PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 21: Kinetic energy and work

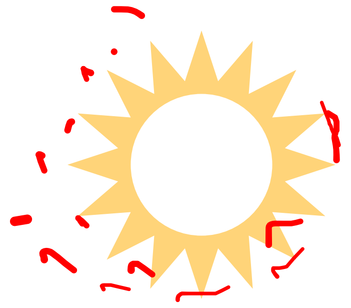
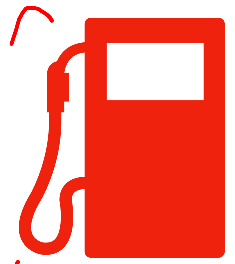
CALIFORNIA STATE UNIVERSITY
FULLERTON

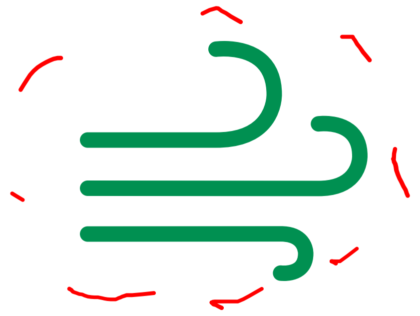
Learning goals for today

- Kinetic energy
- Work

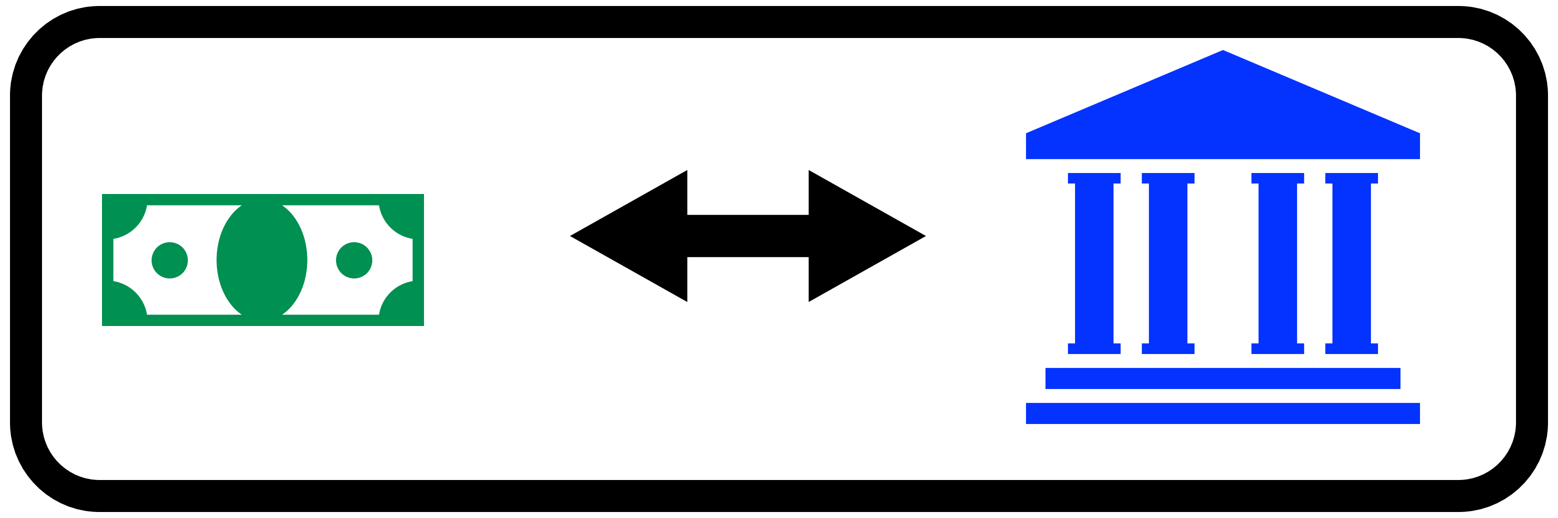
1. Energy

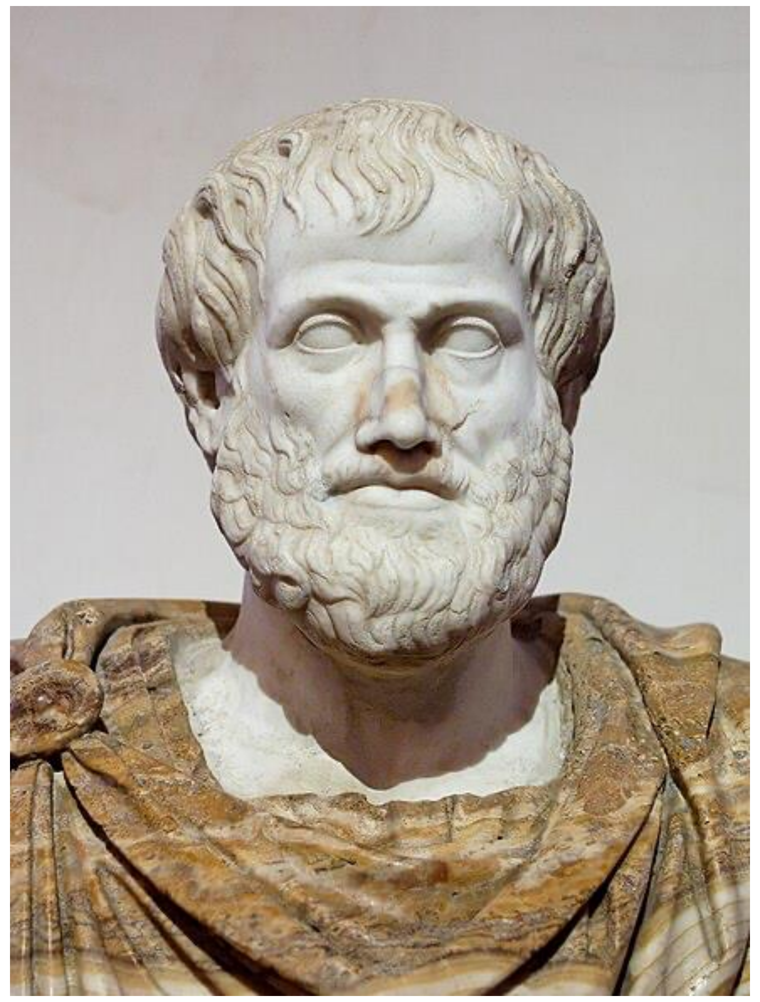
- Energy is the ability to do work
  - Is a scalar quantity
  - Unit: J or N*m
  - Can be changed from one form to another

Solar   Petroleum

Wind   Hydrolic

  - Is conserved in a closed system





“Energy” was first introduced as a philosophical concept by Aristotle [1]

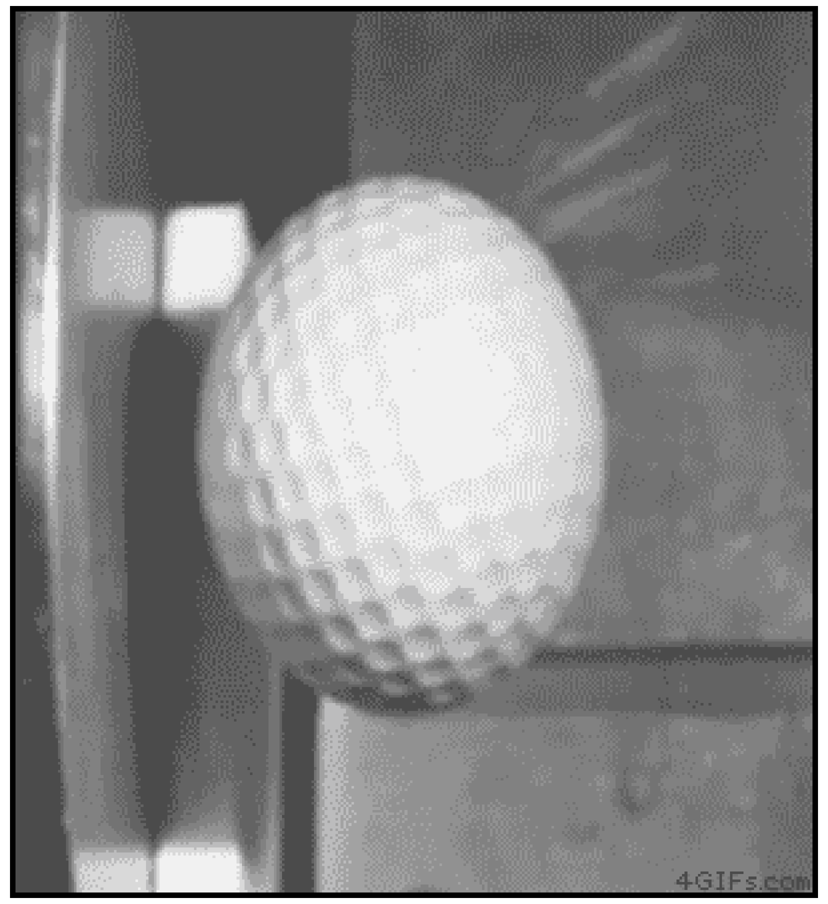
[1] *Bergmann, Gustav. "Logic and reality." Foundations of Language 3, no. 4 (1964).*

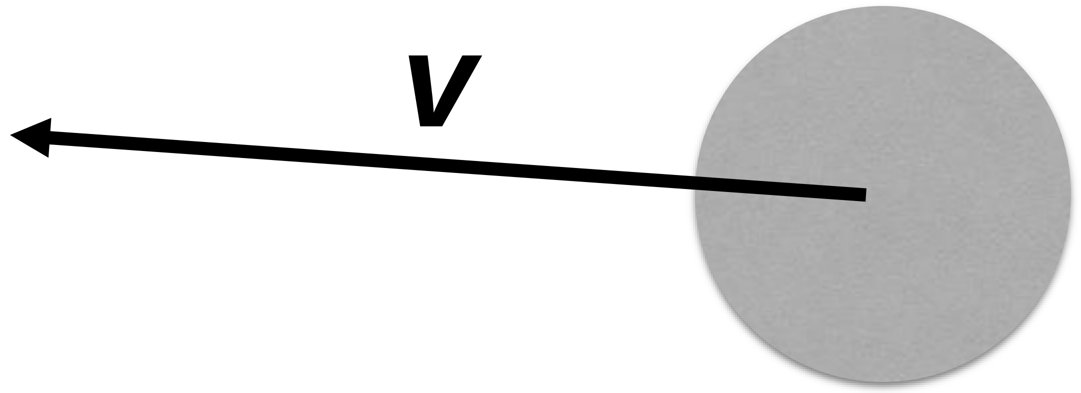
Cannot create or destroy energy.

Analogy

Can transfer or transform E.

1.1. Kinetic energy





mass speed

$$K = \frac{1}{2} m v^2$$

Early understandings of kinetic energy can be attributed to [Gaspard-Gustave Coriolis](), 1829. [1]

velocity

$$v^2 = \vec{v} \cdot \vec{v}$$

Dot product

- Energy associated with the motion of an object
- The faster it moves, the greater is its kinetic energy
- Here we limit ourselves to far below the speed of light

[1] *Du Calcul de l'Effet des Machines*

Clicker Question 1

kg

$$K = \frac{1}{2}mv^2$$

$m \cdot s^{-1}$

- What are the equivalent units of kinetic energy?

A Newtons * m

B Joules

C kg * m^2 * s^{-2}

D All above.

Newton's 2nd law

$\vec{F}_{net} = m\vec{a}$

$N \sim kg \; m \; s^{-2}$

Clicker question 2

$$K = \frac{1}{2}mv^2$$

1 kg

2m s^{-1}

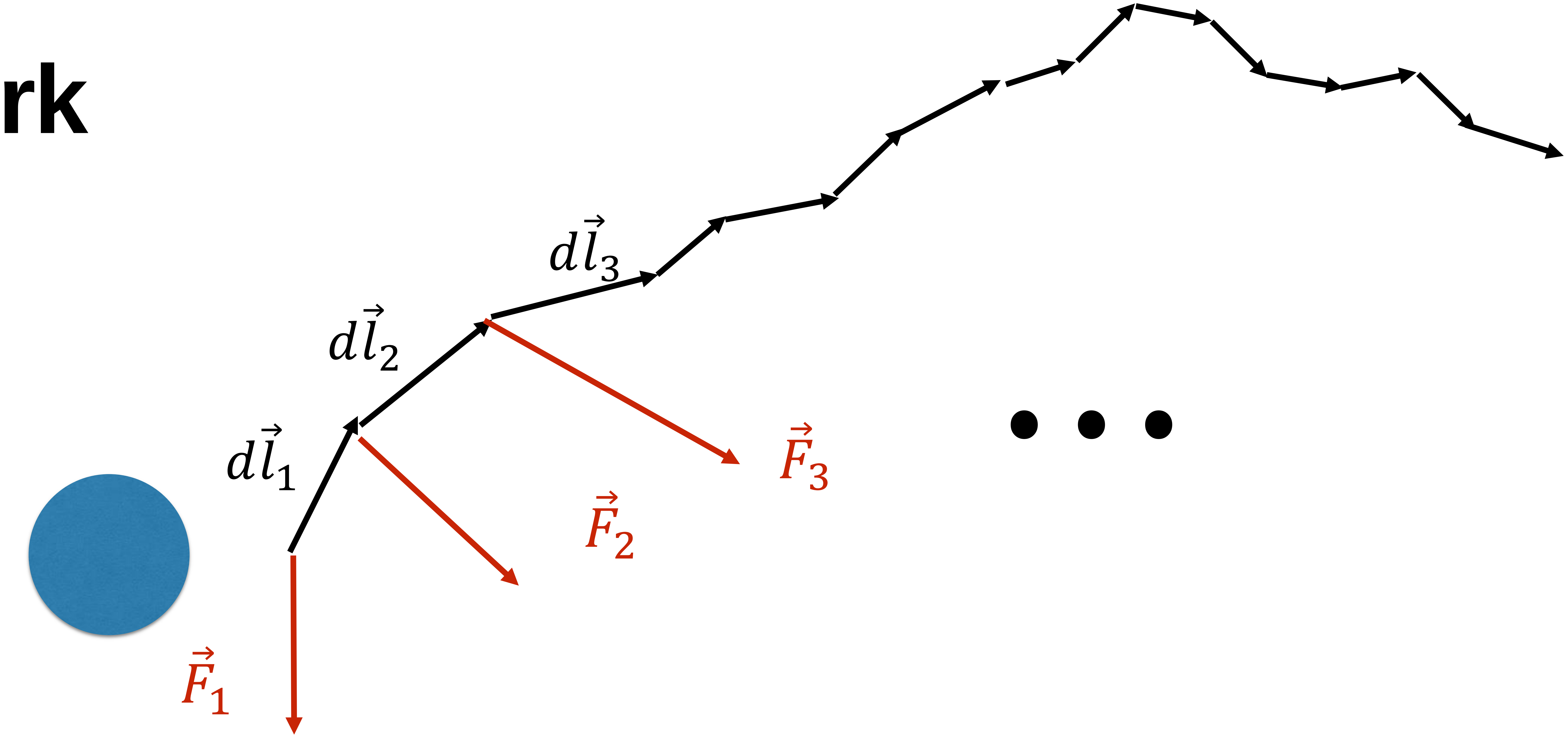
- A 1 kg ball is moving at a speed of 2 m/s. What's its instantaneous kinetic energy?

A 2 J

B 1 J

C 4 J

2. Work



Force Disp.

- Work is the energy transferred to or from an object by a force acting on it: $W = \int \vec{F} \cdot d\vec{l}$
- If energy is transferred to the object, **positive** work is done on the object;
  If energy is transferred from the object, **negative** work is done on the object.

Clicker Question 3

Energy transferred by a force acting on an obj.

$$W = \int \vec{F} \cdot d\vec{l}$$

N m

- What are the equivalent units of work?

  √ A Newtons * m

  √ B Joules

  √ C kg * m^2 * s^{-2}

  D All above.

Recap: Dot product

- **Dot Product** (or scalar product): A product between two vectors that creates a new **scalar.**
  - In terms of vector components:

    1. Decompose
    2.

$$\underbrace{\boxed{\vec{a} \cdot \vec{b}}}_{\text{Scalar}} = a_x b_x + a_y b_y + a_z b_z$$

vector vector scalar scalar scalar

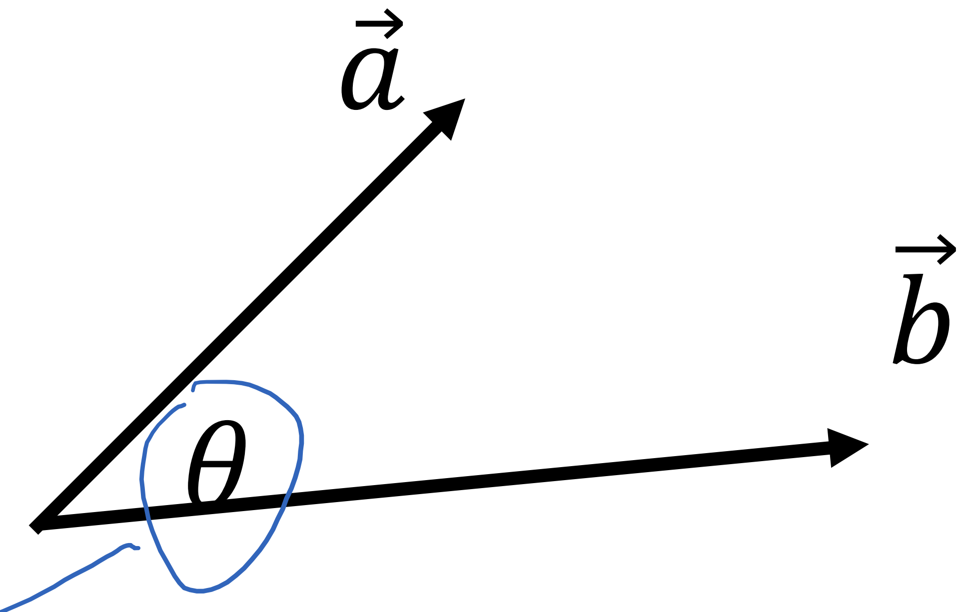
  - In terms of geometry:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

Magnitude of $\vec{b}$

Magnitude of $\vec{a}$

angle between $\vec{a}$ & $\vec{b}$



If $0 < \theta < 90^\circ$ then $\vec{a} \cdot \vec{b} > 0$

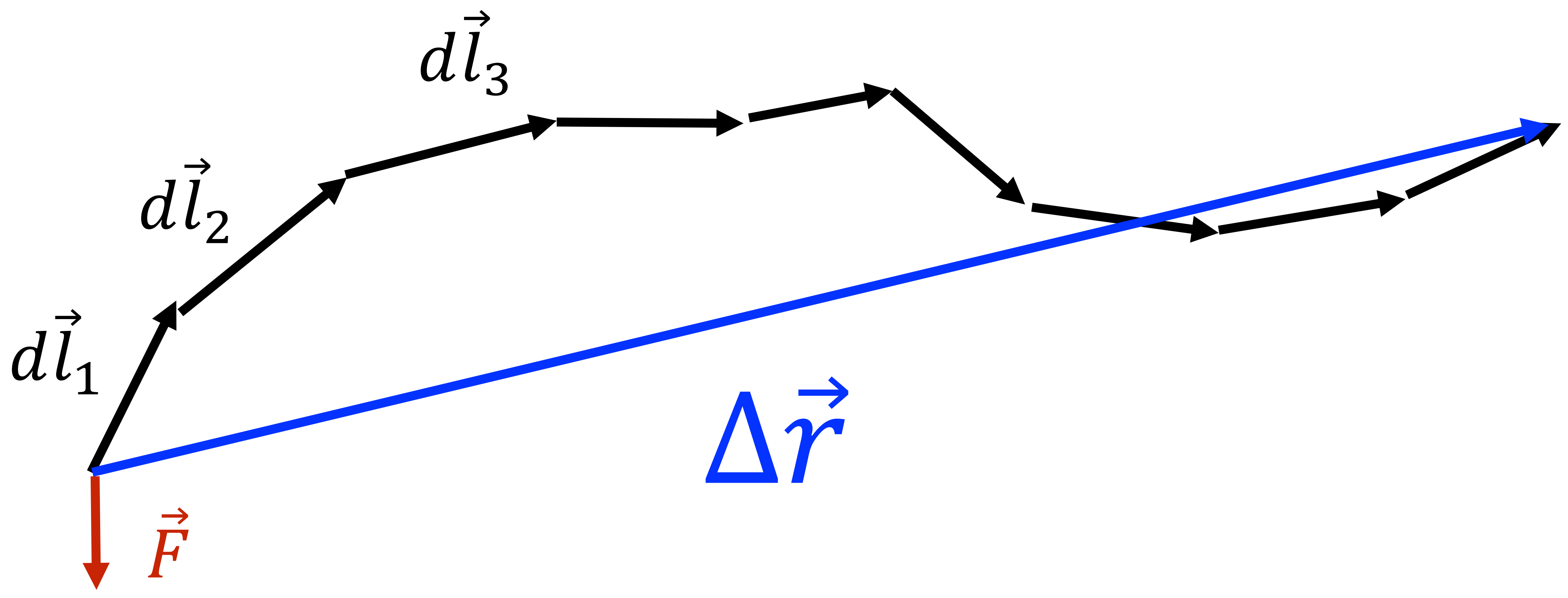
If $90^\circ < \theta < 180^\circ$ then $\vec{a} \cdot \vec{b} < 0$

if $\theta = 90^\circ$ then $\vec{a} \cdot \vec{b} = 0$

Note: The sign of $\vec{a} \cdot \vec{b}$ is determined by the angle between them

2.1. Work by a constant force

- When force is a constant vector, $\vec{F}$, work is the dot product of $\vec{F}$ and the displacement, $\Delta\vec{r}$:



Decompose $\left\{ \begin{array}{l} \vec{F} \\ \Delta\vec{r} \end{array} \right.$

For a constant force, $\vec{F}$,

$$W = \vec{F} \cdot \Delta\vec{r}$$

$$= F_x\Delta x + F_y\Delta y + F_z\Delta z$$

Clicker question 4

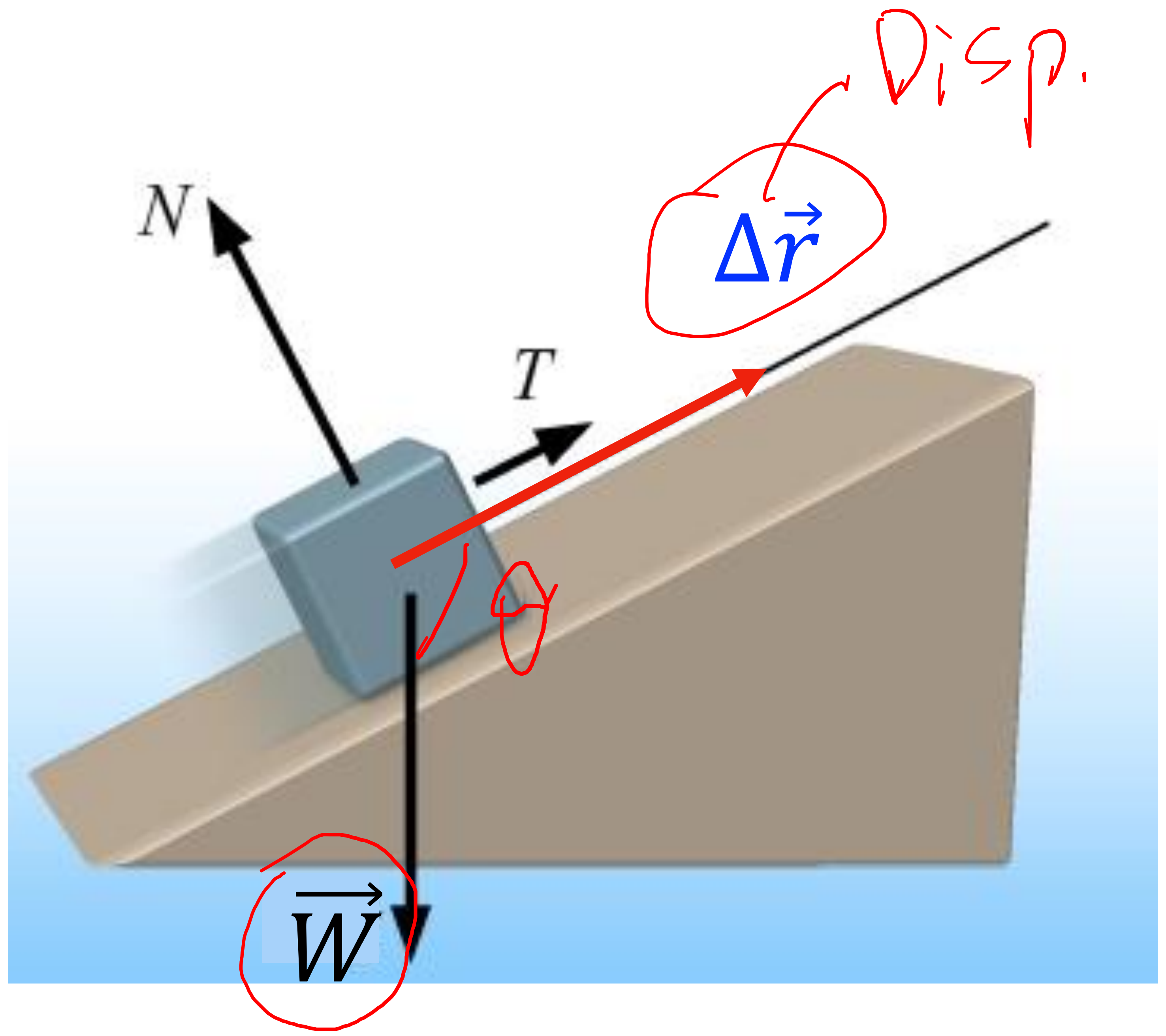
B.C. $\vec{W}$ = const

$\text{Work} = \vec{W} \cdot \Delta\vec{r} = |\vec{W}| \cdot |\Delta\vec{r}| \cos\theta$

- What is the sign of work done by the gravitational force if the block moves ***up*** the slope?

$90^\circ < \theta < 180^\circ$

$\cos\theta < 0$



A –

B *0*

C +

D **not enough information to know**

Clicker question 5

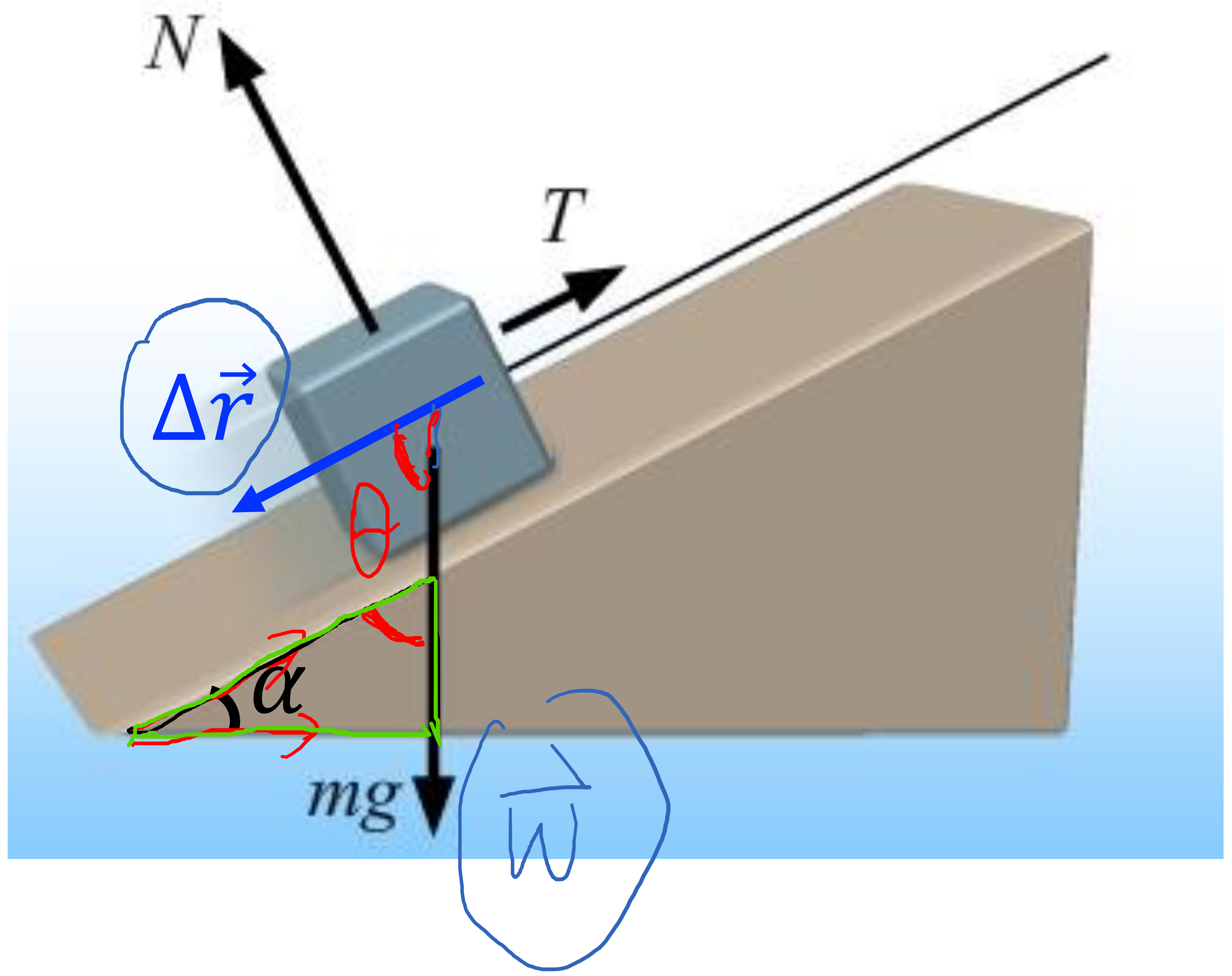
$\vec{W}$ is a const. force

Work $= \vec{W} \cdot \Delta\vec{r} = |\vec{W}||\Delta\vec{r}|\cos\theta$

$= |\vec{W}||\Delta\vec{r}|\cos(90^\circ - \alpha)$

- What is the work done by the gravitational force on the block if the block moves ***down*** the slope of incline angle α by $\Delta\vec{r}$?

$= |\vec{W}||\Delta\vec{r}|\sin\alpha$



A) $-mg|\Delta\vec{r}|\sin\alpha$

B) 0

C) $mg|\Delta\vec{r}|\sin\alpha$

D) $mg|\Delta\vec{r}|\cos\alpha$

$0 < \theta < 90^\circ$

$\theta = 90^\circ - \alpha$

Clicker question 6

$\vec{N}$ is a const. force

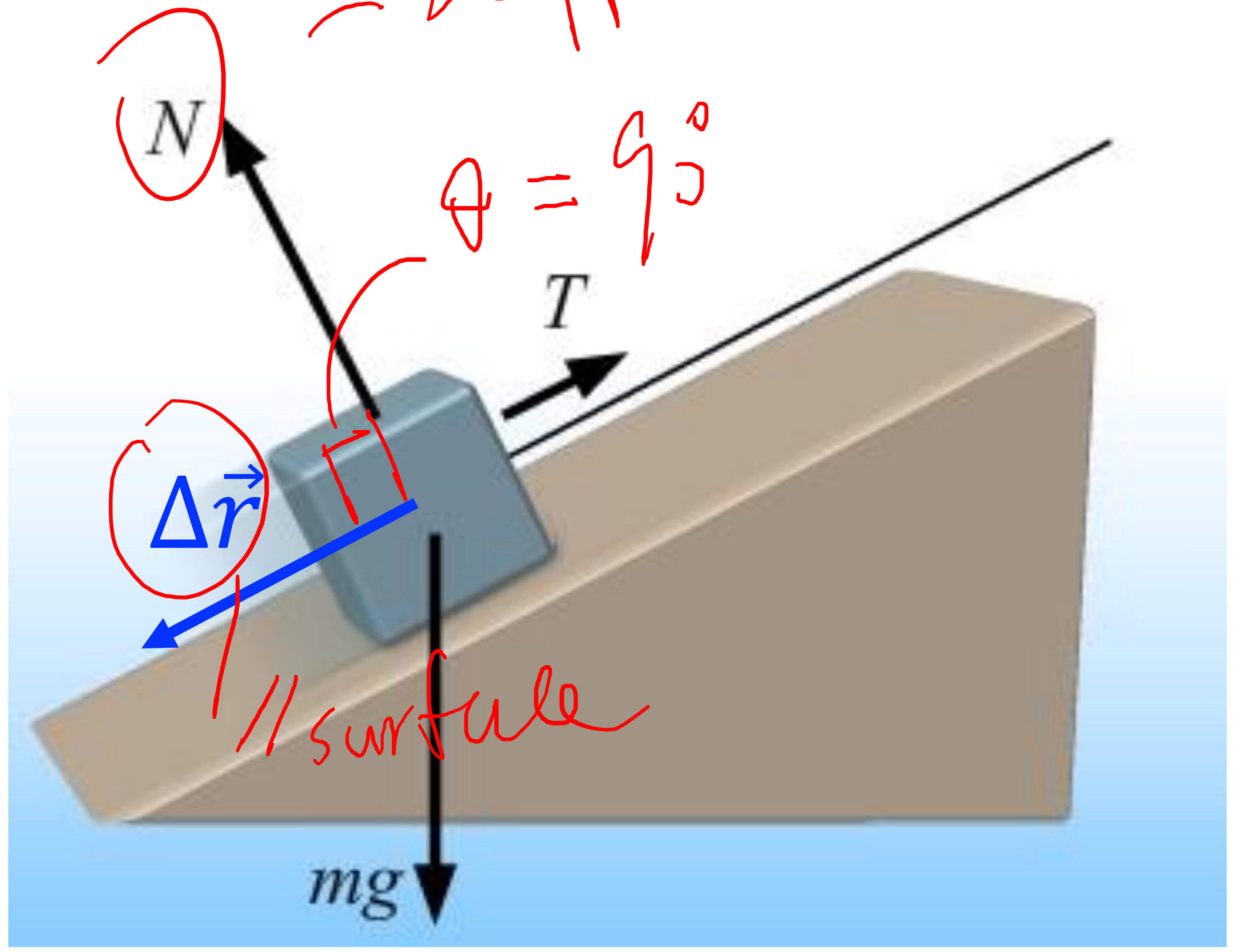
$\text{Work} = \vec{N} \cdot \Delta\vec{r} = |\vec{N}||\Delta\vec{r}| \cos\theta = 0$

90°

- What is the sign of work done by the normal force $\vec{N}$ if the block moves down the slope?

⊥ surface

— supporting force



A –

B 0

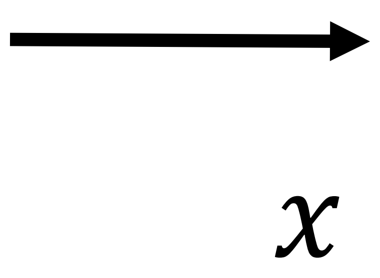
C +

D not enough information to know

Example 1

Given: $\vec{F}$, $\vec{d}$

Goal: Work



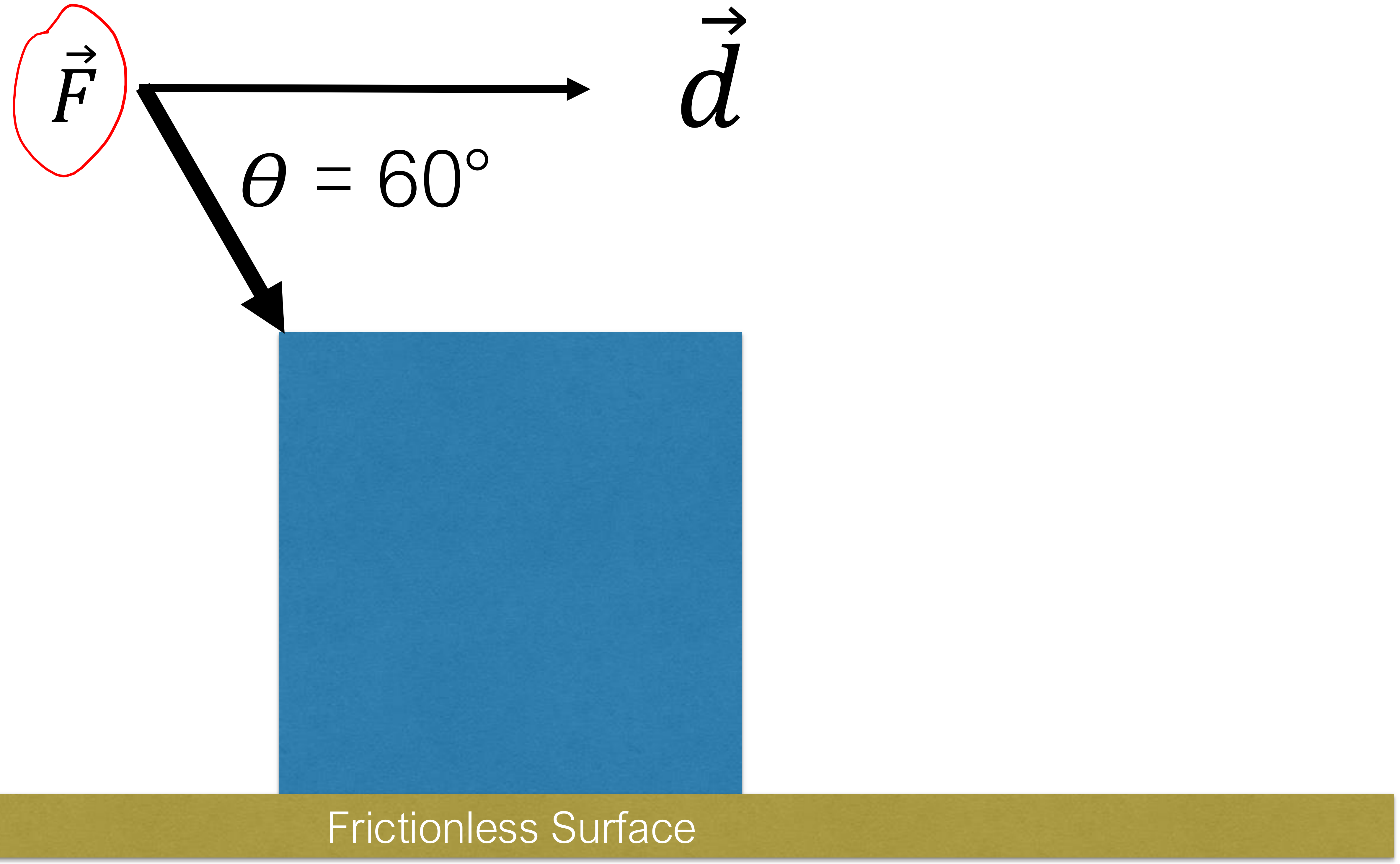
- What is the work done by $\vec{F}$ on the box if it is pushed for a displacement $\vec{d}$ = 10m to the right? (|**F**|= 100 N, and 60° below the +x direction).

$$\text{Work} = \vec{F} \cdot \vec{d}$$

$$= |\vec{F}||\vec{d}| \cos\theta$$

$$= 100\,\text{N} \cdot 10\,\text{m} \cos 60^\circ$$

$$= 500\,\text{J}$$



Work-kinetic energy theorem

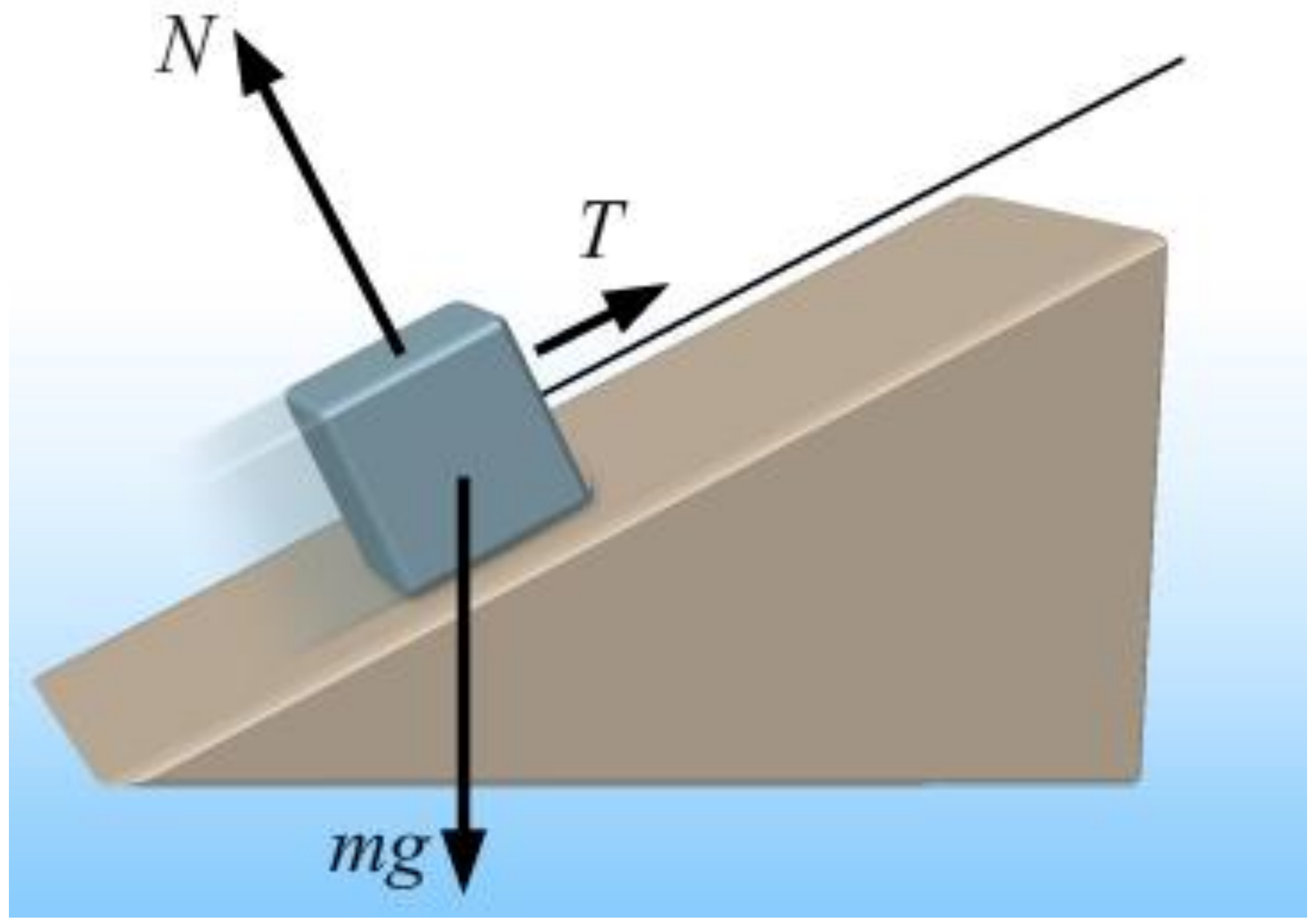
- The ***net*** work done by the net (total) force on an object, is the change of total kinetic energy, ΔK, of the object:

$$W_{net} = K_f - K_i$$

$$W_{NET} = \sum_i W_i = W_1 + W_2 + \cdots$$

You can just add up the work done by each force

- Two ways to calculate work on an object!



Example 2

Given: m, v_0, v_f

Goal: W_{net}

- A pitcher throws a baseball of 0.5 kg from rest to a exit speed of 50 m/s. What is the ***net*** work done on the baseball?

v_f

Step 1: W-K theorem:

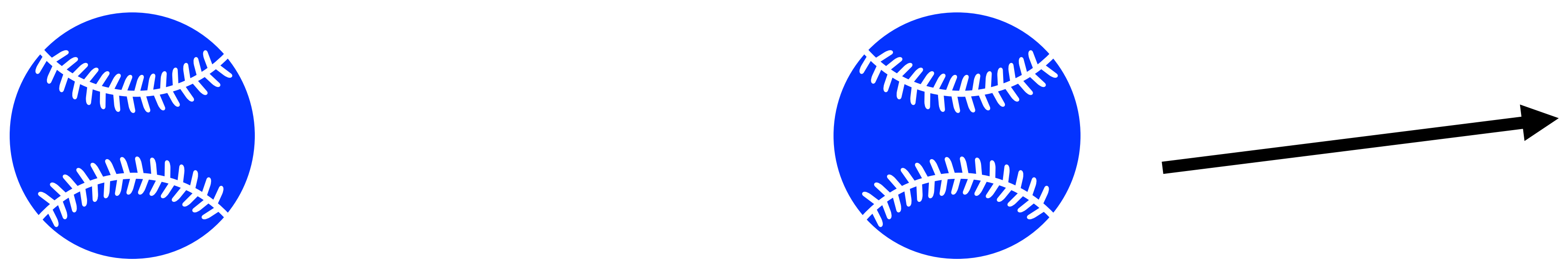
$$W_{net} = K_f - K_i$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= \frac{1}{2} m (v_f^2 - v_i^2)$$

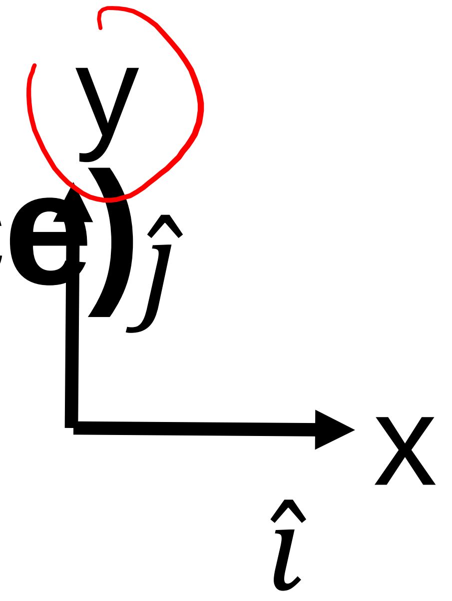
$$= \frac{1}{2} \times 0.5\,kg\,((50\,ms^{-1})^2 - 0)$$

$$= 625\,J$$



$\vec{v}_0 = 0$ $\qquad$ $\vec{v}_e = 50 m/s$

Example 2: Work done by weight (gravity near earth surface)



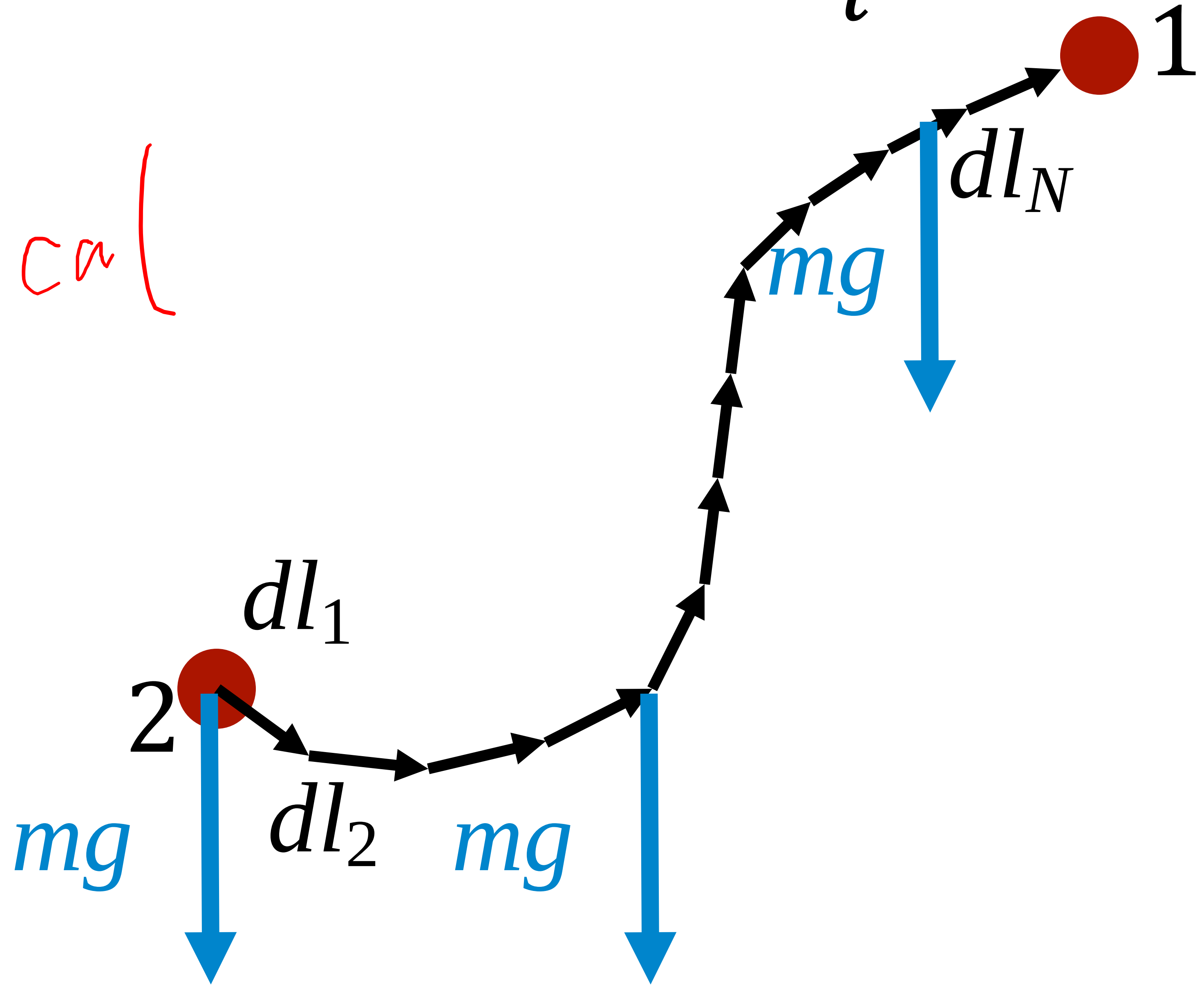
- Weight, i.e., gravity on the earth surface
  - Weight is a constant, $\vec{F} = -mg\hat{\jmath}$ — Along vertical

    $F_x = 0$, $F_y = 0$
  - Work by weight from position 1 to position 2, $W_{1\to 2}$:

$$W_{1\to 2} = \vec{F} \cdot \Delta\vec{r}$$

$$= -mg(y_2 - y_1)$$

final y pos. Initial y pos.



Assume the motion is only in the x-y plane.

  - *Therefore, **work by weight only depends on the initial and final height!***

Clicker question 7

$$\text{Work} = -mg(y_2 - y_1) = -mg(-H) = mgH$$

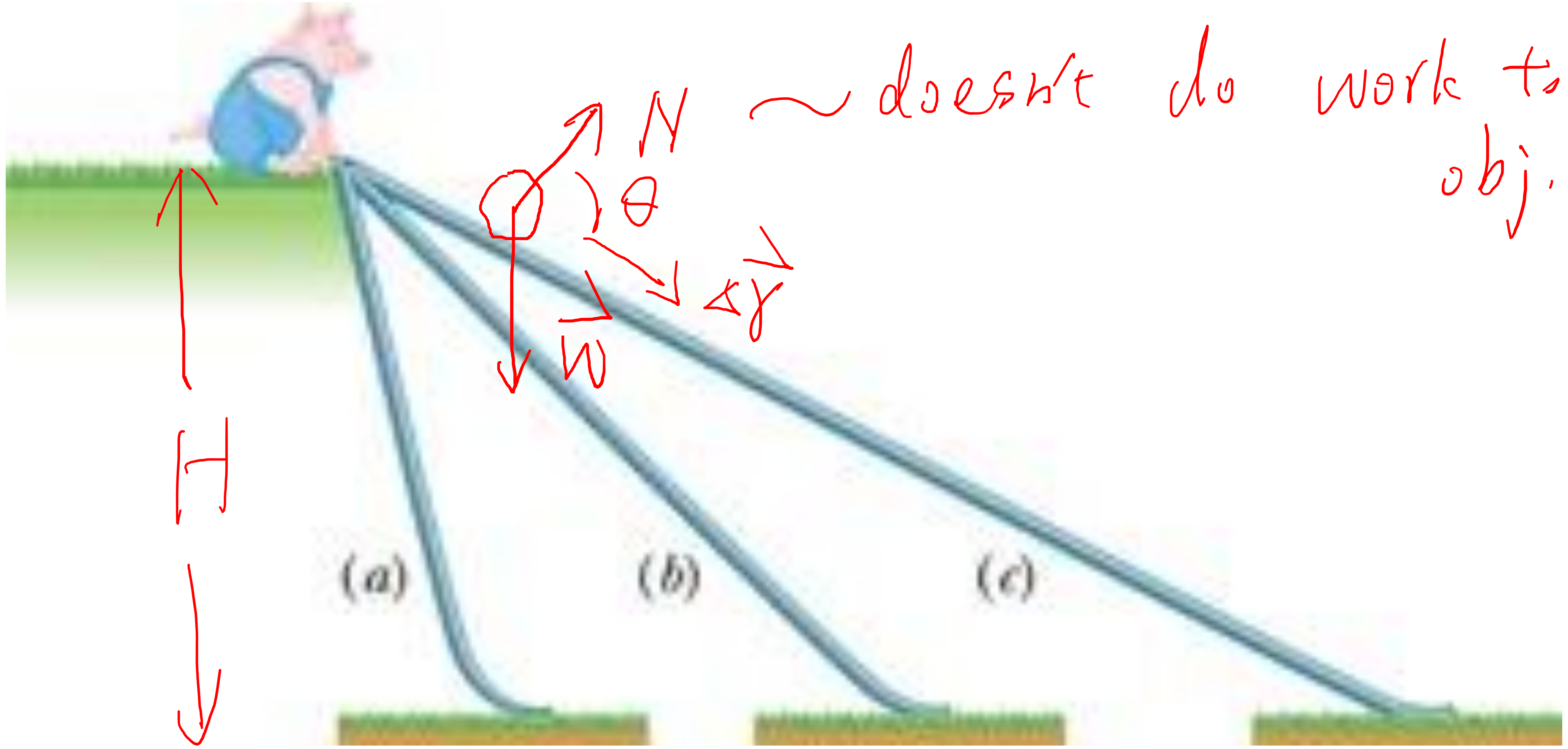
same for all 3 paths

- In the figure below, a piglet has a choice of three ***frictionless*** slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first.

A $a > b > c$

B $c > b > a$

C $a = b = c$

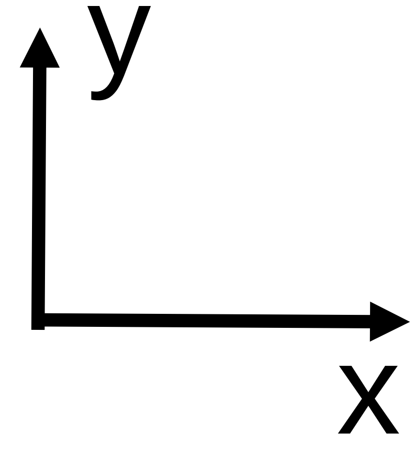


N ~ doesn't do work to obj.

Example 3

Given: m, $\vec{r}_1$, $\vec{r}_2$

Goal: Work



- A climber of mass 65kg climbs on a vertical wall from $\vec{r}_1$ to $\vec{r}_2$. The displacement $\vec{r}_2 - \vec{r}_1 = (2\,m\,\hat{\imath} + 5\,m\,\hat{\jmath})$ .What is the work done on the climber by the gravitational force?

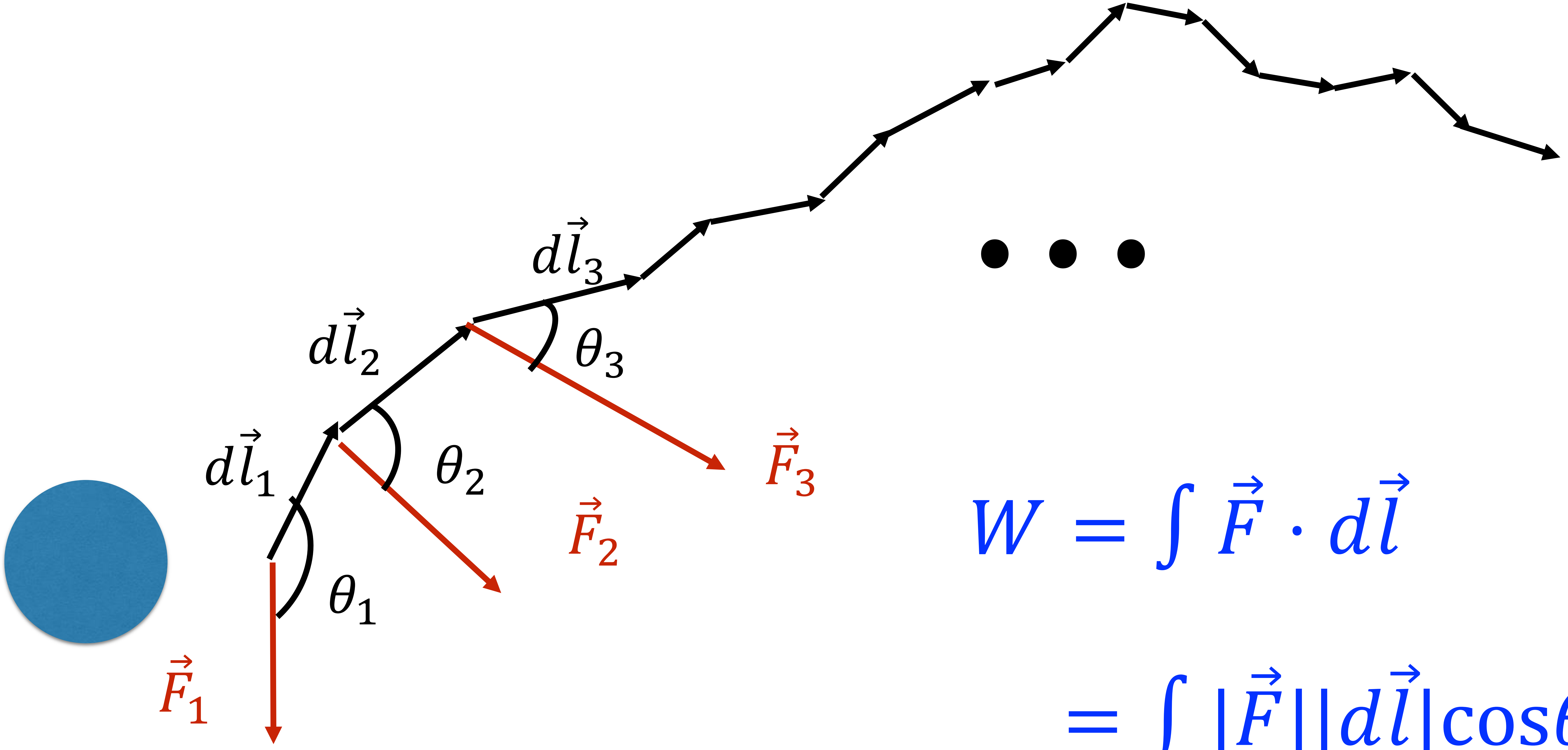
Step1: Work by $\vec{W} = -mg(y_2 - y_1)$

$= -65kg \times 9.8 m s^{-2} \times 5m$

$= -3185 J$

Climbing up, Work by $\vec{W}$ is $-$

2.2. Work of a non-constant force



• • •

$$W = \int \vec{F} \cdot d\vec{l}$$

$$= \int |\vec{F}||d\vec{l}|\cos\theta$$

Decompose the integral:

$$= \int F_x dx + F_y dy + F_z dz$$

Example: Work done by a spring force

Spring Force

+x

from relaxed pos.

- The spring force is linear with the displacement

$$\vec{F}_s = -k\vec{x}$$

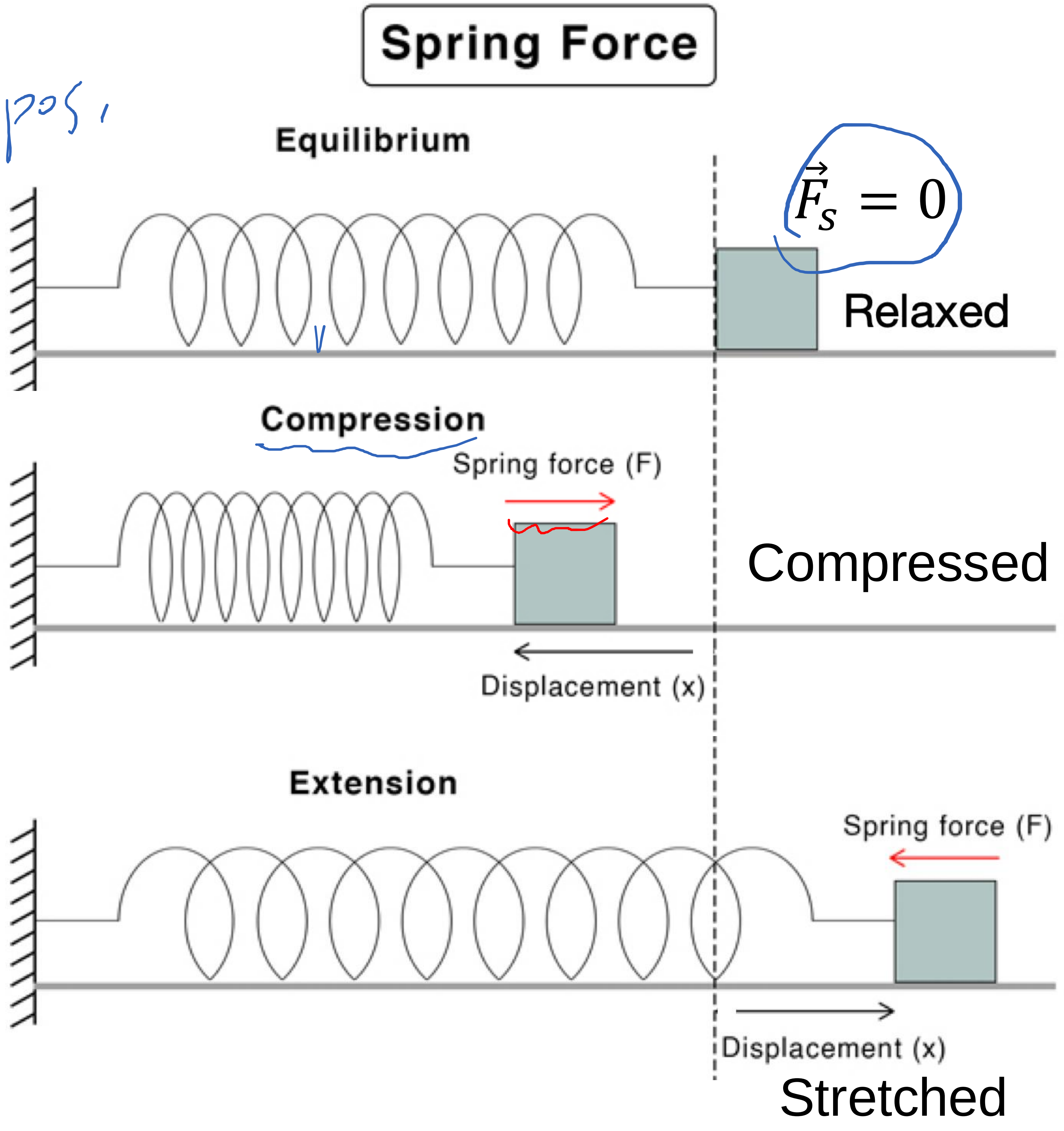
A function of disp.

spring constant

displacement from the relaxed position

Not a constant force!

- Also known as **Hooke's Law**



https://www.sciencefacts.net/spring-force.html

Work done by a spring

force

Hooke's law

$F_s = -kx$

$\rightarrow x$ Axis

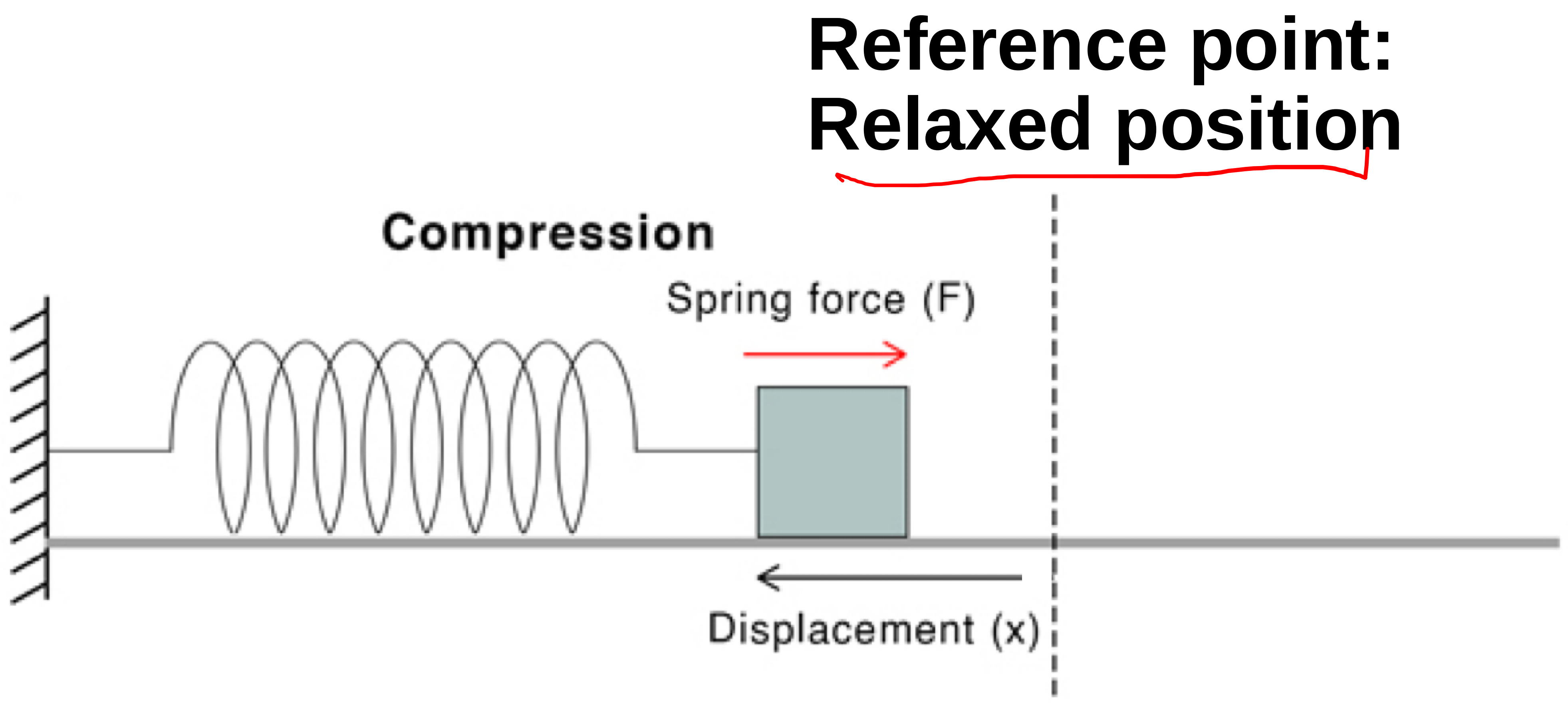
- Work done by a spring:

$$W_s = \int_{x_i}^{x_f} F_s dx$$

$$= \int_{x_i}^{x_f} -kx\, dx$$

$$= -\frac{1}{2}kx^2 \Big|_{x_i}^{x_f}$$

$$= \frac{1}{2}k(x_i^2 - x_f^2)$$

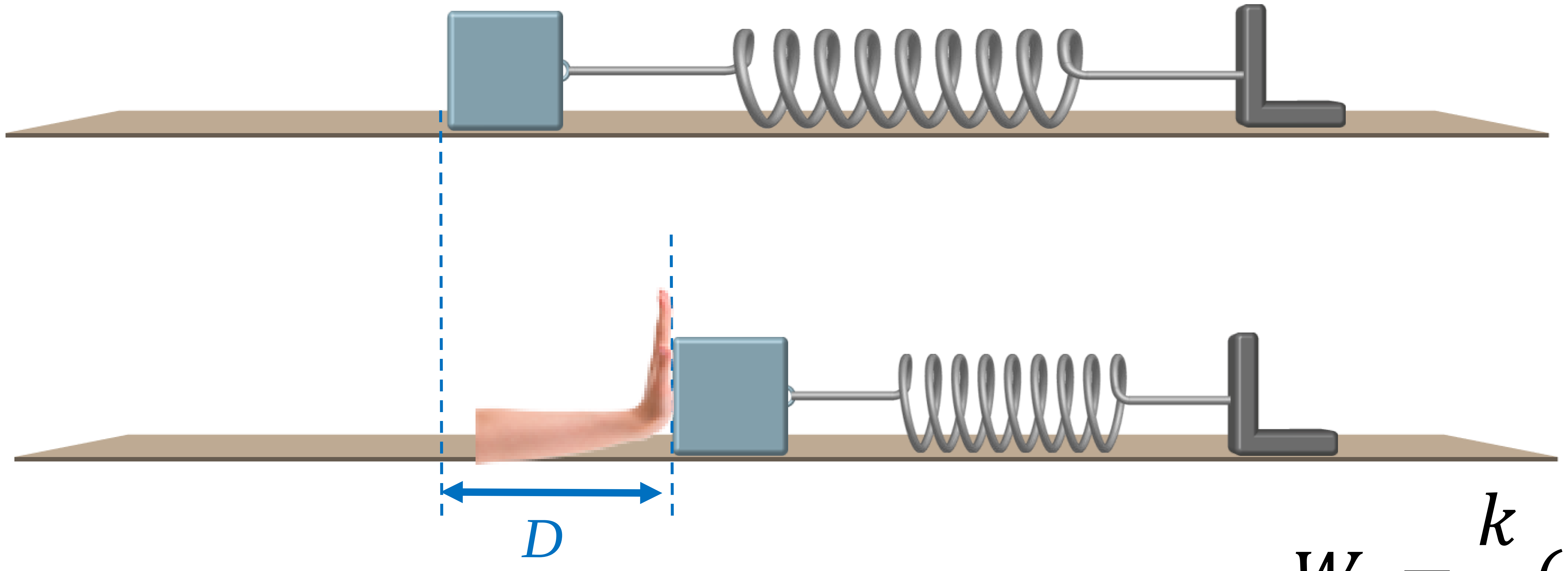


$$\boxed{W_s = \frac{1}{2}k(x_i^2 - x_f^2)}$$

Work by spring force only depends on the ***initial and final positions*** *with respect to the relaxed length!*

Clicker Question 8

A box attached at rest to a spring at its relaxed length. You now push the box with your hand so that the spring is compressed a distance D. During this motion, the spring does:



$$W_s = \frac{k}{2}\left(x_i^2 - x_f^2\right)$$

$x_i = 0$ $x_f \neq 0$

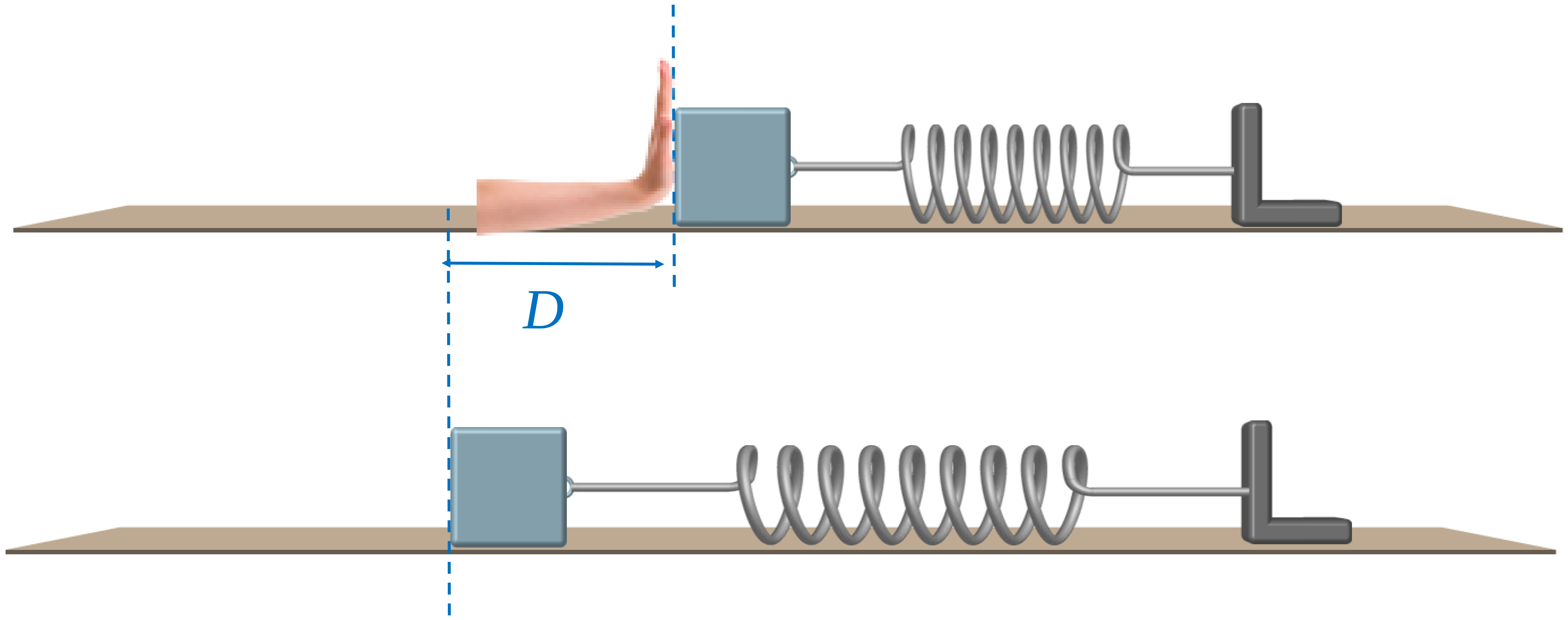
A Positive Work

B Negative Work

C Zero work

Clicker Question 9

A box attached at rest to a spring, which is compressed a distance D from its relaxed length. You **release** the box and the box moved to its relaxed position.



$$W_s = \frac{k}{2}(x_i^2 - x_f^2)$$

During this motion, the spring does:

A Positive Work

B Negative Work

C Zero work