PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 26: Collision

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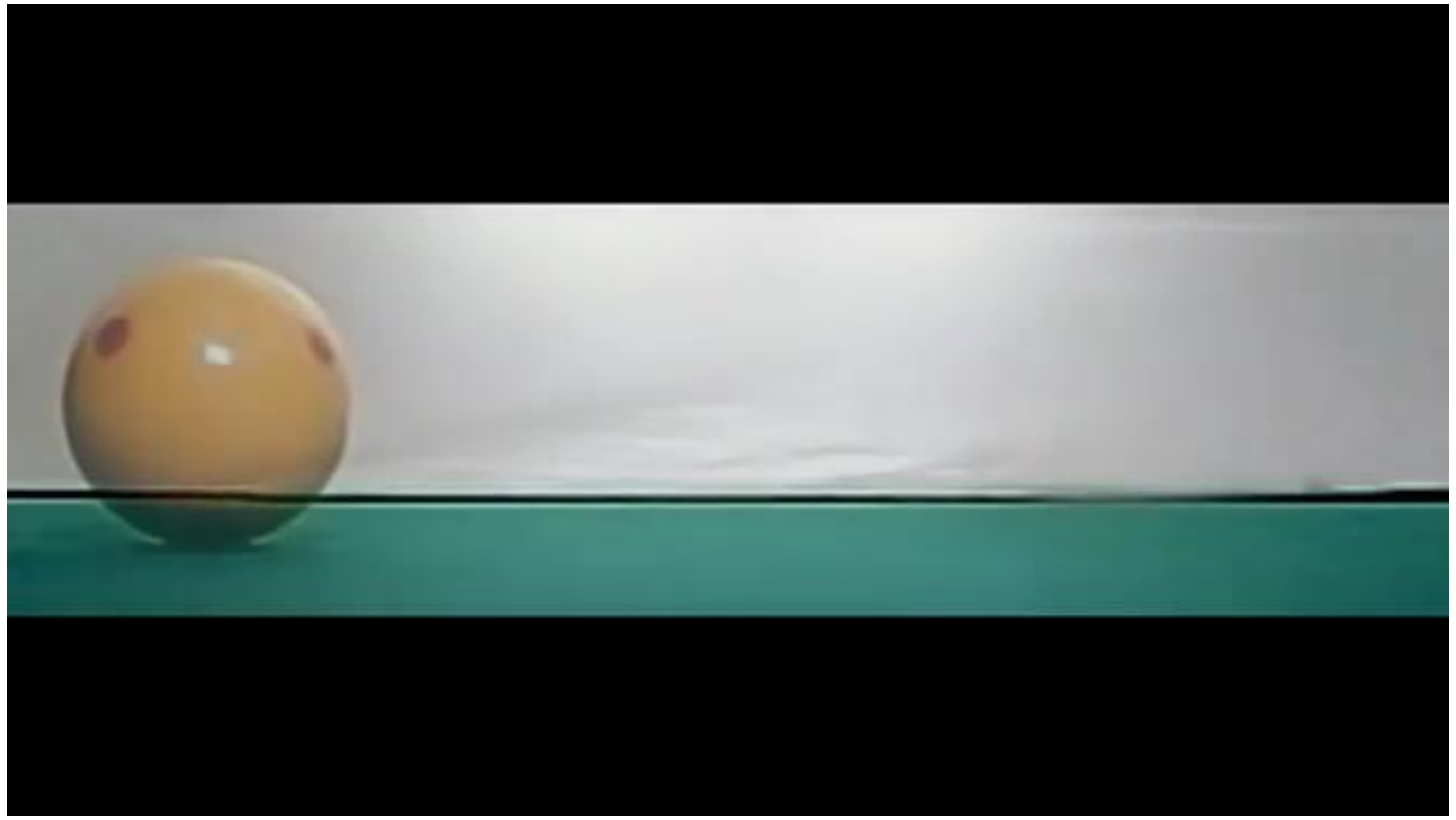
Learning goals for today

- Collision
  - Inelastic collision
  - Elastic collision

Recap:

- Impulse: $\vec{I} = \int_{t_i}^{t_f} \vec{F} dt \quad \approx \vec{F}_{ave} \Delta t$
- Impulse and change of linear momentum: $\vec{I}_{net} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$
- Conservation of linear momentum: If $\vec{I}_{net} = 0$,then $\vec{P}_{tot,final} = \vec{P}_{tot,init}$.

Chapter 9.3: Collision



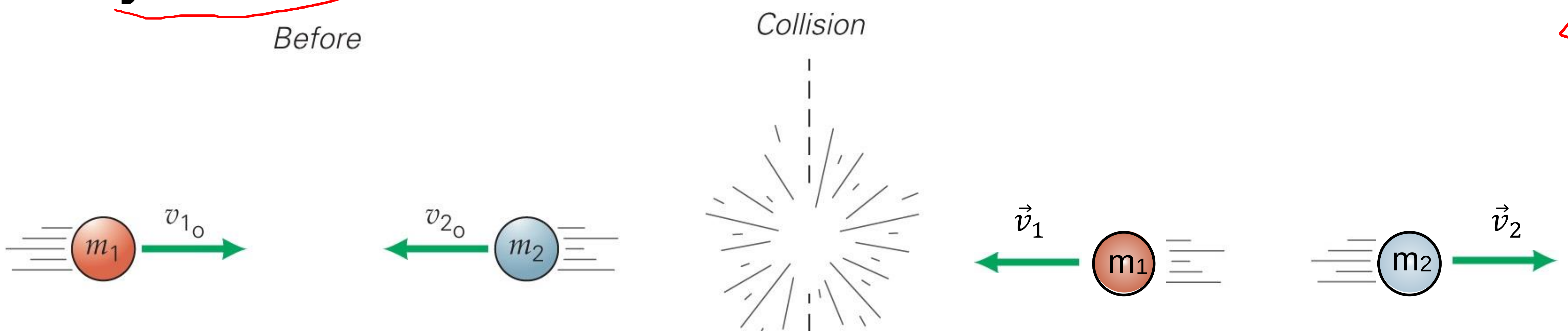
1. Collisions

x

- Collision: An event in which two or more bodies exert forces on each other in a relatively **short time**.

$\Delta t \sim 0$

$\vec{I}_{net} \sim 0$



Since $\Delta t \to 0,$ Therefore, $\vec{I}_{net} = 0$

- Collision happens so quickly that the **total momentum is conserved:**

$$\vec{P}_{tot,f} - \vec{P}_{tot,i} = \vec{I}_{net} \approx 0$$

$\vec{P}_{f,tot} = \vec{P}_{i,tot}$

Clicker question 1

Given: m_A, m_B, $\vec{v}_{A0}=0$, $\vec{v}_{B0}=-5\,ms^{-1}\hat{i}$, $\vec{v}_{fA}=-4\,ms^{-1}\hat{i}$

Goal: $\vec{v}_{f,B}$

x

- In the top figure (before part), car *A* (mass m_A = **1000kg**) is stopped at a traffic light, and *car B* (mass m_B = **1000kg**) is moving with an initial velocity $\vec{v}_0$ at **5 m/s** to the left. In bottom figure (after part), the rear end of *car A* is hit by car *B.* Immediately after the collision, *Car A* gains a velocity of velocity $\vec{v}_{f,A}$ of **4 m/s** to the left. What is the velocity of *Car B* immediately after collision?

A 1 m/s, to the left.

B 1 m/s, to the right.

C 0

Step 1: $\vec{P}_{f,tot}=\vec{P}_{i,tot}$

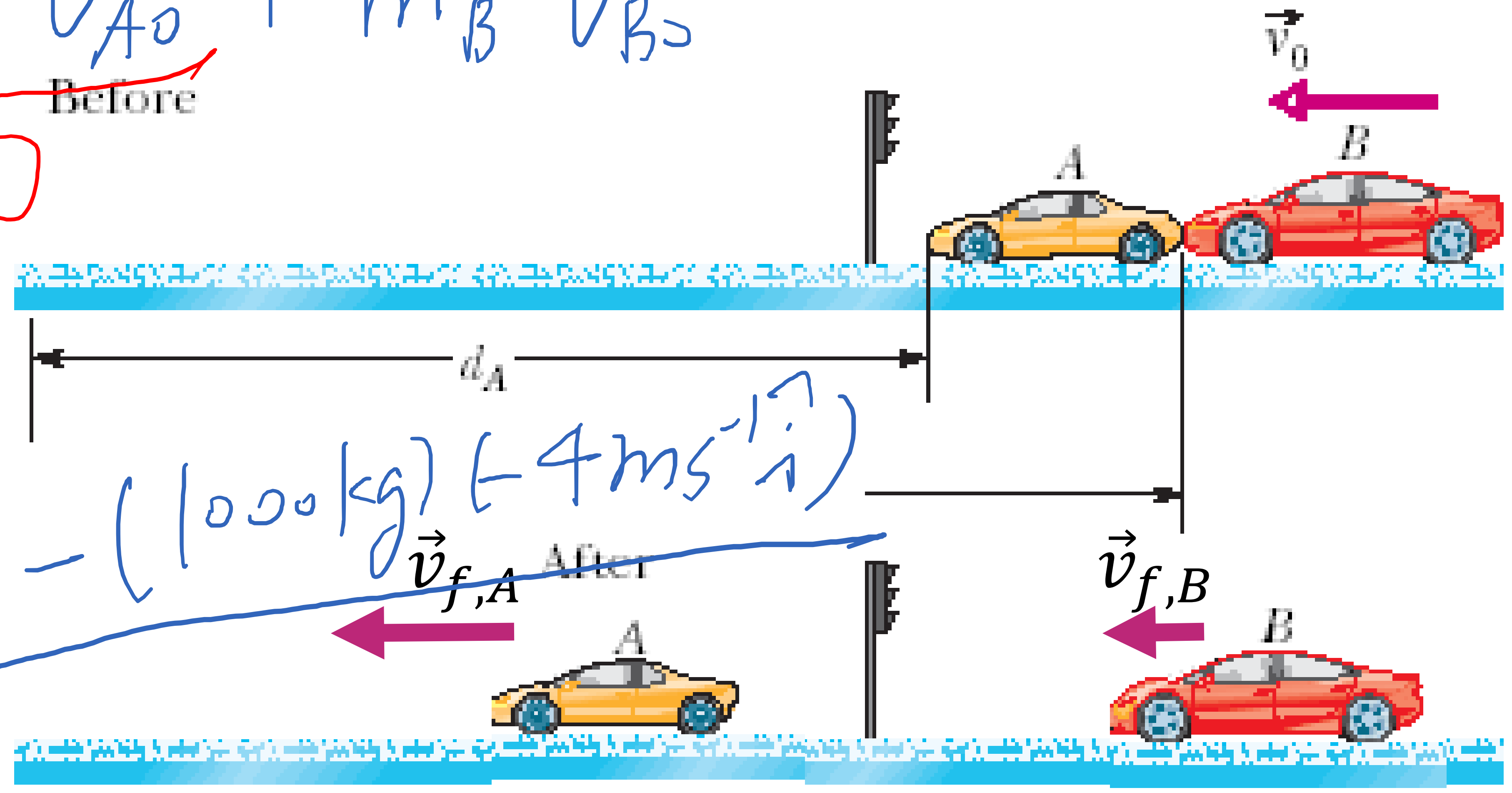
$$m_A\vec{v}_{Af}+m_B\vec{v}_{Bf}=m_A\vec{v}_{A0}+m_B\vec{v}_{B0}$$

Step 2: $$\vec{v}_{Bf}=\frac{m_B\vec{v}_{B0}-m_A\vec{v}_{Af}}{m_B}$$

$m_A=m_B=1000kg$

$$=\frac{1000kg(-5ms^{-1}\hat{i})-(1000kg)(-4ms^{-1}\hat{i})}{1000kg}$$

$$=-1ms^{-1}\hat{i}$$



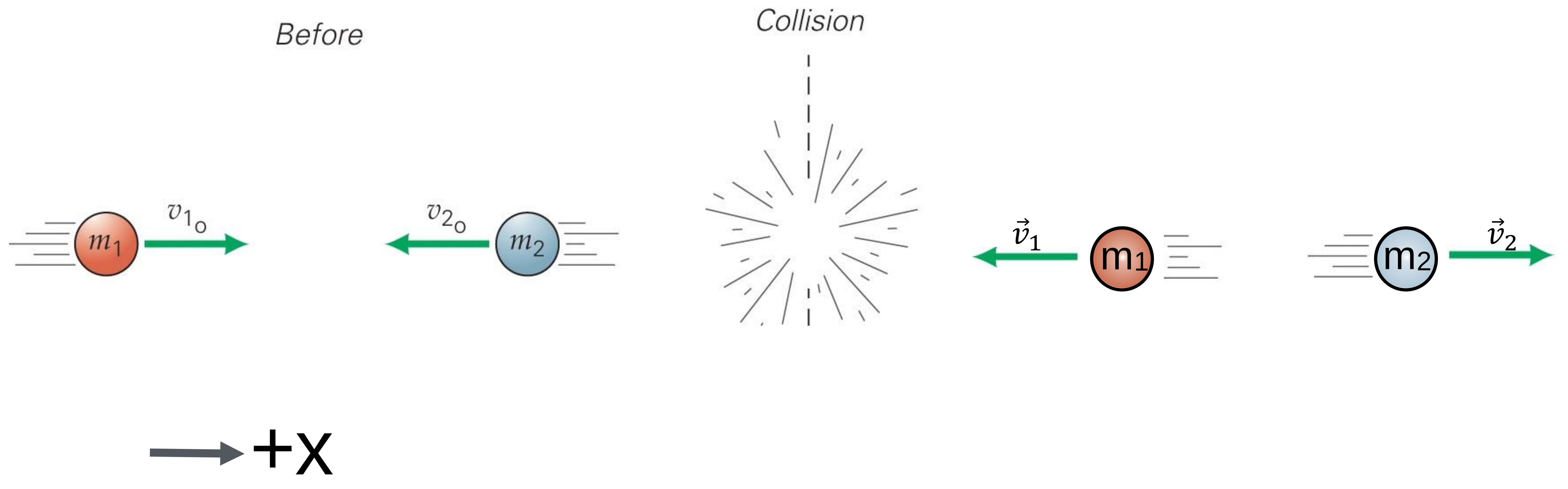
Demo

https://www.tiktok.com/@rhettallain0/video/7223476245500038442

2. Two types of collisions: Elastic and inelastic collision

- **1. Elastic collision**
  - Linear momentum is conserved:
    $$P_{i,tot} = P_{f,tot}$$
  - Kinetic energy is conserved:
    $$K_{i,tot} = K_{f,tot}$$

- **2. Inelastic collision**
  - Linear momentum is conserved:
    $$P_{i,tot} = P_{f,tot}$$
  - Kinetic energy is **NOT** conserved:
    $$K_{i,tot} \neq K_{f,tot}$$



Clicker question 2

Given: m_A, m_B, $\vec{v}_{A0}=0$, $\vec{v}_{B0}$, $\vec{v}_{Af}$, $\vec{v}_{Bf}$

$\longrightarrow x$

Goal: $K_{i,tot} \overset{?}{\neq} K_{f,tot}$

- In the "before" part of the figure, car *A* (mass m_A = **1000kg**) is stopped at a traffic light, and *car B* (mass m_B = **1000kg**) is moving with an initial velocity $\vec{v}_0$ to the left at **5 m/s**. In the “after” part of the figure, the rear end of *car A* is hit by car *B.* Immediately after the collision, *Car A* moves at a velocity of velocity $\vec{v}_{A,f}$ of **4 m/s** to the left, and *Car B* moves at a velocity of $\vec{v}_{B,f}$ of **1 m/s** to the left. What type of collision is it?

$K_{i,tot} = \frac{1}{2} m_A \underbrace{v_{A0}^2}_{0} + \frac{1}{2} m_B v_{B0}^2 = \frac{1}{2} \times 1000\,kg \times (-5\,ms^{-1})^2$

$= 12500\,J$

A It’s an elastic collision.

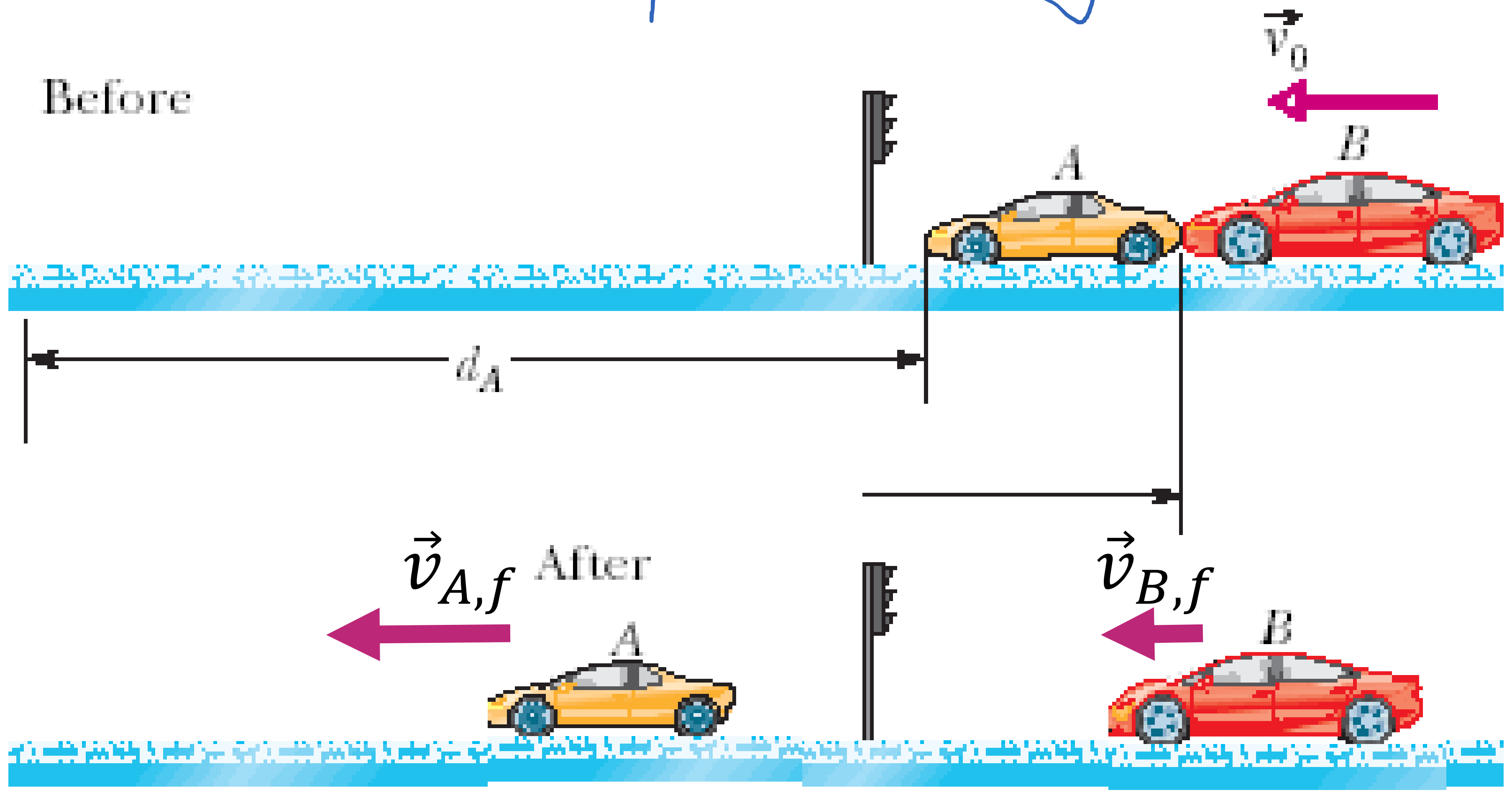
B It’s an inelastic collision.

$K_{f,tot} = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2$

$= \frac{1}{2} \times 1000\,kg\,(-4\,ms^{-1})^2 + \frac{1}{2} \times 1000\,kg\,(-1\,ms^{-1})^2$

$= 8500\,J$

$K_{i,tot} \neq K_{f,tot}$

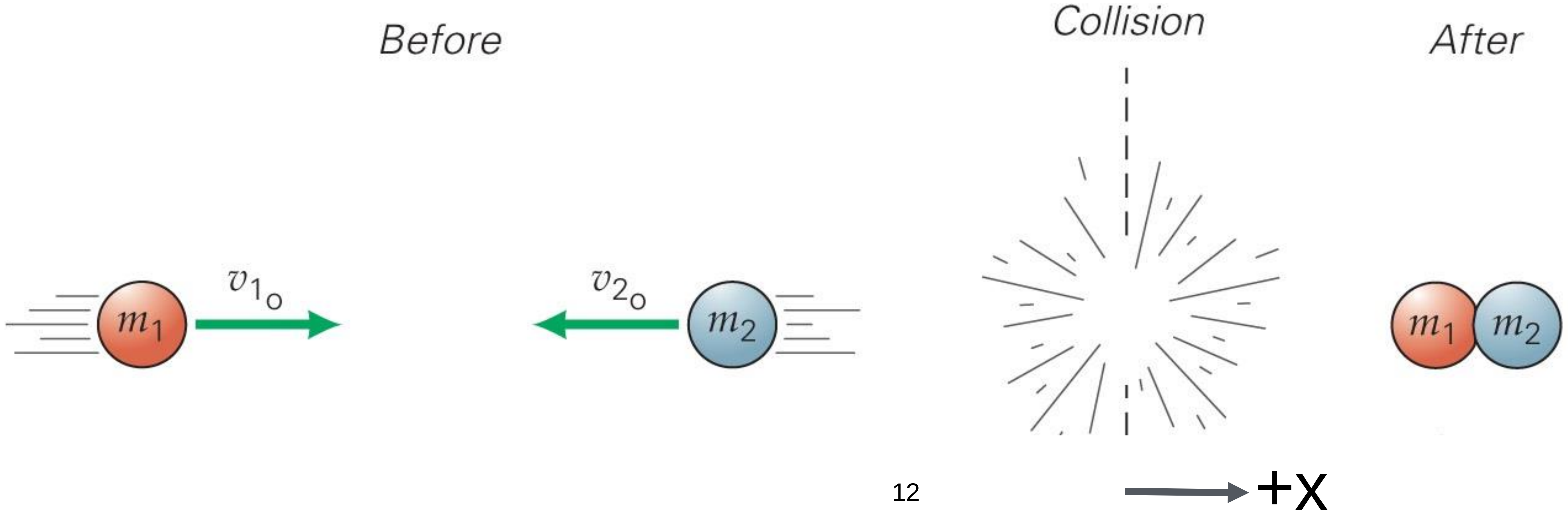


2.1. Inelastic collision and totally inelastic collision

x

- **Inelastic collision** in general
  - Linear momentum is conserved: $\vec{P}_{tot,i} = \vec{P}_{tot,f}$
  - Kinetic energy is NOT conserved $K_{tot,i} \neq K_{tot,f}$
- **Totally inelastic collision:**
  - After the collision, the two objects move at the same velocity: $\underbrace{\vec{v}_{1f}}_{\text{obj.1}} = \underbrace{\vec{v}_{2f}}_{\text{obj.2}}$

Stick together after collision



Clicker question 3

x

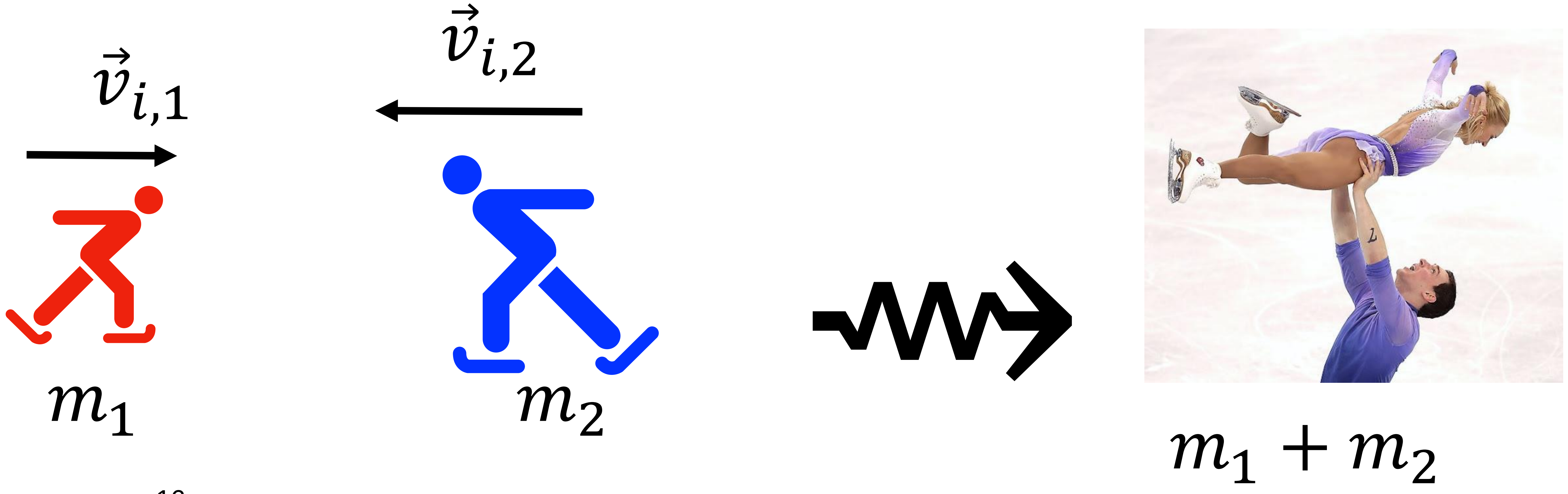
- Skater 1 and Skater 2 are moving towards each other during figure skating. Initially, Skater 1 (mass 50kg) is skating to the right at 2 m/s, and Skater 2 (70 kg) is skating to the left at 5 m/s. After they collide with each other, they **skate together** at the same velocity. Neglect the net impulse on them, what type of collision is it?

$\vec{v}_{f1} = \vec{v}_{f2}$

A **Elastic collision**

B **Totally inelastic collision**

C **Undetermined.**



Example 1

Given: m_C, $\vec{v}_{C0}$, m_J, $\vec{v}_{J0}$, $\vec{v}_{Cf} = \vec{v}_{Jf} = \vec{v}_f$

Goal: $\vec{v}_f$

$\xrightarrow{\quad}$ x

- Two athletes, John and ~~Jane~~ Carley are moving towards each other during figure skating. Initially, Carley (mass 50kg) is skating to the right at 2 m/s, and John (70 kg) is skating to the left at 5 m/s. After they collide with each other, they skate together at the same velocity. Neglect the net impulse on them, please calculate their common velocity after the collision.

Step 1: $\vec{P}_{i,tot} = \vec{P}_{f,tot} \rightarrow m_C\vec{v}_{C0} + m_J\vec{v}_{J0} = m_C\vec{v}_f + m_J\vec{v}_f$

$$\vec{v}_f = \frac{m_C\vec{v}_{C0} + m_J\vec{v}_{J0}}{m_C + m_J}$$

Step 2:

$$\vec{v}_f = \frac{50\,kg(2\,ms^{-1}\hat{i}) + 70\,kg(-5\,ms^{-1}\hat{i})}{50\,kg + 70\,kg}$$

$$\approx -2.08\,ms^{-1}\,\hat{i}$$

$\vec{v}_{i,1}$ $\longrightarrow$

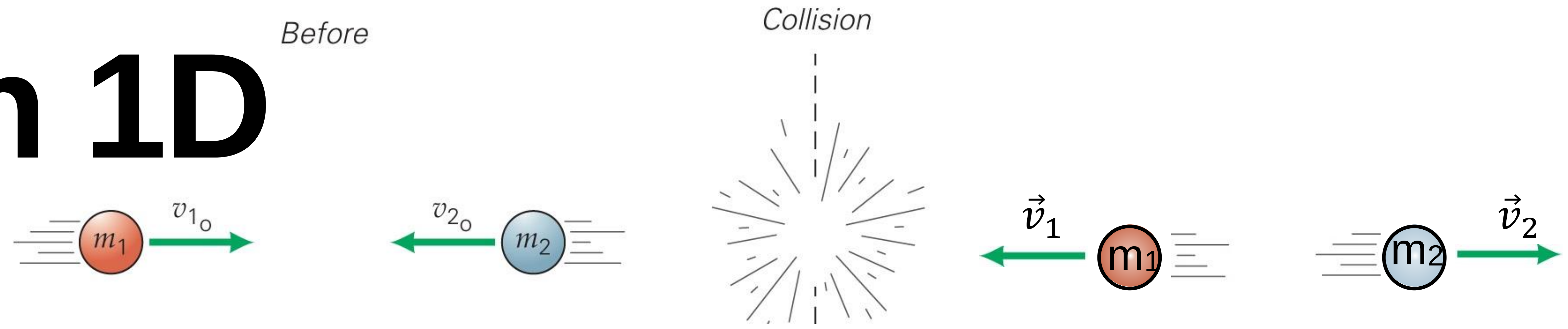
m_1

$\vec{v}_{i,2}$ $\longleftarrow$

m_2

$m_1 + m_2$

2.2. Elastic collisions in 1D



- **Elastic collisions: Both linear momentum and kinetic energy are conserved**

  - Conservation of momentum

$$\boxed{\vec{\mathbf{p}}_{1,i} + \vec{\mathbf{p}}_{2,i} = \vec{\mathbf{p}}_{1,f} + \vec{\mathbf{p}}_{2,f}}$$

  - Conservation of kinetic energy

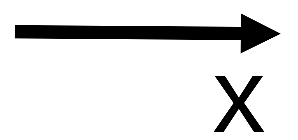
$K_{i,tot} = K_{f,tot}$

$$\frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

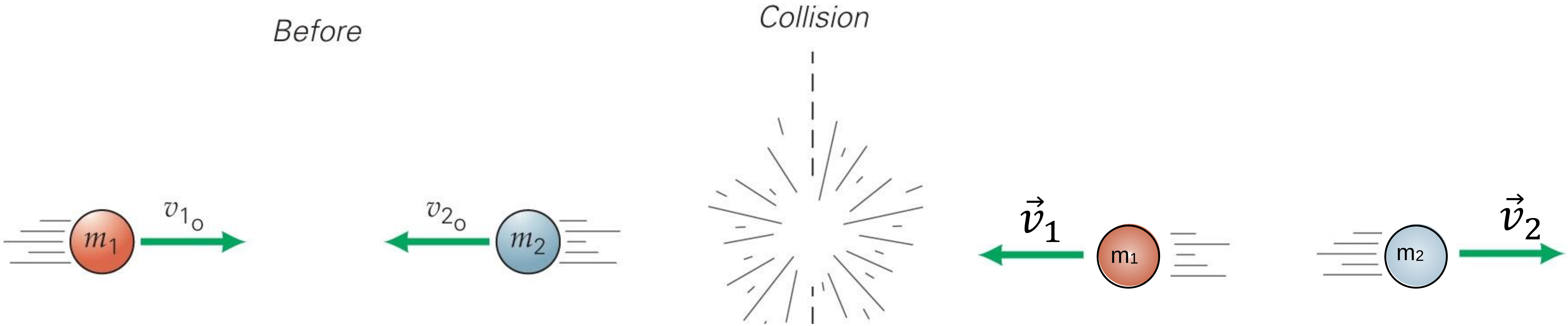
In 1D Elastic Collisions

If Given m_1, m_2, $\vec{v}_{1,i}$, $\vec{v}_{2i}$, then $\begin{cases} \vec{P}_{i,tot} = \vec{P}_{f,tot} \\ K_{i,tot} = K_{f,tot} \end{cases}$

Clicker question 4



A ball of mass m_1 and initial velocity $\vec{v}_{10} = v_{10}\hat{\imath}$ collides **elastically** with another ball of mass m_2 and initial velocity $\vec{v}_{20} = v_{20}\hat{\imath}$ .What principles would you use to find the final velocities, $\vec{v}_1$ and $\vec{v}_2$?



A Conservation of momentum

B Conservation of kinetic energy

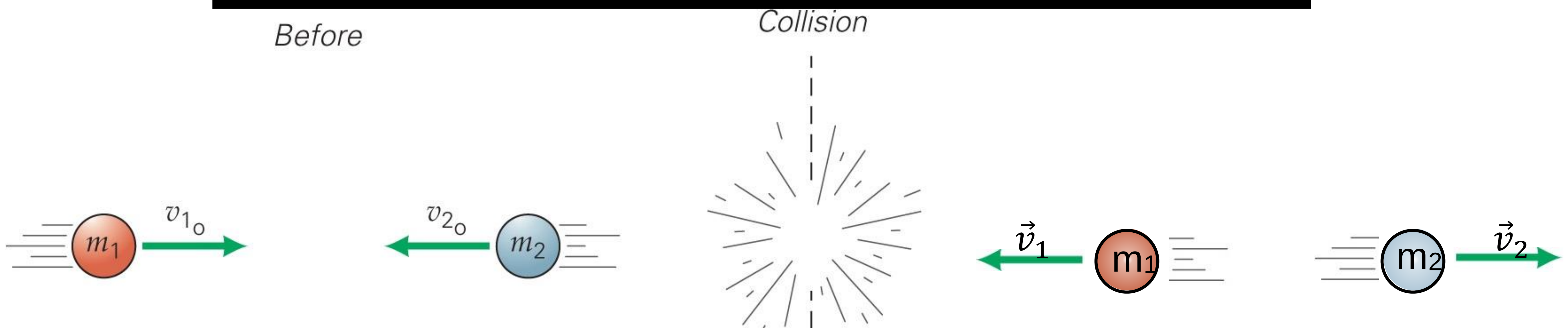
C Both

Elastic collisions in 1D: Special conditions

Given: m_1 ,m_2 ,$\vec{v}_{10}$,$\vec{v}_{20}$,elastic collision

Goal: $\vec{v}_1$,$\vec{v}_2$

v_1, v_2, v_{10} and v_{20} are vector components.



- 1. Final velocities for 1D elastic collision:

In general

$$\begin{cases} \vec{\mathbf{p}}_{1,i} + \vec{\mathbf{p}}_{2,i} = \vec{\mathbf{p}}_{1,f} + \vec{\mathbf{p}}_{2,f} \\ \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2 \end{cases}$$

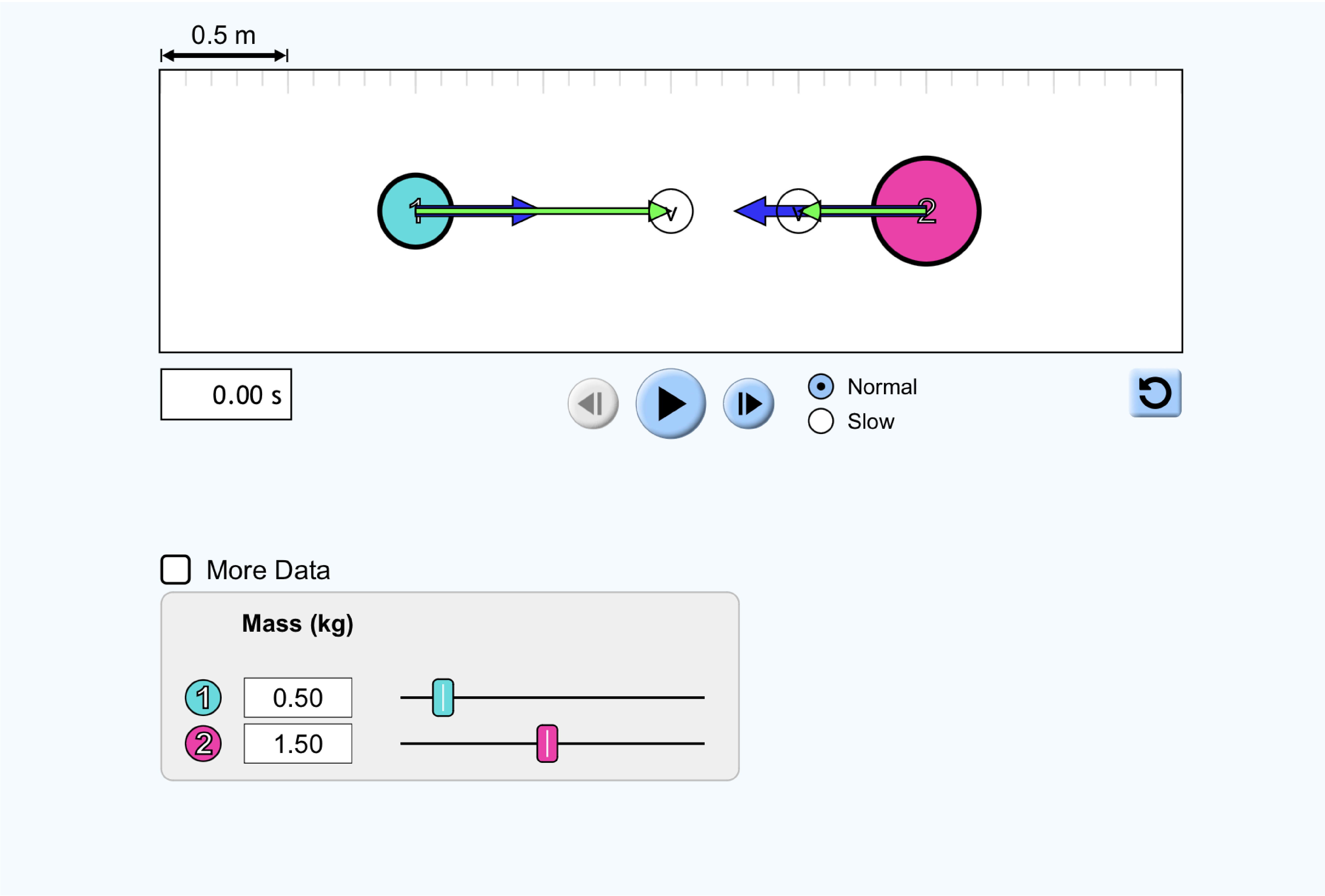
Equal masses, $m_2 = m_1$ $\quad v_{1f} = v_{2i}$ & $v_{2f} = v_{1i}$

Massive m_2, $m_2 \gg m_1$ $\quad v_{1f} \approx -v_{1i} + 2v_{2i}$ & $v_{2f} \approx v_{2i}$

If Head-on elastic collision, $m_2 >> m_1$, lighter obj. is speed up by $2|\vec{v}_{2i}|$

v_{1i} is opposite to v_{2i}

Demo



https://phet.colorado.edu/sims/html/collision-lab/latest/collision-lab_all.html

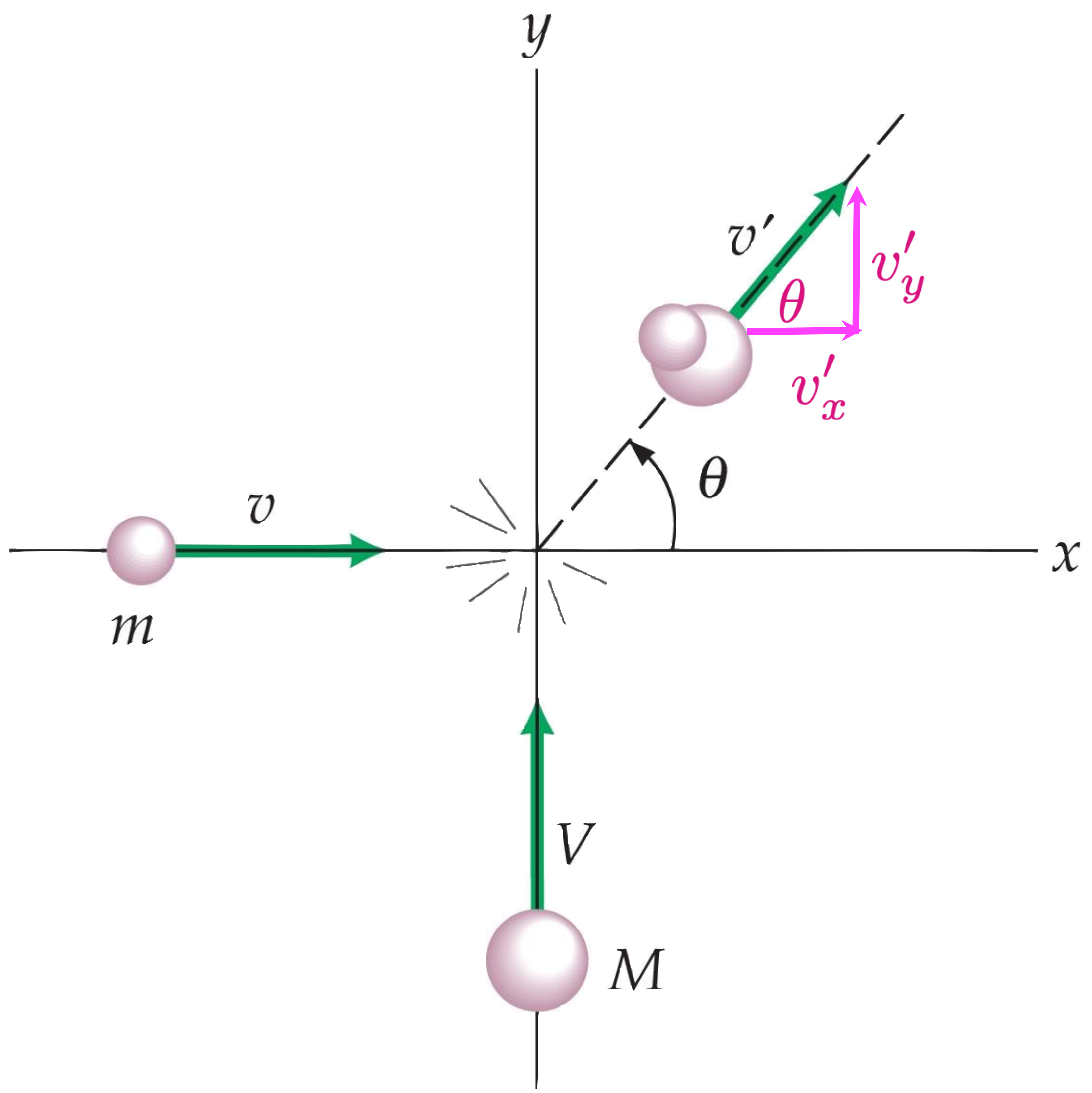
More examples of collision

Example 2: Totally inelastic collision in 2D

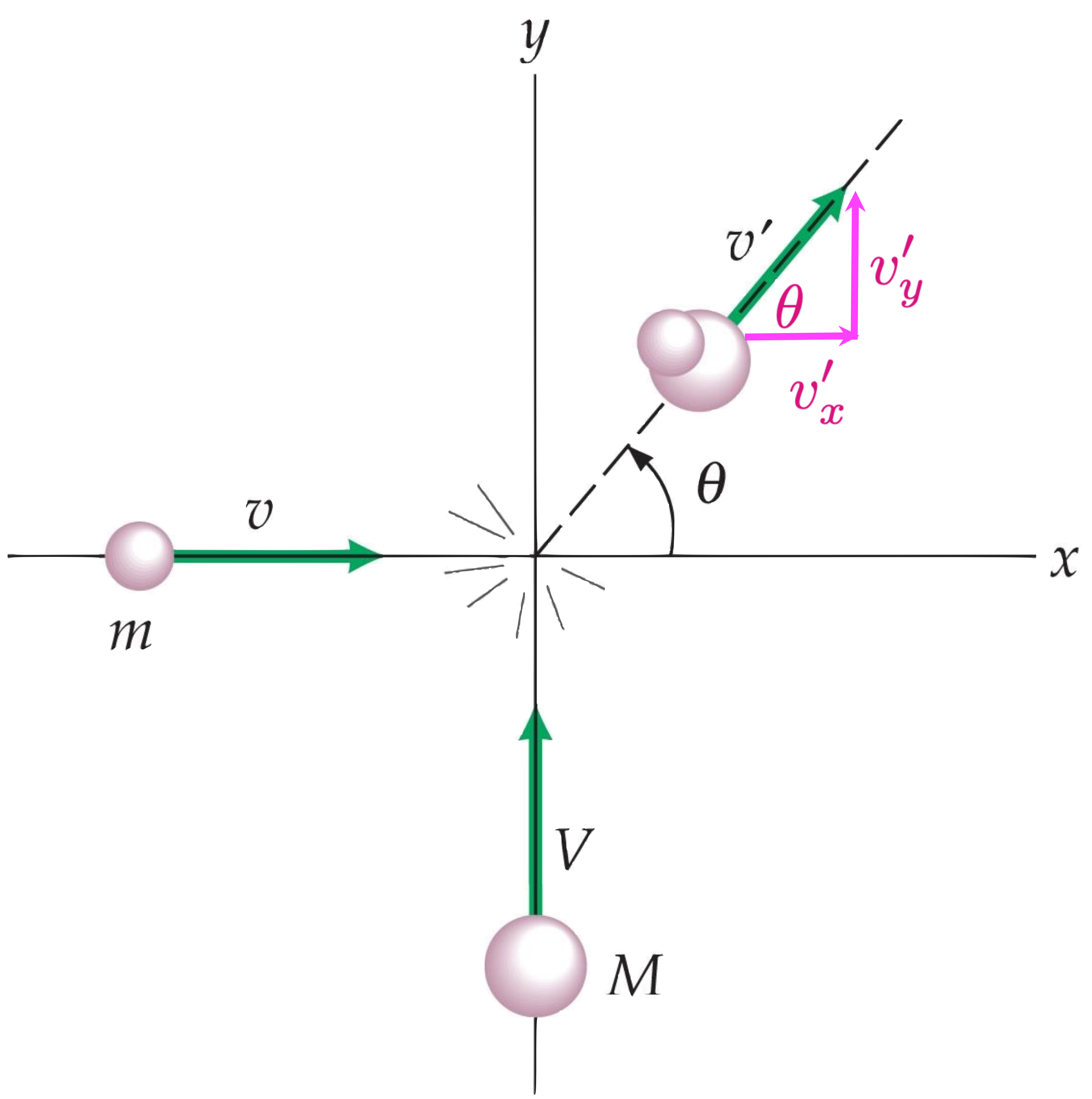
Ex.: Ball 1 has a mass of m, Ball 2 has a mass of M. Ball 1 moves to the +x direction at v, ball 2 moves in the +y direction at V. They collide and ***stick together***. What is their common final velocity, $\vec{v}' = \vec{v}_f$?

$$\vec{P}_{i,tot} = \vec{P}_{f,tot}$$

$$\vec{v}_{mf} = \vec{v}_{Mf} = \vec{v}_f$$



Clicker question 5



Given: m, M; initial velocities: $v\hat{\imath}$, $V\hat{\jmath}$; totally inelastic collision

Goal: Final velocity, $\vec{v}'$

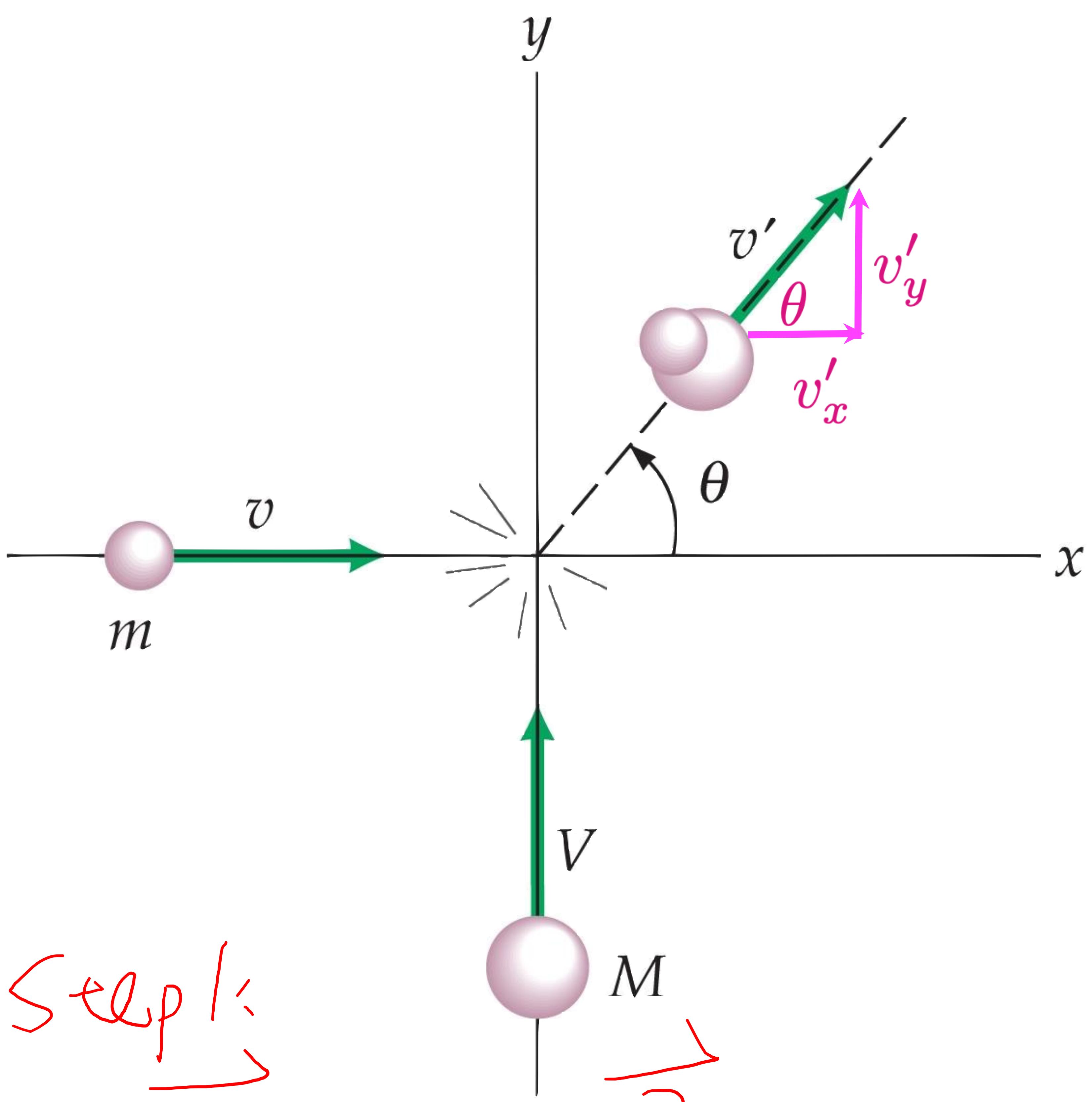
Question: What principle(s) to use?

A **Conservation of momentum**

B **Conservation of kinetic energy**

C **Both**

Clicker question 6



Given: totally inelastic collision in 2D

Given:

Mass	Before collision	After collision
m	$\vec{v}_{10} = v\hat{\imath}$	$\vec{v}_{1f} = \vec{v}_f = ?$
M	$\vec{v}_{20} = V\hat{\jmath}$	$\vec{v}_{2f} = \vec{v}_f = ?$

$\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f$

Goal: Final velocity, $\vec{v}_{fx}$

Question: What is the x component of final velocity, $\boldsymbol{v_{fx}}$ **?**

A $v_{fx} = v$

B $v_{fx} = V$

C $v_{fx} = \dfrac{mv}{m+M}$

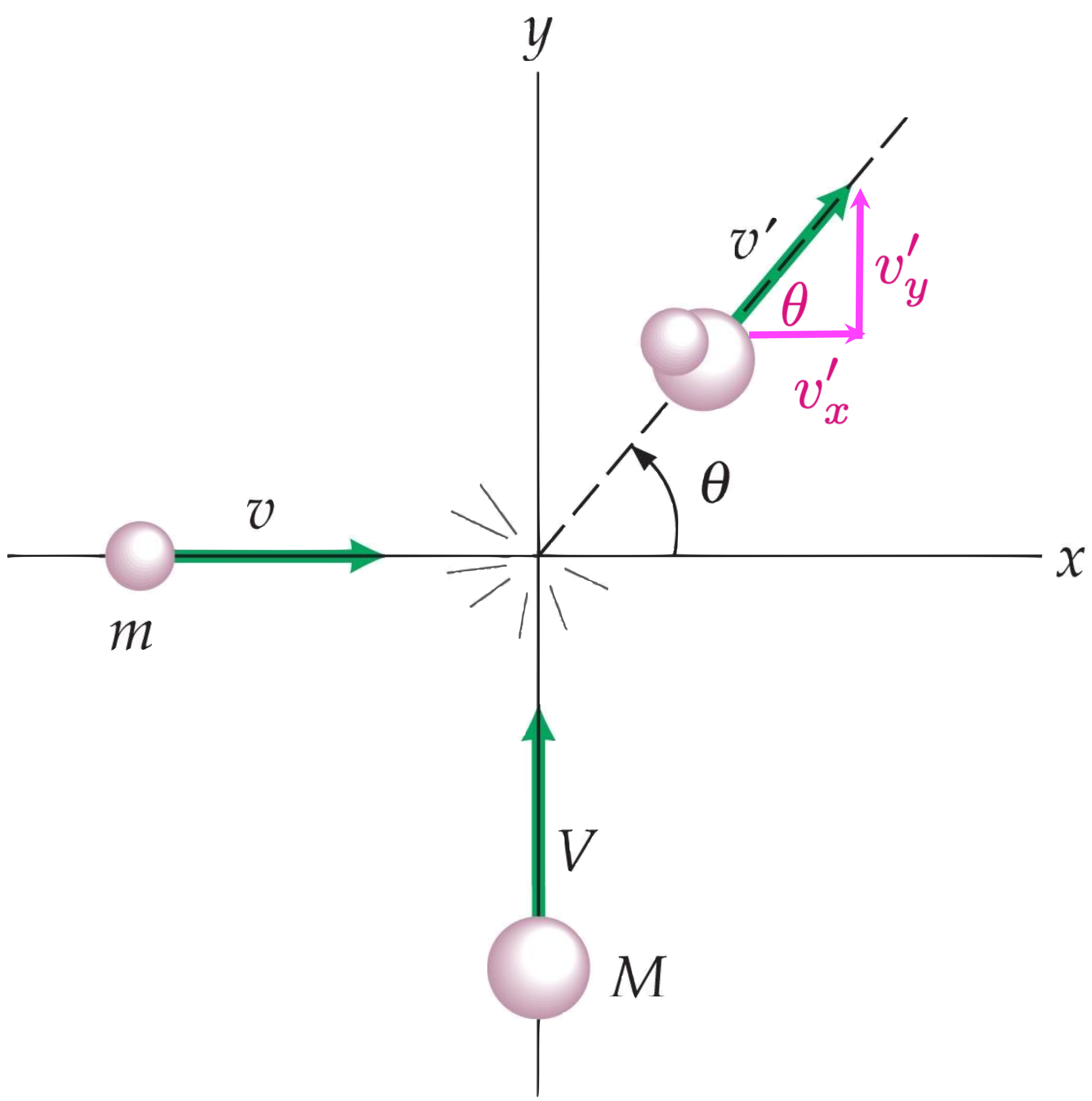
Step 1:

$\vec{P}_{xi,tot} = \vec{P}_{xf,tot}$

$m v \hat{i} + M \cdot 0 = (m+M)\vec{v}_{fx}$

$\rightarrow \vec{v}_{fx} = \dfrac{mv}{m+M}\hat{i}$

Clicker question 7



$P_{yi,tot} = P_{yf,tot}$

$m \cdot 0 + M \cdot V\hat{j} = (m+M)\vec{v}_{fy}$

$\rightarrow \vec{v}_{fy} = \frac{MV}{m+M}\hat{j}$

Given: totally inelastic collision

Mass	Before collision	After collision
m	$\vec{v}_{10} = v\hat{\imath}$	$\vec{v}_{1f} = \vec{v}_f = ?$
M	$\vec{v}_{20} = V\hat{\jmath}$	$\vec{v}_{2f} = \vec{v}_f = ?$

Goal: Final velocity, $\vec{v}_{fy}$

Question: What is the y component of final velocity, v_{fy}?

A $v_{fy} = v$

B $v_{fy} = V$

C $v_{fy} = \frac{MV}{m+M}$

Strategy: Collision

- 1. Read problem carefully
- 2. Sketches (before, after)
- 3. Given? Goal?
- 4. Principles and equations

  - **Collision?** Then the **total momentum is conserved** during the collision
    - ❖ Totally **inelastic collision?** Then all objects share the same final velocity, but $K_{tot,f} \neq K_{tot,i}$
    - ❖ **Elastic collision?** The sum of the kinetic energy is also conserved, $K_{tot,f} = K_{tot,i}$.

Summary of chapter 9

- Learning objectives
  - Concepts:
    - Center of mass, for point masses: $\vec{r}_{com} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$, $\vec{v}_{com}$, $\vec{a}_{com}$
    - Linear momentum: $\vec{P} = m\vec{v}$;
    - Impulse: $\vec{I} = \int \vec{F} dt$; Linear momentum and impulse: $\vec{P}_f - \vec{P}_i = \vec{I}_{net}$
    - Newton's 2nd law for a system: $\sum_i \vec{F}_{external,i} = \sum_i \frac{d\vec{P}_i}{dt}$, $\vec{F}_{net,external} = M_{tot}\vec{a}_{com}$
    - ~~Conservation of linear momentum:~~ If $\sum_i \vec{F}_{external,i} = 0$, then $\sum_i \vec{P}_i = const$

      Conditional
  - Practice:
    - Collision problems: Elastic collision, inelastic collision, totally inelastic collision

Homework 9.5

- Due in a week.

Prelecture 10.1

- Before the next class.