PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 12: Relative motion and reference frames

CALIFORNIA STATE UNIVERSITY FULLERTON

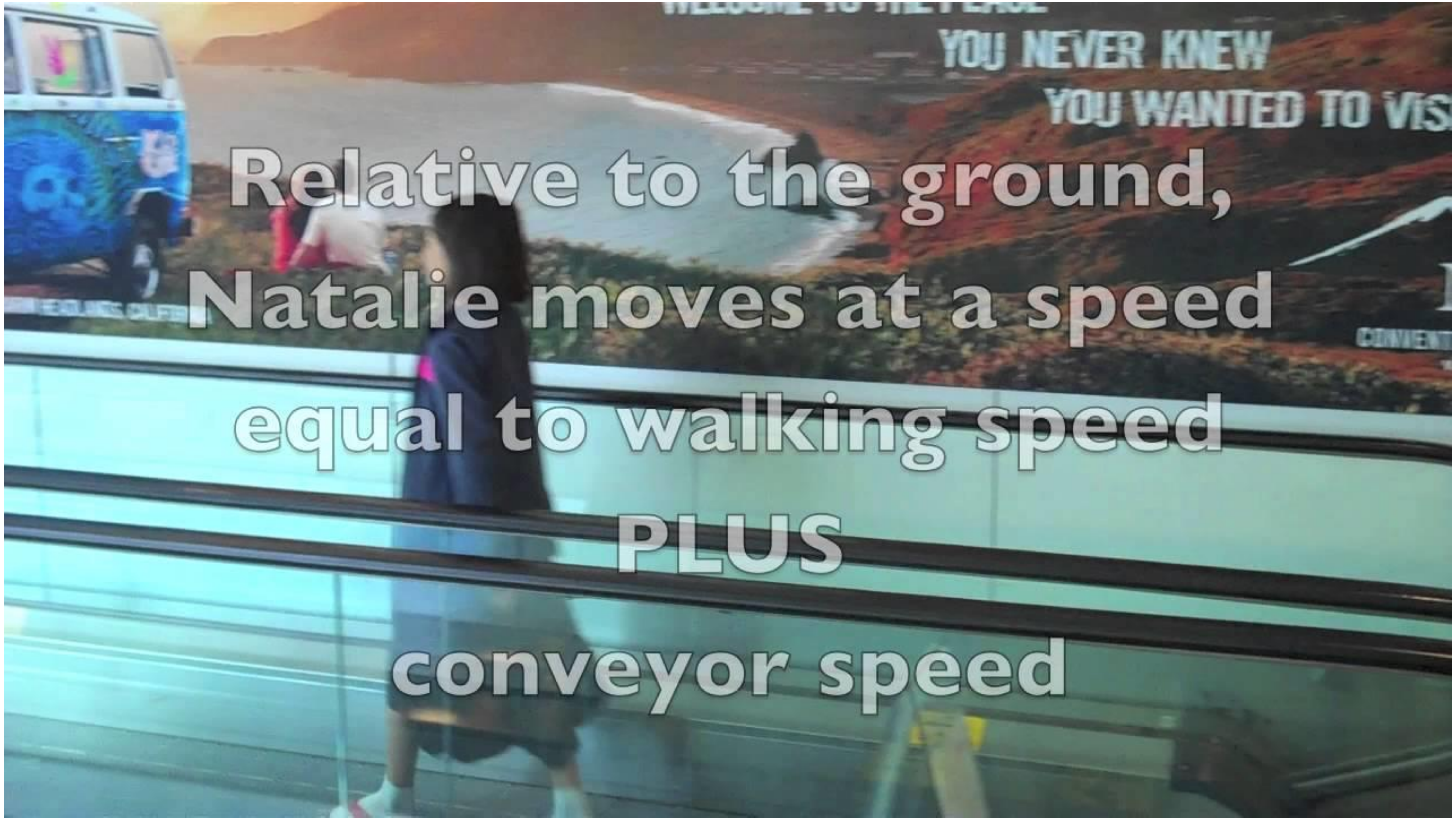
Learning goals for today

- Relative motion and reference frames

Chapter 4.3: Relative motion and reference frames

- **Take home message**
  - RV - Velocity depends on reference frame of observer

Example: Relative motion and a moving belt



Standing:

$\vec{v}_{Ng} = \vec{v}_{bg}$

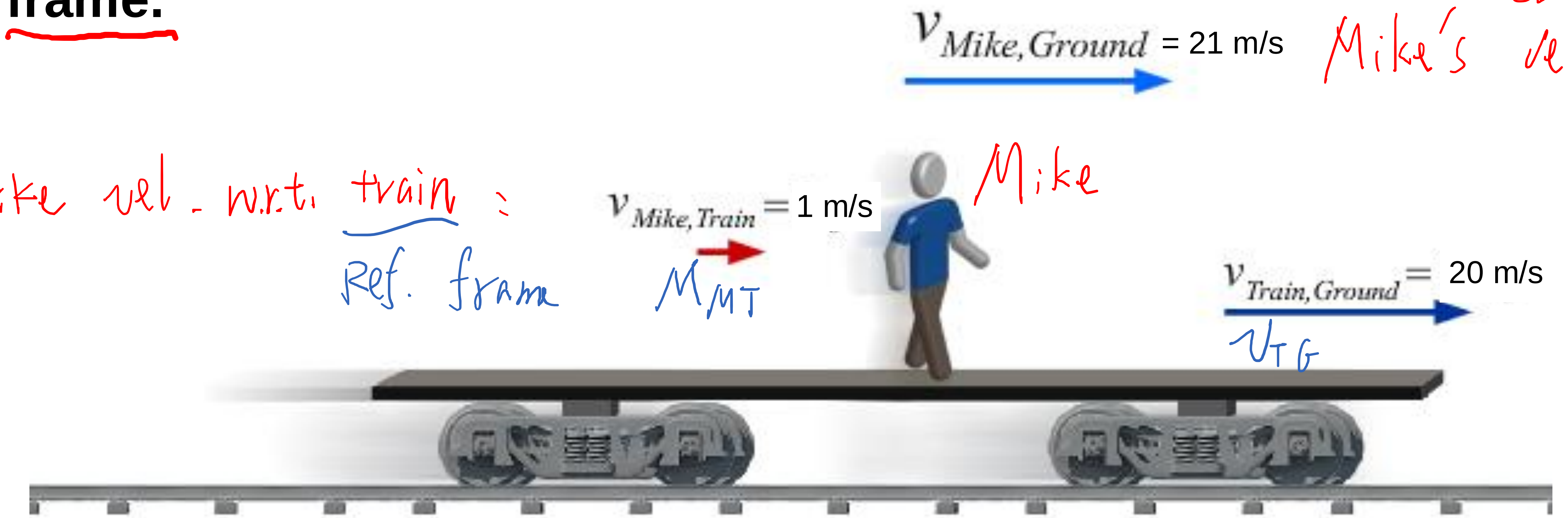
N is moving forward

$\vec{v}_{Ng} \neq \vec{v}_{Nb}$

https://youtu.be/KWhmbYHb7EU

Reference frame and relative motion

- **Reference frame:** The object and the coordinate system to which the observer is attached.
- **Relative motion:** The measurement of the velocity of an object relative to the **reference frame.**



Vector addition: $\vec{v}_{Mike,Ground} = \vec{v}_{Mike,Train} + \vec{v}_{Train,Ground}$

In general: $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

Clicker question 1

A person stands on a moving sidewalk belt that moves to the right at 2 m/s relative to the ground. A dog runs to the left toward the person along the belt at a speed of 8 m/s relative to the belt.

What is the speed of the dog relative to the ground?

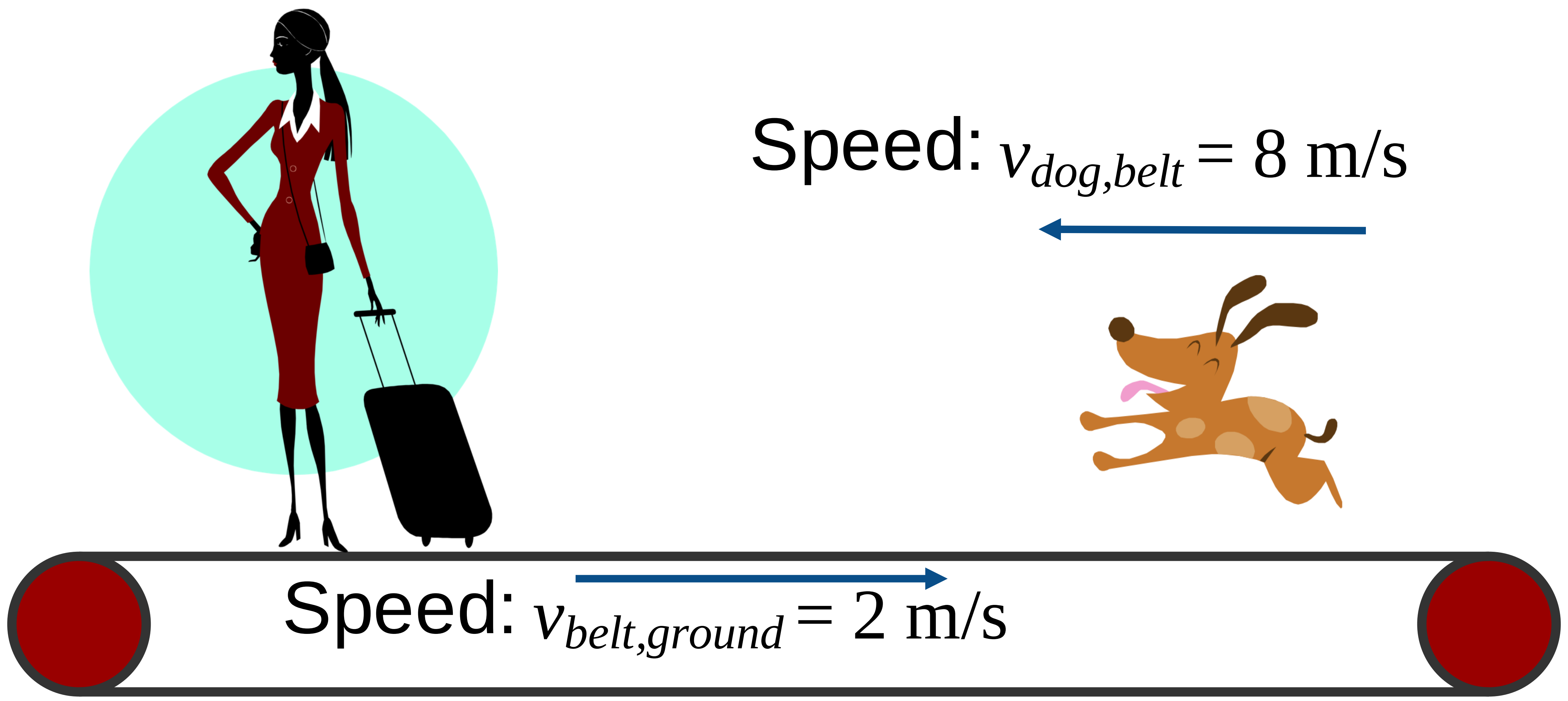
Given: $\vec{v}_{bg} = 2\,m\,s^{-1}\,\hat{i}$

$\vec{v}_{db} = -8\,m\,s^{-1}\,\hat{i}$

Goal: $\vec{v}_{dg} = \vec{v}_{db} + \vec{v}_{bg}$

$= -8\,m\,s^{-1}\,\hat{i} + 2\,m\,s^{-1}\,\hat{i}$

$= -6\,m\,s^{-1}\,\hat{i}$



$$\vec{v}_{dg} = \vec{v}_{db} + \vec{v}_{bg}$$

$|\vec{v}_{dg}| = 6\,m\,s^{-1}$

+x

A 6 m/s

B 8 m/s

C 10 m/s

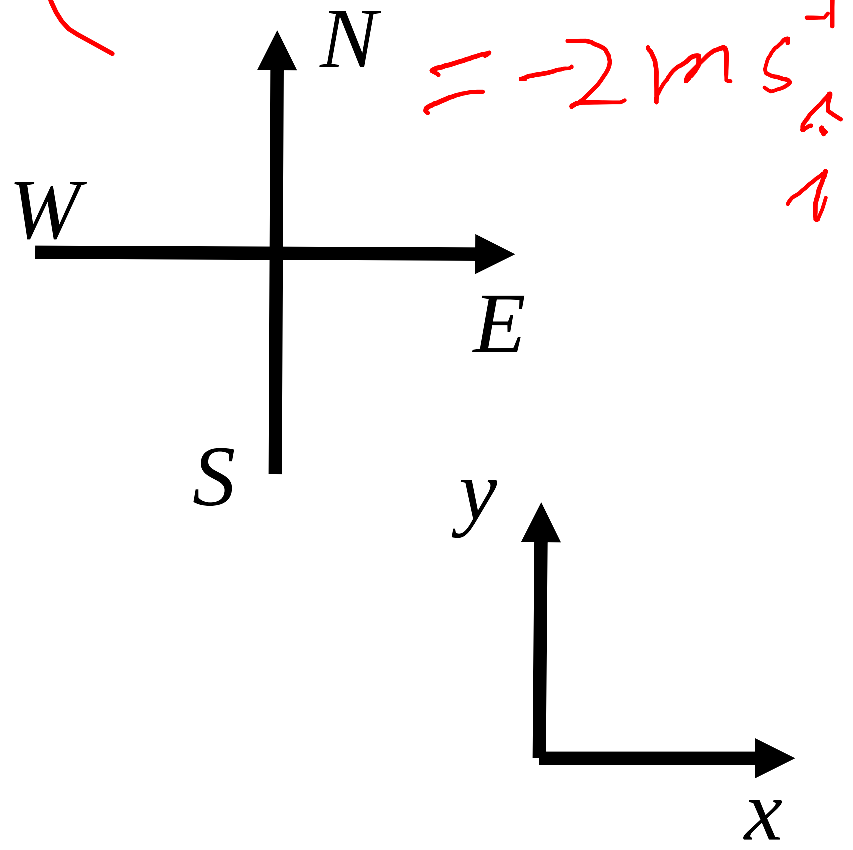
Clicker question 2

Given: $\vec{v}_{wg} = 2\,m/s\,\hat{i}$, $\vec{v}_{cw} = -4\,m\,s^{-1}\,\hat{i}$

Goal: $\vec{v}_{cg}$

$$\vec{v}_{cg} = \vec{v}_{cw} + \vec{v}_{wg} = (-4\,m\,s^{-1} + 2\,m\,s^{-1})\,\hat{i} = -2\,m\,s^{-1}\,\hat{i}$$

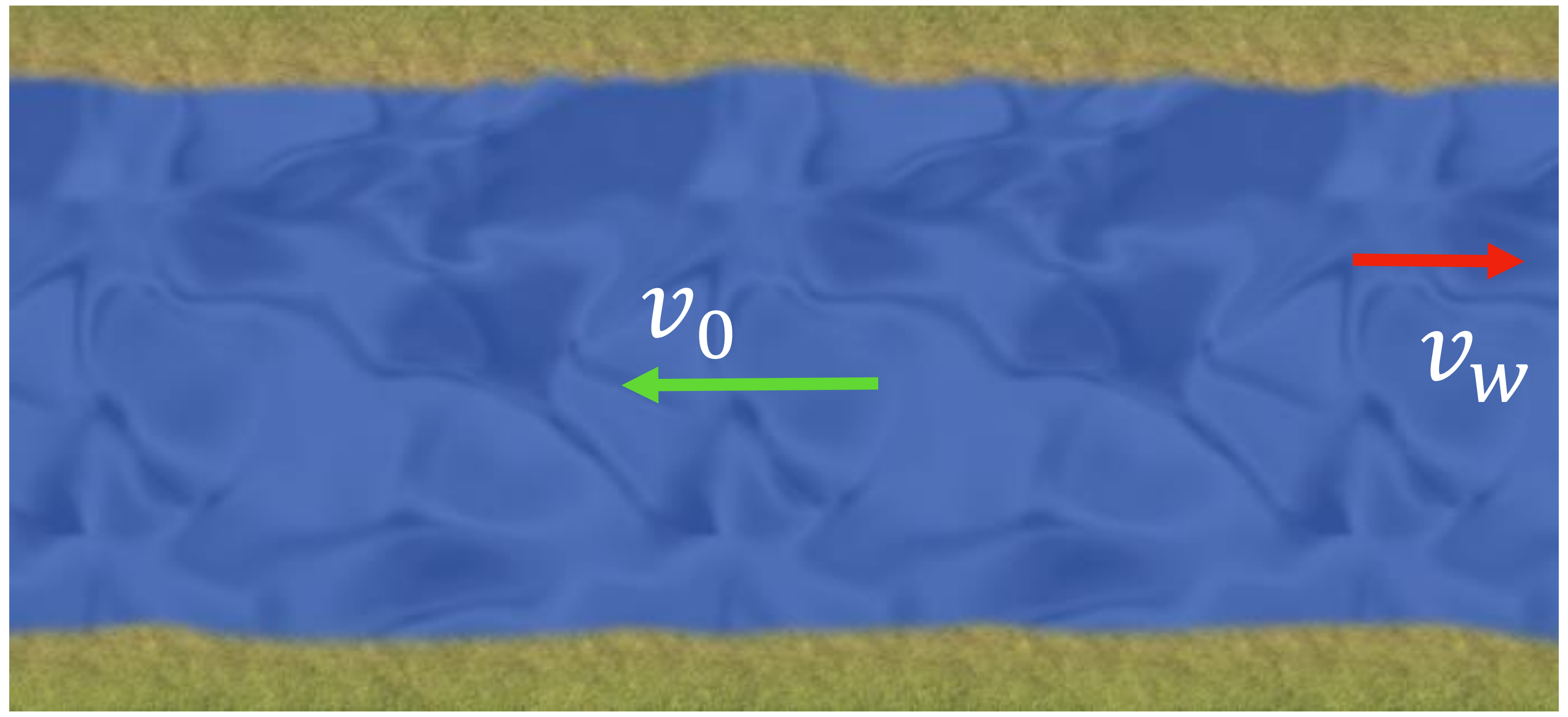
- A river flows due east at a uniform speed of v_w = 2m/s. Carly swims upstream (upstream means opposite to the water flow) with a speed of v_c = 4 m/s with respect to water. If +x points to the east, what is the magnitude and direction of the swimmer's velocity with respect to the ground?



A 6 m/s, upstream

B 2 m/s, upstream

C 2 m/s, downstream

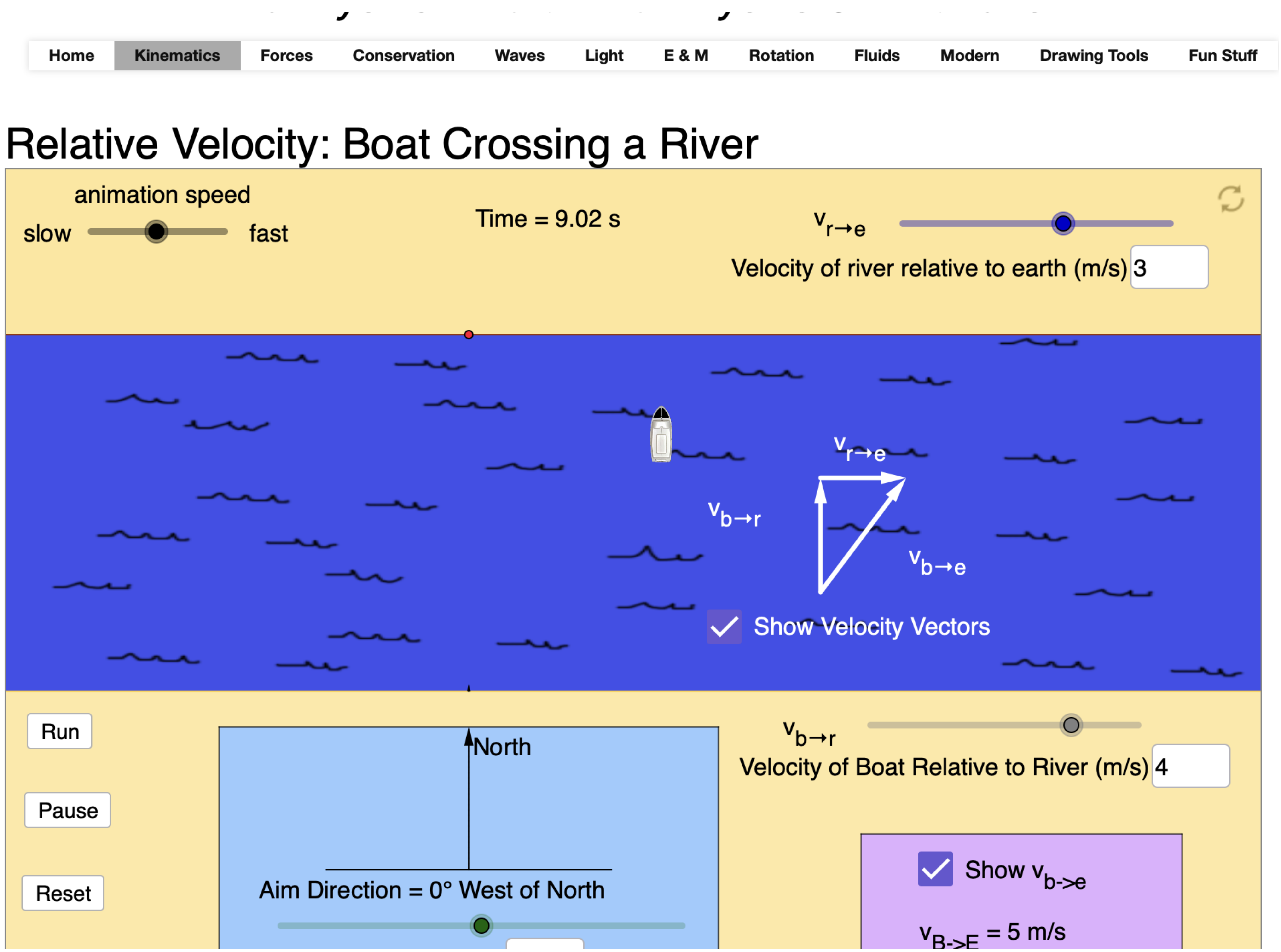


Downstream: Same direction as water velocity w.r.t. ground

Upstream: Opposite direction to water velocity w.r.t. ground

Demo in 2D

Simulation demo in 2D



https://ophy sics.com/k11.html .

Clicker Question 3

Given: $|\vec{v}_{sw}|$, Δy the same for all

Goal: t

$|v_{Aw}| = |v_{Bw}| = |v_{Cw}| = |v|$

$\vec{v}_{sg} = \vec{v}_{sw} + \vec{v}_{wg}$

Three swimmers can swim **equally fast** relative to the water. Relative to the water, Beth swims perpendicular to the flow, Ann swims upstream at 30 degrees, and Carly swims downstream at 30 degrees. They start the same time.

$\vec{v}_{wg} = v_{wg,x}\hat{i} + \underbrace{v_{wg,y}\hat{j}}_{0}$

$v_{sg,y} = v_{sw,y} + \underbrace{v_{wg,y}}_{0}$

Who crosses the river first?

A) Ann B) Beth C) Carly

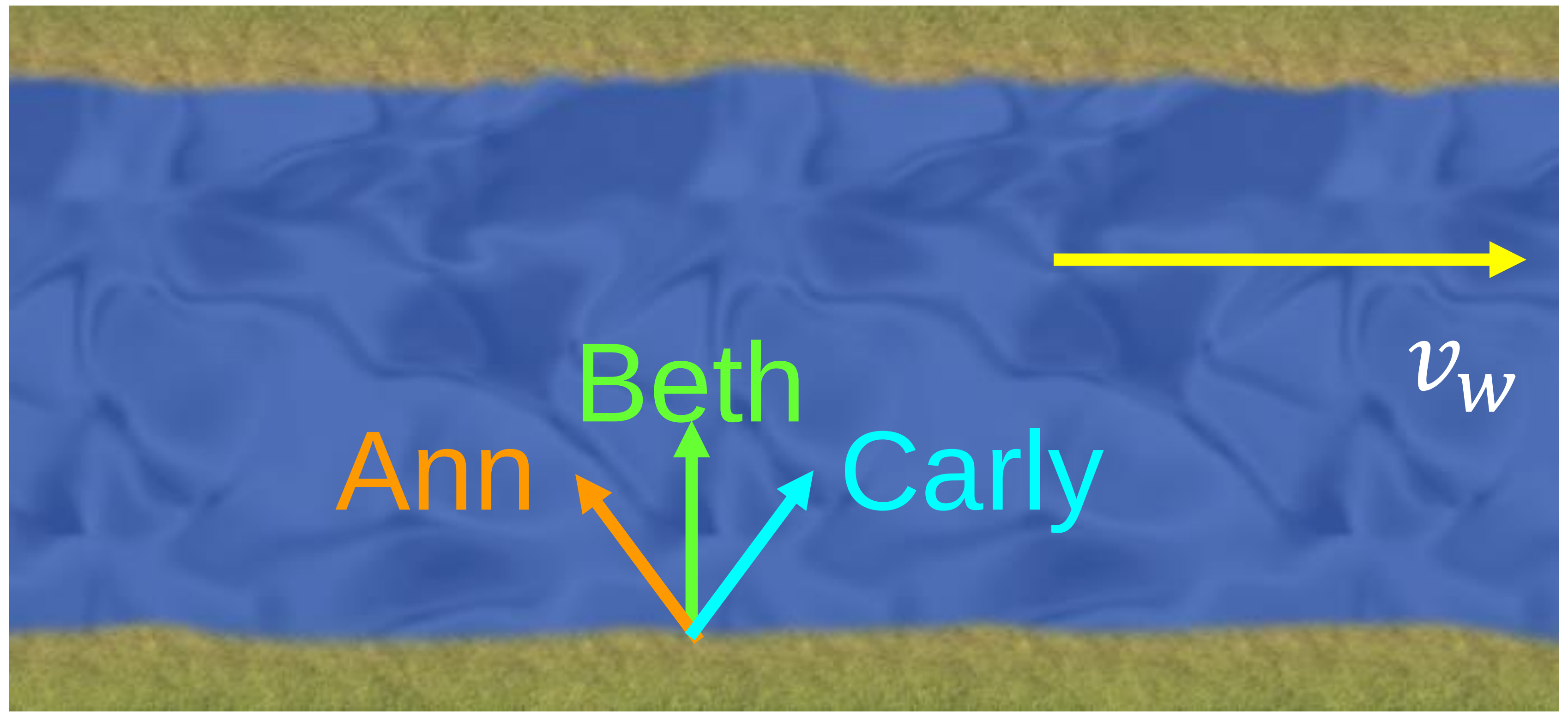
$\Delta y = v_y t$

$t = \dfrac{\Delta y}{v_y}$

$v_{Aw,y} = |v|\cos 30°$

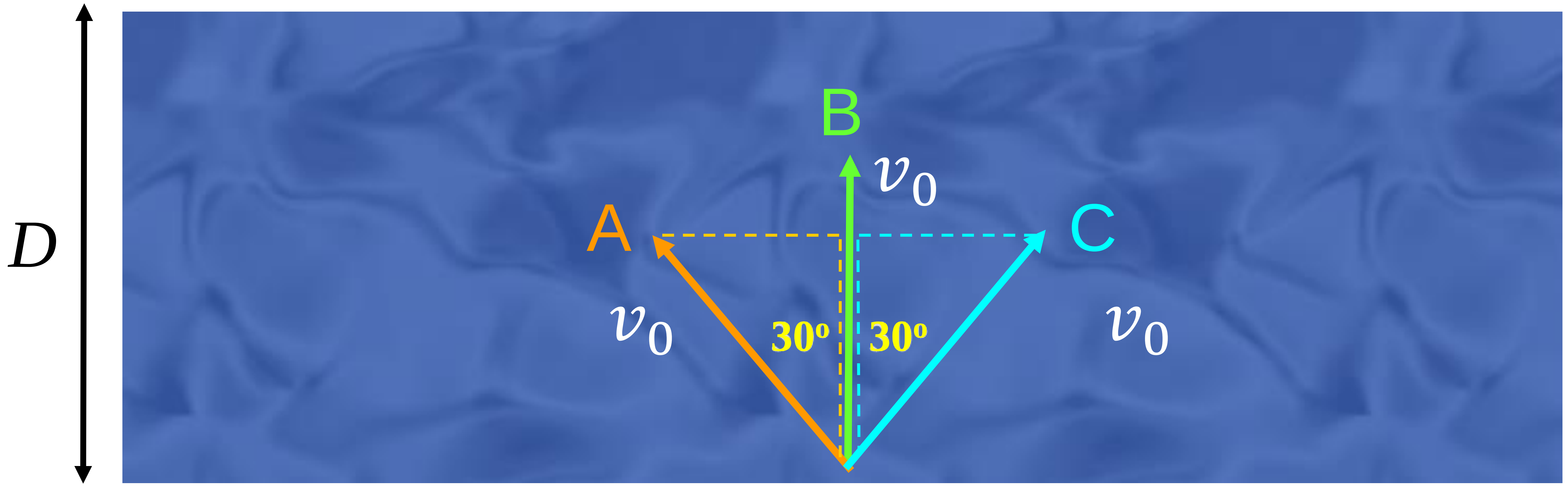
$v_{Cw,y} = |v|\cos 30°$

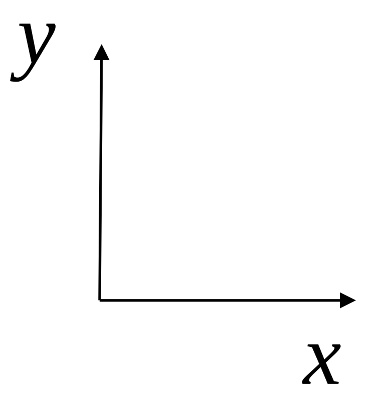
$v_{Bw,y} = |v|$



Time to get across = D / V_y

Look at just water & swimmers





$$V_{y,Beth} = V_o$$

$$V_{y,Ann} = V_o\cos(30^o)$$

$$V_{y,Carly} = V_o\cos(30^o)$$

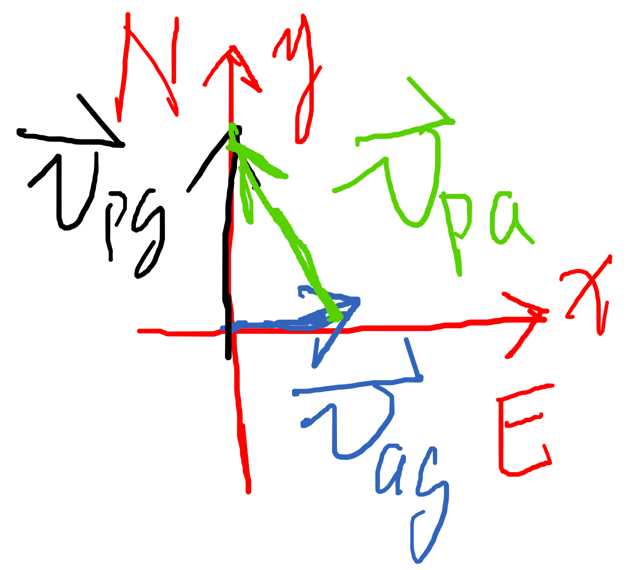
Width of river, Δy is the same for all.

$$t = \frac{\Delta y}{V_y}$$

Example

Given: $|\vec{v}_{pa}| = 550\,mph$, $\vec{v}_{ag} = 50\,mph\,\hat{i}$, $\Delta y = 1000\,mile\,\hat{j}$

Goal: $\vec{v}_{pg}$ $\qquad \vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$

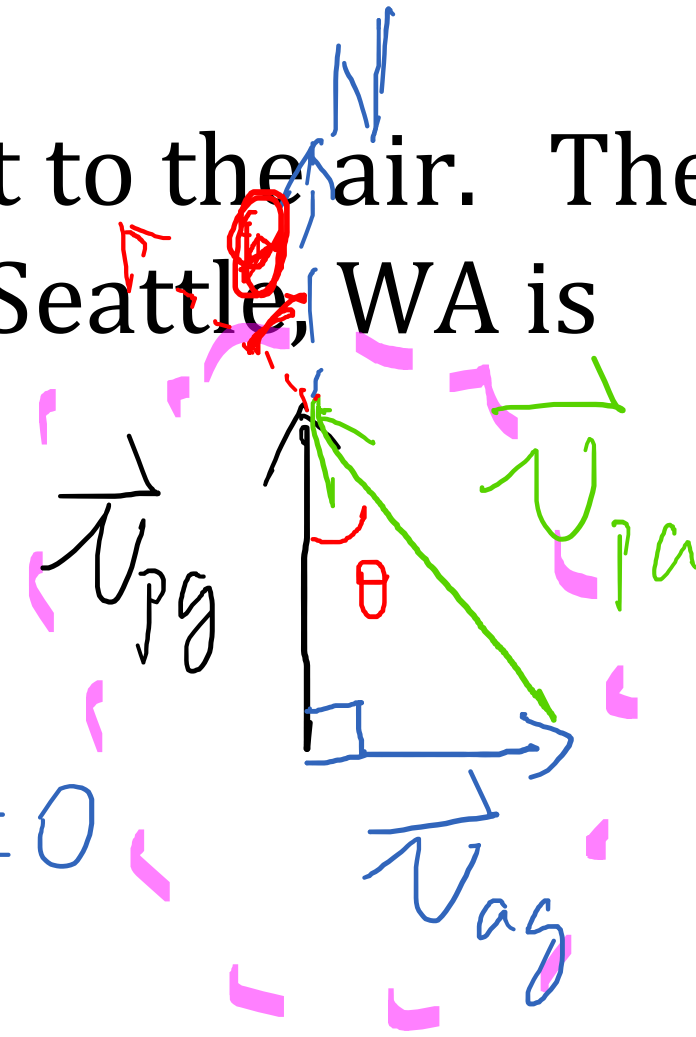


- A jet plane has an airspeed of 550 mph, i.e., it can travel 550 mph with respect to the air. The jet stream is blowing at 50 mph from west to east. The displacement from LA to Seattle, WA is approximately 1000 miles south to north.

air

$\vec{v}_{pa}$

Step1: $v_{pg,x} = 0$



$|\vec{v}_{pa}| = 550\,mph$

$= 50\,mph\,\hat{i}$

  - What heading should the pilot take to fly to Seattle w.r.t due north?
    Hint: make east-west velocity with respect to ground vanish:

  - Approximately how long does it take to fly from LA to Seattle?

$$\sin\theta = \frac{|\vec{v}_{ag}|}{|\vec{v}_{pa}|}$$

$$\theta = \arcsin\frac{|\vec{v}_{ag}|}{|\vec{v}_{pa}|} = \arcsin\frac{50\,mph}{550\,mph} \approx 5.2^\circ \text{ W of N}$$

Step2: $\Delta y = v_{pg,y}\, t \rightarrow t = \dfrac{\Delta y}{v_{pg,y}}$

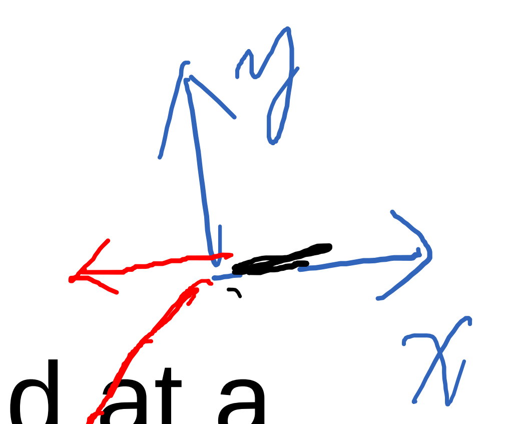
$$v_{pg} = \sqrt{(|\vec{v}_{pa}|)^2 - |\vec{v}_{ag}|^2} \approx 541\,mph$$

$$t = \frac{1000\,miles}{541\,mph} \approx 1.83\,hrs$$

Comprehensive example

Given: $\vec{v}_{hg} = 6.10\,m\,s^{-1}\,\hat{i}$, $\vec{y}_0 = 8.2\,m\,\hat{j}$

$v_{ph} = -12.0\,m\,s^{-1}\,\hat{i}$



Goal: $\vec{v}_{pg,0}$, Δx_{ph}

- A helicopter is flying in a straight line over a level field at a constant speed of 6.10 m/s and at a constant altitude of 8.20 m. A package is ejected horizontally from the helicopter with an initial velocity of 12.0 m/s relative to the helicopter and in a direction opposite the helicopter's motion.
  - **(a)** Find the initial speed of the package relative to the ground.
  - **(b)** What is the horizontal distance between the helicopter and the package at the instant the package strikes the ground?

Step1: $\vec{v}_{pg} = \vec{v}_{ph} + \vec{v}_{hg} = -12.0\,m s^{-1}\,\hat{i} + 6.10\,m s^{-1}\,\hat{i} = -5.9\,m s^{-1}\,\hat{i}$

$|v_{pg}| = 5.9\,m s^{-1}$

Step2: $v_{pg,y0} = 0$, $\Delta y = \underbrace{v_{pg,y0}}_{0}\, t - \frac{1}{2} g t^2 \rightarrow t = \sqrt{\frac{2\Delta y}{g}}$

$= \sqrt{\frac{-2(-8.2\,m)}{9.8\,m s^{-2}}} \approx 1.29\,s$

$\Delta x = |v_{pg,x}|\,t + |v_{hg,x}|\,t = (5.9\,m s^{-1} + 6.1\,m s^{-1}) \times 1.29\,s \approx 15.5\,m$

Step3: or: $\Delta x = v_{ph,0}\, t = 12.0\,m\,s^{-1} \times 1.29\,s \approx 15.5\,m$

Summary of chapter 4

- Practice on the concepts
  - Kinematics in two and three dimensions
  - Projectile motion: Use time, t, to connect motions in x- and y- dimensions
  - Uniform circular motion:
    - Tangential speed and angular speed: $v_t = \omega r$
    - Centripetal acceleration: $a_c = \omega^2 r = \frac{v^2}{r}$
  - Relative motion and reference frame: $\vec{v}_{AT} = \vec{v}_{AB} + \vec{v}_{BT}$

Homework

- Homework assignment for Chapter 4 in module 4.5: homework comprehensive, due in a week.