

## Unit EU

Mensuration is a system to quantify measurements in real life. Measures like perimeter, area, surface area and volume help us solve optimisation problems involving 2D or 3D spaces.

## Unit EQs

- What is perimeter?
- What is area?
- What is volume?
- What is surface area?

## Unit Objectives

- Calculate perimeter and area of composite plane figures, including parallelogram and trapezium
- Calculate volume and surface area of composite solids, including cube, cuboid, prism, cylinder, pyramid, cone and sphere
- Convert between  $\text{cm}^2$  and  $\text{m}^2$ , and between  $\text{cm}^3$  and  $\text{m}^3$

## GCE O-Level Syllabus

<b>G5</b>	<b>Mensuration</b>	<ul style="list-style-type: none"><li>• area of parallelogram and trapezium</li><li>• problems involving perimeter and area of composite plane figures</li><li>• volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere</li><li>• conversion between <math>\text{cm}^2</math> and <math>\text{m}^2</math>, and between <math>\text{cm}^3</math> and <math>\text{m}^3</math></li><li>• problems involving volume and surface area of composite solids</li><li>• arc length, sector area and area of a segment of a circle</li><li>• use of radian measure of angle (including conversion between radians and degrees)</li></ul> <p><b>Will be covered in Sec 3 and 4.</b></p>
-----------	--------------------	--

Plane figures are two-dimensional (**2-D**).



Square



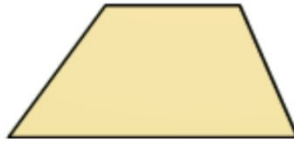
Rectangle



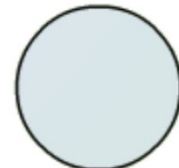
Triangle



Parallelogram

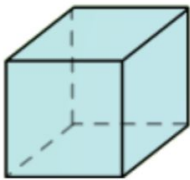


Trapezium

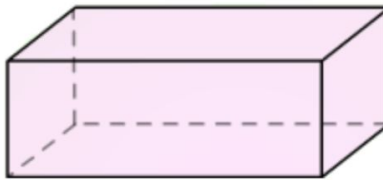


Circle

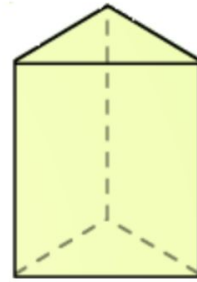
A solid is three-dimensional (**3-D**).



Cube



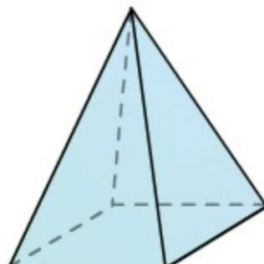
Cuboid



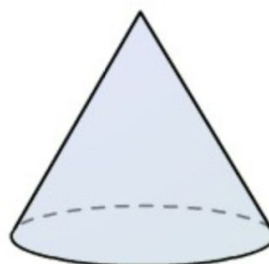
Prism



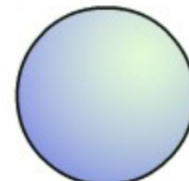
Cylinder



Pyramid

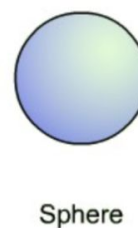
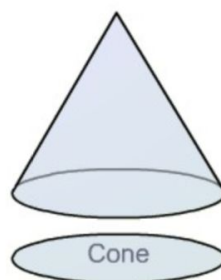
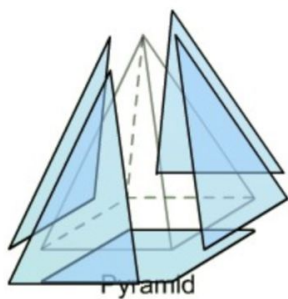
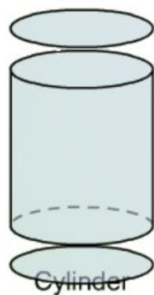
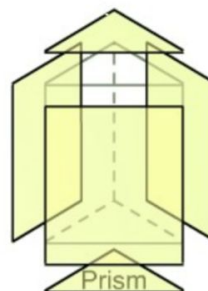
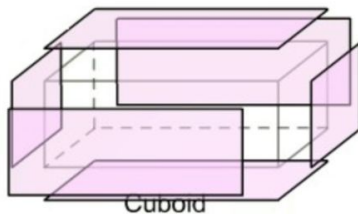
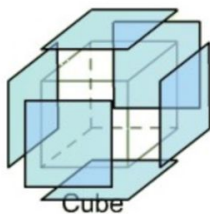


Cone

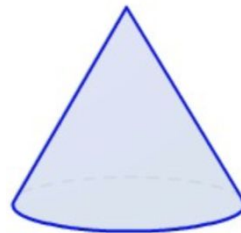
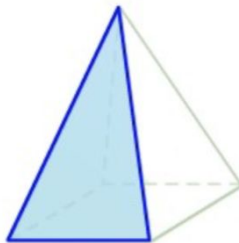
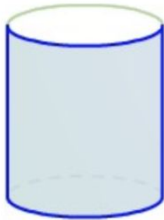


Sphere

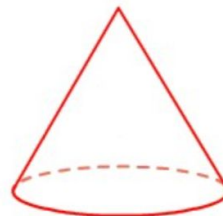
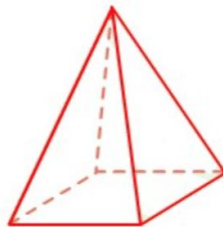
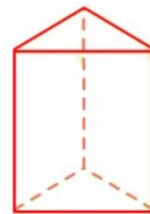
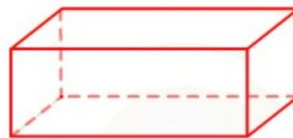
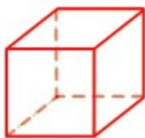
Each of these plane surfaces is called a **face** of the solid.



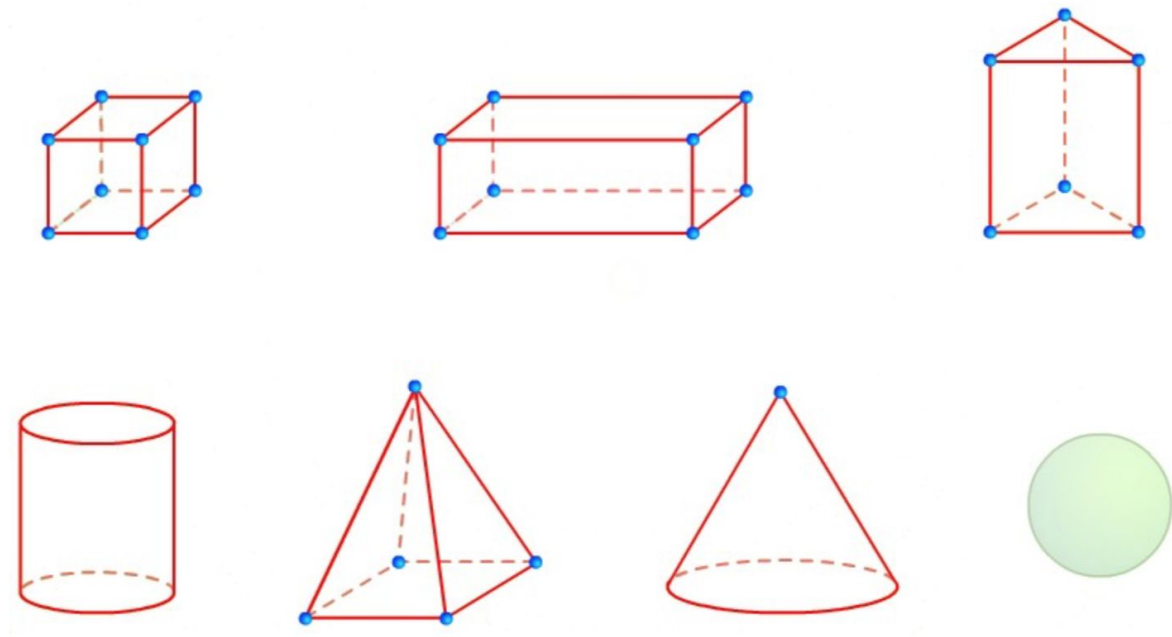
The face of a solid on its sides, that is, any face that is not the end faces, is known as the **lateral face**.



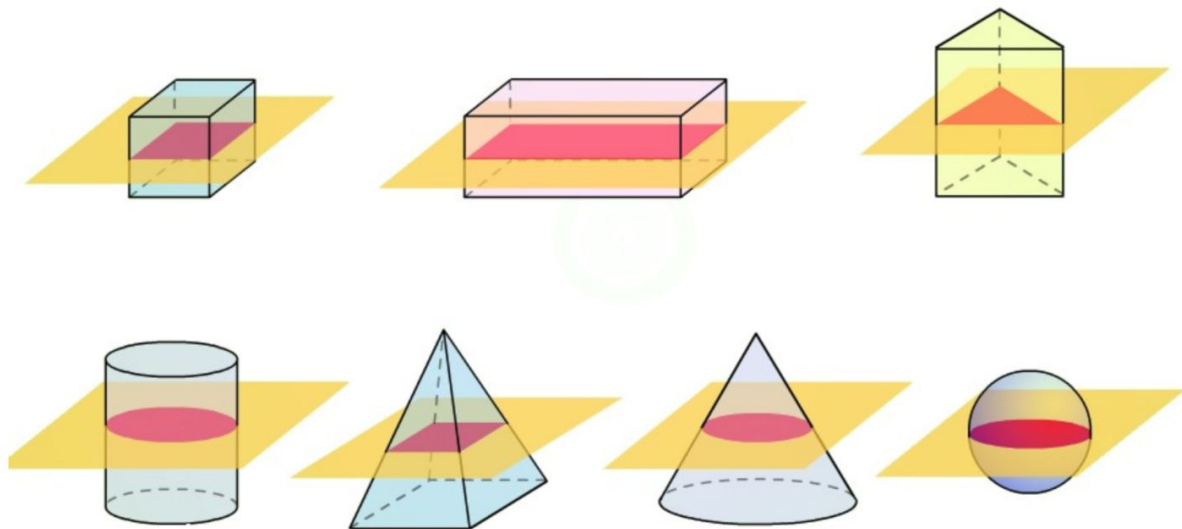
The **edge** is the line where 2 faces meet.



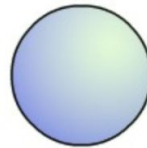
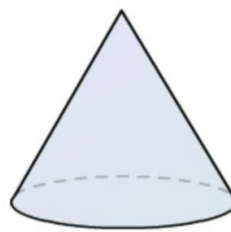
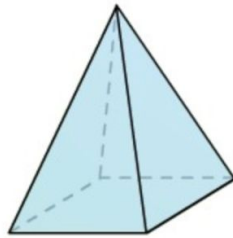
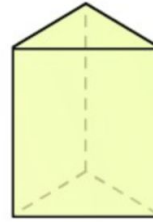
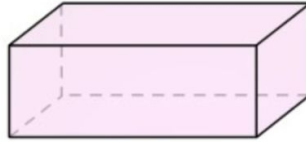
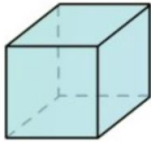
The **vertex** is the point where edges meet.



The **cross-section** is obtained by the intersection of that solid with a plane.

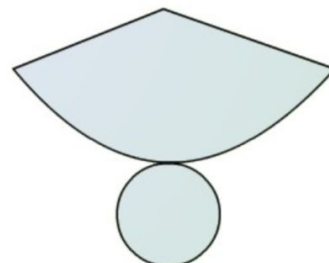
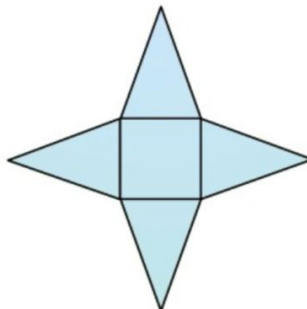
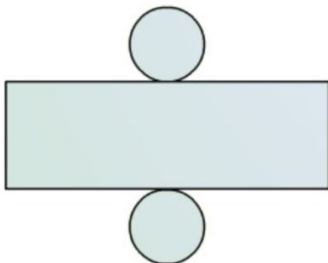
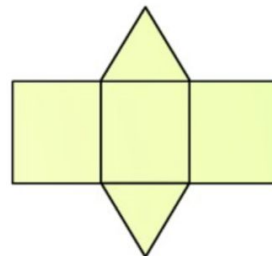
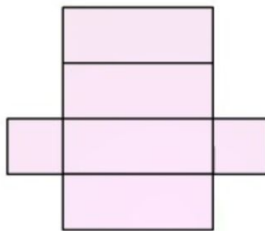
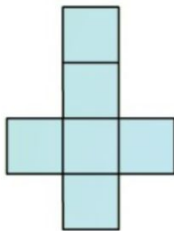


The **amount of space** a solid occupies is its **volume**.

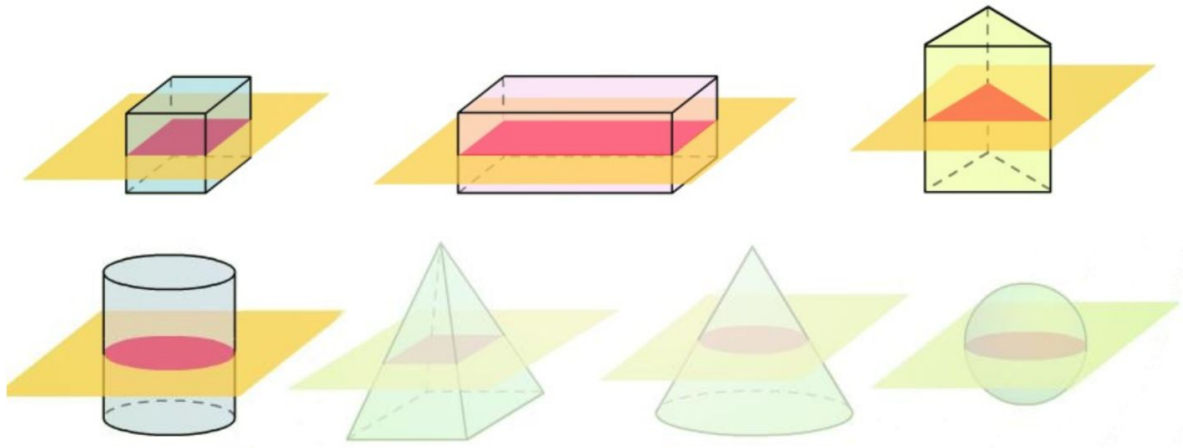


The **total area of the net** of a solid is the solid's **total surface area**.

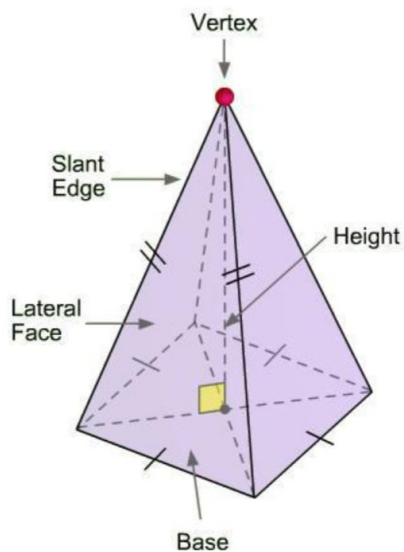
The net of a solid is a two-dimensional figure that can be cut out and folded up to form a solid.



A solid is said to have **uniform cross-section** if the cross-section is identical to its two parallel ends.



### Parts of a pyramid



1. **Base:** Its shape defines the name of the pyramid.
2. **Lateral face:** The triangular faces between the base and the vertex.  
In a right pyramid, the lateral face is an isosceles triangle.
3. **Vertex (Apex):** Point where all the lateral faces meet.
4. **Height:** Perpendicular distance between the base and the vertex.
5. **Slant edge:** The common edge between two adjacent lateral faces.
6. **Slant height:** The distance between the vertex and the mid-point of an edge of the base.

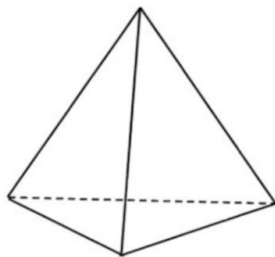
A tetrahedron has the following properties:

- it is a special pyramid with a triangle base (also called triangular pyramid)
- the net consists of all triangles only
- a regular tetrahedron has all sides as equilateral triangles

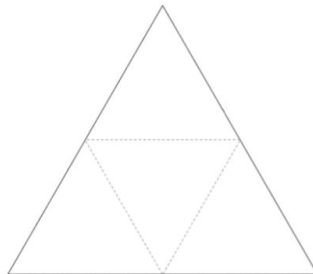
Volume and surface area is calculated in the same way as a pyramid.

For a regular tetrahedron, the surface area can be easily obtained by find the area of 1 side (triangle) and multiplying by 4.

Examples of tetrahedrons:



Solid



Net



4-sided die

#### Real Life Applications

5 stones



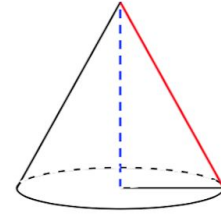


A cone has the following properties:

- it is a special pyramid with a circle base (also called circular pyramid)
- it has 2 sides (base and curved side)

Volume is calculated using the formula: \_\_\_\_\_

(Derived from volume of pyramid)



Surface area is calculated using the formula: \_\_\_\_\_

(Derived from the base and the curved side. Refer to handout)

### Real Life Applications



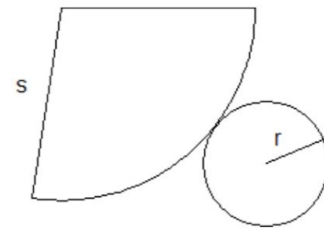
Ice-cream cone



Traffic cone

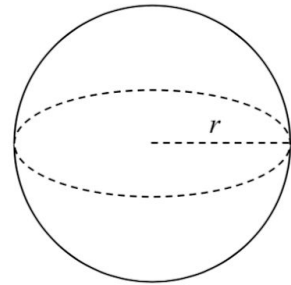


Wheelock Place



A sphere has the following properties:

- equal radius /diameter throughout



Let's find the formula for the volume of a sphere by conducting an experiment.

3. Get a sphere with radius,  $r$  units, and an open cylinder (no lid) with base radius,  $r$  units, and a height of  $2r$  units
4. Fill the cylinder with water.
5. Place the sphere into the cylinder; observing that there is a displacement of water.
6. Remove the sphere. Observe that the height of the water in the cylinder is only one-third left.

$$\begin{aligned}\text{Volume of a sphere} &= \frac{2}{3} \times (\text{Volume of a cylinder}) \\ &= \frac{2}{3} \times \text{Base area} \times \text{Height} \\ &= \frac{2}{3} \times \pi r^2 \times 2r \\ &= \frac{4}{3} \pi r^3 \text{ units}^3\end{aligned}$$