PHYS 225
Fundamentals of Physics: Mechanics

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Lecture 23: Potential energy and conservation of energy

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FULLERTON

Learning goals for today

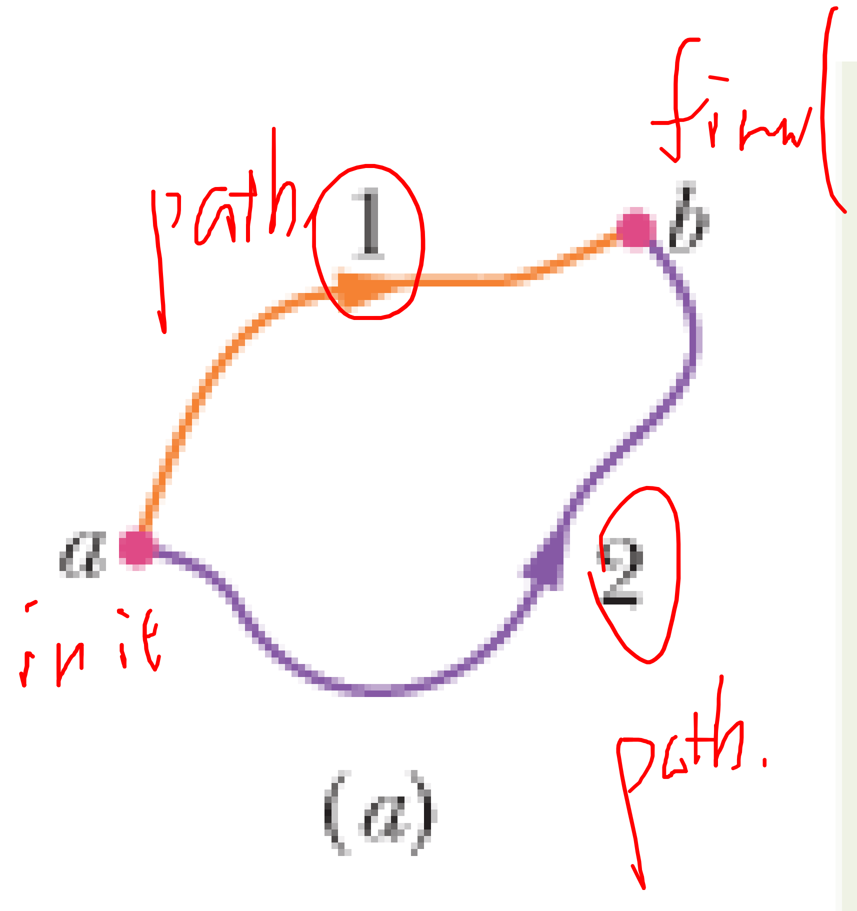
- Conservative forces and potential energy
- Conservation of mechanical energy
- Conservation of total energy

Chapter 8.1: Conservative forces and potential energy

Recap: 1. Conservative forces

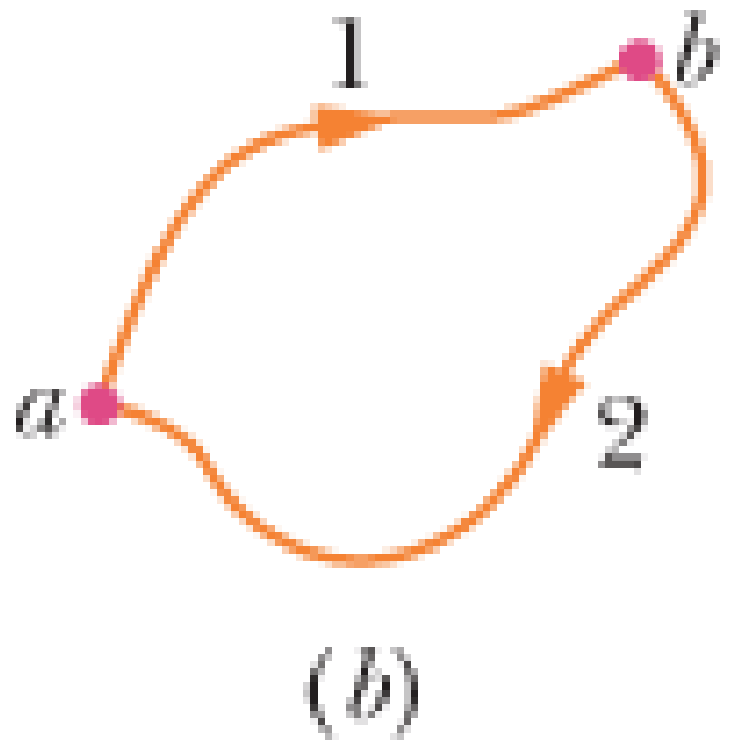
- **Definition of a conservative force**: If the work done by a force ***only*** depends on the initial and final positions, then it is a conservative force.

but not path depend



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

I. Open loops: $W_{a\to b,1} = W_{a\to b,2}$

II. Closed loop: $W_{a\to b,1} + W_{b\to a,2} = 0$

2. Potential energy

- **Potential energy:** Energy of position, U
  - The measure of capability for a conservative force to do work.

- Changes in potential energy: $\Delta U = U_f - U_i = -W_{cons}$

of work done by $\vec{F}_{conservative}$

i.e., The change of potential energy is the *negative of* work done by a conservative force.

Potential energy of the weight

- **Example 1**: Gravitational force near earth surface, weight

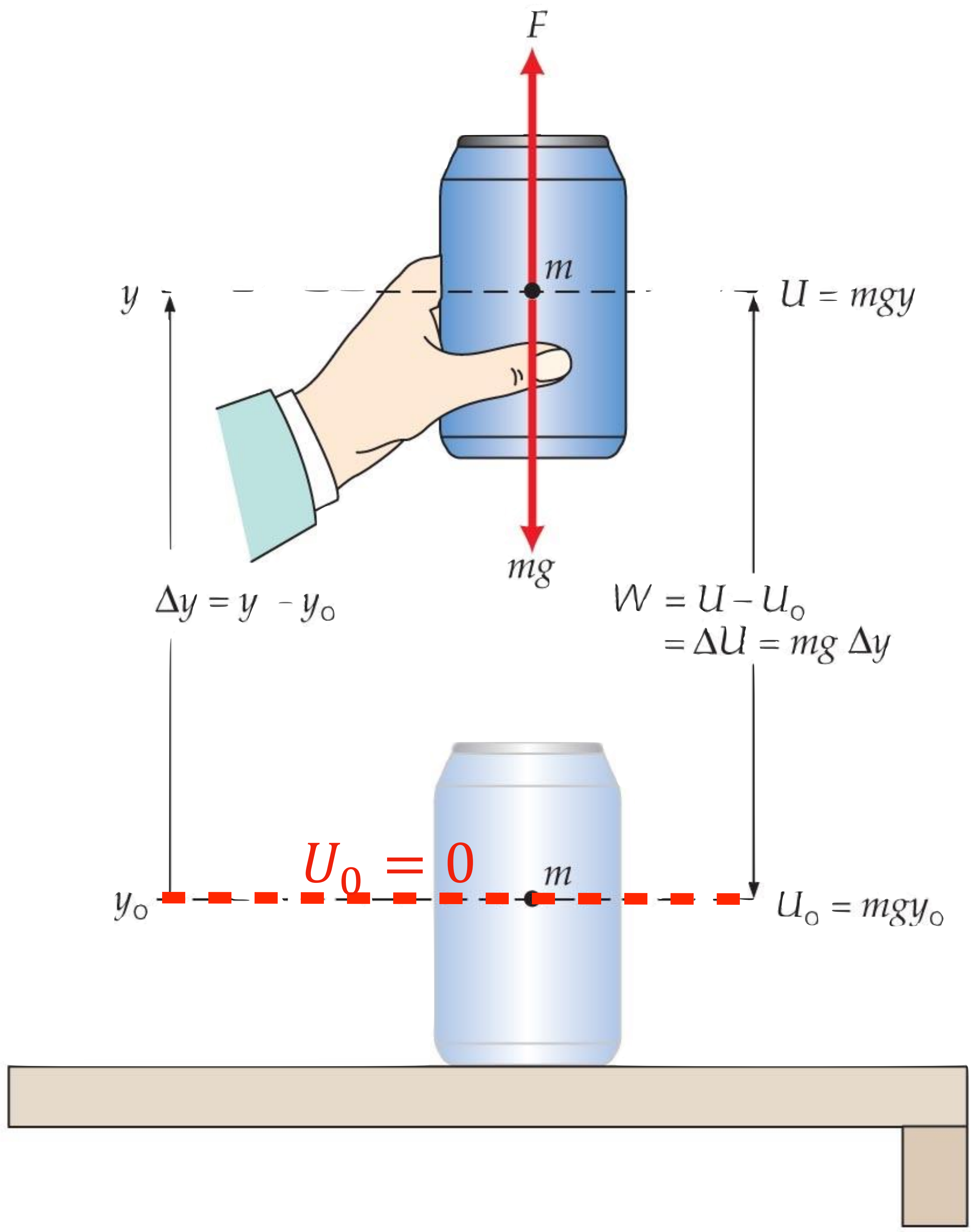
Work from y_0 to y: $W = -mg(y - y_0)$

initial final

Potential energy change from y_0 to y: $\Delta U = -W = mg(y - y_0)$

final initial

If $U(y=0)=0$, then $U(y) = mgy$



Clicker question 4

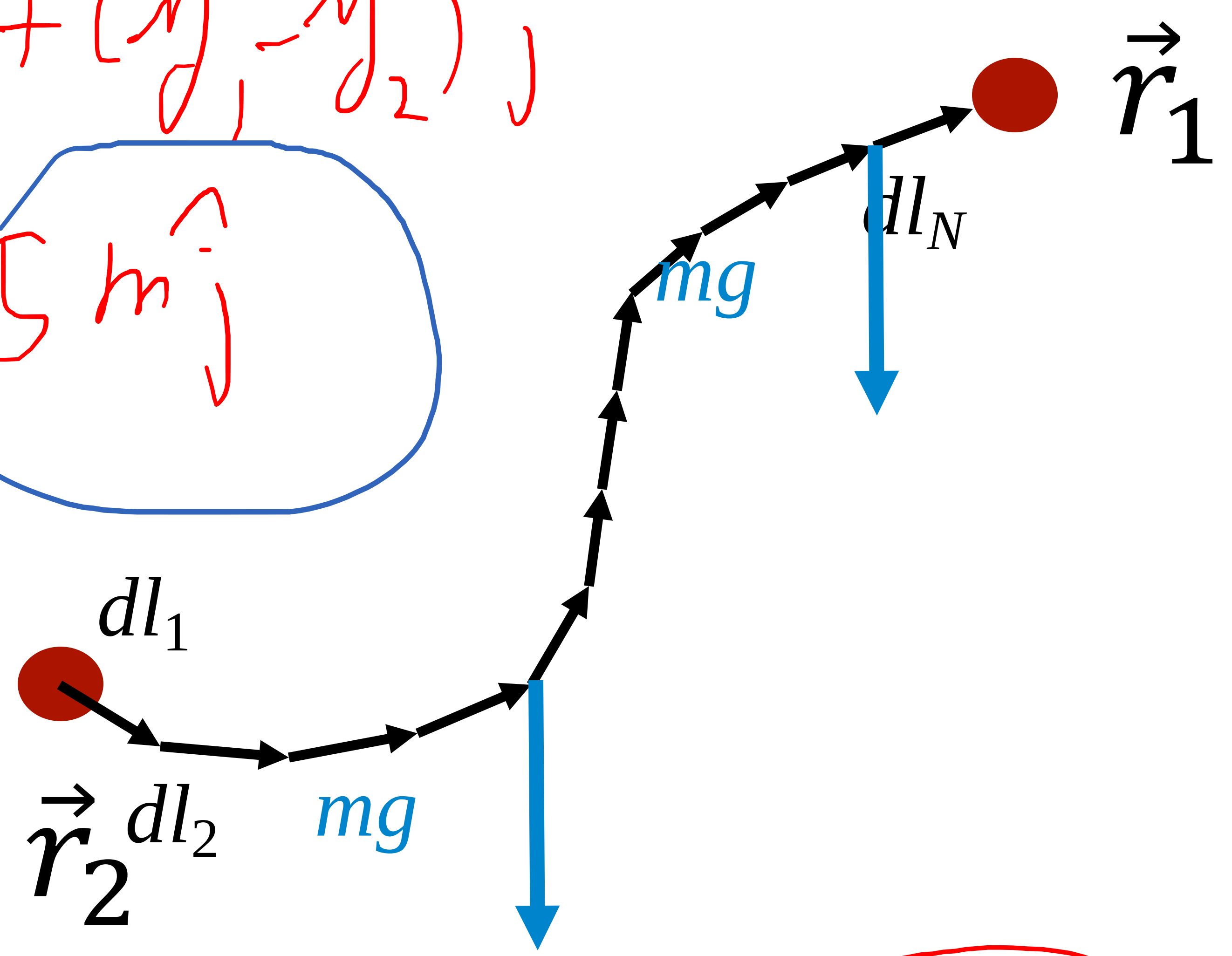
$\Delta U = -W_{cons} = mg(y_f - y_i)$

$= mg(y_1 - y_2)$

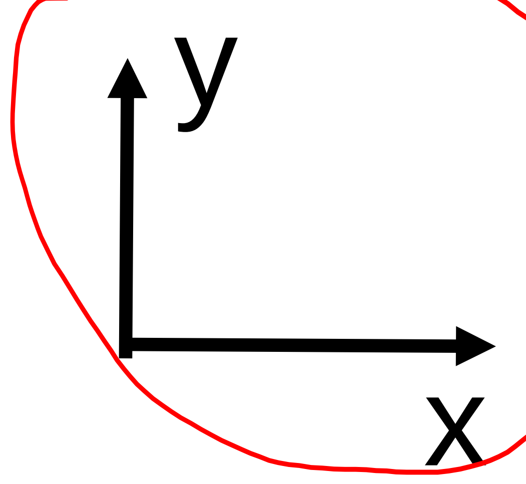
- A climber works on a vertical wall. He/she climbs from $\vec{r}_2$ to $\vec{r}_1$ .The length of the path is $l = 10$m. Besides, $\vec{r}_1 - \vec{r}_2 = (2\hat{\imath} + 5\hat{\jmath})$m. The mass of the climber is 65kg. What is the change of the potential energy, ΔU, from $\vec{r}_2$ to $\vec{r}_1$?

$\vec{r}_1 - \vec{r}_2 = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$

$= 2m\hat{i} + 5m\hat{j}$



A	B	C	D
1274J	-6370J	3185 J	-3185J



Potential energy of the $\underbrace{\text{spring force}}_{\vec{F}_s}$

- **Example 2:** Spring force

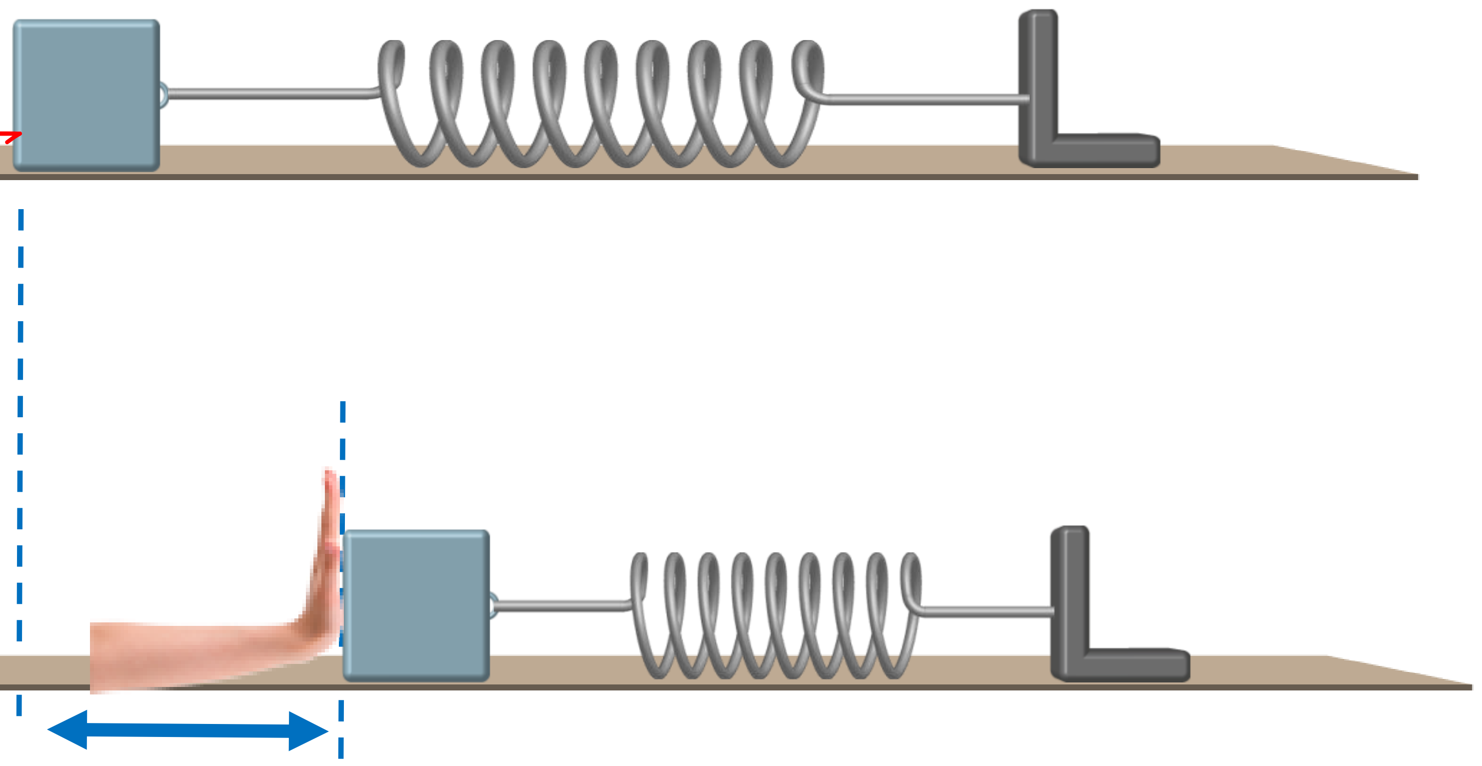
spring const.

Work from x_i to x_f: $W_s = \frac{k}{2}(x_i^2 - x_f^2)$

Potential energy difference from x_i to x_f: $\Delta U = -W_s = \frac{k}{2}(x_f^2 - x_i^2)$

If $U_0 = 0$ *when* $x_i = 0$

Then $U_f = \frac{k}{2}x_f^2$ for x_f



$x_f = D$

Calculating conservative force from potential energy

- Calculating potential energy from a conservative force:

$$U = -W_{cons} = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{cons} \cdot d\vec{r}$$

- Calculating a *conservative* force from potential energy:

$$\vec{F}_{cons} = -\nabla U = -\frac{\partial U}{\partial x}\hat{\imath} - \frac{\partial U}{\partial y}\hat{\jmath} - \frac{\partial U}{\partial z}\hat{k}$$

gradient

Clicker question 5

- Which of the following forces can be directly calculated from a potential energy?

A Gravitational force

B Normal force

C Spring force

D Both A & C

$$\vec{F}_{cons} = -\vec{\nabla} U$$

Demo

Checkpoint: Two types of mechanical energy

- Kinetic energy: $K = \frac{1}{2} m v^2$
- Potential energy: $\Delta U = -W_{cons}$

Chapter 8.2. Conservation of energy

3. Conservation of mechanical energy

Condition

- When only **conservative forces do work** on the system, then the **mechanical energy, $K + U$, is conserved:**

$K+U$

$$K + U = const$$

If only $\vec{F}_{cons}$'s do work, then

$W_{net} = W_{cons}$ But $\Delta U = -W_{cons}$ → $W_{cons} = -\Delta U = -(U_f - U_i)$

W-K theorem: $W_{net} = \Delta K$ → $W_{cons} = K_f - K_i$

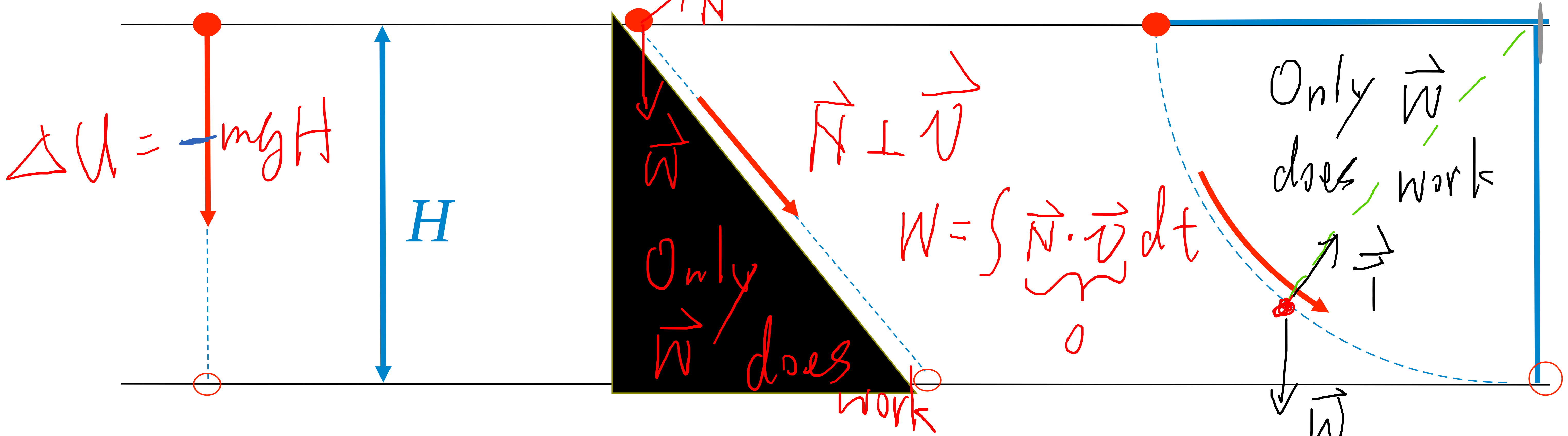
→ $-(U_f - U_i) = K_f - K_i$ → Rewrite: $U_i + K_i = U_f + K_f$ Q.E.D.

That is, $$K_i + U_i = K_f + U_f$$

Clicker question 6

- Three objects having the same mass begin **at rest** at the same height, and all move down the same vertical distance H. One falls straight down, one slides down a ***frictionless*** inclined plane, and one swings on the end of a string.

In which case does the object gain the biggest kinetic energy during its motion?



Free Fall   Frictionless incline   String

A Free Fall   B Incline   C String   D All the same

$$\underbrace{K_i}_{0} + U_i = K_f + U_f$$

$$\vec{T} \perp \vec{v}_t$$

$$W_{\vec{T}} = \int \underbrace{\vec{T}\cdot\vec{v}_t}_{0}\, dt$$

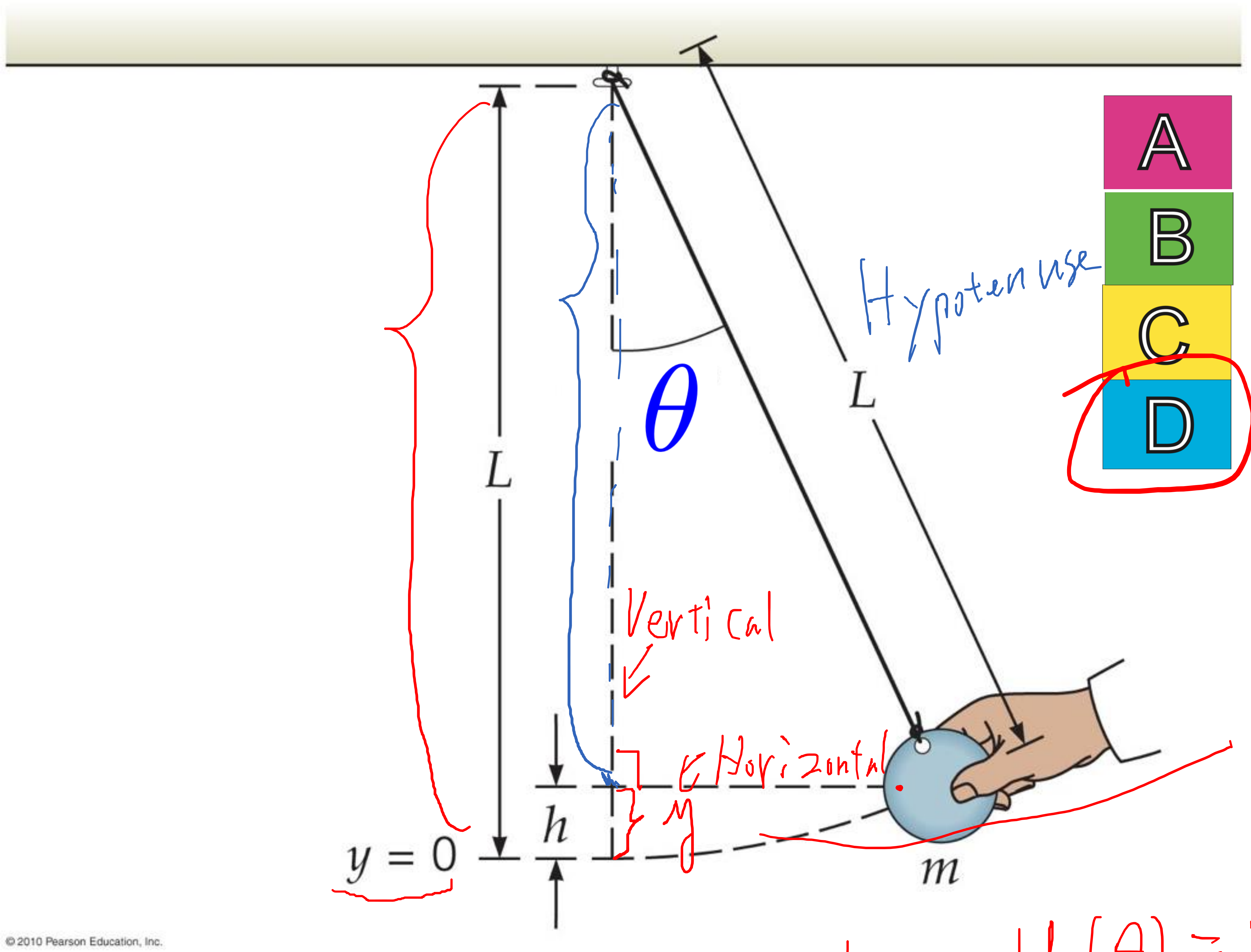
$$K_f = U_i - U_f$$
$$= -\Delta U$$
$$= -(-mgH)$$
$$= mgH$$

Case study 1: Pendulum

Clicker question 7

$\uparrow y$

- Where is the potential energy the **smallest**?



A $\theta = 90°$

B $\theta = 45°$

C $\theta = 30°$

D $\theta = 0°$

$y = L - L\cos\theta = L(1-\cos\theta)$

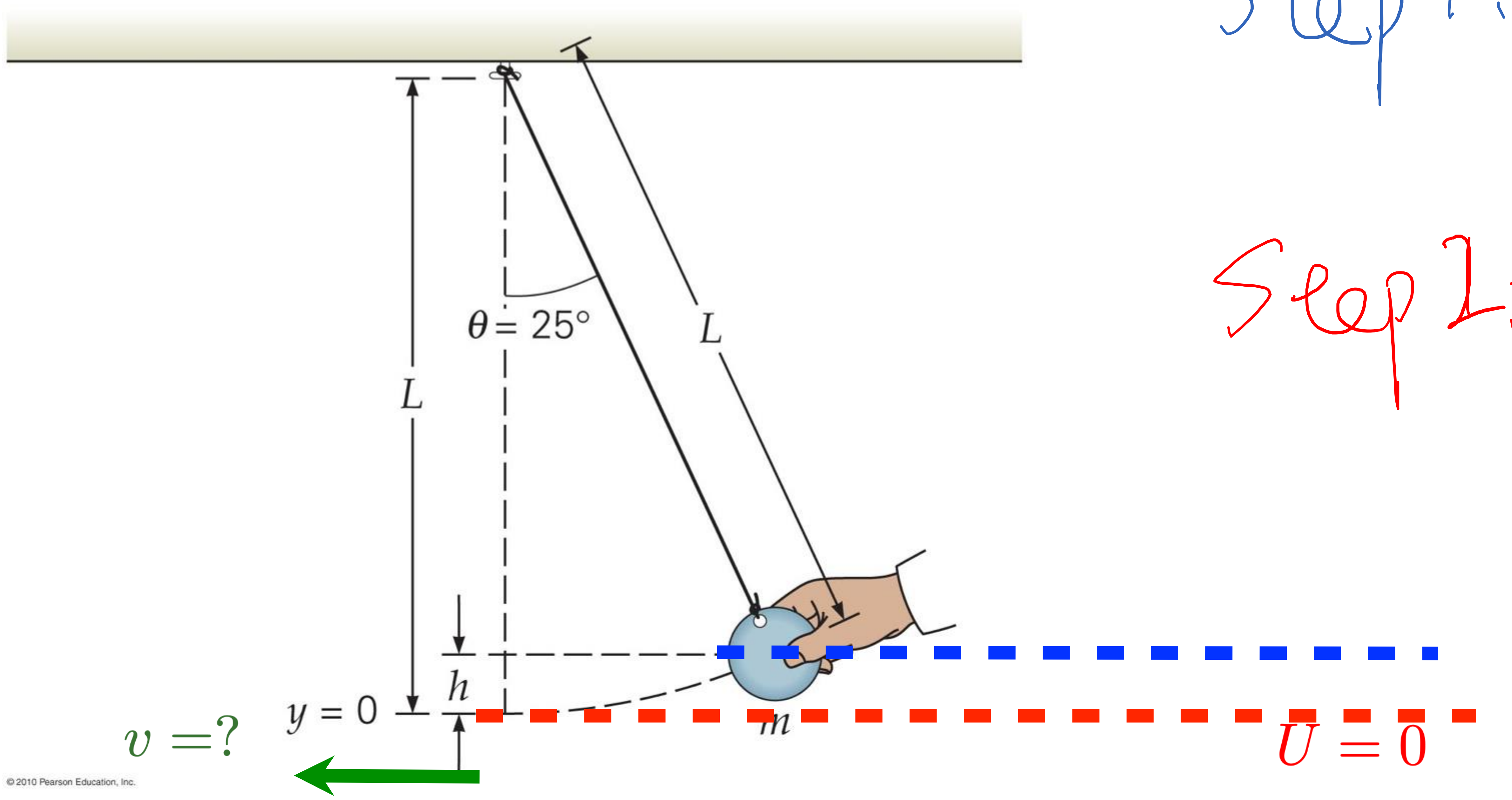
Def. $U(y=0) = U(\theta=0) = 0$, then $U(\theta) = mgy = mgL\underbrace{(1-\cos\theta)}_{\geq 0}$

Example 1: Pendulum

Given: $m, L, \theta_i, v_0, \theta_f = 0$

Goal: v_f

- A pendulum is formed by attaching a ball of mass 1 kg to a 1 m long string of negligible mass. The other end of the string is fixed to the ceiling. The ball is pulled to an angle $\theta = 25°$ from vertical down direction and released from rest. Neglect the air friction. What is the speed of the ball at the lowest point?



Step 1: Conservation of mech. E

$$U_i + \underbrace{K_i}_{0} = \underbrace{U_f}_{0} + K_f$$

Step 2: For, θ, $U(\theta) = mgL(1-\cos\theta)$

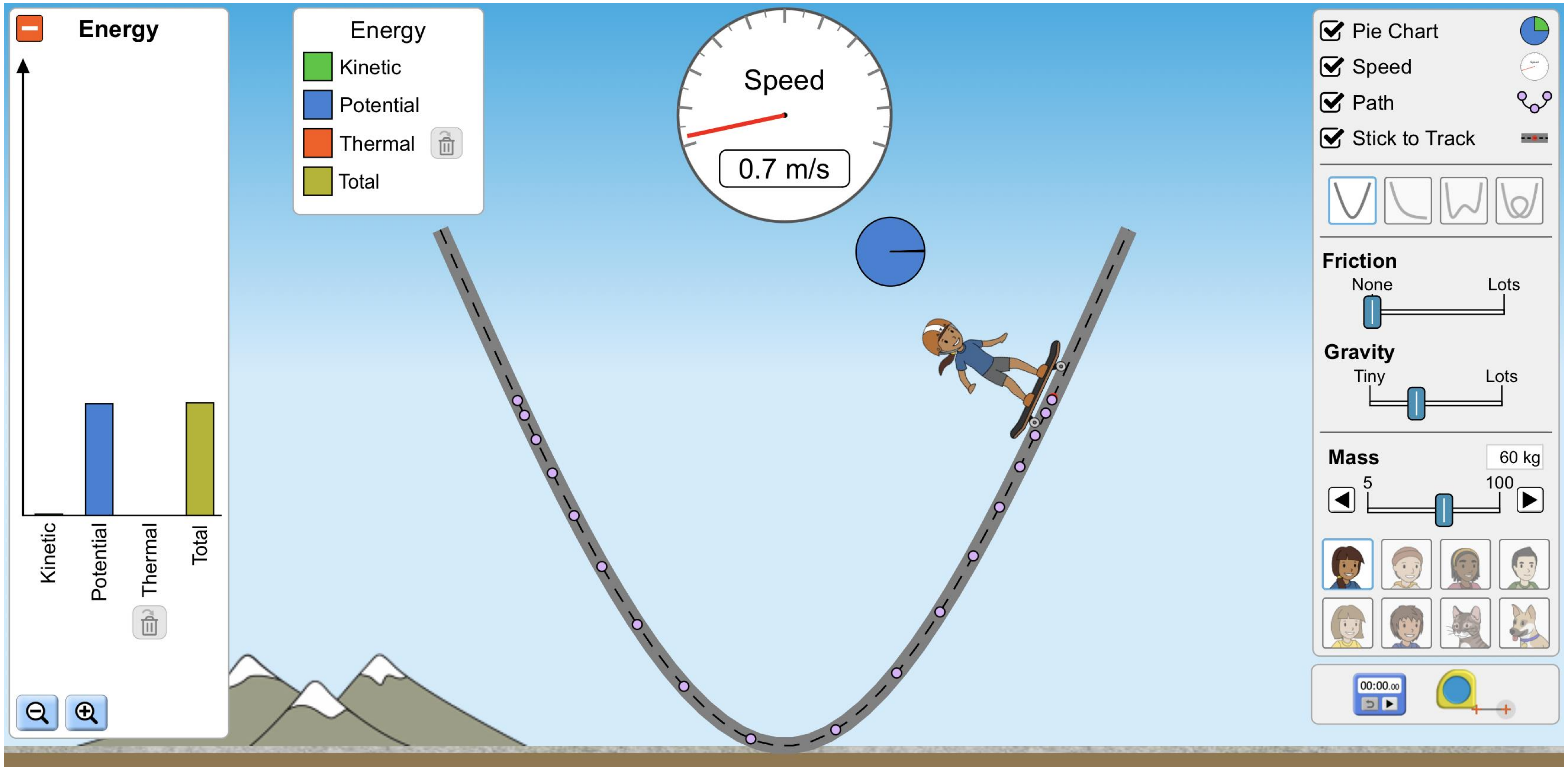
$$\cancel{m}gL(1-\cos\theta_i) = \frac{1}{2}\cancel{m}v_f^2$$

Step 3: $v_f = \sqrt{2gL(1-\cos\theta_i)}$

$$= \sqrt{2 \times 9.8\,\mathrm{m\,s^{-2}} \times 1\,\mathrm{m}\,(1-\cos 25°)}$$

$$\approx 1.36\,\mathrm{m\,s^{-1}}$$

Demo



https://phet.colorado.edu/sims/html/energy-skate-park/latest/energy-skate-park_all.html .

Case study 2: Roller coaster

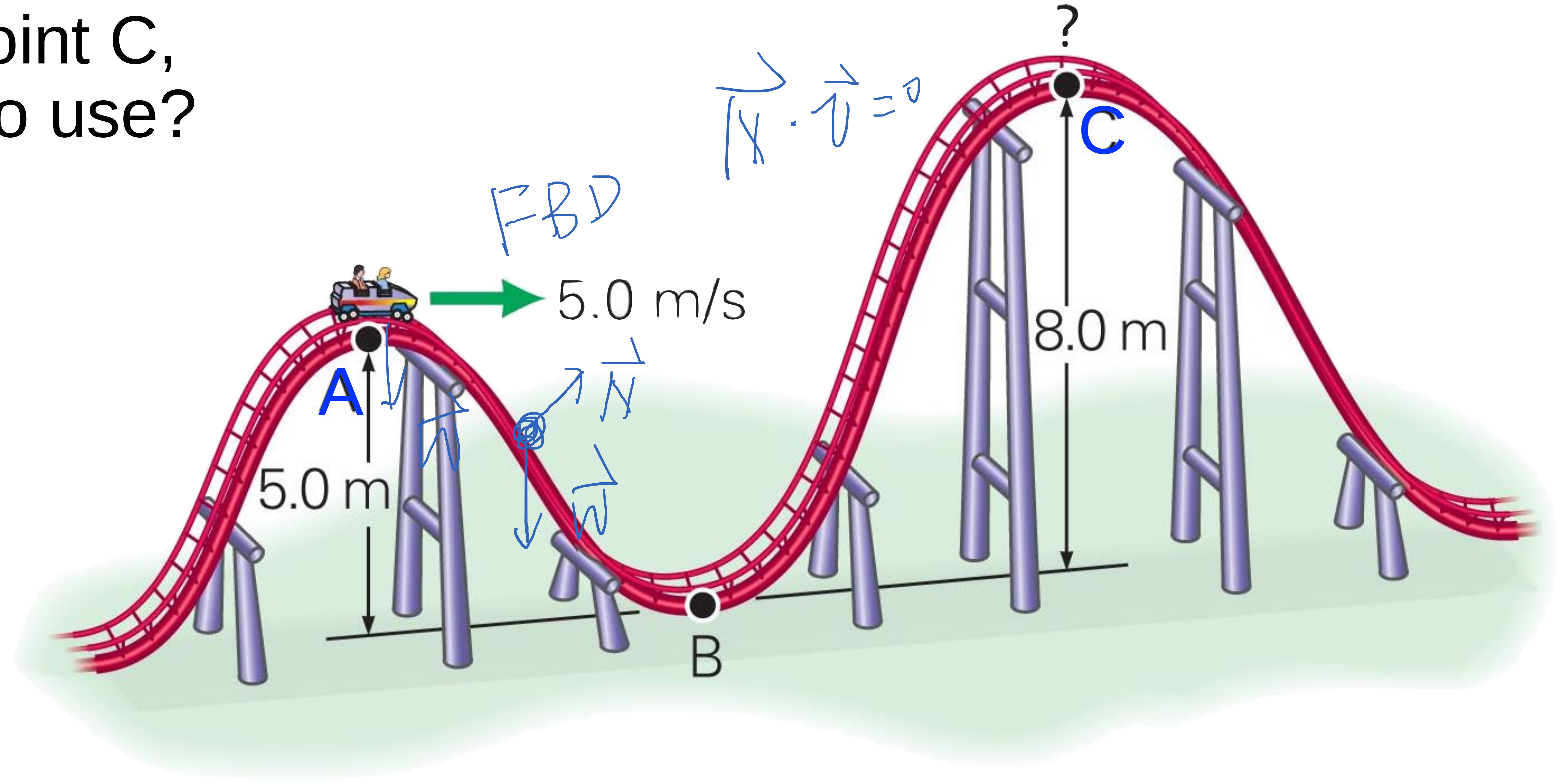
Clicker question 8

- A cart moves freely along a ***frictionless*** roller coaster track. The cart's speed at A is $v_A = 5.00$ m/s, $h_A = 5.00m$, and $h_C = 8.00m$.

  To find out if the cart reaches point C,
  What principles and equations to use?

  A 1D Kinematic equations

  B Conservation of energy



Example 2: Roller coaster

Given: v_A, h_A, g

Goal: $h_{max} \gtreqless h_C$

- A cart moves freely along a ***frictionless*** roller coaster track. The cart's speed at A is $v_A = 5.00$ m/s, $h_A = 5.00m$, and $h_C = 8.00m$. Does the cart reach C?

... max h

if $U(h=0)=0$

Step 1: $U_i + K_i = U_f + \cancel{K_f} = 0$

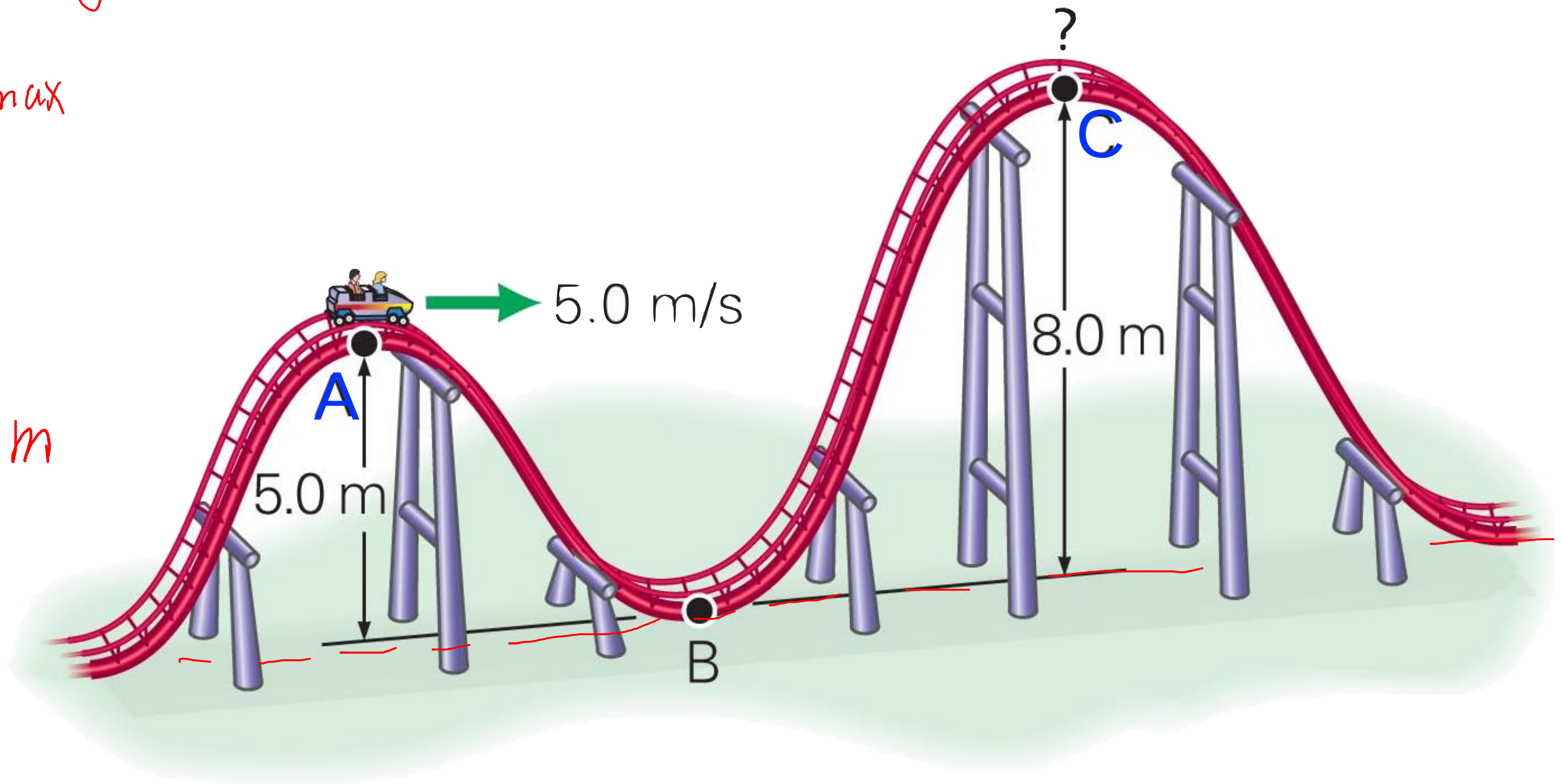
$\cancel{m}gh_A + \frac{1}{2}\cancel{m}v_A^2 = \cancel{m}gh_{max}$

$h_{max} = \frac{v_A^2}{2g} + h_A$

$= \frac{(5\,ms^{-1})^2}{2\times 9.8\,ms^{-2}} + 5.00\,m$

$\approx 6.28\,m < h_C$

Step 2: No



Case study 3: Spring

Example 3

Given: m, v_i, x_i, k
$\quad\quad\quad\quad\quad 0$

Goal: x_f when $v_f = 0$

- A 1.0 kg box is moving with an initial speed of 1.0 m s^{-1} towards a relaxed spring on a frictionless table. The spring constant is 1.0 N m^{-1}. What is the magnitude of the spring compression when the box is stopped?

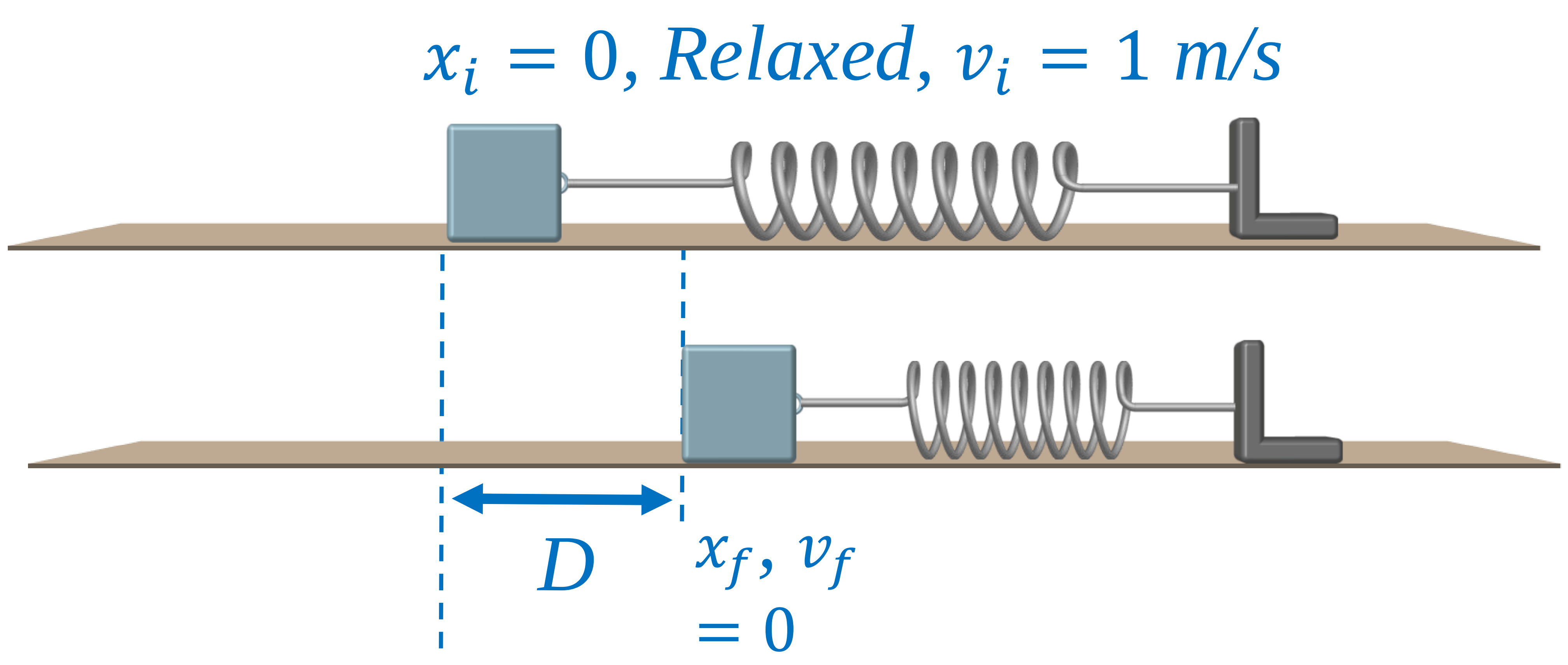
Step 1: $U_i + K_i = U_f + K_f$

$$0 + \frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 + 0$$

Step 2: $x_f = \sqrt{\frac{m v_i^2}{k}}$

$$= \sqrt{\frac{1\,kg \cdot (1\,m s^{-1})^2}{1.00\,N \cdot m^{-1}}}$$

$$= 1\,m$$



Conservation of total energy in general

- When there is work by **friction**, inelastic deformation, chemical reaction, etc.
  - Then the **mechanical energy** is not conserved
- However, **energy** can't be created or destroyed, so…
  - **The *total* energy is conserved**
    i.e., $E = K + U_{pot} + U_{thermal} + \cdots = const$
  - But energy can be *converted* between different forms
- Demo

https://youtu.be/ZvgJ7mVxeg0

Homework 8

- Due in a week

Pre-lecture 9.1.1

- Please complete Pre-lecture in Module 9.1.1 before the next lecture.