Congruency and Similarity

Congruency

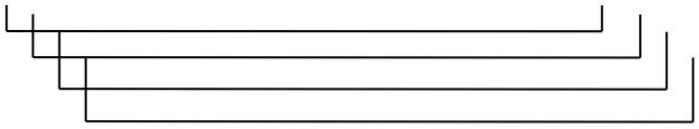
To name polygons,

- **Write a different letter at each vertex (corner) and list consecutive vertices in order.**
- **Two sides that have the same length are called corresponding congruent sides.**

NOTE

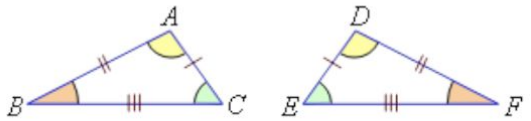
1. The matching diagram shows how the corresponding vertices must match.

Quadrilateral ABCD is congruent to Quadrilateral A'B'C'D'

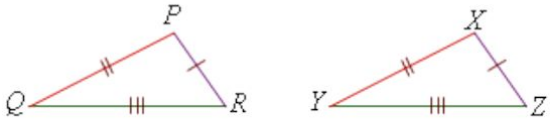
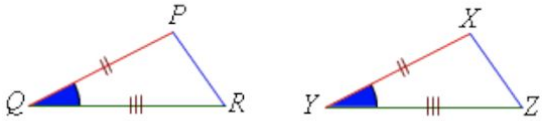
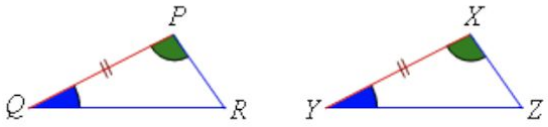
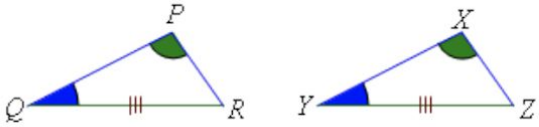


The corresponding vertices **must be named in the correct order.**

Summary (Congruent Triangles)

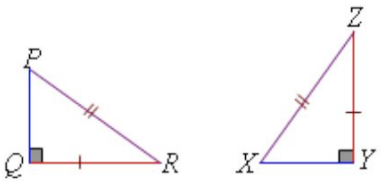
Two triangles are congruent when all corresponding sides and angles are equal.	 ΔABC is congruent to ΔDFE

Congruency Test

1. SSS Congruency If the three sides of one triangle are equal to the three sides of the other triangle, then the two triangles are congruent.	 $PQ = XY$ (S) $QR = YZ$ (S) $RP = ZX$ (S) $\therefore \Delta PQR$ is congruent to ΔXYZ.
2. SAS Congruency If two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, then the two triangles are congruent.	 $PQ = XY$ (S) $P\hat{Q}R = X\hat{Y}Z$ (A) $QR = YZ$ (S) $\therefore \Delta PQR$ is congruent to ΔXYZ.
3. ASA Congruency If the two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then the two triangles are congruent.	 $R\hat{P}Q = Z\hat{X}Y$ (A) $PQ = XY$ (S) $P\hat{Q}R = X\hat{Y}Z$ (A) $\therefore \Delta PQR$ is congruent to ΔXYZ.
4. AAS Congruency If two angles and one side (not included) of one triangle are equal to two angles and the corresponding side of the other triangle, then the two triangles are congruent.	 $R\hat{P}Q = Z\hat{X}Y$ (A) $P\hat{Q}R = X\hat{Y}Z$ (A) $QR = YZ$ (S)

5. RHS Congruency

If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and one side of the other right-angled triangle, then the two right-angled triangles are congruent.



$P\hat{Q}R = X\hat{Y}Z = 90°$ (R)

$PR = XZ$ (H)

$QR = YZ$ (S)

$\therefore$ ΔPQR is congruent to ΔXYZ.

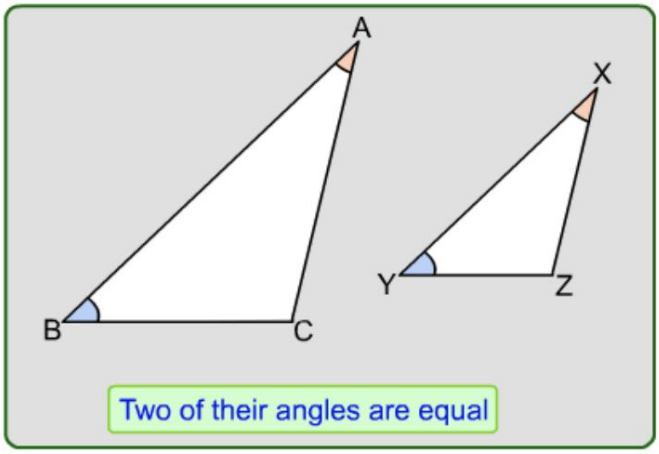
Similarity

Tests:

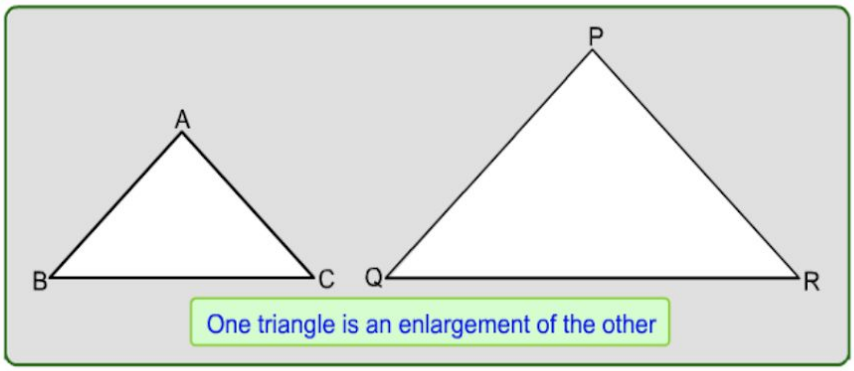
(1) All the corresponding angles are equal (AA TEST – Similarity)
Essentially, we only need 2 corresponding pairs of angles. Why?

Statement

If 2 angles of one triangle are equal to the corresponding 2 angles in the other triangle, then all the corresponding angles are equal, as a result of the total sum of interior angles being 180 degrees.



(2) Ratio of all the corresponding sides are proportional (SSS TEST – Similarity)

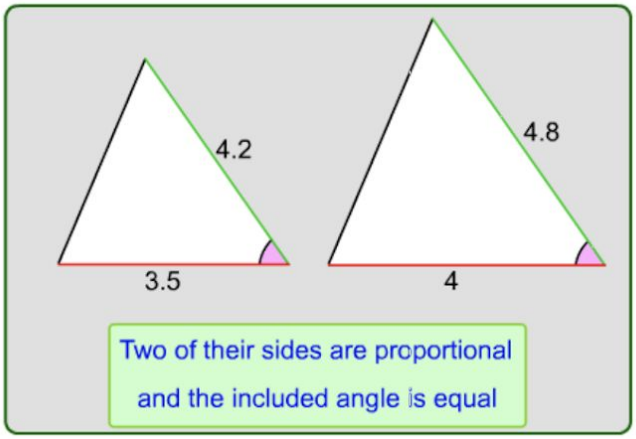


Statement

If the ratios of the corresponding sides are equal, then one triangle is an enlargement of the other. The proportion of the corresponding sides is known as the **enlargement factor**.

Consequently, the two triangles are similar.

(3) Two corresponding sides are proportional and the included angle is equal (SAS – Similarity)

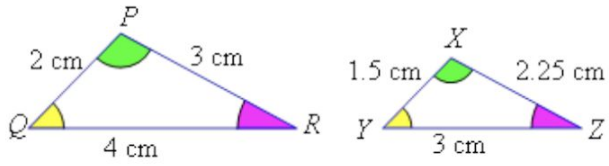


Statement

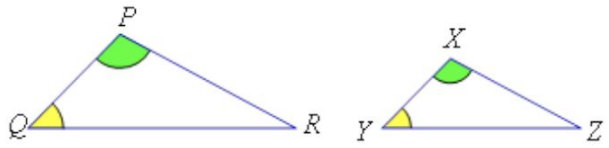
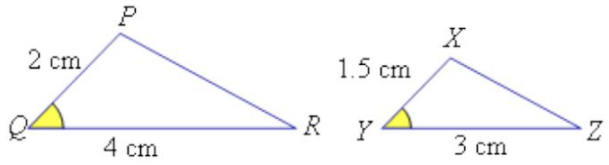
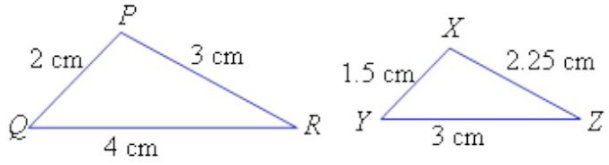
Two triangles are similar if an angle of one triangle is equal to an angle of the other triangle, and the sides which include the equal angle of both triangles are proportional.

Consequently, the two triangles are similar.

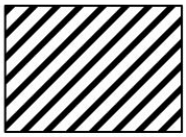
Summary (Similar Triangles)

Two triangles are similar if the corresponding angles are equal and the corresponding sides are proportional.	 ΔPQR is similar to ΔXYZ

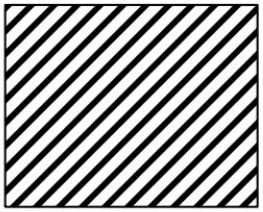
Similarity Test

1. If two angles of one triangle are equal to the corresponding two angles in the other triangle, then the two triangles are similar.	 $R\hat{P}Q = Z\hat{X}Y$ $P\hat{Q}R = X\hat{Y}Z$ $\therefore \Delta PQR$ is similar to ΔXYZ .
2. If two pairs of corresponding sides of two triangles are proportional and their included angles are equal, then the two triangles are similar.	 $\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{4}{3}$ $P\hat{Q}R = X\hat{Y}Z$ $\therefore \Delta PQR$ is similar to ΔXYZ .
3. If all pairs of corresponding sides of two triangles are proportional, then the two triangles are similar.	 $\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{RP}{ZX} = \frac{4}{3}$ $\therefore \Delta PQR$ is similar to ΔXYZ .

Areas of Similar Figures



dimensions
l_1 by w_1



dimensions
l_2 by w_2

$$\frac{A_1}{A_2} = \frac{l_1 \times w_1}{l_2 \times w_2}$$

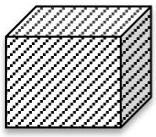
since

$$\frac{l_1}{l_2} = \frac{w_1}{w_2}$$

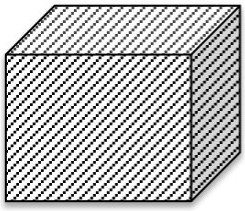
then

$$\frac{A_1}{A_2} = \frac{l_1 \times l_1}{l_2 \times l_2} = \left(\frac{l_1}{l_2}\right)^2$$

Volumes of Similar Figures



dimensions
l_1 by w_1 by h_1



dimensions
l_2 by w_2 by h_2

$$\frac{V_1}{V_2} = \frac{l_1 \times w_1 \times h_1}{l_2 \times w_2 \times h_2}$$

since

$$\frac{l_1}{l_2} = \frac{w_1}{w_2} = \frac{h_1}{h_2}$$

then

$$\frac{V_1}{V_2} = \frac{l_1 \times l_1 \times l_1}{l_2 \times l_2 \times l_2} = \left(\frac{l_1}{l_2}\right)^3$$