PHYS 225
Fundamentals of Physics: Mechanics

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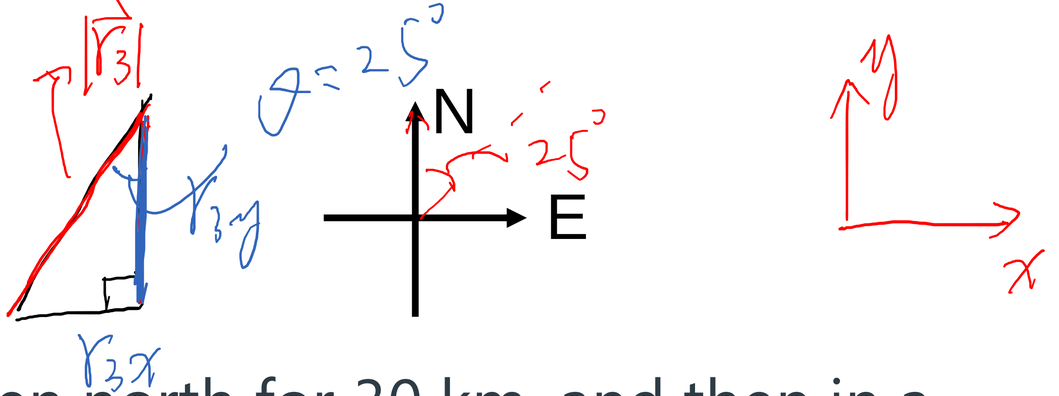
Fall 2024

Lecture 10: Projectile motion: Examples II

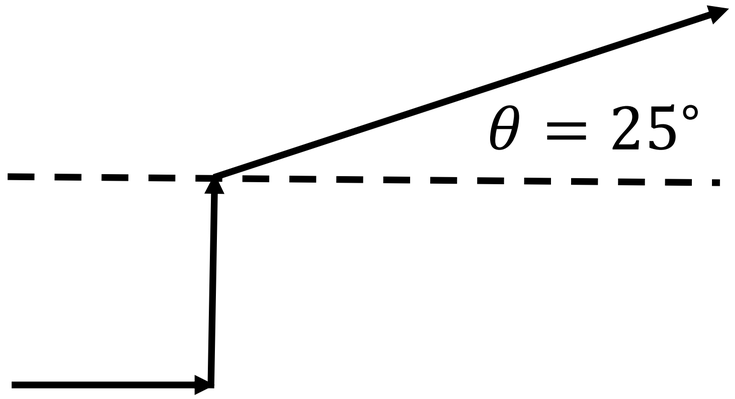
CALIFORNIA STATE UNIVERSITY
FULLERTON

Homework 3.4 Question 8

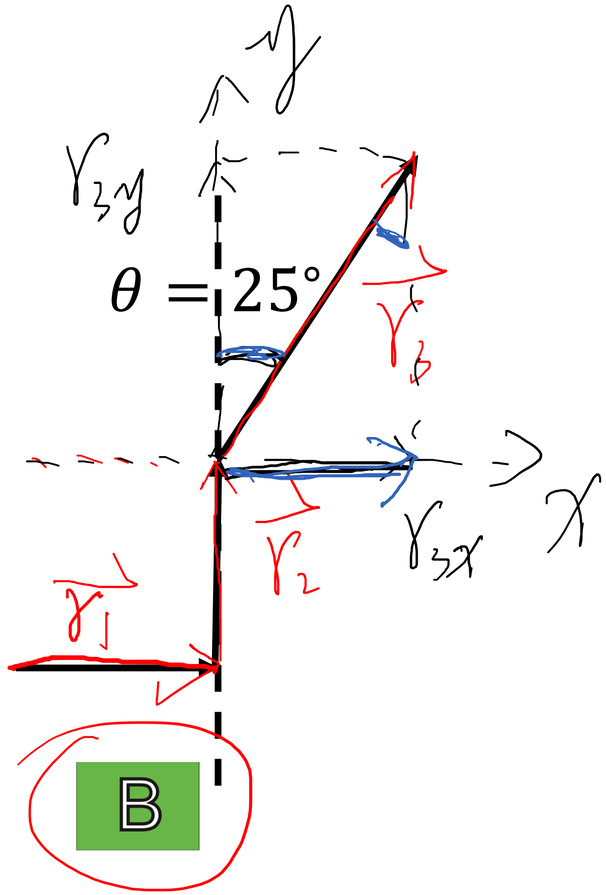


- A car is driven east for a distance of 46 km, then north for 30 km, and then in a direction 25° east of north for 26 km. Which of the following correctly sketch the car's trajectory?

N : Reference



A



B

In this case:

$$r_{3x} = |\vec{r}_3| \sin\theta$$

$$r_{3y} = |\vec{r}_3| \cos\theta$$

Learning goals

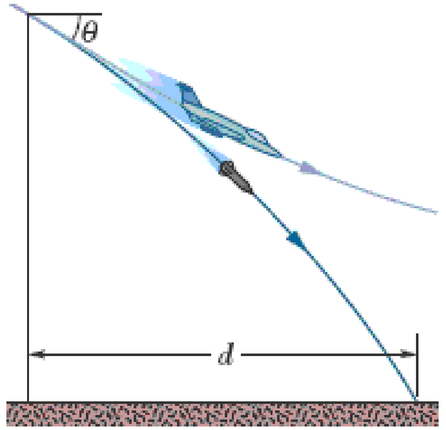
- Practice on projectile motion
- Uniform circular motion

Clicker question 7

Given: $|v_0|$, θ, $\vec{a}$, Δx

Goal: t

- A certain airplane has a speed of 298.2 km/h and is diving at an angle of θ = 26.0° below the horizontal when the pilot releases a radar decoy (see the figure). The horizontal distance between the release point and the point where the decoy strikes the ground is d = 667 m. To find how long (in time) the decoy is in the air, which of the following equation is used?



A y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B x- motion: $\Delta x = v_{0x}t$

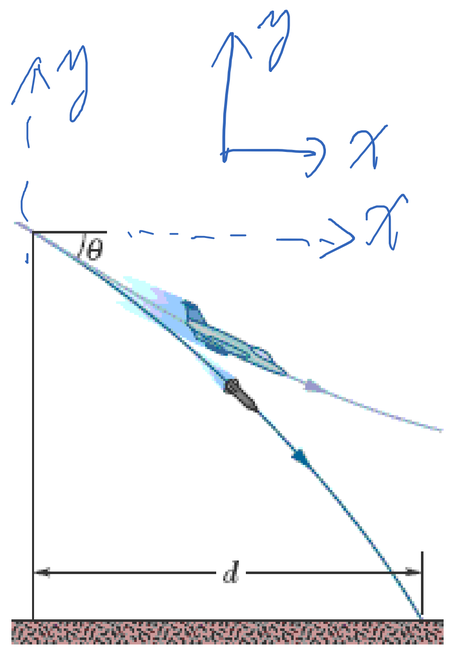
Example 4

Given: $\vec{v}_0$ ($|\vec{v}_0|, \theta$), $\vec{a}$, Δx

Goal: a) t b) $h = -\Delta y = ?$

Step 0: $|v_0| = 298.2 \frac{\text{km}}{\text{h}} \times \frac{1000\,\text{m}}{1\,\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} \approx 82.8\,\text{m s}^{-1}$

- A certain airplane has a speed of 298.2 km/h and is diving at an angle of 26.0° below the horizontal when the pilot releases a radar decoy (see the figure). The horizontal distance between the release point and the point where the decoy strikes the ground is d = 667 m. **(a)** How long is the decoy in the air? **(b)** How high was the release point?



Step 1: Horizontal motion: $\Delta x = v_{x0} t = |v_0| \cos\theta \, t$

$$\rightarrow t = \frac{\Delta x}{v_0 \cos\theta} = \frac{667\,\text{m}}{82.8\,\text{m s}^{-1} \cos 26^\circ} \approx 8.96\,\text{s}$$

Step 2: $\Delta y = v_{y0} t - \frac{1}{2} g t^2 = -|v_0| \sin 26^\circ \cdot 8.96\,\text{s} - \frac{1}{2} \times 9.8\,\text{m s}^{-2} \cdot (8.96)^2$

$$\approx -719\,\text{m}$$

$$h = -\Delta y \approx 719\,\text{m}$$

Math review: Root of a quadratic equation

- A quadratic equation: $ax^2 + bx + c = 0$

Given: $a \neq 0$, b and c are constants

Goal: x

How to solve for the variable x?

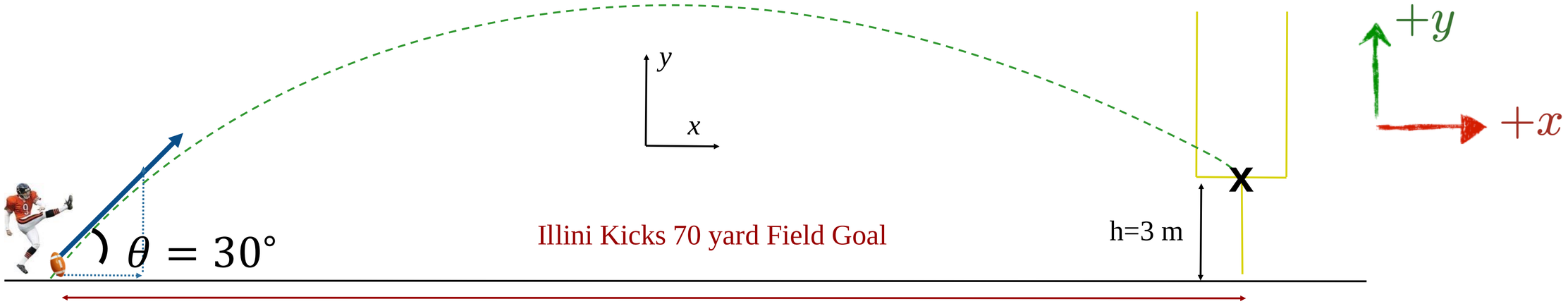
Solution: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Clicker question 8

Given: $|\vec{v}_0|, \theta, \vec{a}, \Delta y$

Goal: Δx

A field goal kicker kicks the ball at a speed of v_0 = 30 m/s and at an angle of 30 degrees w.r.t. the ground. If the crossbar of the goal post is 3m above the ground, from how far away can the kicker kick a field goal (have the ball arrive just the crossbar)? **Which principles to use?**



A

$$x = x_0 + v_{0x}t$$
$$v_y = v_{0y} - gt$$

C

$$x = x_0$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

B

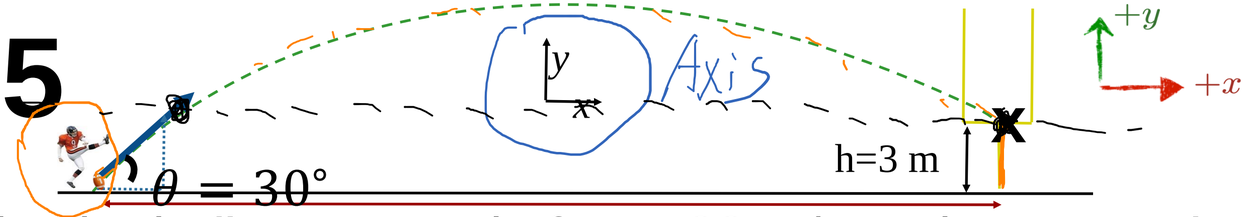
$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$\Delta y = v_{0y}t - \frac{1}{2}gt^2$

D

$$x = x_0 + v_{0x}t$$
$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

Example 5



Given: $|v_0|, \theta, \vec{a}, \Delta y$

Goal: Δx

A field goal kicker kicks the ball at a speed of v_0 = 30 m/s and at an angle of 30 degrees w.r.t. the ground. If the crossbar of the goal post is 3m above the ground, from how far away can the kicker kick a field goal (have the ball arrive just the crossbar)?

Step 1: Decompose $\vec{v}_0$: $v_{x0} = |v_0|\cos\theta$; $v_{y0} = |v_0|\sin\theta = 30\,m\,s^{-1} \times 0.5 = 15\,m\,s^{-1}$

Step 2: Find t: $\Delta y = v_{y0}t - \frac{1}{2}gt^2$

Rewrite: $\frac{1}{2}gt^2 - v_{y0}t + \Delta y = 0$, $t = \frac{v_{y0} \pm \sqrt{v_{y0}^2 - 2g\Delta y}}{g}$

(a, b, c)

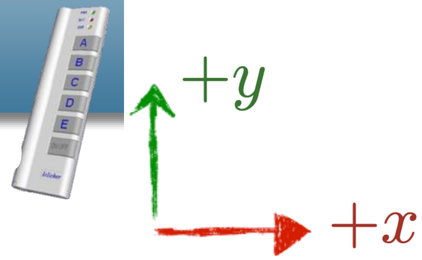
$= \frac{15\,m\,s^{-1} \pm \sqrt{(15\,m\,s^{-1})^2 - 2 \times 9.8\,m\,s^{-2} \cdot 3\,m}}{9.8\,m\,s^{-2}}$

$t_1 \approx 0.21\,s$ or $t_2 \approx 2.84\,s$

Step 3: $\Delta x = v_{x0}t \approx 30\,m\,s^{-1} \cdot \cos 30^\circ \cdot 2.84\,s$

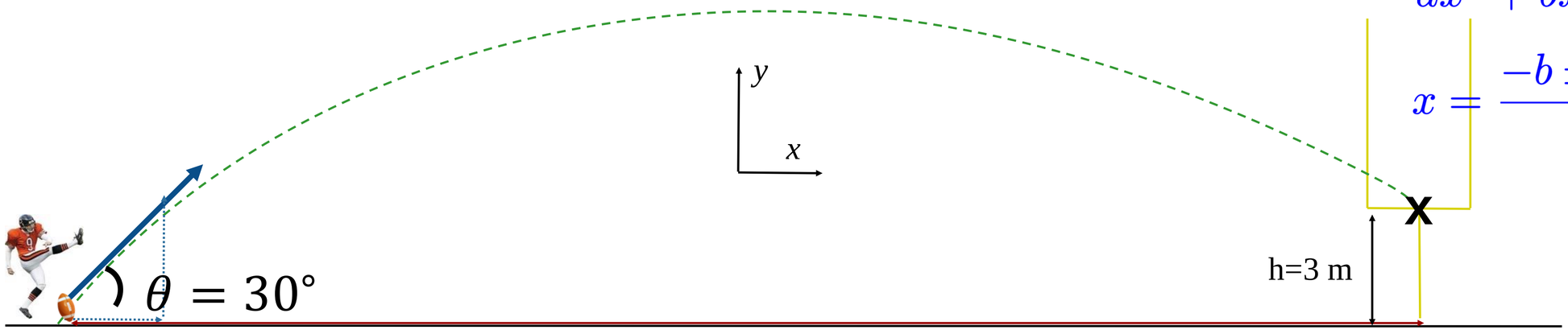
$\approx 74\,m$

Animation: Field Goal Activity



$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



y-direction

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$v_{oy} = v_o \sin(30^o) = 15$ m/s

$\frac{1}{2}\, g t^2 - v_{oy} t + y - y_o = 0$

$\frac{1}{2}$ (9.8 m/s^2) t^2 - (15 m/s) t + 3 m - 0 m = 0

t ≈ 2.85 s or t ≈ 0.22 s.

x-direction

$x = x_0 + v_{0x}t$

$v_{ox} = v_o \cos(30^o) = 26$ m/s

$D = x = x_o + v_{ox}\, t$

= 0 m + (26 m/s)(2.85 s)

≈ 74 m

Given: v_0, θ, h and $-g$

Goal: Δx.

Illini Kicks 70 yard Field Goal

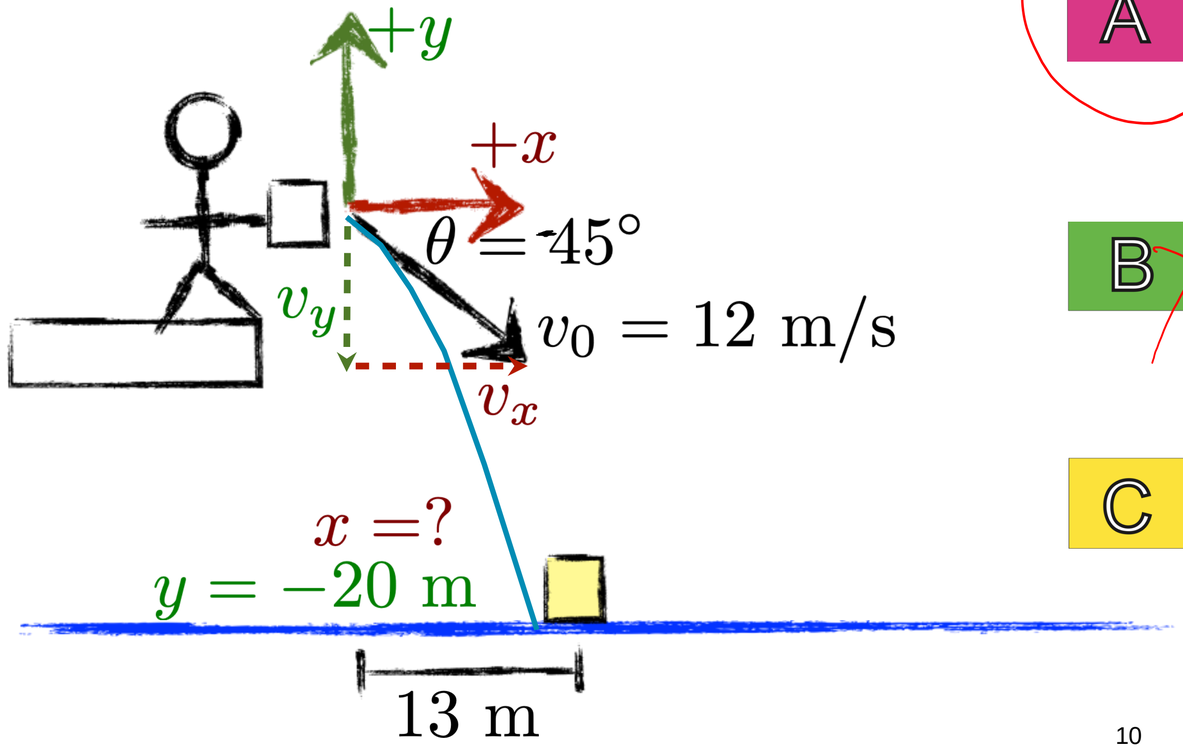
Clicker question 9

- A girl on a bridge (height 20 m) throws a stone at 12 m/s, 45° below horizontal. Assuming water is still. Which equation to use to calculate the time the stone is in air?

Given: $x_0 = y_0 = 0$ $\quad y = -20\ \text{m}$

$v_0 = 12\ \text{m/s}$ $\quad \theta = 45°$

Goal: $x = ?$ **Concept:** Projectile



A: y- motion: $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

B: x- motion: $\Delta x = v_{0x}t$

C: Neither is sufficient. Both are needed.

Group activity (2-3 people in a group)

- A girl on a bridge (height 20 m) throws a stone at 12 m/s, 45° below horizontal. Does the block hit a target on the water that is 13 m from where it goes under the bridge? Assuming water is still. Neglect air resistance.

Given: $x_0 = y_0 = 0 \quad y = -20\ \text{m}$

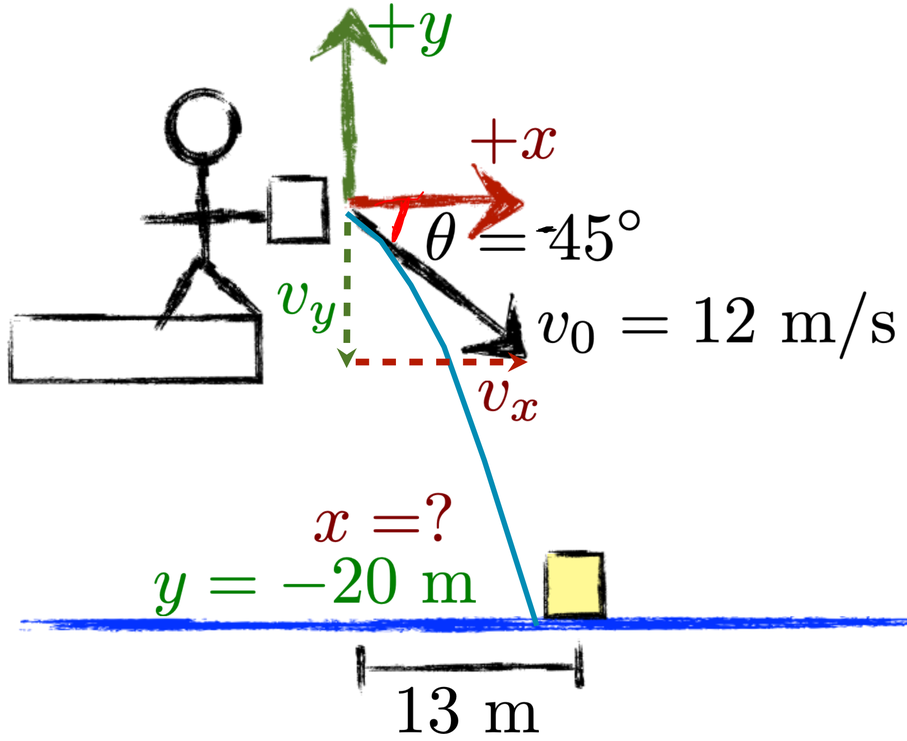
$v_0 = 12\ \text{m/s} \quad \theta = 45°$

Goal: $x = ?$ **Concept:** Projectile

Step 1: Decompose $\vec{v_0}$

Step 2 v_y Find t from y-motion

Step 3: Find Δx

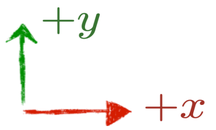


$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = -v_0 \sin\theta$$

1. Read carefully
2. Draw a sketch
3. Given? Goal?
4. Brainstorm
5. Calculate

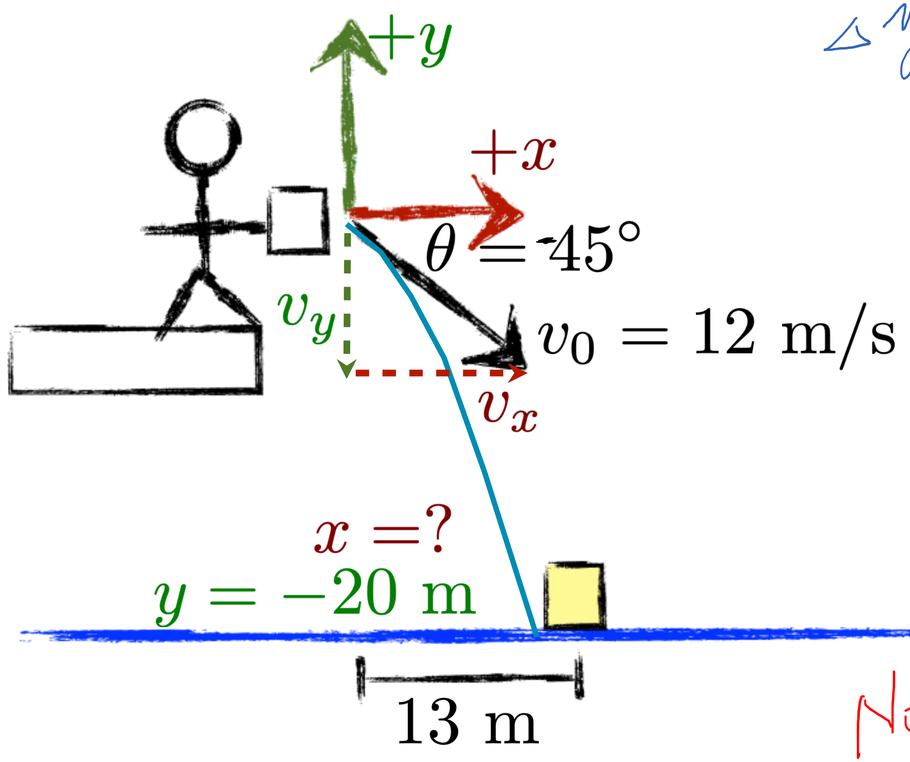
Group activity (2-3 people in a group)



- A girl on a bridge (height 20 m) throws a stone at 12 m/s, 45° below horizontal. Does the block hit a target on the water that is 13 m from where it goes under the bridge? Assuming water is still. Neglect air resistance.

Given: $x_0 = y_0 = 0$ $\quad y = -20$ m
$v_0 = 12$ m/s $\quad \theta = 45°$

Goal: $x = ?$ **Concept:** Projectile



Step 1: $v_{x_0} = |v_0| \cos\theta = 12\,ms^{-1} \cos 45° \approx 8.49\,ms^{-1}$

$v_{y_0} = -|v_0| \sin\theta = -12\,ms^{-1} \sin 45° \approx -8.84\,ms^{-1}$

Step 2: Find t from y-motion

$\Delta y = v_{y_0} t - \frac{1}{2} g t^2 \rightarrow \frac{1}{2} g t^2 - v_{y_0} t + \Delta y = 0$

a b c

Step 1

$t = \frac{v_{y_0} \pm \sqrt{v_{y_0}^2 - 2g\Delta y}}{g}$ $\quad -20$ m

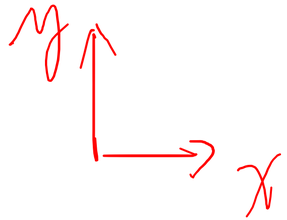
$\sim t_1 \approx -3.06$ s or $t \approx 1.33$ s

Step 3: $\Delta x = v_{x_0} t \approx 8.49\,ms^{-1} \cdot 1.33$ s

≈ 11.3 m

No

Summary of Projectile strategy



- 1. Read the problem carefully
- 2. Draw a sketch
  - Draw and label any velocities or accelerations that you know
  - Choose origin (where is x=0, y=0?)
  - Choose +x, +y directions, and draw these on your sketch
  - Break any vectors into x,y components
- 3. Given? Goal?
- 4. Brainstorm: principles and equations? Horizontal/vertical motions are connected by t
- 5. Calculate
  - Use chosen axes, origin, vector components
  - Solve algebraically
- 6. Plug in numbers, be careful about unit conversion.
- 7. Is your answer reasonable?

Reminders

- Homework 4.4: Due in a week
- Midterm 1:
  - Oct. 17
  - Time and location: Same as a regular class
  - Closed book, closed notes. However, a single page and single side cheat sheet is allowed.
  - Calculators are allowed.