

1Algebraic Expressions and Formulae

UNIT 1.1: Recall from last year

An algebraic expression is an expression built up from constants, variables and operations. It has no equal sign.

A linear expression (in one variable x) is of the form $ax + b$, where

- x is the variable
- a and b are constants, where $a \neq 0$

A quadratic expression (in one variable x) is of the form $ax^2 + bx + c$ where

- x is the variable
- a , b and c are constants, where $a \neq 0$

UNIT 1.2: Expansion of Algebraic Expressions

To recapitulate on the rules of expansion,

- $a(b \pm c) = ab \pm ac$
- $(a + b)(c + d) = ac + ad + bc + bd$
- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)(a - b) = a^2 - b^2$

Factorization of Algebraic Expressions

Factorization is the process of writing an algebraic expression as a product of two or more other algebraic expressions.

The various methods of factorization are:

Grouping Common Factors

$$- ab \pm ac = a(b \pm c)$$

Grouping Terms

$$- ax + ay + bx + by = (x + y)(a + b)$$

Special Identities

$$- a^2 + 2ab + b^2 = (a + b)^2$$

$$- a^2 - 2ab + b^2 = (a - b)^2$$

$$- a^2 - b^2 = (a + b)(a - b)$$

Using Cross-Method

$$- x^2 + x - 6 = (x - 2)(x + 3)$$

x	-2	$-2x$
x	$+3$	$+3x$
x^2	-6	$+x$

UNIT 1.3 Algebraic Fractions

Algebraic fractions are expressions of the form $\frac{a}{b}$ where a and/or b are algebraic expressions, and $b \neq 0$.

Examples of algebraic fractions include

$$\frac{2a}{5}, \frac{16}{4b+c}, \frac{7d}{(2d+1)(3d-1)}, \frac{3f^2}{f^2-1}, \text{ etc.}$$

The rules for performing operations on algebraic fractions are the same as those for numerical fractions. One important rule is as follows:

The value of a fraction remains **unchanged** if both its numerator and denominator are **multiplied or divided by the same non-zero number or expression**.

$$\frac{a}{b} = \frac{a \times c}{b \times c} = \frac{a \div c}{b \div c} \text{ where } b, c \neq 0$$

Multiplication and Division of Algebraic Fractions

The procedure for the multiplication and division of algebraic fractions is similar to that of the multiplication and division of numeric fractions, except that now we have to consider the variables.

When we **multiply** $\frac{c}{d}$ by $\frac{a}{b}$, we have

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

where $b, d \neq 0$.

When we **divide** $\frac{a}{b}$ by $\frac{c}{d}$, we have

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

where $b, c, d \neq 0$.

Adding and Subtraction of Algebraic Fractions

Addition and subtraction of algebraic fractions can only be done when all the fractions have the same denominator.

UNIT 1.4: Manipulation of Algebraic Formula

The subject of a formula is the single variable on the left-hand side of a formula. Changing the subject of an algebraic formula involves applying same operations to both sides of the formulae. The aim is to reduce the formula to a form where the only the required subject remains the left-hand side, without it appearing again on the right-hand side.