PHYS 225
Fundamentals of Physics: Mechanics

Prof. Meng (Stephanie) Shen

Fall 2024

Lecture 5: Motion along a straight line

1D motion

CALIFORNIA STATE UNIVERSITY FULLERTON

Learning goals

- Practice on applying the 4 kinematic equations for <u>constant acceleration 1D motion</u>

$$\boxed{\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ x &= x_0 + \frac{1}{2}(v_0 + v)\,t \\ x &= x_0 + \frac{v^2 - v_0^2}{2a} \end{aligned}}$$

- Free fall

The 4 kinematic equations for constant $\vec{a}$

If a is a constant, (and $t_0 = 0$):

$$a = \frac{v - v_0}{t} \quad (1.1)$$

$$v = v_0 + at \quad (1)$$

$$t = \frac{v - v_0}{a} \quad (1.2)$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)$$

Substitute a in (2) by (1.1)

Eliminate $\vec{a}$: $x = x_0 + \frac{1}{2}(v_0 + v)\, t$ (3)

Substitute t in (3) by (1.2)

Eliminate t: $x = x_0 + \frac{v^2 - v_0^2}{2a}$ (4)

Equivalent: $v^2 = v_0^2 + 2a(x - x_0)$

The 4 kinematic equations

- Why do we need **4** equations to describe constant acceleration?
- We don't!… but additional equations may be useful to solve certain problems.

If a is a constant: $(and\, t_0 = 0)$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Eliminate $\vec{a}$:

$$x = x_0 + \frac{1}{2}\left(v_0 + v\right) t$$

Eliminate t:

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

(4) When time is not needed

Clicker question 2

Draw a sketch

Given: $\vec{v}_0, \vec{a}, \vec{v}=0$

Goal: $\Delta \vec{x}$

t is not needed!

- A Tesla Cybertruck is traveling at an initial velocity of 30 m s^{-1} to the east. It slows down at a constant acceleration of 10 m s^{-2} to the west. How far does it travel before it stops? Which equation is the most convenient to use for this question?

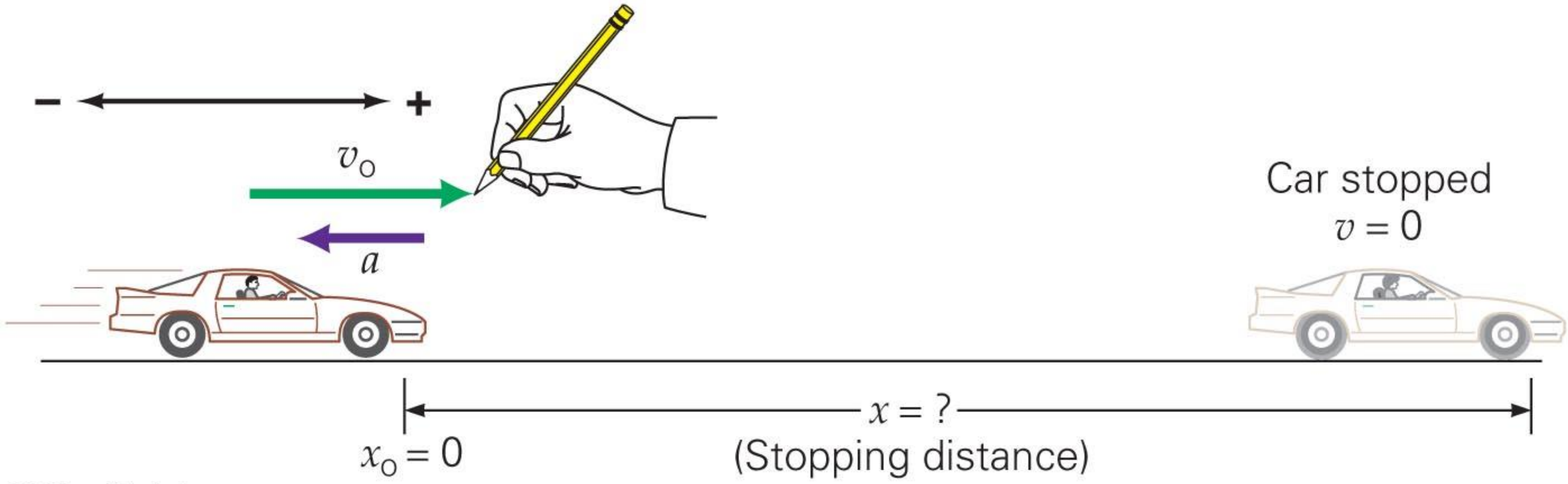
https://www.caranddriver.com/tesla/cybertruck

A $$v = v_0 + at$$

B $$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

C $$x = x_0 + \frac{1}{2}(v_0 + v)\,t$$

D $$x = x_0 + \frac{v^2 - v_0^2}{2a}$$



© 2010 Pearson Education, Inc.

Example 1

Given: $\vec{v}_0$, $\vec{a}$, $\vec{v}=0$

Stopping distance Q

Goal: Δx

https://www.caranddriver.com/tesla/cybertruck

- A Tesla Cybertruck is traveling at a velocity of 30 m s^{-1} to the east. It slows down at a constant acceleration of 10 m s^{-2} to the west. What's its displacement when it stops?

$\longrightarrow +x$

Axis

Step 1: $\Delta x = x - x_0 = \dfrac{(v^2 - v_0^2)}{2a}$

Final initial

Step 2: Plug in numbers

$$\Delta x = \frac{0-(30\,\mathrm{m\,s^{-1}})^2}{2(-10\,\mathrm{m\,s^{-2}})} = 45\ \mathrm{m}$$

Stopping distance: A real-life example

- “5 k here = 27 k there”

Difference in final speeds $\gg$ Difference in initial speeds

$$v_w - v_b \gg v_{w0} - v_{b0}$$

Ref.: http://www.youtube.com/watch?v=WjvVbXDy20w

Example 2

Given: v_{b0}, v_{w0}, $\Delta x_w = \Delta x_b = d$, $a_w = a_b = a$, $v_b = 0$

Goal: v_w

- The speed limit of a highway is 60 miles per hour (mph). A black car travels at the speed limit (v_{b0} =60 mph), and a white car travels at a speed of v_{w0} =65 mph. At the moment when the two cars are side by side, an oversized truck stopped some distance in front of the cars. Both drivers hit the break immediately with the same acceleration $a_w = a_b = a \leq 0$,and the black car managed to stop exactly behind the truck without hitting it. What is the speed of the white car where it reaches the truck?

Step 1: Choose the kinematic equation

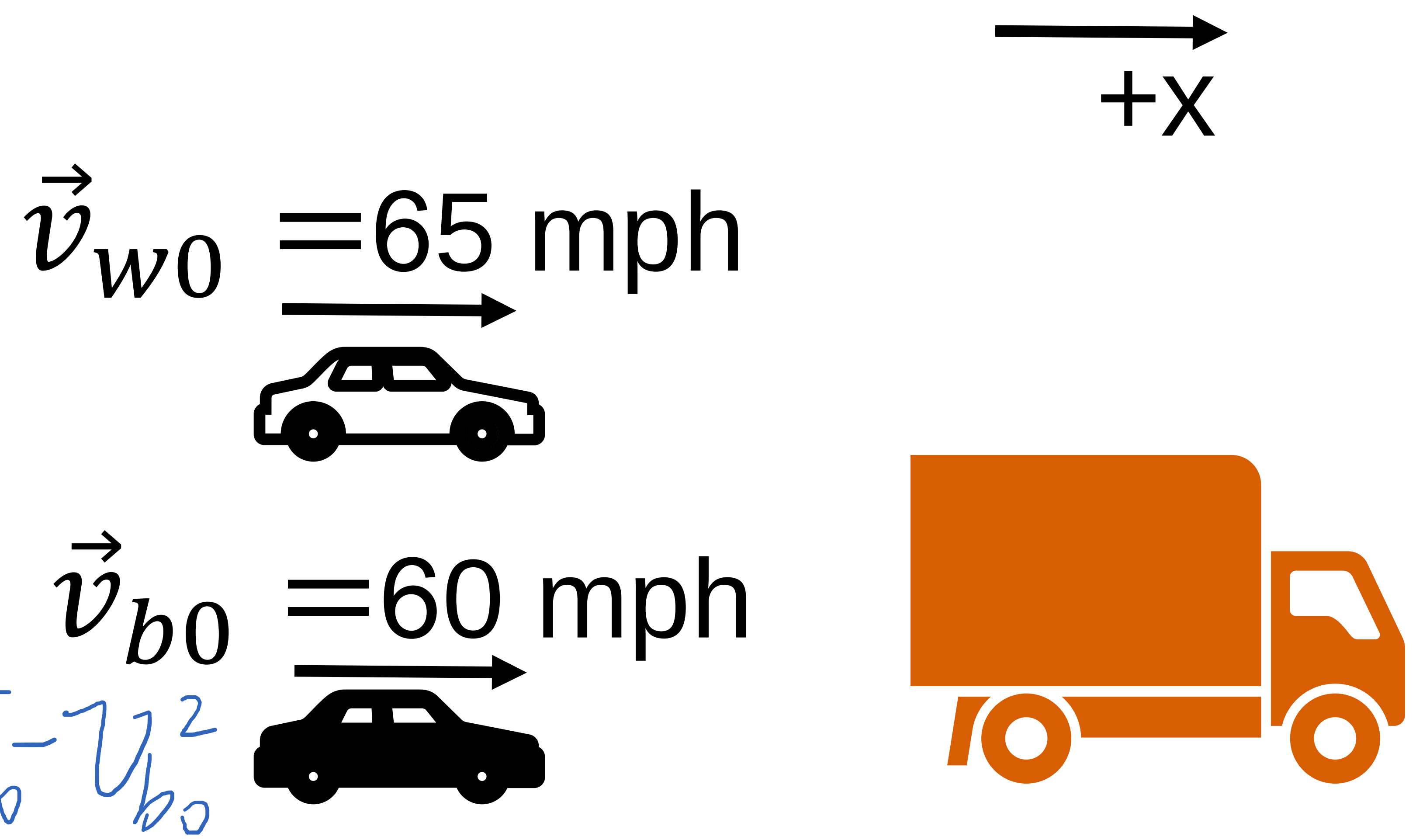
$$\Delta x = x - x_0 = \frac{v^2 - v_0^2}{2a}$$

Step 2: Since $a_w = a_b = \boldsymbol{a}$,and $v_b = 0$,

$$\frac{v_w^2 - v_{w0}^2}{\cancel{2a}} = \frac{v_b^2 - v_{b0}^2}{\cancel{2a}} = \frac{-v_{b0}^2}{\cancel{2a}} \rightarrow v_w^2 = v_{w0}^2 - v_{b0}^2$$

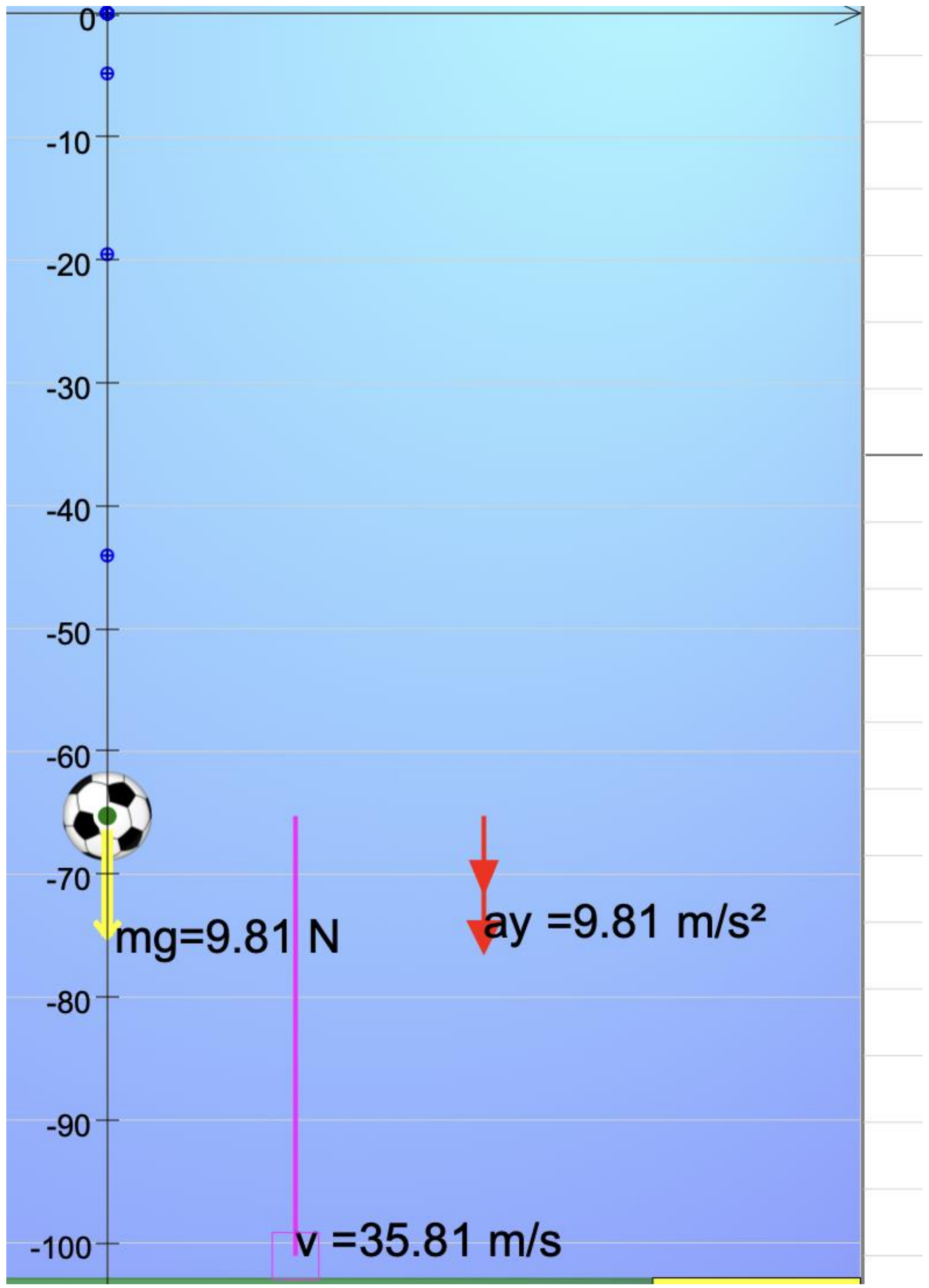
Step 3: Plug in numbers,

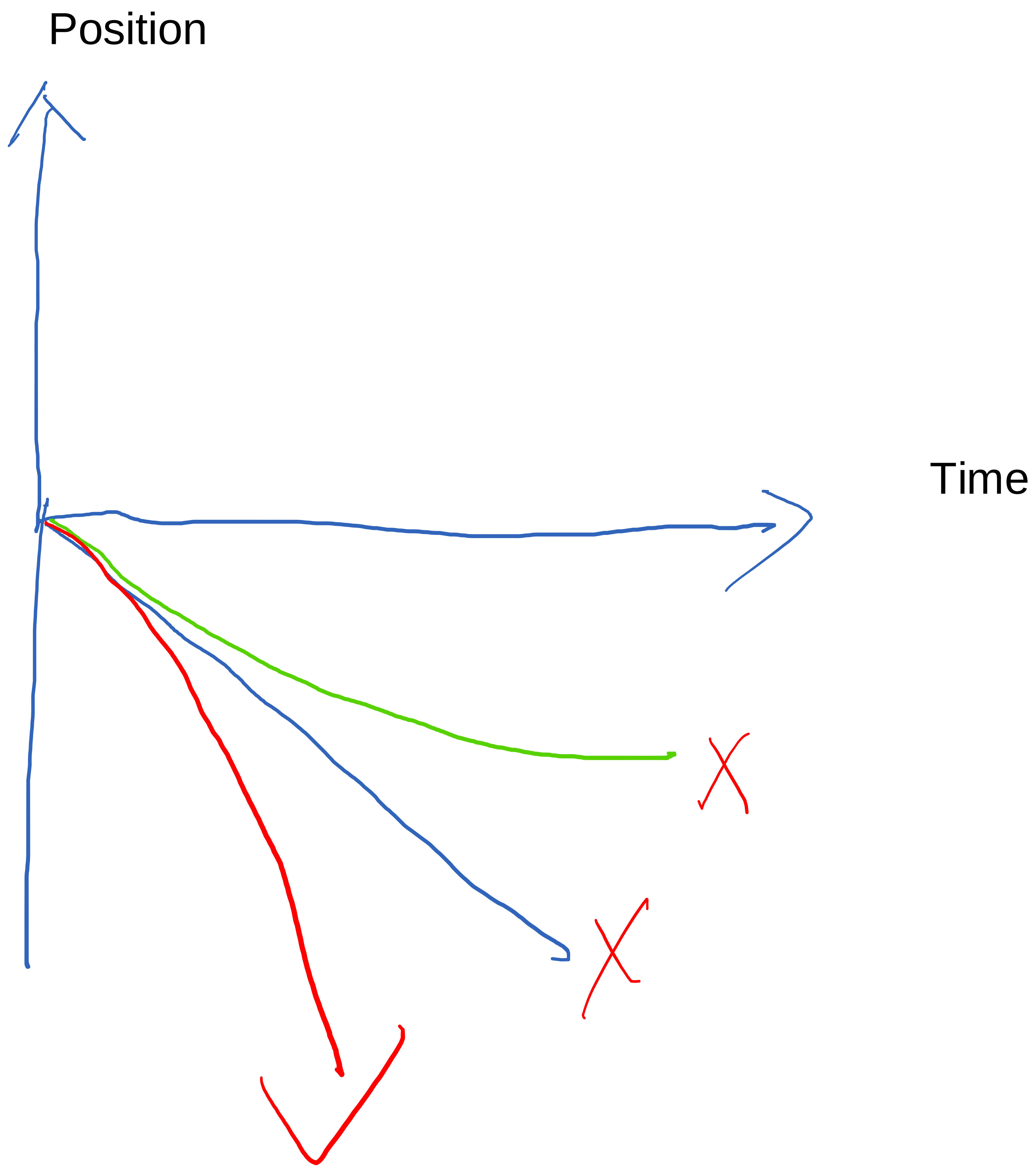
$$v_w = \sqrt{v_{w0}^2 - v_{b0}^2} = \sqrt{(65\ \text{mph})^2 - (60\ \text{mph})^2} = 25\ \text{mph}$$



$t_i = 0$

Free fall simulation

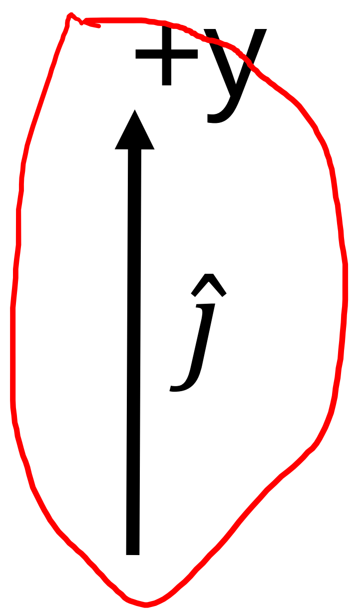




https://iwant2study.org/lookangejss/02_newtonianmechanics_2kinematics/ejss_model_freefall01/freefall01_Simulation.xhtml .

Free fall

Axis



- Near Earth's surface: *all* freely falling objects accelerate downward because of gravity:
  - If positive direction is pointing up, then the acceleration for free fall is $a = -g$

$g = 9.8\ ms^{-2}$

- We can use the 4 kinematic equations for constant acceleration 1D motion for free fall

$a = -g$ for free fall

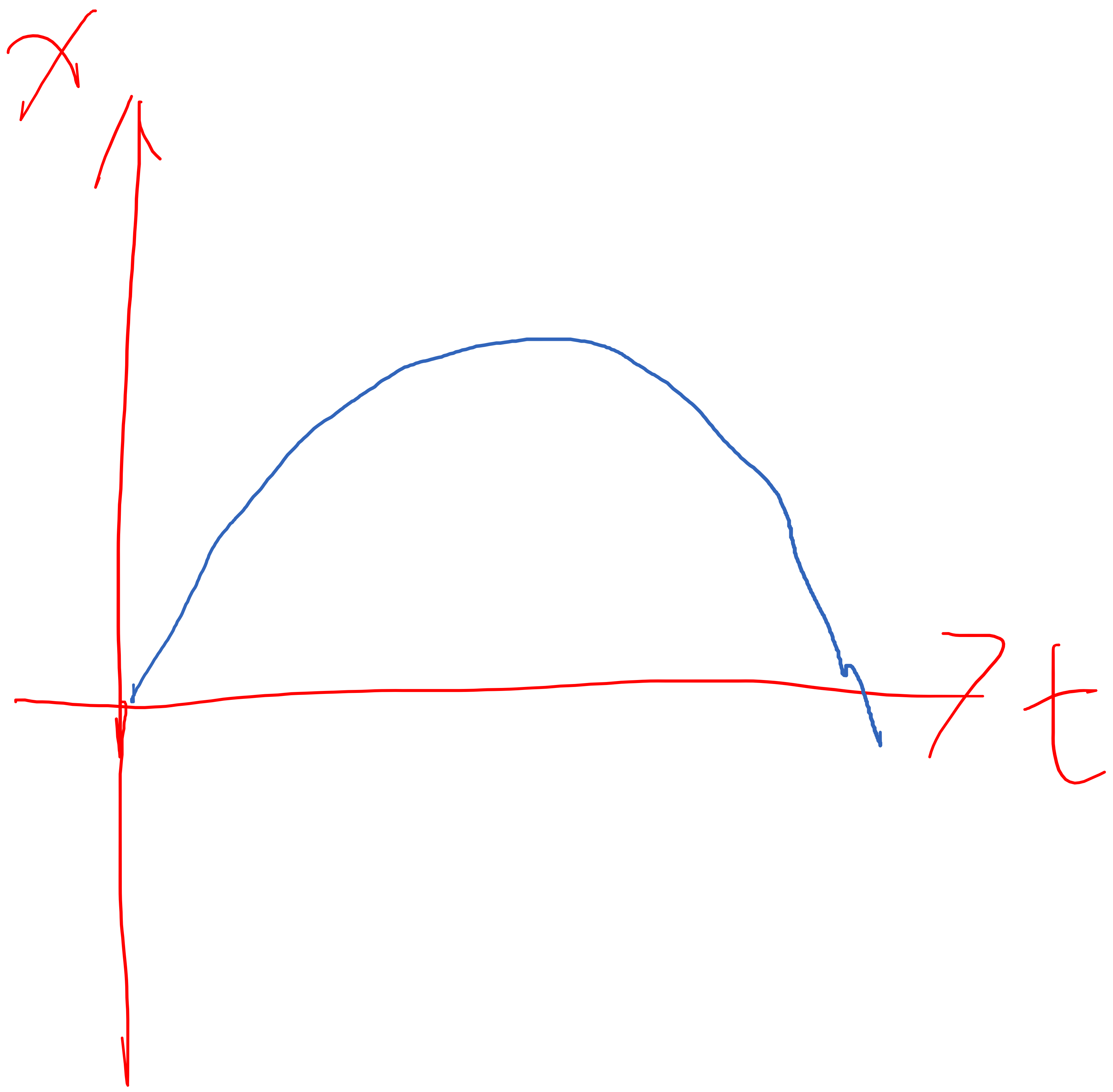
Constant acceleration 1D kinematics

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$x = x_0 + \frac{1}{2}(v_0 + v)\, t$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

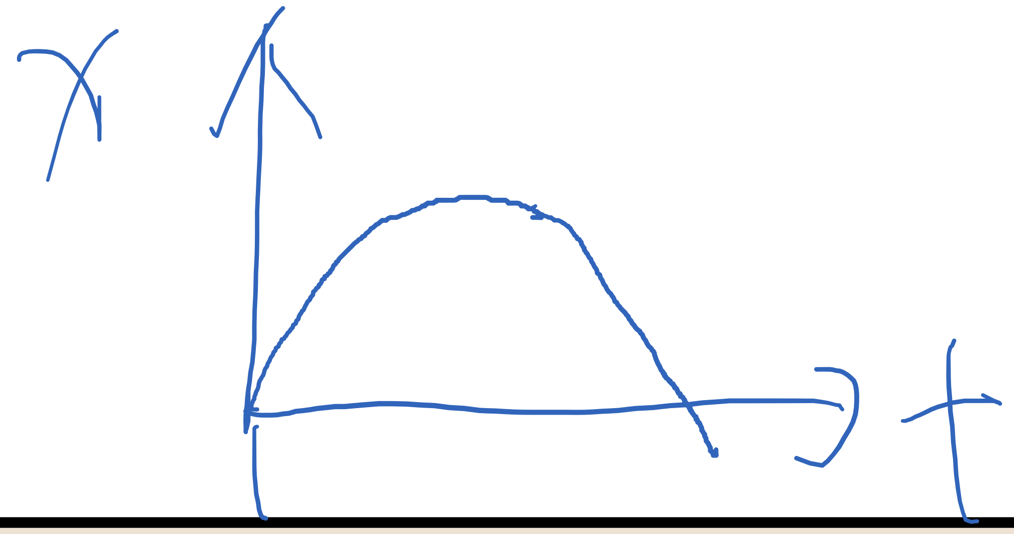
Free fall

$$v = v_0 - gt$$
$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$
$$y = y_0 + \frac{1}{2}(v_0 + v)\, t$$
$$v^2 = v_0^2 - 2g(y - y_0)$$

Demo



Clicker question 3



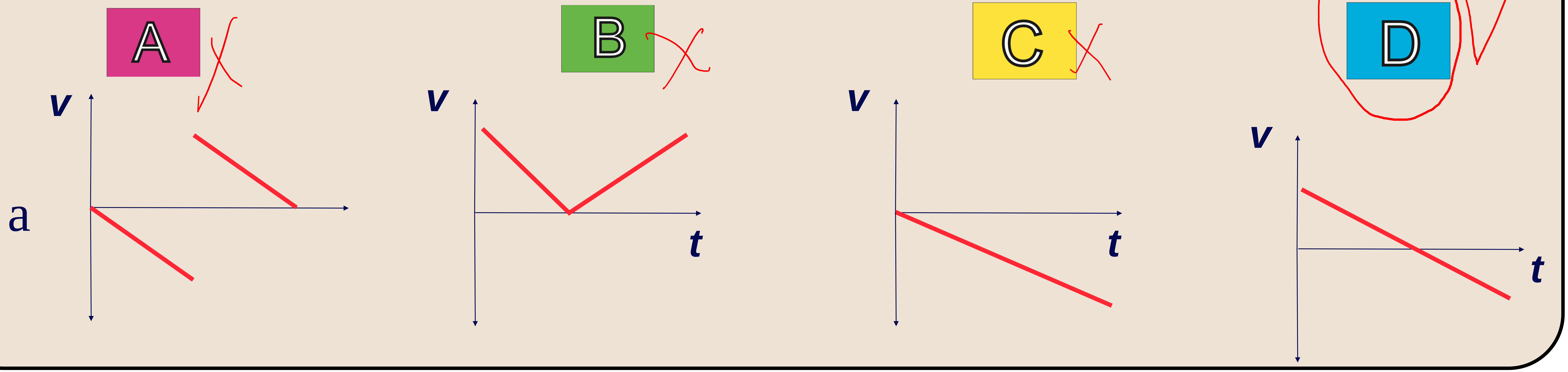
Free fall: $a = -g\,\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt}$

+y

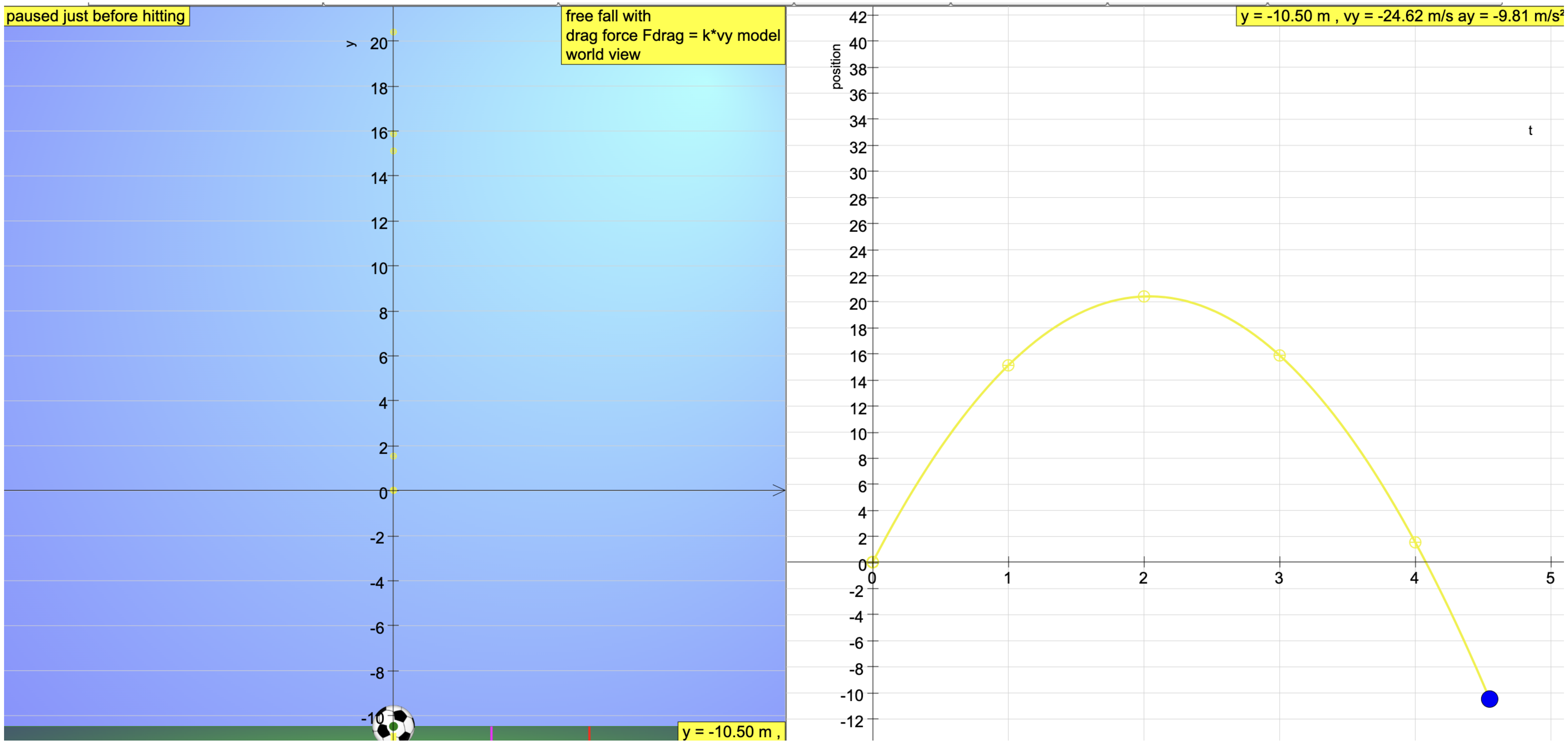
Axis

Question 2.15b Rubber Balls II



You toss a ball straight up in the air and catch it again. Right after it leaves your hand and before you catch it, which of the above plots represents the ***v* vs. *t*** graph for this motion? (Assume your y-axis is pointing up).

Simulation 2



https://iwant2study.org/lookangejss/02_newtonianmechanics_2kinematics/ejss_model_freefall01/freefall01_Simulation.xhtml

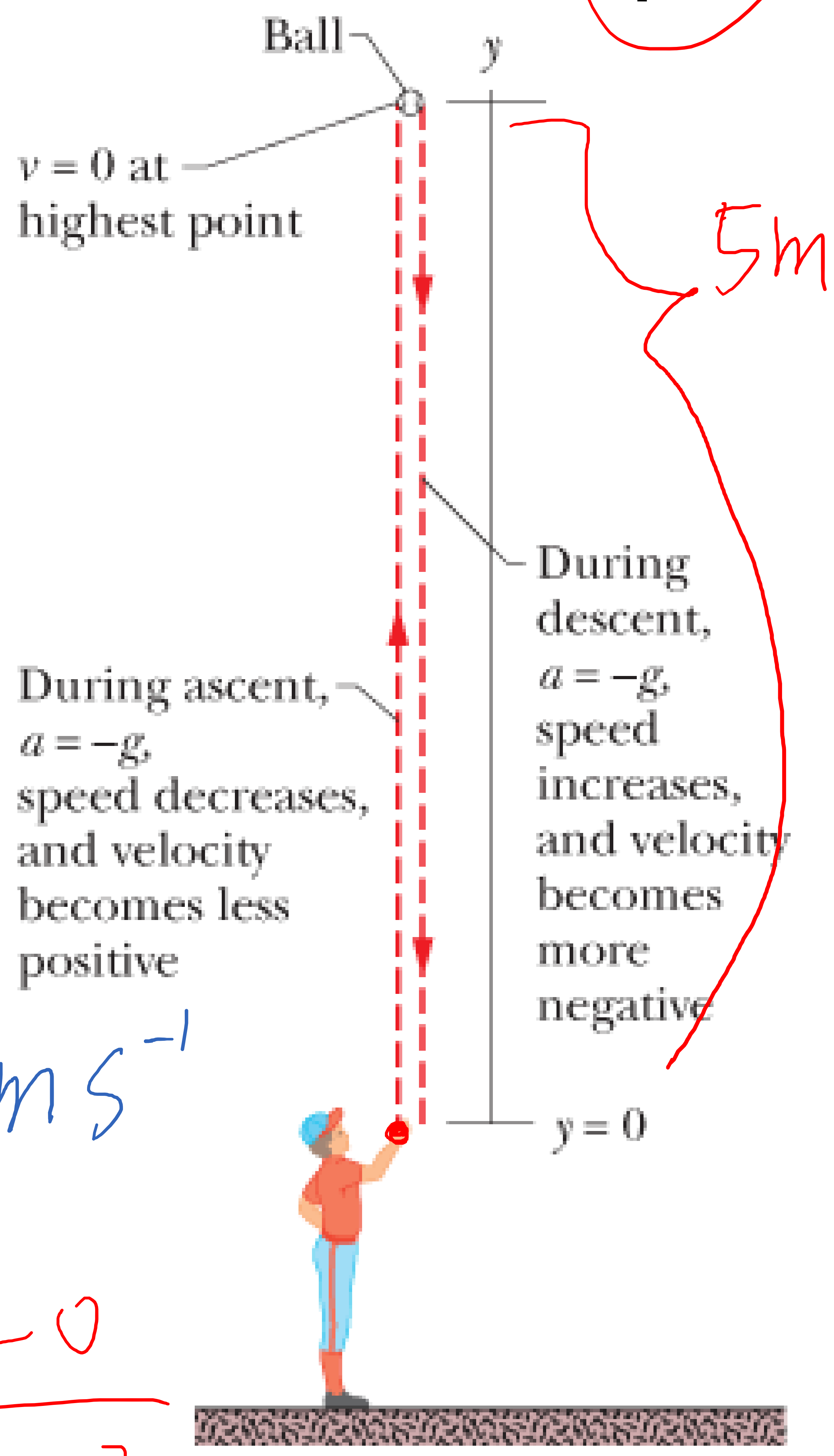
Example 3

Given: Δy, $a = -g$, $v = 0$

Goal: v_0, t

Axis $+y$

- A pitcher throws a ball up in the air. The ball reached a maximum height of 5 m above the pitcher's hand. (The +y axis points up.)
  - What's the initial velocity of the ball?
  - How long does it take for the ball to reach the maximum height?



5m

Step 1: $\Delta y = \dfrac{v^2 - v_0^2}{2a} = \dfrac{-v_0^2}{-2g}$

Rewrite: $v_0 = \sqrt{2g\Delta y} = \sqrt{2 \times 9.8\,\mathrm{ms^{-2}} \times 5\,\mathrm{m}} \approx 9.9\,\mathrm{ms^{-1}}$

Step 2: $v = v_0 - gt \rightarrow t = \dfrac{v_0 - v}{g} = \dfrac{9.9\,\mathrm{ms^{-1}} - 0}{9.8\,\mathrm{ms^{-2}}}$

$\approx 1.01\,\mathrm{s}$

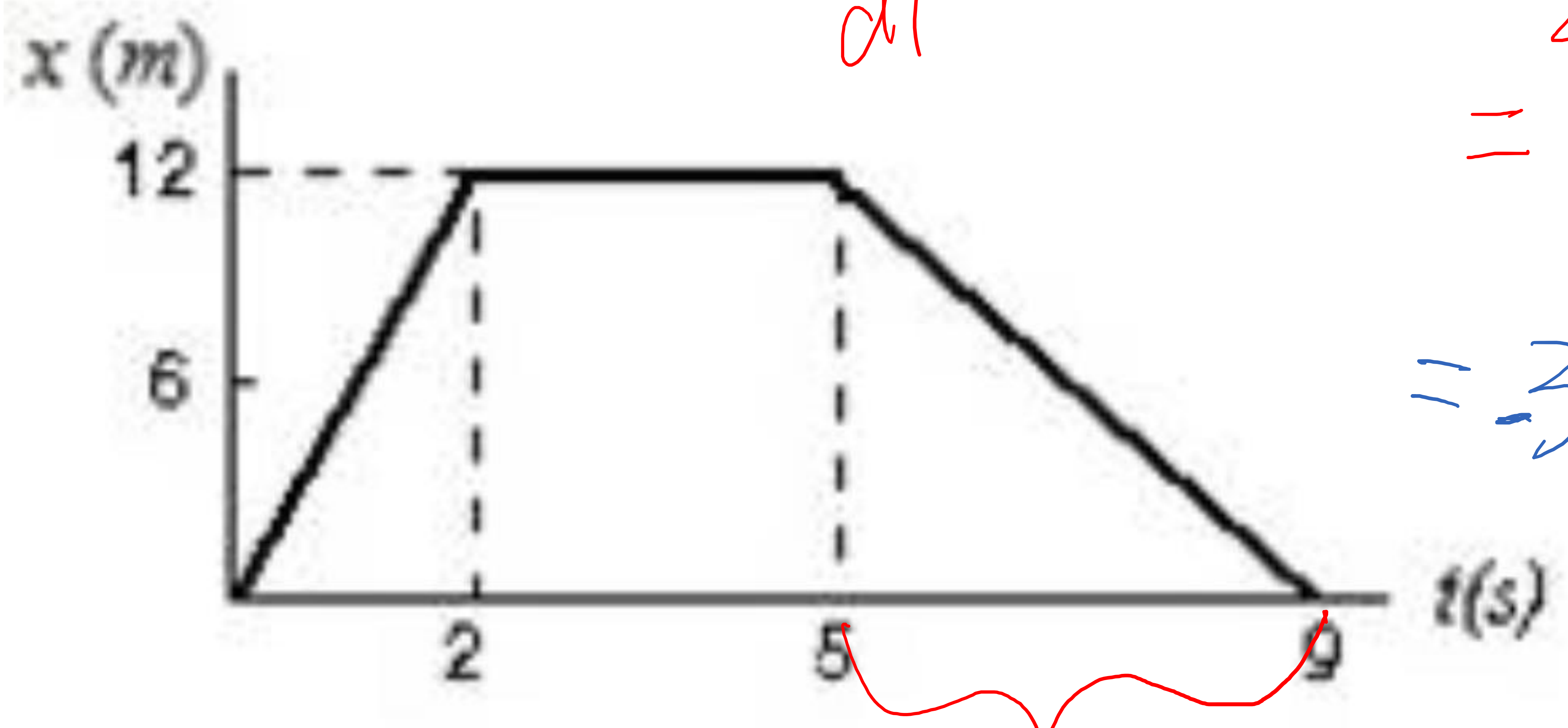
Clicker question 4: Kinematics from graph

- The graph shows the position of a particle as a function of time. What's its instantaneous velocity at t = 6s?

Goal

$$\vec{v} = \frac{d\vec{x}}{dt} \xrightarrow{\text{Straight line}} \frac{\Delta \vec{x}}{\Delta t} = \frac{-12\,m}{4\,s} = -3\,m\,s^{-1}$$

A	3 m s^{-2}
B	-3 m s^{-2}
C	0 m s^{-2}



Clicker question 5

Δx

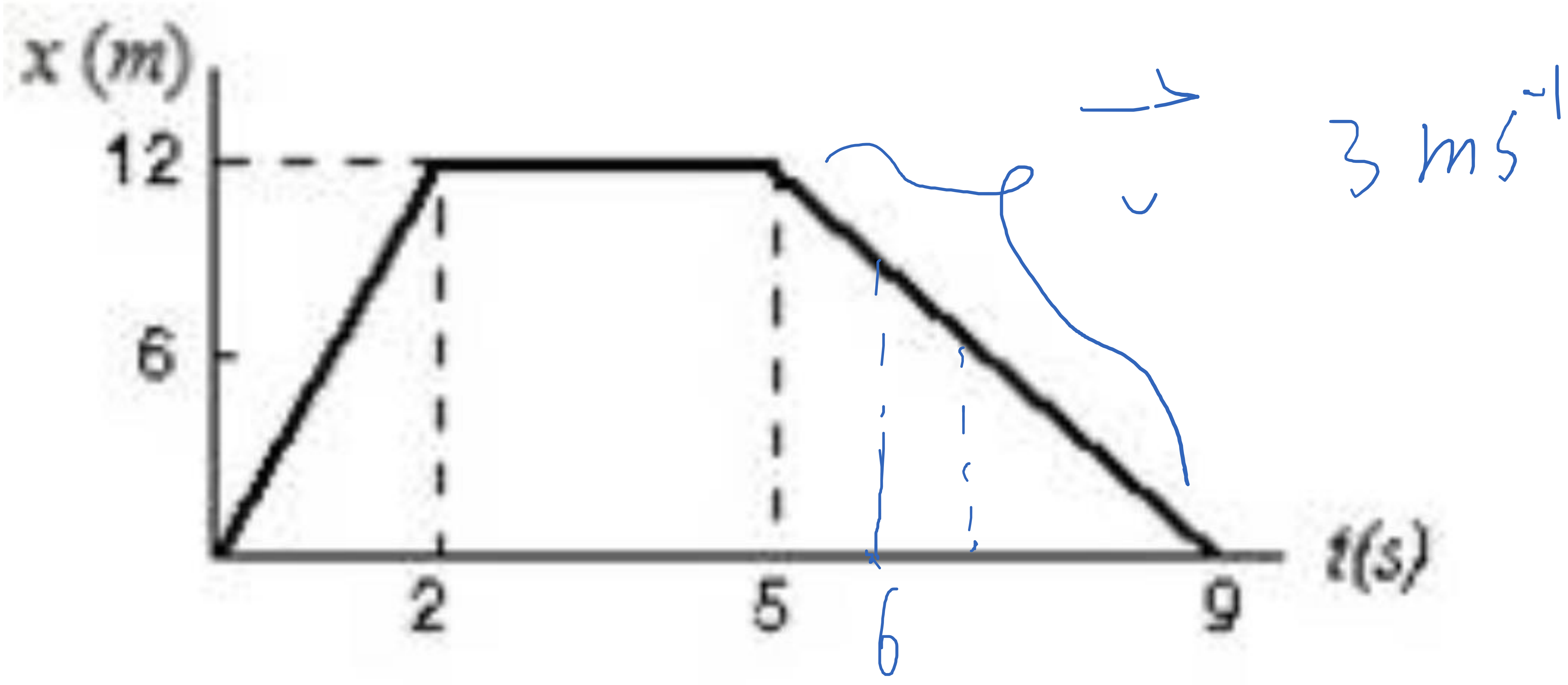
$\vec{v} = \frac{d\vec{x}}{dt}$ $\vec{a} = \frac{d\vec{v}}{dt}$

- The graph shows the position of a particle as a function of time. What's its instantaneous acceleration at t = 6s?

A 3 m s^{-2}

B -3 m s^{-2}

C 0 m s^{-2}



Practice question: A general 1D motion

Given: $\vec{x}(t)$

Goal: $\vec{a}$

$\vec{v} = \frac{d\vec{x}}{dt}$, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$

- The 1D displacement of an object is given by $x = c_1 t^3 + c_2 t^2$, where t is time, and the constant parameters are: $c_1 = 1.0\, m\, s^{-3}$ and $c_2 = 1.0\, m\, s^{-2}$. Please find the acceleration at a) $t = 1.0\, s$ and b) $t = 2.0\, s$. Is the acceleration a constant in this example?

Step 1: $\vec{a} = \frac{d^2\vec{x}}{dt^2} = \frac{d^2(c_1 t^3 + c_2 t^2)}{dt^2} = 6c_1 t + 2c_2$

Step 2: Plug in #s: $\vec{a}(t = 1\,s) = 8\ ms^{-2}$

$\vec{a}(t = 2\,s) = 14\ ms^{-2}$

Summary of Chapter 2

- Vector (1D): Magnitude and direction
- Concepts: Describe 1D motion of an object (with point particle approximation): **Displacement**, **velocity** and **acceleration**
- Practices: Extract the motion of an object from a 1D diagram: Graphic integration and derivative
- Calculation: **Four kinematic equations** for 1D motion with **a constant acceleration** (free fall a=-g)

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$x = x_0 + \frac{1}{2}\left(v_0 + v\right)t$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Assignments

- Chapter 2 homework: Module 2.4: Assignment
  - due in a week.
  - Please start early to give yourself enough time to solve the problems.
  - Note for Homework 2 magnitude of gravitational acceleration: $g \approx 9.8$ m/s^2
- Pre-lecture survey 3.1: Due before the next class

More practice questions

- Canvas
  - End of Chapter 2: eTextbook -> Chapter 01 Student Solutions Manual