

## Set Language and Notation

### **Set Language and its notation**

A set is a collection of well-defined and distinct objects such as letters, numbers, people, etc

### **Greatest and Least Values of Subsets**

Since the letter 'a' is an element of  $S = \{a, e, i, o, u\}$ , we write  $a \in S$ .

Since the letter 'b' is not an element of  $S$ , we write  $b \notin S$ .

Since there are 5 elements in  $S$ , we write  $n(S) = 5$

$\in$  denotes 'is an element of' and  $\notin$  denotes 'is not an element of'.

$n(A)$  denotes the number of elements in set  $A$

### **A set can be defined by**

(a) listing its elements within braces, e.g.  $A = \{1, 2, 3, 4\}$ ,

(b) stating its characteristic in words, e.g.  $B = \{x : x \text{ is a prime number}\}$ ,

In set-builder notation:

It says "the set of all  $x$ 's, such that  $x$  is greater than zero".

With the use of braces " $\{ \}$ " which stand for the words "the set of", the elements of a set may be specified in a few ways.

For example, the set of prime numbers less than 10 can be presented as

i.  $A = \{2, 3, 5, 7\}$

or ii.  $B = \{x : x \text{ is a prime number less than } 10\}$

We use upper case letters to denote a set. Elements of a set may consist of upper case or lower case letters. We may also use lower case letters for variables representing elements of a set.

### **Some Definitions**

A finite set is a set which contains a countable number of elements.

e.g.  $A = \{\text{months in the year with 31 days}\}$

- An infinite set is a set which contains an uncountable number of elements.

e.g.  $B = \{\text{even numbers}\}$

- The universal set  $\epsilon$  is a set which contains all the available elements.

- The empty set or null set  $\{ \}$  is a set which contains no elements.

e.g.  $C = \{x : x \text{ is a real number and } 2x + 1 = 0\}$

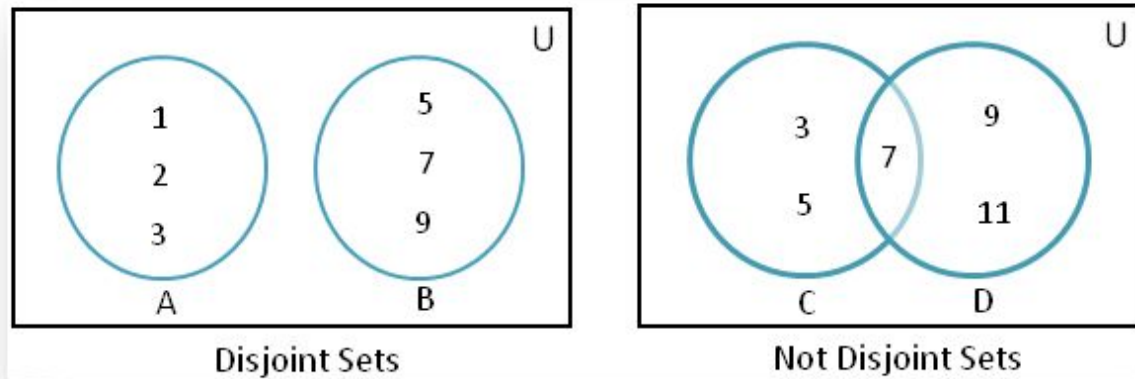
### **Equal sets**

If two sets contain the exact same elements, we say that the two sets are equal sets.

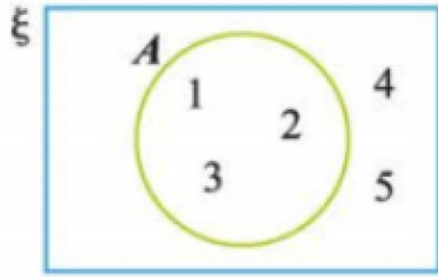
E.g. Consider 3 sets,  $A = \{1, 2, 3\}$ ,  $B = \{3, 2, 1\}$  and  $C = \{a, b, c\}$ .

$A$  and  $B$  are equal sets but  $A$  and  $C$  are not equal sets.

**Disjoint sets** are sets with no common elements at all



A **Venn Diagram** is a diagrammatic representation of sets. It is commonly used to illustrate relationships among sets.



The rectangle represents the set of all the elements that are under consideration for this particular situation, i.e.  $\{1, 2, 3, 4, 5\}$ . This is called the **universal set** and is denoted by the symbol  $\xi$  (pronounced 'xi'), i.e.  $\xi = \{1, 2, 3, 4, 5\}$ .

In the diagram,  $A = \{1, 2, 3\}$ .

**Complement of a set** contains elements belonging to the universal set but not belonging to another given set.

The complement of a set  $A$  is denoted by  $A'$ . That is:  $A' = \{x : x \in \xi \text{ and } x \notin A\}$

A rectangle is used to denote a universal set and circles or ovals to represent its subsets.

### Subsets and Proper Subsets

$P$  is a subset of  $Q$  if each element of  $P$  is also an element of  $Q$ .

If every element of  $B$  is also an element of  $A$ ,  $B$  is said to be a subset of  $A$ .

- In symbols, we use the notation  $B \subseteq A$ .
- However, not every element in  $B$  can be found in  $C$ , thus  $B \not\subseteq C$ .
- By definition of subset, we find that  $B \subseteq D$  and  $D \subseteq B$ .
- When this happens, the two sets are equal sets, or,  $B = D$ .
- In general, any set is a subset of itself.
- By definition, the empty set is a subset of any set.
- Therefore,  $\subseteq A, \subseteq B, \subseteq C$  and  $\subseteq D$ .

Each element of  $B$  is also an element of  $A$  but  $A$  has at least one element not found in  $B$ .

We can be more specific and say that:

$B$  is a proper subset of  $A$ , denoted by  $B \subset A$ , or  $A \supset B$ .

## Notation

## Examples

$\in$  means "is an element of"

$$2 \in \{1, 2, 3, 4\}$$

$\notin$  means "is not an element of"

$$2 \notin \{1, 3, 5\}$$

$\cup$  means "union of set"

$$\{1, 2, 3\} \cup \{3, 4, 8\} = \{1, 2, 3, 4, 8\}$$

$\cap$  means "intersection of set"

$$\{1, 2, 3\} \cap \{3, 4, 8\} = \{3\}$$

$\subset$  means "subset of set"

$$\{1, 2, 3\} \subset \{1, 2, 3, 4, 8\}$$

$\supset$  means "superset of set"

$$\{1, 2, 3, 4, 8\} \supset \{1, 2, 3\}$$

$\not\subset$  means "not a subset of set"

$$\{10, 20, 30\} \not\subset \{1, 2, 4, 8\}$$

$\emptyset$  or  $\{\}$  denotes the empty set

### Intersection of Sets

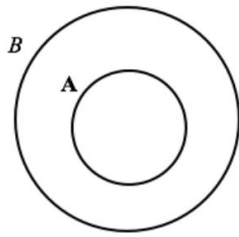
The intersection of sets A and B, is the set of elements which are common to both A and B. It is denoted by  $A \cap B$ .

### Union of Sets

The union of sets A and B is the set of elements which are in set A, or set B, or both set A and set B. It is denoted by  $A \cup B$ .

## Venn Diagrams

Using Venn diagrams to show relationships between sets



$$A \subset B$$

or

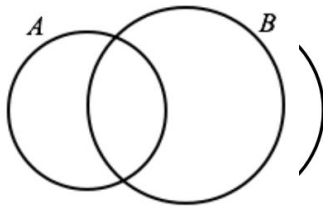
$$A \cap B = A$$

$$n(A \cap B) = n(A)$$

or

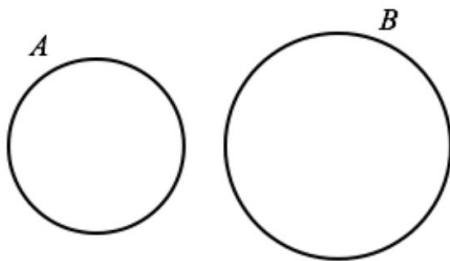
$$A \cup B = B$$

$$n(A \cup B) = n(B)$$



$$A \cap B \neq \phi$$

$$n(A \cap B) > 0$$



$$A \subset B'$$

$$B \subset A'$$

or

$$A' \cap B = B$$

$$B' \cap A = A$$

or

$$A \cap B = \phi$$

$$n(A \cap B) = 0$$

If it is given that  $B \subseteq A$  but the elements are not known, it is usual to represent the relation as  $B \subset A$  in the Venn diagram.