

PHYS 225

Fundamentals of Physics: Mechanics

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Fall 2024

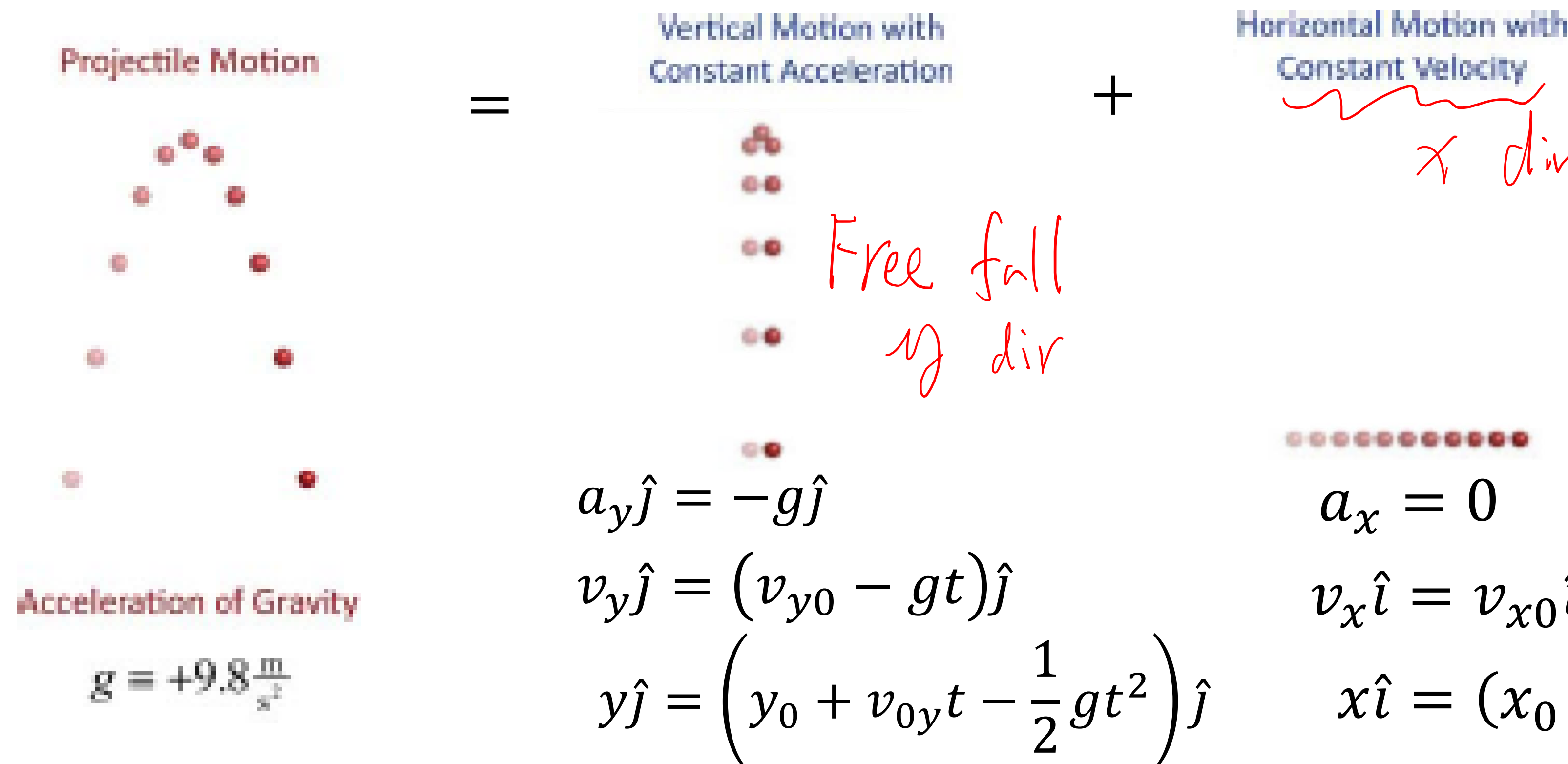
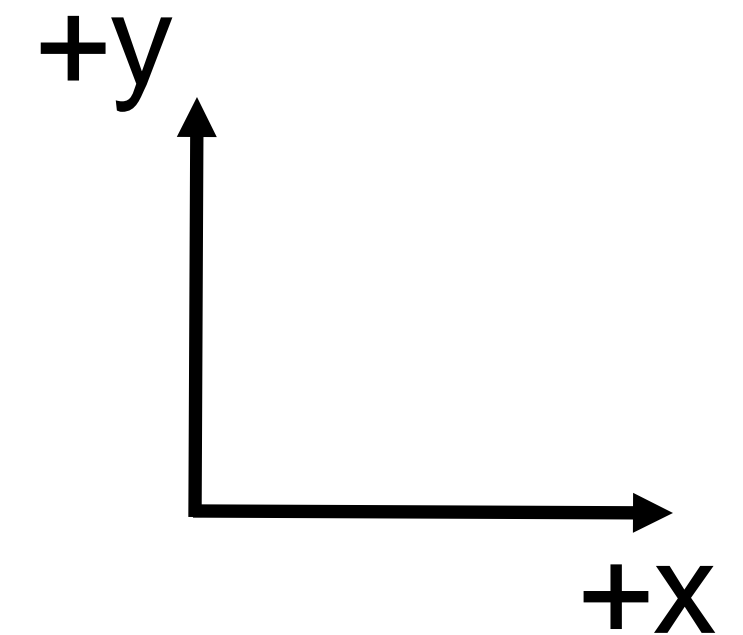
Lecture 9: Projectile motion: Examples

Learning goals

- Practice on projectile motion

Projectile motion — Decomposed

- Projectile motion is the superposition of two independent motions:
 1. Vertical motion: constant acceleration
 2. Horizontal motion: constant velocity



Vertical and horizontal motions are connected by the time!

Clicker question 4

- How are the horizontal and vertical components of the projectile motion related to each other?

A

The vertical component of the acceleration is the same as the horizontal component of the acceleration.

B

The two components of the motion share the same initial and final time.

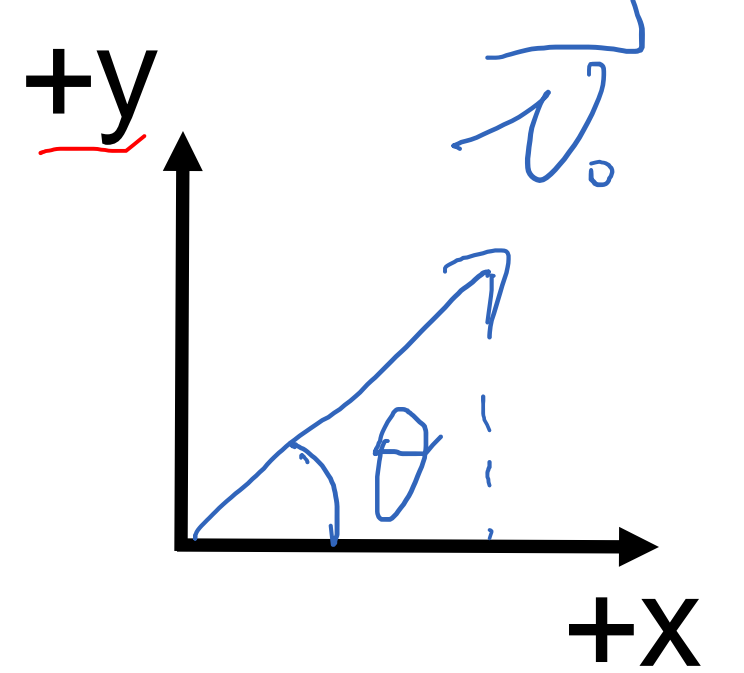
Example 2 (more details in Lecture 8)

Given: $|\vec{v}_0|$, \vec{a} , $\Delta y = 0$

Goal: θ s.t. $\Delta x = \max(\Delta x)$

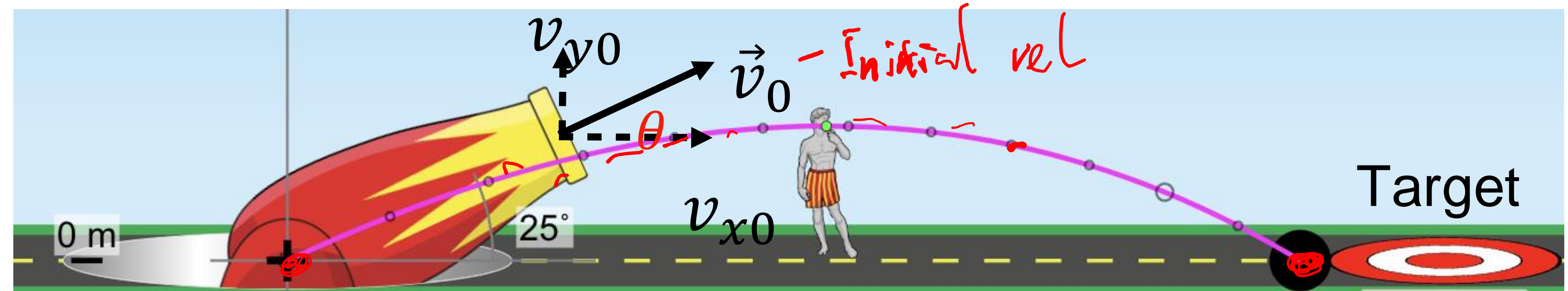
Δx ? θ

$$v_{y0} = |\vec{v}_0| \sin \theta$$



- The cannon is aimed at an angle θ above the +x direction at an initial speed of $|\vec{v}_0|$. Suppose the initial speed of the cannon, $|\vec{v}_0|$ is fixed. What is the aiming angle θ (the angle between \vec{v}_0 and +x direction) to make it farthest in the horizontal direction? (Assume the initial and final y-coordinates are the same.)

Step 1: $\Delta x = v_{x0} \Delta t$



Step 2: $\Delta y = v_{y0} \Delta t - \frac{1}{2} g \Delta t^2$

$\Delta y = 0$ Factor RHS

Step 3: Substitute Δt in Step 2 to Step 1,

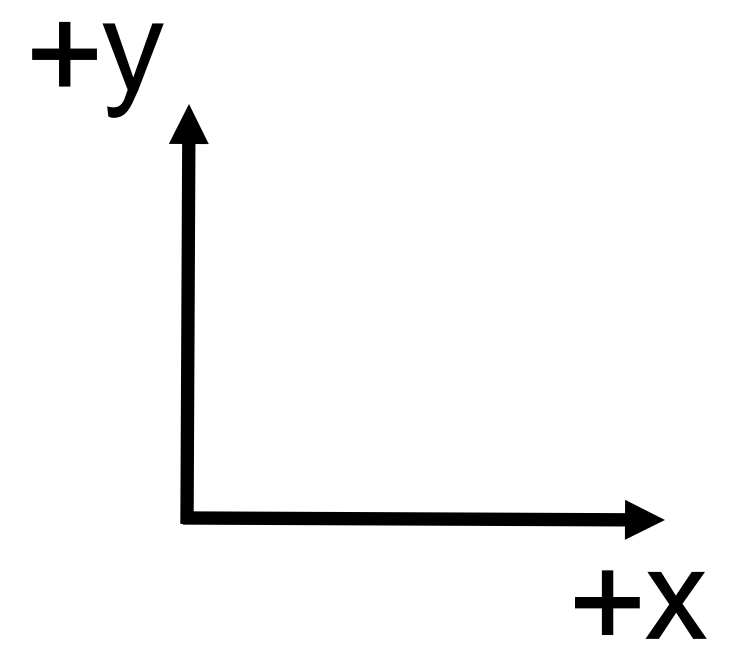
$$\Delta x = \frac{|\vec{v}_0|^2 \sin 2\theta}{g}$$

Combine Step 1 & 2

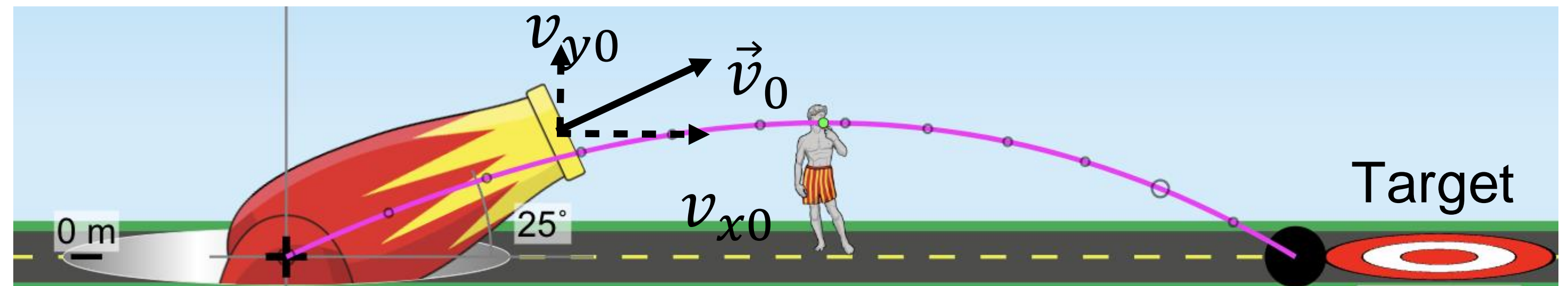
$\Delta x \propto \sin 2\theta$ when $\theta = 45^\circ$

For maximum Δx at a fixed $|\vec{v}_0|$, $\theta = 45^\circ$

Clicker question 5



- The cannon is aimed 25° above the $+x$ direction at an initial speed of $|\vec{v}_0|$, but it didn't make it to the target. Suppose the initial speed of the cannon, $|\vec{v}_0|$ is fixed. How would you adjust the aiming angle (the angle between \vec{v}_0 and $+x$ direction) to make it further in the horizontal direction? (Assume the initial and final y -coordinates are the same.)



A

Decrease the aiming angle

B

Increase the aiming angle

Clicker question 6



Given: Δx , \vec{a} , θ , $\Delta y = 0$

Goal: \underline{t}

$|\vec{v}_0|$?

- The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 13.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, which of the following equations is sufficient to find the time that the athlete is in air?

A

y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B

x- motion: $\Delta x = v_{0x}t$

C

Neither is sufficient. Both are needed.

Practice question (work in group of 2-3 people)

Given: Δx , \vec{a} , θ , $\Delta y = 0$

Goal: $|\vec{v}_0|$, t - Assume $t_0 = 0$

$|\vec{v}_0|$ unknown

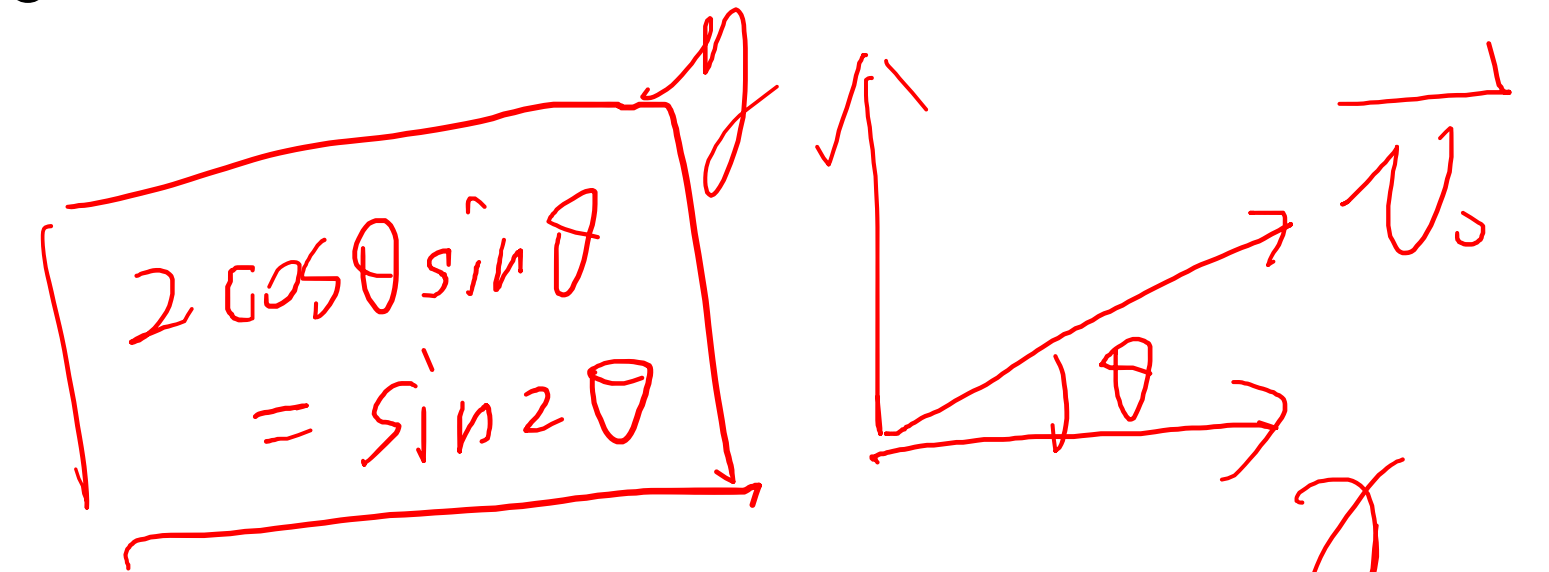
$$\vec{v}_0 = v_{x0} \hat{i} + v_{y0} \hat{j} = |\vec{v}_0| \cos \theta \hat{i} + |\vec{v}_0| \sin \theta \hat{j}$$

v_{x0} v_{y0}

- The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 13.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, a) determine his take-off speed, v_0 . b) how long (in time) is the jump?

Step 1: Set up the equation for the x-dir motion.

$$\Delta x = v_{x0} t = |\vec{v}_0| \cos \theta t$$



Step 2: Set up the equation for the y-dir motion.

$$\Delta y = v_{y0} t - \frac{1}{2} g t^2 \rightarrow 0 = v_{y0} t - \frac{1}{2} g t^2 = (v_{y0} - \frac{1}{2} g t) t$$

$$v_{y0} - \frac{1}{2} g t = 0 \rightarrow t = \frac{2 v_{y0}}{g} = \frac{2 |\vec{v}_0| \sin \theta}{g}$$

Step 3: Eliminate time and calculate the initial speed.

Substitute t to step 1

$$\Delta x = |\vec{v}_0| \cos \theta t = |\vec{v}_0| \cos \theta \frac{2 |\vec{v}_0| \sin \theta}{g} = \frac{|\vec{v}_0|^2 \sin 2 \theta}{g}$$

$g = 9.8 \text{ m s}^{-2}$

$$|\vec{v}_0| = \sqrt{\frac{\Delta x g}{\sin 2 \theta}} \approx 41.5 \text{ m s}^{-1}$$

$$t = \frac{\Delta x}{|\vec{v}_0| \cos \theta} \approx 1.91 \text{ s}$$

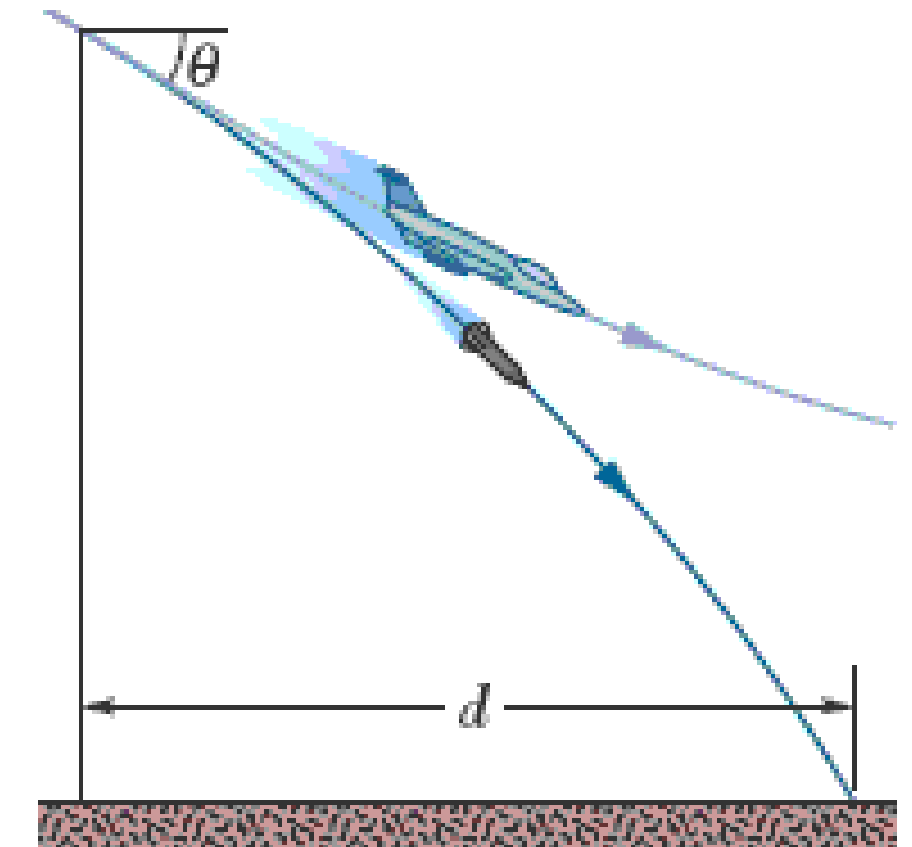
Practice question

- The current world-record motorcycle jump is 77.0 m, set by Jason Renie. Assume that he left the take-off ramp at 13.0° to the horizontal and that the take-off and landing heights are the same. Neglecting air drag, a) determine his take-off speed, v_0 . b) how long (in time) is the jump?

Clicker question 7

Given: $|v_0|$, θ , \vec{a} , Δx
Goal: t

- A certain airplane has a speed of 298.2 km/h and is diving at an angle of $\theta = 26.0^\circ$ below the horizontal when the pilot releases a radar decoy (see the figure). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 667$ m. To find how long (in time) the decoy is in the air, which of the following equation is used?



A

y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B

x- motion: $\Delta x = v_{0x}t$

Reminders

- Pre-lecture survey 4.1.2 before the next class
- Midterm 1:
 - Date: Oct. 17
 - Chapters: 1 to 5, any content covered before the exam
 - Same time and location as regular classes