Unit 1: Prime Numbers, Highest Common Factor, Lowest Common Multiple

Prime Numbers and Sieve of Eratosthenes

Essential Understanding

Numbers are used to represent quantities in real life.
Numbers help us make meaning of the world.
A symbol that represents a number is a numeral

Essential Questions

What is a number?
How do numbers interact with each other?
How do numbers interact with the real world?

Key Points (Learning Outcomes)

Understand the definition of prime numbers
Use the sieve of Eratosthenes to generate a small list of prime numbers

Difficult Point

Why the sieve of Eratosthenes works to generate a list of prime number
Why is 1 not a prime?

Critical Point

Through and complete understanding of the definition of prime numbers (Eg: A prime number is a positive integer greater than 1

Definition of a prime number

An integer greater than one is called a prime number if its only positive divisor factors are one and itself.
Give examples and non-examples of prime numbers.
* From the definition, clearly one is left out; but this does not really address the question "why?"
Why do you think 1 is not a prime number?

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Divisibility Test, Prime Factorization and Index Form

Essential Understanding

-Numbers are used to represent quantities in real life.
-Numbers help us make meaning of the world.
-A symbol that represents a number is a numeral

Essential Questions

-What is a number?
-How do numbers interact with each other?
-How do numbers interact with the real world?

Key Points (Learning Outcomes)

• Divisibility Tests for divisors 2, 3, 5, 7 and 11
• Determining if a given number (<169) is prime
• Prime Factorization of Composite Numbers
• Index Form for repeated multiplications

Difficult Point

• Divisibility test for 7 and 11
• “Uniqueness” of prime factorization of a composite number
• “Usefulness” of prime factorization of a composite number

Critical Point

• A fixed and secured method to prime factorize a composite number

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Highest Common Factor and Lowest Common Multiple

Essential Understanding

- Numbers are used to represent quantities in real life.
- Numbers help us make meaning of the world.
- A symbol that represents a number is a numeral

Essential Questions

- What is a number?
- How do numbers interact with each other?
- How do numbers interact with the real world?

Key Points (Learning Outcomes)

- Definition of Highest Common Factor (HCF) of 2 or more numbers
- Procedure to obtain the HCF of 2 or more numbers
- Definition of Lowest Common Multiple of 2 or more numbers
- Procedure to obtain the LCM of 2 or more numbers
- Solving word problems by applying HCF or LCM concept Difficult Point
- Translate a requirement of a word problem and deciding whether to use HCF or LCM.

Critical Point

- Applying the HCF or LCM concept to solve word problem

Highest Common Factor

Definition: The highest common factor (HCF), of two or more non-zero integers (numbers), is the largest positive integer that divides the numbers without a remainder.
The drawback of this method is the working needed increases with the value of the numbers given in the question. To find the Highest Common Factors (HCF), we can use prime factorisation.

Lowest Common Multiple

To find the Lowest Common Multiple (LCM), we will use prime factorisation as well.
Example
Find the LCM of 1080 and 336.
Earlier we have done the prime factorisation for 1080 and 336. 1080 = 23 x 33 x 5
336 = 24 x 3 x 7
Select the prime factors with the highest power/index from each set and multiply them to get LCM.
LCM = 24 x 33 x 5 x 7 = 15120

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Squares and Square roots, Cubes and Cube roots

Essential Understanding

• Numbers are used to represent quantities in real life.
• Numbers help us make meaning of the world.
• A symbol that represents a number is a numeral

Essential Questions

• What is a number?
• How do numbers interact with each other?
• How do numbers interact with the real world?

Key Points (Learning Outcomes)

• Squaring and taking square root of a number
• Cubing and taking cube root of a number
• Using prime factorization to find the square roots of perfect squares and cube roots of perfect Cubes

Difficult Point

• Using mathematical concept of index notation to observe patterns (in the index) when taking square root or cube root.

Critical Point

• To verbalize and internalize the difficult point above

Finding the square root of a number is the inverse operation of squaring that number. The square root of a number, n, is the number that gives n when squared (multiplied by itself).

Finding the cube root of a number is the inverse operation of cubing that number. The cube root of a number m, is the number that gives m when cubed.

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