Probability

Definition of Probability and Sample Space

Many events can't be predicted with total certainty before it happens. The best we can say is how likely they are to happen, using the idea of probability.

• Probability is the chance that something will happen - how likely it is that some event will happen.

• We call 'something happening' an event.

• Probability should always be written as a fraction, decimal or percentage.

• If we call a particular event 'E', the probability of E happening is:

$$\mathrm{P}(E) = \frac{\textbf{number of different ways Event can happen}}{\textbf{Total number of outcomes}} = \frac{n(E)}{n(S)}$$

• The probability of something happening must be between 0 (impossible) and 1 (certain) inclusive, i.e. $0 \leq P(E) \leq 1$.

• The sum of the probabilities of every possible outcome is 1.

• The probability of something not happening (E') is 1 minus the probability of it happening, i.e $P(E') = 1 - P(E)$.

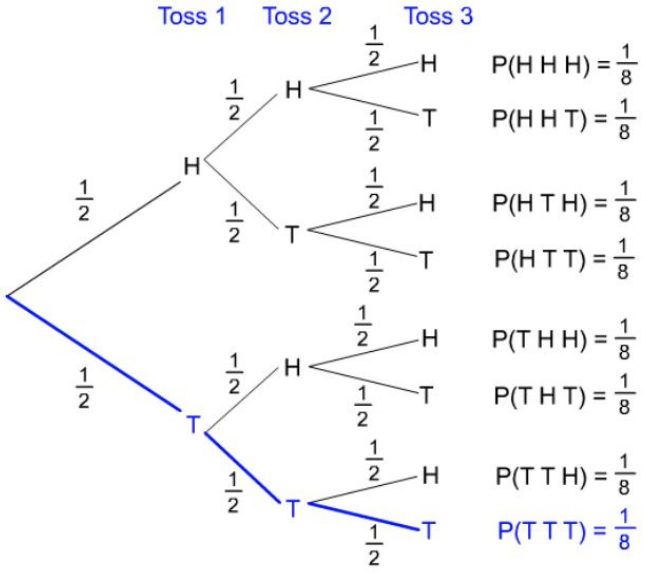
Experiment	an action where the result is uncertain. e.g. Tossing a coin, throwing dice, seeing what pizza people choose are all examples of experiments.
Sample Space	The set of all possible outcomes of an experiment is called the sample space or probability space. It is usually denoted by $\boldsymbol{S}$ and we put all the outcomes in braces { }. e.g. Choosing a card from a deck of 52 cards (not including Jokers). Sample Space is all 52 possible cards: $\boldsymbol{S}$ ={Ace of Hearts, 2 of Hearts, etc... } The Sample Space is made up of Sample Points.

Sample Point	just one of the possible outcomes Example: Deck of Cards • the 5 of Clubs is a sample point • the King of Hearts is a sample point "King" is not a sample point. As there are 4 Kings, these are 4 different sample points.
Event	a single result of an experiment e.g. Events: • Getting a Tail when tossing a coin is an event • Rolling a "5" is an event. An event can include one or more possible outcomes: • Choosing a "King" from a deck of cards (any of the 4 Kings) is an event • Rolling an "even number" (2, 4 or 6) is also an event

Tree Diagrams

A coin is tossed three times.

Let's draw a tree diagram showing the possible outcomes.



$P(H\ H\ H) = \frac{1}{8}$

$P(H\ H\ T) = \frac{1}{8}$

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$P(T\ T\ T) = \frac{1}{8}$

$P(3\ Heads) = P(H\ H\ H) = \frac{1}{8}$

$P(2\ Heads) = P(H\ H\ T) + P(H\ T\ H) + P(T\ H\ H)$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= \frac{3}{8}$

$P(1\ Head) = P(H\ T\ T) + P(T\ H\ T) + P(T\ T\ H)$

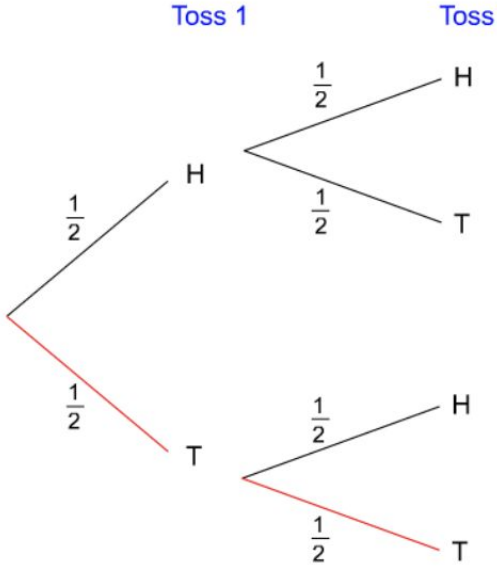
$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= \frac{3}{8}$

$P(No\ Heads) = P(T\ T\ T) = \frac{1}{8}$

A coin is tossed two times.

Let's draw a tree diagram showing all the possible outcomes.



H H **Probability(H and H) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$**

H T **Probability(H and T) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$**

T H **Probability(T and H) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$**

T T **Probability(T and T) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$**

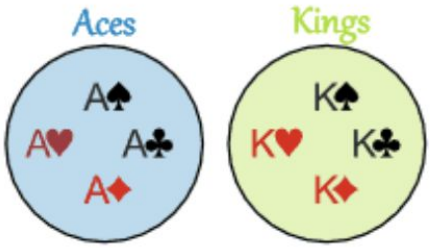
These 4 outcomes are equally likely.

They all have a probability of $\frac{1}{4}$

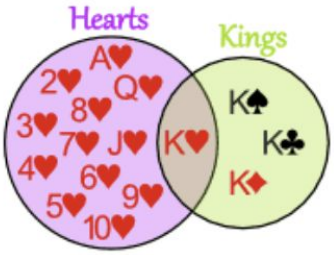
Types of Events

Mutually Exclusive Events

Mutually Exclusive means you can't get both events at the same time. It is either one or the other, but not both.
Two mutually exclusive events A and B cannot occur together. A and B are known as mutually exclusive events because if A occurs, B does not and vice versa.
In this case, P(E) = P(A) + P(B)
In general, we have P(A or B) = P(A) + P(B) where A and B are mutually exclusive events.
e.g.
• Turning left or right are mutually exclusive
• Heads and Tails are mutually exclusive
• Kings and Aces are mutually exclusive
• Kings and Hearts are not Mutually Exclusive, because you can have a King of Hearts.



Aces and Kings are mutually exclusive.



Hearts and Kings are **not** mutually exclusive.

Independent Events
• Two events are independent events if the outcome of one event does not affect the probability of the outcome of the other event.
Two or More Independent Events
When two independent events happen together, the probability of the combined event is the product of the probabilities of the two single events.
You can calculate the chances of two or more independent events by multiplying the chances.
P(A and B) = P(A) × P(B)

Dependent Events
When 2 events occur one after another in such a way that the first event affects the second, these events are called dependent events.

Complementary Events
If A and B are complementary events to each other, then
• P(A) = 1 – P(B) and
• P(B) = 1 – P(A)