Unit 3: Estimation and Approximation

Lesson 1: Estimation and Approximation

Essential Understanding

• Numbers are used to represent quantities in real life.
• There are more types of number other than the natural counting numbers
• Numbers can be operated on and there is an order to which operations can be carried out.

Essential Questions

•How can arithmetic be performed in quick and simplified manner to obtain estimated values instead of exact ones?
•How do we approximate to a given degree of accuracy?
• What is the difference between estimation and approximation?

Key Points (Learning Outcomes)

• What is estimation? What is approximation?
• Why do we estimate?
• How do we estimate?

Difficult Point

• To use different techniques of estimating based on the needs of situations

Critical Point

• Approximation is estimation to a given degree of accuracy.
• Understand that there is need to have a system to ascertain the relative degree of accuracy of an estimation, hence the need to modify estimation by rounding to a given decimal place

Definition to Estimation

Estimation
ɛstɪˈmeɪʃ(ə)n/
noun
noun: **estimation**; plural noun: **estimations**
1. a rough calculation of the value, number, quantity, or extent of something.
"estimations of protein concentrations"

So why do we estimate?

-Estimation is often enough to solve a problem, so why the need for an exact answer?
If a hamburger costs $4.25, fries cost $0.99 and you've got a five dollar note, a two-dollar note and some coins, you don't need to count the notes and coins to know if you can afford it.

-Sometimes all you need to know is "about how much"
f you're painting the living room, you want to know about how much paint you need. No need to figure it out to the teaspoon.

-Sometimes you don't have any paper or pencil or a calculator with you, or it's too hard to calculate an exact answer mentally.

-Sometimes it's not even possible to get an exact answer.
You've promised to pick up a friend at the airport. The vagaries of traffic prevent you from knowing precisely how long it will take to get there. But you must still decide when to leave home.

-You can check whether an answer is reasonable.
You've started a new exercise programme and you want to know how many extra calories you've burned this week. Because your interest is so keen, you gather calories-per-minute figures and add them. The sum seems low. Did you add correctly?

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Significant Figures

Essential Understanding

• Numbers are used to represent quantities in real life.
• There are more types of number other than the natural counting numbers
• Numbers can be operated on and there is an order to which operations can be carried out.

Essential Questions

• How can we evolve the natural number systems to model real-life situations?
• How do we denote and perform operations on real numbers?

Key Points (Learning Outcomes)

• Definition of Significant figures Difficult Point
• Rules for counting significant figures

Critical Point

• Why is rounding to a given significant figure necessary when we can already round to a given decimal point?

The exact thickness of a piece of glass is 0.004 503 m.
The thickness of the piece of glass in metre correct to 2 decimal places is 0.01cm.
• Is this figure going to be useful to him?
• How else can you describe the length to him?

RULES FOR WRITING SIGNIFICANT FIGURES

Rules	Example	Number of Significant Figures
All non-zero figures are significant.	846.381 5	7
Zeros at the **end** of a decimal are significant.	182.00 3.660	5 4
Zeros that come before the first non-zero figure are **not** significant.	0.000 256 0	4
Zeros that lie between significant figures are significant.	310.008 45000.0	6 6
The final zeros in a whole number may or may not be significant depending on how the estimation is made.	94 000	2 (if the estimation is made correct to the nearest 1 000) 3 (if the estimation is made correct to the nearest 100) 4 (if the estimation is made correct to the nearest 10) 5 (if the estimation is made correct to the nearest 1)

Remember not to confuse the number of significant figures with the number of decimal places!

Round and Truncation Errors

Essential Understanding

• Numbers are used to represent quantities in real life.
• There are more types of number other than the natural counting numbers
• Numbers can be operated on and there is an order to which operations can be carried out.

Essential Questions

• How can we evolve the natural number systems to model real-life situations?
• What are rounding errors and how to avoid them in multi-steps workings

Key Points (Learning Outcomes)
• understand the concepts of rounding and truncation errors for multi-steps workings

• know the meta-language such as
- evaluate
- estimate
- round .. to x decimal places
- round .. to x significant figures

Difficult Point
• How to avoid rounding errors

Critical Point
• Premature rounding of figures in workings will lead to rounding errors

When trying to **evaluate**, **calculate** or **solve** a question, **DO NOT** round off before the end of the calculation even if you are asked to round off the answer.

E.g. **Evaluate** $2.44 + 6.93$ to 2 significant figures.

$2.44 + 6.93 \approx 2.4 + 6.9 = 9.3$ **(WRONG!)** 

$2.44 + 6.93 = 9.37 \approx 9.4$ (2 s.f.) **(RIGHT!)** 

2. When asked to **estimate** the calculation, always work to one more significant figure than you are required to give.

E.g. **Estimate** 18.456×25.923 to 1 significant figure.

 $18.456 \times 25.923 \approx 20 \times 30$ (incorrect to estimate each number to 1 s.f.)

$= 600$ **(WRONG!)**

 $18.456 \times 25.923 \approx 18 \times 26$ (estimate each number to 2 s.f.)

$= 468$

≈ 500 (1 s.f.) **(RIGHT!)**

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When numbers in an expression is rounded off too early, we may not get an accurate numerical value for the expression.
In Practice... If a problem requires an answer correct to 3 significant figures, we should store the intermediate working values in the calculators or round them off to at least 4 significant figures. Storing and using stored calculator values is the preferred method though.