PHYS 225
Fundamentals of Physics: Mechanics

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Fall 2024

Lecture 24: Conservation of energy | Center of mass

CALIFORNIA STATE UNIVERSITY FULLERTON

Learning goals for today

- Conservation of energy
- Center of mass – Ch. 9

Chapter 8.2. Conservation of energy

3. Conservation of mechanical energy — Conditional.

- When only **conservative forces do work** on the system**,** then the **mechanical energy,** $K + U$**, is conserved:**

When only $\vec{F}_{cons}$'s do work

$$K + U = const$$

Kinetic potential

That is, $$K_i + U_i = K_f + U_f$$

initial final

Conservation of total energy in general

- When there is work by **friction**, inelastic deformation, chemical reaction, etc.
  - Then the **mechanical energy** is not conserved
- However, **energy** can’t be created or destroyed, so…
  - **The *total* energy is conserved**
    i.e., $E = \underbrace{K + U_{pot}}_{E_{Mech}} + U_{thermal} + \cdots = const$
  - But energy can be *converted* between different forms
- Demo

https://youtu.be/ZvgJ7mVxeg0

Clicker question 9

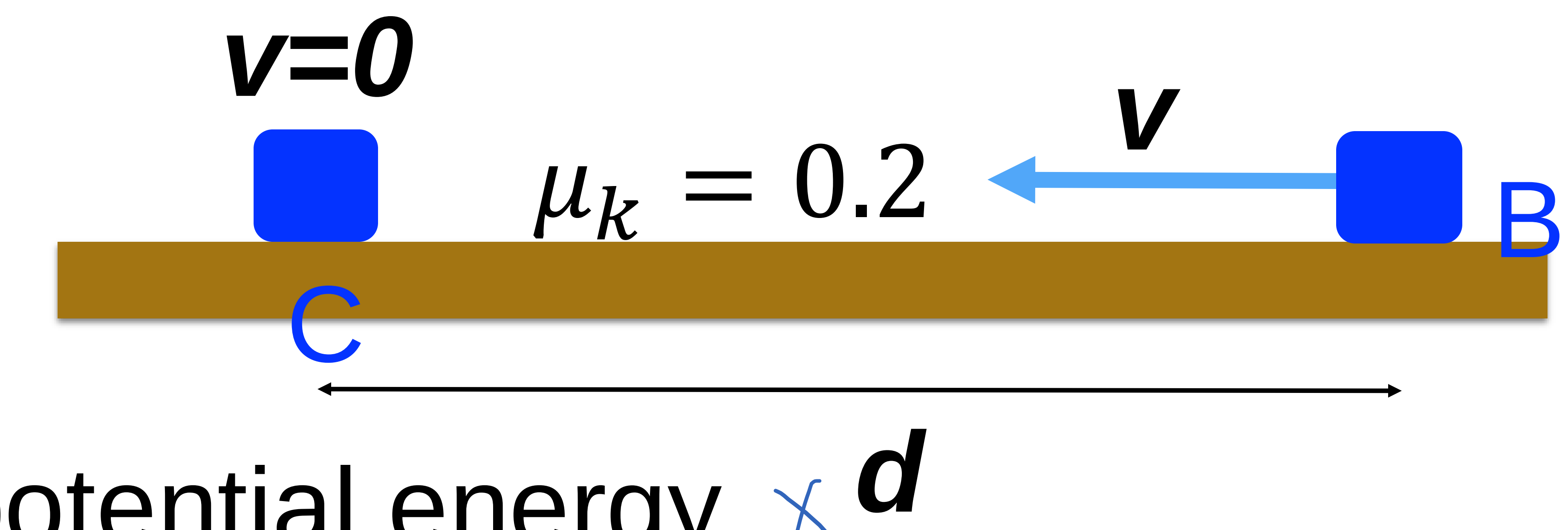
- A box of initial speed $v > 0$ slides on a horizontal surface with kinetic friction coefficient μ_k for a distance d and stops. Which of the following is true?

A The mechanical energy is conserved.

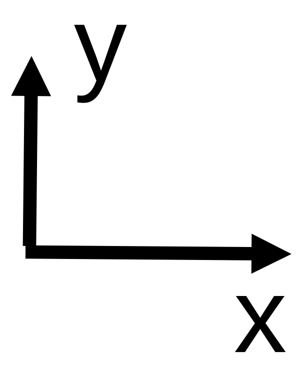
B The total energy is not conserved.

C The kinetic energy is transformed to the potential energy.

D The kinetic energy is transformed to the thermal energy.



Clicker question 10

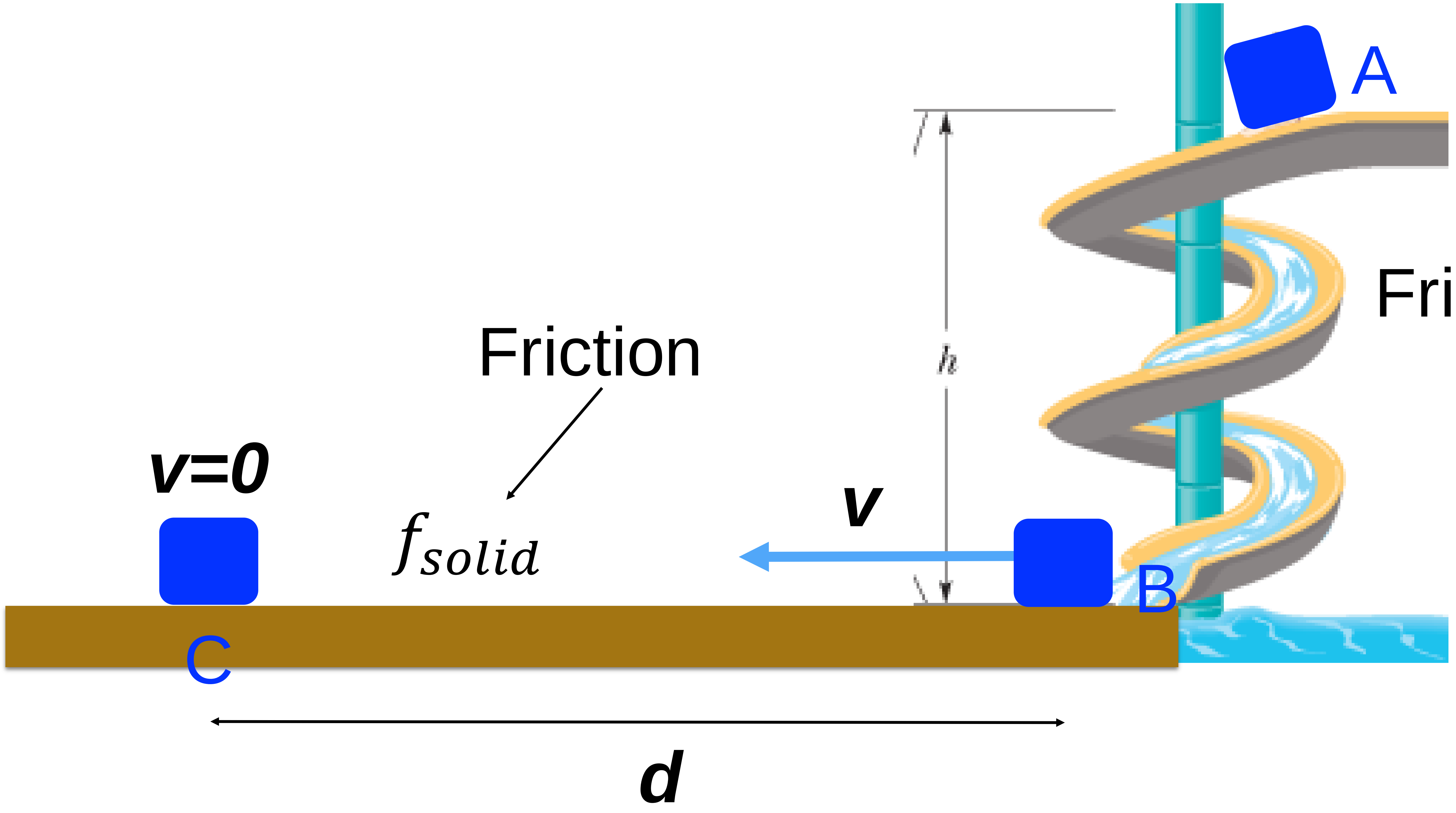


- The blue box of mass m begins at rest at A, then slides down to the end of the ~~***frictionless***~~ spiral, B, continues to move in a flat surface with friction $\vec{f}_{solid}$, and stops at C. In which part is the **mechanical energy** conserved?

A AB

B BC

C AC



$$E_{mechA} = mgh$$

Example 4

Given: $m, h_{BA} = h, \mu_{AB} = 0, \mu_{k,BC} = \mu_k, |v_C| = 0, v_A = 0$

Goal: $|v_B|, |\Delta x_{BC}| = d, U_{th,BC}$

$\xrightarrow{\hspace{1cm}} x$ $\quad +y, +x$

- The box of mass m = 1kg begins at rest at A of height h = 2 m from ground, then slides down to the end of the **frictionless** spiral, B, continues to move in a ***horizontal*** surface with friction coefficient $\mu_k = 0.2$, and stops at C, a distance of d from B. Please find **1)** the speed of the box at the bottom of the spiral, v; **2)** the distance d; and **2)** how much energy is transferred to thermal energy from B to C.

Step1: AB: $U_A + \underset{0}{K_A} = \underset{0}{U_B} + K_B \rightarrow U_A = K_B \rightarrow mgh = \frac{1}{2} m v_B^2$.

$v_B = \sqrt{2gh} = \sqrt{2 \times 9.8\,m s^{-2} \times 2m} \approx 6.26\ m s^{-1}$

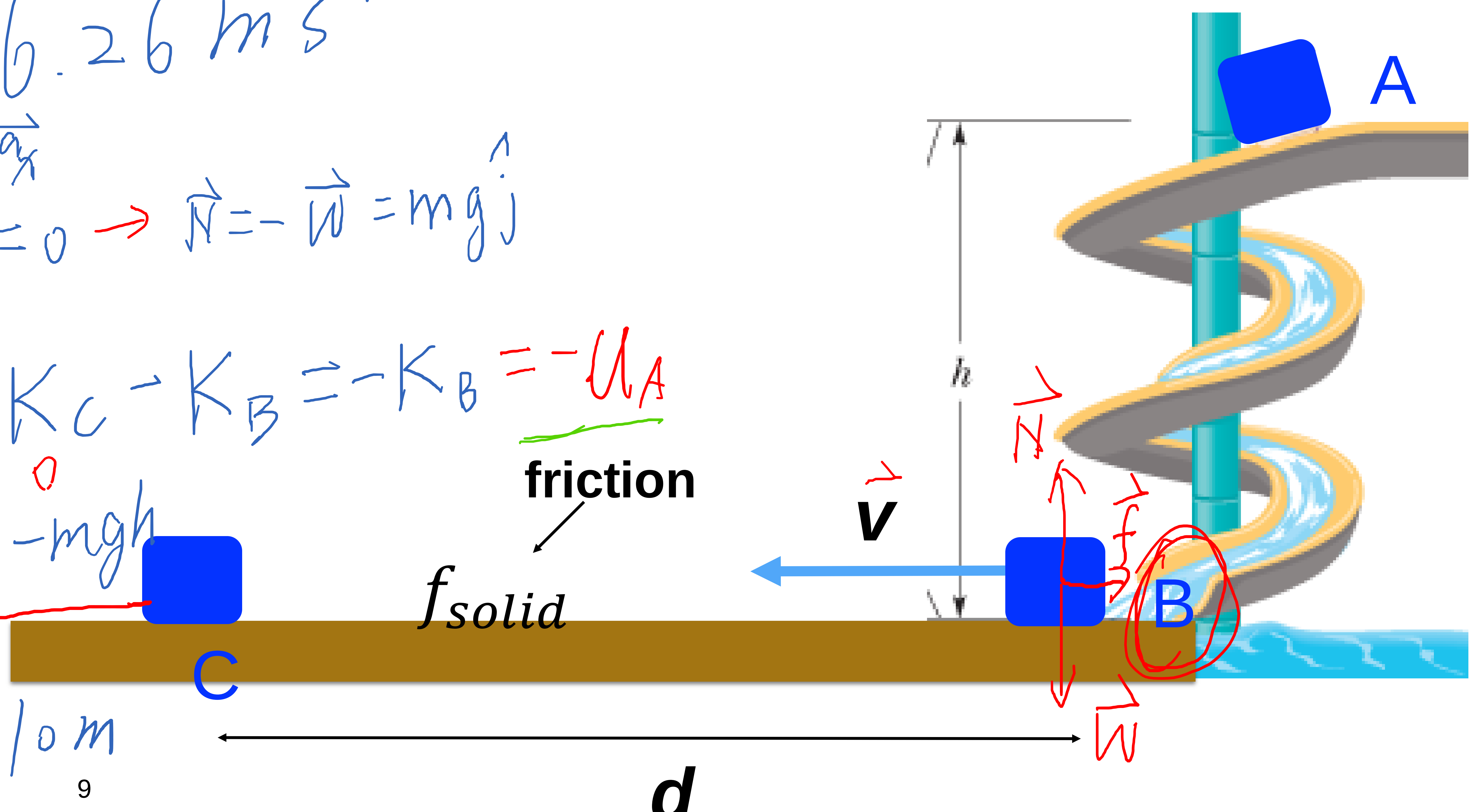
Step2, FBD, Newton's 2nd law: $\begin{cases} \vec{f} = m \vec{a}_x \\ \vec{W} + \vec{N} = 0 \rightarrow \vec{N} = -\vec{W} = mg\hat{j} \end{cases}$

$|\vec{f}| = \mu_k |\vec{N}| = \mu_k mg$

Step3, W-K theorem: $W = \Delta K = \underset{0}{K_C} - K_B = -K_B = -U_A$

$W = \vec{f} \cdot \Delta\vec{x} = |\vec{f}|\, d \cos\theta = -|\vec{f}|\, d = -mgh$

$\rightarrow \mu_k \cancel{m}\cancel{g}\, d = \cancel{m}\cancel{g}\, h \rightarrow d = \frac{h}{\mu_k} = \frac{2m}{0.2} = 10\,m$



Example 4: continued

x

- The box of mass m = 1kg begins at rest at A of height h = 2 m from ground, then slides down to the end of the **frictionless** spiral, B, continues to move in a ***horizontal*** surface with friction coefficient $\mu_k = 0.2$, and stops at C, a distance of d from B. Please find **1)** the speed of the box at the bottom of the spiral, v; **2)** the distance d; and **3s)** how much energy is transferred to thermal energy from B to C.

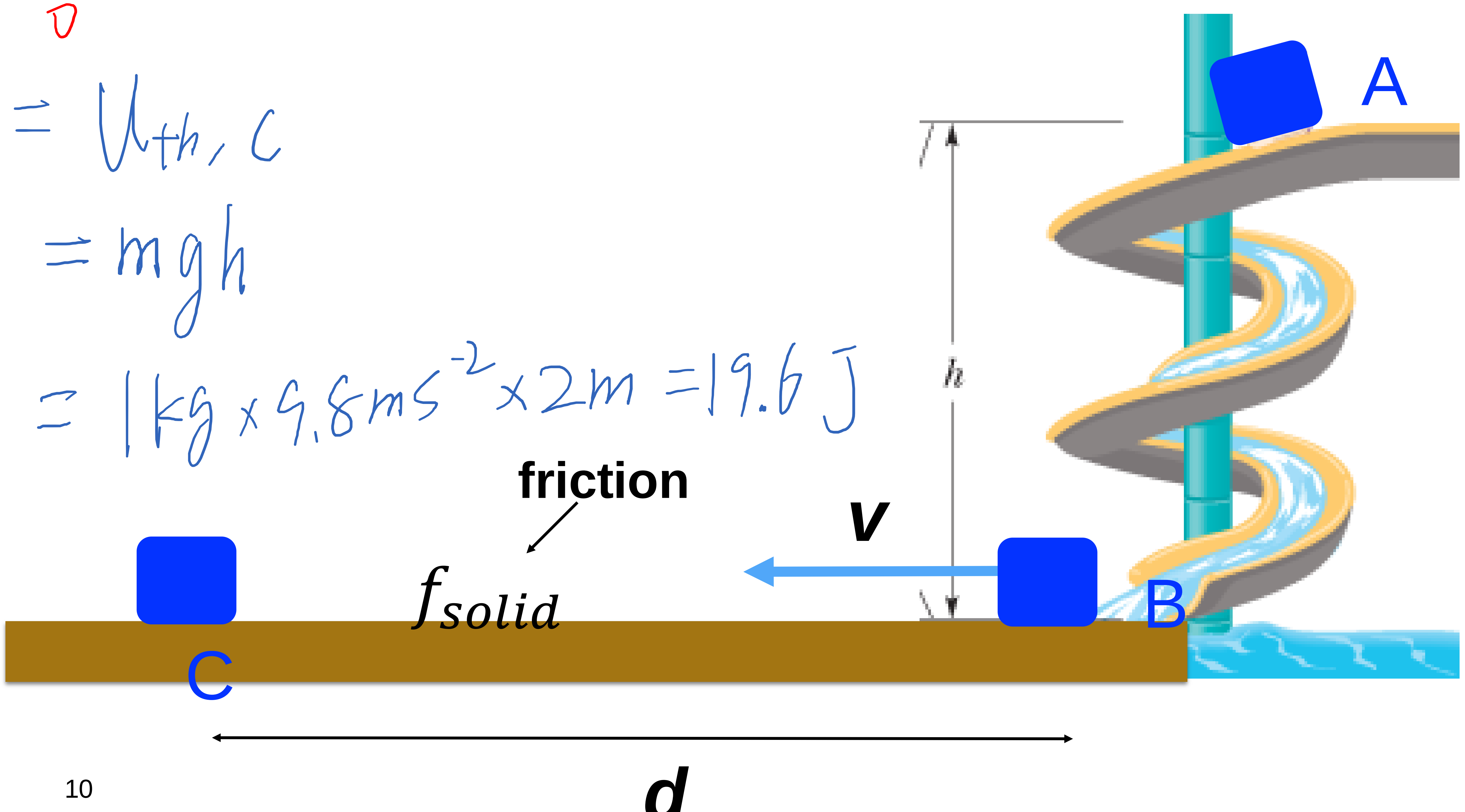
Step 4: $K_B + \underset{0}{U_B} + U_{th,B} = \underset{0}{K_C} + \underset{0}{U_C} + U_{th,C}$ $\quad U_{th,BC}$

Step 1: $U_A = K_B \rightarrow U_A + U_{th,B} = U_{th,C}$

$\Delta U_{th} = U_{th,C} - U_{th,B} = U_A = mgh$

$= 1kg \times 9.8ms^{-2} \times 2m = 19.6 J$

Step 5:



c

Summary of chapter 8

- Learning objectives
  - Concepts:
    - Conservative force: Work done by $\vec{F}_{cons}$ is path independent: $W_{path1} = W_{path2}$
    - potential energy: Change: $\Delta U(\vec{r}) = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{conservative} \cdot d\vec{r} = -W_{conservative}$
    - Conservation of mechanical energy (when only conservative forces do work):

$$K_f + U_f = K_i + U_i$$

    - Conservation of energy (in general): $E_{i,tot} = E_{f,tot}$

Homework 8

- Due this Friday

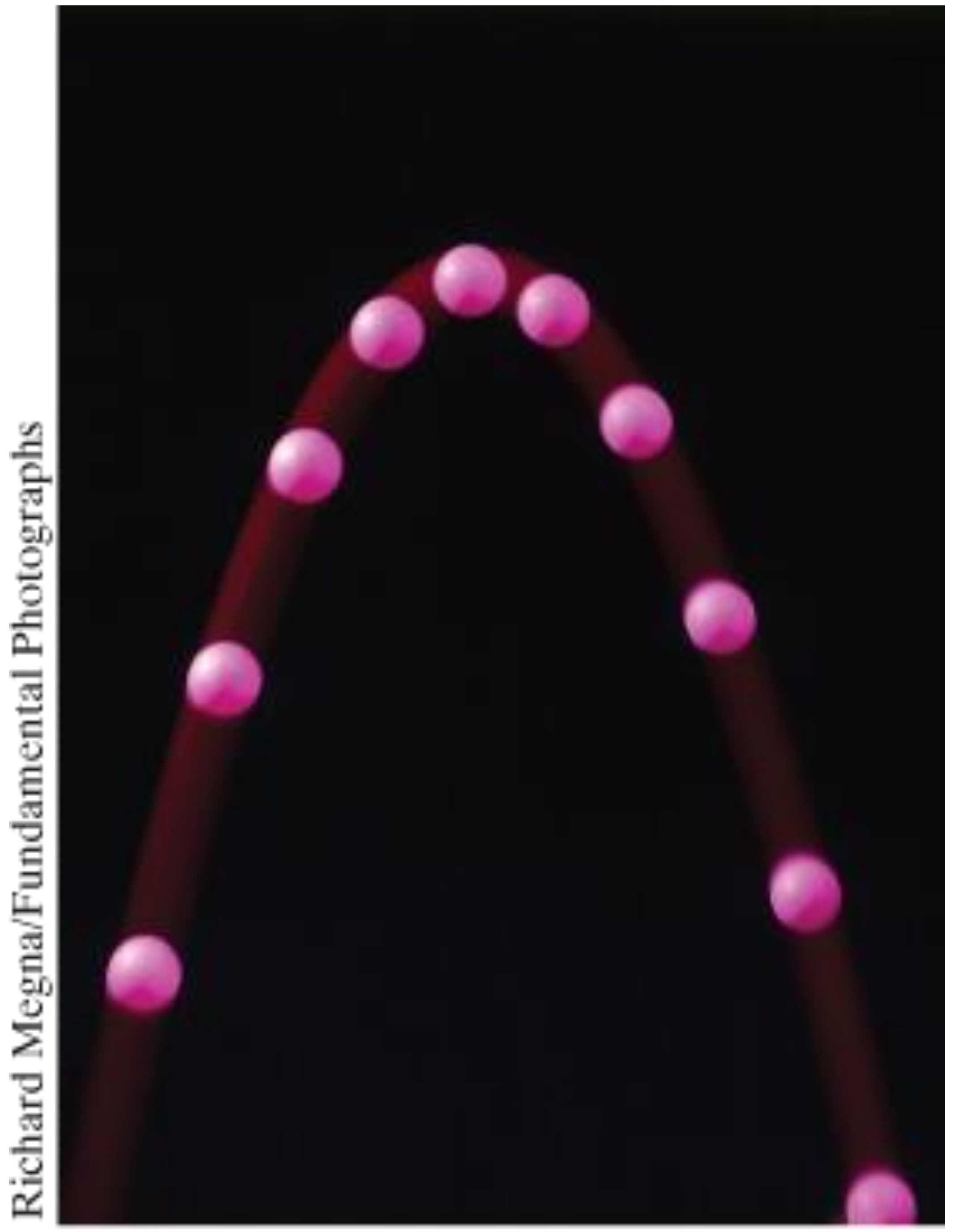
Pre-lecture 9.1.1

- Please complete Pre-lecture in Module 9.1.1 before the next lecture.

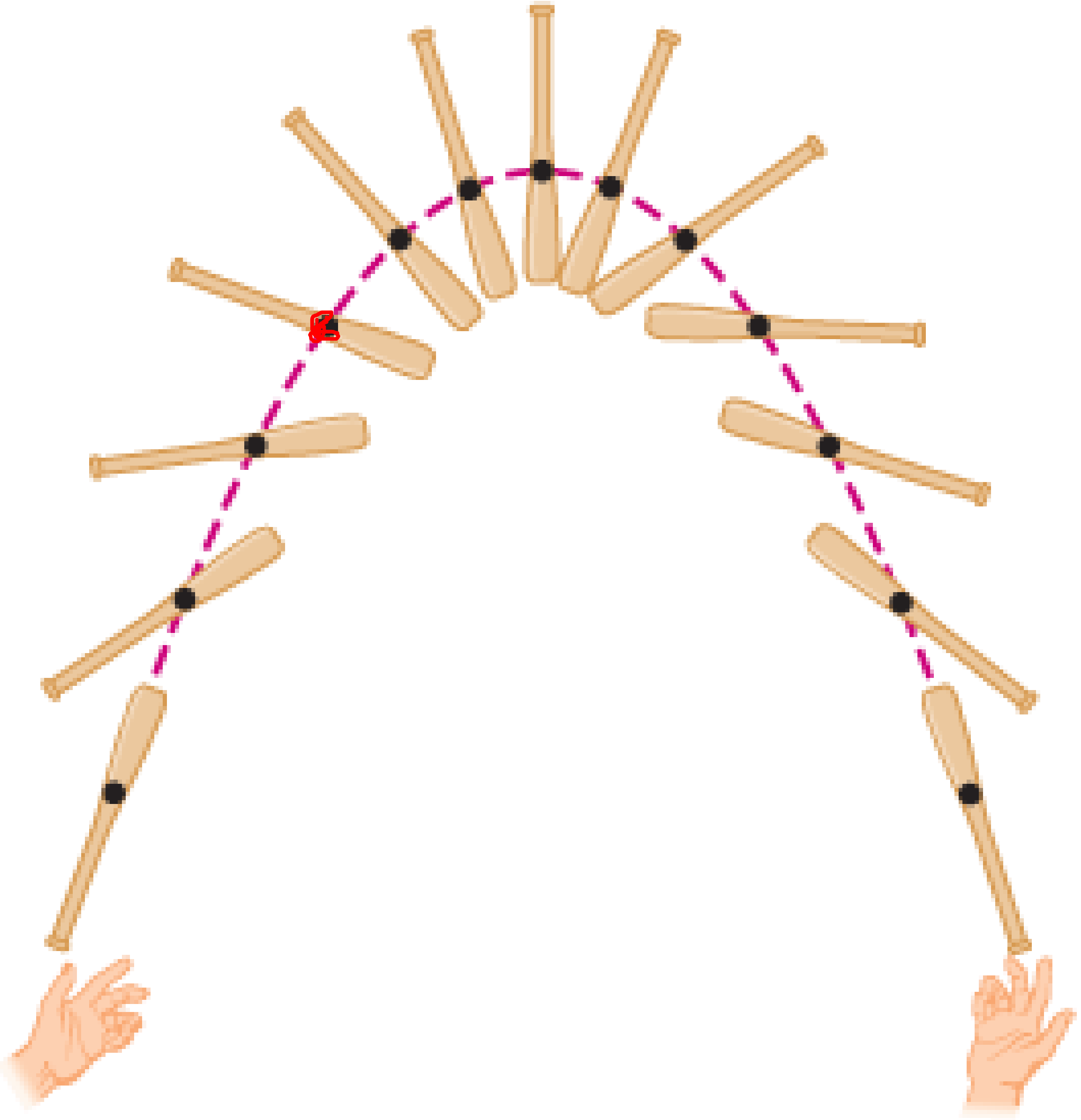
Chapter 9: Center of mass and linear momentum

- Learning objectives
  - Concepts:
    - Center of mass
    - Linear momentum and impulse
    - Conservation of linear momentum
    - Collision

When an object doesn’t move like a point particle



(a)



(b)

Com
Proj. motion

What we learned:
All points in the ball
have the same velocity

What to learn next:
Every point of the bat
has its own velocity

watch the video on WileyPlus

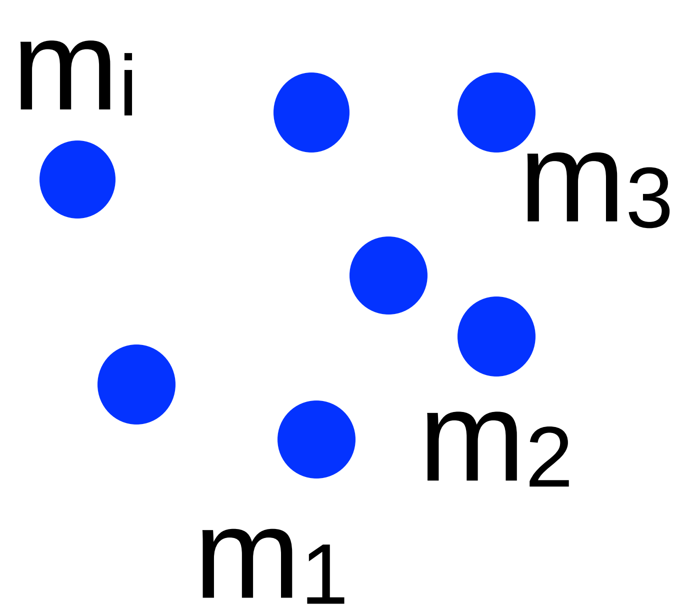
Demo

1. Center of mass (COM) of a system of point particles

- **Definition** of **center of mass (com)**:

  - The mass-weighted average of the **positions** of the objects in the system:

$$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_i\vec{r}_i + \dots}{M_{tot}}$$



A system of point particles

$$\vec{R}_{com} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i}^{N} m_i \vec{r}_i}{M_{tot}}$$

- **SI unit:** meters

Archimedes of Syracuse introduced the concept of center of mass in the 3rd century b.c.e.[1]

[1] Baron, Margaret E., The origins of the infinitesimal calculus, (2004)

Center of mass (COM) of a system of point particles

$$\overrightarrow{r_{COM}} = \frac{1}{M_{tot}} \sum_{i=1}^{n} m_i \overrightarrow{r_i}$$

Components

$$x_{COM} = \frac{1}{M_{tot}} \sum_{i=1}^{n} m_i x_i$$

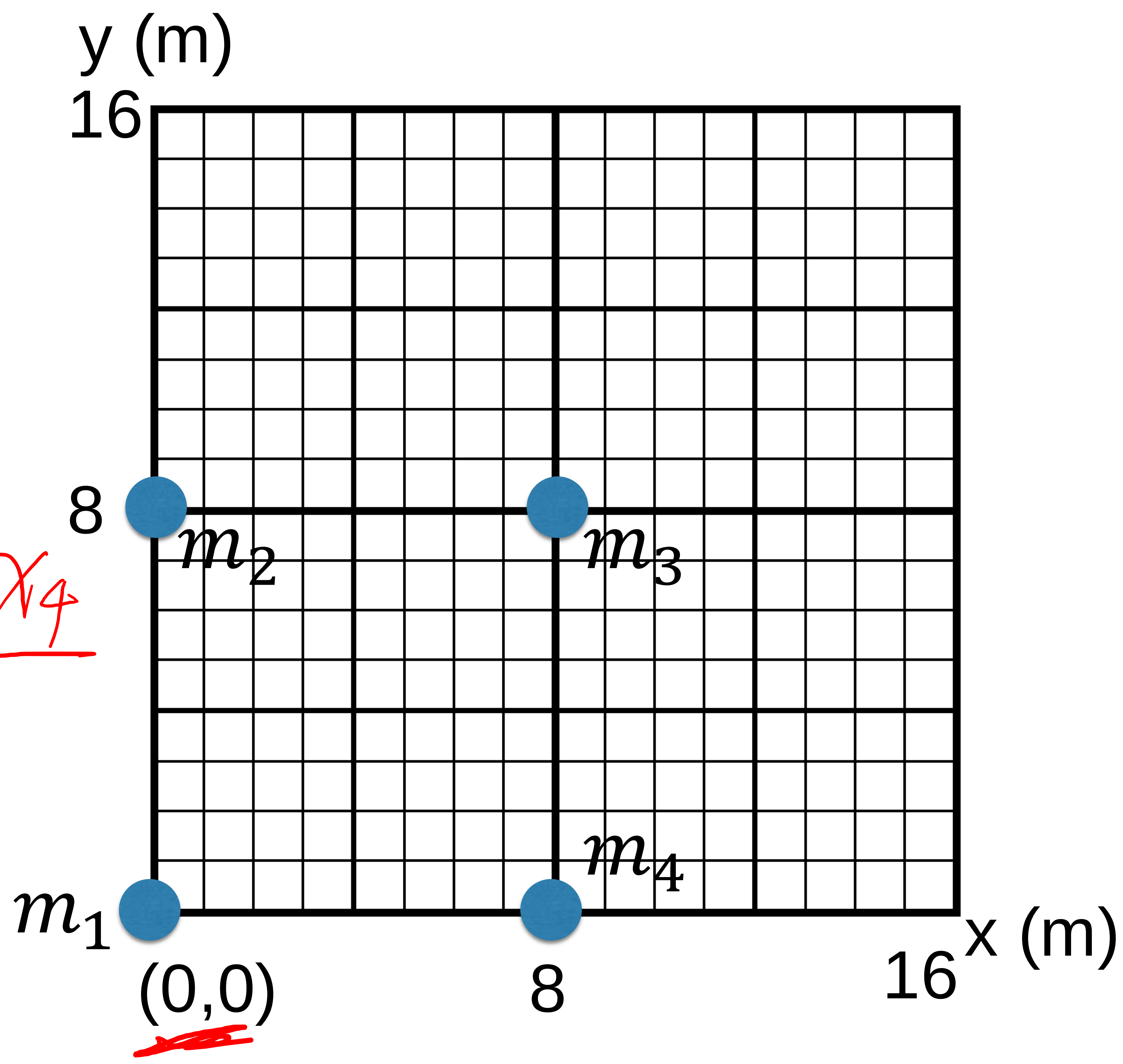
$$y_{COM} = \frac{1}{M_{tot}} \sum_{i=1}^{n} m_i y_i$$

$$z_{COM} = \frac{1}{M_{tot}} \sum_{i=1}^{n} m_i z_i$$

Example 1: 2D COM

2D

- What is the center of mass of the 4-particle system, if $m_1 = m_2 = m_3 = m_4 = 1kg$?



Step1: $\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j}$

Step2: $x_{com} = \dfrac{\sum_{i=1}^{4} m_i x_i}{M_{tot}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$= \dfrac{1kg \times 0 + 1kg \times 0 + 1kg \times 8m + 1kg \times 8m}{1kg \times 4}$

$M_{tot} = 4kg$

$= 4m$

Step3: $y_{com} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{M_{tot}} = \dfrac{1kg(0 + 8m + 8m + 0)}{4kg} = 4m$

Step4: $\vec{r}_{com} = 4m\hat{i} + 4m\hat{j}$

Clicker question 1

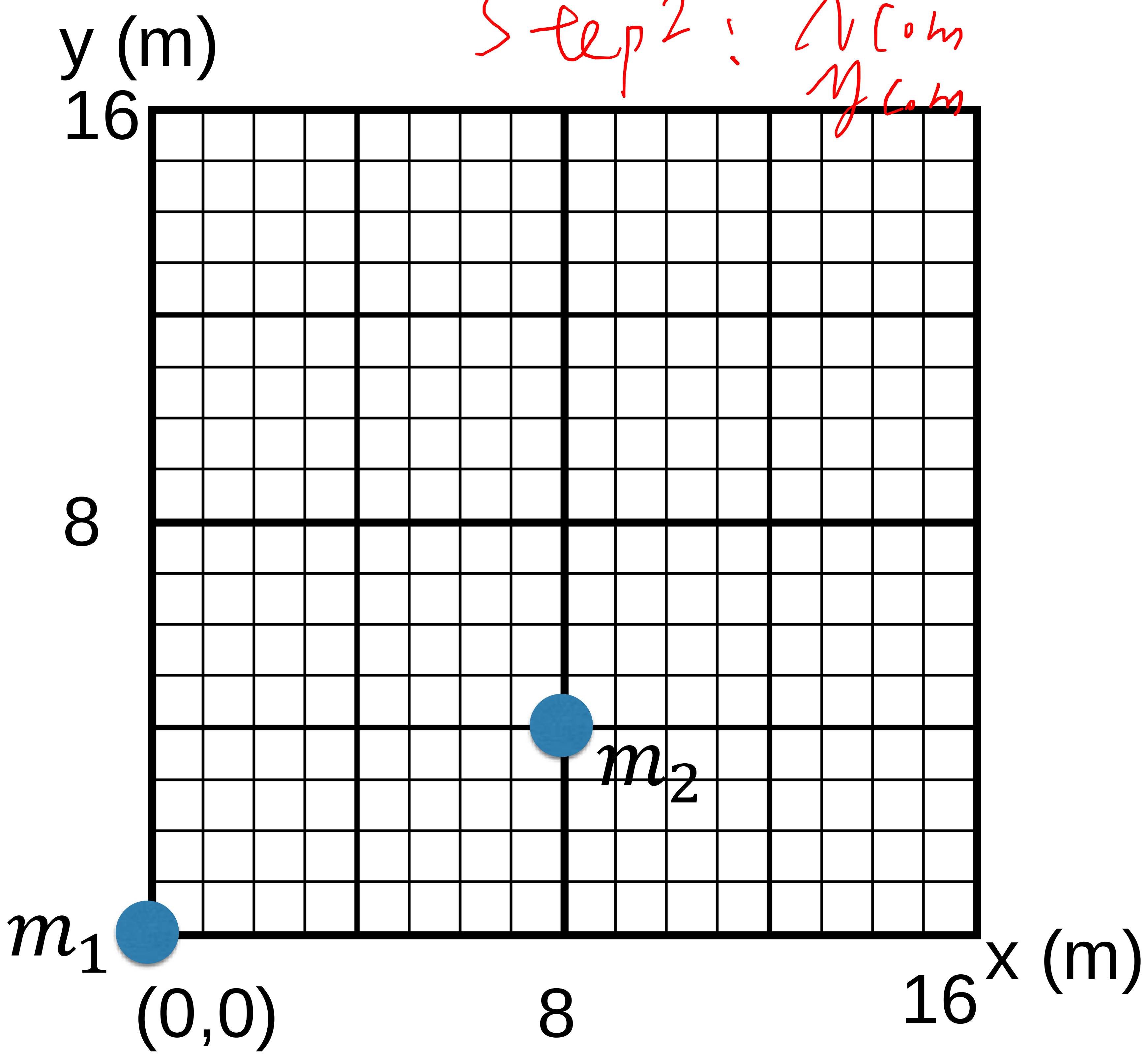
2D case: $\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j}$

$\vec{r}_{com} = \dfrac{\sum_i m_i \vec{r}_i}{M_{tot}}$

Step 1: M_{tot}

Step 2: x_{com}, y_{com}

- What is the center of mass of the 2-particle system, if $m_1 = 1kg, m_2 = 3kg$?



Step 3:

A $\vec{r}_{com} = 2m\hat{i} + 2m\hat{j}$

B $\vec{r}_{com} = 4m\hat{i} + 2\ m\hat{j}$

C $\vec{r}_{com} = 6m\hat{i} + 3m\hat{j}$

4. Center of mass (COM) velocity and acceleration

$\vec{v}_i$

$$\vec{v}_{com} = \frac{d}{dt}\vec{r}_{com} = \frac{\frac{d}{dt}\sum_i m_i \vec{r}_i}{M_{tot}} = \frac{\sum_i m_i \frac{d\vec{r}}{dt}}{M_{tot}} = \frac{\sum_i m_i \vec{v}_i}{M_{tot}}$$

- Center of mass velocity:

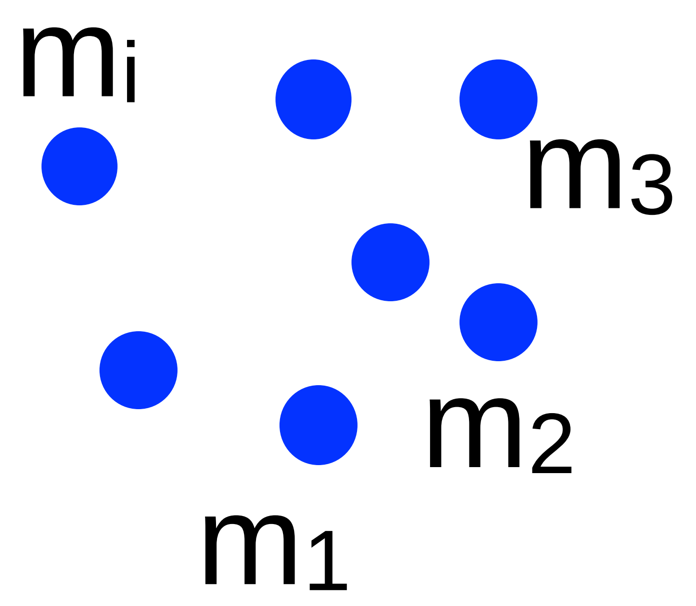
$$\vec{r}_{com} = \frac{\sum_i m_i \vec{r}_i}{M_{tot}}$$

$$\vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{\sum_i m_i \vec{v}_i}{M_{tot}}$$

$\vec{a}$

- Center of mass acceleration:

$$\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_i m_i \frac{d\vec{v}_i}{dt}}{M_{tot}} = \frac{\sum_i m_i \vec{a}_i}{M_{tot}}$$

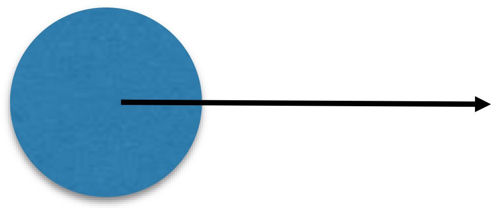


$$\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_i m_i \vec{a}_i}{M_{tot}}$$

Newton’s 2nd Law for a system of particles

- We already saw, for a single particle:

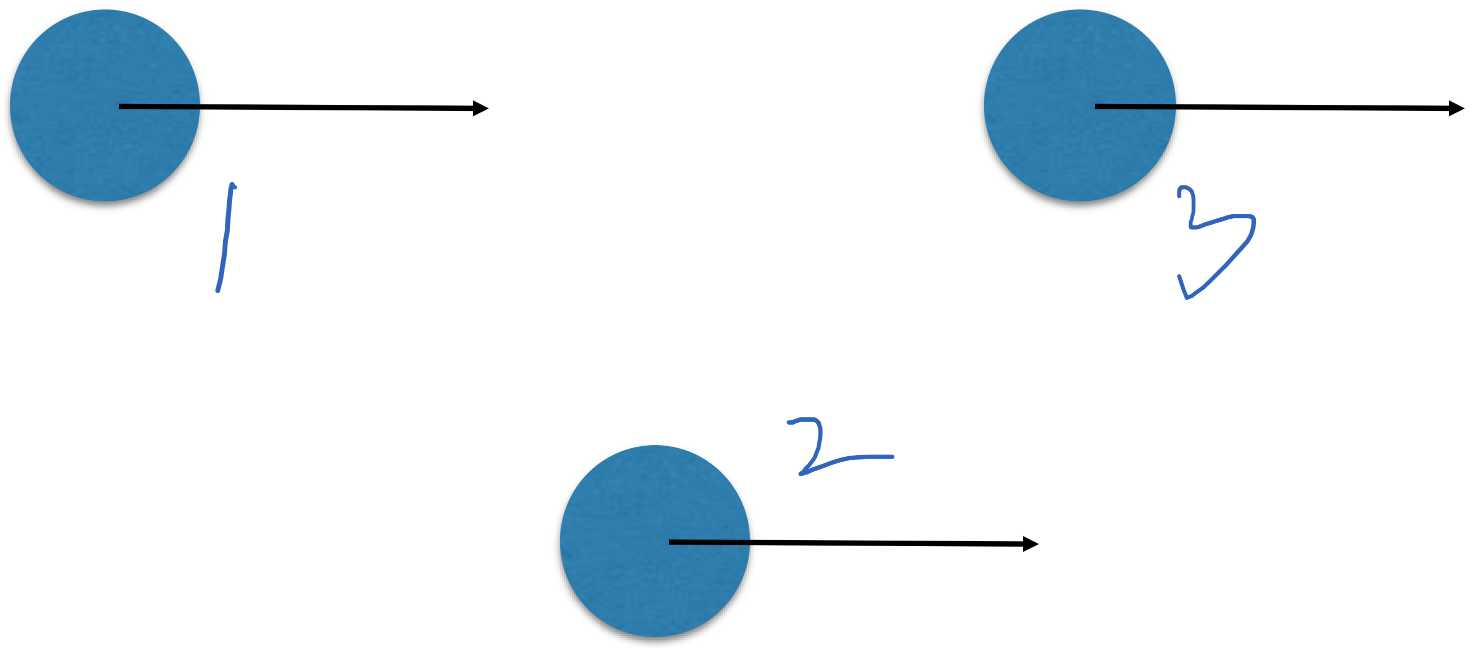
$$\vec{F}_{net} = m\vec{a}$$



- What about for many particles?

$$\vec{F}_{external,net} = m_{tot}\vec{a}_{com}$$

Newton's 2nd law for many particles.



Clicker Question 2

$\longrightarrow$ **+x**

$$\vec{F}_{external,tot} = m_{tot}\vec{a}_{com}$$

F₁ = -10 N

F₂ = 15 N

m₁ = 2 kg

m₂ = 3 kg

$$\vec{a}_{com} = \frac{\sum m_i \vec{a}_i}{M_{tot}}$$

What is the acceleration of the **COM** of the system containing ***m₁*** and ***m₂***? (+x direction is to the right)

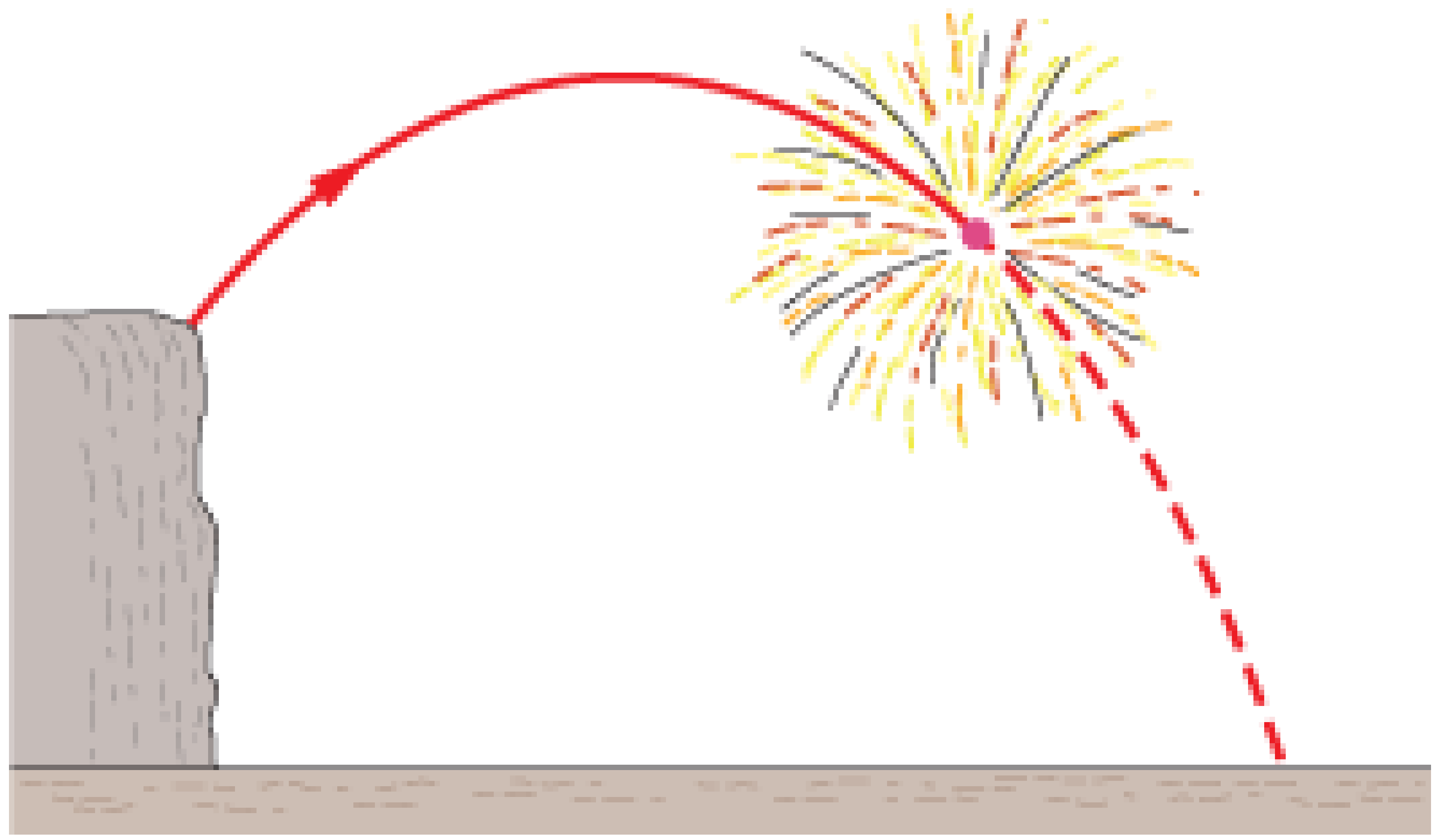
A $\quad \vec{a}_{COM} = \vec{a}_1 + \vec{a}_2$ & $\vec{a}_{COM} = 0$

B $\quad \vec{a}_{COM} = \frac{\vec{F}_1 + \vec{F}_2}{m_1 + m_2}$ & $\vec{a}_{COM} = 1.0\ \text{m/s}^2\ \hat{\imath}$

Clicker Question 3

+y

- The firework explodes, what is the acceleration of the COM?



How is the trajectory of the COM affected?

A $\mathbf{a}_{COM} = 0$

B $\mathbf{a}_{COM} = -g\,\hat{j}$

C $\mathbf{a}_{COM} = mg\,\hat{j}$

D $\mathbf{a}_{COM}$ points

COM Quantities and dynamics for a system of particles

Displacement of Center of Mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M_{Total}}$$

Velocity of Center of Mass

$$\vec{V}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M_{Total}}$$

Acceleration of Center of Mass

$$\vec{A}_{CM} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M_{Total}}$$

$$\vec{A}_{CM} = \frac{\vec{F}_{Net,External}}{M_{Total}}$$

Key idea: center of mass obeys Newton's laws of motion for the total mass.

Note: Only *external* forces contribute to F_{net} of the system in Newton's second law.

Review for Chapters 5 - 6

- Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$
- Various forces: $\vec{N}$, $\vec{W}$, $\vec{T}$, $\vec{f}$riction, $\vec{F}_s$
- Friction: Opposes sliding or opposes tendency to slide
  - Magnitude of kinetic friction force: $|\vec{f}_k| = \mu_k |\vec{N}|$
  - Magnitude of static friction force: $|\vec{f}_s| \leq \mu_s |\vec{N}|$
- Drag force: $|\vec{F}_{drag}| = \frac{1}{2} C \rho A v^2$
- Centripetal force: Net force in UCM

Review for Chapter 7-8

- Kinetic energy: $K = \frac{1}{2} m v^2$
- Work: $\int \vec{F} \cdot d\vec{r}$
  - Work by a constant force: $\vec{F} \cdot \Delta\vec{r}$
  - Work by a variable force:
- Work-kinetic energy theorem: $W_{net} = \Delta K$
- Potential energy: Change: $\Delta U = -W_{cons}$
- Conservation of energy, see today's lecture.