PHYS 225
Fundamentals of Physics: Mechanics

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Fall 2024

Lecture 19: Force and motion II: Frictions on inclines

Learning goals for today

- Examples for friction:
  - Last class: Flat surfaces
  - Today: Inclines

Steps to calculate a friction force

If $\vec{v}_0 \neq 0$, $\vec{f}_k$

- **Step 1:** Determine whether it's static friction or kinetic friction
  - Static friction: **Tendency** to slide over each other, but **don't slide** $\vec{f}_s$
  - Kinetic friction: **Slide** over each other $\vec{f}_k$

If $\vec{v}_0 = 0$,

1. Driving force $> |f_{s,max}|$ then sliding then $\vec{f}_k$
2. Driving force $\leq |f_{s,max}|$ then $\vec{f}_s$

- **Step 2**: Direction
  - The direction of friction force **opposes** sliding or the tendency of sliding
- **Step 3**: Magnitude
  - Static friction: $|\vec{f}_s| \leq \mu_s |\vec{N}|$ or Newton's laws
  - Kinetic friction: $|\vec{f}_k| = \mu_k |\vec{N}|$

Clicker question 8

Emily of mass m sits on a surface with incline angle θ, static friction coefficient, μ_s, and kinetic friction coefficient, μ_k. Which of the following is true?

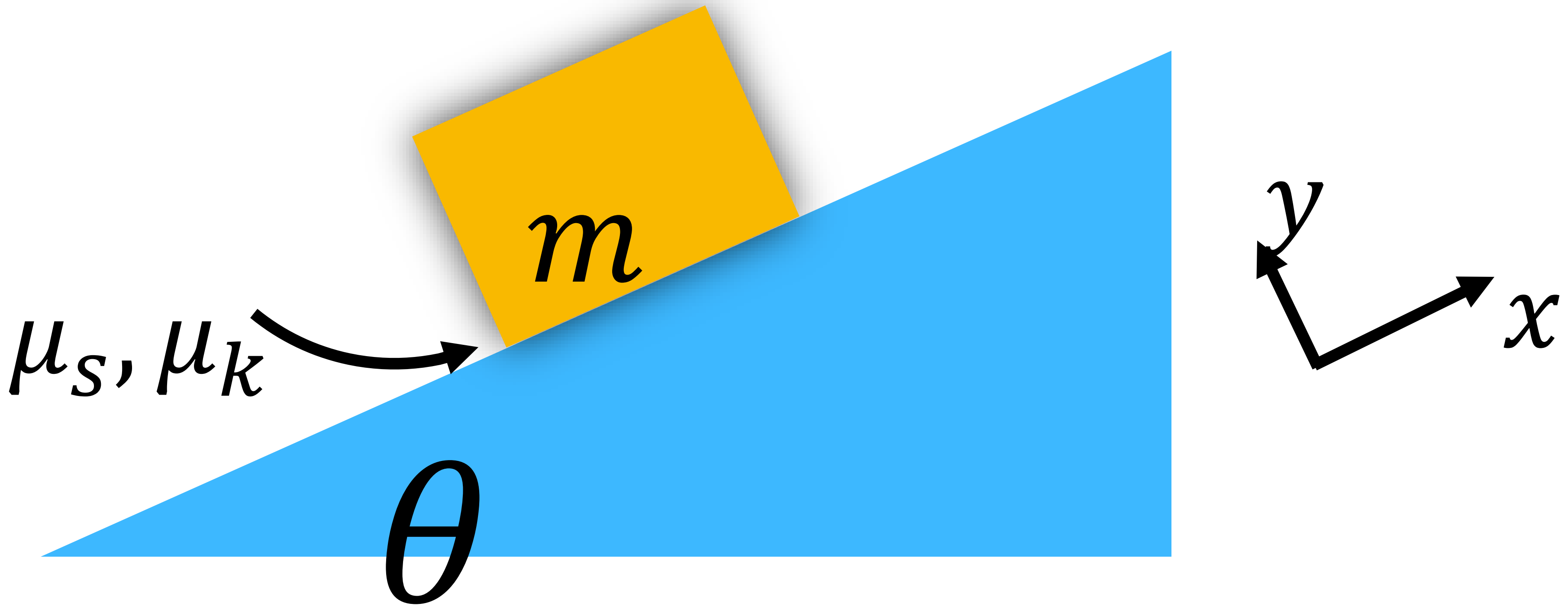
A She will slide uphill.

B She will slide downhill.

C Whether she will slide uphill, downhill or remain at rest depends on her mass.

D Whether she will slide uphill, downhill or remain at rest depends on θ, μ_s and her initial velocity.

https://www.gettyimages.dk/photos/



Incline examples 1 & 2

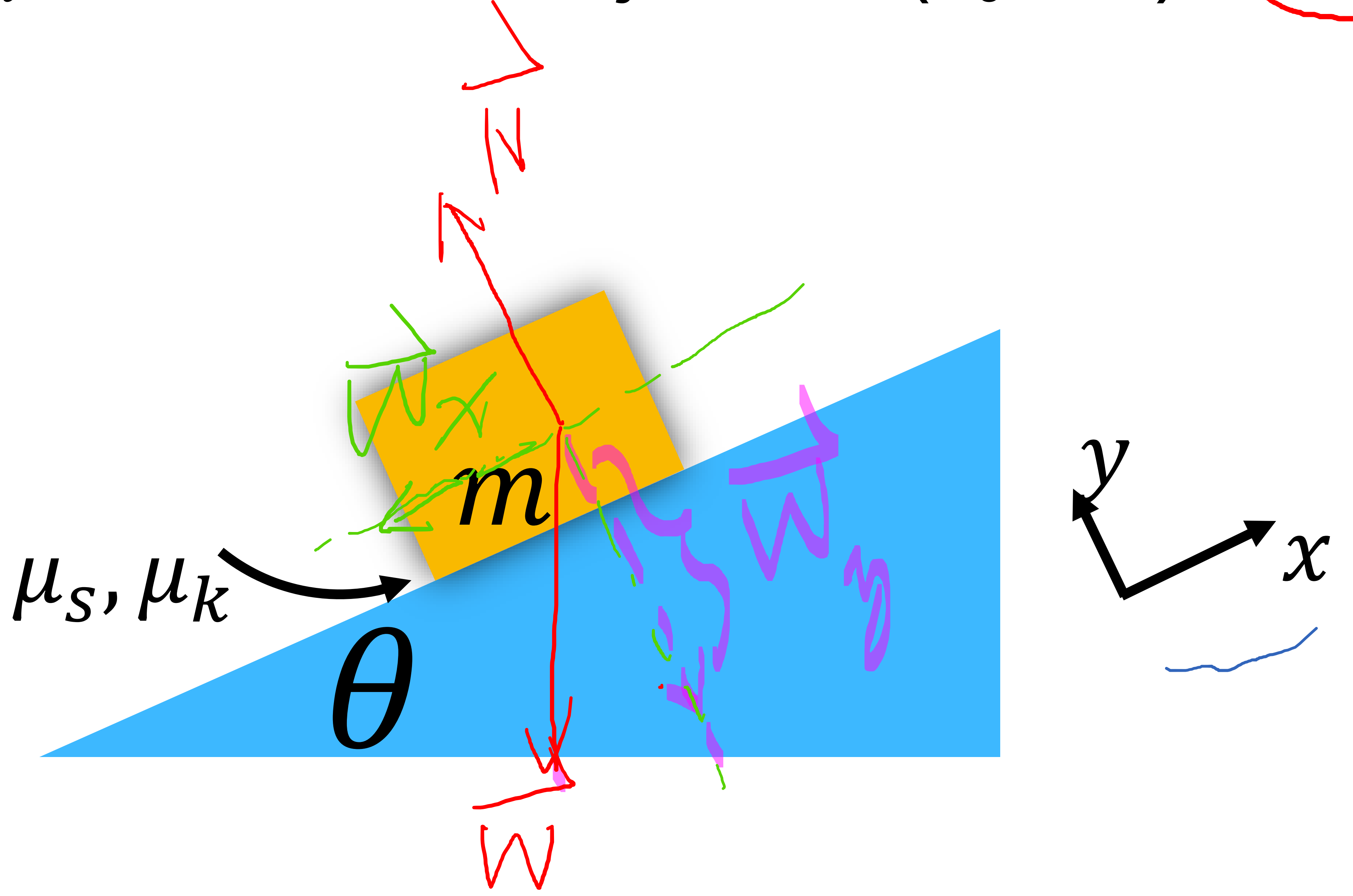
A box of mass m is laid on a surface with incline angle θ, static friction coefficient, μ_s, ~~and~~ kinetic friction coefficient μ_k , ~~the box~~ is initially at rest ($v_0 = 0$). ~~Let's work~~ on the friction.

FBD

Step 1: Axes

Step 2: Forces

Step 3: W_x & W_y



Driving force $\vec{W_x}$

Example 1:

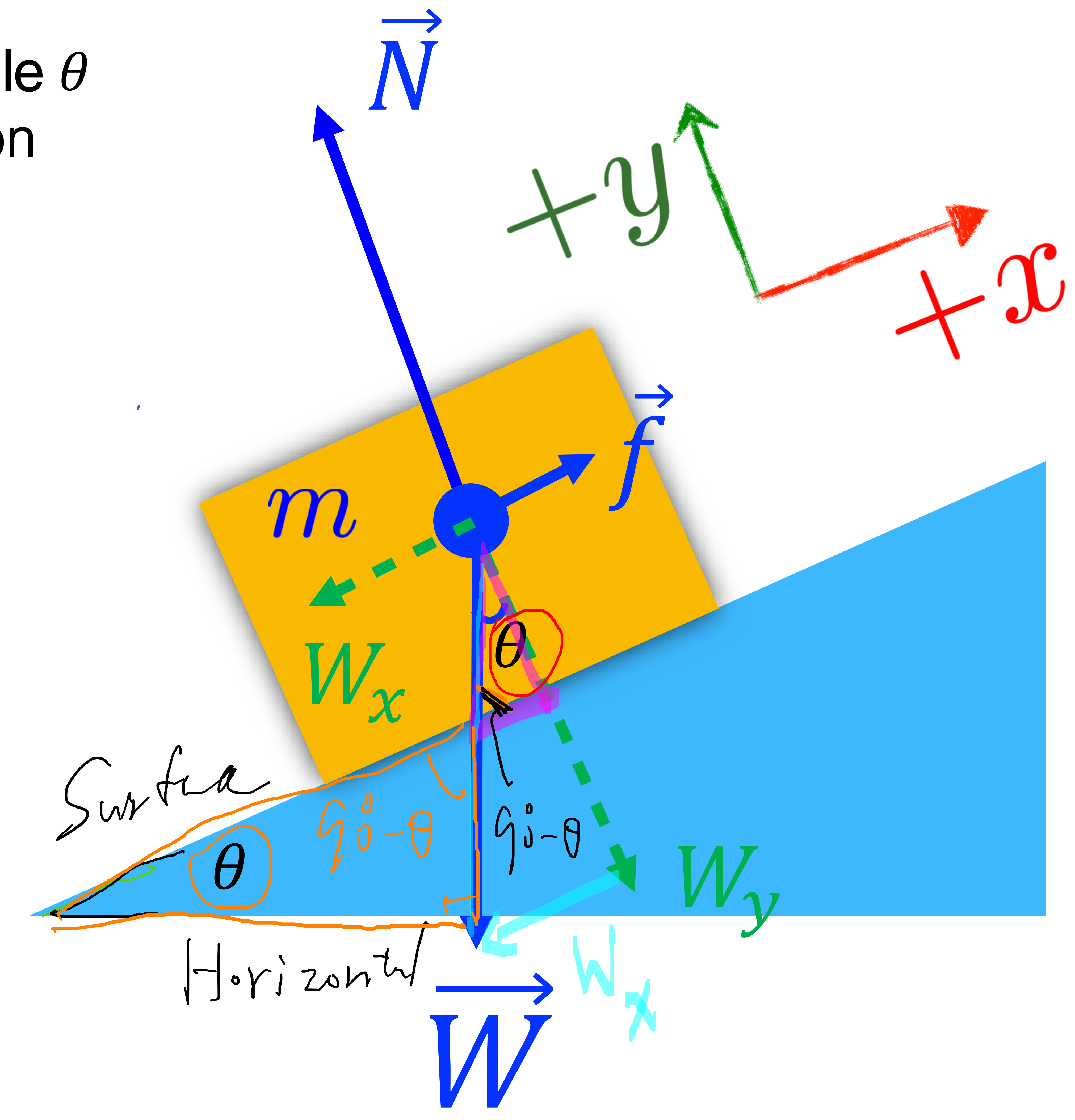
$\theta = 15°$, $\mu_s = 0.3$, $v_0 = 0$

Example 2:

$\theta = 30°$, $\mu_s = 0.3$, $\mu_k = 0.2$, $v_0 = 0$

Clicker question 9

A box of mass $m = 1$kg is laid on a surface with incline angle $\theta = 30°$, static friction coefficient, $\mu_s = 0.3$, and kinetic friction coefficient $\mu_k = 0.2$, If the box is initially at rest ($v_0 = 0$),
Which of the following is true regarding the x- and y-components of weight, $\vec{W}$?



A $W_x = |\vec{W}|\cos\theta,\ W_y = |\vec{W}|\sin\theta$

B $W_x = -|\vec{W}|\sin\theta,\ W_y = -|\vec{W}|\cos\theta$

C $W_x = -|\vec{W}|\cos\theta,\ W_y = -|\vec{W}|\sin\theta$

D $W_x = |\vec{W}|\sin\theta,\ W_y = -|\vec{W}|\cos\theta$

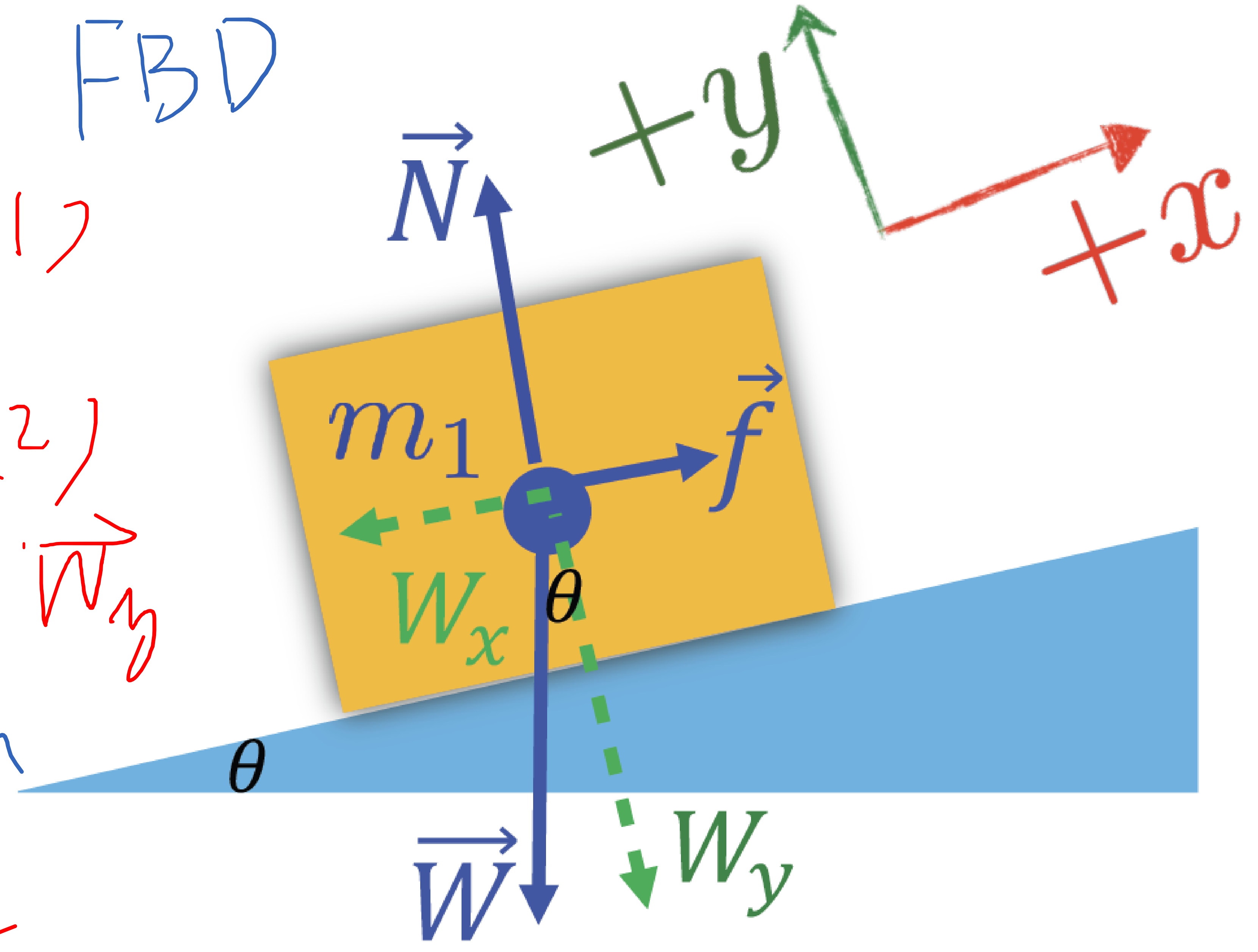
Example 2: Incline 1

In p.6

$W_x = -mg\sin\theta$
$W_y = -mg\cos\theta$

A box of mass $m = 1kg$ is laid on a surface with incline angle $\theta = 15°$, static friction coefficient, $\mu_s = 0.3$, and kinetic friction coefficient $\mu_k = 0.2$, If the box is initially at rest ($v_0 = 0$), will it start sliding? What is the friction force on the box?

Step 0: FBD



Given:

$\theta = 15°$, $\mu_s = 0.3$, $\mu_k = 0.2$, $v_0 = 0$

Step 1: Newton's 2nd law:

$$x: \vec{f} + \vec{W}_x = m\vec{a}_x \quad (1)$$
$$y: \vec{N} + \vec{W}_y = 0 \quad (2)$$

$\vec{N} = -\vec{W}_y$

Step 2: $|\text{Driving force}| \geq = < |f_{s,max}|$

$\vec{W}_x = -mg\sin\theta\,\hat{i} = -mg\sin 15°\,\hat{i} \approx -0.259\,mg\,\hat{i}$

$|\vec{f}_{s,max}| = \mu_s|\vec{N}| = \mu_s|-mg\cos\theta| = 0.3\cdot mg\cos 15° \approx 0.290\,mg$

$|\vec{W}_x| < |\vec{f}_{s,max}|$ $\therefore$ No sliding $\therefore \vec{f} = \vec{f}_s$

$a = 0$

Step 3: (1) $\vec{f}_s + \vec{W}_x = 0 \rightarrow \vec{f}_s = -\vec{W}_x = -(-mg\sin\theta\,\hat{i}) = mg\sin\theta\,\hat{i} = 1kg\cdot 9.8ms^{-2}\sin 15°\,\hat{i} \approx 2.54N\,\hat{i}$

Example 2: Incline 2

$W_x = -mg\sin\theta$

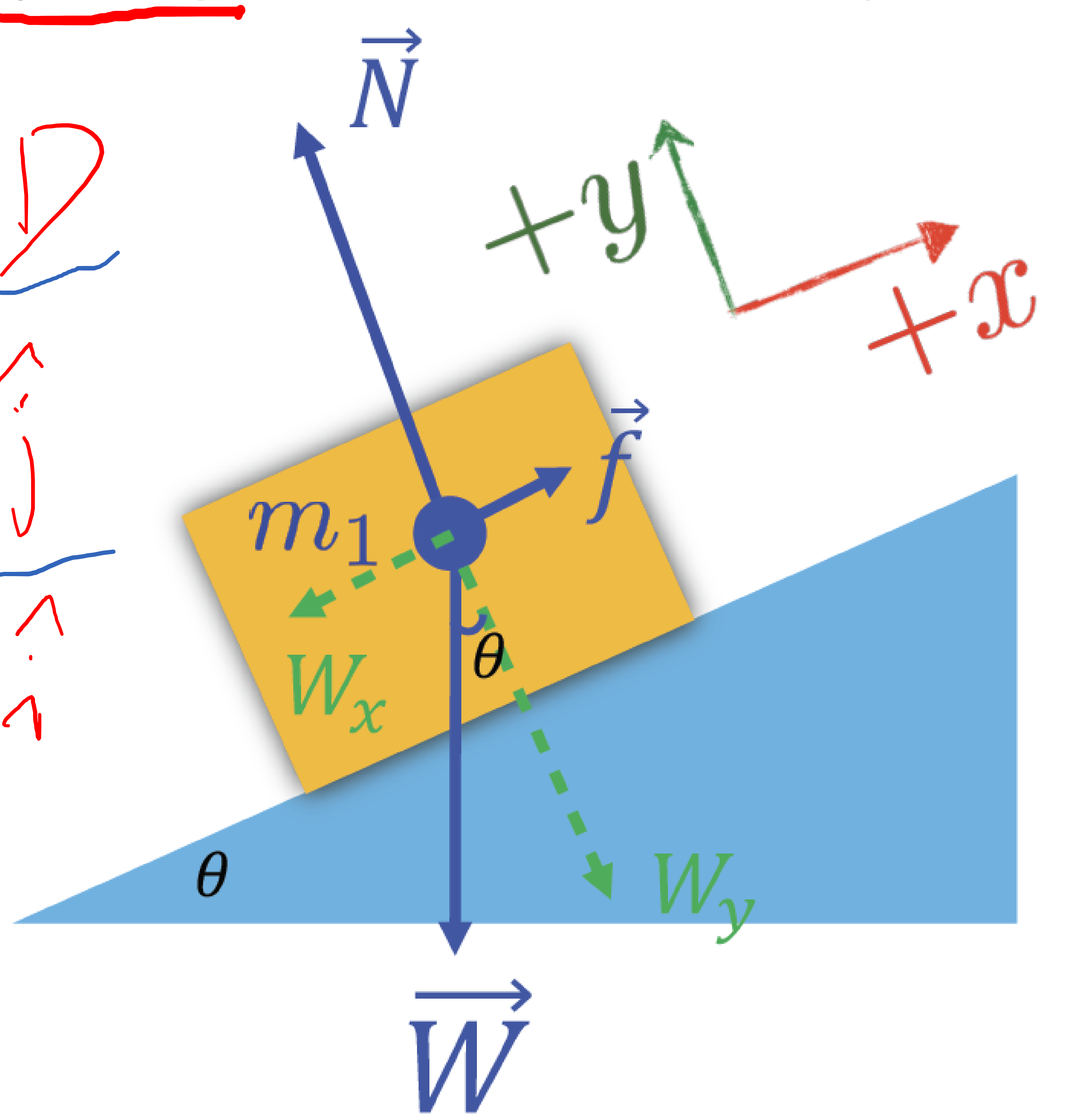
$W_y = -mg\cos\theta$

A box of mass $m = 1kg$ is laid on a surface with incline angle $\theta = 30°$, static friction coefficient, $\mu_s = 0.3$, and kinetic friction coefficient $\mu_k = 0.2$, If the box is initially at rest ($v_0 = 0$), will it start sliding? What is the friction force on the box?

Given:

$\theta = 30°$, $\mu_s = 0.3$, $\mu_k = 0.2$, $v_0 = 0$

Step 0: FBD



Step 1: 2nd law: x: $\vec f + \vec W_x = m\vec a_x$ (1)

y: $\vec N + \vec W_y = 0$ (2) $\rightarrow$ $\vec N = -\vec W_y = mg\cos\theta\,\hat j$

Driving force $\vec W_x = -mg\sin\theta\,\hat i$

$= -mg\sin 30°\,\hat i$

$= -0.5\,mg\,\hat i$

Step 2: $|W_x| \gtreqless |f_{s,max}|$

$|f_{s,max}| = \mu_s|\vec N| = \mu_s mg\cos\theta = 0.3\,mg\cos\theta \approx 0.26\,mg$

$|W_x| > |f_{s,max}|$ Sliding $\therefore \vec f = \vec f_k$

Step 3: $|\vec f_k| = \mu_k|\vec N| = \mu_k mg\cos\theta = 0.2\times 1kg\times 9.8ms^{-2}\cos 30° \approx 1.7N$

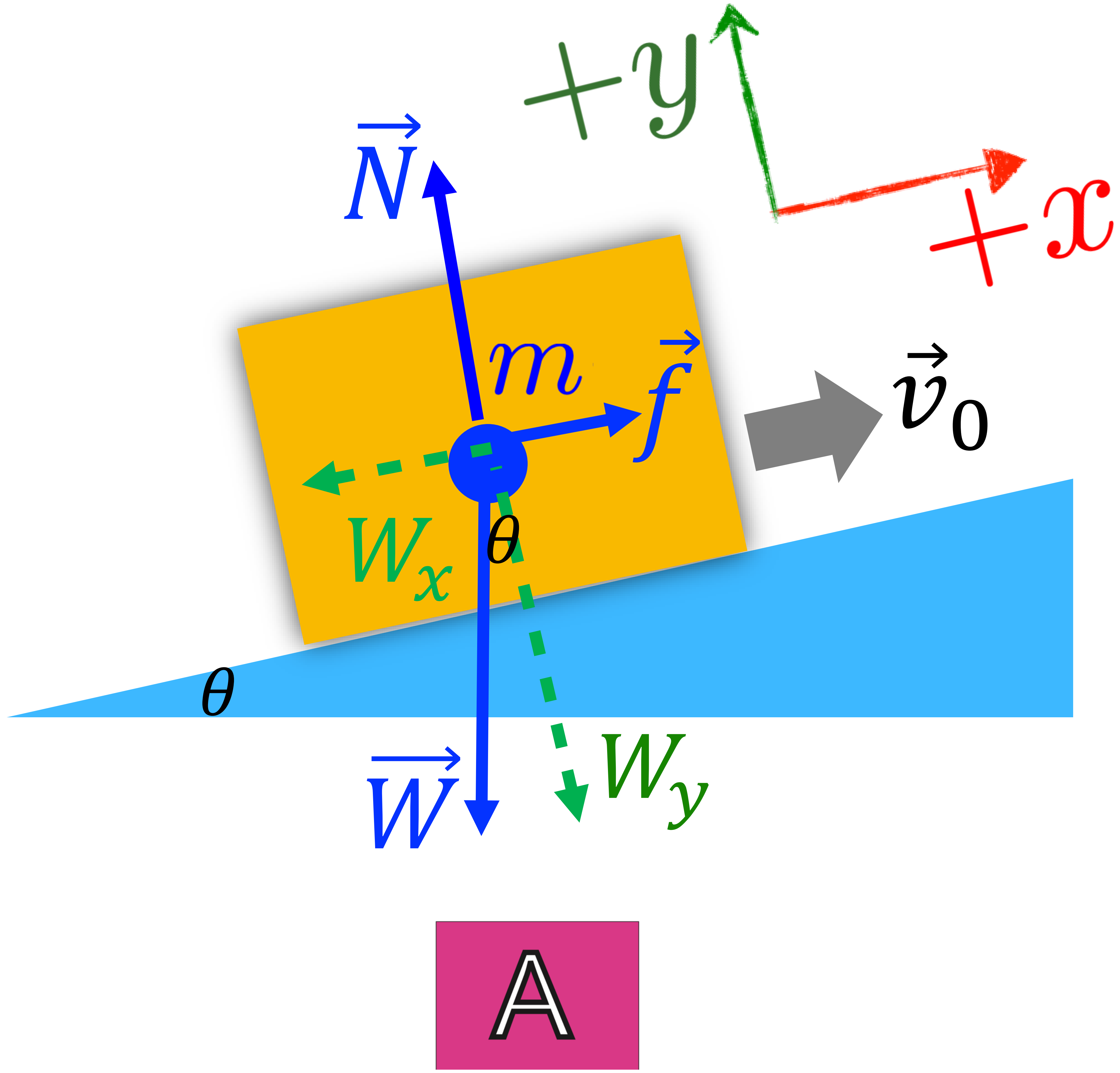
$\vec f_k = 1.7N\,\hat i$

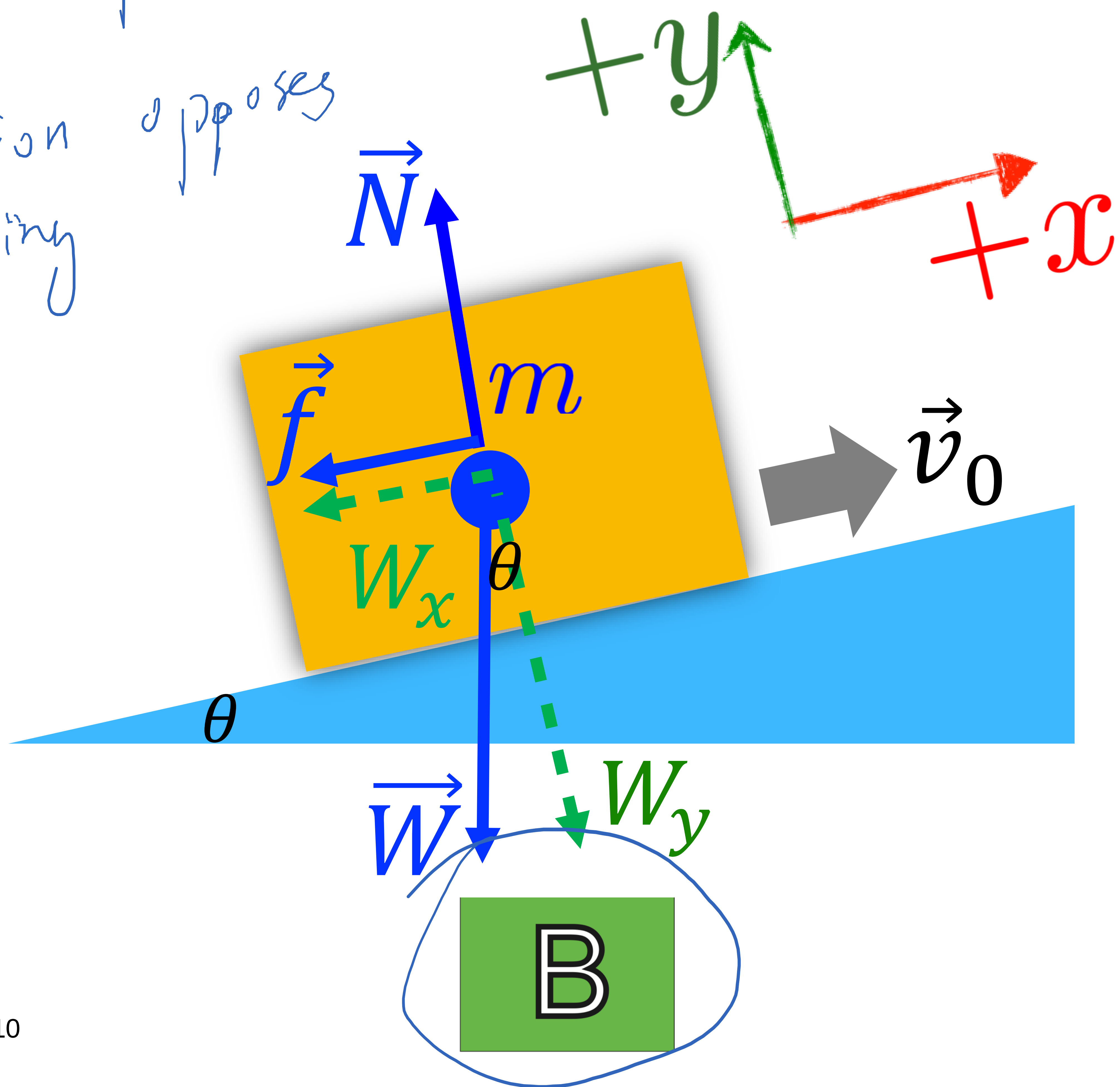
Clicker question 10 (Example 3)

A box of mass $m = 1\text{kg}$ is moving up ($v_0 > 0$) along a surface with incline angle $\theta = 15°$, static friction coefficient, $\mu_s = 0.3$, and kinetic friction coefficient $\mu_k = 0.2$. **What is the free body diagram on the box?**

Sliding up

friction opposes sliding





Example 3 (group activity): Incline 1, $v_0 > 0$

A box of mass $m = 1$kg is moving up ($v_0 > 0$) along a surface with incline angle $\theta = 15°$, static friction coefficient, $\mu_s = 0.3$, and kinetic friction coefficient $\mu_k = 0.2$. What is the friction force on the box?

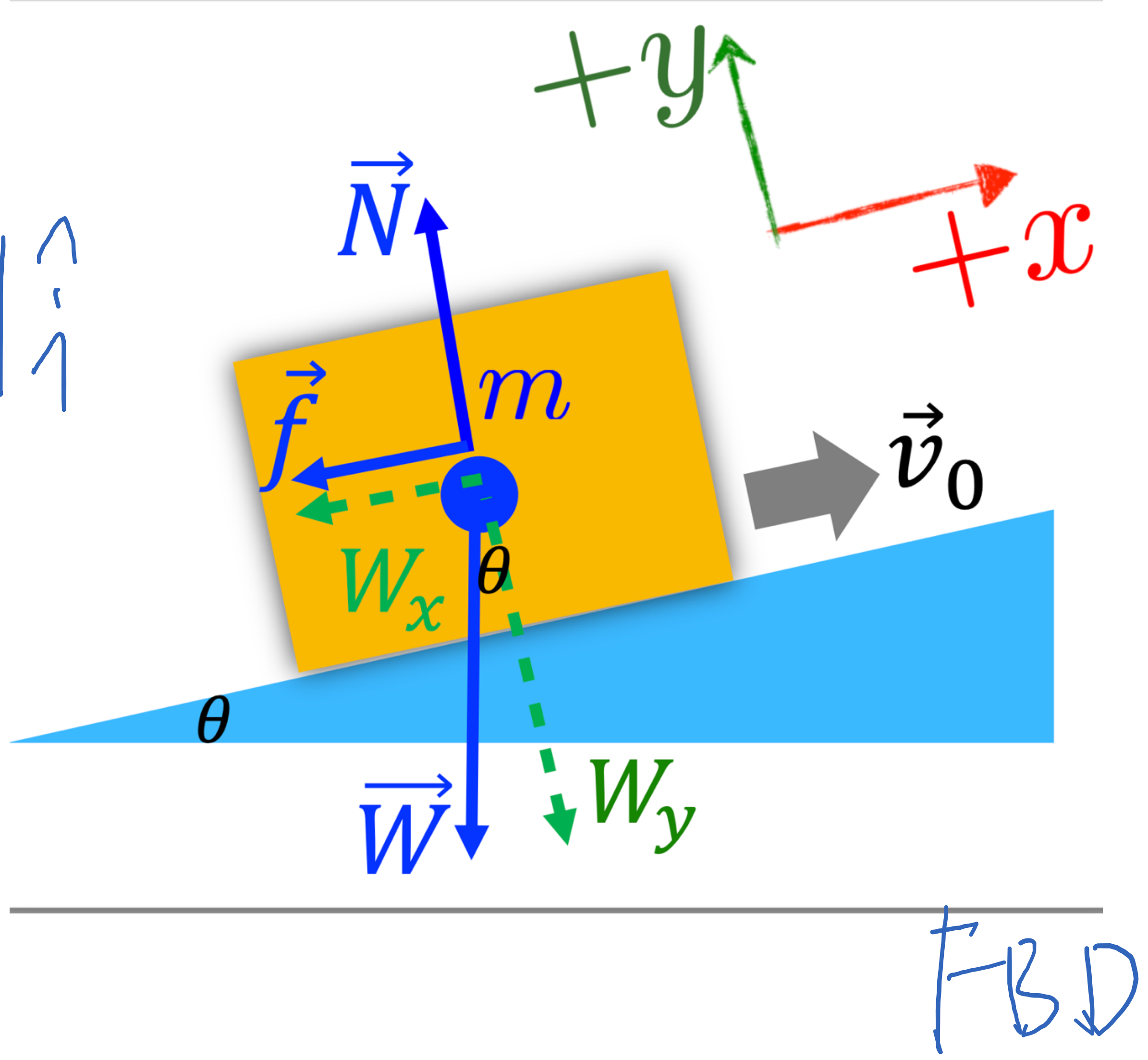
Given:

$\theta = 15°$,$\mu_s = 0.3$, $\mu_k = 0.2$,$v_0 > 0$

Sliding up

Step 1: Given $v_0 > 0$, is the friction $\vec{f}_s$ or $\vec{f}_k$? What's its direction?

$\vec{f}_k = -|\vec{f}_k|\hat{i}$



Step 2: Newton's 2nd law in x- and y- directions:

$$\begin{cases} \vec{f} + \vec{W}_x = m\vec{a}_x \\ \vec{N} + \vec{W}_y = 0 \end{cases} \rightarrow \vec{N} = -\vec{W}_y = -(-mg\cos\theta\hat{j}) = mg\cos\theta\hat{j}$$

Step 3: Calculate the normal force:

$$\vec{N} = 1\text{kg} \cdot 9.8\text{m s}^{-2} \cos 15° \hat{j} \approx 9.46\text{N}\hat{j}$$

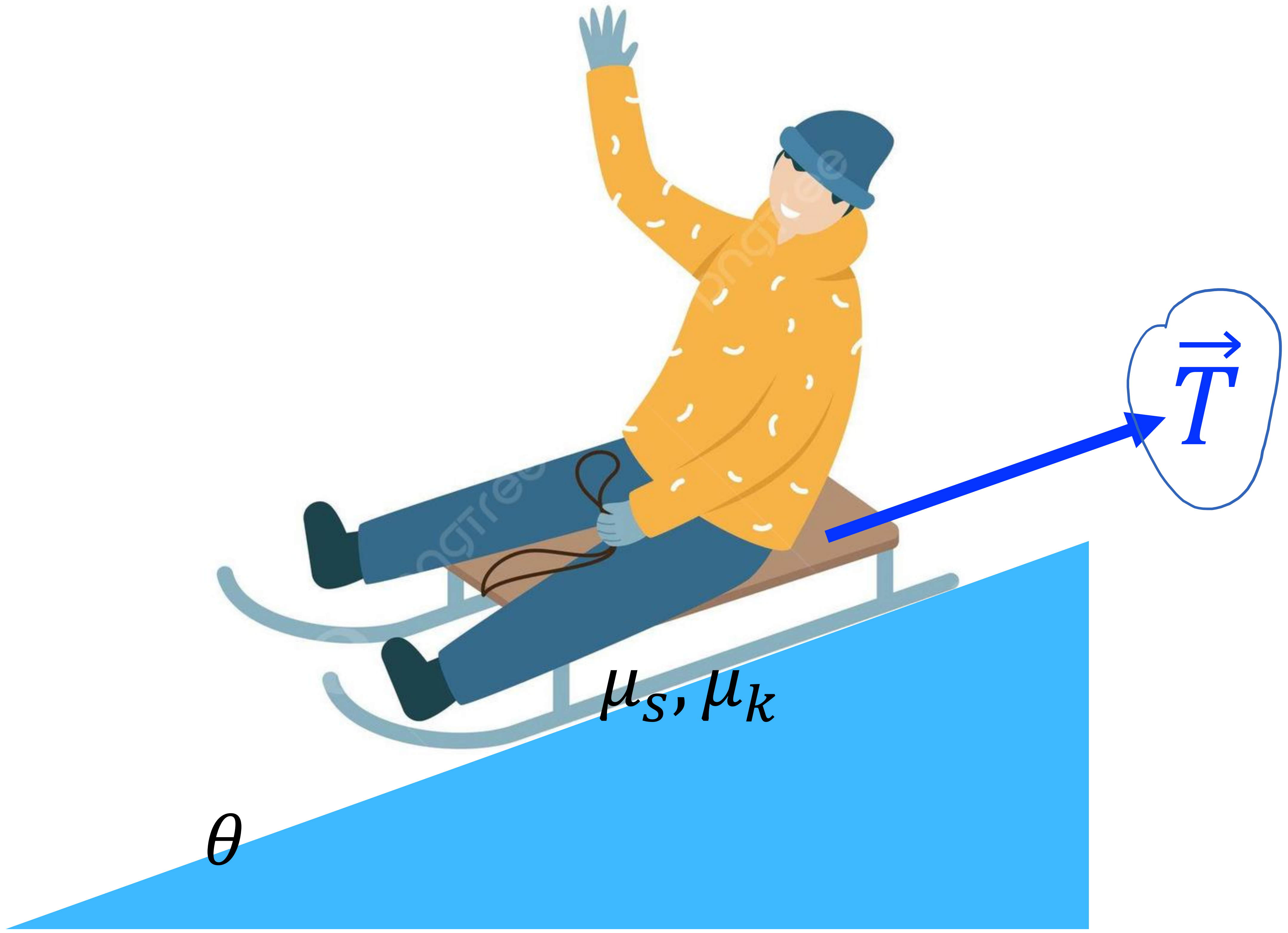
Step 4: Calculate the friction force.

$$|\vec{f}_k| = \mu_k|\vec{N}| = 0.2 \cdot 9.46\text{N} \approx 1.89\text{N}$$

$$\vec{f}_k = -1.89\text{N}\hat{i}$$

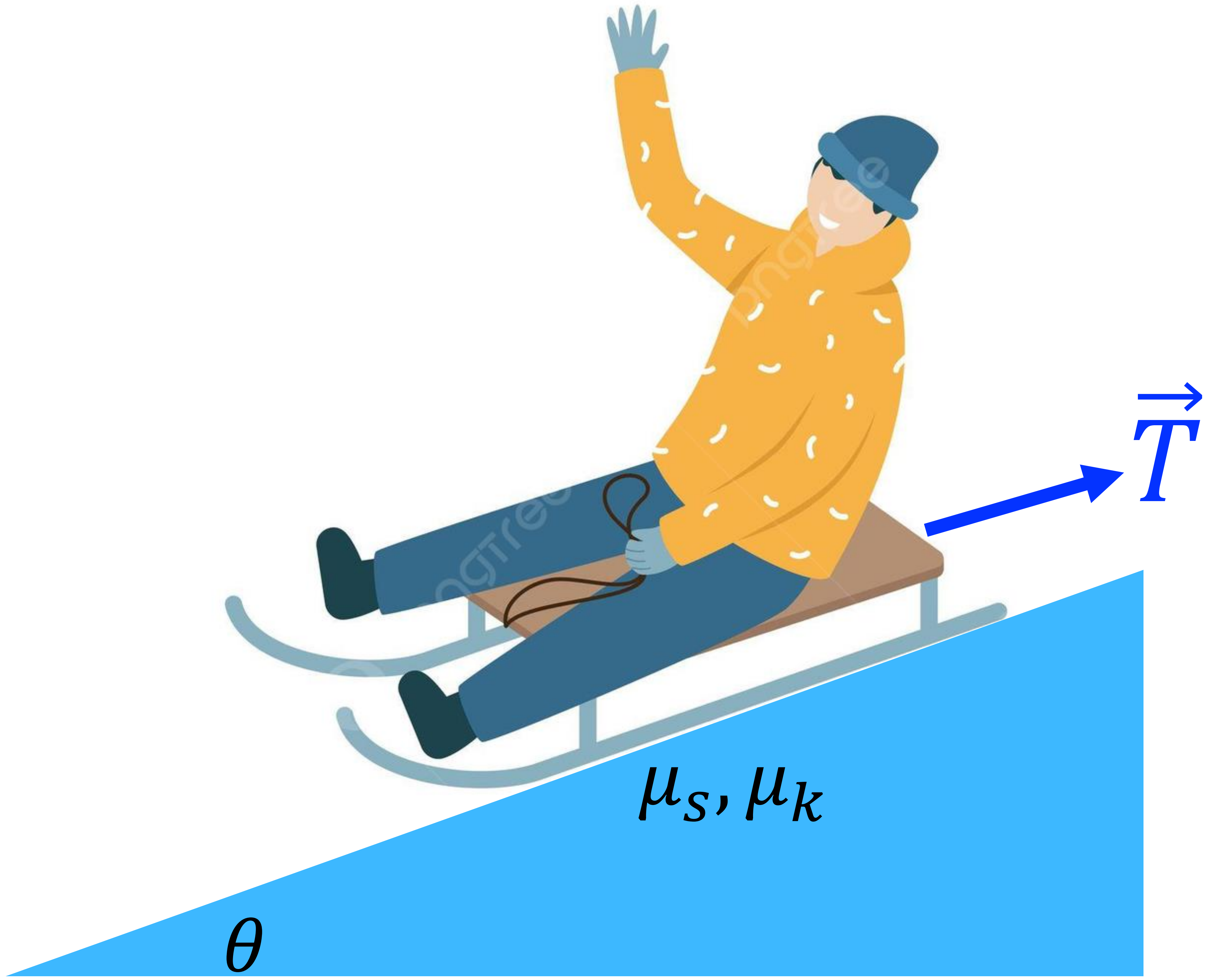
Example 4: More forces on the incline

- A sledge on slope is pulled by a tension force along the slope.



Clicker question 11

- A sledge (including the loading) on is pulled by a tension force. What drives the sledge to slide?



A The tension force, $\vec{T}$.

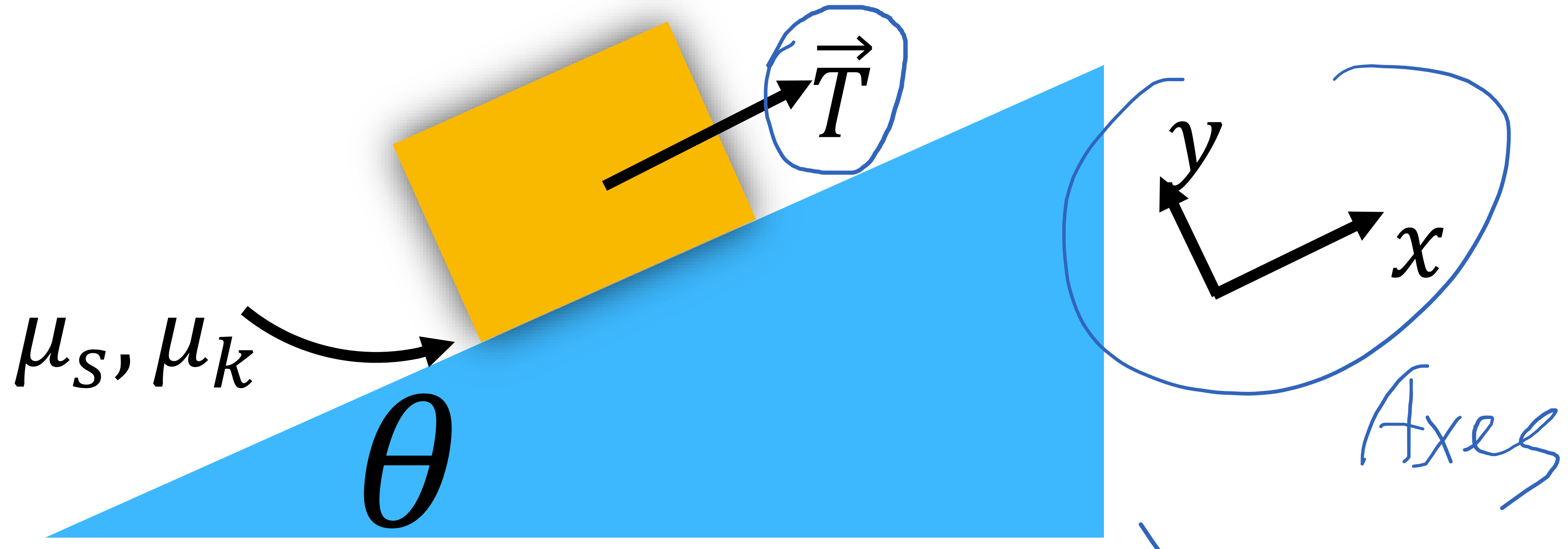
B The component of the weight along the slope, $\overrightarrow{\mathrm{W}}_{\mathrm{x}}$.

C The vector sum of $\vec{T}$ and $\overrightarrow{\mathrm{W}}_{\mathrm{x}}$.

Incline example 4

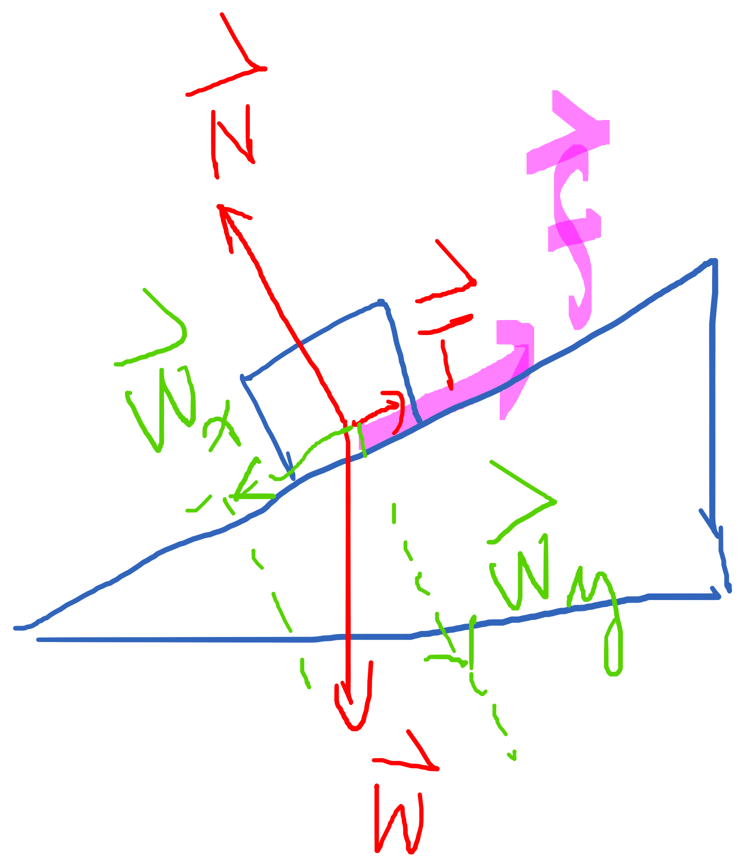
Given: $m = 1kg, \mu_s, \mu_k, \theta, v_0 = 0, \vec{T}$...... ,

Goal: Friction, $\vec{f}$, does it slide?



- FBD

Condition 1: Small $\vec{T}$



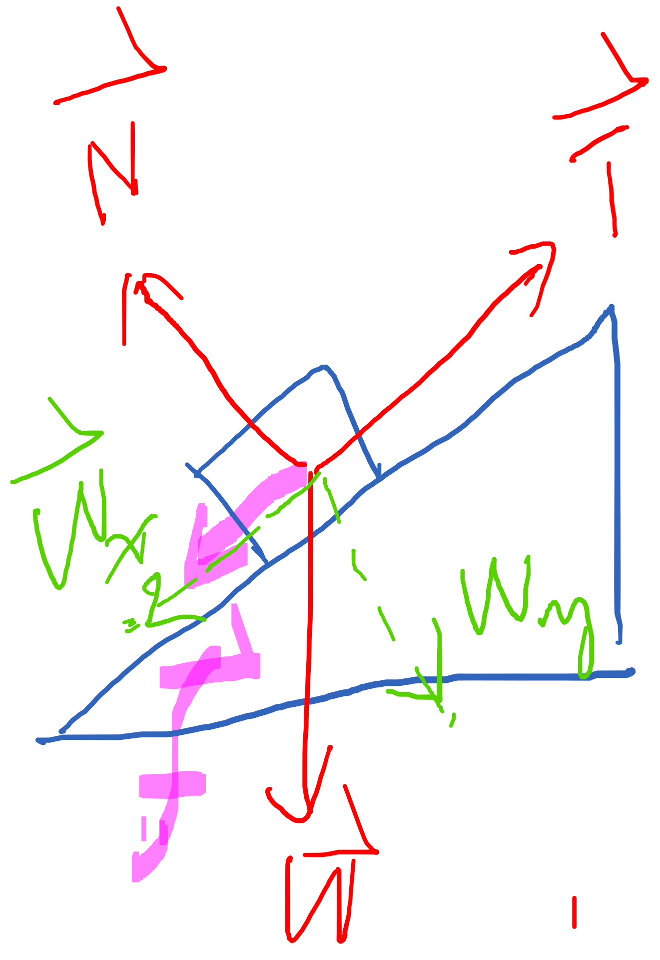
$\vec{W}_x + \vec{T} < 0$

Tending to slide down.

Friction opposes sliding

$\vec{f} > 0$

Cond. 1: Large $\vec{T}$

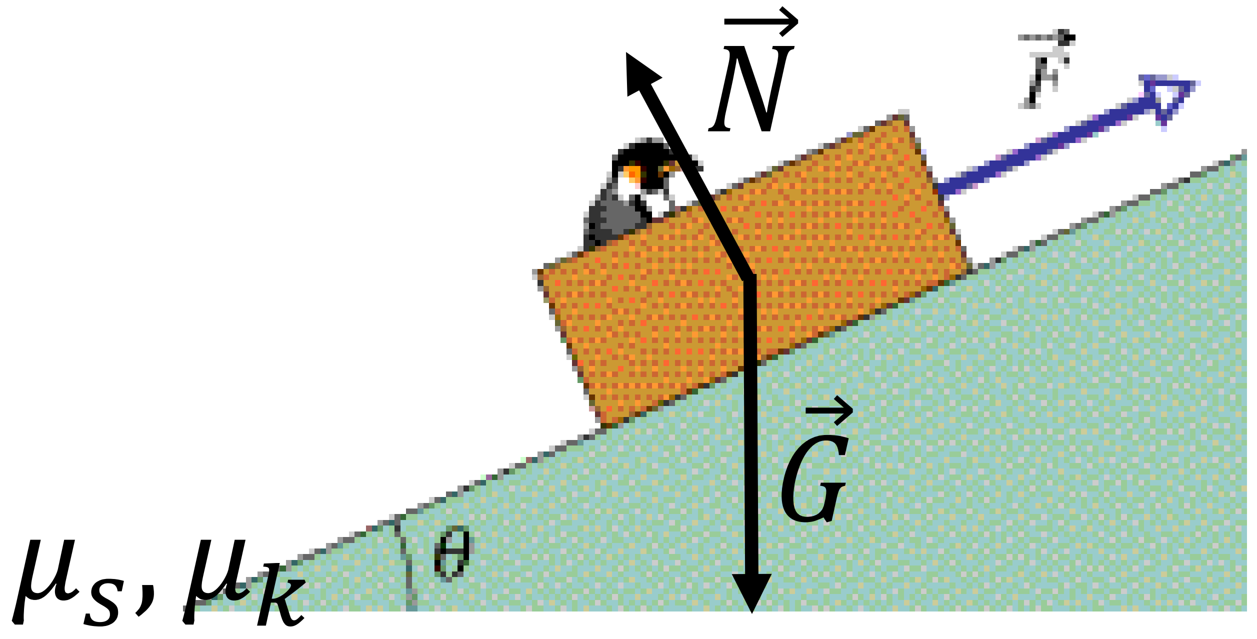


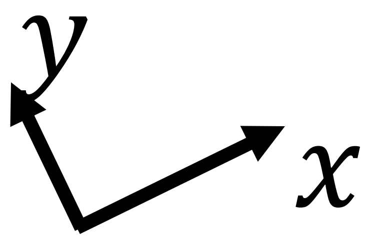
$\vec{W}_x + \vec{T} > 0$

Tending to slide up

$\therefore \vec{f} < 0$

Clicker question 12

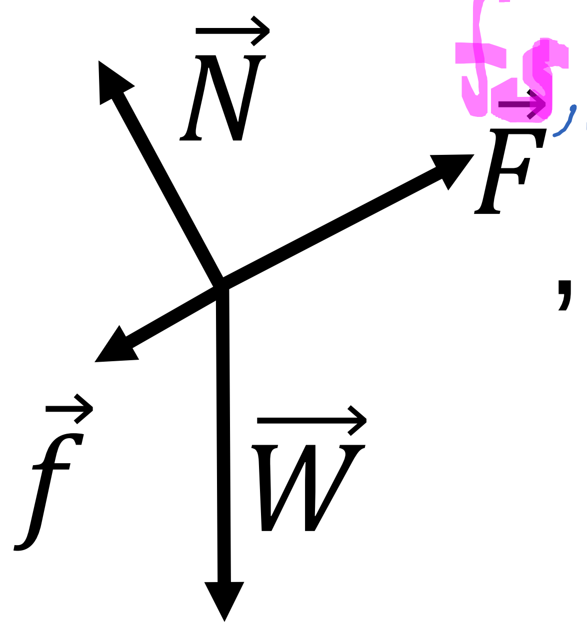




- A loaded penguin sled weighing 70.0 N rests on a plane inclined at angle θ = 21.0° to the horizontal (see the figure). Between the sled and the plane, the coefficient of static friction is $\mu_s = 0.290$,and the coefficient of kinetic friction is μ_k =0.200. At the **minimum** magnitude of $\vec{F}$ to **prevent** the sled from sliding **down**, which of the following best describes the forces on the sled? (x- and y- directions are shown above)

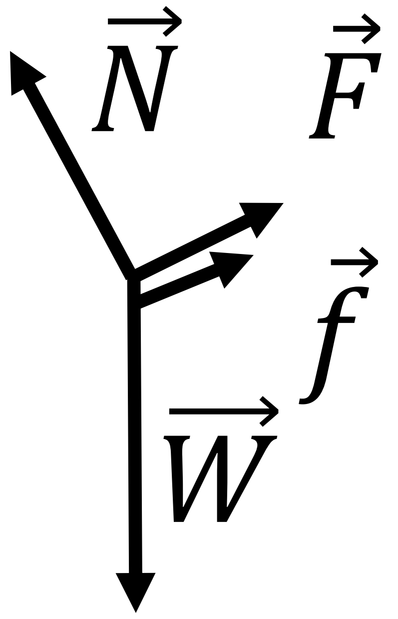
$\vec{f}_{s,max}$ up

A



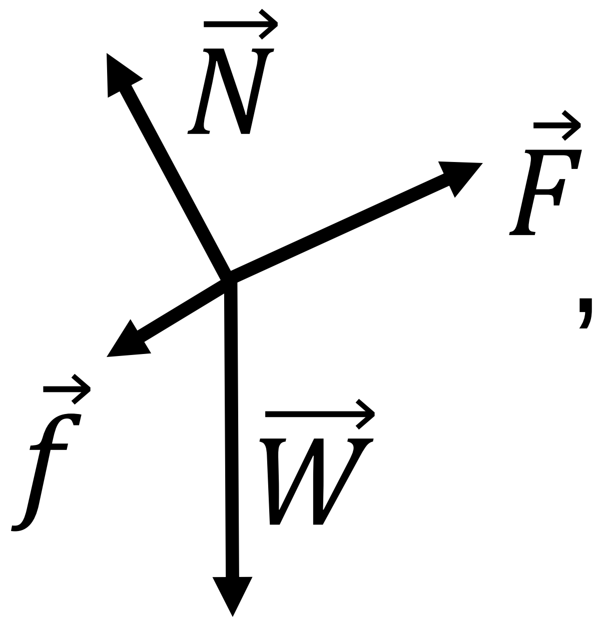
, $|\vec{f}| = \mu_k |\vec{N}|$

B



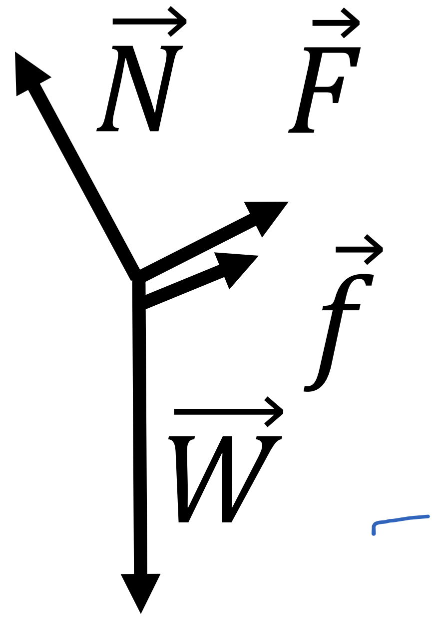
, $|\vec{f}| = \mu_k |\vec{N}|$

C



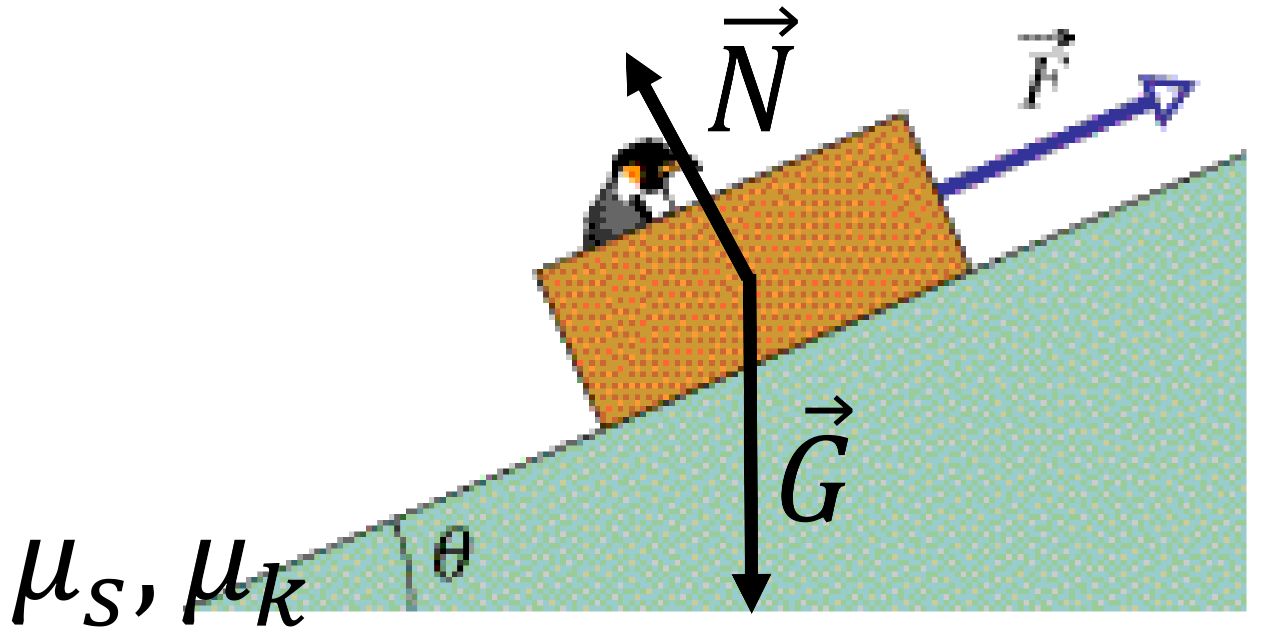
, $|\vec{f}| = \mu_s |\vec{N}|$

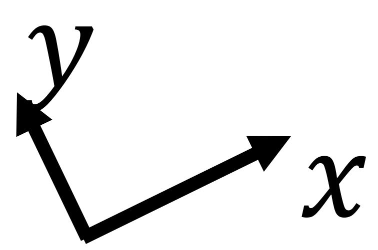
D



, $|\vec{f}| = \mu_s |\vec{N}|$

Clicker question 13

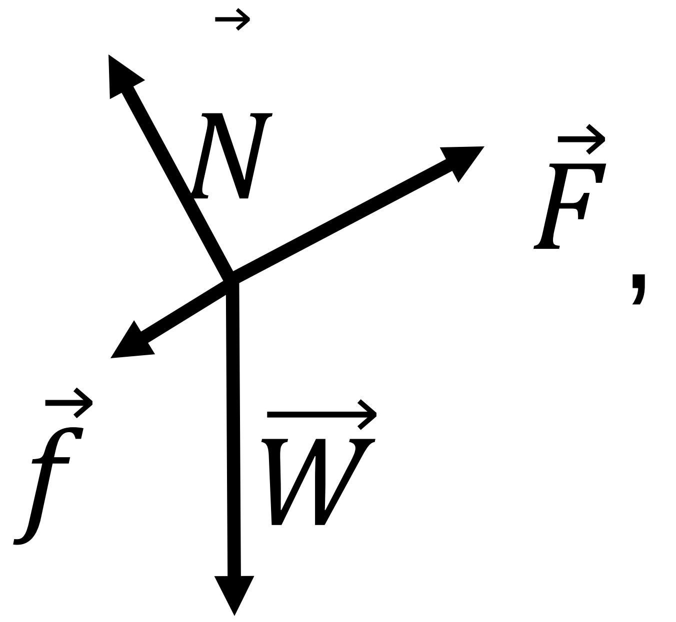




- A loaded penguin sled weighing 70.0 N rests on a plane inclined at angle θ = 21.0° to the horizontal (see the figure). Between the sled and the plane, the coefficient of static friction is $\mu_s = 0.290$, and the coefficient of kinetic friction is $\mu_k = 0.200$. At the **maximum** magnitude of $\vec{F}$ that sled **won't** slide **up**, which of the following best describes the forces on the sled? (x- and y- directions are shown above)

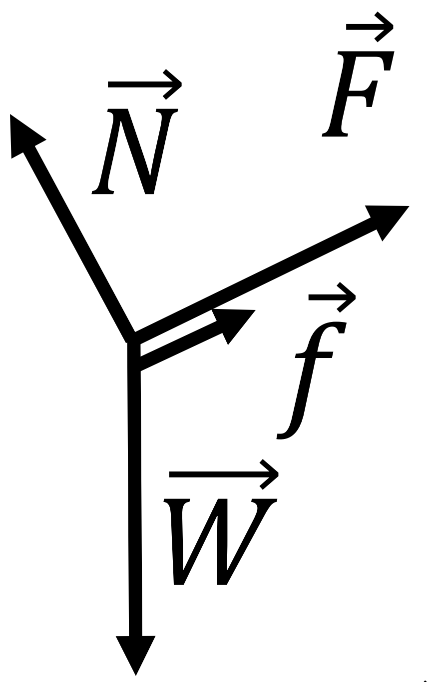
$f_{s,max}$ down

A



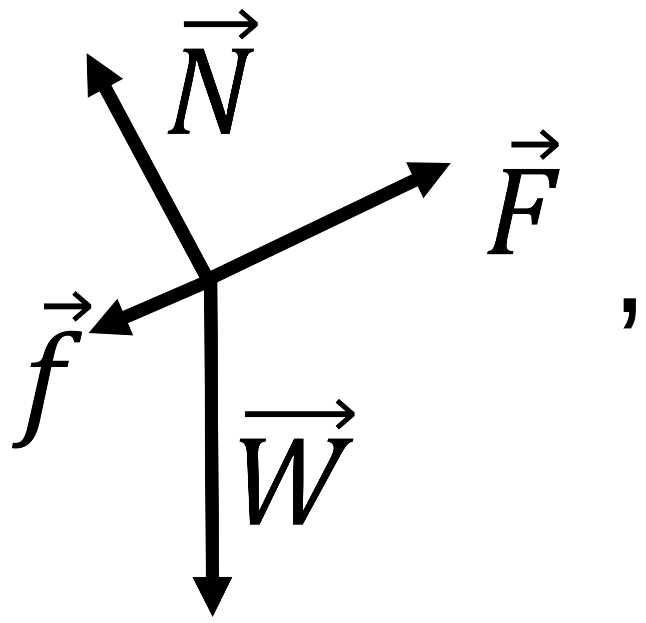
, $|\vec{f}| = \mu_k|\vec{N}|$

B



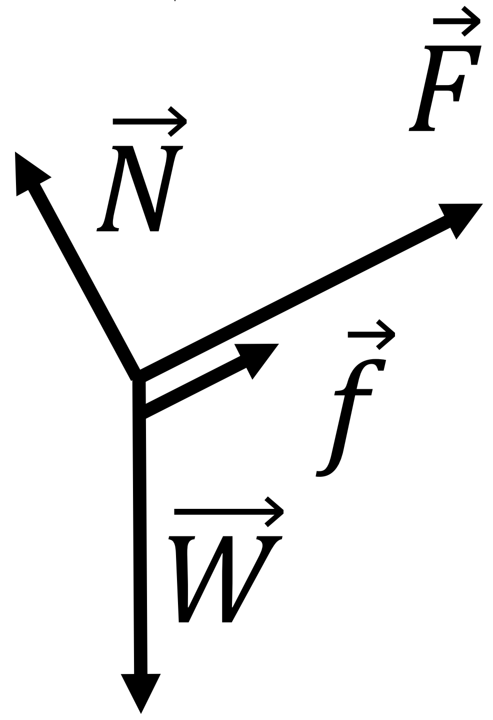
, $|\vec{f}| = \mu_k|\vec{N}|$

C



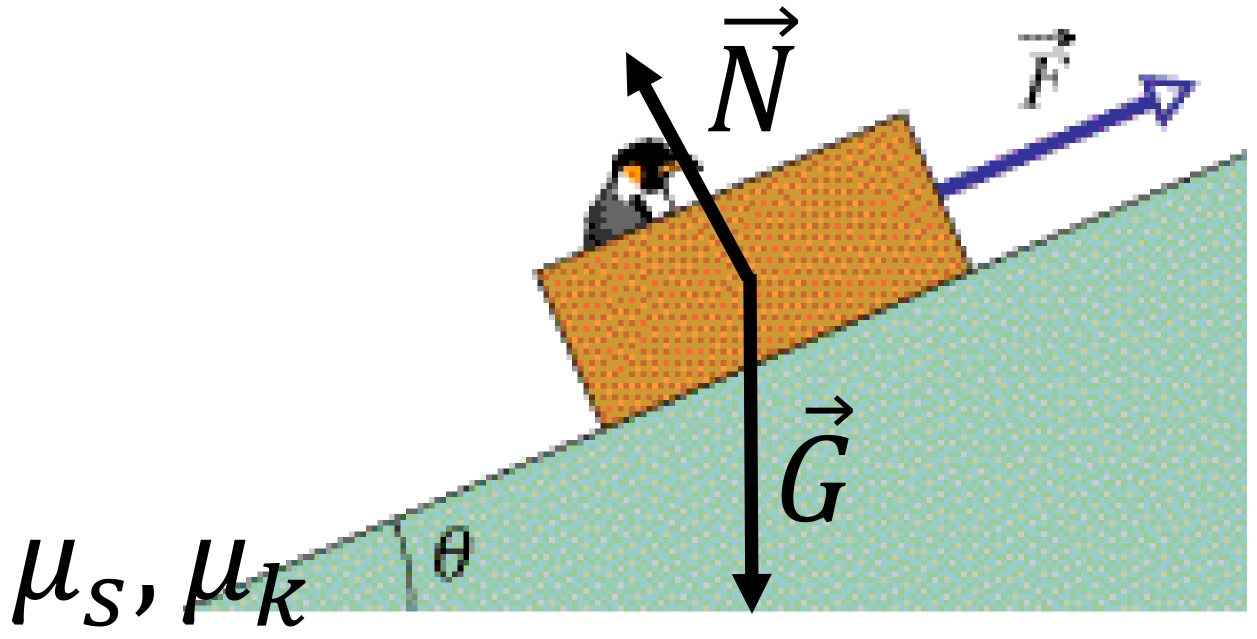
, $|\vec{f}| = \mu_s|\vec{N}|$

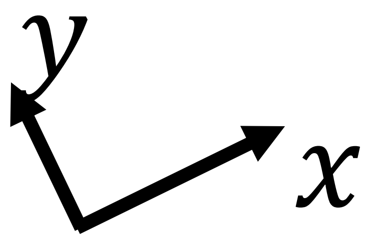
D



, $|\vec{f}| = \mu_s|\vec{N}|$

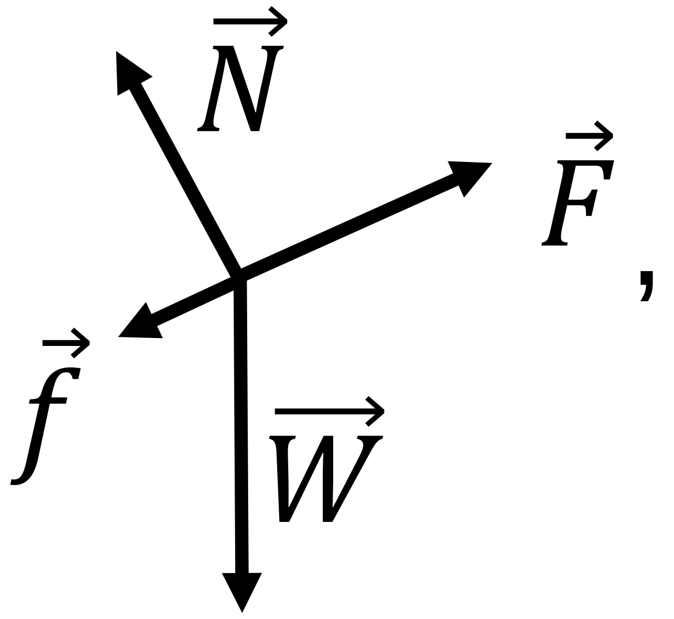
Clicker question 14





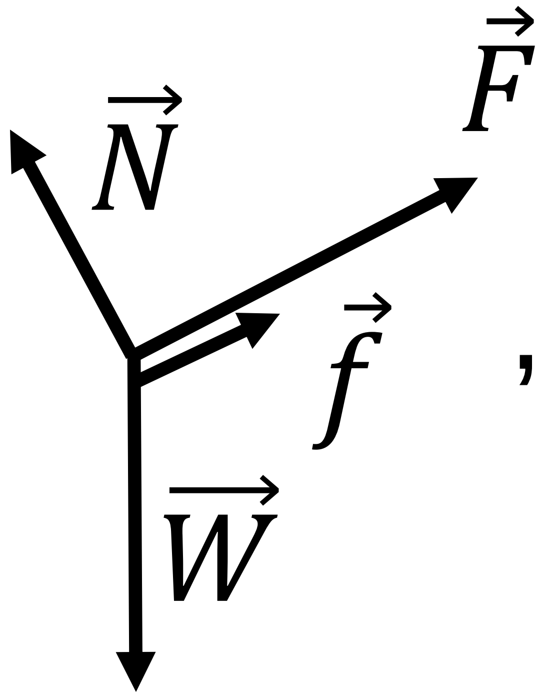
- A loaded penguin sled weighing 70.0 N rests on a plane inclined at angle θ = 21.0° to the horizontal (see the figure). Between the sled and the plane, the coefficient of static friction is μ_s =0.290, and the coefficient of kinetic friction is $\mu_k = 0.200$. At the **minimum** magnitude of $\vec{F}$ to **start** the sled moving **up**, which of the following best describes the forces on the sled? (x- and y- directions are shown above)

A



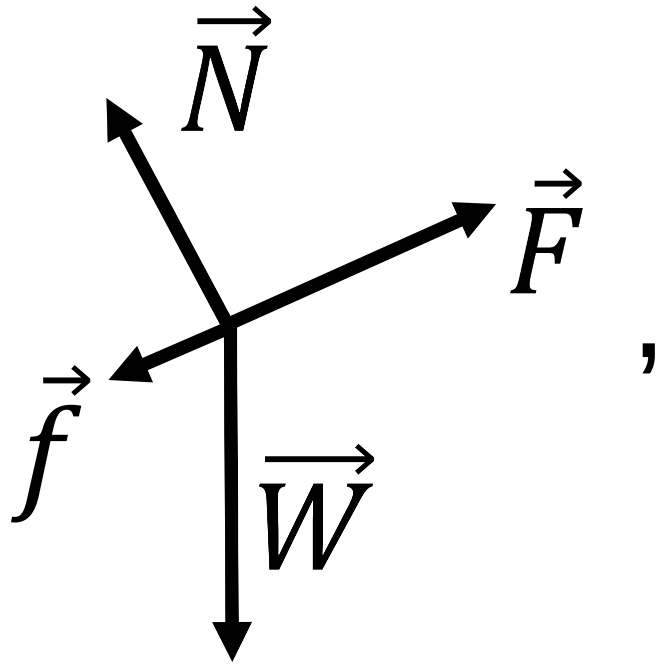
, $|\vec{f}| = \mu_k |\vec{N}|$

B



, $|\vec{f}| = \mu_k |\vec{N}|$

C



, $|\vec{f}| = \mu_s |\vec{N}|$

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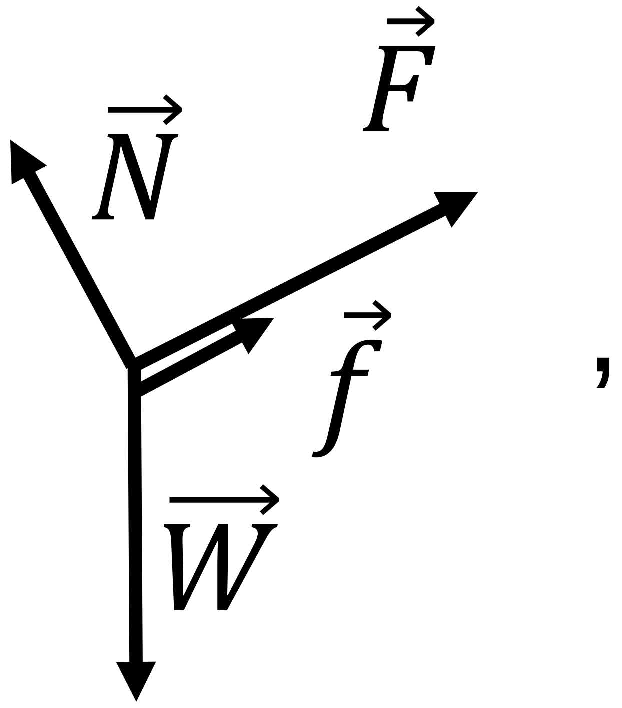
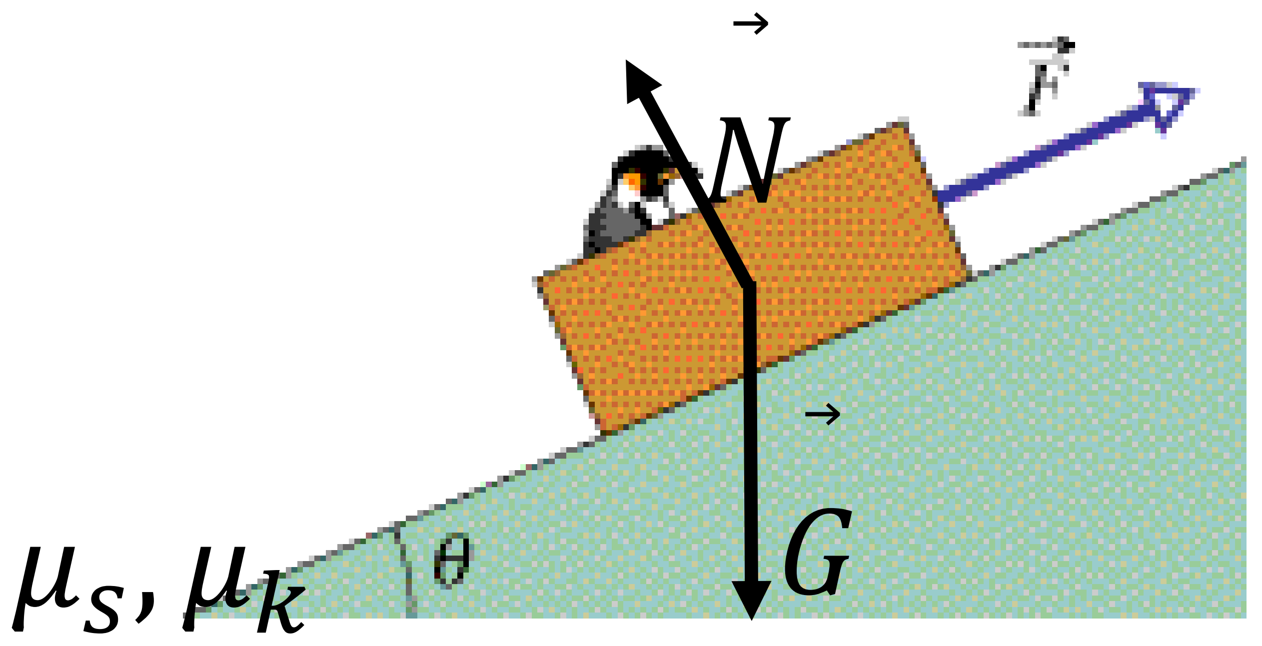
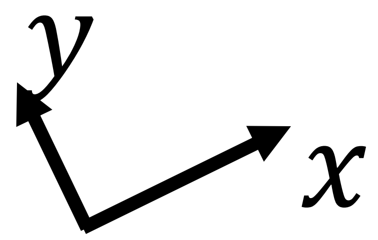


, $|\vec{f}| = \mu_s |\vec{N}|$

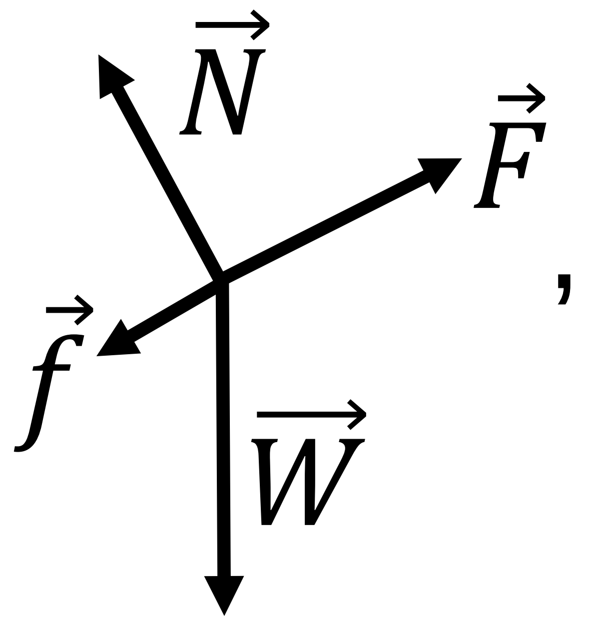
Clicker question 15





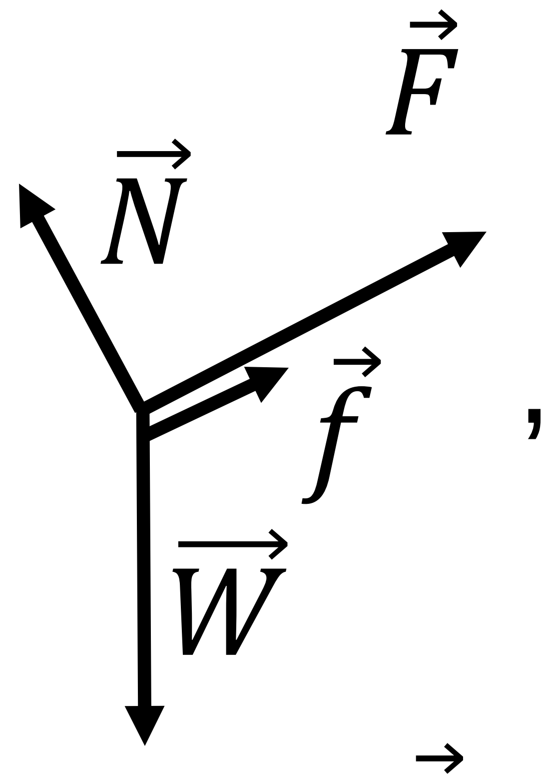
- A loaded penguin sled weighing 70.0 N rests on a plane inclined at angle θ = 21.0° to the horizontal (see the figure). Between the sled and the plane, the coefficient of static friction is μ_s =0.290, and the coefficient of kinetic friction is μ_k =0.200. When the sled **is sliding up** the slope at a constant velocity, which of the following best describes the forces on the sled? (x- and y- directions are shown above)

A



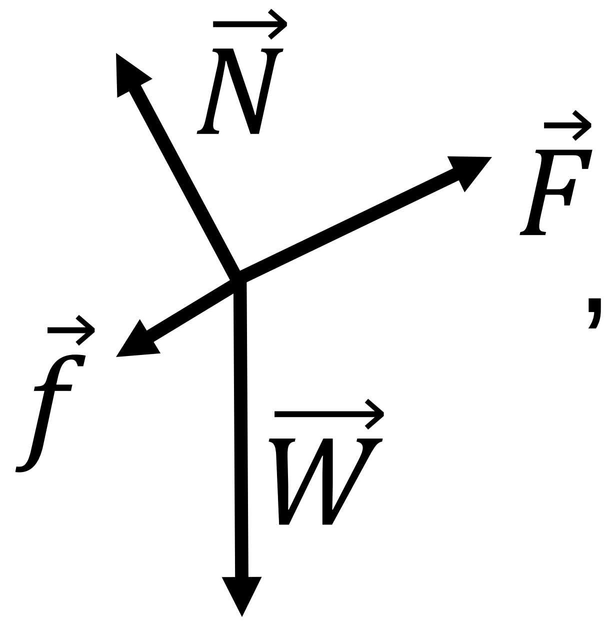
, $|\vec{f}| = \mu_k |\vec{N}|$

B



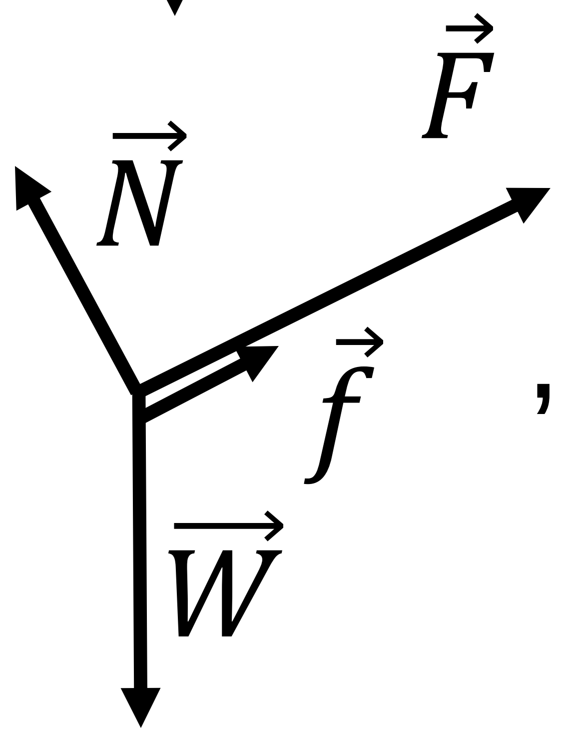
, $|\vec{f}| = \mu_k |\vec{N}|$

C



, $|\vec{f}| = \mu_s |\vec{N}|$

D



, $|\vec{f}| = \mu_s |\vec{N}|$

Pre-lecture for the next lecture

- Please complete Module 6.1.2: Pre-lecture survey before the next lecture