Unit 2: Integers, Rational and Real Numbers

Prime Numbers, Highest Common Factor, Lowest Common Multiple

Essential Understanding

• Numbers are used to represent quantities in real life.
• There are more types of number other than the natural counting numbers
• Numbers can be operated on and there is an order to which operations can be carried out.

Essential Questions

• How can we evolve the natural number systems to model real-life situations?
• How do we denote and perform operations on real numbers?

Key Points (Learning Outcomes)

• Know how to recognize situations that need the use negative numbers
• Represents integers on a real number line

Difficult Point

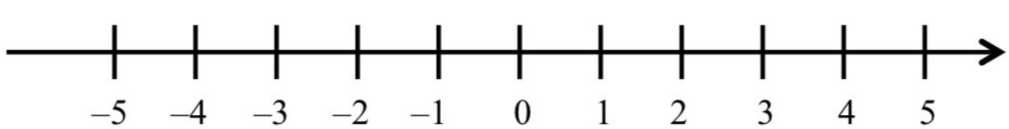
• Understanding the existence of other types of numbers besides the counting numbers

Critical Point

• Able to relate the names rational, irrational and real numbers to its type and understanding why these numbers are created.

Terminology

1. Numbers with the sign " – " are called negative numbers. " – "is read as "negative". E.g. –1, –2, –3, ...
2. Integers (Z) are positive or negative whole numbers. E.g. ., –3, –2, –1, 0, 1, 2, 3, ...
3. Positive integers (Z+) are numbers that are greater than zero. E.g. 1, 2, 3, 4, 5, ...
4. Zero is an integer that is neither positive nor negative.
5. A number line representing integers is given below



Every number is greater than the number on its left.

E.g. –4 is greater than –3, 0 is greater than –1, 5 is greater than 4

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6. <, ≤, > and ≥ are inequality signs.
- “<” means “smaller than”,
- “>” means “greater than”,
- “ ” means “smaller than or equals to”,
- “ ” means “greater than or equals to”.

7. **Rational numbers** (Q) are numbers that can be expressed in the form ba , where a and b are integers and b ≠ 0 . E.g. All fractions and whole numbers.

8. **Irrational number**s are numbers that are not rational. E.g. π, 2 , −3 4 . More on irrational numbers on next lesson.

9. **Real Numbers** (R) are numbers that can be found on the number line. They are made up of rational and irrational numbers.

10. The numerical or absolute value of a number, x, written as |x| , is its distance from zero on the number line. The absolute value of an integer is always positive since distance is never negative.

E.g. | 1 | = 1, | -2 | = 2. | -5 | = 5

Summary: Important number systems

$\mathbb{Z}$	**Integer**	$\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	**Positive Integer**	$\{1, 2, 3, \ldots\}$
$\mathbb{Q}$	**Rational**	$\frac{a}{b}$ where a and b are integers and b is not zero
$\mathbb{Q}^+$	**Positive Rational**	$\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{R}$	**Real**	All rational and irrational numbers.
$\mathbb{R}^+$	**Positive Real**	$\{x \in \mathbb{R} : x > 0\}$

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Operations (+, −,×,÷) with Negative Numbers

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Essential Questions

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Key Points (Learning Outcomes)
• Perform addition, subtraction, division and multiplications on negative numbers

Difficult Point
• Wrong interpretation of operations on negative numbers, essentially in the order of execution of the operations.

Critical Point
• There are certain number properties that governs the basic operations of numbers (including negative) for the whole number system to make sense.

Rules for adding two integers
Case 1: To add two positive numbers, add their absolute values. The result is positive.
$(+x)+(+y)=+(x+y)$
Case 2: To add two negative numbers, add their absolute values and place a negative sign before the result.
$(-x)+(-y)=-(x+y)$
Case 3: To add a positive and a negative number, find the difference of their absolute values by subtracting the smaller absolute value from the larger absolute value. Place the sign of the number having the larger absolute value before the result.
For any $x > 0$, $y > 0$,

$(+x)+(-y)=+(x-y)$, if $x>y$
$(+x)+(-y)=-(y-x)$, if $y>x$
$(-x)+(+y)=-(x-y)$, if $x>y$

$(-x)+(+y)=+(y-x)$, if $y>x$

Note:

Definition: The reciprocal of a number x is 1/x.

To divide integers, find the reciprocal of the integer being divided (divisor) and multiply the first integer according to the rules for multiplication of integers.

You may think of dividing a number as an abbreviation of multiplying its reciprocal!

$x \div y = x \times 1/y$

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Number Laws

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Essential Questions

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•How do we denote and perform operations on real numbers?

Key Points (Learning Outcomes)

• Commutative, Associative and Distributive laws governs the arithmetic operations of addition and multiplication. Subtraction and division are derived operations from addition and multiplication.

Difficult Point

• Name of numbers laws might sound intimidating, but they are just simple rules.

Critical Point

• To apply this laws to speed up or simplify mental arithmetics

- Commutative and Associative Laws
- Distributive Law

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Rational, Irrational and the Real Number System

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Essential Questions

• How can we evolve the natural number systems to model real-life situations?
• How do we denote and perform operations on real numbers?

Key Points (Learning Outcomes)
• Definition of Rational and Irrational Numbers
• Able to quote examples of irrational numbers such as 2 and π .
• Evaluate correctly and quickly compound numerical operations involving negative numbers

Difficult Point
• Evaluate correctly and quickly compound numerical operations involving negative numbers

Critical Point
• To reason through each step when evaluating compound numerical operations involving negative numbers, so as to avoid “careless mistakes”.

A Closer Look at the Definition of a Rational Number
1. A rational number is any number that can be written as a ratio of two integers i.e. it can be
written as a fraction where both the numerator and denominator are integers.

Rational numbers are numbers of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

2. All integers are rational numbers as each integer, x ,can be written as x/1 .
=E.g. 5 = 5/1 and -7 = − 1/7
3. Most decimals can also be expressed as rational numbers.
4.Irrational numbers are numbers that cannot be written in the form a/b.
e.g. π , √2 and the Euler’s constant, e , which is 2.7€18…