PHYS 225
Fundamentals of Physics: Mechanics

Prof. Meng (Stephanie) Shen

Fall 2024

Lecture 22: Work and power

CALIFORNIA STATE UNIVERSITY FULLERTON

Learning goals for today

- Work done by
  - A spring force
  - A general variable force
- Power
- Conservative forces and potential energy

Recap: Work

- Work: Energy transferred to or from an object by a force acting on an object
  - $W = \int \vec{F} \cdot d\vec{l} = \int F_x dx + \int F_y dy + \int F_z dz$

    dot

    displacement
- Work done by a **constant force** along a displacement, $\Delta\vec{r}$:

  If $\vec{F} = \text{const}$
  - $W = \vec{F} \cdot \Delta\vec{r} = F_x \Delta x + F_y \Delta y + F_z \Delta z$

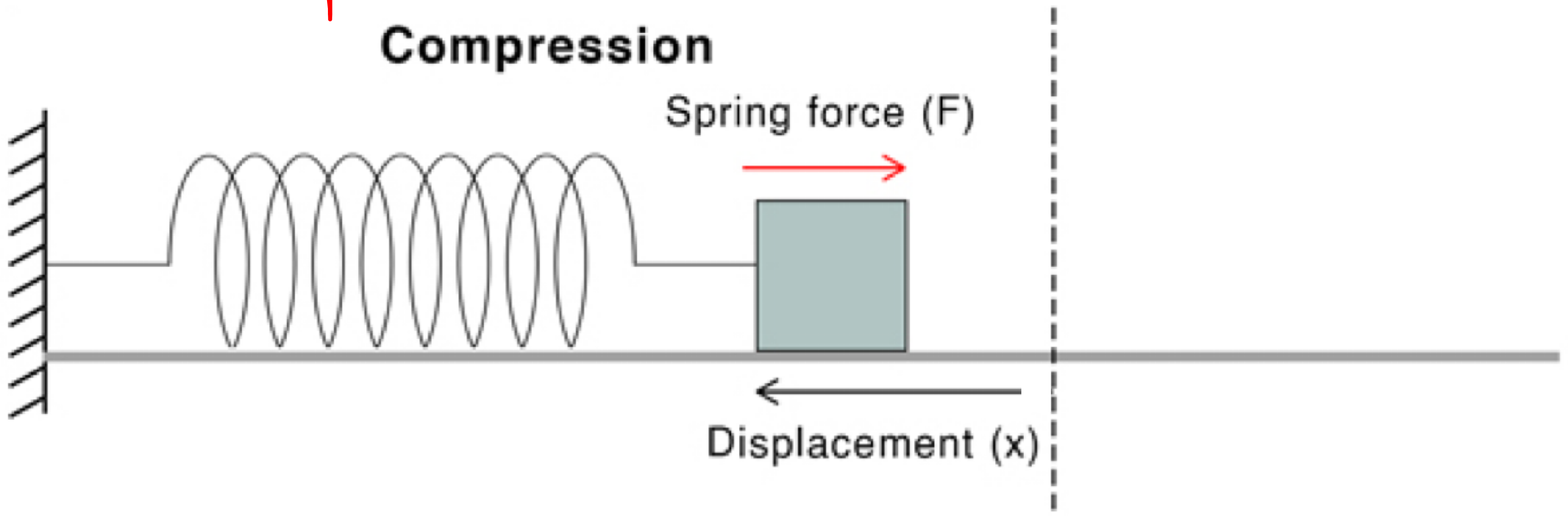
    displacement
- Work done by a spring force:

  spring const

  $\vec{F}_s = -k\vec{x}$ — displacement from relaxed pos.
  - $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$

    sp. const

Reference point: Relaxed position



Example 4

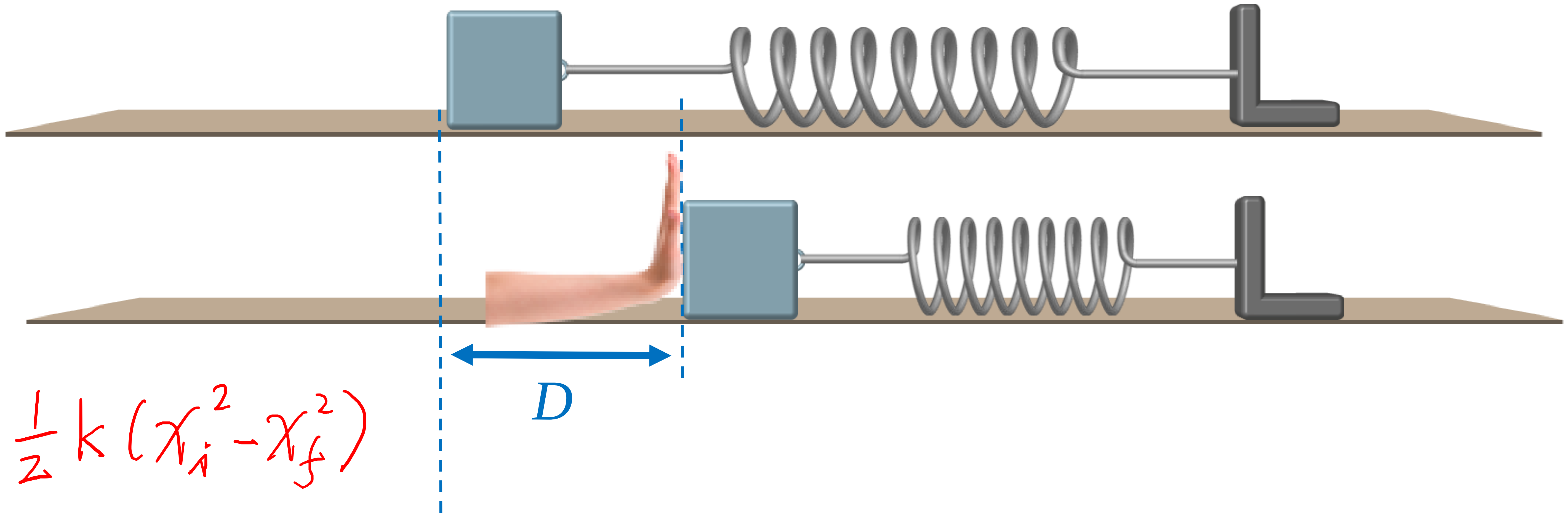
Given: x_i, x_f, k

Goal: W_s

$\rightarrow +x$

$x_i = 0$

- A box is attached at rest to a spring at its relaxed length. You now push the box with your hand so that the spring is compressed by a distance *D = 0.5 m. The spring constant is k = 1.0 N/m.* Please calculate the work done by the spring on the box during this motion.



$$W_s = \frac{k}{2}(x_i^2 - x_f^2)$$

Step1: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$

Step2: $= \frac{1}{2} \times 1.0\ N\,m^{-1}\,(0^2 - (0.5m)^2)$

$= -0.125\ J$

Clicker question 10

- The position of a toy car can be expressed as $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, in meters. What principle to use to calculate the work done by a variable force $\vec{F} = (3.00N/m\, x)\ \hat{\imath} + 5.00N\ \hat{\jmath}$ that moves the toy car from a position $\vec{r}_i = (7.00\text{ m})\ \hat{\imath} + (6.00\text{ m})\hat{\jmath}$ to a position $\vec{r}_f = (-4.00\text{ m})\ \hat{\imath} - (3.00\text{ m})\hat{\jmath}$?

$\vec{F} \neq$ const

A $W = \vec{F} \cdot \vec{d} = \vec{F} \cdot (\vec{r}_2 - \vec{r}_1)$

B $W = \int F_x dx + \int F_y dy + \int F_z dz$

C $W = -mg(y_f - y_i)$

Example 5

Given: $\vec{F}$, $\vec{r}_i$, $\vec{r}_f$

Goal: W

- The position of a toy car can be expressed as $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, in meters. What work is done by a variable force $\vec{F} = (3.00\frac{N}{m})\,x\ \hat{\imath} + 5.00N\,\hat{\jmath}$ that moves the toy car from a position $\vec{r}_i = (7.00\text{ m})\,\hat{\imath} + (6.00\text{ m})\hat{\jmath}$ to a position $\vec{r}_f = (-4.00\text{ m})\,\hat{\imath} - (3.00\text{ m})\hat{\jmath}$?

Step1: $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \cancel{\int_{z_i}^{z_f} \underbrace{F_z}_{0} dz}$

$$= \int_{x_i}^{x_f} 3.00(Nm^{-1})\,x\,dx + \int_{y_i}^{y_f} 5.00N\,dy$$

x_f: −4m, x_i: 7m, y_f: −3m, y_i: 6m

$$= \frac{1}{2}3.00(Nm^{-1})\,x^2\Big|_{7m}^{-4m} + 5.00N\,y\Big|_{6m}^{-3m}$$

Step2:

$$= \frac{1}{2}3.00\,N\,m^{-1}\left((-4m)^2 - (7m)^2\right) + 5.00\,(-3m - 6m)$$

$$= -94.5\text{ J}$$

2.3. Work, velocity and power

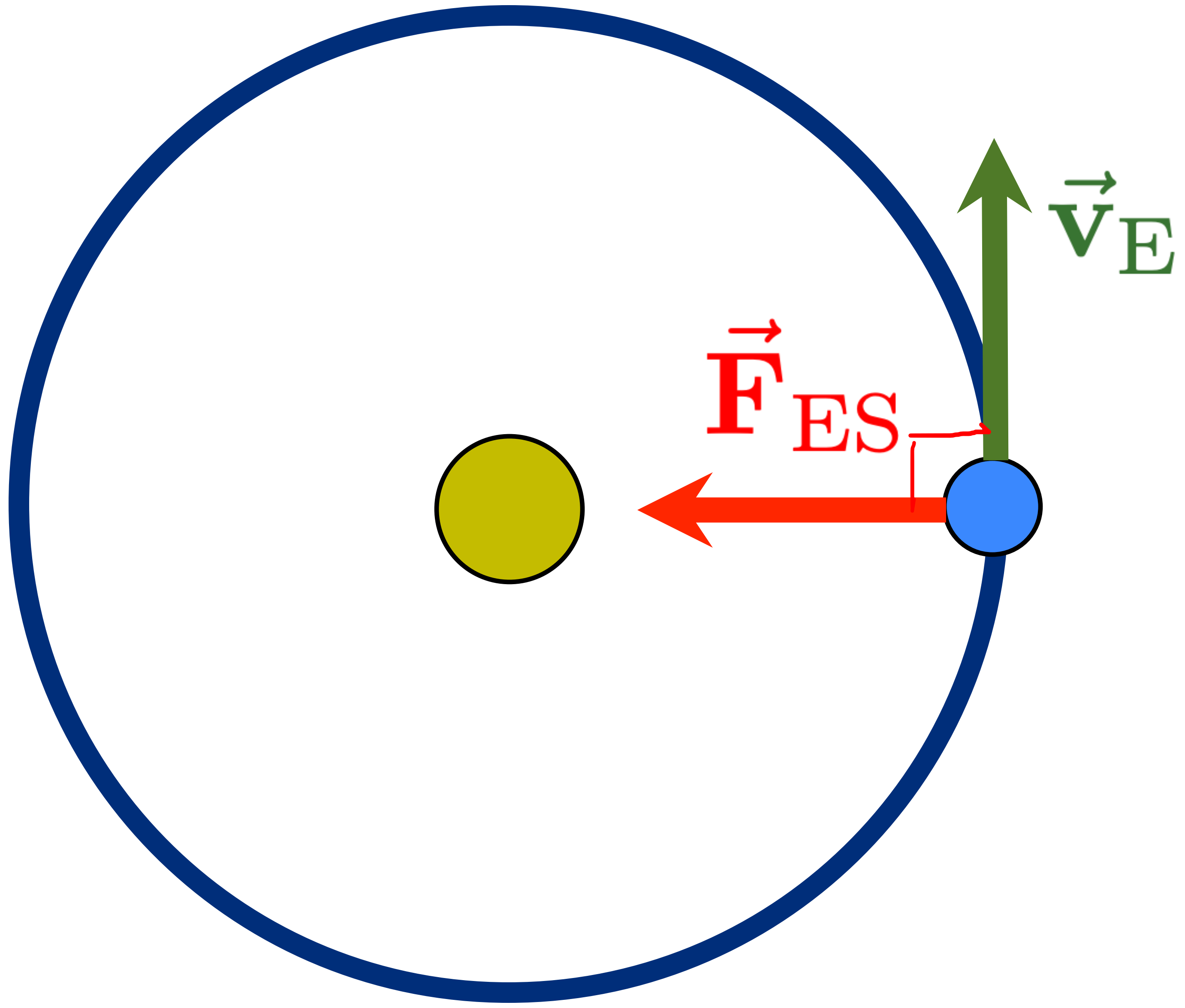
$\vec{v} = \frac{d\vec{l}}{dt} \rightarrow d\vec{l} = \vec{v}\,dt$

Velocity

- $W = \int \vec{F} \cdot d\vec{l} = \int \underbrace{(\vec{F} \cdot \vec{v})}_{\text{Power}} dt$
- Power: Rate of energy transfer, or rate of work
- Average power = work / time: $\overline{P} = \frac{W}{\Delta t}$ — work; — time duration
- Instantaneous power: $P = \frac{dW}{dt}$, $P = \vec{F} \cdot \vec{v}$ dot prod.
- Units: Watt (1 W = 1 J/s)

Clicker question 11

- If the earth orbits the sun in a circular orbit and tidal friction can be neglected, what is the work done on the earth by the sun?

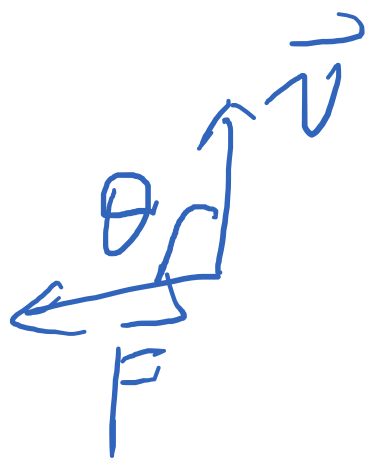


A – $|W|$

B 0

C + $|W|$

D not enough information to know



90°

$\vec{F}\cdot\vec{v} = \underline{|\vec{F}||\vec{v}|\cos\theta}$

Recall that $W = \int \underbrace{\vec{F}\cdot\vec{v}}_{\text{Power}} dt$

Clicker question 12: Power

Time for Work II

Mike performed 5 J of work in 10 secs. Joe did 3 J of work in 5 secs. Who produced the greater average power?

A Mike produced more power

B Joe produced more power

C both produced the same amount of power

$$\bar{P}_M = \frac{W_M}{\Delta t_M} = \frac{5J}{10s} = 0.5 \text{ Watt}$$

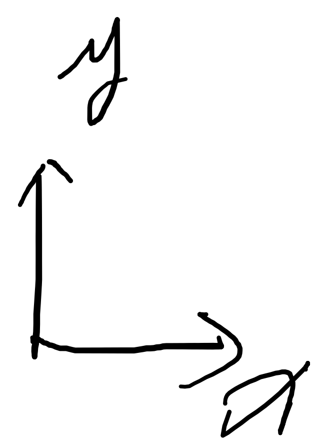
$$\bar{P}_J = \frac{W_J}{\Delta t_J} = \frac{3J}{5s} = 0.6 \text{ Watt}$$

$$\bar{P} = \frac{W}{\Delta t}$$

Example 6

Given: m, $\vec{F}_{net}$, $v_0 = 0\,ms^{-1} = 0$, Δt

Goal: W, $\bar{P}$



- A box of mass $m = 5kg$ is pulled by a **net** force $\vec{F} = 10N\,\hat{\imath} + 10N\hat{\jmath}$ from rest for a time of $\Delta t = 5s$. **1)** Please calculate the work done by the force on the box; **2)** Please calculate the average power of the force on the mass.

Step 1: $\vec{F}$ = const $\rightarrow W = \vec{F}\cdot\underbrace{\Delta\vec{r}}_{?}$

Step 2: 1D kinematics for Δx, Δy

Newton's 2nd law: $\underbrace{\vec{F}_{net}}_{\vec{F}} = m\vec{a} \rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{10N\hat{i} + 10N\hat{j}}{5kg} = \underbrace{2\,ms^{-2}}_{a_x}\hat{i} + \underbrace{2\,ms^{-2}}_{a_y}\hat{j}$

Step 3: $\because v_0 = 0$, $\Delta x = v_{x0}\Delta t + \frac{1}{2}a_x(\Delta t)^2 = 0 + \frac{1}{2}\times 2ms^{-2}\times(5s)^2 = 25\,m$

$\Delta y = v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2 = 0 + \frac{1}{2}\times 2ms^{-2}\times(5s)^2 = 25m$

Step 4: $\rightarrow \Delta\vec{r} = 25m\,\hat{i} + 25m\,\hat{j}$

$W = F_x\Delta x + F_y\Delta y = 10N\cdot 25m + 10N\cdot 25m = 500\,J$

Step 5: $\bar{P} = \frac{W}{\Delta t} = \frac{500J}{5s} = 100$ Watt

Homework

- Homework assignment for Chapter 7 in Module 7.4, due in a week.

Pre-lecture survey 8.1

- Please complete Module 8.1: Pre-lecture survey by next lecture

Check point for Chapter 7

- Kinetic energy: $K = \frac{1}{2}mv^2$
- Work in general: W $= \int \vec{F} \cdot d\vec{l}$ $= \int F_x dx + \int F_y dy + \int F_z dz$
- Work by a constant force, $\vec{F}$: $W = \vec{F} \cdot \Delta\vec{r}$
- **Work-kinetic energy theorem:** $W_{net} = \Delta K$ $= K_f - K_i$
- Power, P: Rate of work, $P = \dfrac{dW}{dt} = \vec{F} \cdot \vec{v}$

Chapter 8: Potential energy and conservation of energy

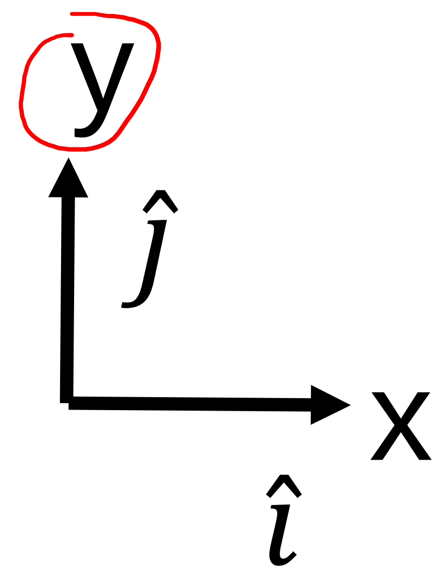
- Learning objectives
  - Concepts:
    - Conservative force
    - potential energy
    - Conservation of energy

W by $\vec{F}_g$ or $\vec{W}$

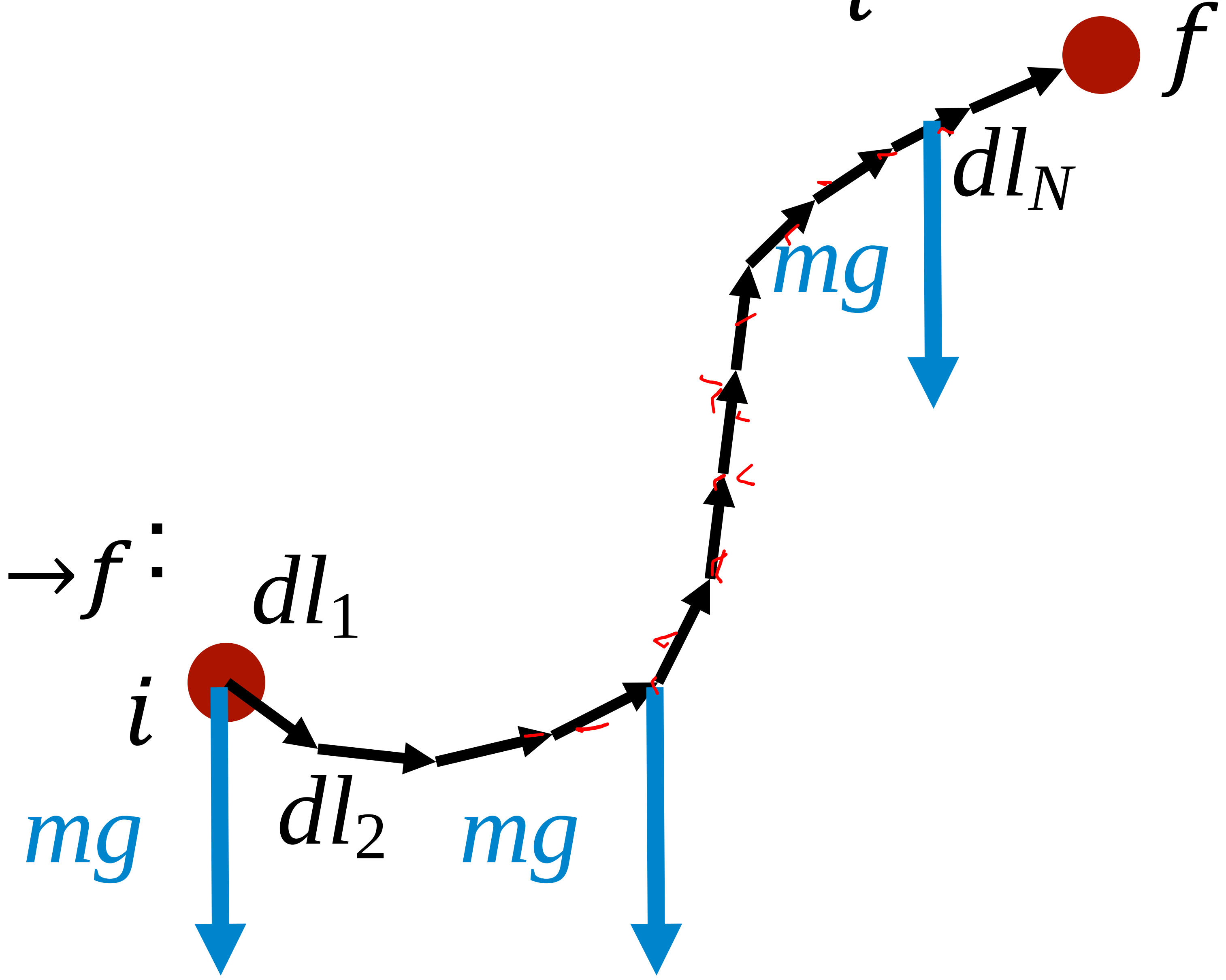
W by $\vec{F}_s$

Chapter 8.1: Conservative forces and potential energy

Recap: Work done by the weight



- Weight, i.e., gravity on the earth surface
  - Weight is a constant, $\vec{F}_g = -mg\hat{\jmath}$ = W
  - Work by weight from initial position to final position, $W_{i\to f}$:



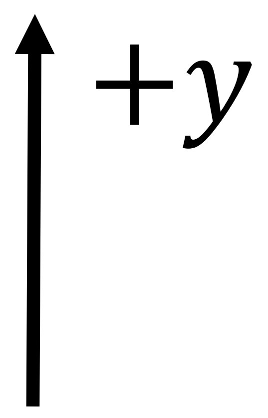
Assume the motion is only in the x-y plane.

$$W_{i\to f} = -mg(y_f - y_i)$$

final height   initial height

  - *Therefore, **work by weight only depends on the initial and final height!***

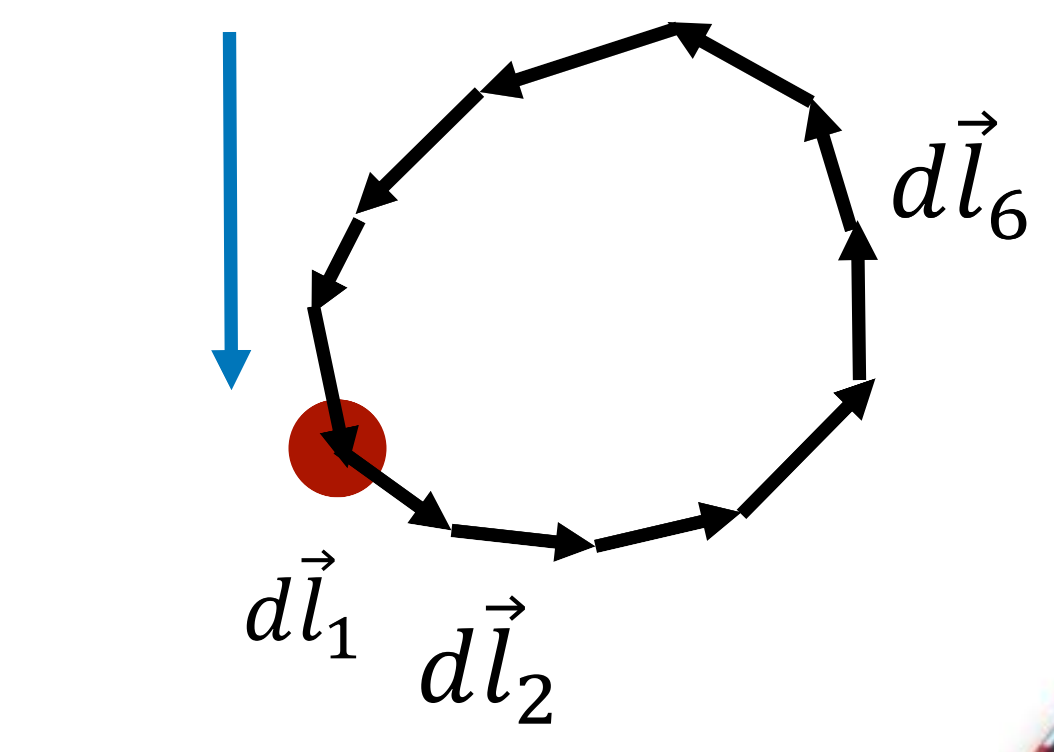
Clicker question 1



- The cart starts sliding from position A in the direction pointed by the arrow, and **returns to A**. The kinetic friction coefficient is 0.2. What is the work done on the car by the **gravitational force**? (The trail is on earth surface.)

A	Positive
B	0
C	Negative

$\vec{W} = -mg\hat{j}$



y_A y_A

$$W_{grav} = -mg(y_f - y_i)$$

0

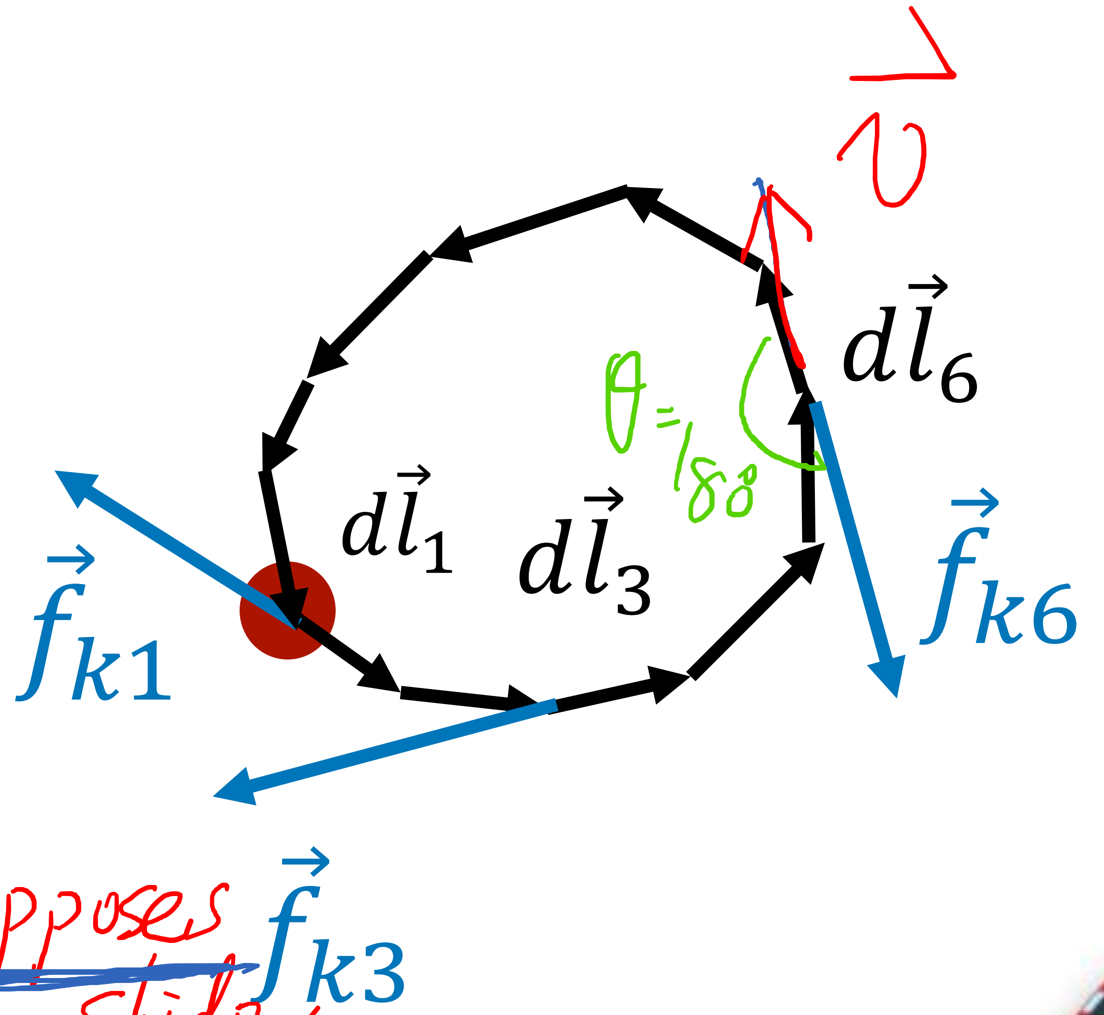
Clicker question 2

- The cart starts sliding from position A in the direction pointed by the arrow, and back to A. The kinetic friction coefficient is 0.2. What is the work done on the car by the **kinetic friction force**?

A Positive

B 0

C Negative



friction opposes sliding

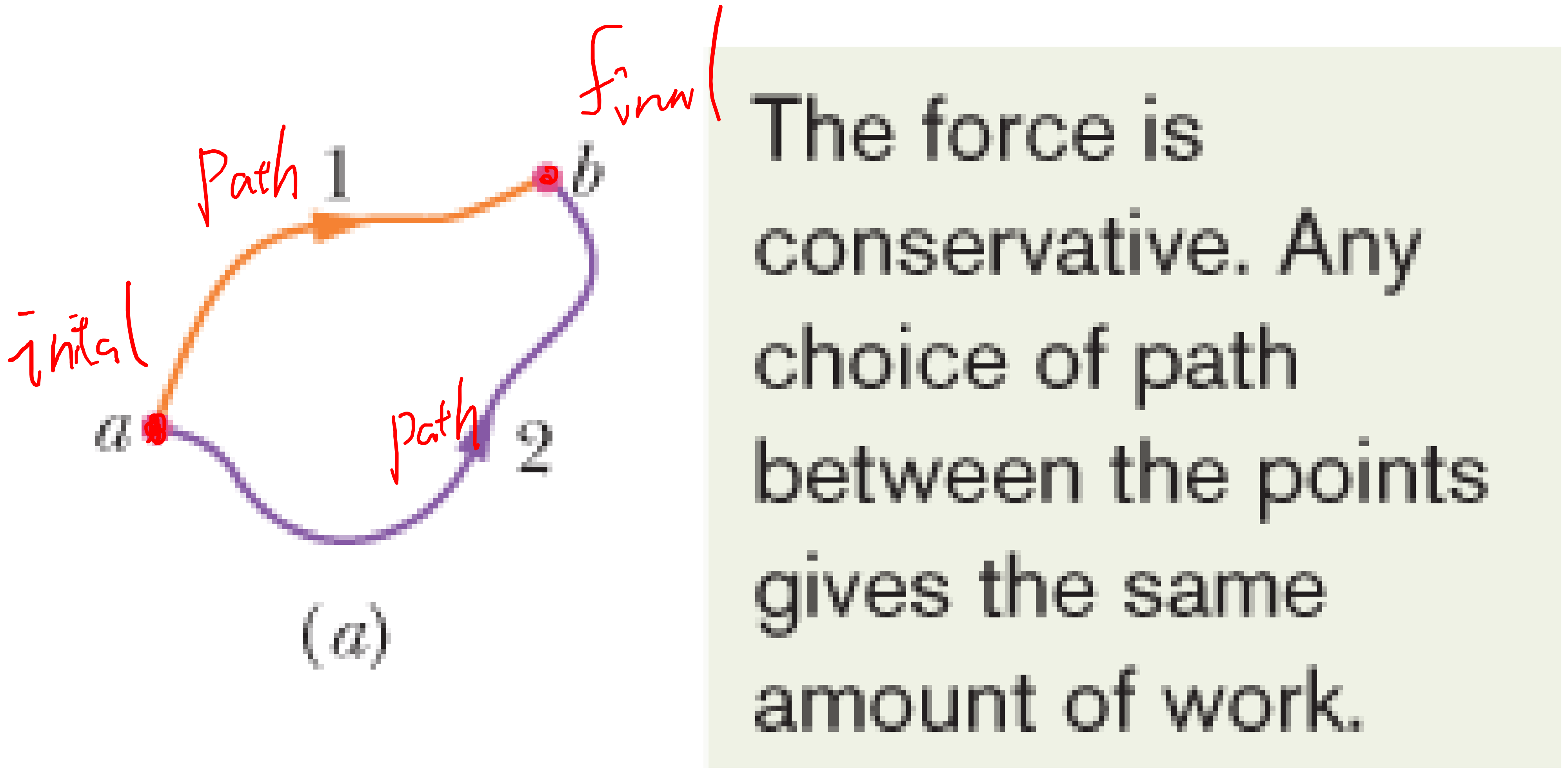
$\vec{F}\cdot\vec{v} = |\vec{F}||\vec{v}|\cos\theta$

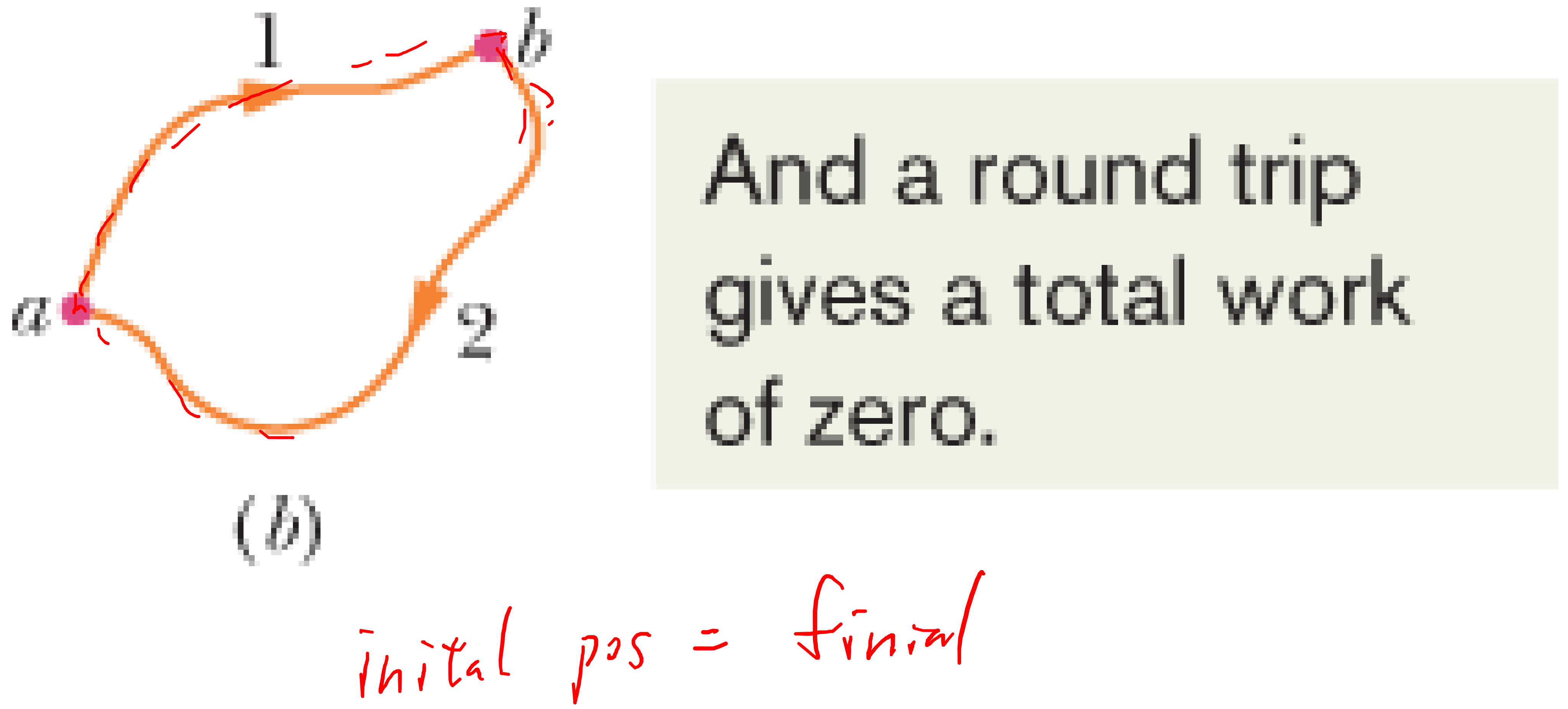
$|\vec{f}_k|$

$$W = \int \underbrace{\vec{F}\cdot\vec{v}}_{\text{power}}\, dt$$

Conservative forces

- **Definition of a conservative force**: If the work done by a force ***only*** depends on the initial and final positions, then it is a conservative force.





I. Open loops: $W_{a\to b,1} = W_{a\to b,2}$

II. Closed loop: $W_{a\to b,1} + W_{b\to a,2} = 0$

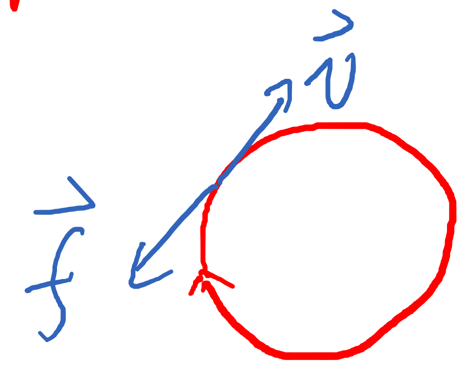
Clicker question 3

- Which of the following is true?

A Gravitational force is a conservative force.

B Friction is a conservative force.

friction is not a conservative force.



$W_f = \int (\underbrace{\vec{f}\cdot\vec{v}}_{<0})\, dt$

Closed loop, $W_f < 0$.

Potential energy

- **Potential energy:** Energy of position, U
  - The measure of capability for a ***conservative force*** to do work.
- Changes in potential energy: $\Delta U = U_f - U_i = -W_{cons}$

i.e., The change of potential energy is the *negative of* work done by a conservative force.