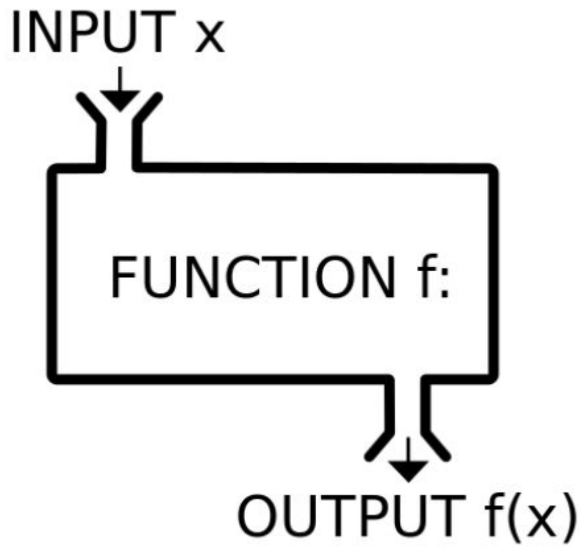
Equations and Inequalities

UNIT 4.1 :
Introduction

A function is a rule, which relates the values of one variable quantity, x, to the values of another variable quantity, f(x), and does so in such a way that the value of the second variable quantity is uniquely determined by the value of the first variable quantity. In simple terms, given an input x in a function f, it produces an output f(x). A function can be thought of as a blending machine. For any values x (fruit) being placed into the function f (fruit juicer), you will get your desired value f(x) (juice).



Functions are essentially made up of three components, mainly
- input (fruits)
- relationship (fruit juicer)
- output (juice)

Writing Equations As Functions
An equation can be written as a function f(x) instead of y. For instance, we can write
$y = 5x - 6$ as $f(x) = 5x - 6$
Functions are not always written using f(x). Other examples include g(x) and T(n).

UNIT 4.2 :
Balancing Algebraic Equations

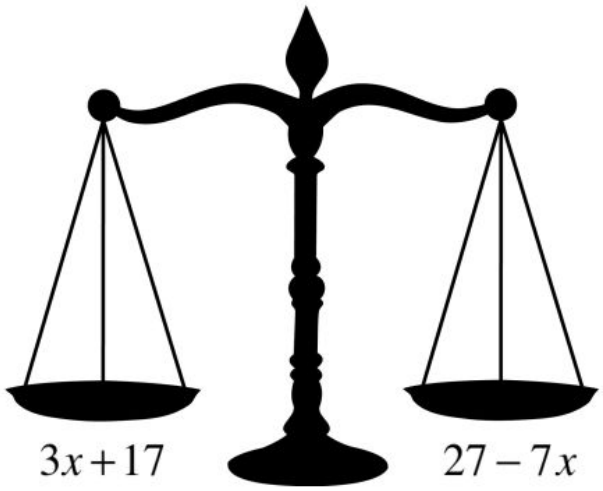
Equations
An equation is a mathematical statement that one expression is equal to another.

Examples of algebraic equations include

$$5x = 1000\ ,\ \frac{x}{4} + 2 = 7\frac{1}{2} - 2x\ , \text{etc.}$$

Balancing Equations

Consider the linear equation $3x + 17 = 27 - 7x$.



The key idea to solving a linear equation is to “shift” variables to the left side and values to the right side. You are advised to pay attention to the operations as they are “shifted”.

$$3x + 17 = 27 - 7x$$
$$3x + 7x = 27 - 17$$
$$10x = 10$$
$$x = 1$$

To recapitulate, the operations change as the terms are “shifted” to the other side of the equal Sign.

UNIT 4.3 :

Understanding Quadratic Equations

Recap:

Quadratic Equation

A quadratic equation is a polynomial equation in which the highest power of the unknown variable is two.

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$. The roots of a quadratic equation are the solutions to the unknown Variable.

UNIT 4.4 :
Solving By Factorization

Factorization

Zero Factor Principle
The product of any real number with a zero is zero.
Hence, if two real numbers A and B are such that A × B = 0, then either A = 0 or B = 0 or both A and B are equal to 0.
We shall apply this principle to solve the following quadratic equations.

Solve $(x+3)(x+7)=0$.

$$(x+3)(x+7)=0$$

By Zero Factor Principle,

$$x+3=0 \qquad \text{or} \qquad x+7=0$$

where the roots are

$$x=-3 \qquad \text{or} \qquad x=-7$$

in order to satisfy the condition(s).

Solve $12y^2+14y=40$.

Rearranging the terms, we get

$$12y^2+14y-40=0$$

Performing factorization, we get

$$2(2y+5)(3y-4)=0$$

and by Zero Factor Principle,

$$2y+5=0 \qquad \text{or} \qquad 3y-4=0$$

$$y=-2.5 \qquad \text{or} \qquad y=1\frac{1}{3}$$

Forming Quadratic Equations From Roots
When we solve $(x + 3)(x + 7) = 0$, we deduce the roots $x = -3$ or $x = -7$. This means that we can deduce that given the roots of a quadratic equation. In other words, we can form the quadratic equation by reversing the procedure.
In general, the roots of a quadratic equation, a and b allow us to form the quadratic equation with these roots, which is $(x - a)(x - b) = 0$, we can expand it if needed.

UNIT 4.5 :
Solving By Taking Square Root

Taking Square Root
In order to make x the subject, we may need to square root both sides of the equation.

UNIT 4.6 :
Solving By Completing The Square

Completing The Square

To complete the square is where we take a quadratic polynomial

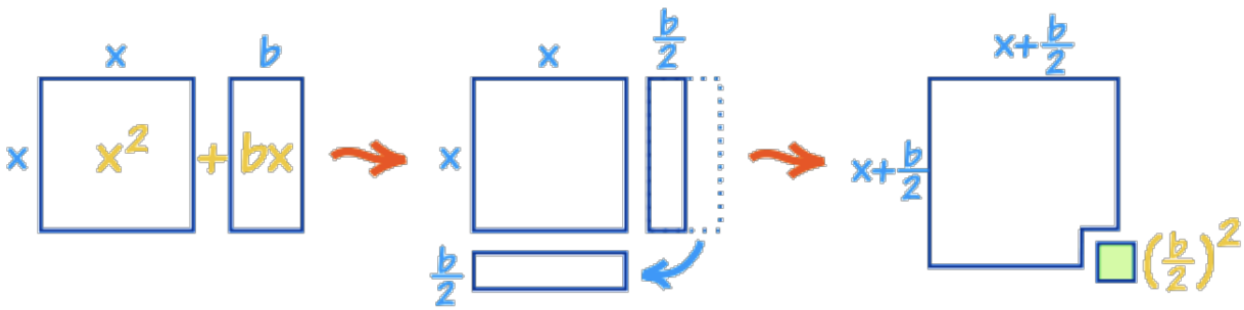
$$ax^2 + bx + c$$

and turn it into

$$a(x+d)^2 + k$$

The question is how do we do that?

First, let's see how we can **complete the square** for $x^2 + bx$ using "model method".



As you can see, $x^2 + bx$ can **almost** be rearranged into a square.

It requires an additional "piece" of $\left(\frac{b}{2}\right)^2$ to make it complete.

To put it algebraically,

$$x^2 \quad + \quad bx \quad + \quad \left(\frac{b}{2}\right)^2 \quad = \quad \left(x+\frac{b}{2}\right)^2$$

By adding $\left(\frac{b}{2}\right)^2$, we can complete the square.

On an important note, this method works only when the coefficient of x^2 is 1. That is to imply that given the quadratic equation $ax^2 + bx + c = 0$, $\boldsymbol{a = 1}$.

Example:

Technique	Equation Form
Given the quadratic equation	$x^2 + 8x - 20 = 0$
We note that the coefficient of x^2 is 1. We move -20 to the right of the equation.	$x^2 + 8x = -20$
We add $\left(\frac{8}{2}\right)^2$ to both sides of the equation so that it will be balanced.	$x^2 + 8x + \left(\frac{8}{2}\right)^2 = -20 + \left(\frac{8}{2}\right)^2$
We convert the expression on the left to a factored form to show "completed square".	$(x+4)^2 = -20 + (4)^2$
Simplify the expression on the right as a single fraction.	$(x+4)^2 = 36$
Take square roots on both sides.	$x + 4 = \pm\sqrt{36}$
We make x the subject of the formula.	$x = -4 \pm 6$
Finally, we derive two solutions from the equation.	$x = -10$ or $x = 2$

UNIT 4.7 :

Solving By Quadratic Formula

Recall that using completing the square method, the solution of a quadratic equation in the form ax2 + bx + c = 0 are related to the coefficients a, b and c by the general formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Technique	Equation Form
Given the quadratic equation	$ax^2 + bx + c = 0$
We first divide every term by a as we require the coefficient of x^2 to be 1.	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
Next, we move $\frac{c}{a}$ to the right of the equation.	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
We add $\left(\frac{b}{2a}\right)^2$ to both sides of the equation so that it will be balanced.	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
We convert the expression on the left to a factored form to show "completed square".	$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$
Simplify the expression on the right as a single fraction.	$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$
Take square roots on both sides.	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$
We make x the subject of the formula.	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
Finally, combine the fractions to get the solutions of equation $ax^2 + bx + c = 0$.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

UNIT 4.9 :

Linear Inequalities

Inequalities enable us to understand that there is no one exact value that will satisfy the equation.