PHYS 225
Fundamentals of Physics: Mechanics

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Fall 2024

Lecture 28: Relating rotation and linear motion | angular momentum

Learning goals for today

- Relating linear and rotational motions
- Angular momentum
- Kinetic energy of rotation

7. Relating angular and tangential variables

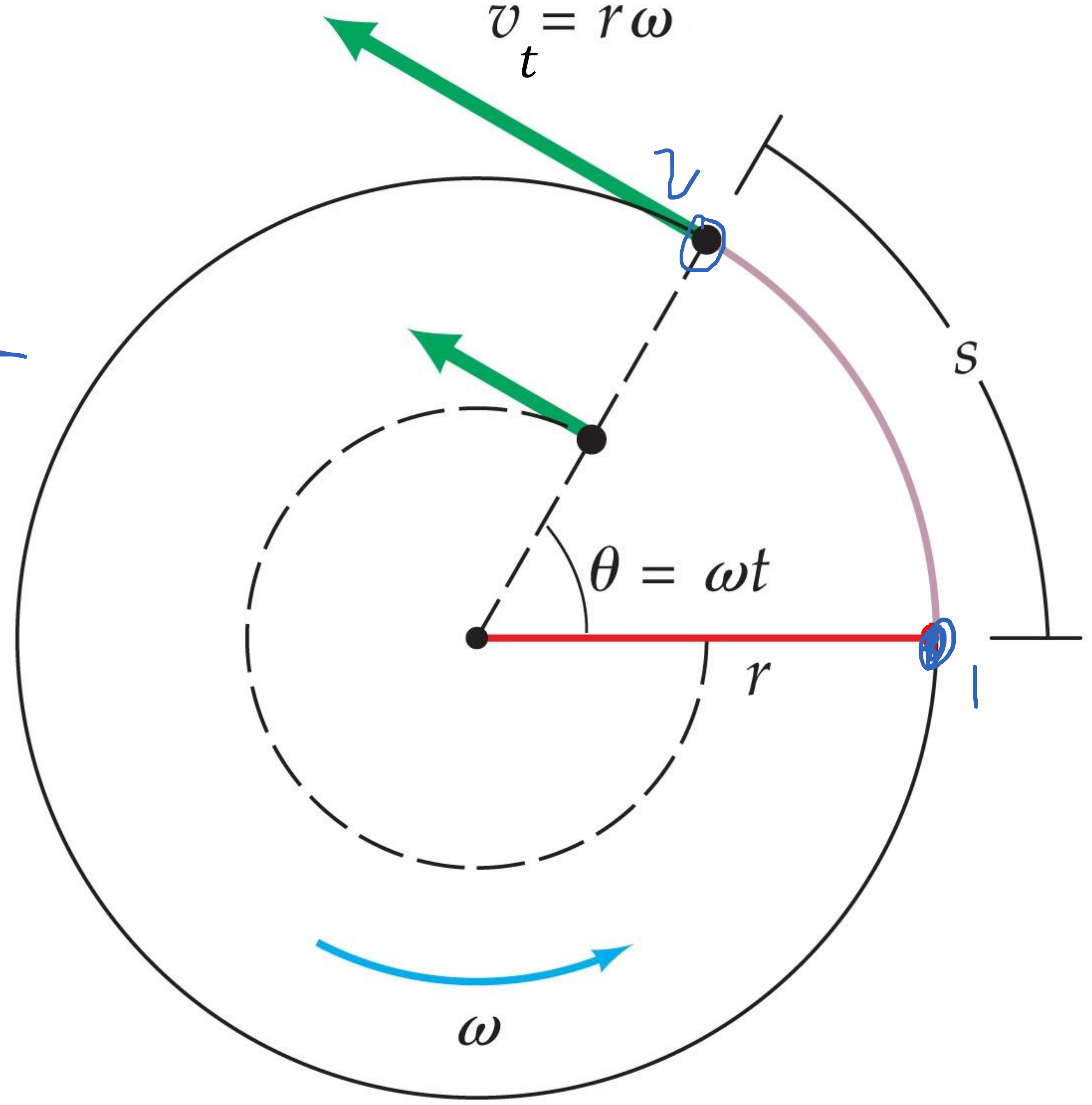
- Distance traveled by a point at a distance r from the axis is related to the angle swept:

$$|S| = |\theta| r$$

Distance traveled

Angle swept

Distance from the rotation axis



- Relation between tangential speed, $|\vec{v}_t|$, and angular speed, $|\vec{\omega}|$

$$|\vec{v}_t| = |\vec{\omega}| r \text{ — radius}$$

Tangential speed

Angular speed

- Relation between tangential acceleration, $|\vec{a}_t|$, and angular acceleration, $|\vec{\alpha}|$

$$|\vec{a}_t| = |\vec{\alpha}| r \text{ — radius}$$

Tangential acceleration

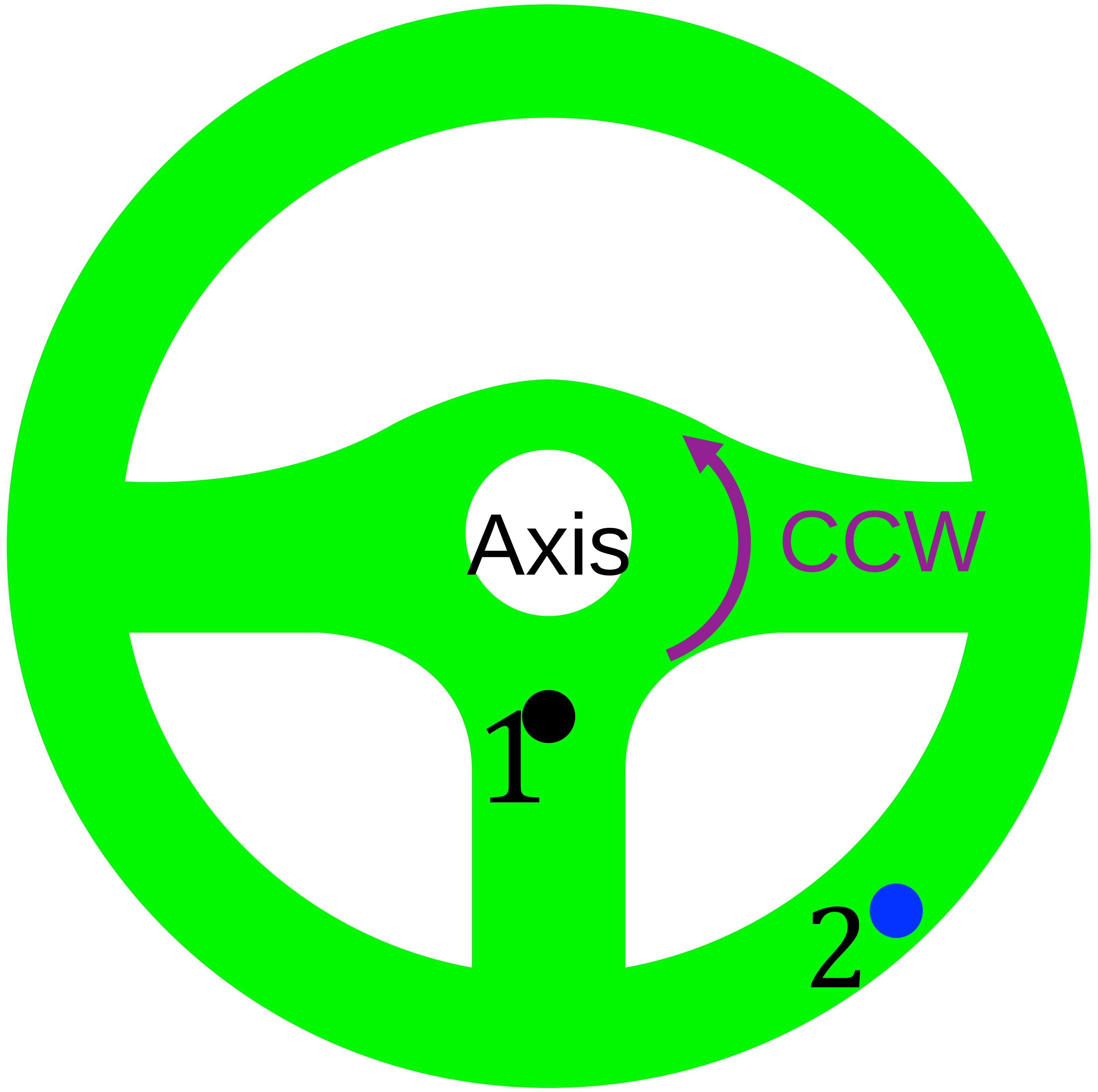
Angular acceleration

Clicker question 7

- The steering wheel is rotating w.r.t. the fixed axis. Which of the following is true?

A Point 1 and point 2 have the same angular speed.

B Point 1 and point 2 have the same tangential speed.



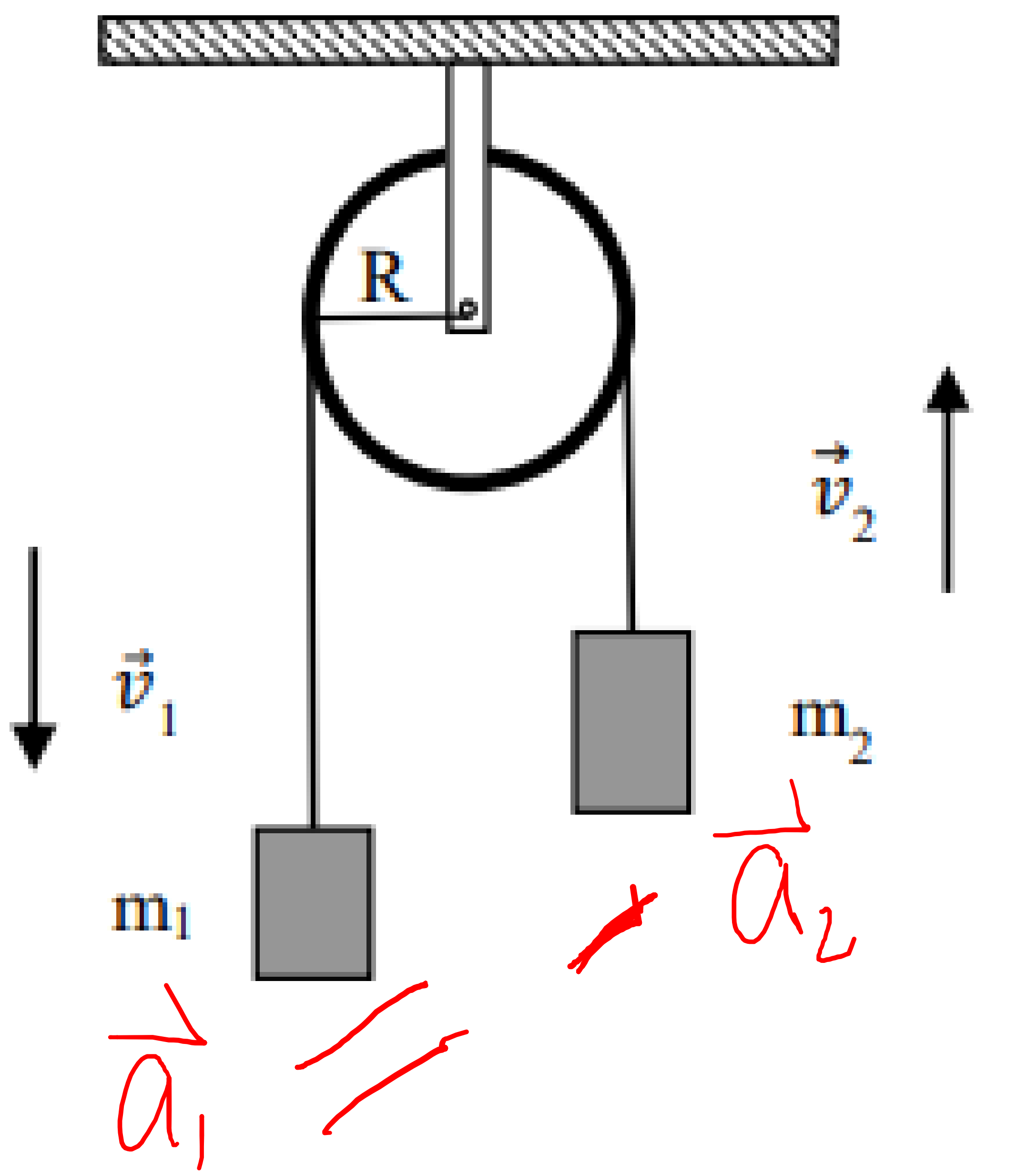
Clicker question 8

- Mass m_1 and m_2) $m_1 > m_2$ (are wrapped by the same string around a pulley of radius R. The system is released from rest and the string can move without slipping on the pulley. The linear acceleration of m_2 is $\vec{a}_2$. Which of the following is true?

A The linear acceleration of m_1 is $-\vec{a}_2$, and the angular acceleration of the pulley is 0.

B The linear acceleration of m_1 is $-\vec{a}_2$,the angular acceleration of the pulley, $\vec{\alpha}$, is out of the screen and $|\vec{a}_2| = |\vec{\alpha}|R$.

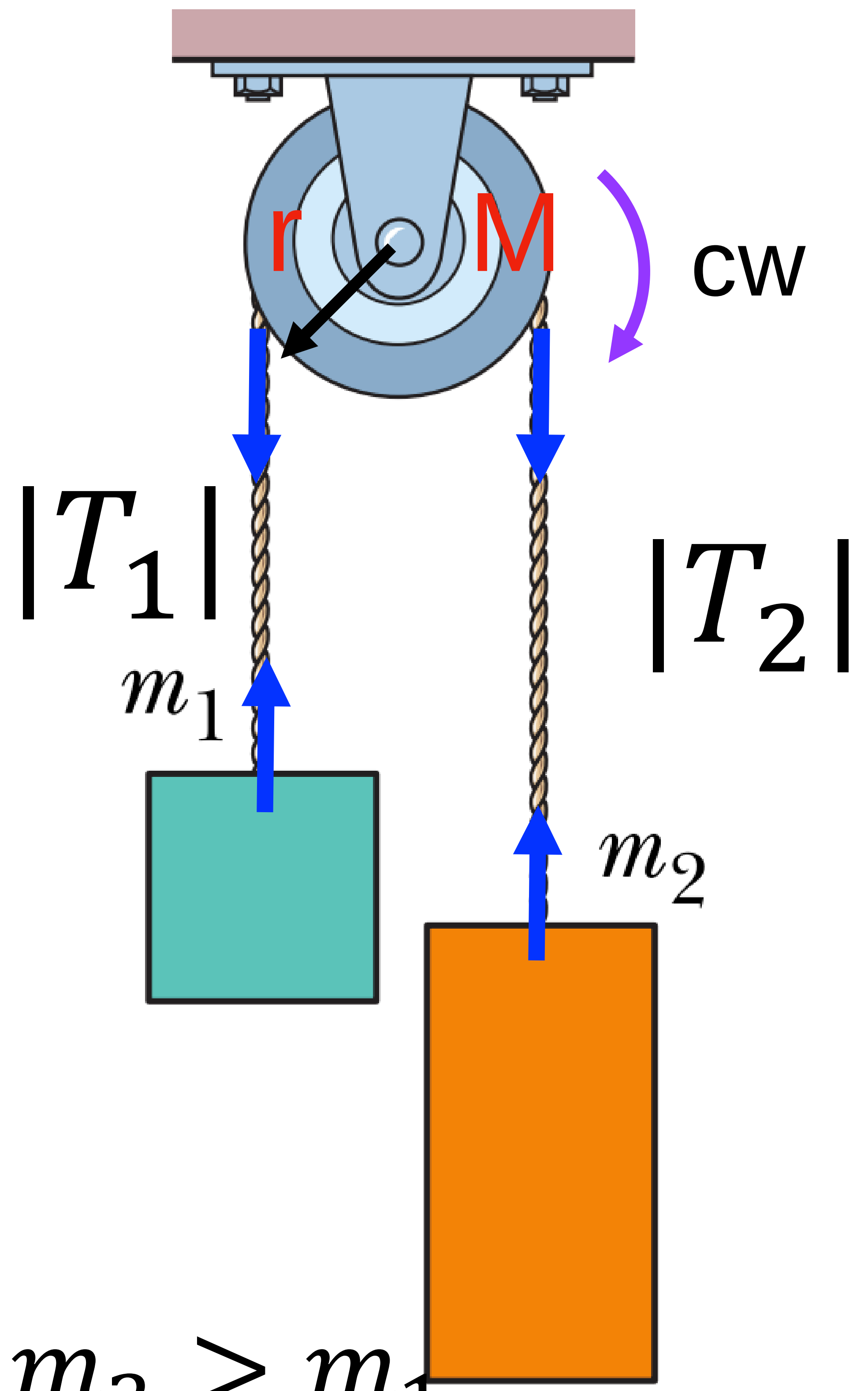
C The linear acceleration of m_1 is $\vec{a}_2$,the angular acceleration of the pulley, $\vec{\alpha}$, is into the screen and $|\vec{a}_2| = |\vec{\alpha}|R$.



Example 3

Given: M, m_1, m_2, r

Goal: $\vec{\alpha}_p$



$m_2 > m_1$

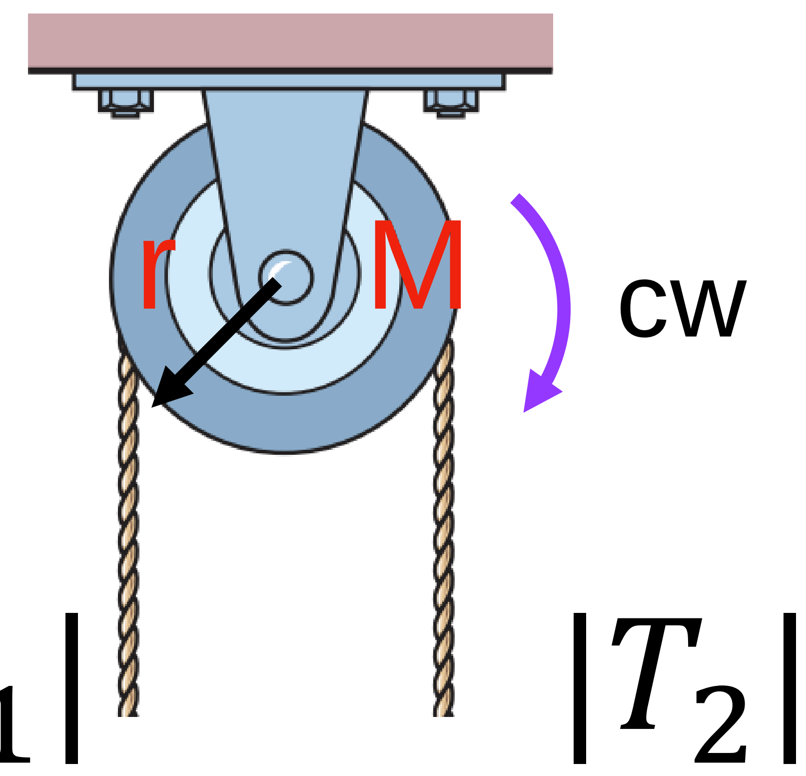
Clicker question 9.1

- The mass of the pulley is M, and the moment of inertia is $\frac{1}{2}Mr^2$. The massless string is wrapped around the pulley and can move without slipping. The string is pulled on both sides, such that the pulley is accelerated **from rest** to **rotating clockwise** with a non-zero angular speed.

RHR for $\vec{\alpha}$

What is the direction of the torque on the pulley?

$\vec{\alpha}$ $\otimes$ into



A Pointing into the screen

B Pointing out of the screen

C Zero

2nd law for rot.

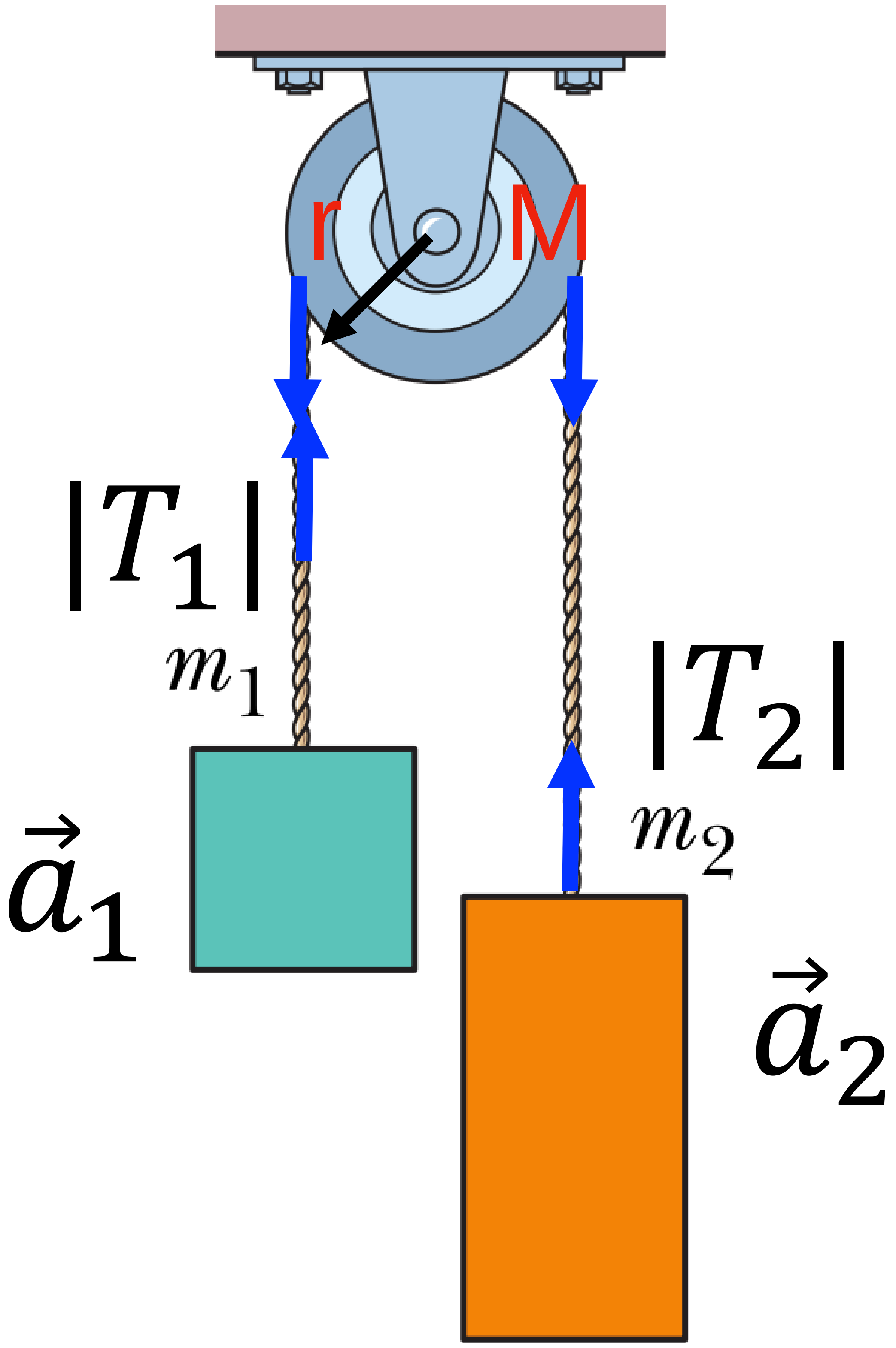
$$\vec{\tau}_{net} = I\vec{\alpha}$$

$\otimes$ $\otimes$

Clicker question 9.2

- The mass of the pulley is M, and the radius of the pulley is r. The massless string is wrapped around the pulley and can move without slipping. Two masses are tethered to each ends of the string. $m_2 > m_1$. The system is released **from rest**.

What is true about the acceleration of m_1 and m_2?



• The mass of the pulley is M, and the radi
around the pulley and can move without s
the string. $m_2 > m_1$. The system is relea

$|\vec{a}_1| > |\vec{a}_2|$ X

pulley is r. The massless string is wrapped
Two masses are tethered to each ends of
n rest.

$\vec{a}_2 = -\vec{a}_1$

$|\vec{a}_1| = |\vec{a}_2| = |\vec{\alpha}|r$,where α is the angular acceleration of the pulley.

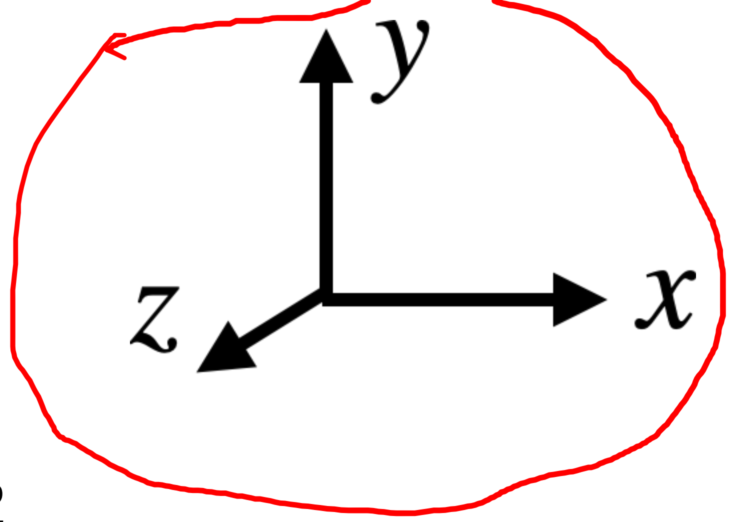
Both B and C

Example

Given: M, m_1, m_2, r

Goal: $\vec{\alpha}$

$\vec{\tau} = \vec{r} \times \vec{F}$



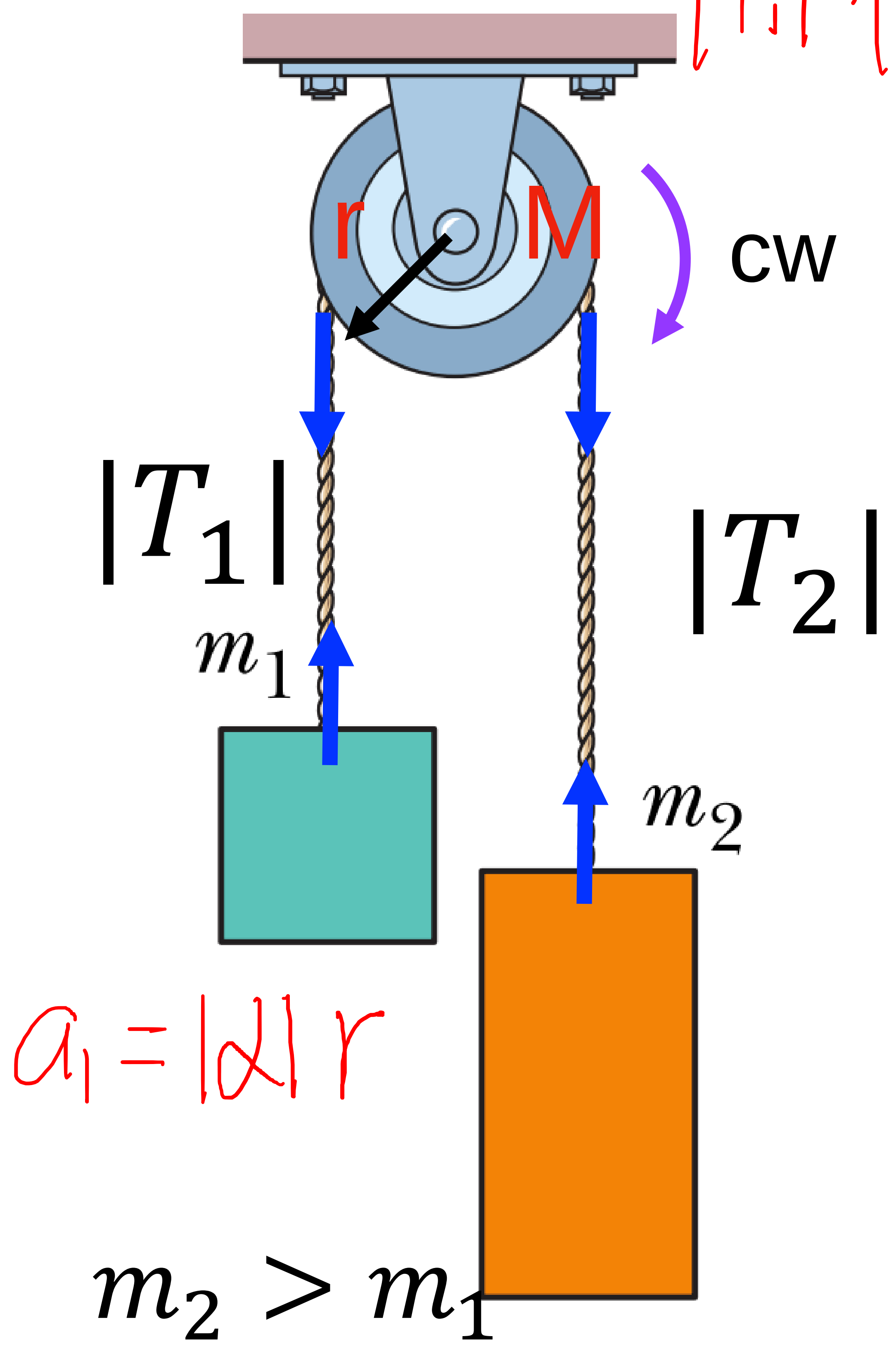
- The mass of the pulley is $M = 0.500kg$, and the radius of the pulley is $r = 0.100m$. $m_2 = 2.00kg$, $m_1 = 1.00kg$. The string attached to m_1 and m_2 wraps around the pulley and is released from rest. The two blocks then move without slipping on the pulley. What's the angular acceleration of the pulley? (The moment of inertia of the pulley here is $I = \frac{1}{2}Mr^2$)

Step 1: 2nd law for rot. pulley: $\vec{\tau}_{net} = I\vec{\alpha}$ (1)

2nd law for translation: $\vec{F}_{net,1} = m_1\vec{a}_1$ (2)

$\vec{F}_{net,2} = m_2\vec{a}_2$ (3)

$|T_1| \neq |T_2|$



Step 2: $\cancel{r}|T_1| - \cancel{r}|T_2| = -\frac{1}{2}Mr^{\cancel{2}}|\vec{\alpha}|$ (4)

$|T_1| - m_1 g = m_1 a_1 = m_1|\alpha| r$ (5)

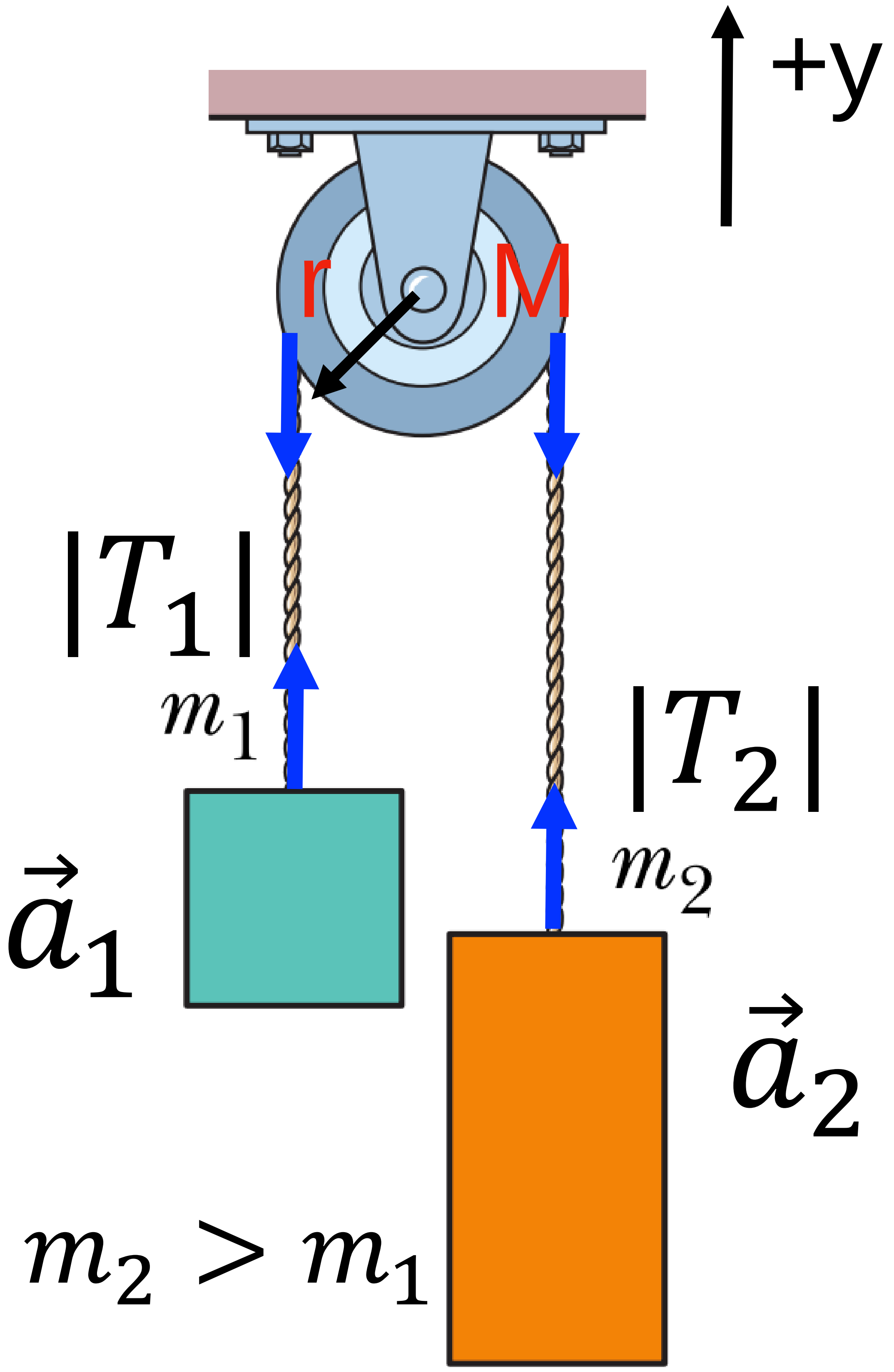
$|T_2| - m_2 g = m_2 a_2 = -m_2|\alpha| r$ (6)

Example: Listing equations to solve the problem

- The mass of the pulley is $M = 0.500kg$, and the radius of the pulley is $r = 0.1m$. $m_2 = 2.00kg$, $m_1 = 1.00kg$. The string attached to m_1 and m_2 wraps around the pulley and is released from rest. The two blocks then move without the string slipping on the pulley.

Given: M, r, m_1 and m_2

Goal: α



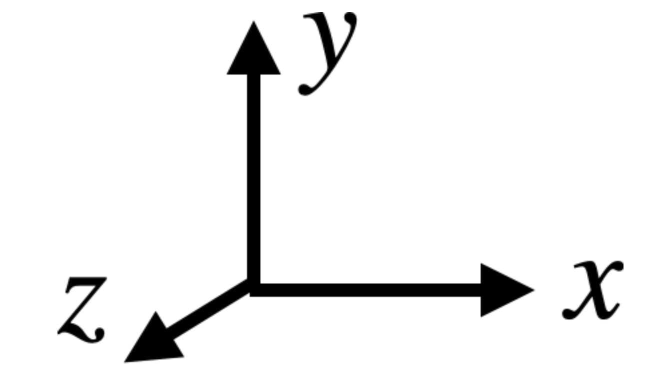
Rotation of the pulley: $$r\left(\left|\vec{T}_1\right| - \left|\vec{T}_2\right|\right) = -\frac{1}{2}Mr^2|\vec{\alpha}| \qquad (1)$$

Rising of m_1: $$\left(\left|\vec{T}_1\right| - m_1 g\right)\hat{j} = m_1\vec{a}_1 = m_1|\vec{\alpha}|r\hat{j} \qquad (2)$$

Falling of m_2: $$\left(\left|\vec{T}_2\right| - m_2 g\right)\hat{j} = m_2\vec{a}_2 = -m_2|\vec{\alpha}|r\hat{j} \qquad (3)$$

Example: Solving for $\vec{\alpha}$

Given: $M = 0.500kg$, $r = 0.1m$, $m_1 = 1.00kg$ and $m_2 = 2.00kg$
Goal: α



Rotation of the pulley: $\cancel{r}\left(|\vec{T}_1| - |\vec{T}_2|\right) = -\frac{1}{2}Mr^{\cancel{2}}|\vec{\alpha}|$ (1)

Rising of m_1: $\left(|\vec{T}_1| - m_1 g\right)\hat{j} = m_1|\vec{\alpha}|r\hat{j}$ (2)

Falling of m_2: $\left(|\vec{T}_2| - m_2 g\right)\hat{j} = -m_2|\vec{\alpha}|r\hat{j}$ (3)

$|T_1| \neq |T_2|$

(1): $|\vec{T}_1| - |\vec{T}_2| = -\frac{1}{2}Mr|\vec{\alpha}|$ .... (4)

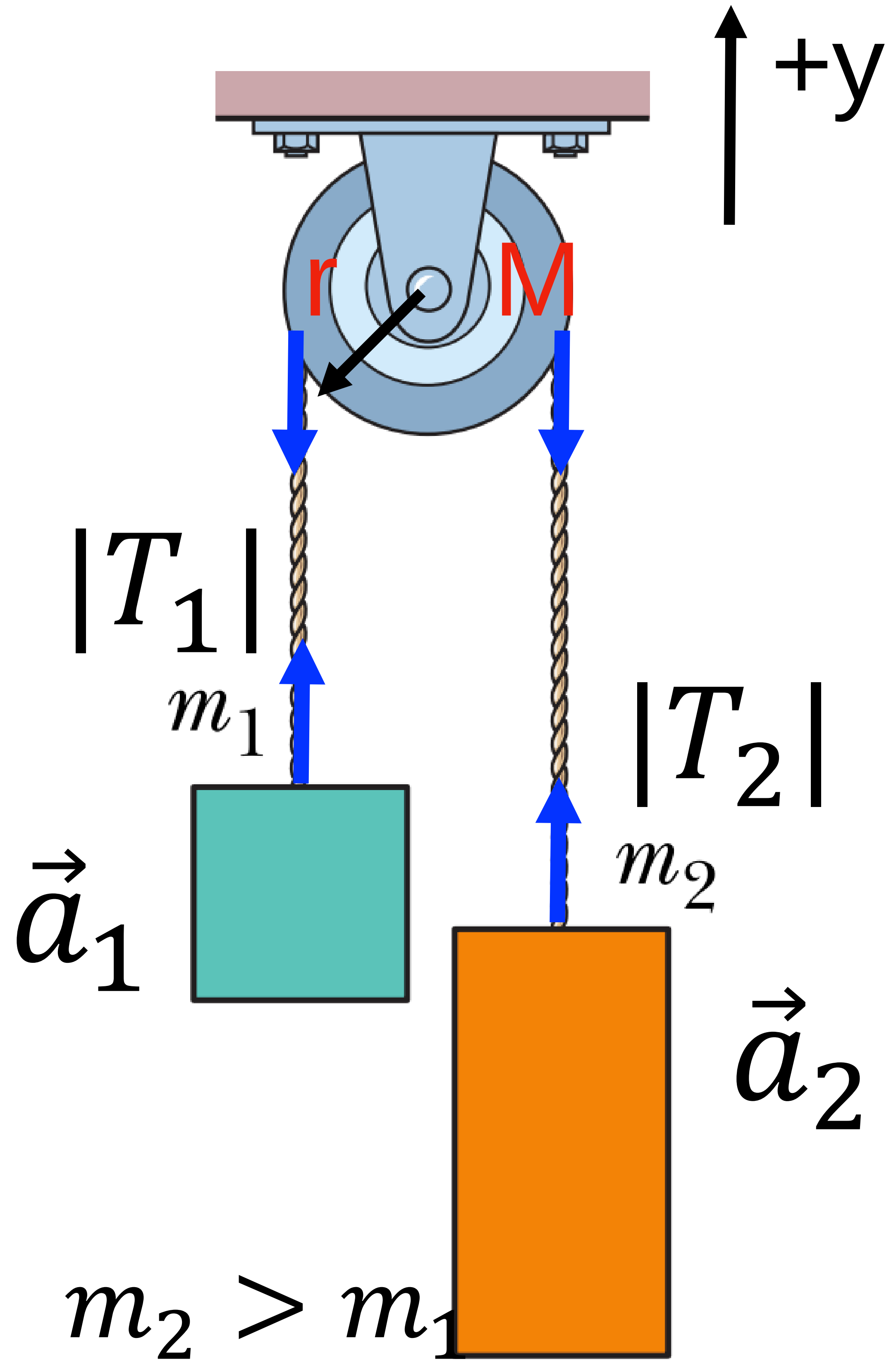
(2) − (3): $|\vec{T}_1| - |\vec{T}_2| + (m_2 - m_1)g = (m_1 + m_2)|\vec{\alpha}|r$ ..... (5)

(5) − (4): $(m_2 - m_1)g = (m_1 + m_2 + \frac{1}{2}M)|\vec{\alpha}|r$ ..... (6)

$$|\vec{\alpha}| = \frac{(m_2 - m_1)g}{(m_1 + m_2 + \frac{1}{2}M)r} = \frac{(2kg - 1kg)\cdot 9.8ms^{-2}}{1kg + 2kg + \frac{1}{2}\times 0.5kg}$$

$$\approx 30.2\ rad\cdot s^{-2}$$

$$\vec{\alpha} = -|\vec{\alpha}| = -30.2\ rad\cdot s^{-2}\,\hat{k}$$



Analogy: Translational/linear motion and rotational motion

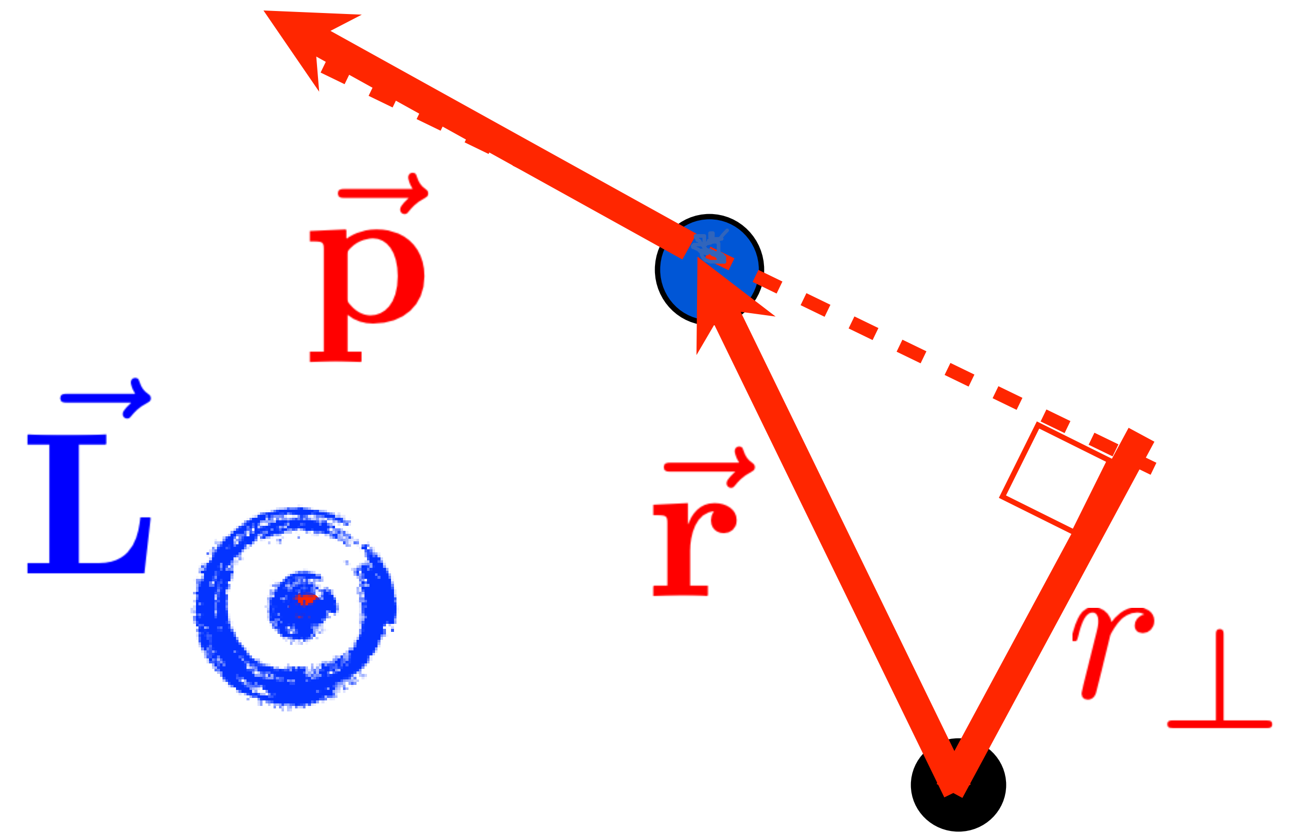
Linear motion	Rotational motion
Velocity, $\vec{v}$	Angular velocity, $\vec{\omega}$
Acceleration, $\vec{a}$	Angular acceleration, $\vec{\alpha}$
Mass, m	Moment of inertia, I
Force, $\vec{F}$	Torque, $\vec{\tau}$
Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$	Newton's 2nd law for rotation: $\vec{\tau}_{net} = I\vec{\alpha}$
Linear momentum: $\vec{P} = m\vec{v}$	Angular momentum: ?
Kinetic energy: $K = \frac{1}{2}mv^2$	Kinetic energy: ?

8. Angular momentum: Two expressions

- **Angular momentum** In terms of $\vec{P}$ (for a point):

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} \qquad (1)$$

Displacement from the axis　　Linear momentum

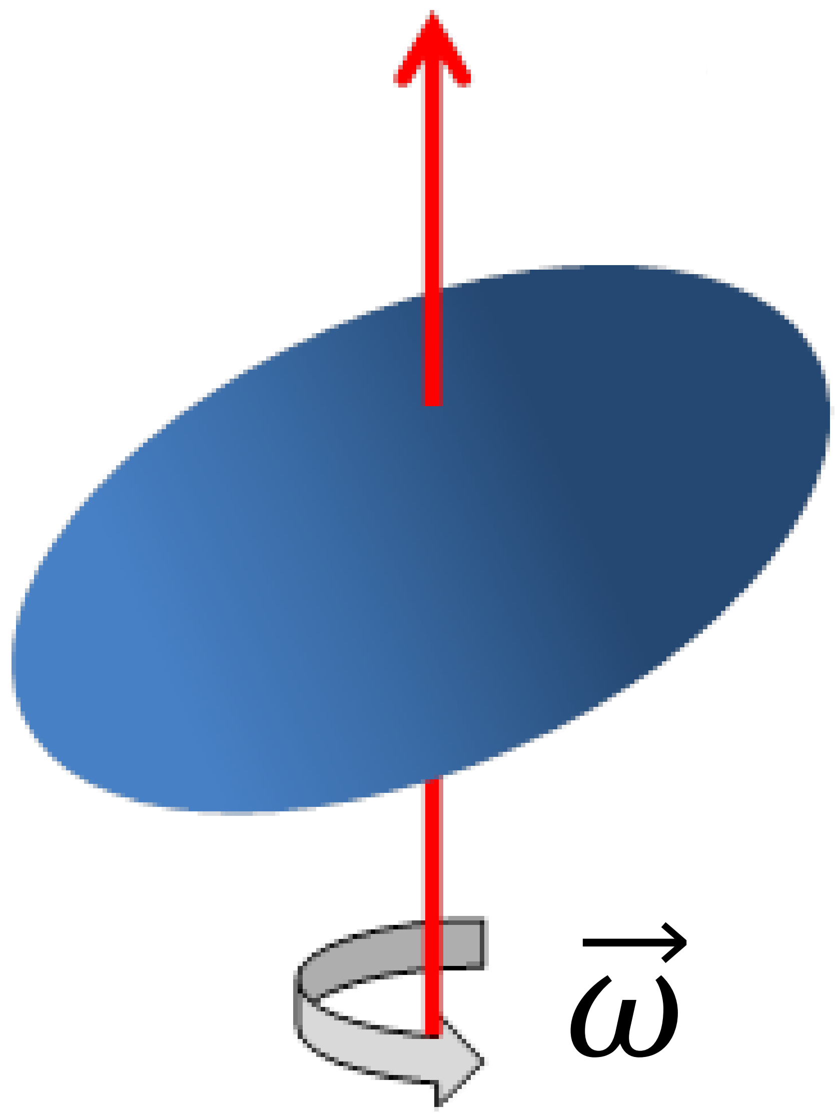


- **Angular momentum** In terms of I:

$$\vec{L} = I\vec{\omega} \qquad (2)$$

Moment of inertia　　Angular velocity

This expression is more convenient for rigid bodies!



Newton’s 2nd law in terms of angular momentum

- Recall Newton’s 2nd law for rotation in terms of **angular acceleration**:

$$\vec{\tau}_{net} = I\vec{\alpha}$$

Torque

Moment of inertia

Angular acceleration

$$= I\frac{d\vec{\omega}}{dt} = \frac{d(I\vec{\omega})}{dt} = \frac{d\vec{L}}{dt}$$

- Newton’s 2nd law for rotation in terms of **angular momentum**:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Angular momentum

Conservation of angular momentum

Condition:

If $\vec{\tau}_{net} = 0$

- Newton's 2nd law for rotation in terms of angular momentum:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Torque

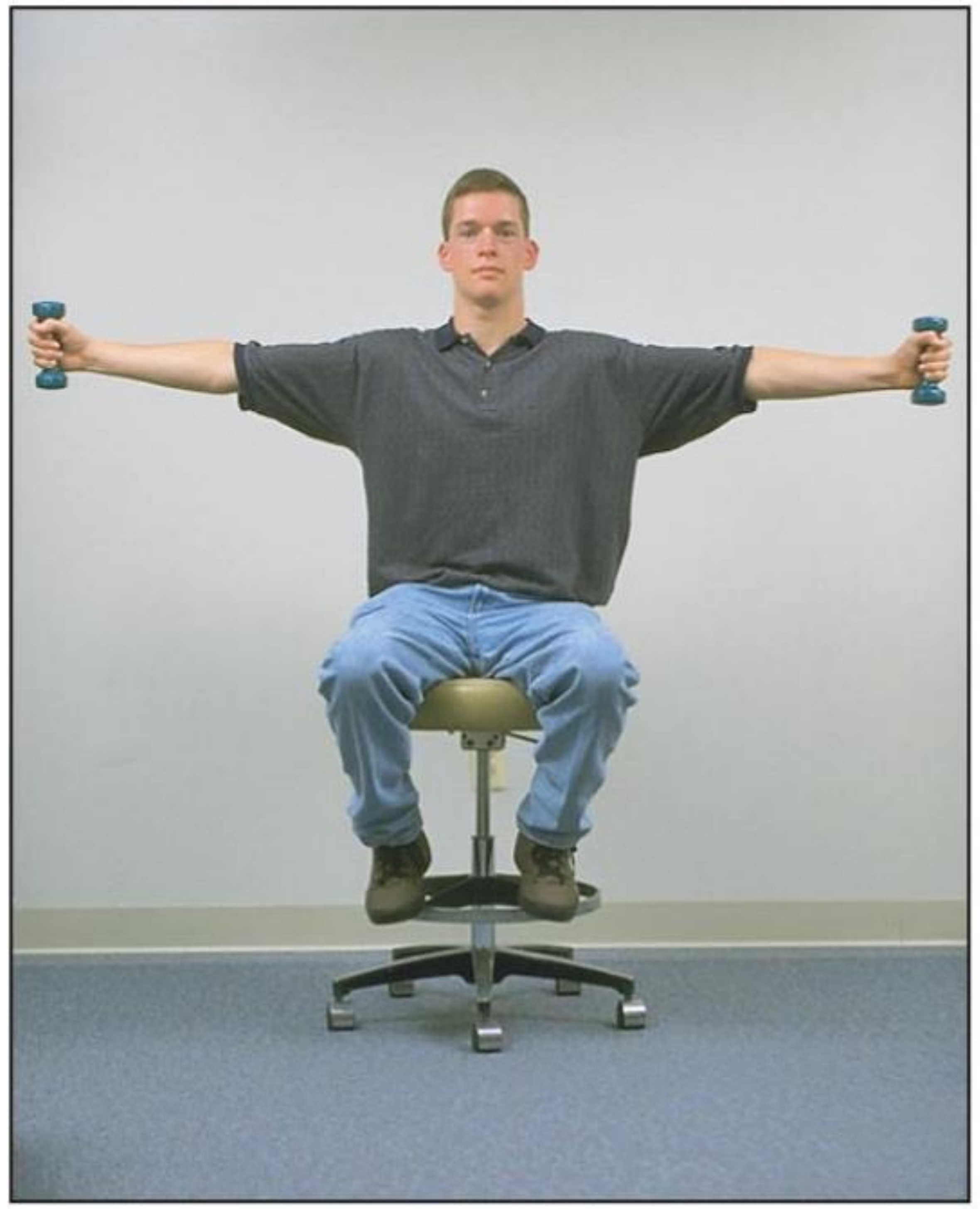
Angular momentum

- Conservation of angular momentum:

If $\vec{\tau}_{net} = 0$,then $\frac{d\vec{L}}{dt} = 0$,i.e., $\vec{L} = const.$

Clicker question 1

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **angular momentum** after pulling the weights **inward**? (Assuming the friction and air resistance is negligible)



A. Increases in magnitude

B. Decreases in magnitude

C. Changes direction

D. Remains constant

$\vec{\tau}_{net} = 0$

then $\vec{L}$ is conserved.

Conservation of angular momentum:

If $\tau_{tot} \sim 0$, then $\vec{L}_f = \vec{L}_0$

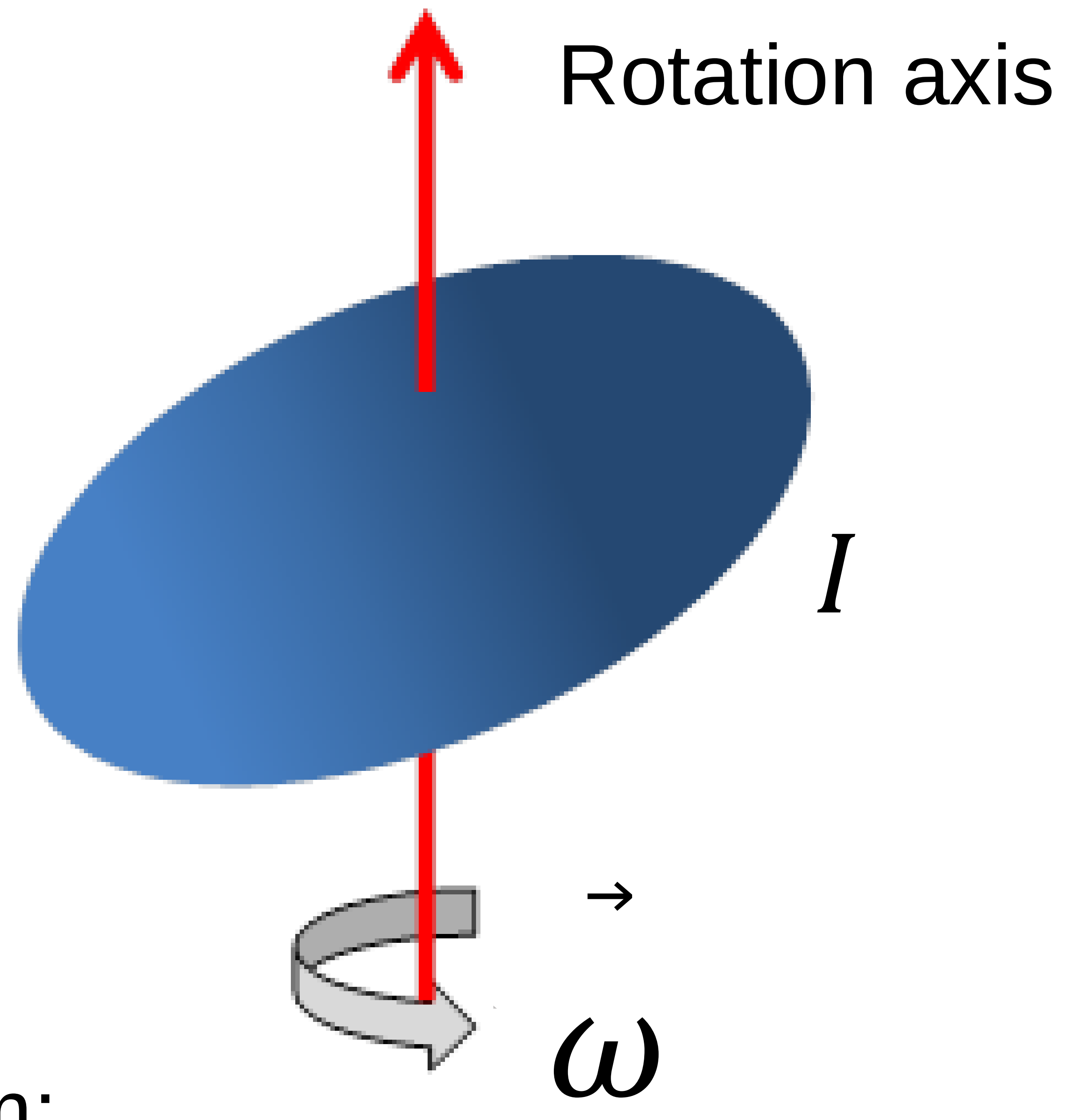
9. Rotational kinetic energy

- Rotational kinetic energy of a rigid body (unit: kg m^2 s^{-2}):

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2}\frac{(I\omega)^2}{I} = \frac{1}{2}\frac{L^2}{I}$$

Moment of inertia

Angular velocity



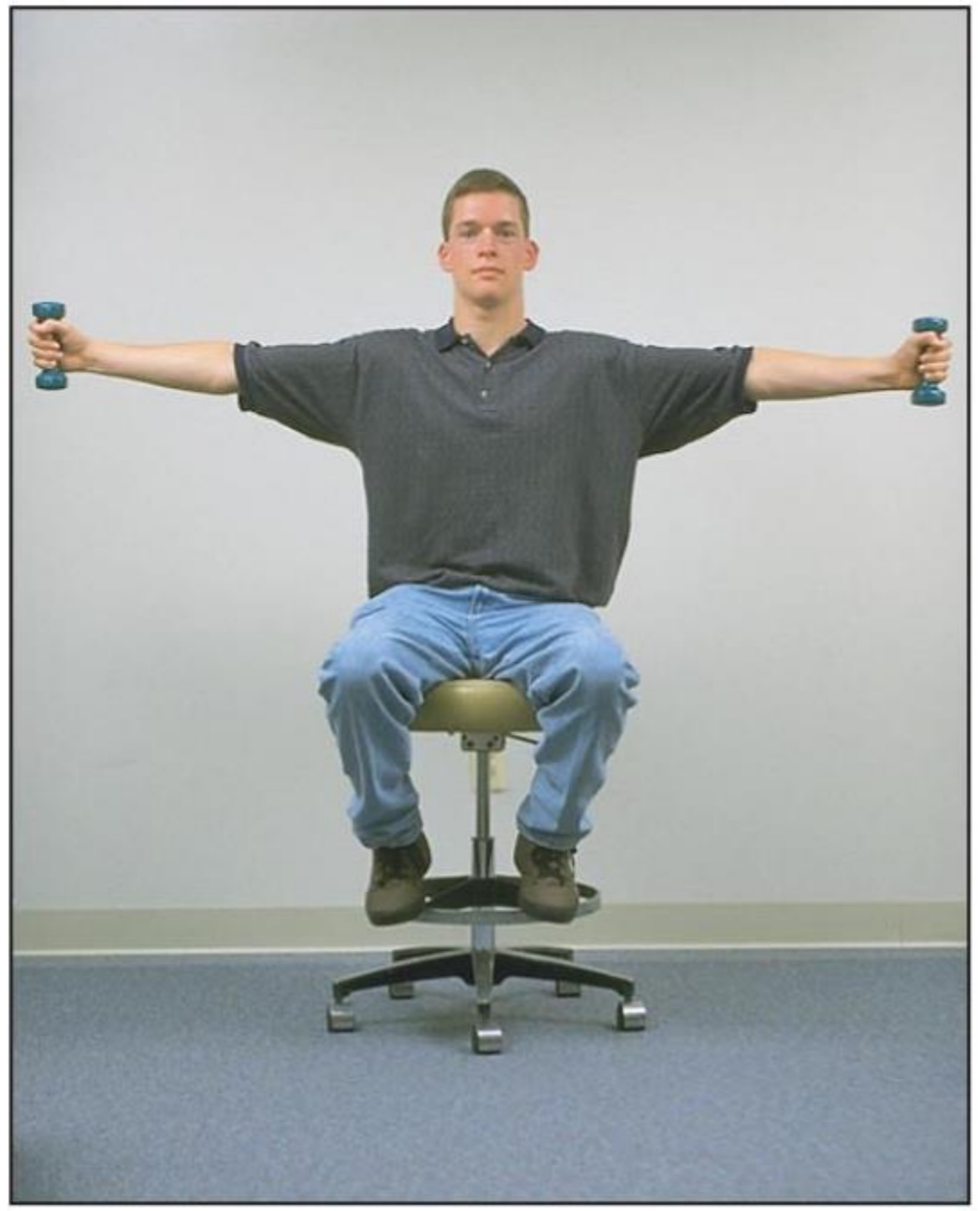
- Rotational kinetic energy of a rigid body in terms of angular momentum:

Angular momentum

$$K = \frac{L^2}{2I}$$

Clicker question 2

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **moment of inertia** after pulling the weights **inward**? (Assuming the friction and air resistance is negligible)



A Increases

B Decreases

C Depends on speed of weights

D Remains constant

$I \downarrow$

$$I = \int r^2 dm$$

$r \downarrow$

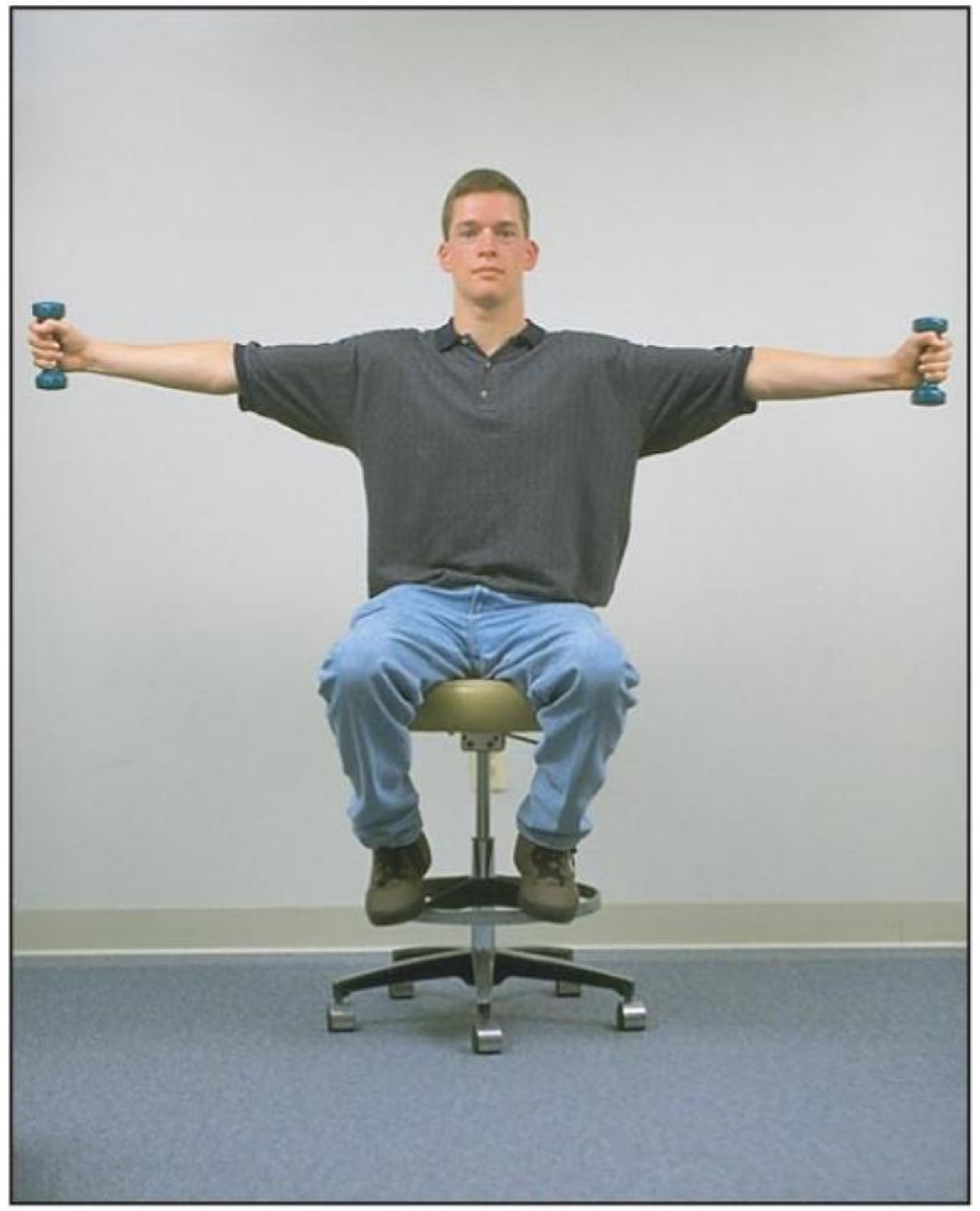
Clicker question 3

$\vec{L}$ ~ conserved

$I \downarrow$

K

- A person sits on a slowly spinning stool holding weights, arms extended. What happens to the person's **kinetic energy of rotation** after pulling the weights inward? (Assuming the friction and air resistance is negligible)



A Increases
B Decreases
C Remains constant

Kinetic energy of rotation: $K = \frac{L^2}{2I}$

$K \uparrow$

Summary: Translational/linear motion and rotational motion

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Velocity, $\vec{v}$	Angular velocity, $\vec{\omega}$
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Kinetic energy: $K = \frac{1}{2}mv^2$	Kinetic energy: $K = \frac{1}{2}I\omega^2$
Linear momentum: $\vec{P} = m\vec{v}$	Angular momentum: $\vec{L} = I\,\vec{\omega}$

Reminder for Final exam

- When: Dec. 19, 5:00 pm-6:50 pm
- Where: In person, SGMH 1506 (the same classroom)
- How:
  - Closed book, closed notes, but a 1-page 2-sided cheat sheet is allowed
  - Calculators are allowed
- What:
  - Chapters 1 to 10