PHYS 225
Fundamentals of Physics: Mechanics

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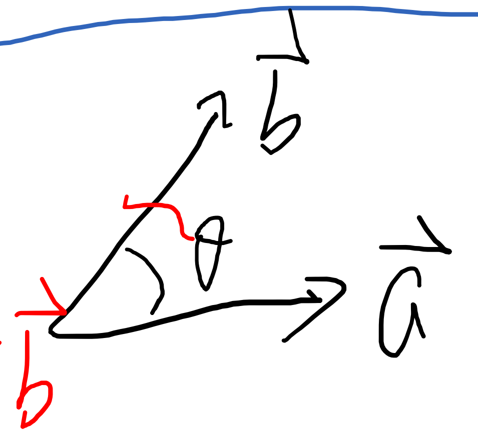
Fall 2024

Lecture 8: Motion in two and three dimensions

CALIFORNIA STATE UNIVERSITY
FULLERTON

Learning goals for today

- Practice on vector algebra
- Learning projectile motion

θ: Angle between $\vec{a}$ & $\vec{b}$ 	Dot product $\vec{a} \cdot \vec{b}$	Cross product $\vec{a} \times \vec{b}$
By components	$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$
Geometrically	$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \lvert\vec{b}\rvert \cos\theta$	$\vec{a} \times \vec{b} = \lvert\vec{a}\rvert \lvert\vec{b}\rvert \lvert\sin\theta\rvert \hat{u}$ by RHR

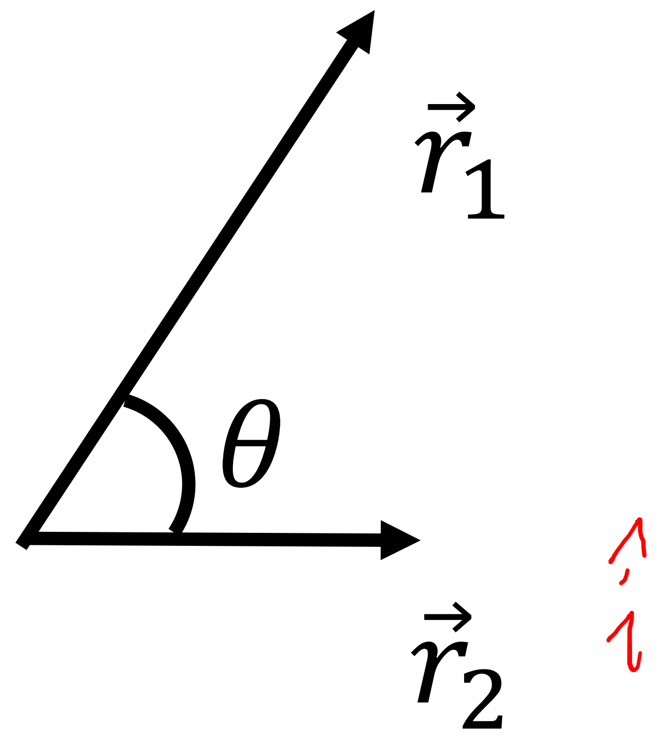
Group activity

Example:

Given: $\vec{r}_1, \vec{r}_2, \underbrace{|\vec{r}_1|, |\vec{r}_2|, \theta}$

Goal: $\vec{r}_1 \times \vec{r}_2$

- The magnitude of $\vec{r}_1$ is $|\vec{r}_1| = 2.0$,and $\vec{r}_1$ is in the xy plane and is $\theta = 60°$ counterclockwise from the x-axis; the magnitude of $\vec{r}_2$ is $|\vec{r}_2| = 1.0$,and $\vec{r}_2$ is along the +x direction.
    - What is the magnitude and direction of $\vec{r}_1 \times \vec{r}_2$?
    - Please express $\vec{r}_1 \times \vec{r}_2$ in unit vector notation.

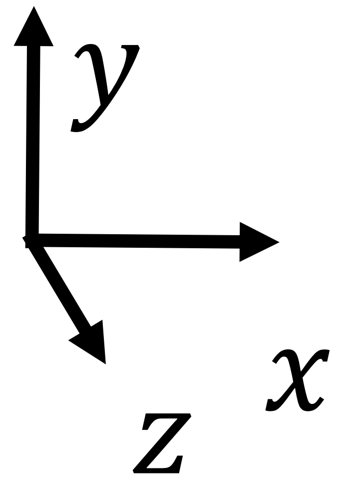


Step 1: $\vec{r}_1 \times \vec{r}_2 = \underbrace{|\vec{r}_1||\vec{r}_2||\sin\theta|}\,\hat{u}$ RHR

Direction: $\hat{u} = -\hat{k}$

$|\vec{r}_1 \times \vec{r}_2| = |\vec{r}_1||\vec{r}_2||\sin\theta| = 2.0 \times 1.0 \cdot |\sin 60°| \approx 1.73$

Step 2: $\vec{r}_1 \times \vec{r}_2 = 1.73(-\hat{k}) = -1.73\hat{k}$

Axis


Properties of vector multiplication

- Vector scaling, dot product and cross product are **distributive** over addition:

Scaling :

$$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$$

Dot prod.

$$\vec{c} \cdot (\vec{a} + \vec{b}) = \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b}$$

Cross prod.

$$\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$$

- Dot product is **commutative**

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

- Cross product is neither commutative or associative

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a},\ but\quad \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

Practice questions

Clicker question 5

- A vector, $\vec{r}$,has a magnitude of 3.50 units, and is in the direction of $300°$ as measured counterclockwise from the positive x axis. Please find the x and y components of $\vec{r}$,r_x and r_y.

A $\vec{r} = 3.03\hat{\imath} + 1.75\hat{\jmath}$

B $\vec{r} = 1.75\hat{\imath} + 3.03\hat{\jmath}$

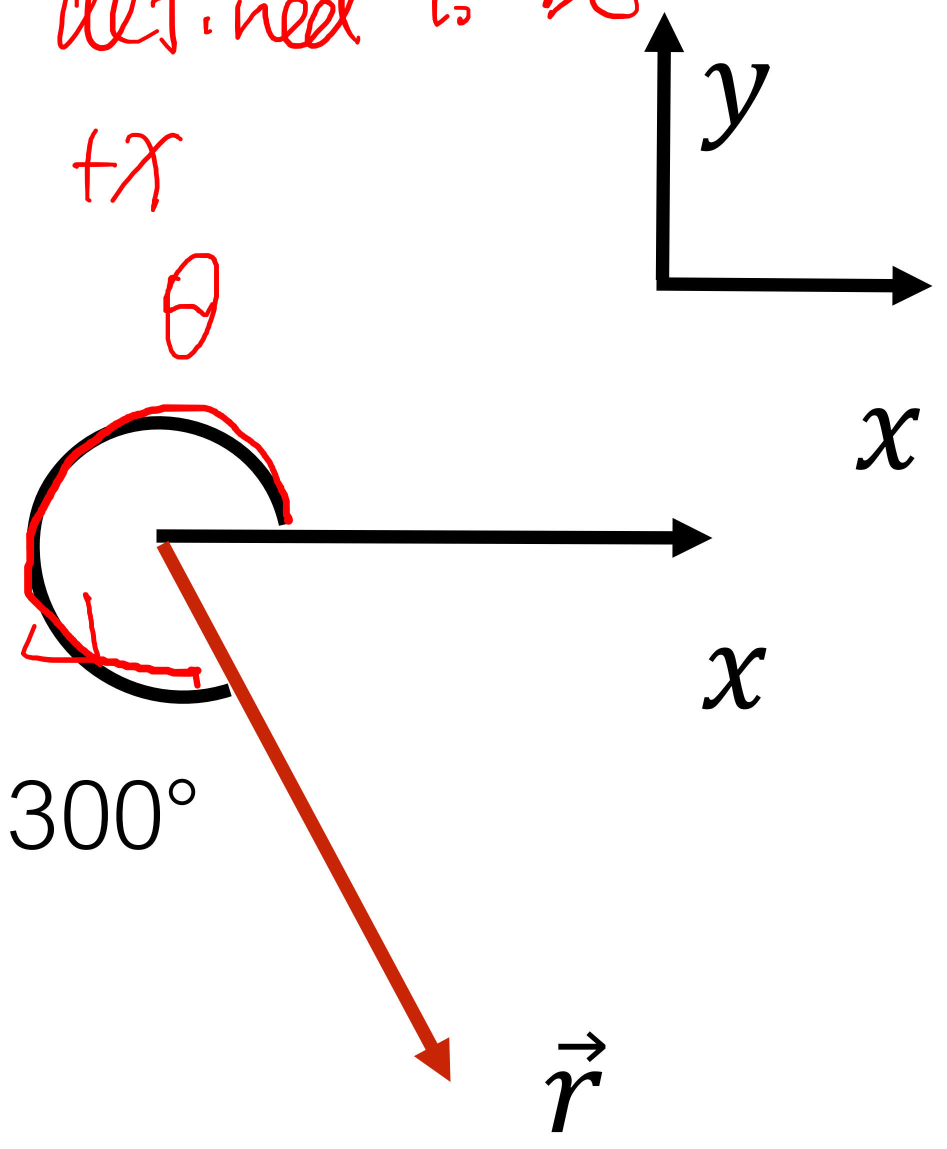
C $\vec{r} = 1.75\hat{\imath} + (-3.03)\hat{\jmath}$

D $\vec{r} = 3.03\hat{\imath} + (-1.75)\hat{\jmath}$

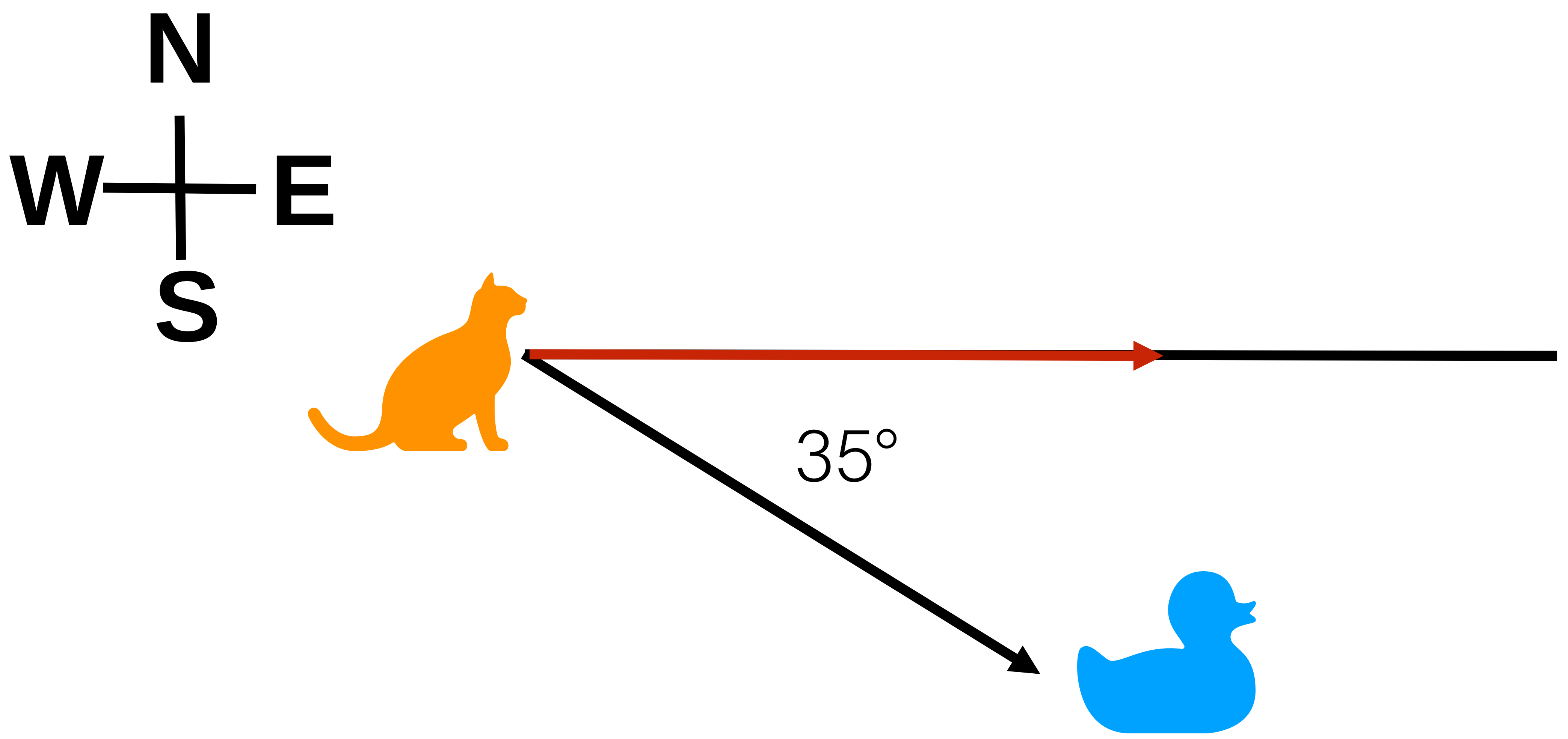
B.C. θ is defined to be between $\vec{r}$ & $+x$

$r_x = |\vec{r}| \cos\theta$

$r_y = |\vec{r}| \sin\theta$



Clicker question 6



- Which of the following is correct?

A The duck is 35° to the east of north from the cat.

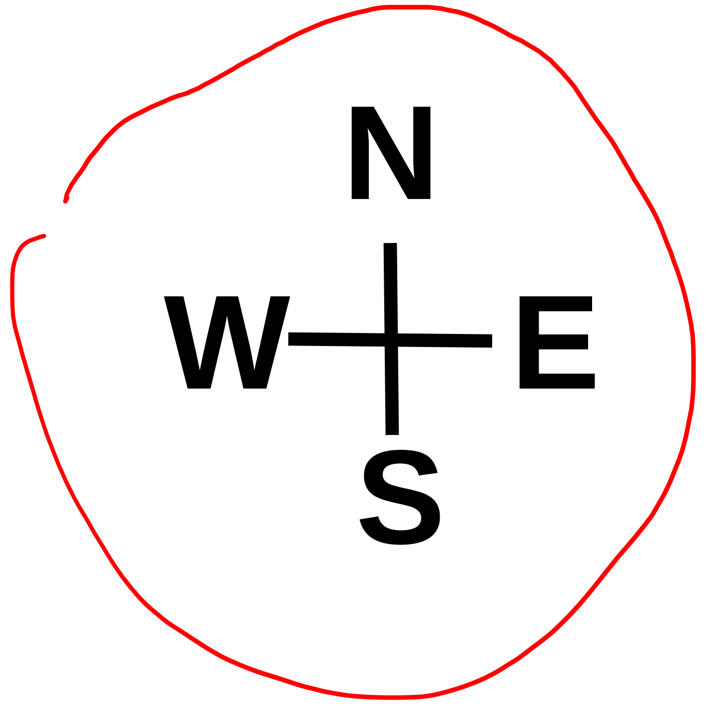
B The duck is 35° to the north of east from the cat.

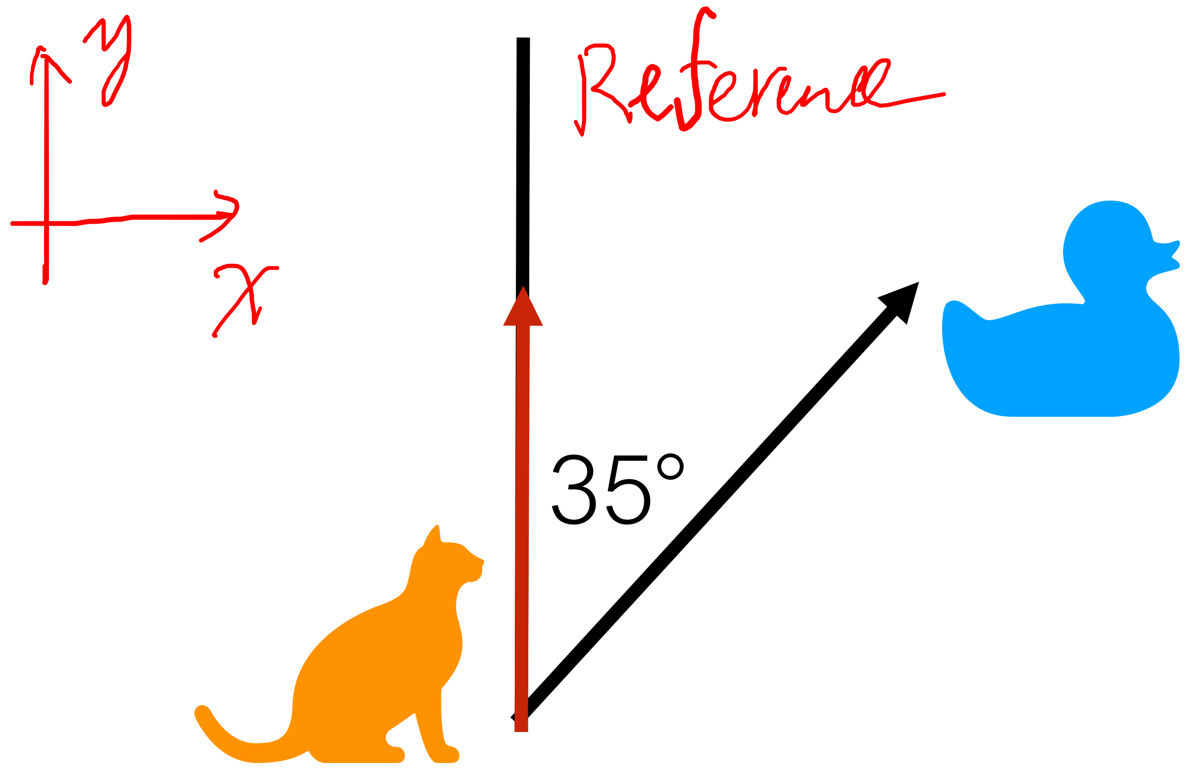
C The duck is 35° to the south of east from the cat.

Reference

D The duck is 35° to the west of north from the cat.

Clicker question 7





- Which of the following is correct?

A The duck is 35° to the south of east from the cat.

B The duck is 35° to the east of north from the cat.

C The duck is 35° to the north of east from the cat.

D The duck is 35° to the west of north from the cat.

Summary of chapter 3

- Learning objectives
  - Vectors: Magnitude (size) and direction
  - Vector decomposition
  - Vector addition, vector scaling
  - Properties of vector addition: Commutative and associative
  - Vector multiplication:
    - ❖Vector scaling, vector multiplied by a scalar;
    - ❖dot product, $\vec{vector_1} \cdot \vec{vector_2}$;
    - ❖cross product, $\vec{vector_1} \times \vec{vector_2}$
  - Properties of dot product: Commutative
  - Properties of cross product: **Anti-commutative, and not associative**

Chapter 4: Motion in two and three dimensions

- Kinematics in two and three dimensions
- Projectile motion
- Uniform circular motion
- Relative motion and reference frames

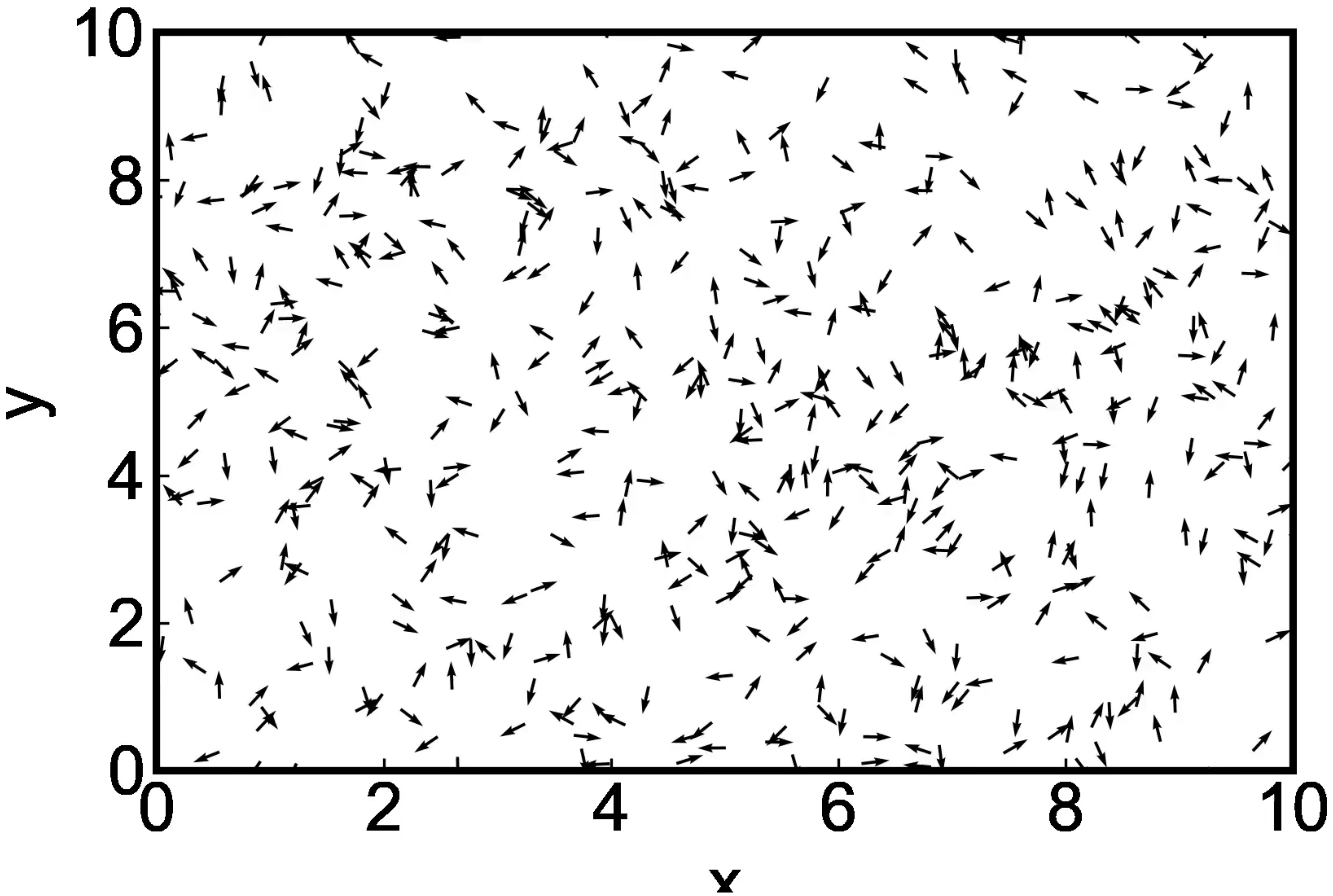
Learning goals for today

- Decompose motions in 2D (& 3D) to 1D motion components
- Projectile motion

Motions in 2D and 3D

- Flocking birds in 3D
- Simulation in 2D

https://youtu.be/0dskCpuxqtl



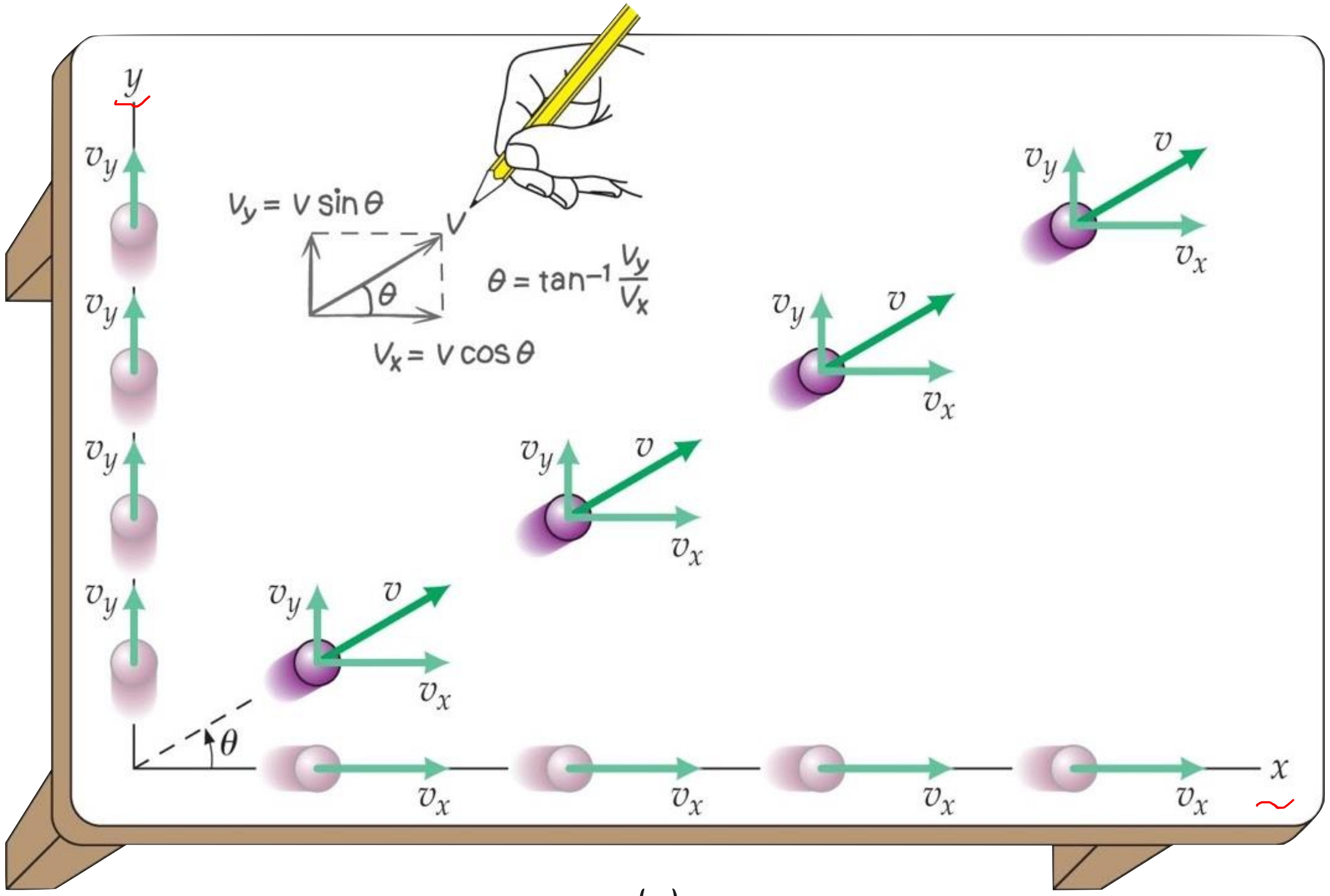
Data from own group

Motion in 2 dimensions

- Horizontal & vertical motions
  - Break vectors into components
  - Treat each component separately

$\vec{v} = v_x \hat{i} + v_y \hat{j}$

Equation depends on where θ is!



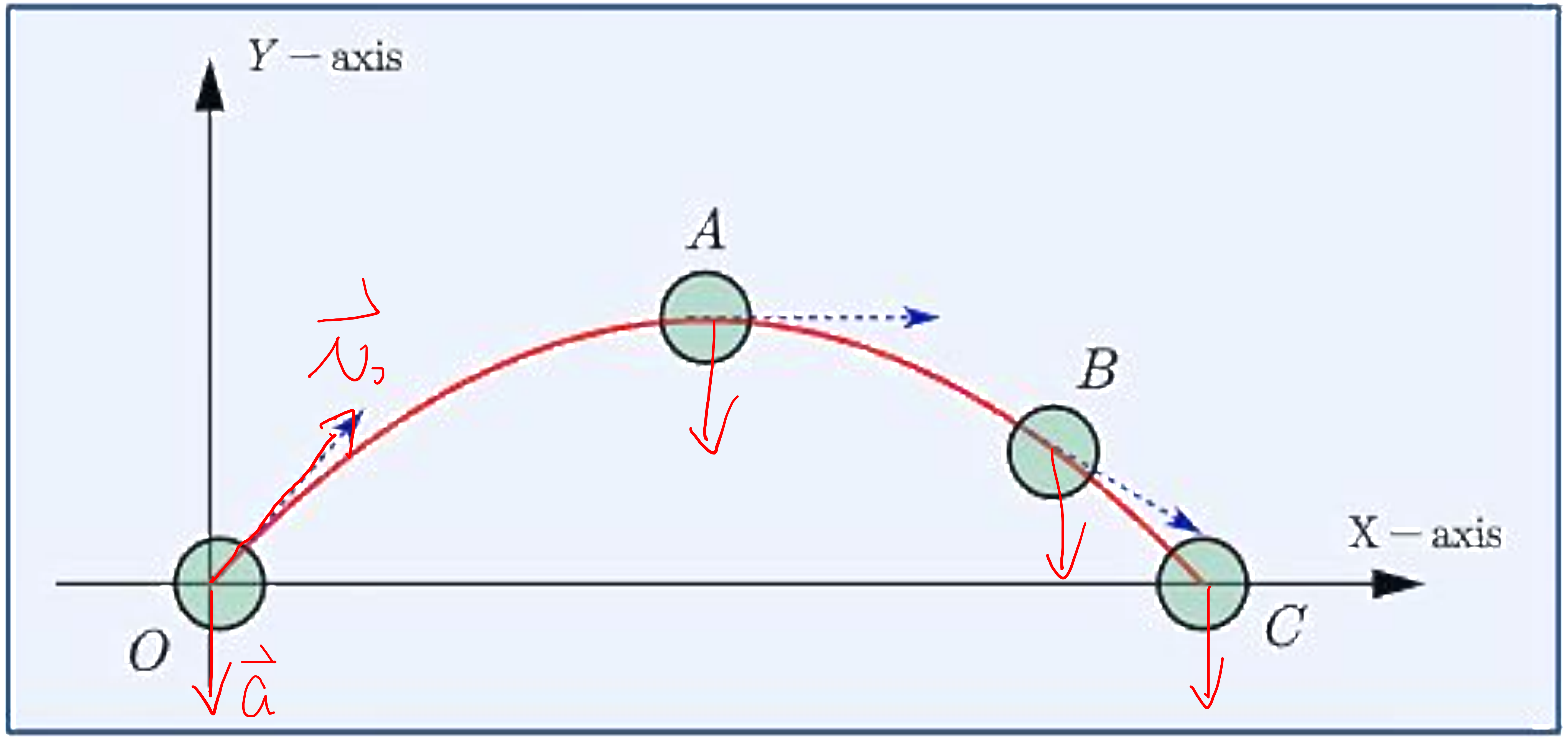
(a)

Curvilinear motion

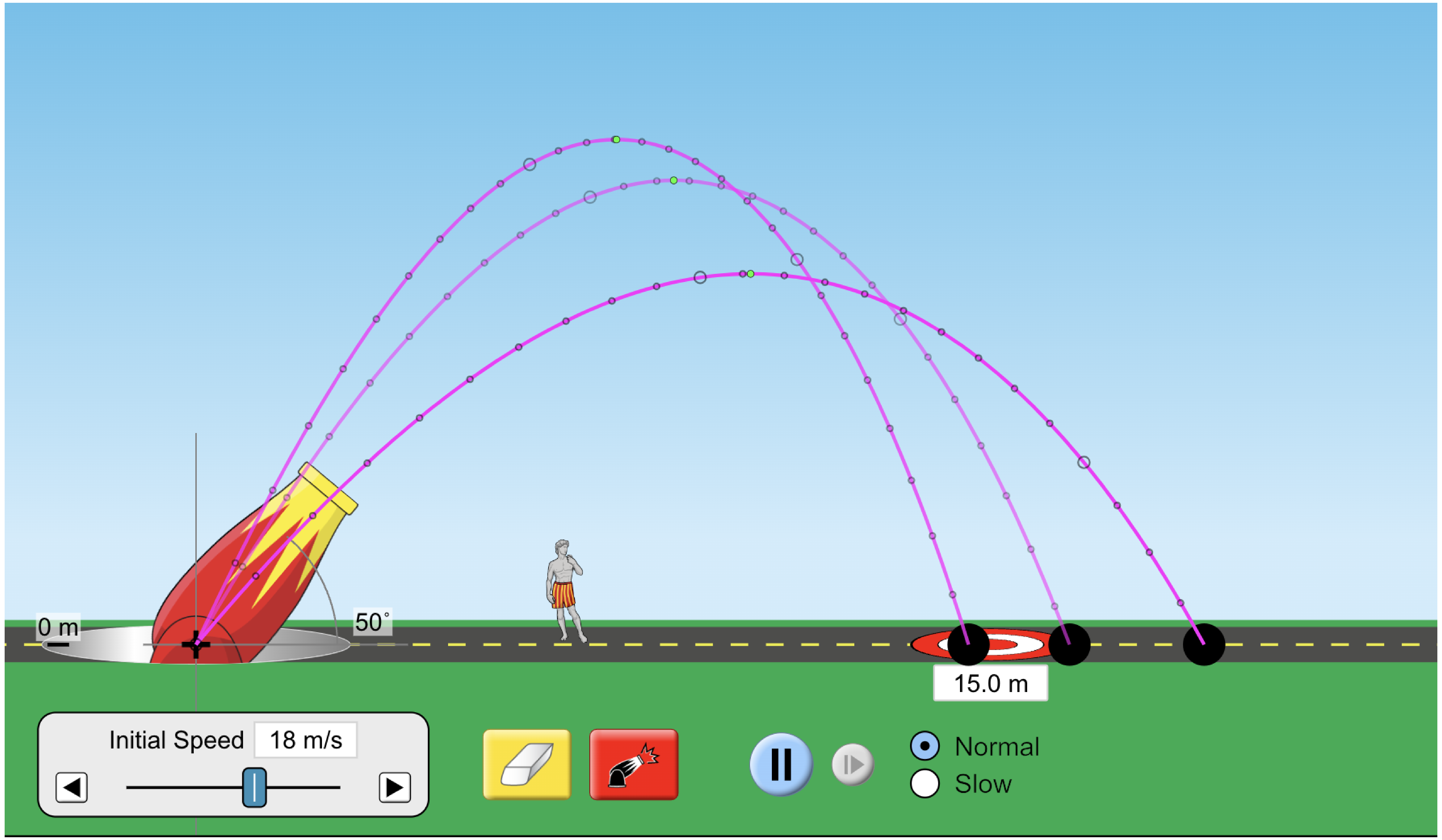
If $\vec{v}_0$ & $\vec{a}$ aren't parallel

$\vec{v}_0$ $\vec{a}$

- If the initial velocity and acceleration are not parallel, then motion is along a curve (“curvilinear motion”)



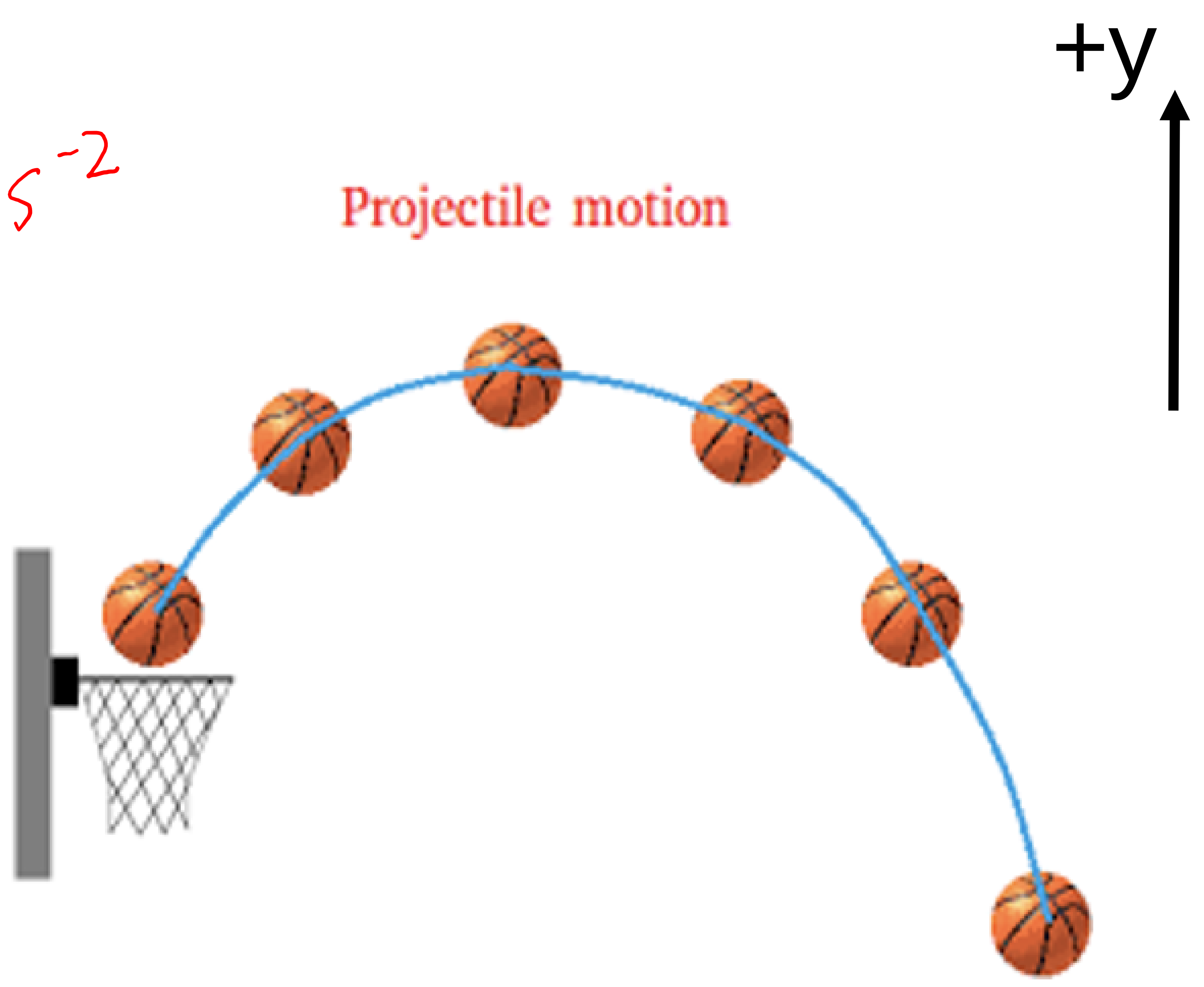
Demo



https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_all.html .

Projectile motion

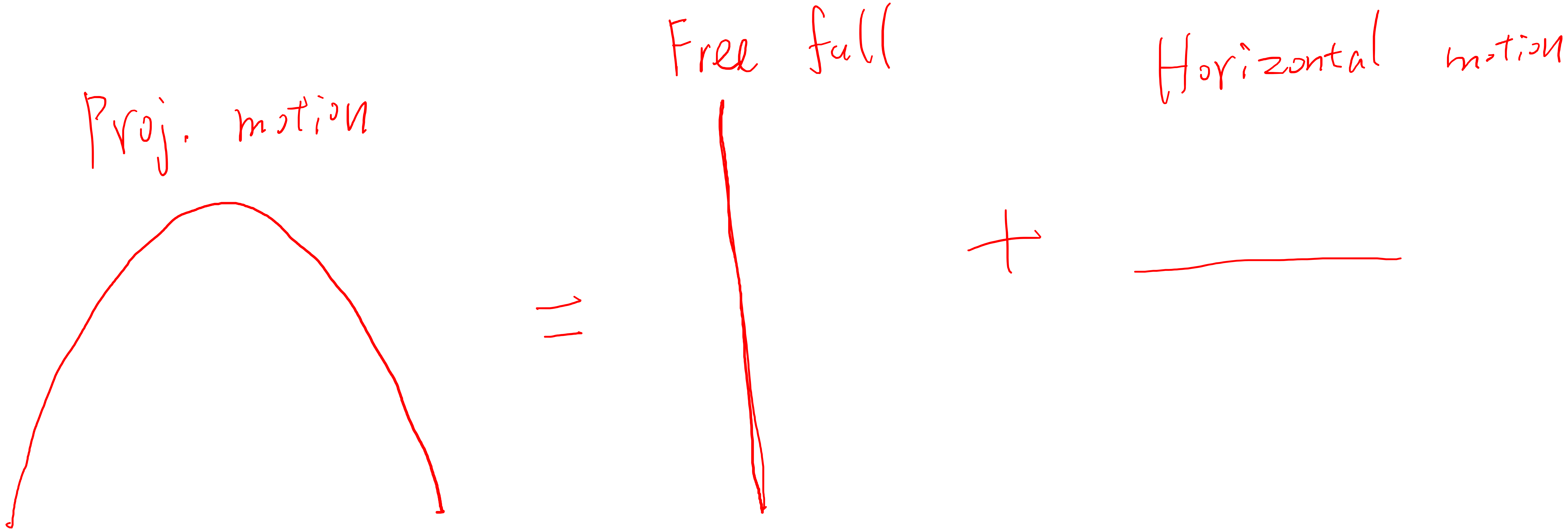
$g = 9.8\ ms^{-2}$



- A projectile motion is defined by the following conditions:
  - The acceleration is a constant: $\vec{a} = -g\hat{j}$
  - The initial velocity is not parallel to the acceleration

$\vec{v}_0$

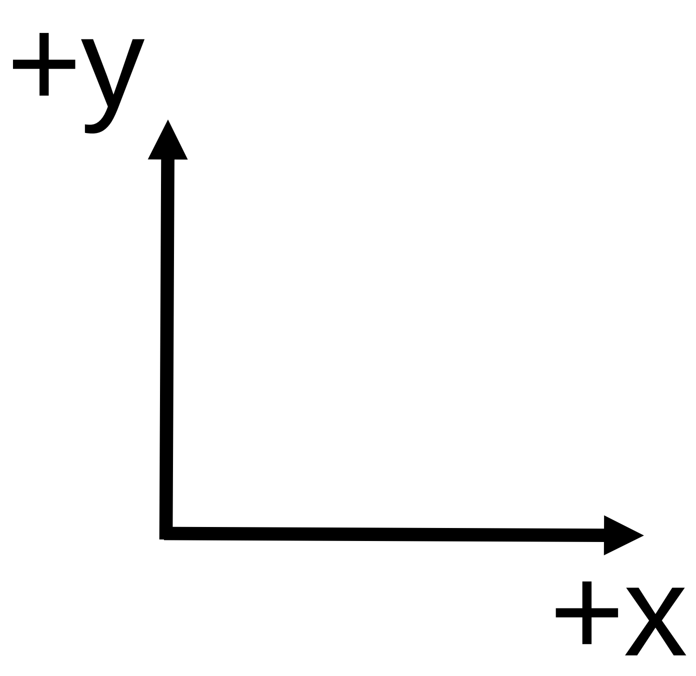
Demo

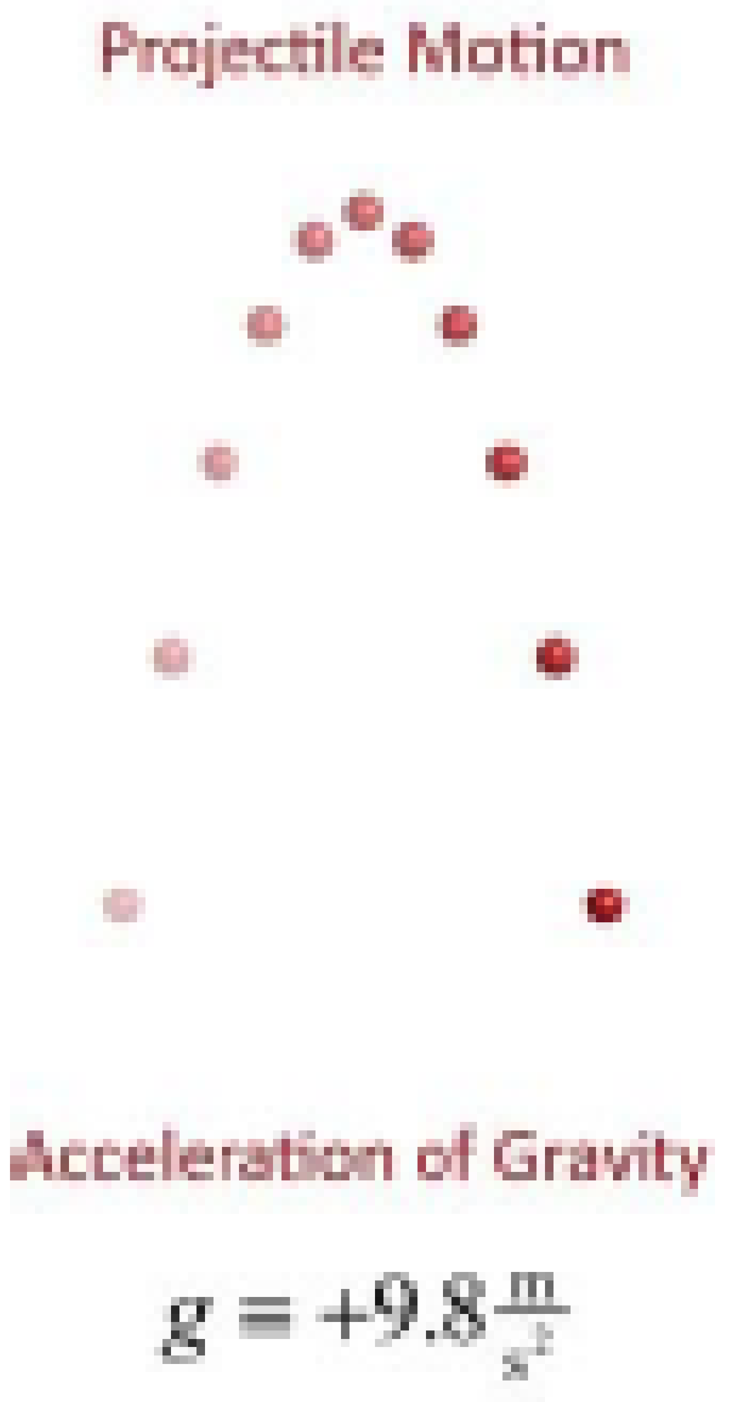


$v_x = const$, $\vec{a}_x = \frac{d\vec{v}_x}{dt} = 0$

Projectile motion — Main points

- Projectile motion is the superposition of two independent motions:
  1. Vertical motion: constant acceleration
  2. Horizontal motion: constant velocity





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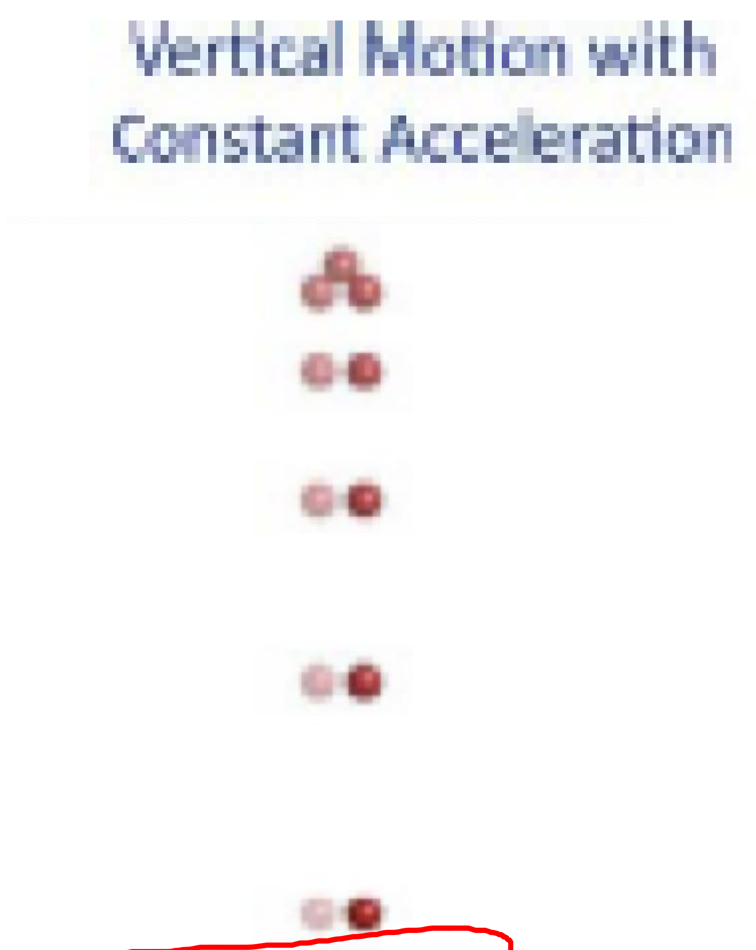

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Vertical and horizontal motions are connected by the time!

t

$$\boxed{a_y\hat{j} = -g\hat{j}}$$

$$v_y\hat{j} = (v_{y0} - gt)\hat{j}$$

$$y\hat{j} = \left(y_0 + v_{0y}t - \frac{1}{2}gt^2\right)\hat{j}$$

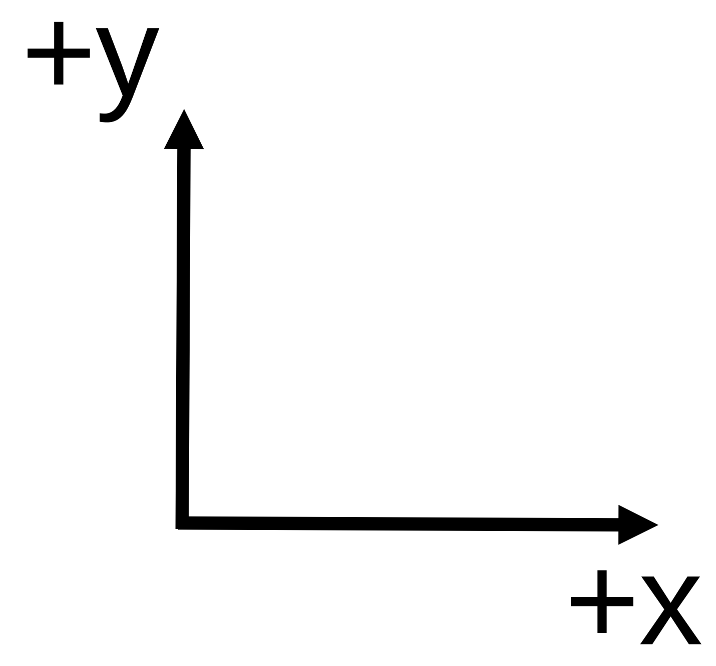
$$\boxed{a_x = 0}$$

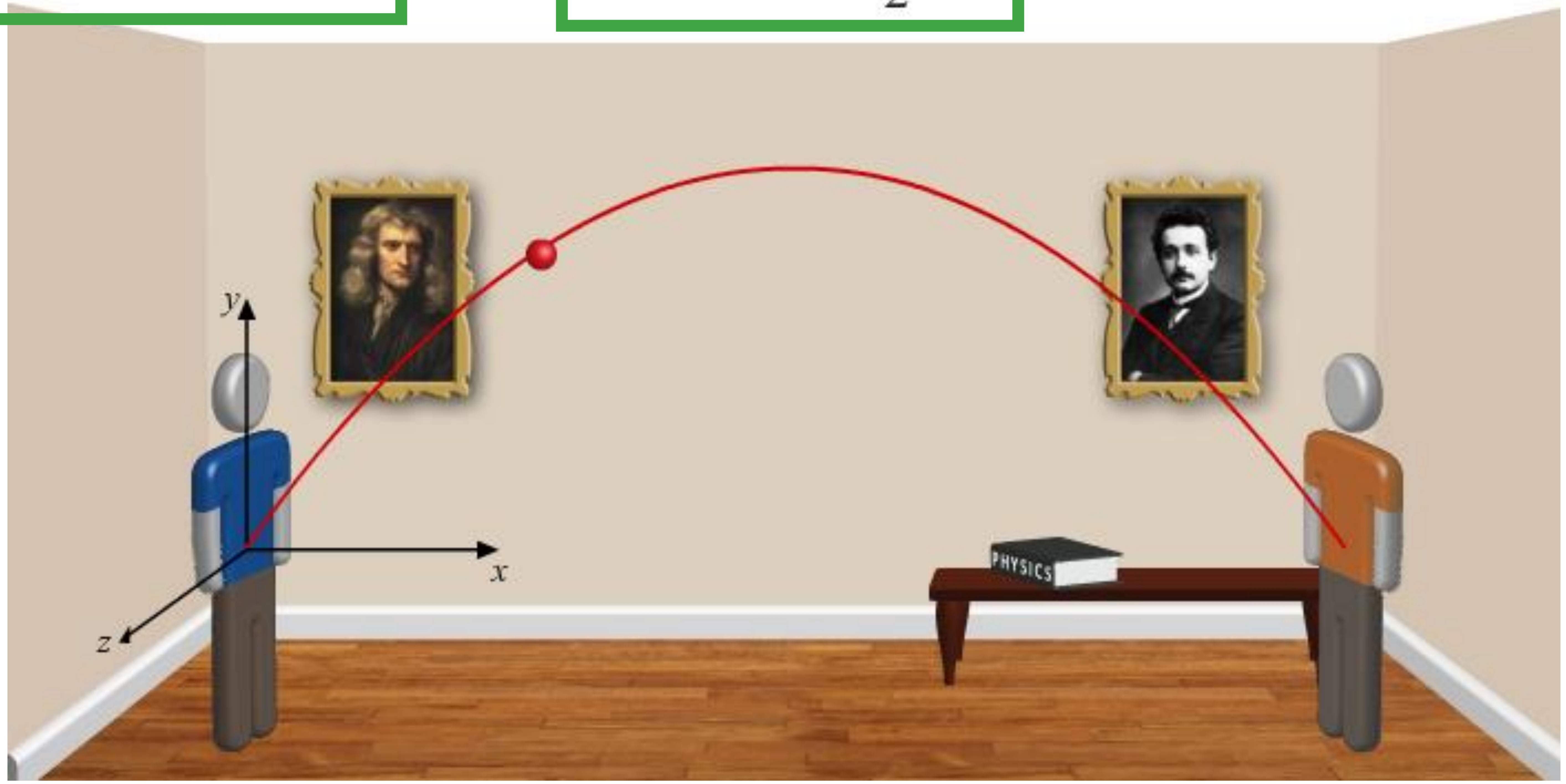
$$v_x\hat{i} = v_{x0}\hat{i}$$

$$x\hat{i} = (x_0 + v_{0x}t)\hat{i}$$

Projectile Motion in 3D

Horizontal	Vertical	Boring
$a_x = 0$	$a_y = -g$	$a_z = 0$
$v_x = v_{o_x}$	$v_y = v_{o_y} - gt$	$v_z = 0$
$x = x_o + v_{o_x} t$	$y = y_o + v_{o_y} t - \frac{1}{2} g t^2$	$z = z_o$





Clicker question 1

Given: Δy, $\vec{a}$, $\vec{v}_{0y}$

Goal: t

- A small ball is released from at rest at 1.17 m high from the ground. How long (in time) is the ball in the air?

  Which equation to use?

A y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B x- motion: $\Delta y = v_{0x}t$

Clicker question 2

Given: Δy, Δx, a_x, a_y, v_{y_0}

Goal: Δt

$v_{y_0} = 0$

- A small ball rolls horizontally off the edge of a tabletop that is 1.17 m high. It strikes the floor at a point 1.61 m horizontally away from the edge of the table. How long (in time) is the ball in the air?

  Which equation to use?

A y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B x- motion: $\Delta y = v_{0x}t$

Example 1

Given: Δy, Δx, a_x, a_y, $v_{y_0} = 0$

Goal: Δt, v_{x_0}

- A small ball rolls horizontally off the edge of a tabletop that is 1.17 m high. It strikes the floor at a point 1.61 m horizontally away from the edge of the table. **(a)** How long is the ball in the air? **(b)** What is its speed at the instant it leaves the table?

a) Step 1: $\Delta y = \underbrace{v_{y_0}}_{=0} \Delta t - \frac{1}{2} g \Delta t^2 \rightarrow \Delta t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2 \times (-1.17\,m)}{9.8\,m\,s^{-2}}} \approx 0.489\,s$

b) Step 2: $\Delta x = v_{x_0} \Delta t \rightarrow v_{x_0} = \frac{\Delta x}{\Delta t} = \frac{1.61\,m}{0.489\,s} \approx 3.29\,m\,s^{-1}$

t connects x & y motions!

Clicker questions 3

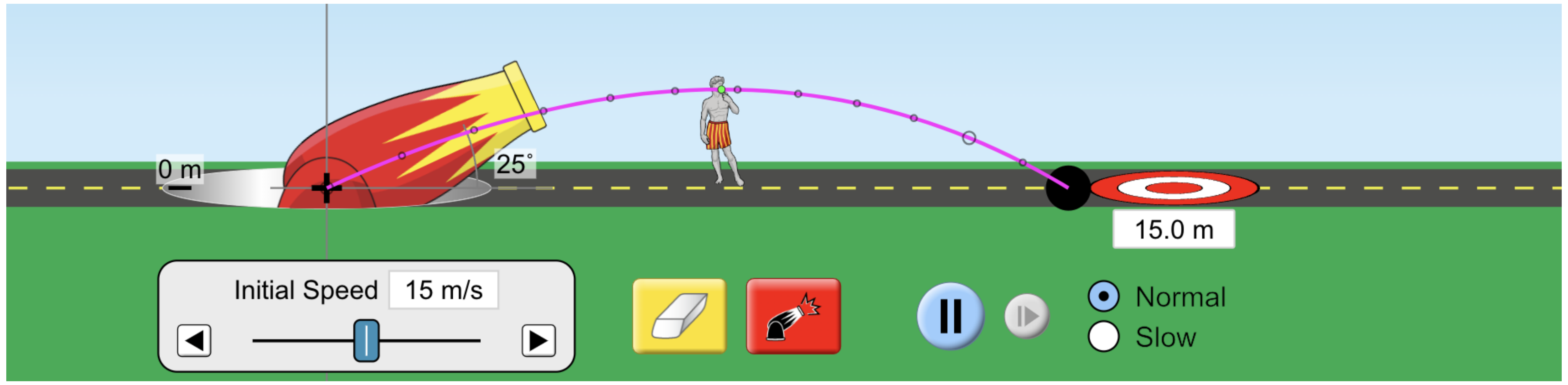
- How are the horizontal and vertical components of the projectile motion related to each other?

A The two components of the motion are not related in any way.

B The two components of the motion share the same initial and final time.

In-class activity (2-3 people)

- Try to make it to the target by adjusting:
  - The initial speed
  - The aiming angle

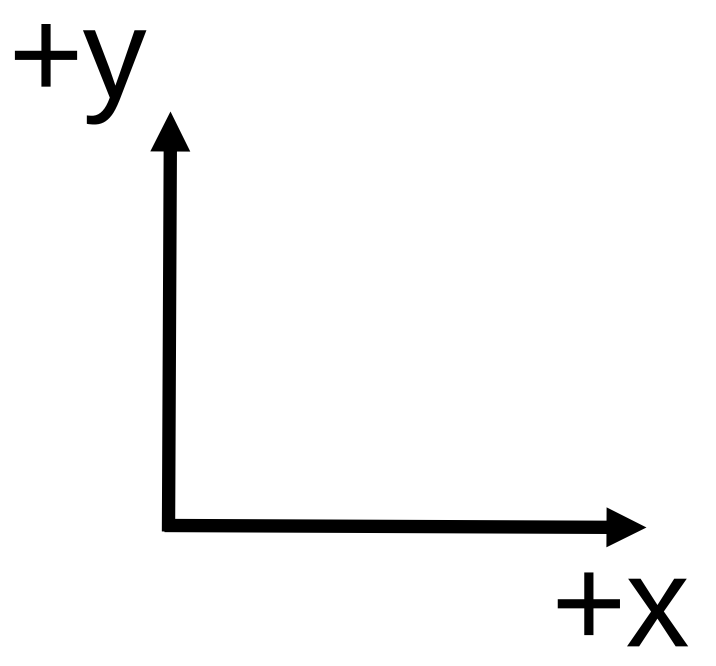


https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_all.html

Example 2

Given: $|\vec{v}_0| = \text{const}$, $\Delta y = 0$, $\vec{a}$

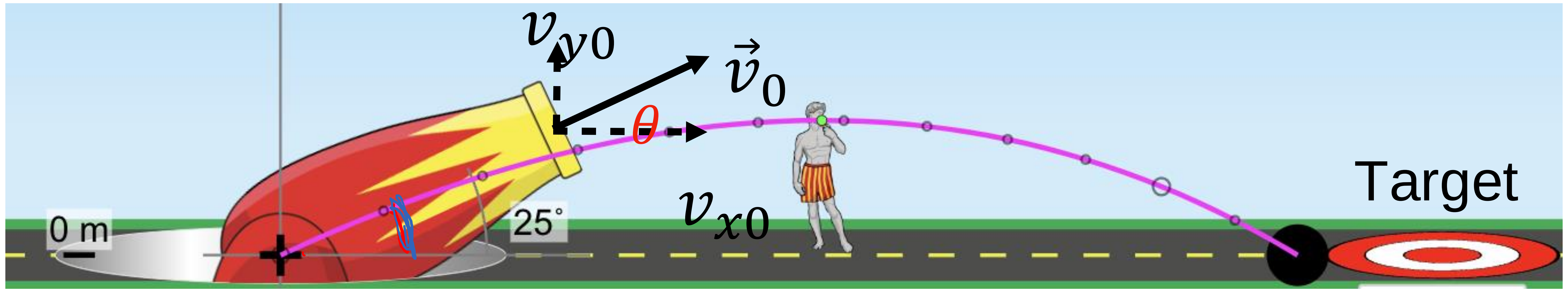
Goal: θ s.t. $\Delta x = \max(\Delta x)$.



- The cannon is aimed at an angle θ above the +x direction at an initial speed of $|\vec{v}_0|$ .Suppose the initial speed of the cannon, $|\vec{v}_0|$ is fixed. What is the aiming angle θ (the angle between $\vec{v}_0$ and +x direction) to make it farthest in the horizontal direction? (Assume the initial and final y-coordinates are the same.)

Step 1: $\Delta x = v_{x0} \Delta t$

$= |\vec{v}_0| \cos\theta \Delta t$



Step 2: $\Delta y = v_{y0} \Delta t - \frac{1}{2} g \Delta t^2$

$\Delta y = 0$, $v_{y0} = |\vec{v}_0| \sin\theta$

$\rightarrow (|\vec{v}_0| \sin\theta - \frac{1}{2} g \Delta t) \Delta t = 0$

$\rightarrow \Delta t = \frac{2 |\vec{v}_0| \sin\theta}{g}$

Step 3: Eliminate Δt from Step 2

$\Delta x = |\vec{v}_0| \cos\theta \Delta t = |\vec{v}_0| \cos\theta \cdot \frac{2 |\vec{v}_0| \sin\theta}{g}$

$\boxed{2\cos\theta \sin\theta = \sin 2\theta}$

$= \frac{|\vec{v}_0|^2 \sin 2\theta}{g}$

$\max(\Delta x)$ means $\max(\sin 2\theta)$, when $\theta = 45^\circ$