PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 13: Uniform circular motion

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Chapter 4.2: Uniform circular motion

- Learning goals:
  - Uniform circular motion
    - Angular speed, angular velocity
    - Centripetal acceleration

Circular motion examples

- Centrifuge

Separate protein

Angular speed

Not the same as speed
Speed: Distance per unit time
Angular speed: Angle per unit time

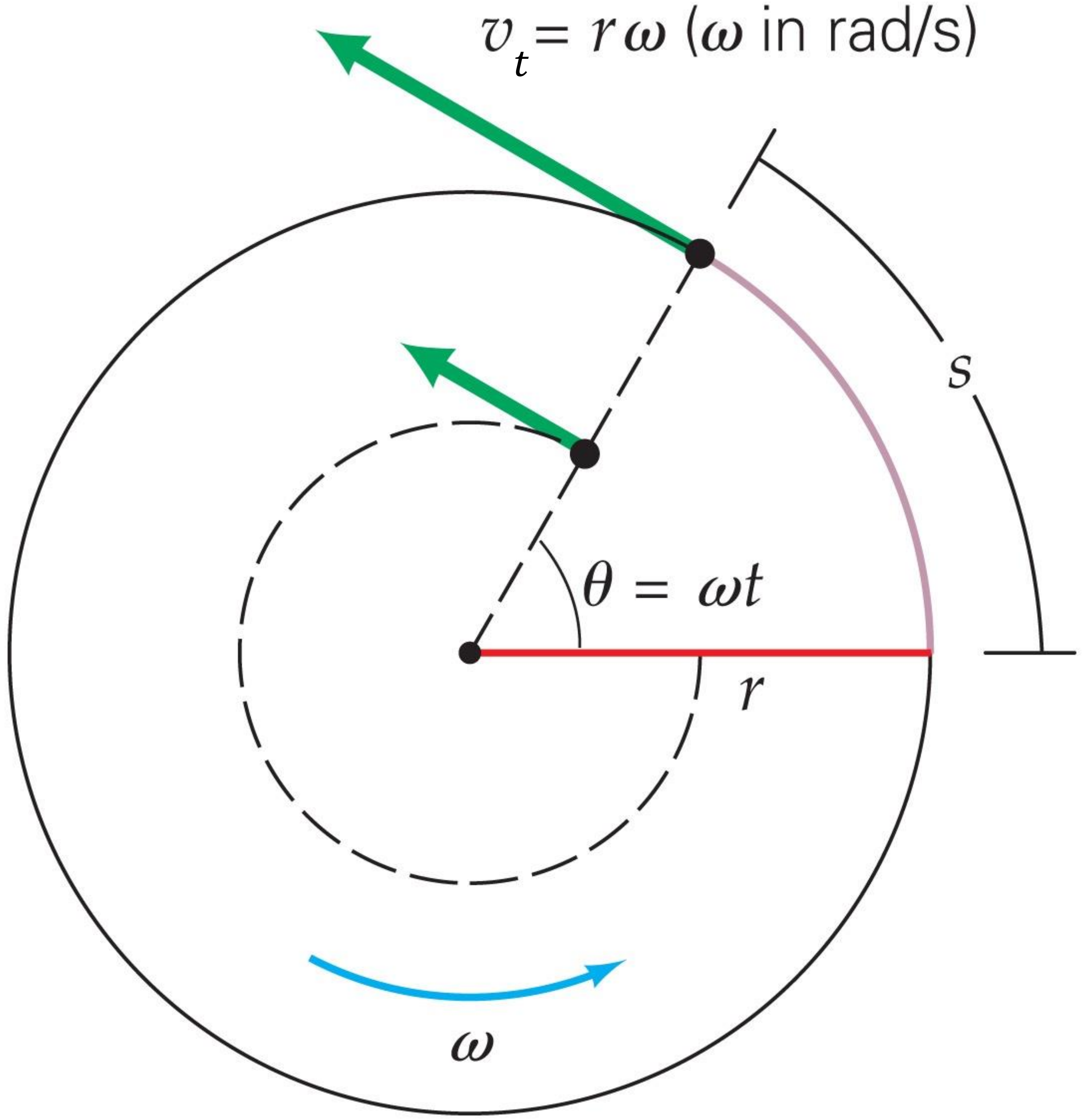
- **Angular speed, ω:** Angle per unit time

$$|\omega| = \left|\frac{d\theta}{dt}\right|$$

Angle

- **Units of angular speed:**
  - SI: rad/s or s^{-1}
  - Revolutions per minute ("RPM")

$$\frac{1 rev}{min} = \frac{2\pi rad}{60 s}$$

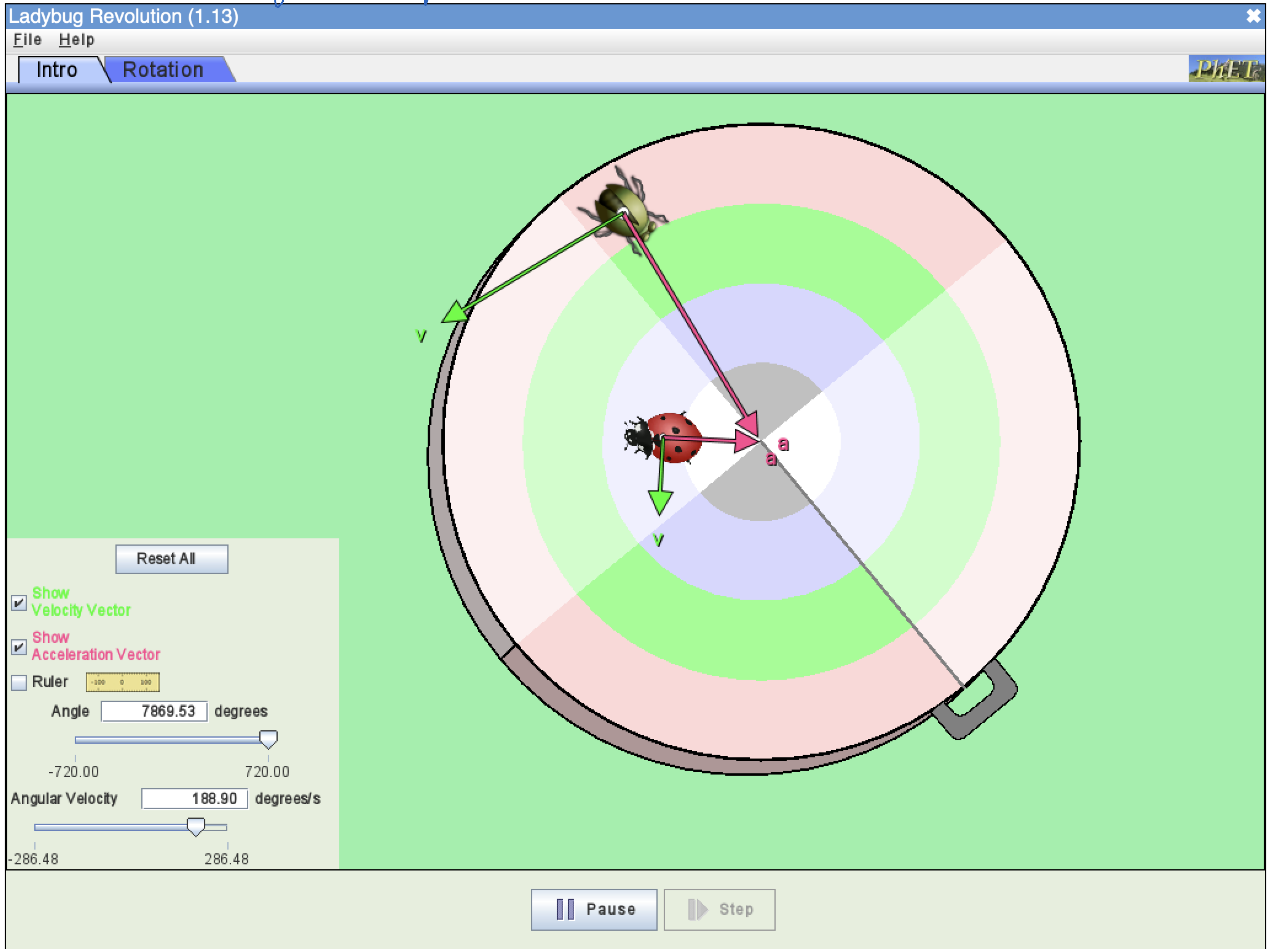


Demo

Tangential speed: $|v_g| > |v_r|$

Angular speed $|\omega_g| = |\omega_r|$

- Step 1: Put the red bug in the center
- Step 2: Rotate the circle. Does the red bug move?
- Step 3: Move the red bug to be off center. How about its motion now?
- Step 4: Place the grey bug farther from the center than the red bug. Compare the speed of the two bugs. Compare the angular speed of the two bugs.



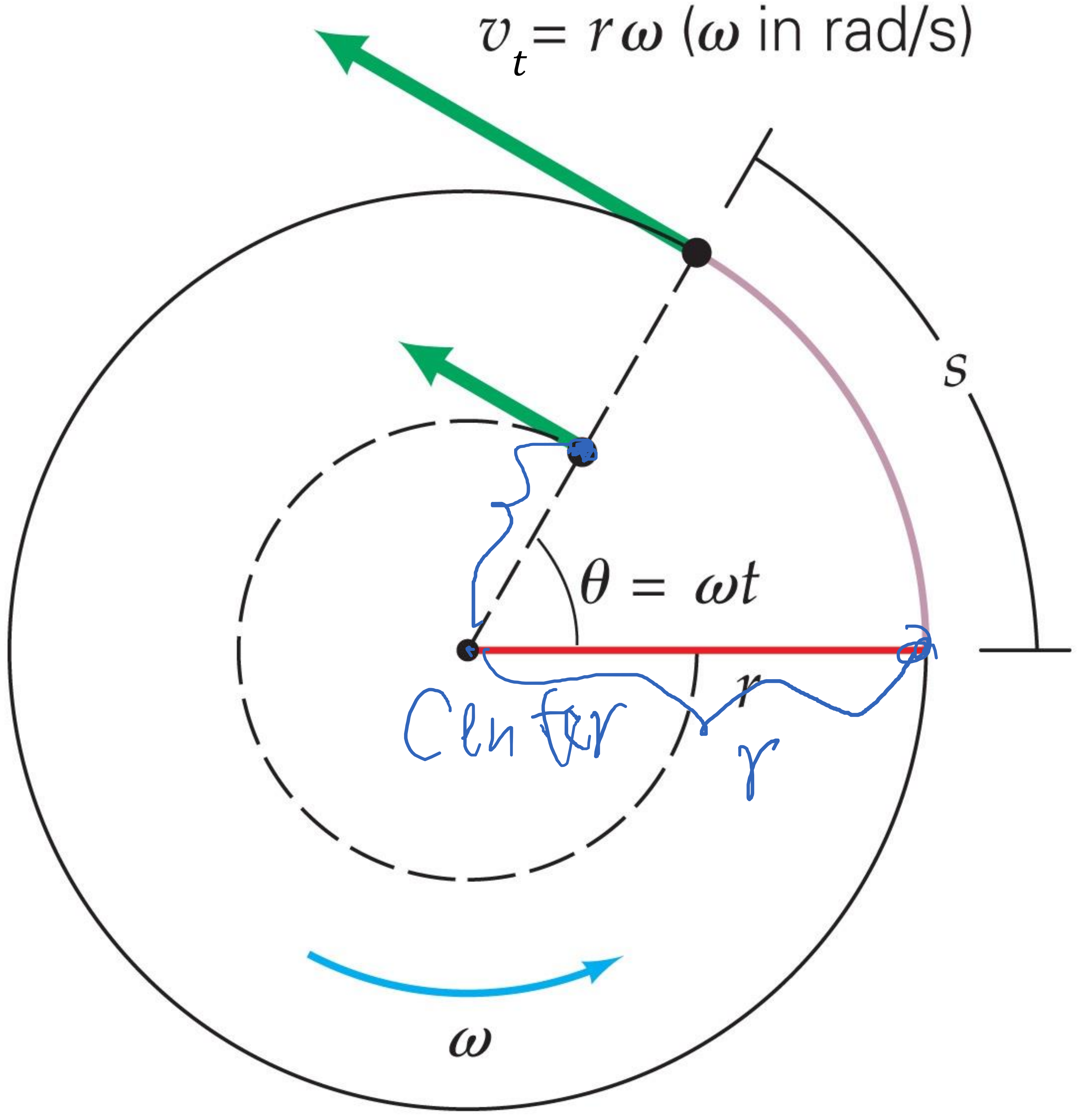
https://phet.colorado.edu/sims/cheerpj/rotation/latest/rotation.html?simulation=rotation

Relation between angular speed and tangential speed

- Angular speed and tangential speed:

$$v_t = \omega r$$

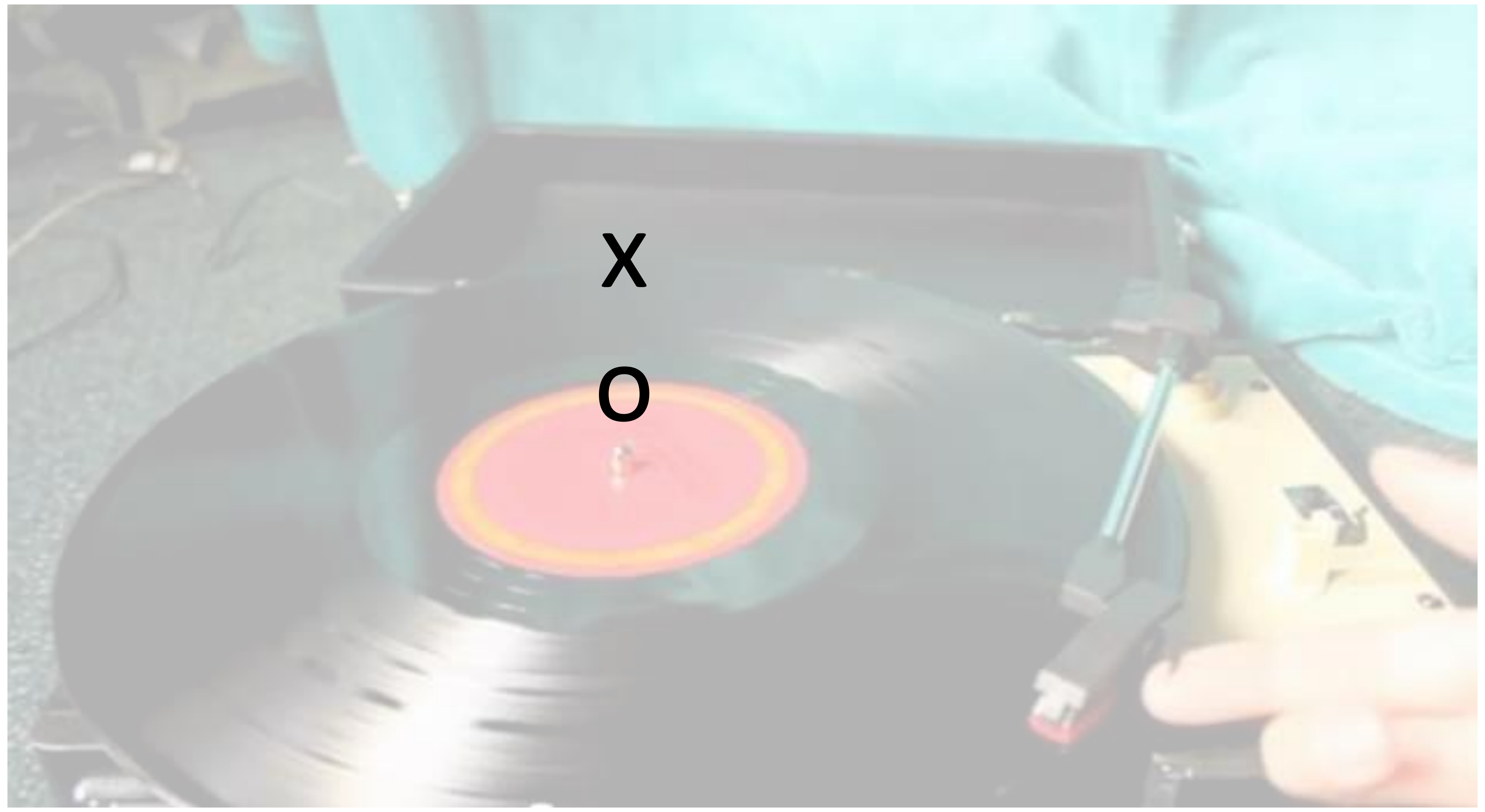
Tangential speed   Angular speed   Radius



Clicker question 1

The record rotates **clockwise**. How are the **angular speeds** related at points x and o?

- A Larger at x
- B Larger at o
- C equal at x and o
- D Depends on how fast the record spins



Clicker question 2

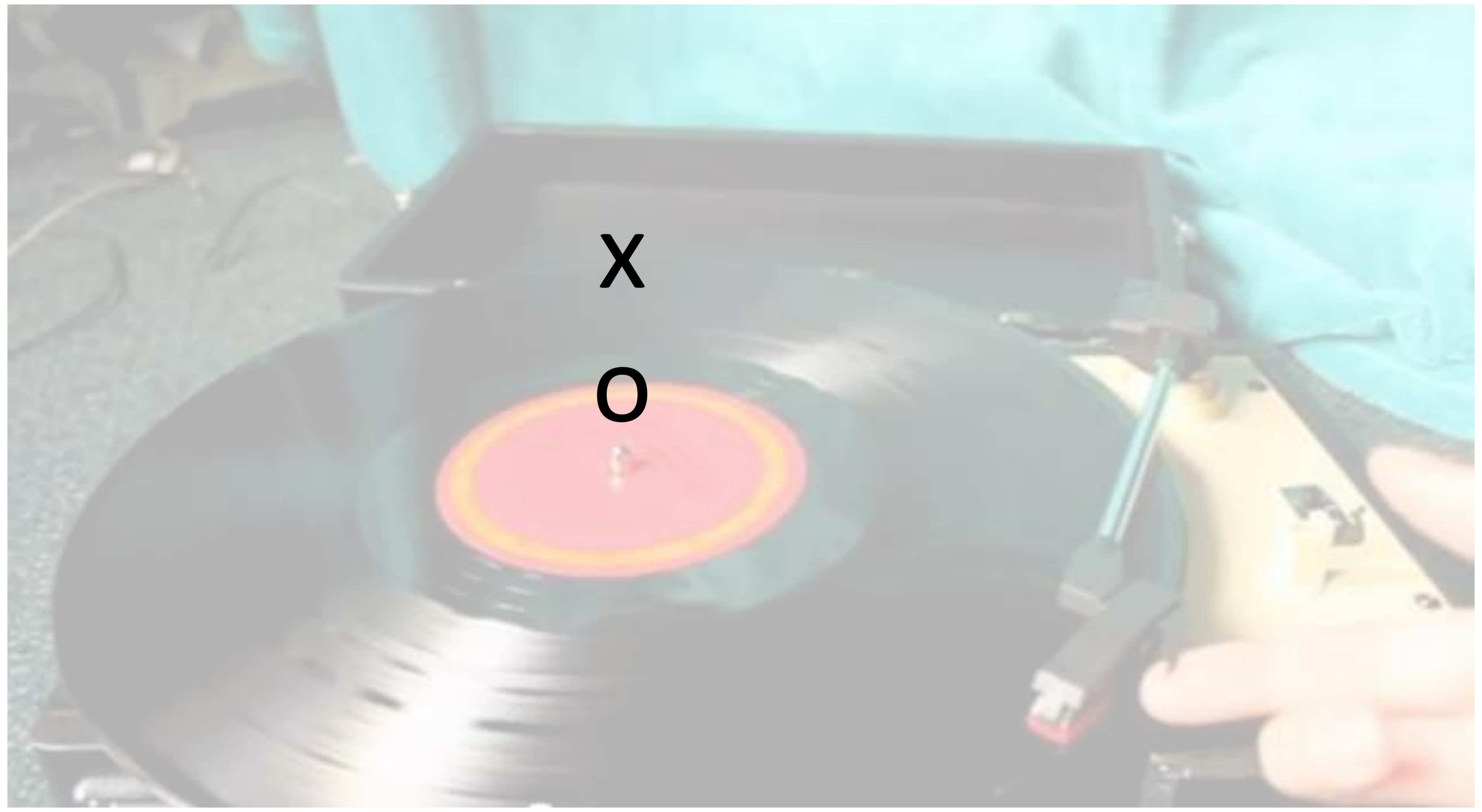
The record rotates **clockwise**. Which point has a faster **tangential speed**?

A Larger at x

B Larger at o

C equal at x and o

D Depends on how fast the record spins



$v_t = \omega r$ radius

Centripetal acceleration

- What happens in a circular motion?
  – The linear velocity **keeps changing the direction**

Changes dir. of vel.

- **Centripetal acceleration** (center-seeking acceleration)

tangential speed

- The **magnitude** is: $|a_c| = \frac{v_t^2}{r} = \omega^2 r$

radius

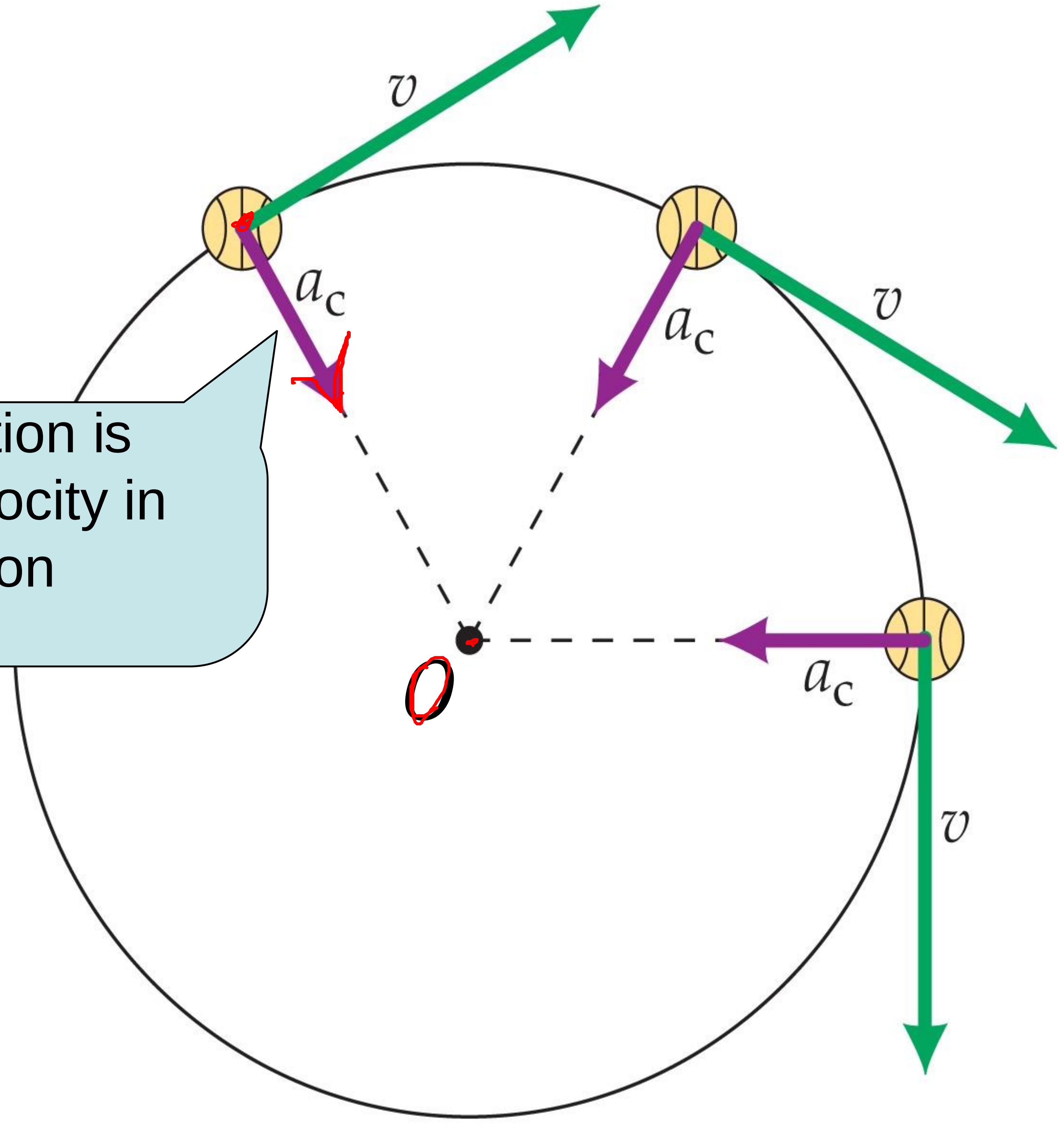
angular speed

$|a_c| = \frac{v_t^2}{r}$

$|a_c| = \omega^2 r$

- The **direction** is: Pointing towards the center of the circle.

Centripetal acceleration is perpendicular to velocity in uniform circular motion



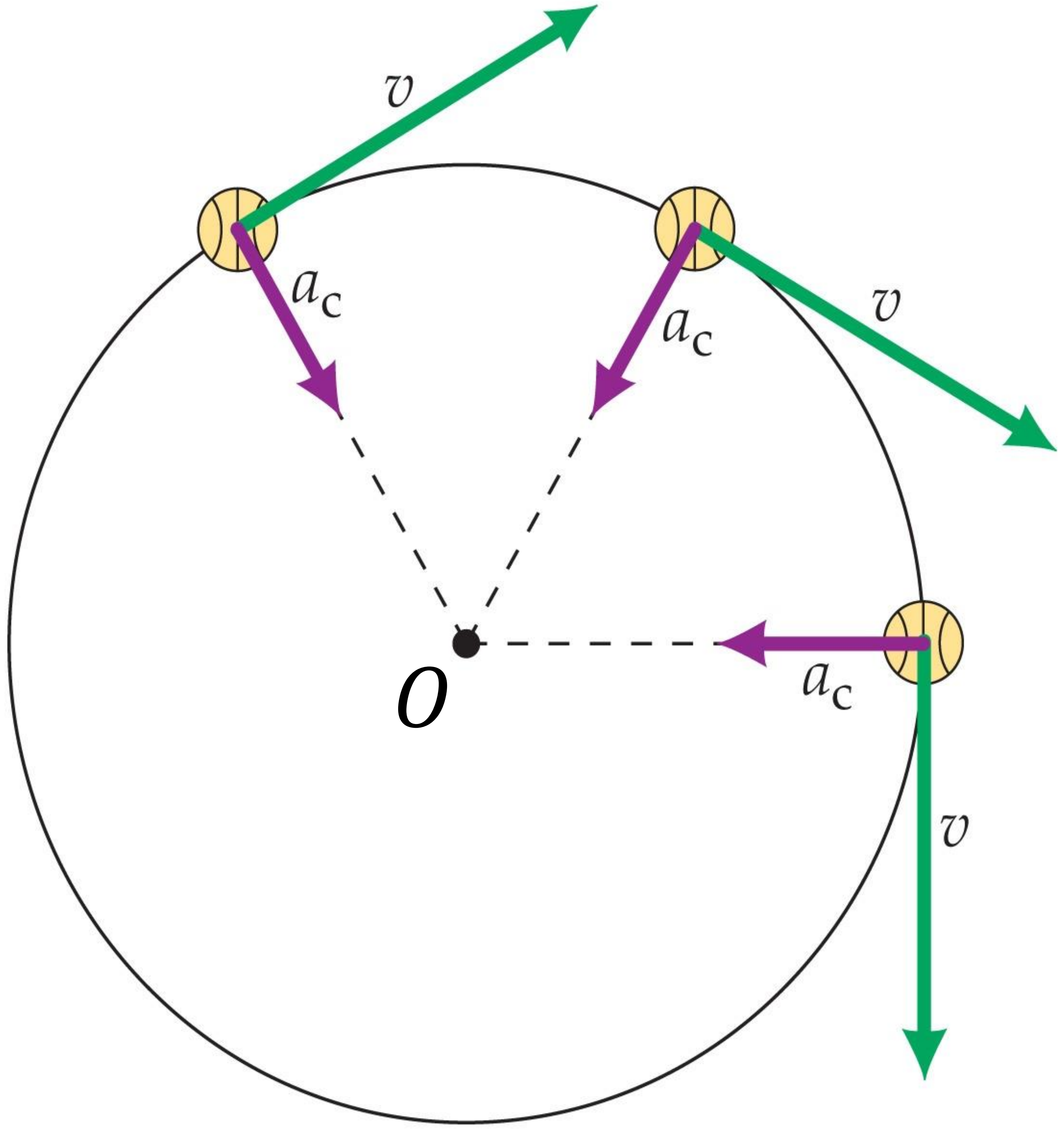
Uniform circular motion (UCM)

- **Uniform circular motion:** A circular motion with a const angular velocity, $\vec{\omega} = const$
- In a uniform circular motion, the acceleration is summarized by the **centripetal acceleration**. $\vec{a}_c$

$$|\vec{a}_c| = \frac{v_t^2}{r}$$

$$\text{or} \quad |\vec{a}_c| = \omega^2 r$$

Direction: Points to center



Clicker question 3

$\vec{a}$ can change $|\vec{v}|$ or direction of $\vec{v}$

vel.: $\vec{v}_t$ changes direction

- Is it possible for the acceleration to be non-zero while traveling at a constant speed?

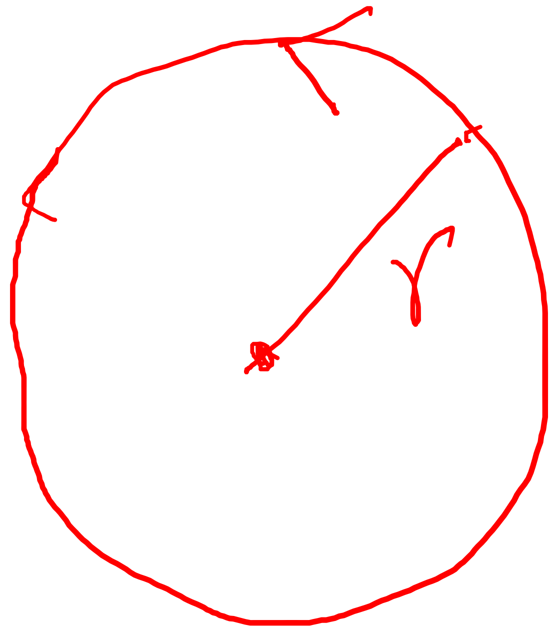
Scalar

A Yes

B No

Example: UCM: $|\vec{\omega}| = \text{const}$, $r = \text{const}$

Tangential $|v_t| = \omega r$



UCM: $|v_t| = \text{const}$

$$|a_c| = \frac{v_t^2}{r} \neq 0$$

Uniform circular motion 4

Question 7.1 Tetherball

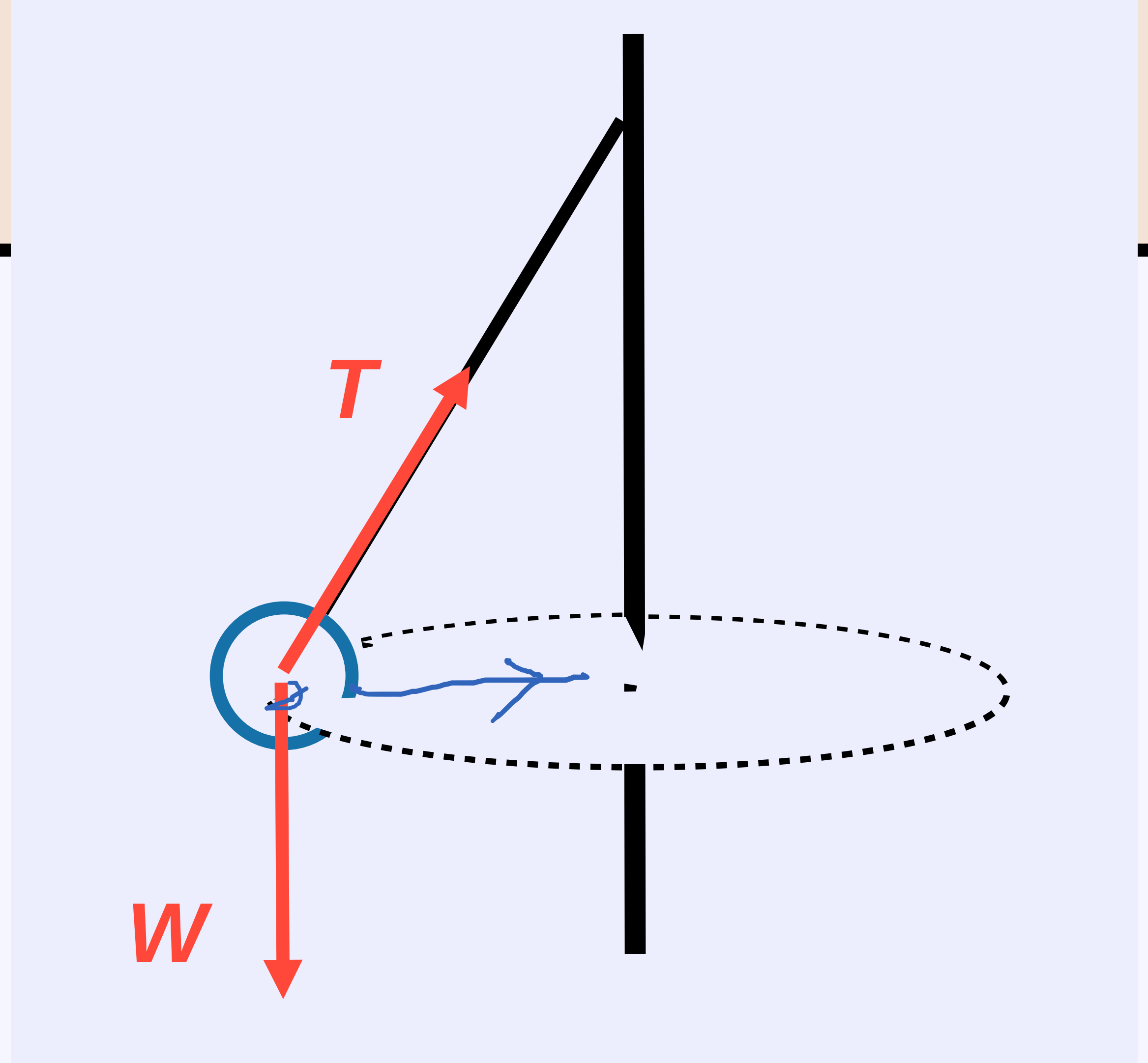
In the game of tetherball, the struck ball whirls around a pole, moving at a constant speed in a circle. In what direction is the acceleration on the ball point?

A toward top of pole along rope

B tangential to the circle

C along the horizontal component of the tension force

D along the vertical component of the tension force

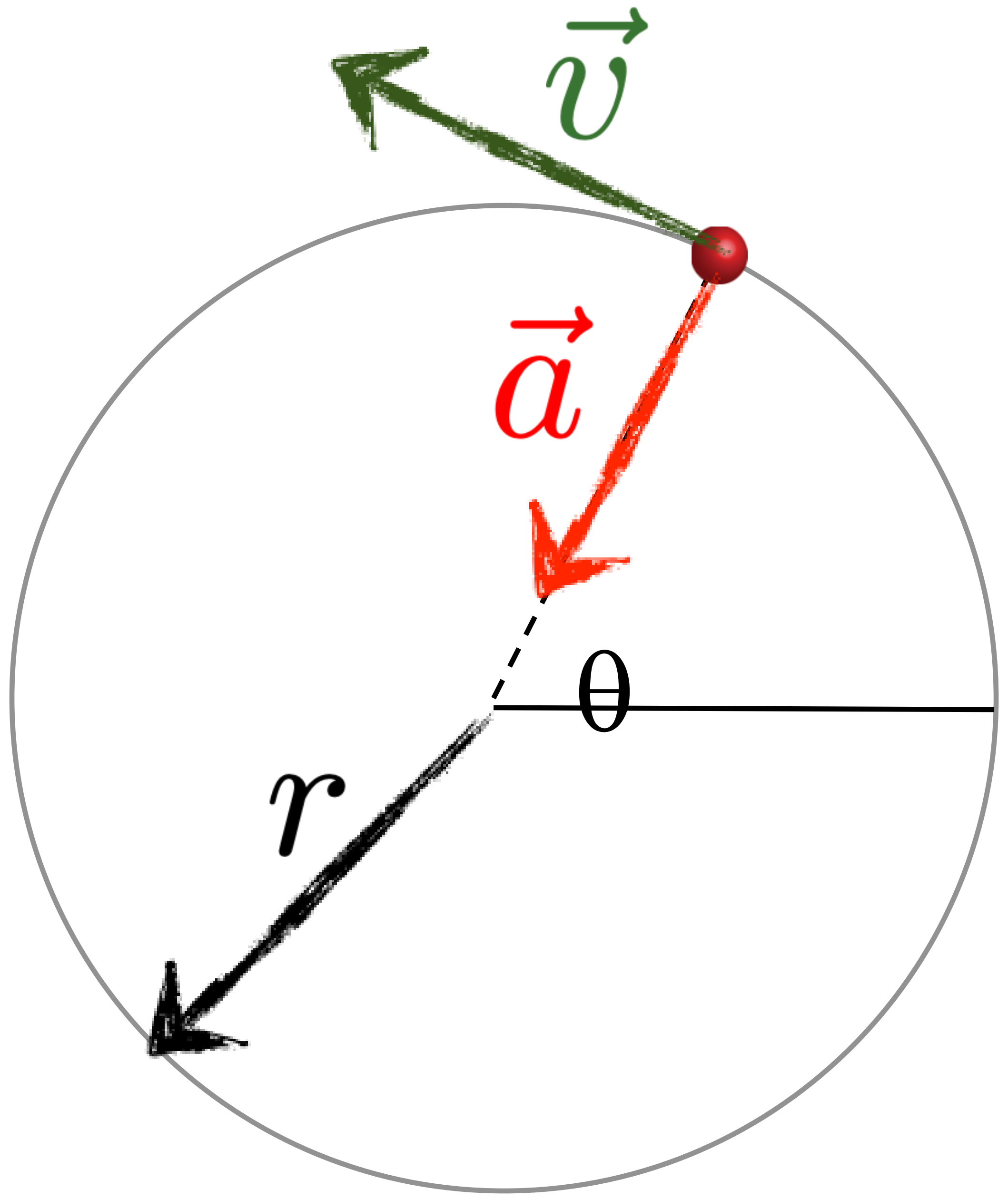


Dir. of $\vec{a}_c$ is from the obj. to center of circle.

Clicker question 5: Emulator of Mission in Space

Gien: |a|, r
Goal: v_t

- In Disney World's "Mission: Space", tourists move in uniform circular motion, experiencing a centripetal acceleration of magnitude $|\vec{a}| = 2.5g$. If the radius of motion is $r = 9.0$ m, to calculate the magnitude of the tourists' speed, v, which principle & eqn for Centripetal acceleration is more convenient?



A $a_c = \omega^2 r$

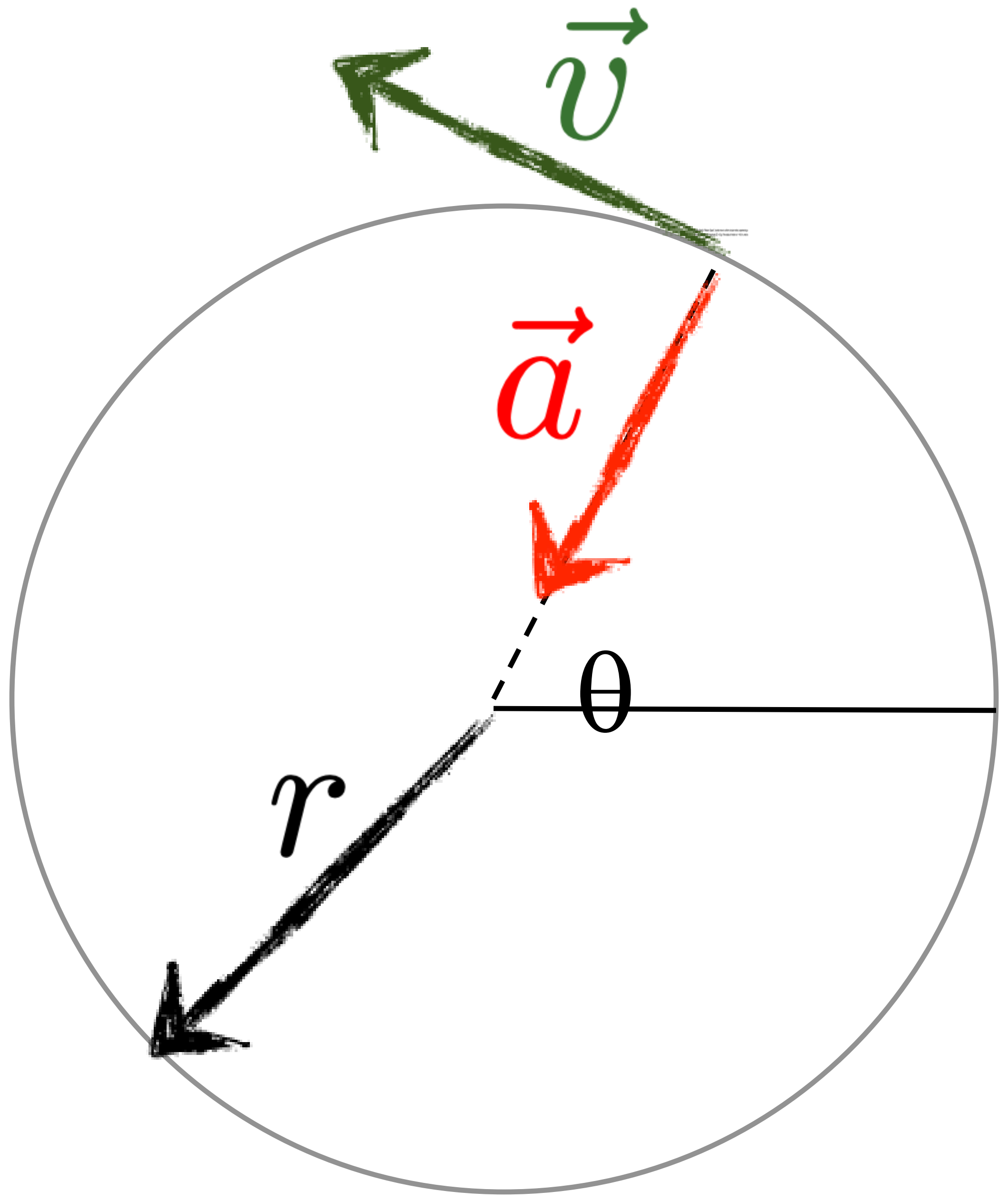
B $a_c = \dfrac{v^2}{r}$

Example 1

Given: $|a_c|$, r

Goal: v_t

- In Disney World's "Mission: Space", tourists move in uniform circular motion, experiencing a centripetal acceleration of magnitude $|\vec{a}| = 2.5g$. If the radius of motion is $r = 9.0$ m, what is the tourists' speed, v?



Step1: $|a_c| = \frac{v_t^2}{r}$

Rewrite: $v_t = \sqrt{|a_c|\,r} = \sqrt{2.5 \times 9.8\,ms^{-2} \cdot 9.0\,m} \approx 14.8\,ms^{-1}$

$\sqrt{2.5\,g\,r}$

$g = 9.8\,ms^{-2}$

Clicker question 2: Earth

$\frac{2\pi}{1\,\text{day}}$

- Earth radius = 6371 km
- What centripetal acceleration (magnitude and direction) do you feel because of the rotation of the earth? (***Hint:*** *Earth rotates 1 rev/day*)

Which principle & eqn. is more convenient?

A $a_c = \omega^2 r$

B $a_c = \frac{v^2}{r}$

Example 2: Earth

- Earth radius = 6371 km $= 6371000\,m$
- What centripetal acceleration (magnitude and direction) do you feel because of the rotation of the earth?

Step1: $\omega = \frac{2\pi}{day} = \frac{2\pi}{\cancel{day} \cdot \frac{24\,\cancel{hr}}{\cancel{day}} \cdot \frac{3600\,s}{\cancel{hr}}}$

$= \frac{2\pi}{24 \times 3600\,s} \approx 0.0000727\,s^{-1}$

Step2: $a_c = \omega^2 r$

$\approx (0.0000727\,s^{-1})^2 \times 6371000\,m$

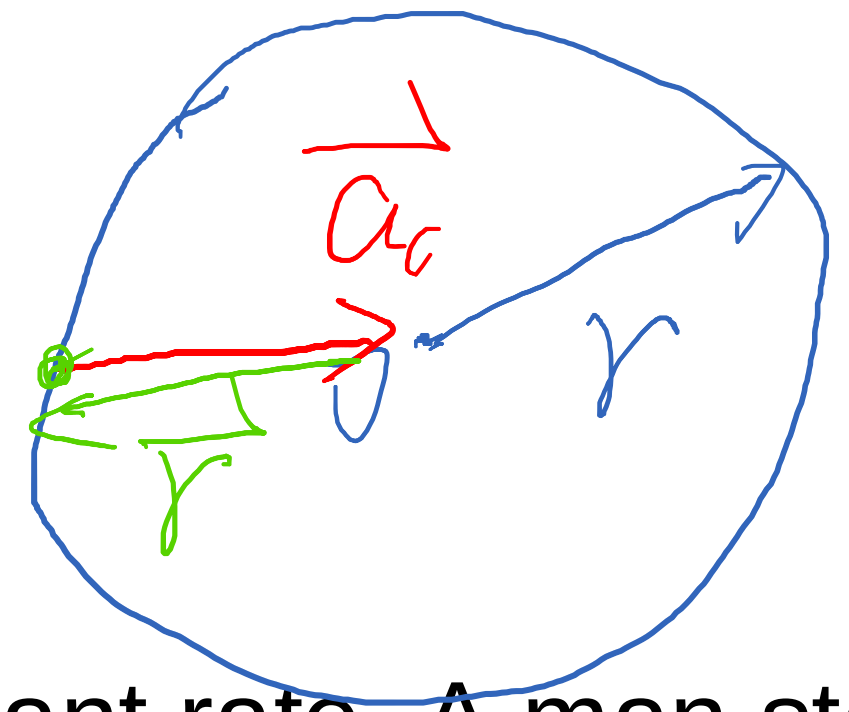
$\approx 0.03369\,m\,s^{-2}$

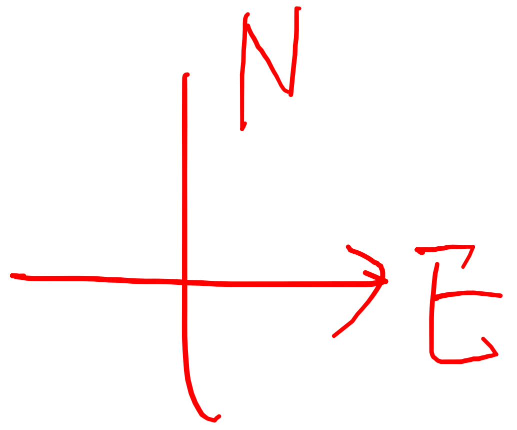
Example 3:

Given: $|v_t|, |a_c|, \vec{a}_c$

Goal: $|\vec{r}|, \vec{r}$

v_t ~ Tangential speed





- A carnival merry-go-round rotates about a vertical axis at a constant rate. A man standing on the edge has a constant speed of 3.04 m/s and a centripetal acceleration of magnitude 1.93 m/s^2. The man is at the position $\vec{r}$ relative to the rotation axis. **(a)** What is the magnitude of $\vec{r}$? **)b)** If $\vec{a}$ is directed due east, What is the direction of $\vec{r}$?

a) $|a_c| = \frac{v_t^2}{r}$ $\longrightarrow |\vec{r}| = \frac{v_t^2}{|a_c|} = \frac{(3.04\,m s^{-1})^2}{1.93\,m s^{-2}} \approx 4.79\,m$

Tangential speed

b) W

Given: r (Radius), Δy, Δx for proj. motion, $\vec{a}$

Comprehensive example

Goal: $|a_c|$

Sketch

UCM x-z

Proj. motion x-y

$\Delta y = -1.99\,m$

- A tetherball is whirled in a horizontal circle of radius 1.46 m and at height 1.99 m above level ground. The string breaks, and the tetherball flies off horizontally and strikes the ground after traveling a horizontal distance of 9.43 m. What is the magnitude of the centripetal acceleration of the tetherball while in circular motion? Use g=9.8 m/s^2. Air friction is neglected.

Step 1: $|a_c| = \frac{v_t^2}{r}$, $v_t = ?$

Step 2: $v_t = v_{x_0}$, $\Delta x = v_{x_0} t \rightarrow v_{x_0} = \frac{\Delta x}{t}$

Step 3: $\Delta y = \underset{0}{v_{y_0}} t - \frac{1}{2} g t^2$, $\rightarrow t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{-2(-1.99m)}{9.8ms^{-2}}} \approx 0.637s$

Step 4: Plug t to Step 2, $v_{x_0} = \frac{\Delta x}{t} = \frac{9.43m}{0.637s} \approx 14.8ms^{-1} = v_t$

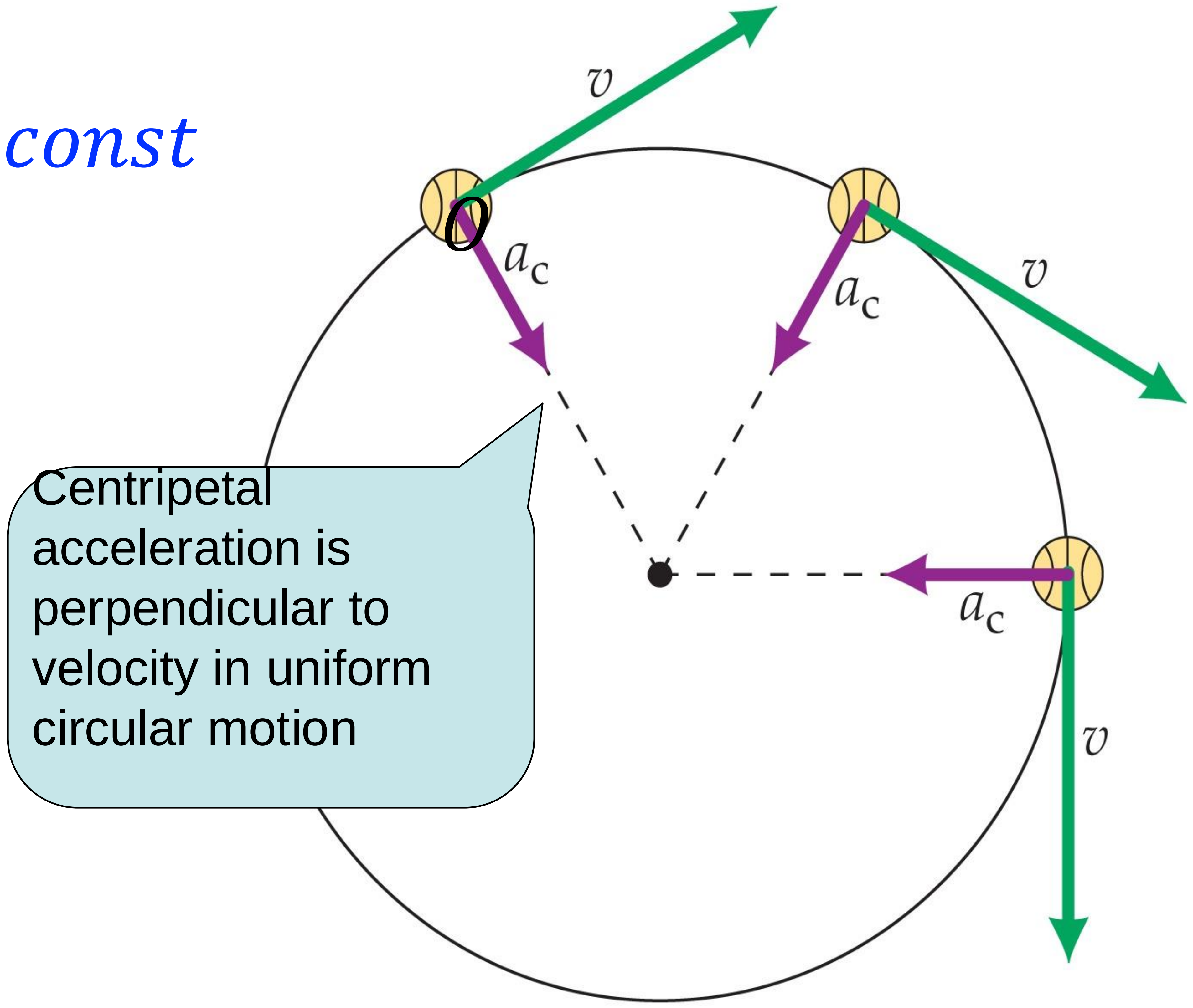
Step 5: Plug v_t to step 1: $|a_c| = \frac{v_t^2}{r} = \frac{(14.8ms^{-1})^2}{1.46m} \approx 150\,ms^{-2}$

Summary: Uniform circular motion

- **Uniform circular motion:** A circular motion with a const angular velocity

- What happens in a uniform circular motion (UCM)?
  - **Angular velocity** is a constant: $\vec{\omega} = const$
  - **Linear speed (tangential speed)** is a constant: $|\vec{v}| = const$
  - **Linear velocity** keeps changing the direction
  - **Centripetal acceleration** points to the center and

$$|a_c| = \frac{v_t^2}{r} = \omega^2 r$$



Pre-lecture survey

- Please complete Module 4.1.3: Pre-lecture Survey on reference frames before the next lecture on Wednesday.