<u>**Matrcies**</u>

Introduction To Matrices

A matrix (plural: matrices) is an array of numbers (and/or algebraic unknowns) presented in brackets.Common usages of matrices in real life includes: Statistics, Computer graphics, Computer Engineering, Robotics, Mechanical engineering.

A matrix is typically written in the following format:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

- "**A**" represents the name of the matrix, usually written as a **single capital letter in bold**.
- Either square or curved brackets can be used. In this case, square brackets are used.
- The numbers/unknowns in the matrix (namely, *a, b, c, d, e, f*) are known as <u>**elements**</u> (or **entries**). They are typically written with sufficient space in between so as not to mix up the terms.

- Each matrix can comprise of different number of <u>**elements**</u>, arranged in different number of rows and columns. In the example above and below, matrix <u>**A** has 2 rows and 3 columns</u>.

$$A = \left.\overbrace{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}}^{n\text{ number of columns}}\right\} m\text{ number of rows}$$

- We call matrix **A** a "2 by 3" matrix, or a "2 × 3" matrix. This represents the <u>**order**</u> (or **dimension**) of the matrix.
  (i.e. Equivalently, one could say that the **order** of the matrix is "2 × 3").

- Conventionally, we can also denote each **element** of the matrix by a variable with two subscripts, instead of using individual letters for each **element**.

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

For matrices with different orders or dimensions, or certain special rules on its elements, there exist special names, such as:

a) Column Matrix (or Column Vectors)

b) Row Matrix (or Row Vectors)

c) Square Matrix

d) Zero Matrix (or null matrix)

e) Identity Matrix
f) Diagonal Matrix
If two matrices have exactly the same elements (and hence the same order), they are known as equal matrices.

Eg. If $\mathbf{A} = \begin{bmatrix} 2 & -8 \\ 4 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & -8 \\ 4 & 3 \end{bmatrix}$, then $\mathbf{A} = \mathbf{B}$.

Addition and Subtraction of Matrices
Only matrices of the same order (or dimension) can be added or subtracted to each other. We simply add or subtract the elements at the same position.

Given that $\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 4 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 2 \\ -8 & 0 \end{bmatrix}$, find $\mathbf{A} + \mathbf{B}$.

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 3 & -1 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ -8 & 0 \end{pmatrix} = \begin{pmatrix} 3+4 & -1+2 \\ 4-8 & 1+0 \end{pmatrix} = \begin{pmatrix} 7 & 1 \\ -4 & 1 \end{pmatrix}$$

Scalar Multiplication of Matrices
Multiplying a matrix with a number or unknown is known as scalar multiplication of matrices. Every element in the matrix is multiplied by the scalar term.

Given that $\mathbf{A} = \begin{bmatrix} -3 & 2 & 5 \\ 10 & 0 & 1 \end{bmatrix}$, find (a) $3\mathbf{A}$ and (b) $\frac{1}{2}\mathbf{A}$.

$$\mathbf{3A} = \mathbf{3}\begin{pmatrix} -3 & 2 & 5 \\ 10 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3(-3) & 3(2) & 3(5) \\ 3(10) & 3(0) & 3(1) \end{pmatrix} = \begin{pmatrix} -9 & 6 & 15 \\ 30 & 0 & 3 \end{pmatrix}$$

Multiplication of Matrices
In matrix multiplication, the elements in the rows of the matrix on the left are multiplied to the elements in the columns of the matrix on the right.

Matrix on Left		Matrix on Right
Row 1	Multiplied by	C1, C2, C3…
Row 2		C1, C2, C3…
Row 3		C1, C2, C3…
Row 4		C1, C2, C3…

When a m × n matrix is multiplied to a n × p matrix, the resultant matrix will have the order m x p

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix}$$

Laws of Matrices

Given the matrices A, B, C, Z (zero matrix) and I (identity matrix) of the same order and the constants p and q, they have the following properties:

Adding zero matrix	$\mathbf{A}+\mathbf{Z}=\mathbf{A}$
Adding two matrices	$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$ (Note $\mathbf{A}-\mathbf{B}\neq\mathbf{B}-\mathbf{A}$)
Commutative law for addition	$\mathbf{A}+\mathbf{B}+\mathbf{C}=(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})$
Distributive law	$p(\mathbf{A}+\mathbf{B})=p\mathbf{A}+p\mathbf{B}$
Distributive law	$(p+q)\mathbf{A}=p\mathbf{A}+q\mathbf{A}$
Scalar multiplication	$p(q\mathbf{A})=pq\mathbf{A}$
Multiplying identity matrix	$\mathbf{AI}=\mathbf{IA}=\mathbf{A}$
Associative law	$(\mathbf{AB})\mathbf{C}=\mathbf{A}(\mathbf{BC})$ (must satisfy order conditions)

Problem-Solving Using Matrices

A real-life application of matrices is its usage in complex statistical calculations. Typically, solving such problems using matrices involves the following processes:

a. Representing the given data in the form of a matrix/matrices

b. Combining two or more matrices into one matrix using addition, subtraction, scalar multiples and matrix multiplication

c. Determine what the resultant matrix represents

d. Sometimes, (b) and (c) are repeated on the resultant matrix from the first combination.

e. Relate the resultant matric to the context of the question.

Determinant of Matrices(Extension)

- Consider a 2×2 square matrix, the **determinant** of this matrix can be obtained by the following process. Given a 2×2 matrix **A**:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- Its **determinant** is given by

$$|\mathrm{A}| = \mathrm{ad} - \mathrm{bc}$$

An easy way to remember how to calculate determinant is to use a “cross”.