PHYS 225
Fundamentals of Physics: Mechanics

Prof. Meng (Stephanie) Shen

Fall 2024

Lecture 18: Force and motion II

CALIFORNIA STATE UNIVERSITY FULLERTON

Learning goals for today

- Calculating two types of friction force on flat and inclined surfaces:
  - Static friction, $\vec{f}_s$
  - Kinetic friction, $\vec{f}_k$

Magnitude of the two types of friction

Newton's laws are needed.

- Static friction: Tendency to slide, but no sliding.

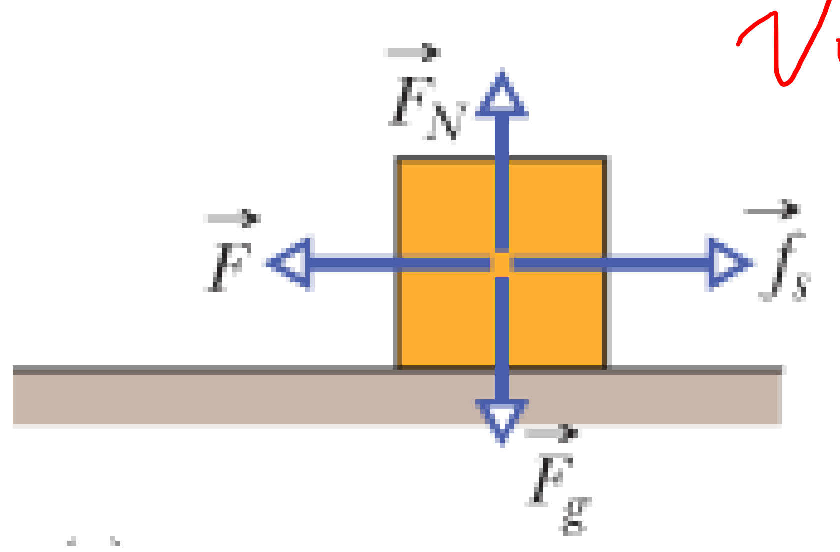
Magnitude $$|f_s| \leq \mu_s |N|$$

μ_s: Static friction coefficient

Normal force

$|f_{s,max}| = \mu_s |N|$

$\vec{v}_0 = 0$

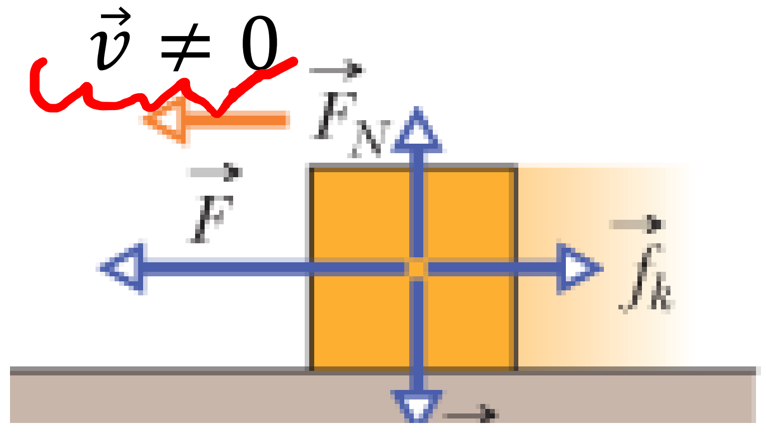


- Kinetic friction: Sliding.

$$|f_k| = \mu_k |N|$$

μ_k: Kinetic friction coefficient

Normal force



Sliding

$\vec{v}_0 = 0$

if $\mu_k \neq 0$ then $\vec{f}_s$

Clicker question 1

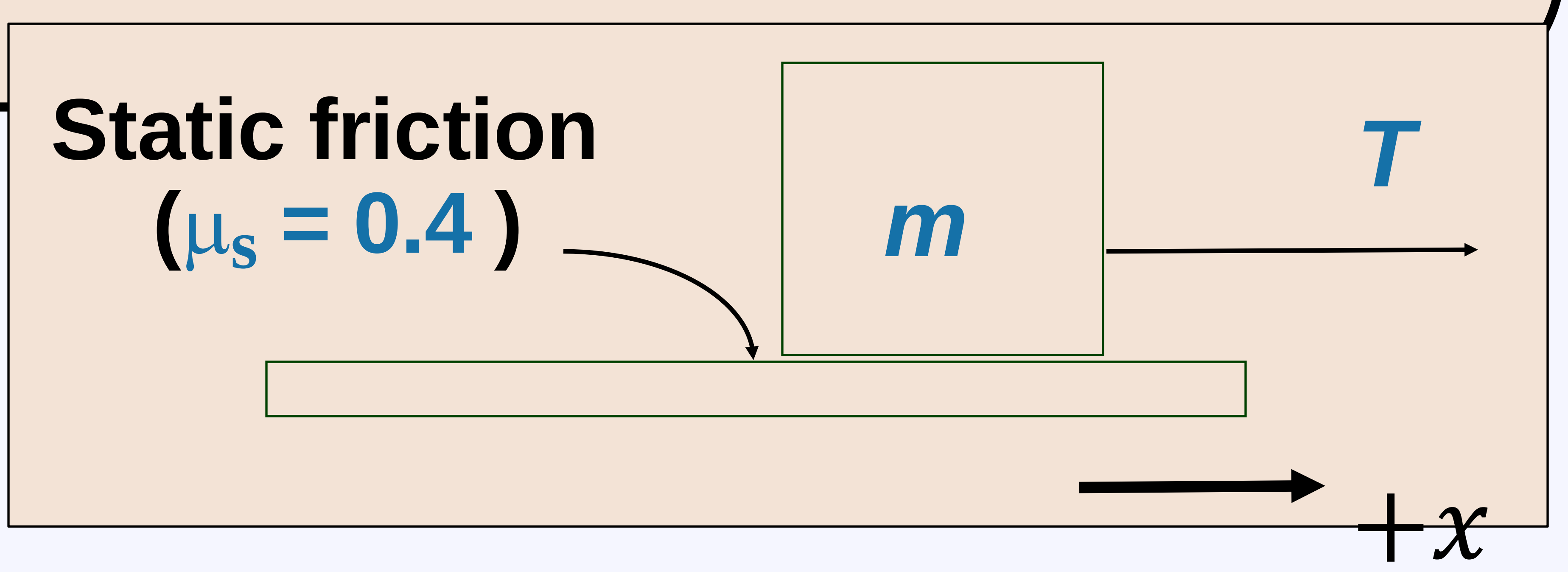
$|f_s| \le \mu_s |\vec{N}|$

Question: **Will it move?**

$|f_{s,max}| = \mu_s |\vec{N}|$

A box of **weight 100 N** in magnitude is at rest on a horizontal floor where $\mu_s = 0.4$. A rope is attached to the box and pulled horizontally with tension. What's the static friction force of maximum magnitude on the box (include the sign)?

- A - 40 N
- B - 30 N
- C 40 N
- D 30 N



Clicker question 2

$|\vec{F}_{driving}| \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} |f_{s,max}|$

Question: Will it move?

$|f_{s\,max}| = \mu_s |N|$

$= \mu_s |W|$

$= 0.4 \times 100\,N$

$= 40\,N$

A box of weight 100 N in magnitude begins at rest on a horizontal floor where μ_s = *0.4*. A rope is attached to the box and pulled horizontally with tension T = 30 N $\hat{\imath}$. Which way does the box move?

a) moves to the left

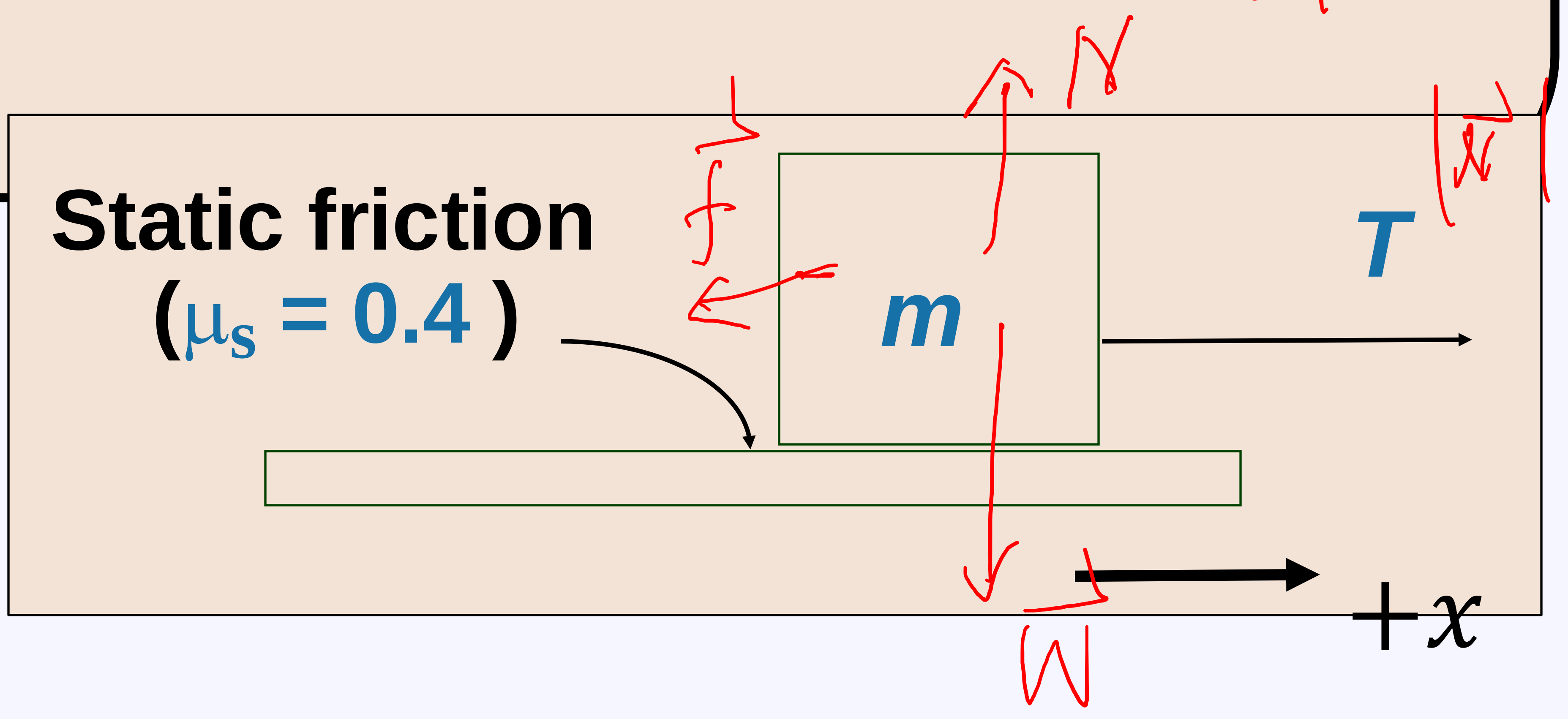
b) moves to the right

c) the box does not move

$|\vec{F}_{driving}| = |\vec{T}| = 30\,N < |f_{s\,max}|$

$|\vec{N}| = |\vec{W}|$

flat



Clicker question 3

Step 1: $|\vec{T}| > |f_{s,max}|$

Question: Will it move?

Step 2: Opposing slide

Step 3: Newton's 1st law

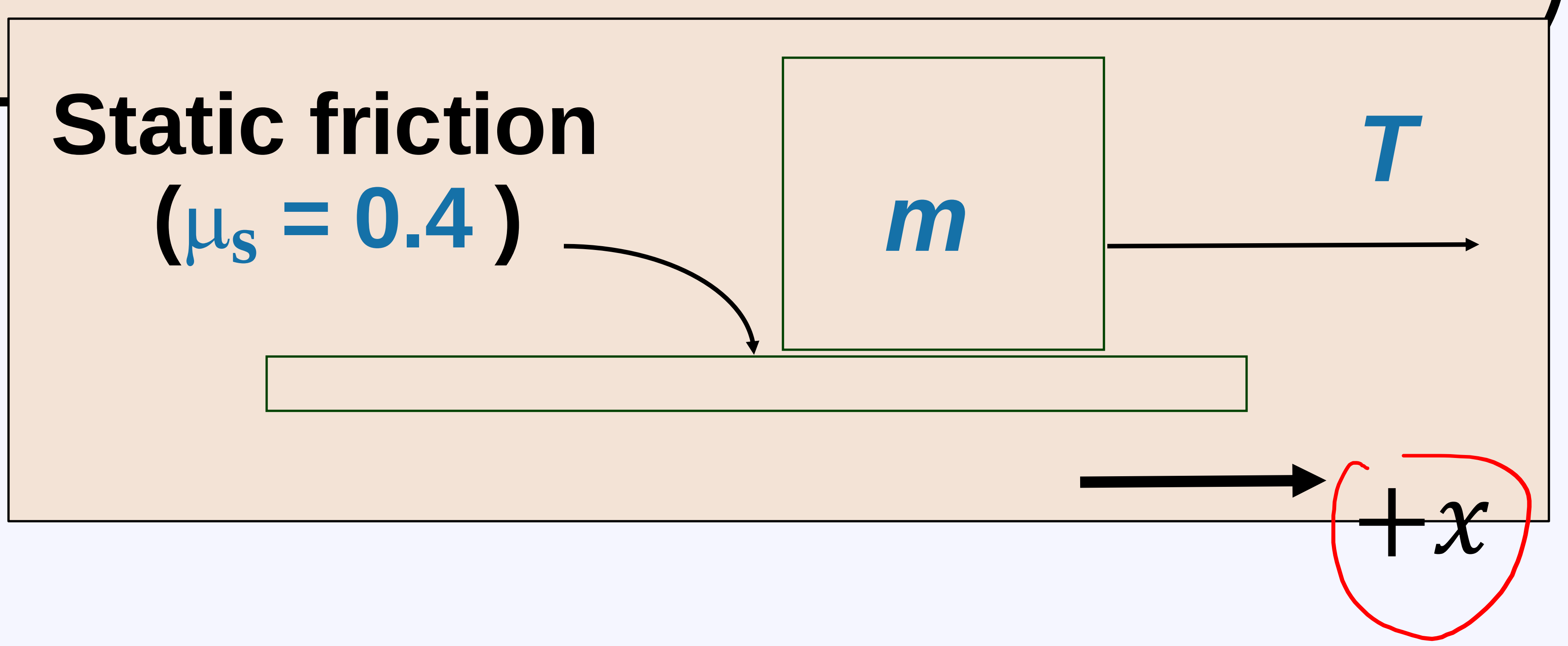
A box of **weight 100 N** in magnitude begins at rest on a horizontal floor where $\mu_s = 0.4$. A rope is attached to the box and pulled horizontally with tension $T = 30\text{ N}\,\hat{\imath}$. What's the friction force on the box?

- A: $-40\text{ N}\,\hat{\imath}$
- B: $-30\text{ N}\,\hat{\imath}$
- C: $40\text{ N}\,\hat{\imath}$
- D: $30\text{ N}\,\hat{\imath}$

$\vec{v} = 0, \quad \vec{a} = 0$

$\vec{F}_{net} = \vec{T} + \vec{f}_s = 0$

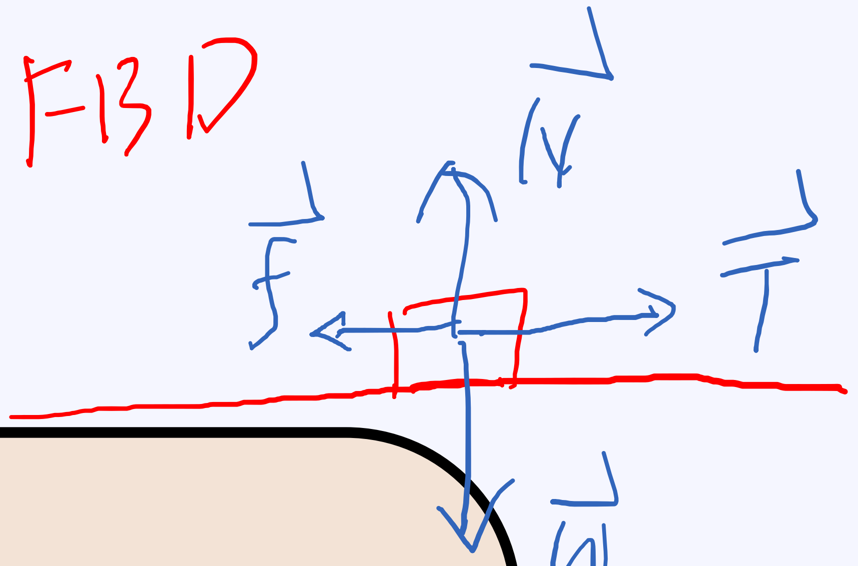
$\vec{f}_s = -\vec{T} = -30\text{ N}\,\hat{\imath}$



Clicker question 4

Step 0: FBD

flat



Question: **Will It move?**

A box of weight 100 N in magnitude is at rest on a horizontal floor where $\mu_s = 0.4$ and $\mu_\kappa = 0.2$. A rope is attached to the box and pulled horizontally with tension $T = 60$ N. What's the friction force on the box after the pulling force is applied?

- **A** - 40 N
- **B** - 20 N
- **C** 20 N
- **D** - 100 N

Step 1: Direction

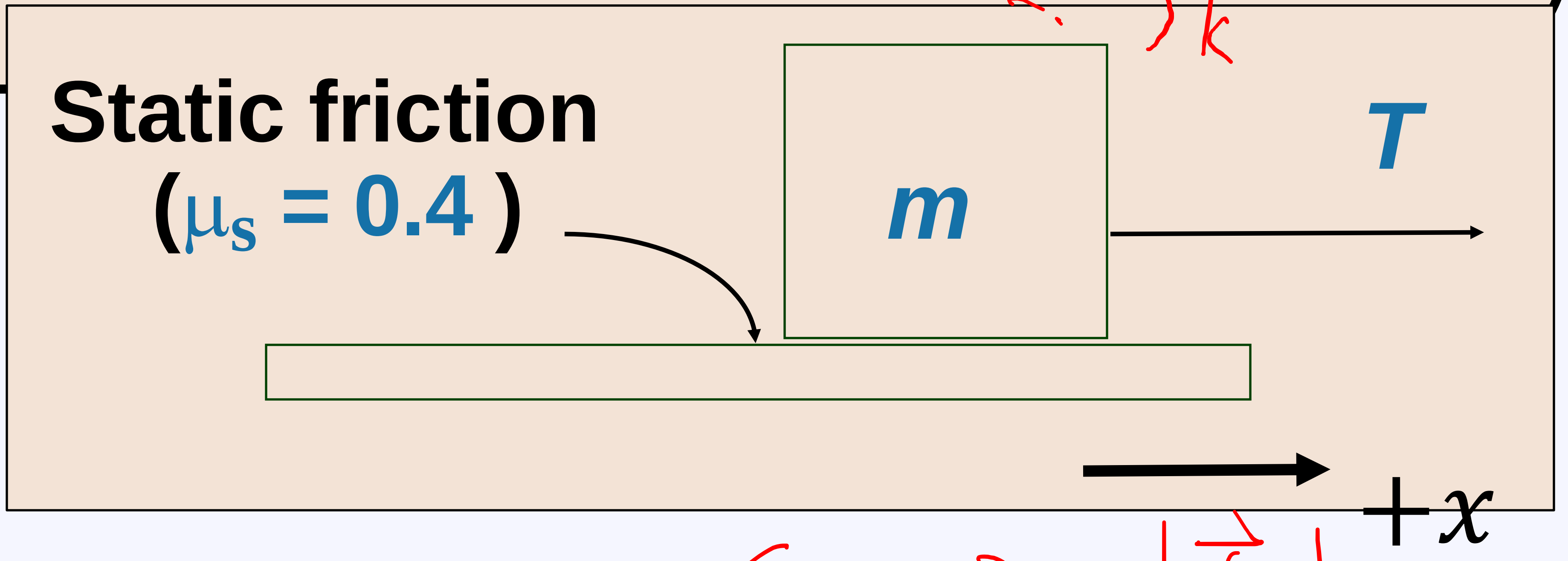
sliding: $\vec{f}$ opposing

Step 2: S or K?

$|T| \gtreqless |f_{s,max}|$

$|f_{s,max}| = \mu_s |N| = 0.4 \times 100\,N = 40\,N$

$|\vec{T}| > |f_{s,max}|$ ∴ $\vec{f}_k$



Step 3: $|\vec{f}_k| = \mu_k |N| = 0.2 \times 100\,N = 20N$

Step 4: $\vec{f} = \vec{f}_k = -20N\,\hat{i}$

Oppose sliding

Static friction and kinetic friction: Takeaway messages

- Static friction, $\vec{f}_s$:
  - Tendency to slide, but no sliding
  - $|\vec{f}_s| \leq \mu_s |\vec{N}|$

    $|\vec{f}_s|$ is usually determined by Newton's laws.
  - $|\vec{f}_{s,max}| = \mu_s |\vec{N}|$
- Kinetic friction, $\vec{f}_k$:
  - Sliding
  - $|\vec{f}_k| = \mu_k |\vec{N}|$

Steps to calculate a friction force

① $\vec{f_s}$: $\begin{cases} 1. \ \vec{v}_{rel,0} = 0 \ \text{(Initial)} \\ 2. \ |\vec{F}_{driving}| \le |f_{s,max}| \end{cases}$

- **Step 1:** Determine whether it's static friction or kinetic friction
  - Static friction: Tendency to slide over each other, but don't actually slide (oppose)
  - Kinetic friction: Slide over each other $\vec{v}_{rel} \neq 0$

② $\vec{f_k}$: $\vec{v}_{rel} \neq 0$

- **Step 2**: Direction
  - The direction of friction force ~~oppresses~~ opposes sliding or the tendency of sliding
- **Step 3**: Magnitude
  - Static friction: $|\vec{f_s}| \le \mu_s |\vec{N}|$ and Newton's laws $|\vec{f}_{s,max}| = \mu_s |N|$
  - Kinetic friction: $|\vec{f_k}| = \mu_k |\vec{N}|$

Clicker question 5

Assumptions $\begin{cases} 1. \vec{f}_k \\ 2. \vec{a} = 0 \end{cases}$

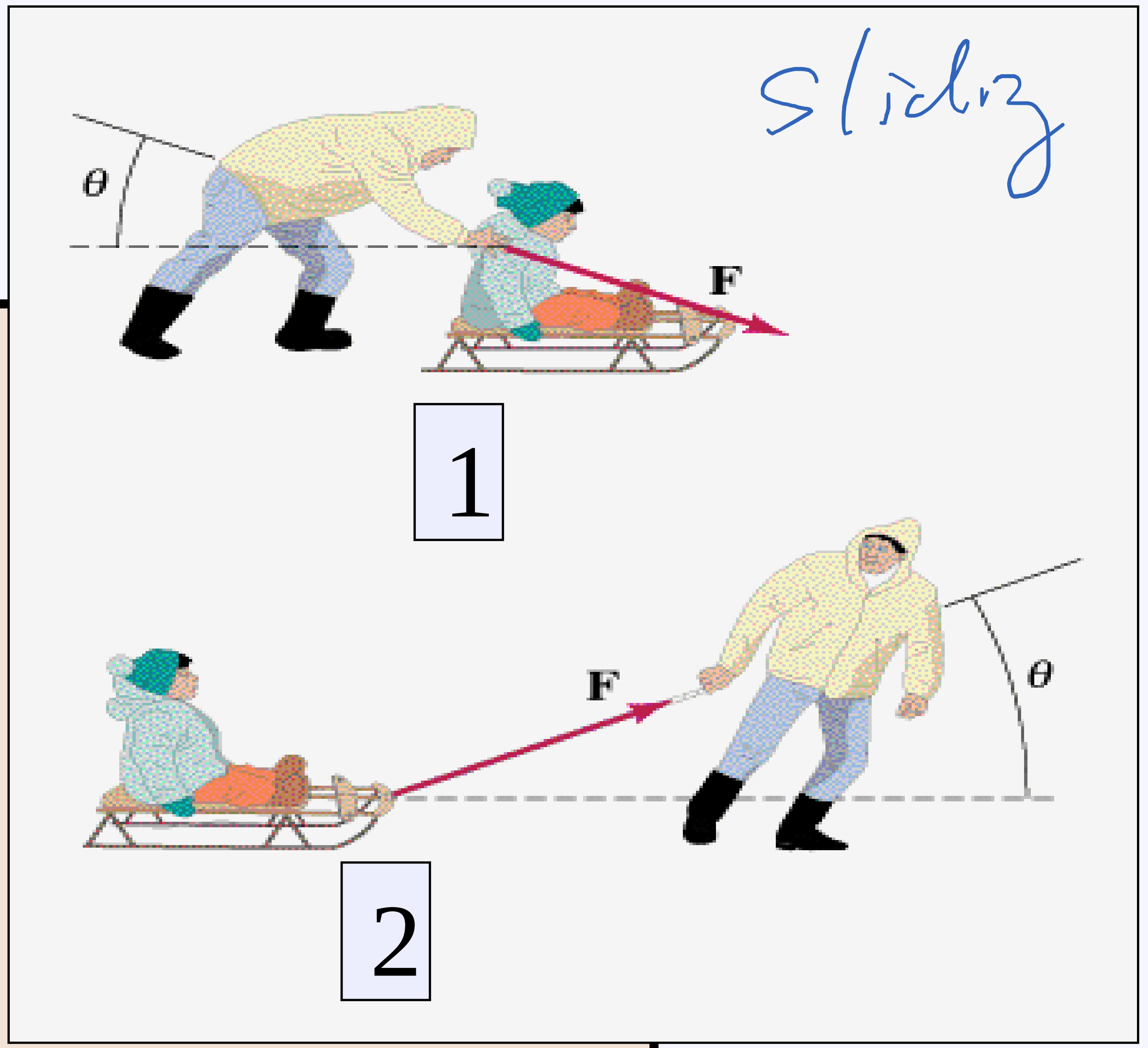
$\uparrow y$, $\rightarrow x$

Question 4.21 Going Sledding

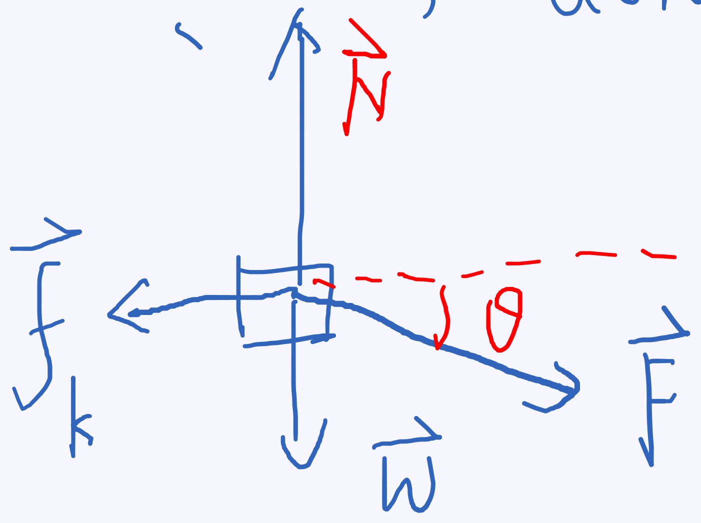
Your little sister wants you to give her a ride on her sled. On level ground, what is the easiest way to accomplish this?

Horizontal

- A Pushing her slightly downward from behind [1]
- B Pulling her slightly upward from the front [2]
- C Both [1] and [2] are equivalent
- D Tell her to get out and walk



Step 0: if down



$|\vec{f}_k| = \mu_k |\vec{N}|$

$= \mu_k (|\vec{F}| \sin\theta + mg) > \mu_k mg$

Newton 2nd law in y-dir.:

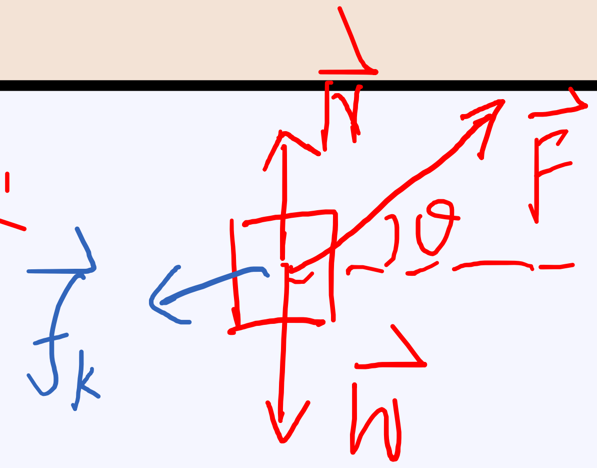
$\vec{F}_{net,y} = m\vec{a}_y = 0$

$\vec{N} + \vec{F}_y + \vec{W} = 0$

$\vec{N} - \vec{F} \sin\theta \hat{j} - mg\hat{j} = 0$

$\vec{N} = (\vec{F} \sin\theta + mg)\hat{j}$

If up:



Newton's 2nd law in y:

$\vec{F}_{net,y} = m\vec{a}_y = 0$

$\rightarrow \vec{N} + |\vec{F}| \sin\theta + \vec{W} = 0$, $\vec{W} = -mg\hat{j}$

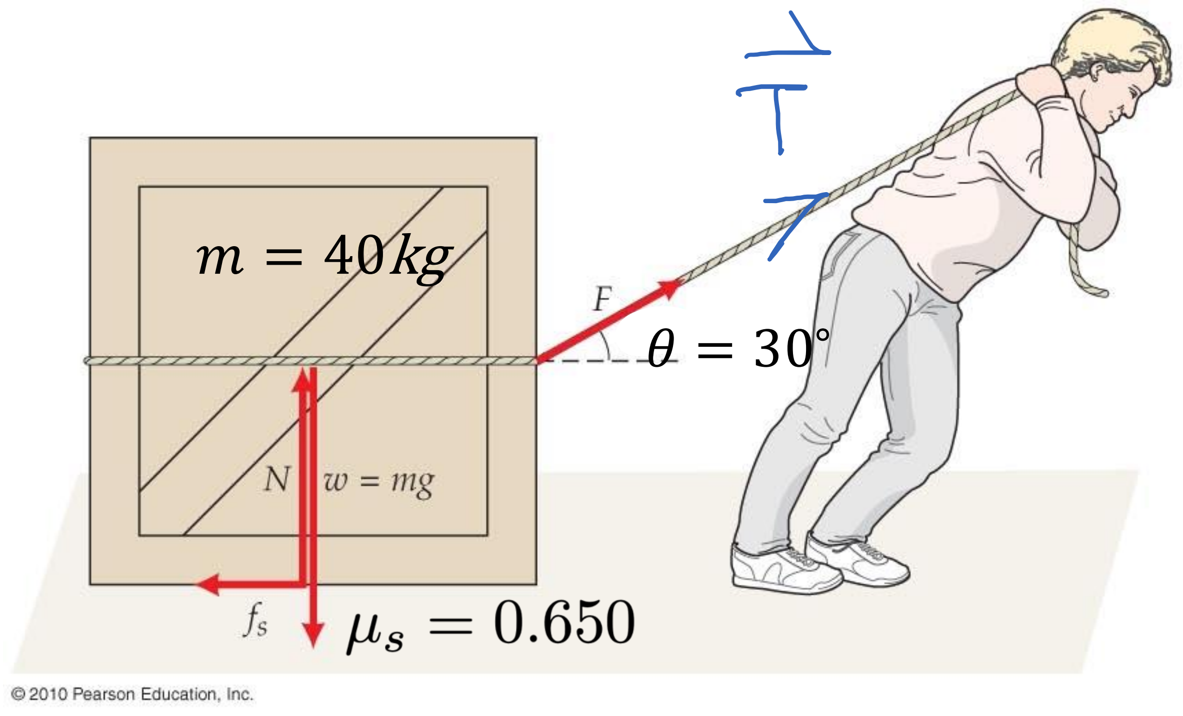
$\rightarrow \vec{N} = mg\hat{j} - |\vec{F}| \sin\theta \hat{j}$

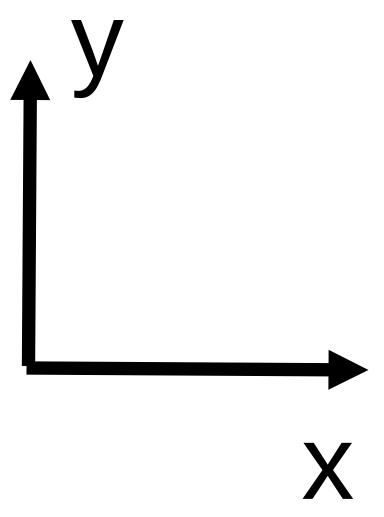
$|\vec{f}_k| = \mu_k |\vec{N}| = \mu_k (mg - \vec{F} \sin\theta) < \mu_k mg$

Clicker question 6

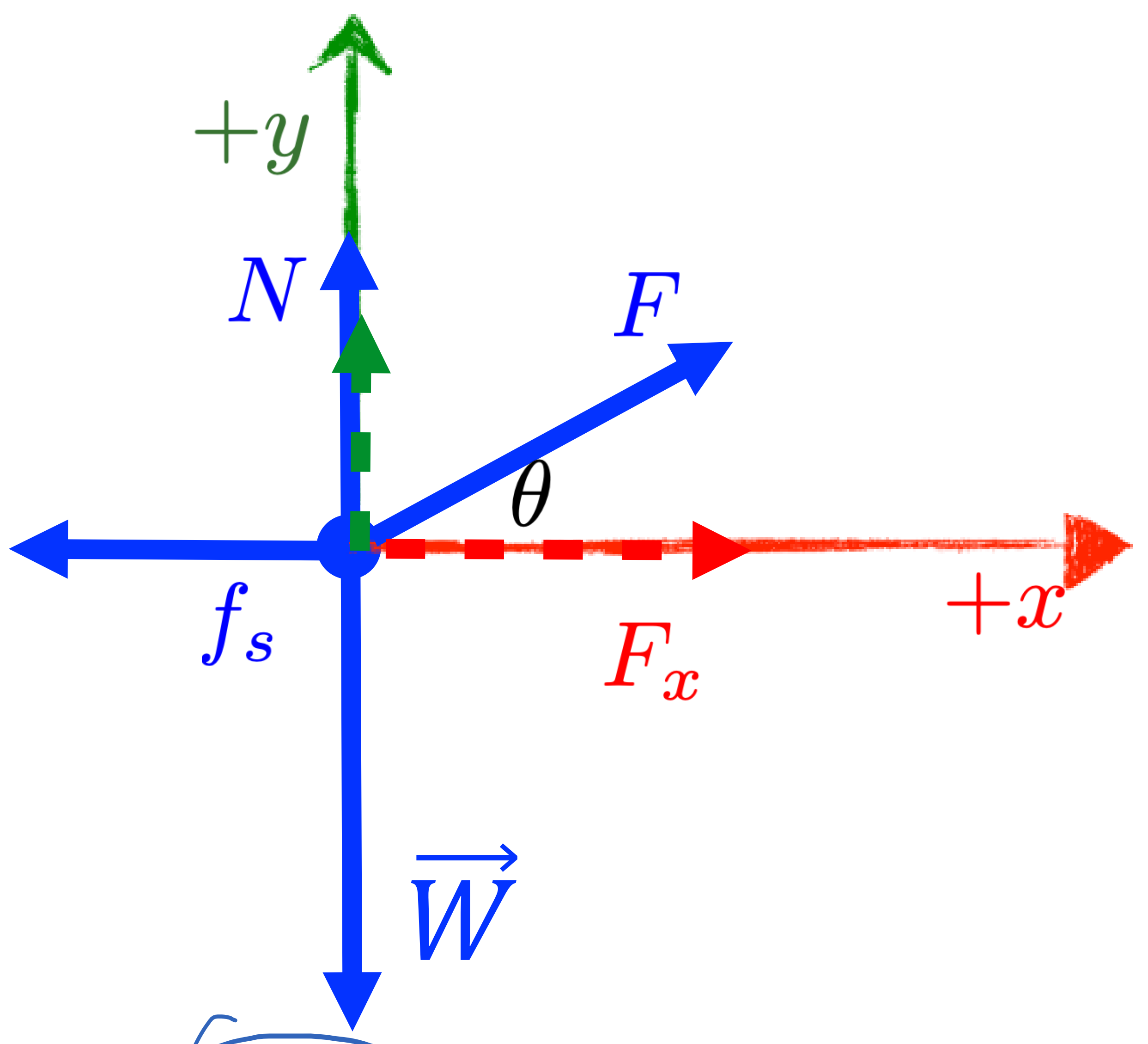
1. Driving force

2. $\vec{\text{friction}}$ opposes sliding

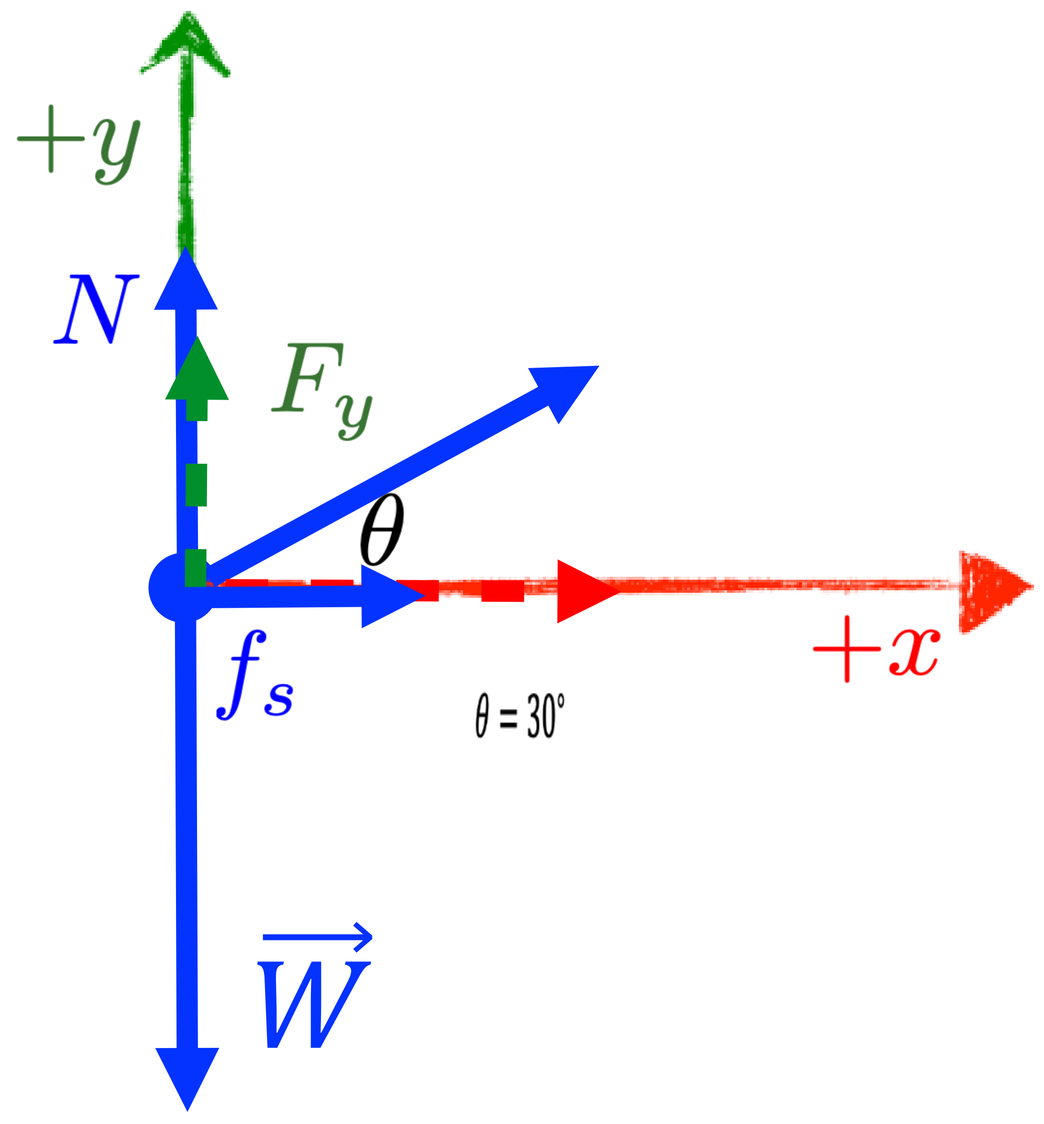




- A box of mass $m = 40kg$ is pulled by a force of magnitude F at $\theta = 30°$ above the horizontal. The static friction coefficient is $\mu_s = 0.650$. Which of the following is the free body diagram of the box?

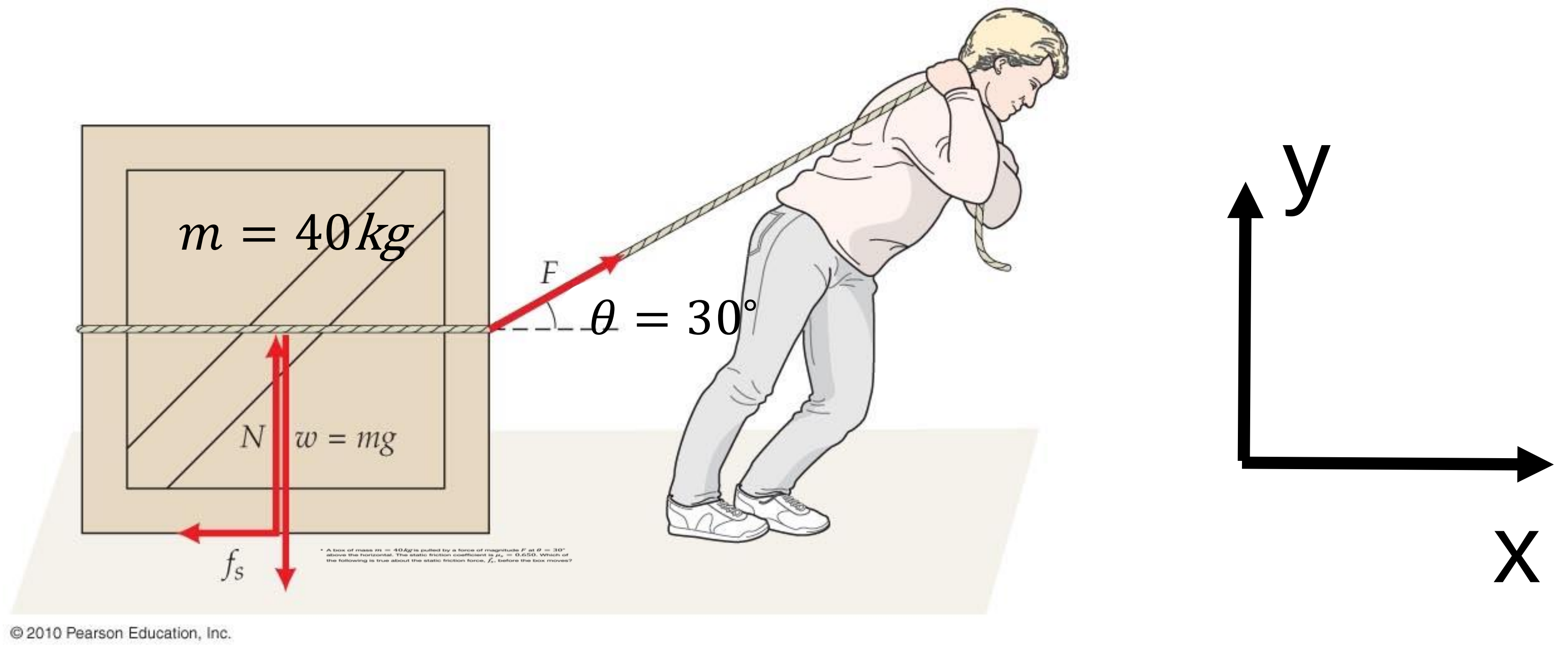


A



B

Clicker question 7



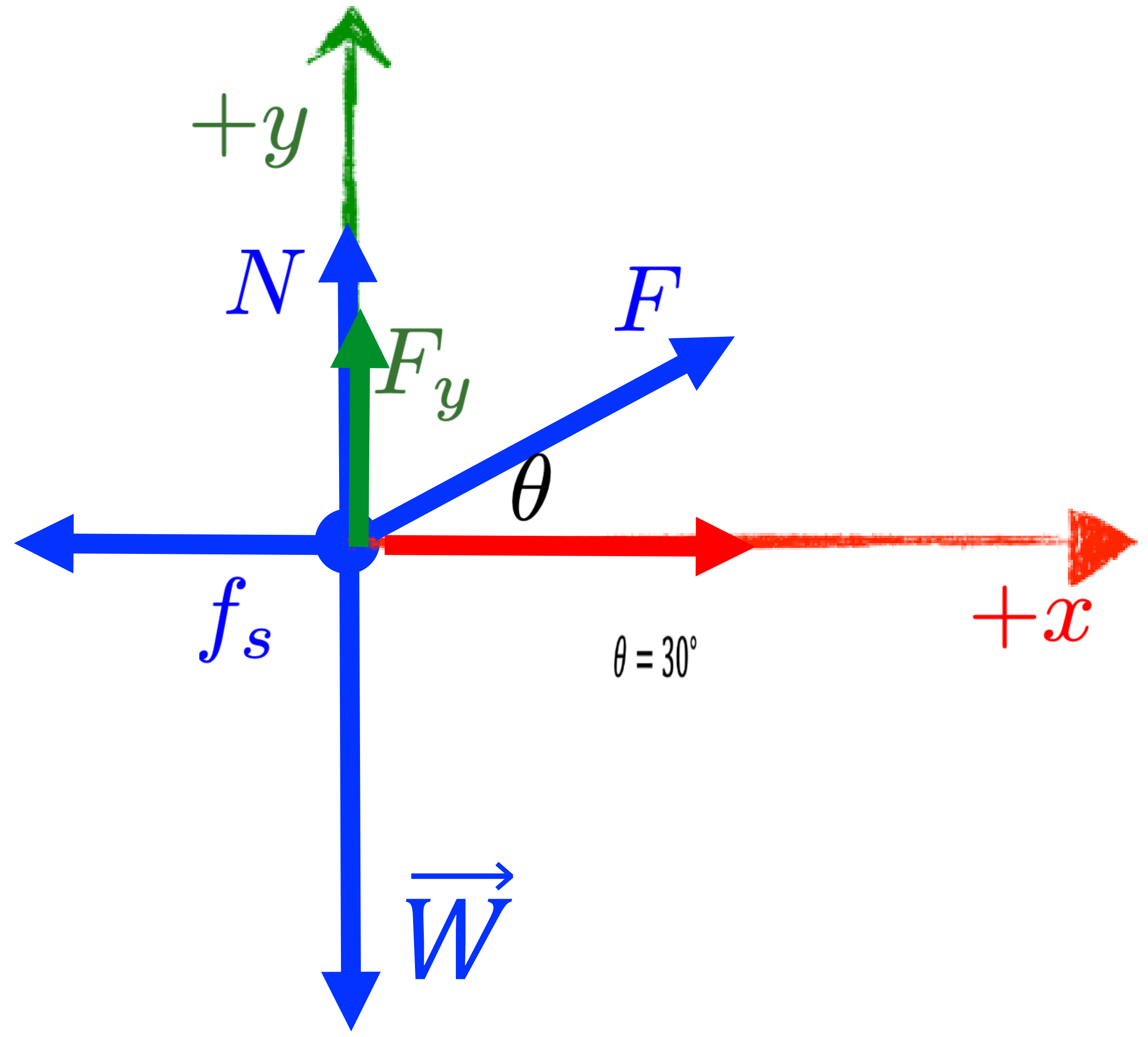
- A box of mass $m = 40kg$ is pulled by a force of magnitude F at $\theta = 30°$ above the horizontal. The static friction coefficient is $\mu_s = 0.650$ .Which of the following is true about the static friction force, $\vec{f_s}$,before the box moves?

A $\vec{f_s} + \vec{F_x} = 0$

Newton's law

B $|\vec{f_s}| \leq |\mu_s \vec{N}|$

C Both A & B

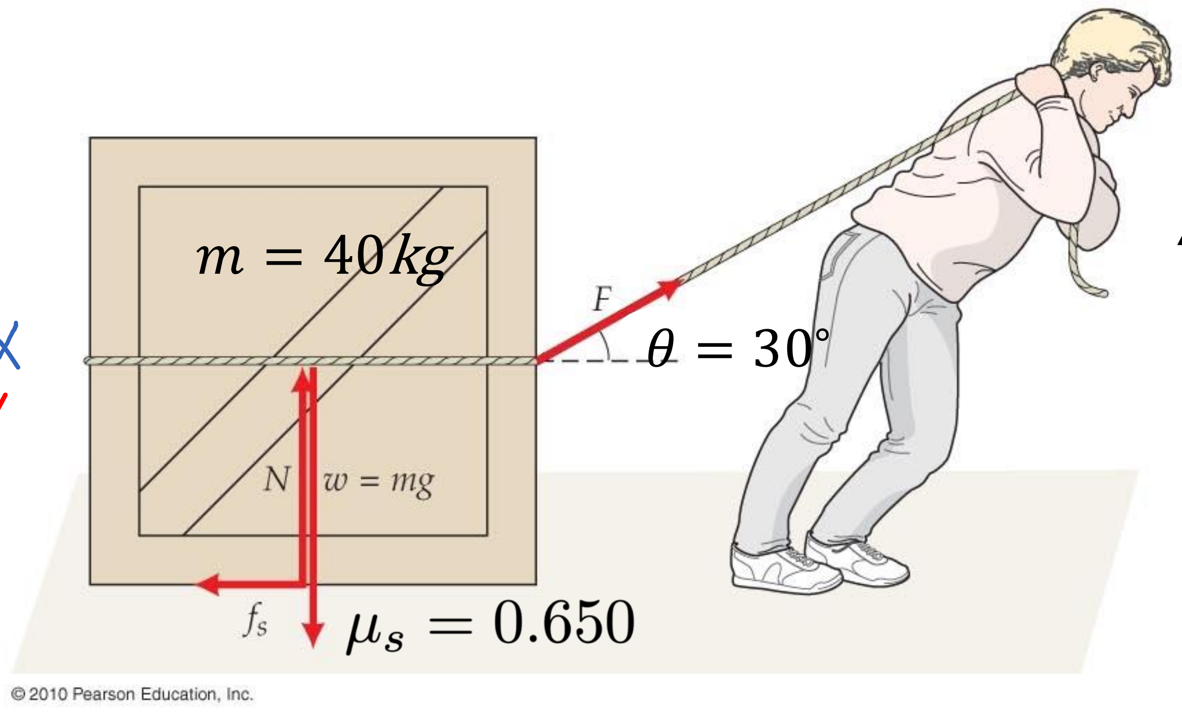


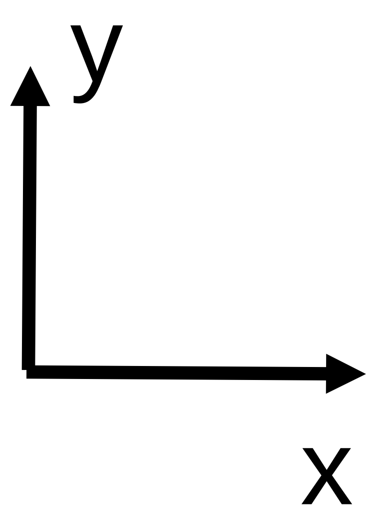
Example 1

Given: m, θ, μ_s

Goal: $\vec{F}_{max}$, such that $\vec{f}_s = \vec{f}_{s,max}$

$$|\vec{f}_{s,max}| = \mu_s |N|$$

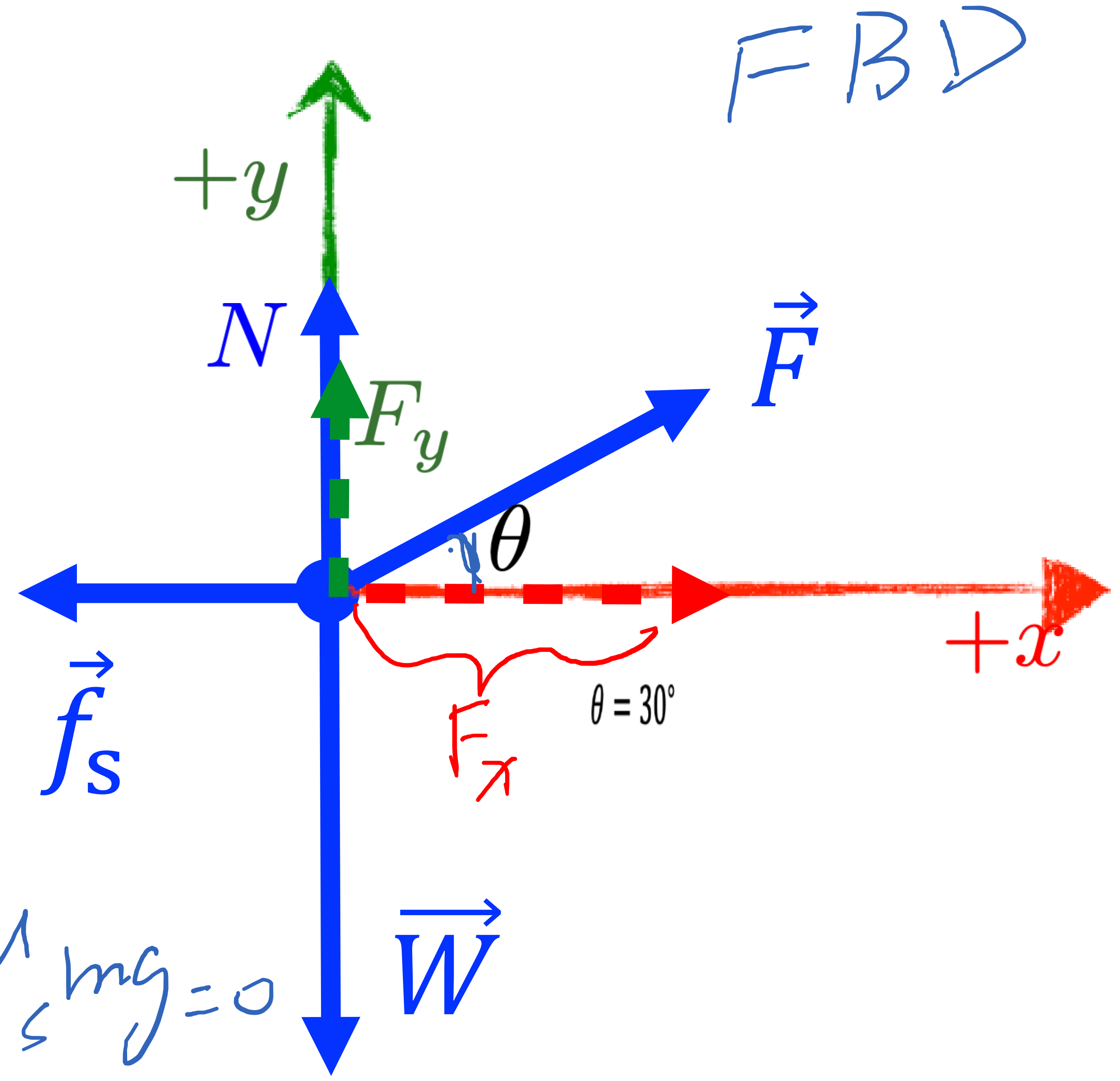




- A box of mass $m = 40kg$ is pulled by a force of magnitude $|F|$ at $\theta = 30°$ above the horizontal. The static friction coefficient is $\mu_s = 0.650$ .What is the maximum F before the box moves?

Step 0: FBD ;

Step 2: Newton's 2nd law: $\vec{F}_{net} = m\vec{a} = 0$



Step 3: $x: \vec{F}_x + \vec{f}_s = 0$; $y: \vec{F}_y + \vec{N} + \vec{W} = 0$ $\rightarrow$

$$\begin{cases} |\vec{F}|\cos\theta\,\hat{x} - \mu_s |N|\,\hat{x} = 0 & (1) \\ |\vec{F}|\sin\theta\,\hat{y} + |N|\,\hat{y} - mg\,\hat{y} = 0 & (2) \end{cases}$$

Step 4: Eliminate $|N|$: $(1) + (2)\cdot\mu_s$

$$|\vec{F}|\cos\theta - \cancel{\mu_s |N|} + |\vec{F}|\mu_s \sin\theta + \cancel{\mu_s |N|} - \mu_s mg = 0$$

$$|\vec{F}| = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \frac{0.650 \times 40kg \times 9.8\,ms^{-2}}{\cos 30° + 0.650 \cdot \sin 30°} \approx 214\,N$$

Homework 5.4 reminder

- As a friendly reminder, Homework 5.4 was released last Tuesday, and due this Thursday.