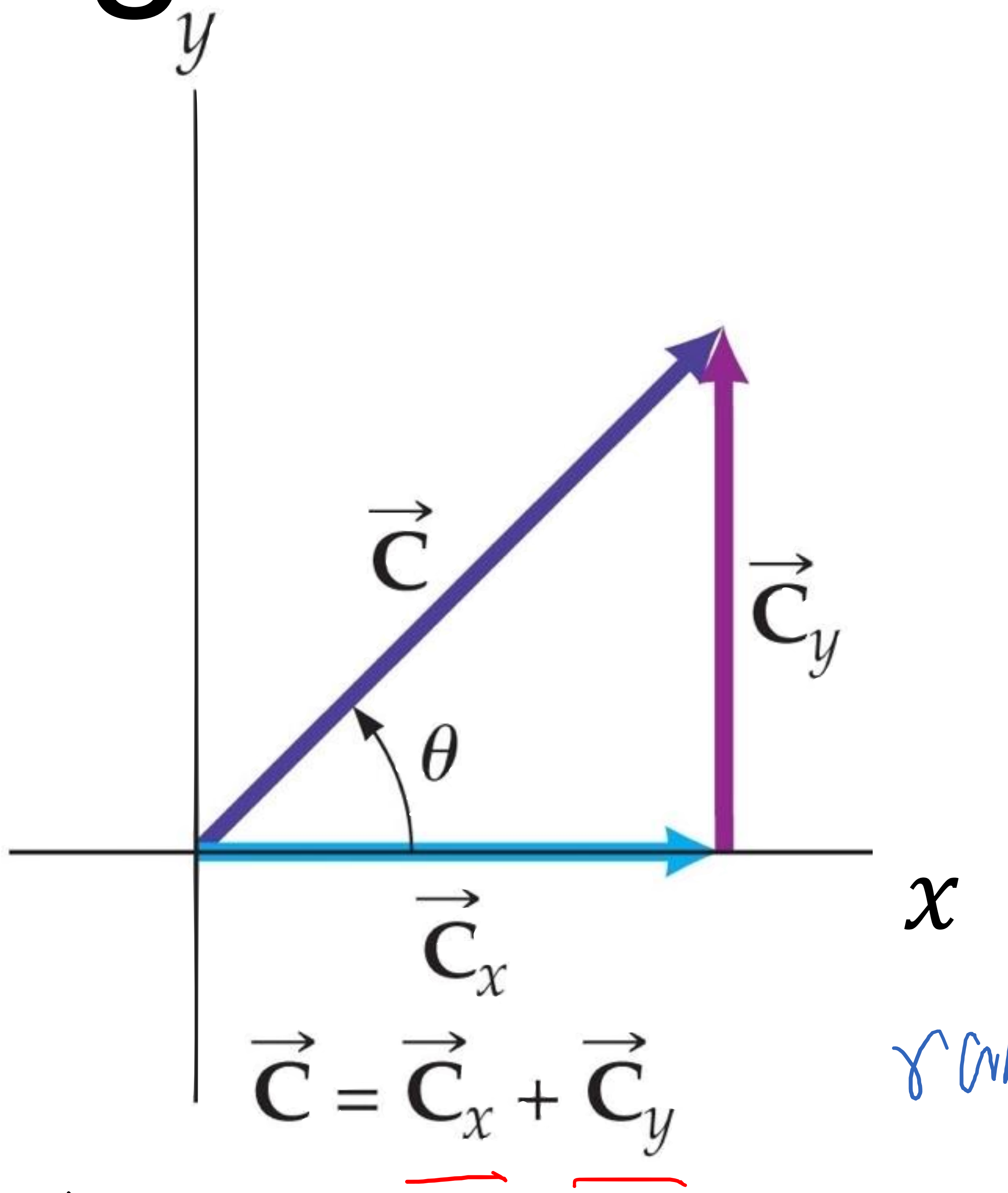
C $\tan\theta$

D θ



$$\frac{\text{oppo}}{\text{Hypo}} = \sin\theta$$

Magnitude and direction in terms of components



Draw a sketch

Magnitude: $C = \sqrt{C_x^{\,2} + C_y^{\,2}}$ ✓

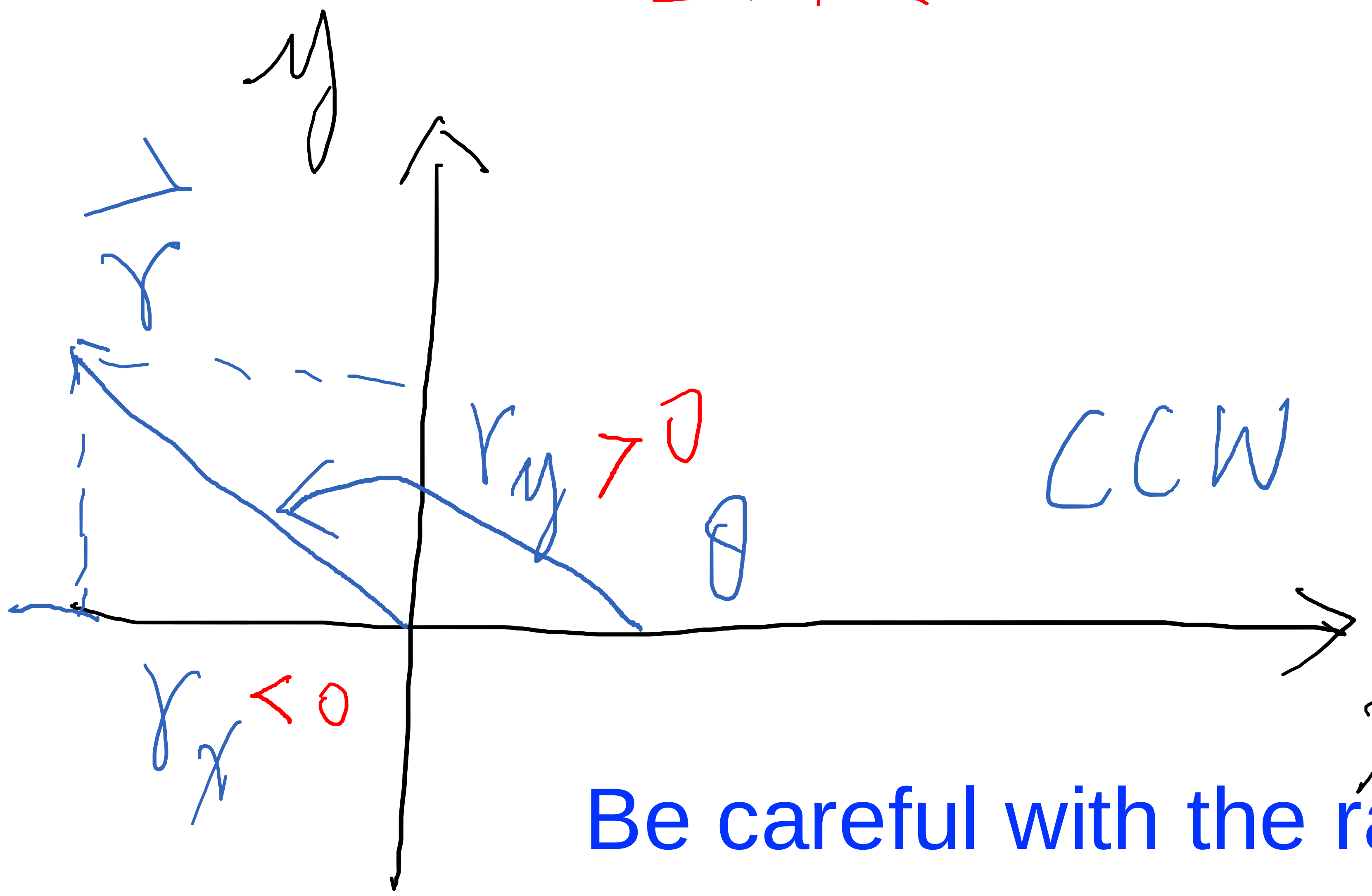
Direction: $\theta = \tan^{-1}(C_y/C_x)$

arctan, atan

Here, θ is the angle counterclockwise from the +x direction to the vector $\vec{C}$

range of arctan $(-90°, 90°)$

$|\vec{r}| = \sqrt{r_x^2 + r_y^2}$



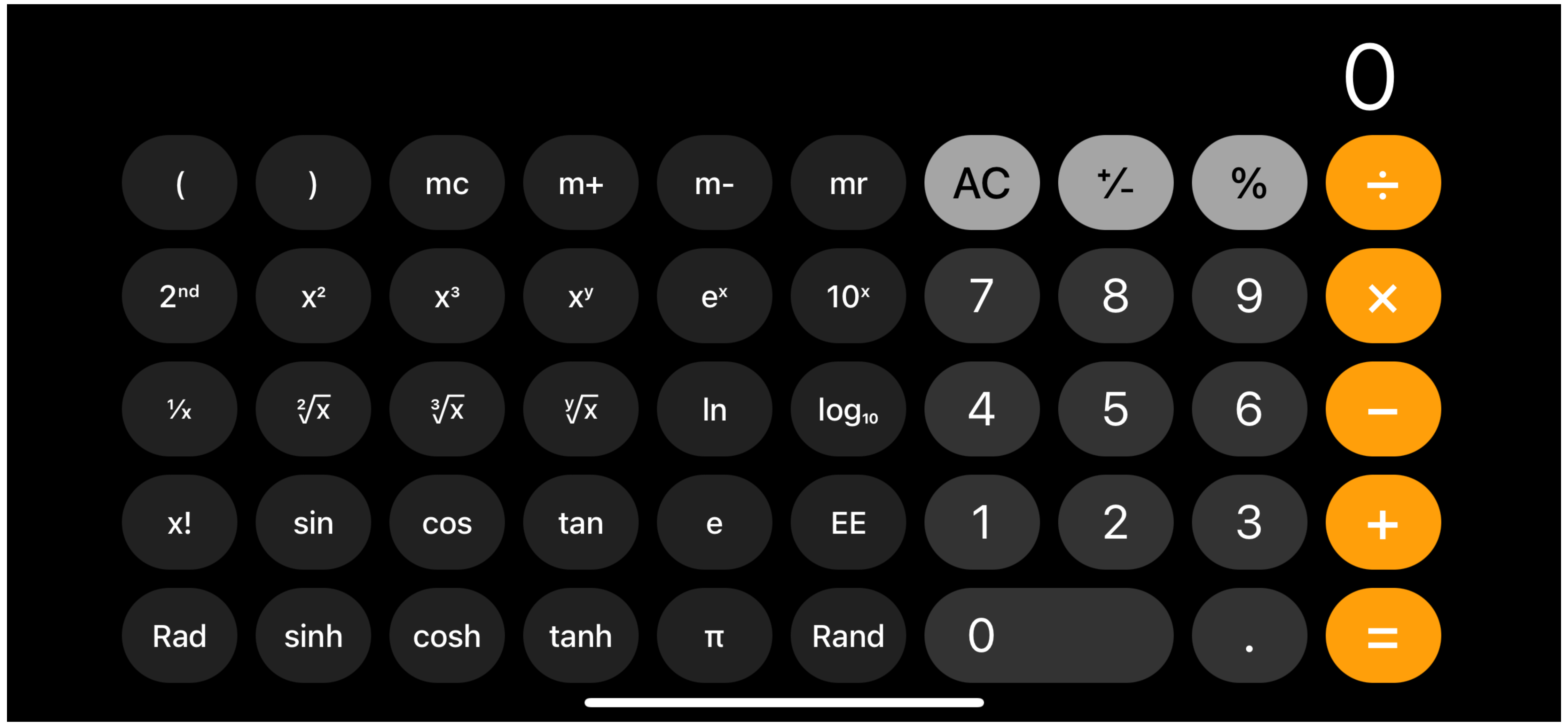
CCW

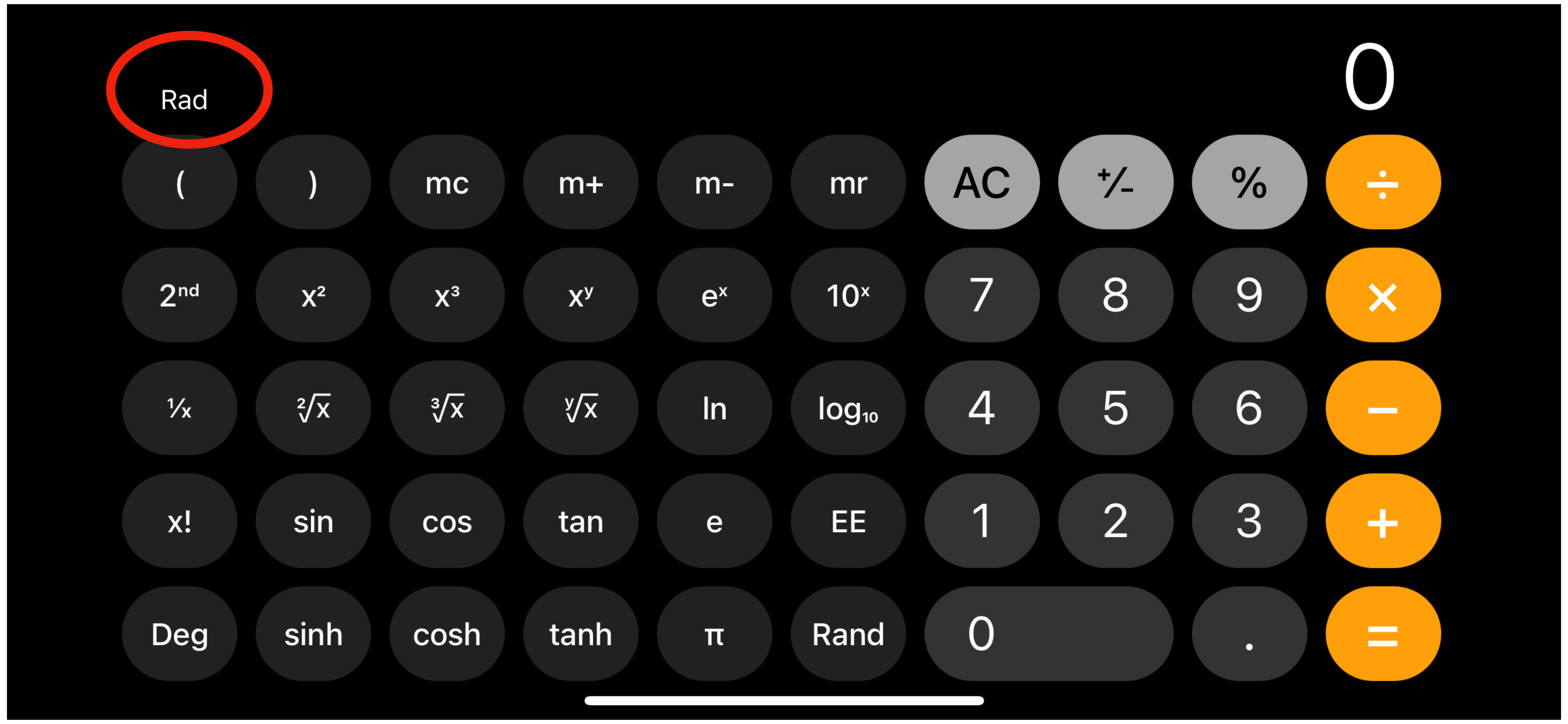
$\theta = \arctan\left(\frac{r_y}{r_x}\right) + 180°$

Be careful with the range of $\tan^{-1}$ function!

Calculator set up: Rad or degree?

- In degrees for Chapter 3



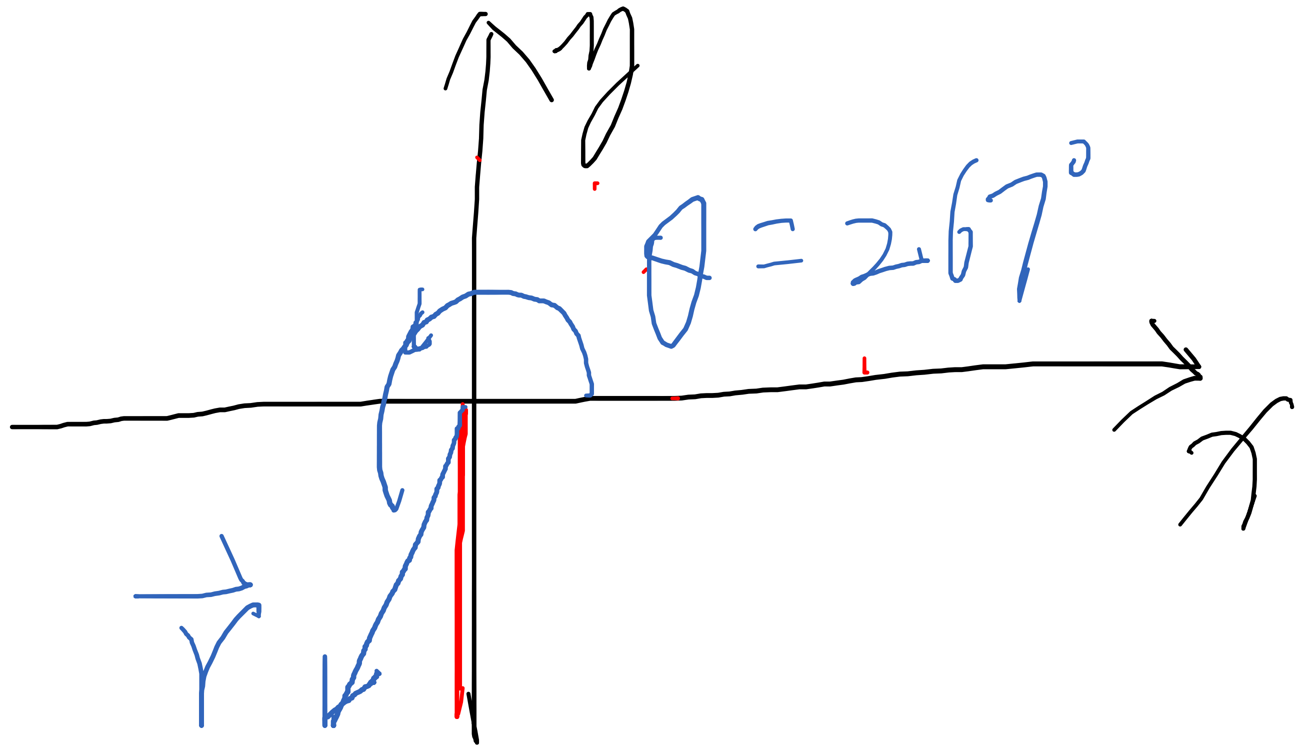


If you are unsure if your calculator is in rad or degree mode, do simple calculations to validate!

Example 1

Given: $\vec{r}$, Goal: r_x, r_y

Step 1: Sketch



CCW

- Please decompose vector $\vec{r}$ in the xy plane if $\vec{r}$'s direction is 267° counterclockwise from the positive direction of the x axis and its magnitude is 5.1 m?

Step 2: Decomposition / Unit vector notation

$$\vec{r} = \underbrace{|\vec{r}|\cos\theta}_{r_x}\,\hat{i} + \underbrace{|\vec{r}|\sin\theta}_{r_y}\,\hat{j}$$

Be careful about how θ is defined!

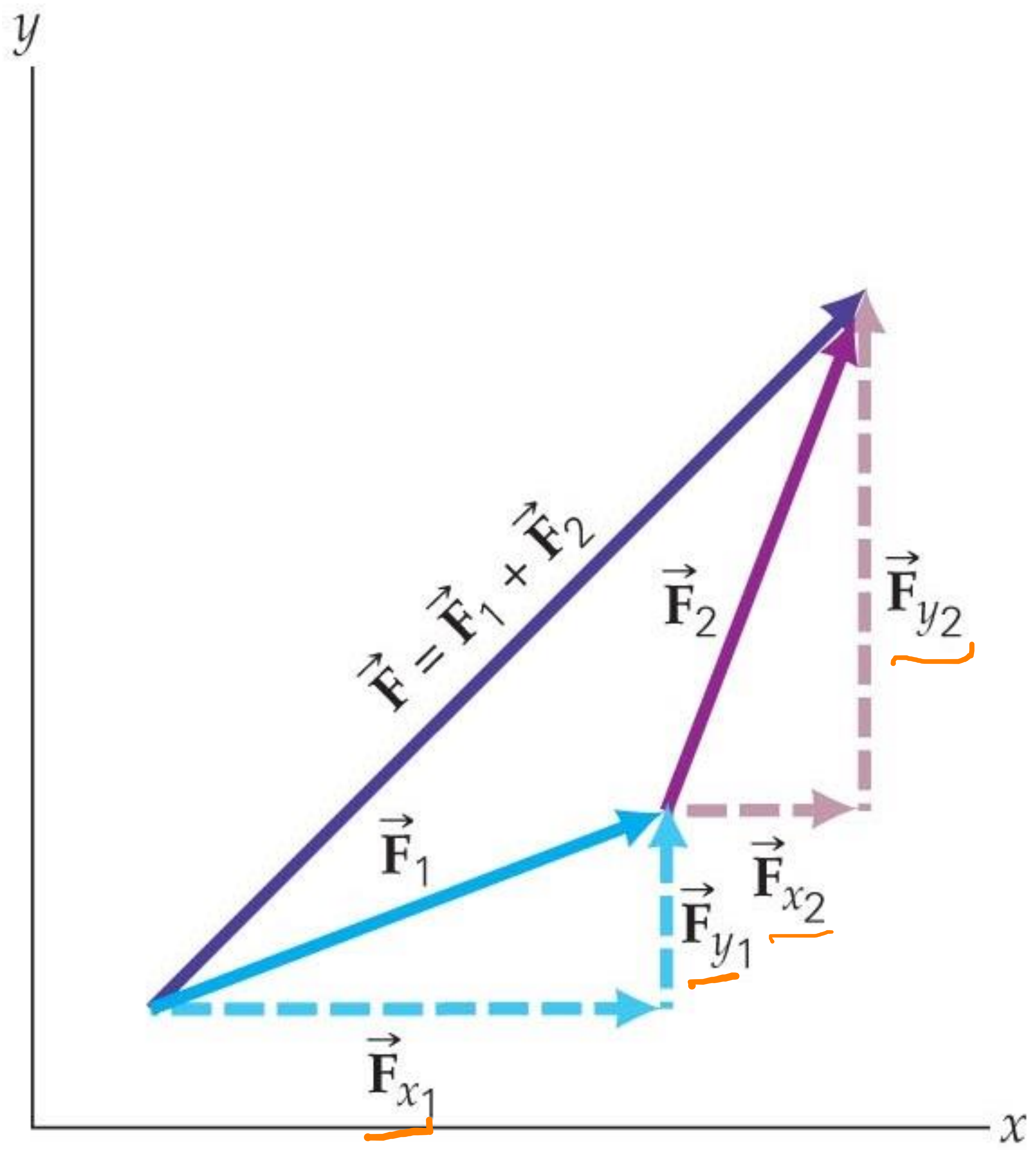
Step 3: Plug in #s:

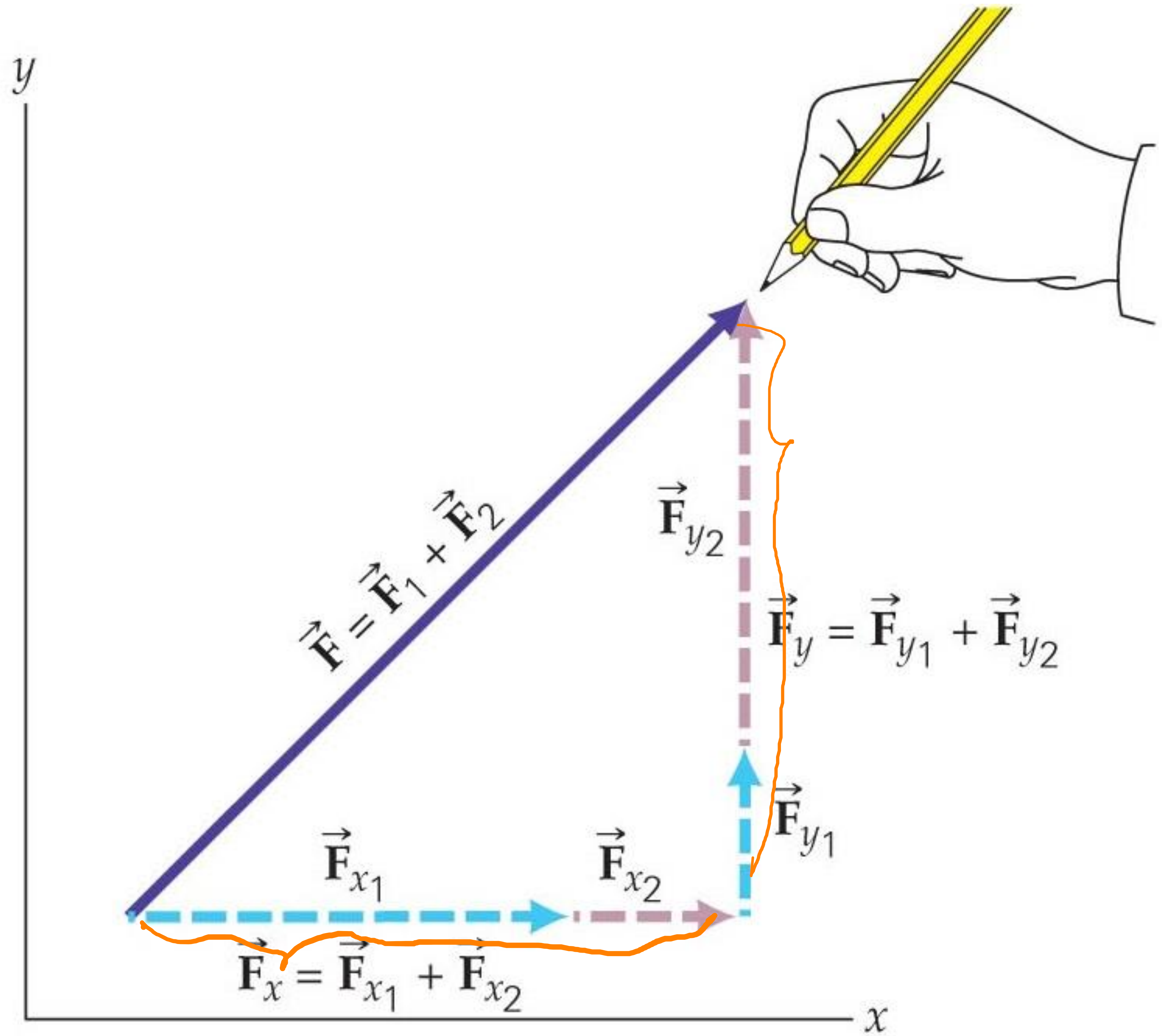
$$r_x = 5.1\,\text{m}\,\cos 267^\circ \approx -0.267\,\text{m}$$

$$r_y = 5.1\,\text{m}\,\sin 267^\circ \approx -5.09\,\text{m}$$

$$\vec{r} = -0.267\,\text{m}\,\hat{i} - 5.09\,\text{m}\,\hat{j}$$

Add vectors by components



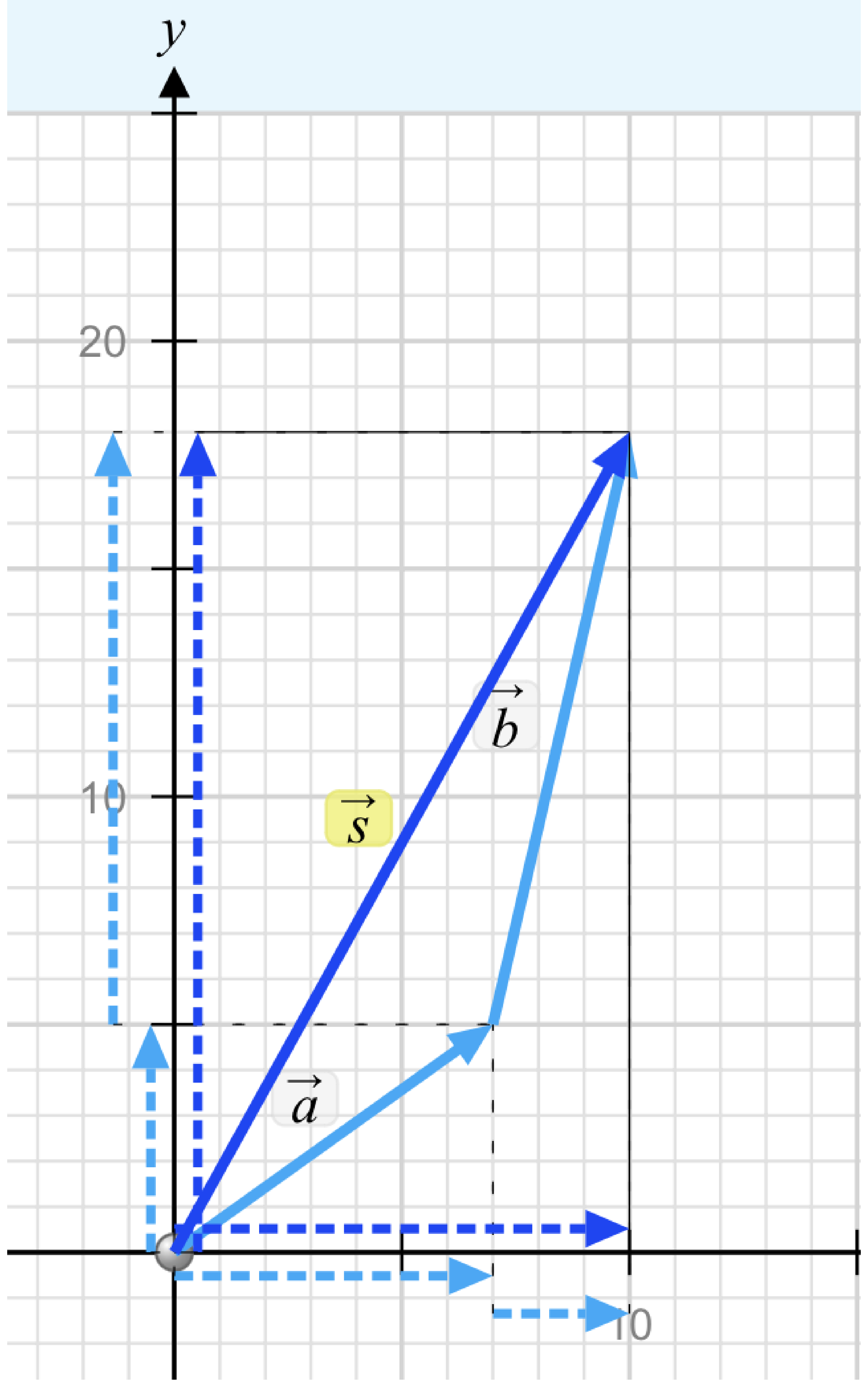


Step 1: Decompose each of the vectors

Step 2: Vector sum the components in each direction

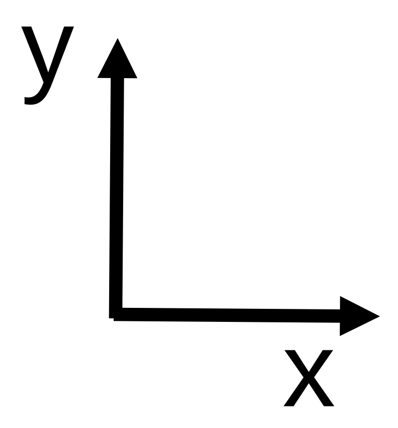
Demo: Add vectors by components

- Demo:

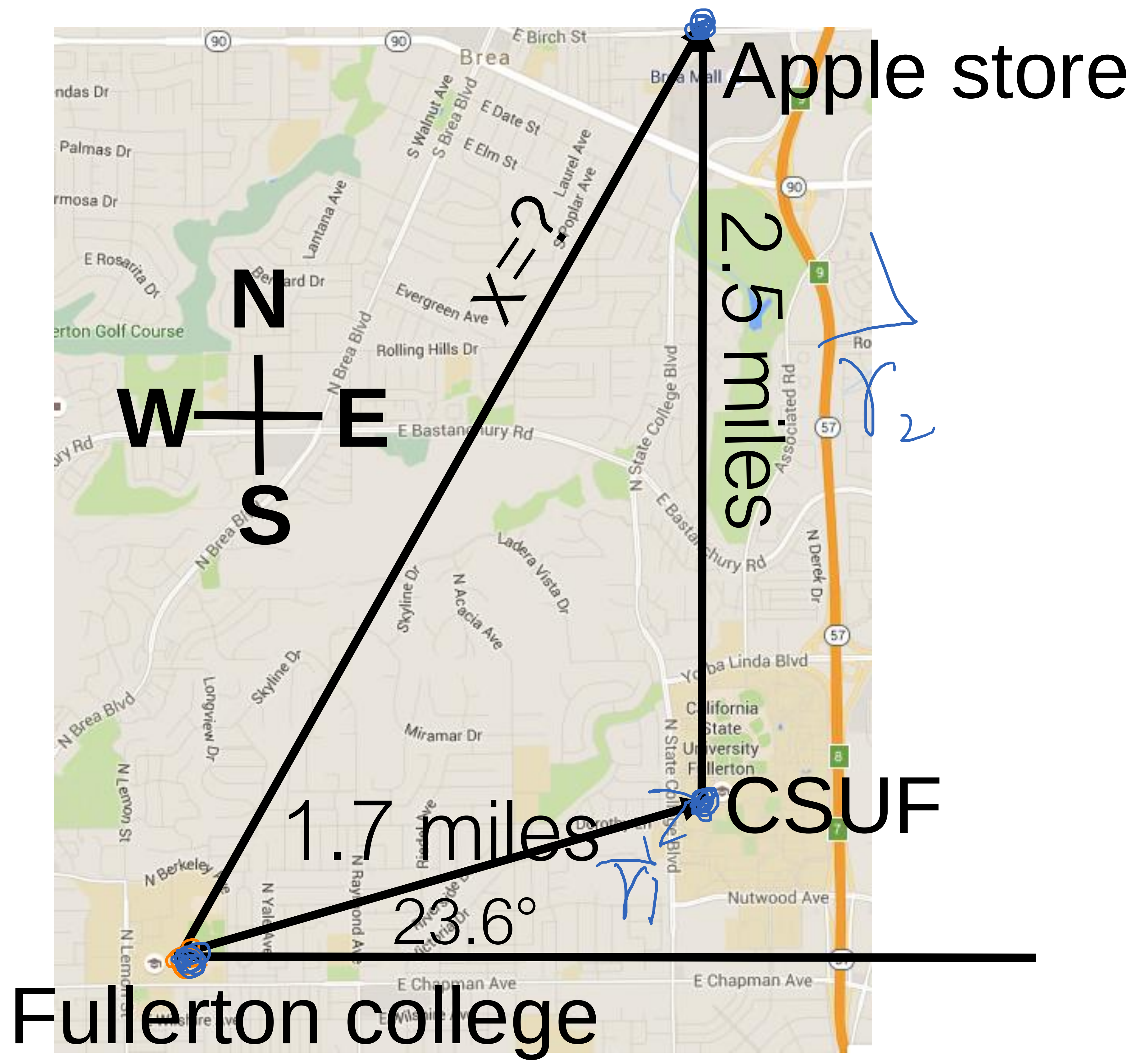


https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_all.html

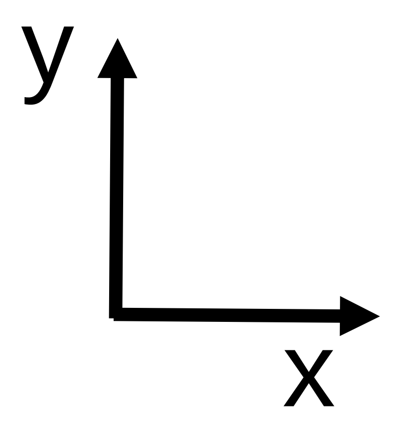
Vector addition: Real-life example



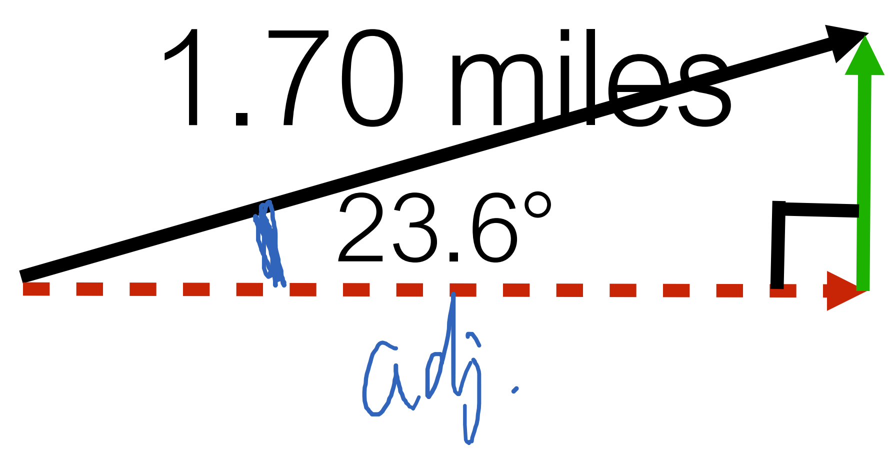
- Cal State Fullerton is 1.70 miles away from Fullerton College, at an angle 23.6° **north of east.** The Apple Store is 2.50 miles due north of Cal State Fullerton. A bird flies from Fullerton College to Cal State Fullerton, and then from Cal State Fullerton to the Apple Store. What is the bird's displacement?



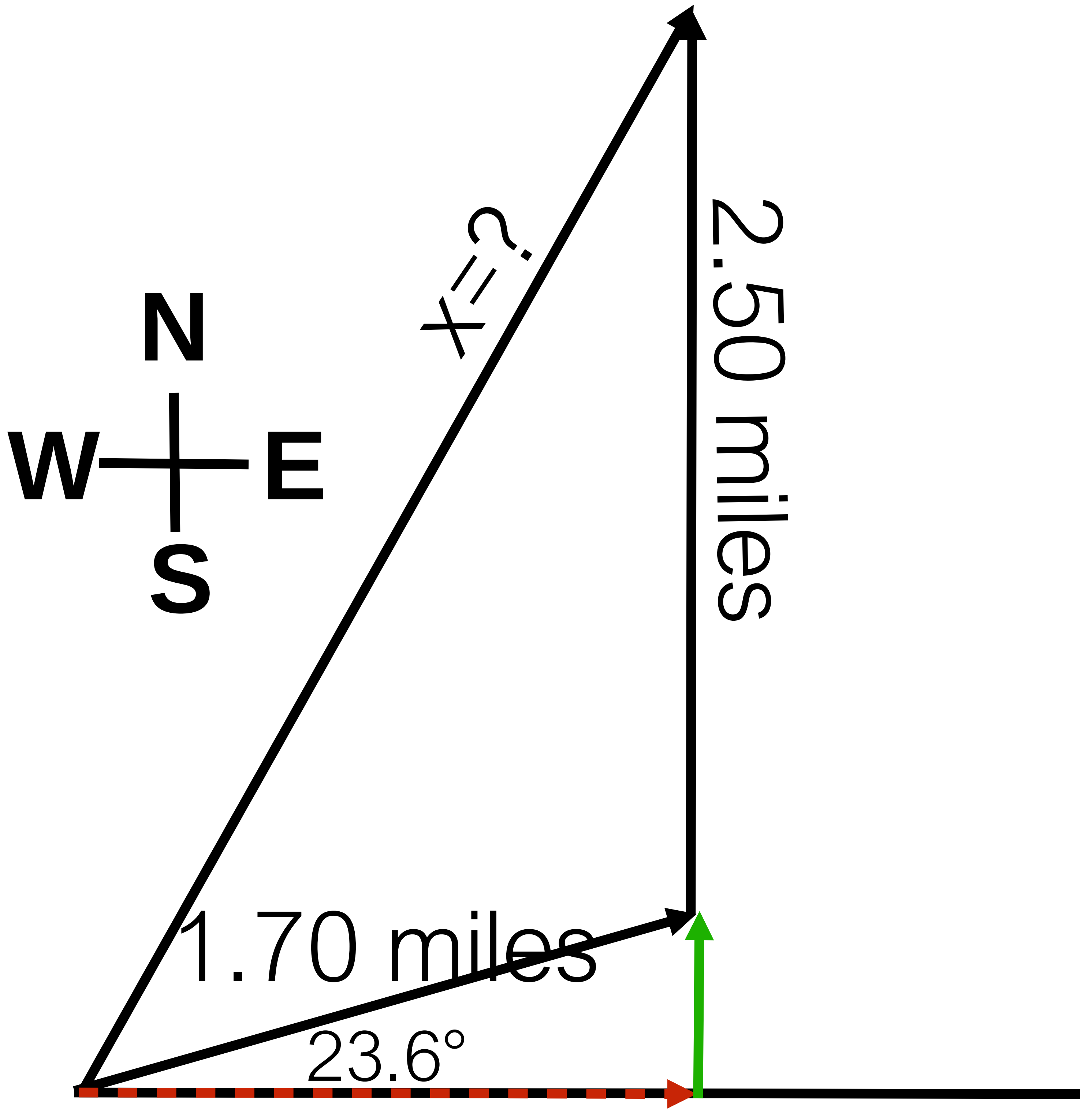
Clicker question 5: Real-life example



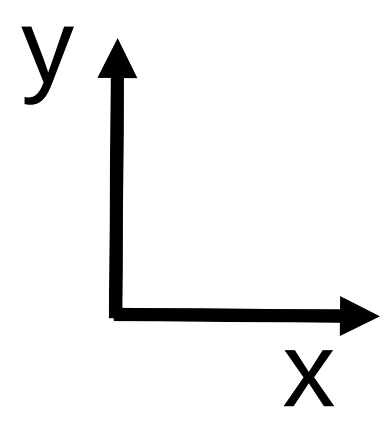
- How long is the dashed arrow?



A	1.70 miles x sin 23.6°
B	1.70 miles x cos 23.6°
C	1.70 miles x tan 23.6°
D	1.70 miles
E	None of ABCD



Clicker question 6: Real-life example



- How long is the dashed arrow?

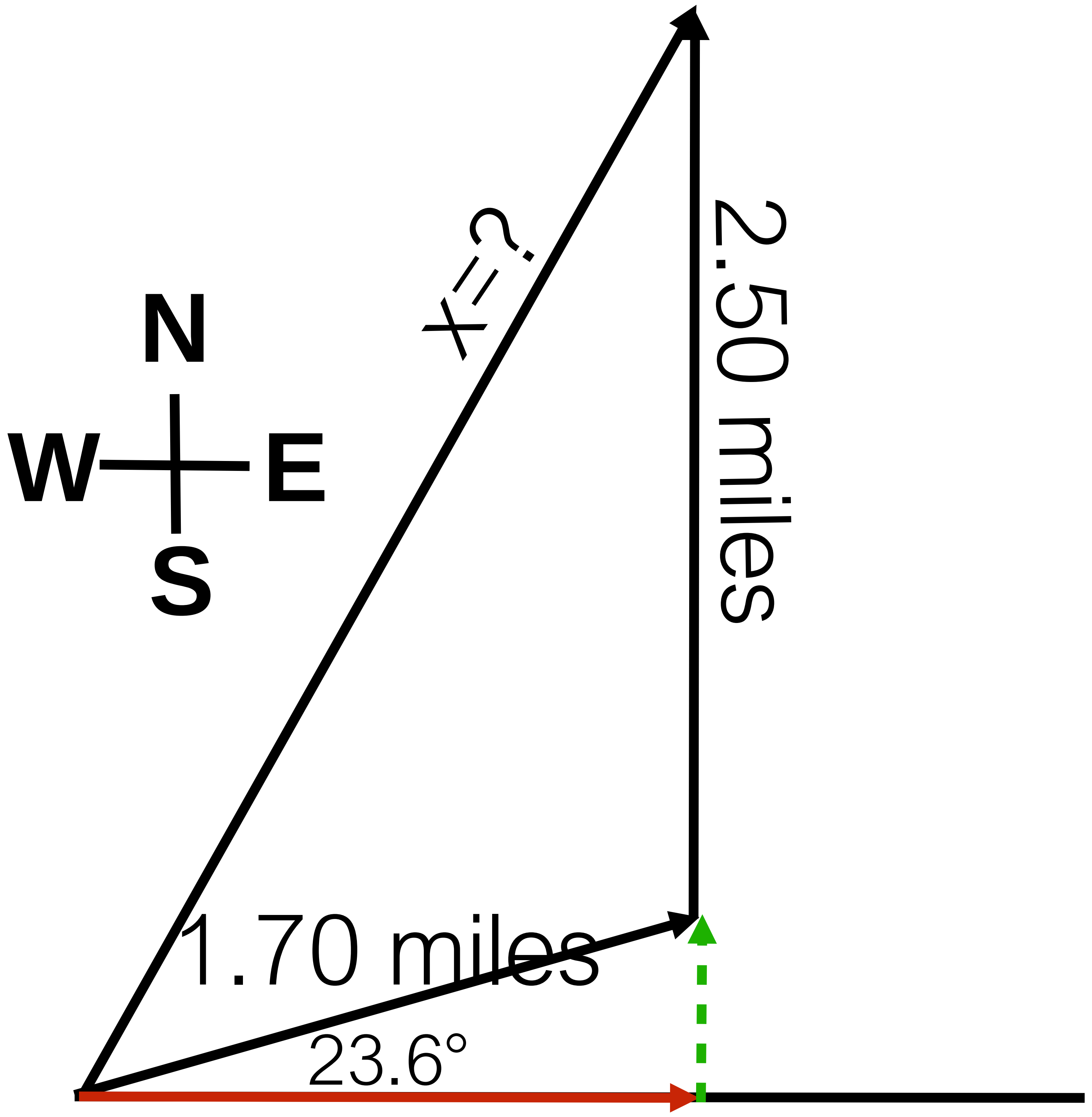
A 1.70 miles x sin 23.6°

B 1.70 miles x cos 23.6°

C 1.70 miles x tan 23.6°

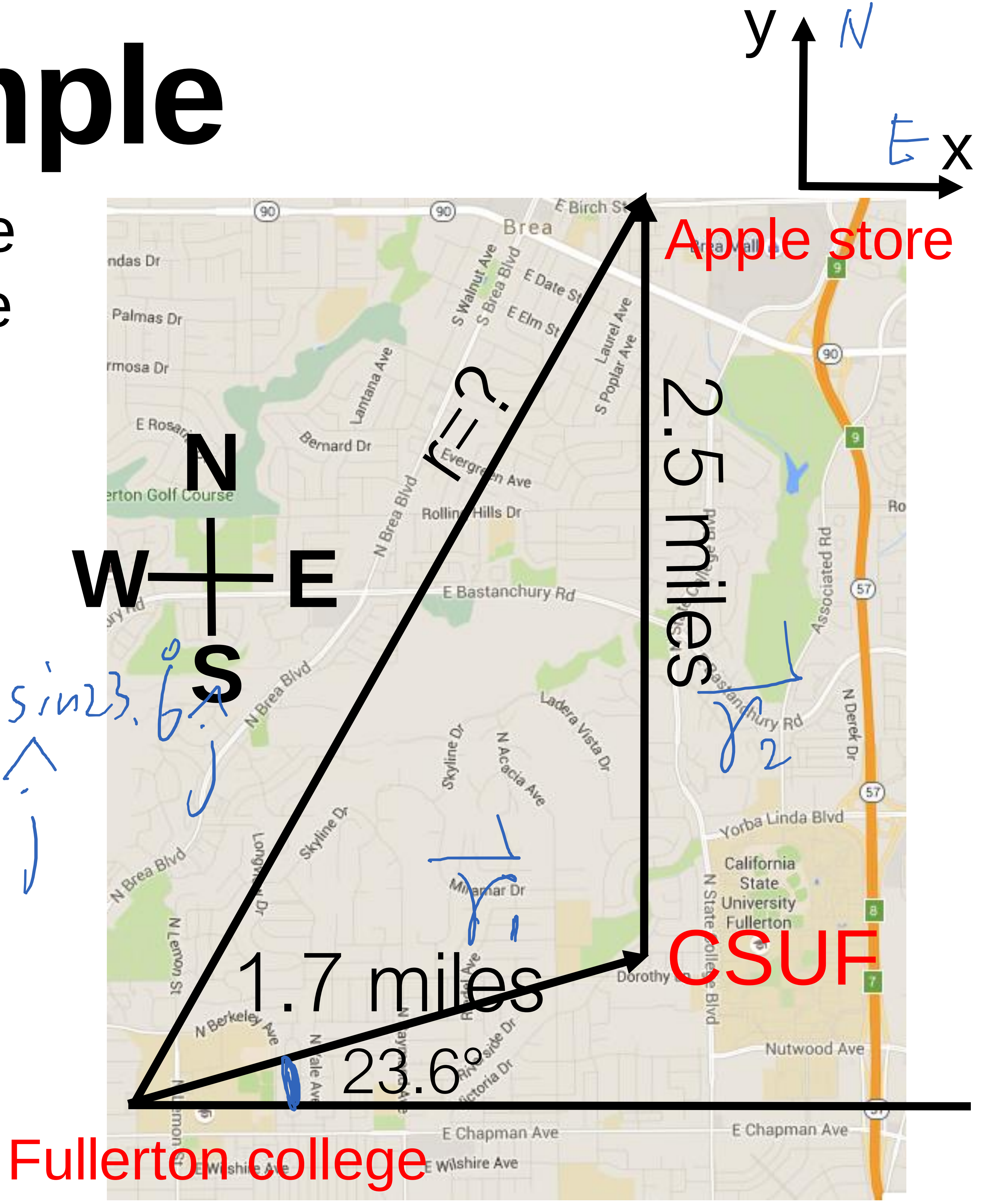
D 1.70 miles

E None of ABCD



Vector addition: A real-life example

- Cal State Fullerton is 1.70 miles away from Fullerton College, at an angle 23.6° **north of east.** The Apple Store is 2.50 miles due north of Cal State Fullerton. A bird flies from Fullerton College to Cal State Fullerton, and then from Cal State Fullerton to the Apple Store. What is the bird's displacement?



Step 1: Vector decomposition:

$$\vec{r}_1 = |\vec{r}_1|\cos\theta\,\hat{i} + |\vec{r}_1|\sin\theta\,\hat{j} = 1.7\,\text{mi}\cos 23.6^\circ\,\hat{i} + 1.7\,\text{mi}\sin 23.6^\circ\,\hat{j}$$

$$= 1.56\,\text{mi}\,\hat{i} + 0.68\,\text{mi}\,\hat{j}$$

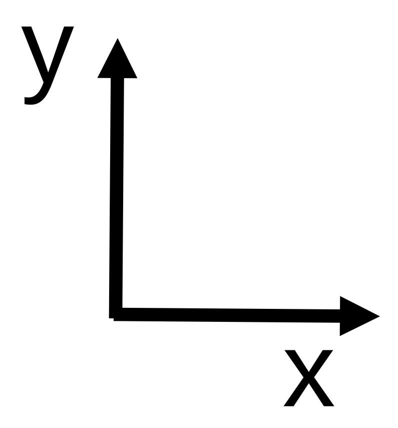
$$\vec{r}_2 = 2.5\,\text{mile}\,\hat{j}$$

Step 2: Add:

$$\vec{r} = r_{1x}\hat{i} + (r_{1y} + r_{2y})\hat{j}$$

$$= 1.56\,\text{mi}\,\hat{i} + 3.18\,\text{mi}\,\hat{j}$$

Vector addition example: Solution



$$\vec{r} = (1.7\text{mi}\cos 23.6^\circ)\ \hat{\imath} + (1.7\text{mi}\sin 23.6^\circ + 2.5\ \text{mi})\ \hat{\jmath}$$

Magnitude of displacement:

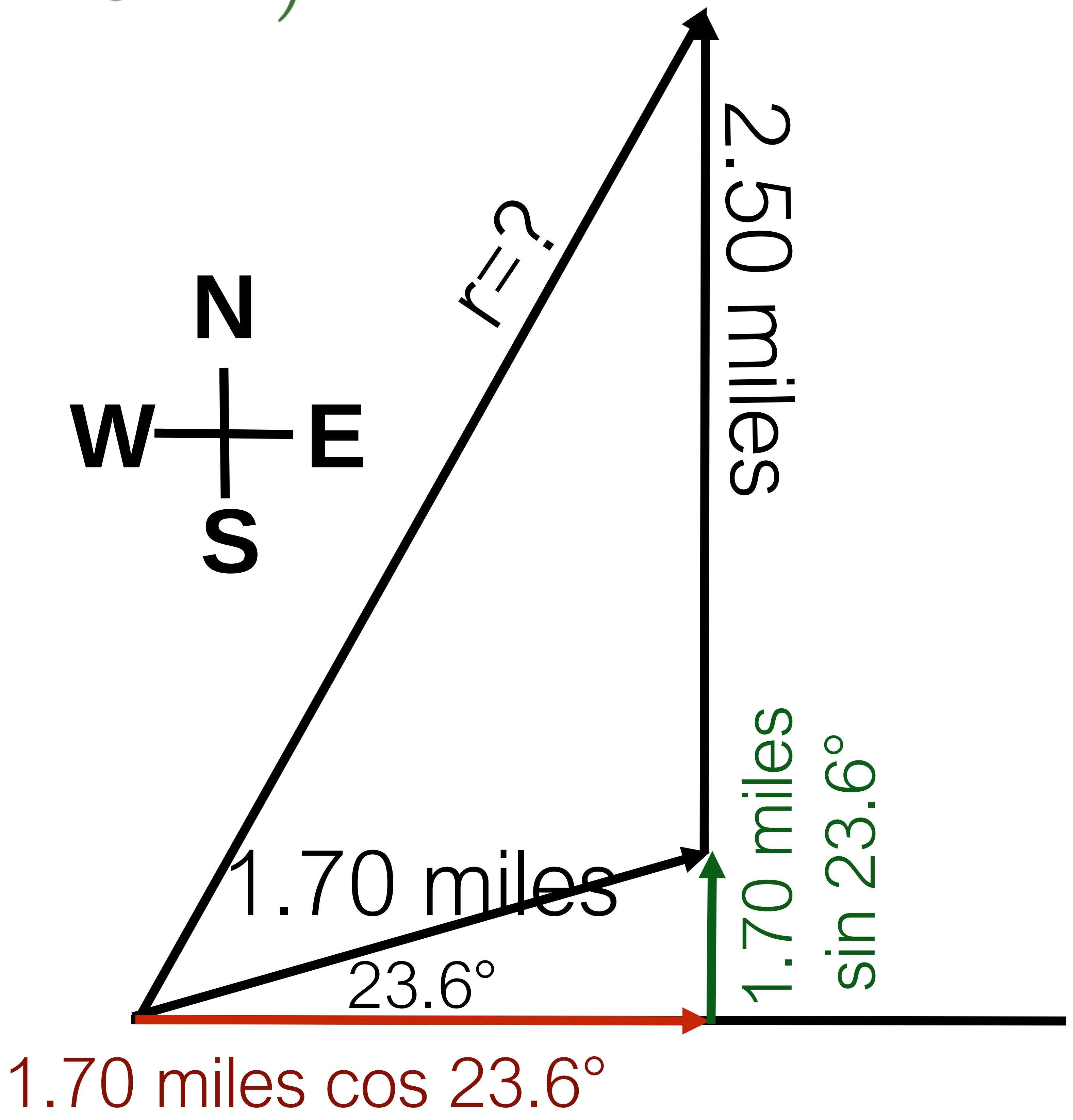
$$|\vec{r}| = \sqrt{(1.7\text{mi}\cos 23.6^\circ)^2 + (1.7\text{mi}\sin 23.6^\circ + 2.5\ \text{mi})^2}$$

= 3.54 mi

Direction of displacement (angle counter-clockwise
= angle “north of east”:

$$\theta = \tan^{-1}\left(\frac{1.7\ \text{mi}\sin 23.6^\circ + 2.5\ \text{mi}}{1.7\ \text{mi}\cos 23.6^\circ}\right)$$

= 63.9°

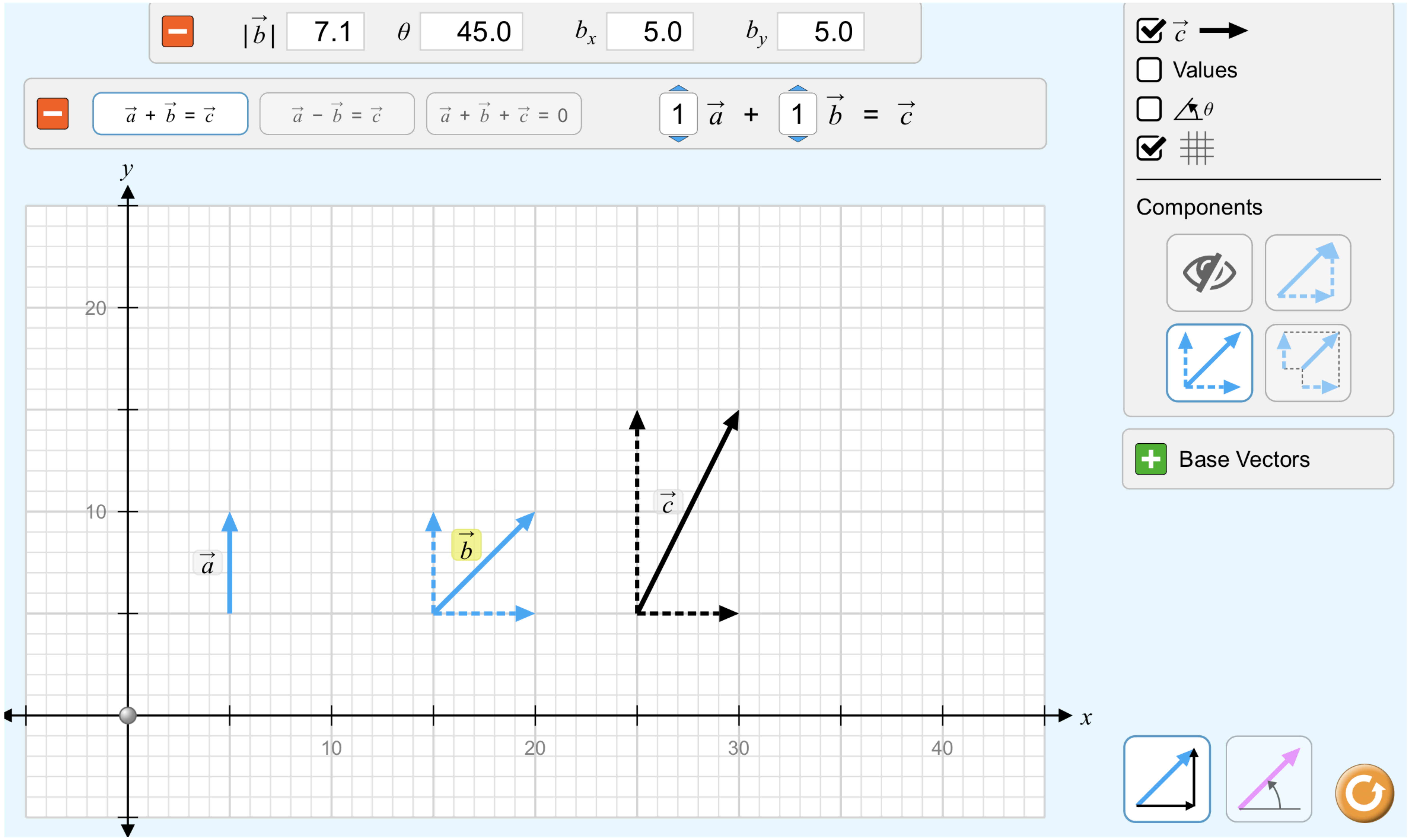


Determine if your calculator accepts radians or degrees

Practice example (work in a group of 2-3)

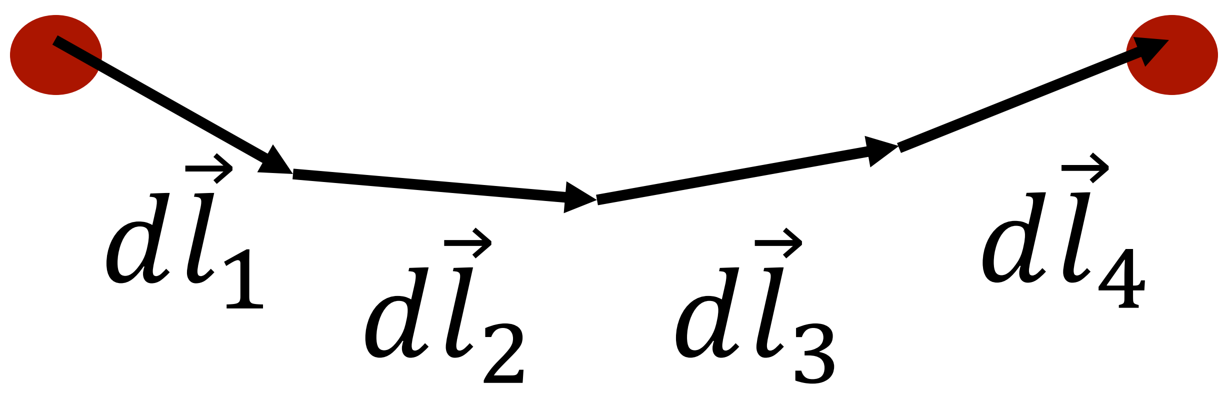
(a) In unit-vector notation, what is the sum of $\vec{a} = (3.7 \text{ m})\hat{i} + (1.7 \text{ m})\hat{j}$ and $\vec{b} = (-12.0 \text{ m})\hat{i} + (6.8 \text{ m})\hat{j}$. What are **(b)** the magnitude and **(c)** the direction of $\vec{a} + \vec{b}$ (relative to $\hat{i}$)?

Practice (try simulations)



https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_all.html

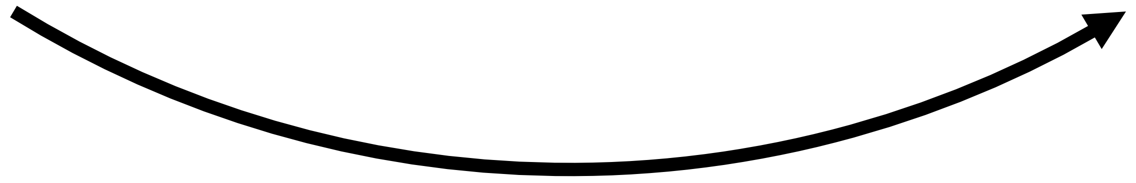
Clicker question 7



- What is the vector sum of vectors $d\vec{l}_1$,$d\vec{l}_2, \cdots, d\vec{l}_4$ above: $\sum_{i=1}^{i=4} d\vec{l}_i$?

A

B



Vector addition summary

- Vector addition by head-tail convention
- Vector addition by components
- Properties of vector addition
  - Commutative: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
  - Associative: $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

Next time: Vector multiplication