Number Theory

Binomial Coefficients

 $\binom{n}{k}$ (read "n choose k") This is number of ways to select k items from n possibilities. The coefficients in a nth power polynomial: $(a+b)^n$ The

akbn-k term is n choose k...

Pascal's Triangle

The (n+1)st row gives the values of n choose i for $0 \le i \le n$.

Computation: n choose k = n!/(n-k)!k!

Since factorials are too hard to compute, can use recurrence relation:

```
= \binom{n-1}{k-1} + \binom{n-1}{k} (remembering that when k=0, n choose k is zero and when k=1, n choose k is n. Dynamic programming solution:
```

```
#define MAXN 100 // largest n or m
long binomial_coefficient(n,m) // compute n choose m
int n,m;
   int i,j;
   long bc[MAXN][MAXN];
   for (i=0; i<=n; i++) bc[i][0] = 1;
   for (j=0; j<=n; j++) bc[j][j] = 1;
   for (i=1; i<=n; i++)
       for (j=1; j<i; j++)
           bc[i][j] = bc[i-1][j-1] + bc[i-1][j];
   return bc[n][m];
```

Catalan Numbers

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} = \frac{1}{n+1} \binom{2n}{n}$$
 The first terms of this sequence are 2, 5, 14, 42, 132, 429, 1430 when $C_0 = 1$. This is the number of ways to build

a balanced formula from n sets of left and right parentheses. It is also the number of triangulations of a convex polygon, the number of rooted binary tress on n+1 leaves and the number of paths across a lattice which do not rise above the main diagonal.

Eulerian numbers

```
\binom{n}{k} = k \binom{n-1}{k} + (n-k+1) \binom{n-1}{k-1} This is the number of permutations of length n with exactly k ascending sequences or runs. (Basis: k=0
has value 1)
#define MAXN 100 // largest n or k
long eularian(n,k)
int n,m;
{
     int i, j;
    long e[MAXN][MAXN];
    for (i=0; i<=n; i++) e[i][0] = 1;
    for (j=0; j<=n; j++) e[0][j] = 0;
    for (i=1; i<=n; i++)
         for (j=1; j<i; j++)
e[i][j] = k*e[i-1][j] + (i-j+1)*e[i-1][j-1];
    return e[n][k];
```

GCD/GCF

```
Greatest Common Denominator - Euclid's Algorithm
```

LCM

```
Least Common Denominator
LCM(A, B) = A* B / GCD(A, B)
```

Partitions of Integers

An integer partition of n is an unordered set of positive integer which adds up to n. The number of integer partitions of n with the largest element is at most k is: f(n,k) = f(n-k,k) + f(n,k-1) where f(1,1) = 1 and f(n,k) = 0 if k > n or k < 0

Partitions of Sets

$$\binom{n}{k} = k \binom{n-1}{k} + \binom{n-1}{k}$$
 counts the number of ways to partition n items into k sets. Basis: when $k=2$, this has a value of $2^{n-1}-1$

Primeness Fast

Fermat's Theorem, which states that a^tot(b) % b = 1 where the GCF of b and a is zero, can be used to determine primeness.

tot(b) = count of numbers smaller than b where the GCF between b and the number is 1. For primes, tot(b) = b - 1.

Therefore, if b is prime, $a^{(b-1)}$ should equal 1 modulus b if a and b are relatively prime. The powermod algorithm takes advantage of the fact that (a % b)(c % b) = (a * c) % b.

```
bool isPrime(long input)
   long cpowmod = 1;
   long cpow;
   long targpwr;
   //Take care of the special freak cases.
   if (input==1)
       return false;
   if (input==2)
       return true;
   // The base I am choosing, two, must be relatively prime to
   // the target.
   if (input%2==0)
       return false;
   //If this is prime, then 2^p-1 will equal 1. If not, dream on.
   targpwr = input - 1;
   //My chosen base to start with is 2.
   cpow = 2;
   //Now split target power into binary. a*b mod n = amodn * bmodn
   while(targpwr > 0)
```

```
//binary 1
         if (targpwr % 2==1)
         {
             targpwr -= 1;
             //Why it's called powermod...
             cpowmod *= cpow;
             cpowmod %= input;
         //Divide targpwr by 2
        targpwr >>=1;
        //Get the next powermod of 2.
        cpow *= cpow;
        cpow %= input;
    //If I'm left with 1, this is prime.
    if (cpowmod==1)
        return true;
    else
        return false;
}
Prime Factorization
Unfortunately, no great way to factor numbers into their prime factors exists. This function will return the smallest prime factor of a number.
function factor(int n)
    int a;
    if (n%2==0)
        return 2;
    for (a=3;a<=sqrt(n);a++++)
        if (n%a==0)
        return a;
    }
    return n;
By putting it in a while loop, you can find the complete prime factorization.
int r;
while (n>1)
    r = factor(n);
    printf("%d ", r);
    n /= r;
Probability
P(event) = (Count desired outcomes) / (Count all possible outcomes)
P(\text{not } x) = 1 - P(x)
If the events x and y do not depend on one another, P(x \text{ and } y) = P(x) * P(y)
```

If the events x and y depend on one another, P(x and y) = P(not x) * P(y given not x) + P(x) + P(y given x)

P(x inclusive or y) = P(x and not y) + P(not x and y) + P(x and y)

P(x exclusive or y) = P(x and not y) + P(not x and y)

Geometry

Angles or Sides of Triangles

```
If a triangle has three sides (a, b, and c) and three angles (A, B, and C) where A is opposite a, etc. these rules apply: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} cosine rule: a^2 = b^2 + c^2 - 2bc\cos A \text{ and } b^2 = a^2 + c^2 - 2ac\cos B \text{ and } c^2 = b^2 + a^2 - 2ba\cos C So, to get the angles from the sides:

1. Use the cosine rule to find the largest angle \cos C = \left(a^2 + b^2 - c^2\right)/2ab where c is the longest side

2. Use the sine rule to find the second angles

3. Use the fact that the sum of the angles is 180 to find the last angle.
```

Area of a Polygon with Vertices on Lattice Points: Pick's Theorem

```
Area = {Number of Lattice Points on the Inside} + {Number of Lattice Points on the Border} / 2-1 Note: Works for ANY polygon, concave or convex.
```

Area of a Polygon:

Green's Theorem: Area = $\frac{1}{2}$ * Sum({Sum of Coordinates Cross Product of each pair of adjacent points. Consistently clockwise or counterclockwise}) Note: Sign of result indicates whether clockwise (-) or counterclockwise (+).

Collinearity

If the dot product of two vectors is zero, they are orthogonal. Take the three points and then create two vectors, then check for collinearity with the dot product.

Geometric Classes

```
class Point
   public:
   double x;
   double v;
   Point(double nx = 0, double ny = 0)
       x = nx; y = ny;
   bool operator == (Point const &pt) const
       return ( (x == pt.x) && (y == pt.y) );
   void print() const
       cout << "(" << x << "," << y << ")" << " ";
};
class Line
   public:
   double a,b,c;
   Line(double na = 0, double nb = 0, double nc = 0)
       a = na; b = nb; c = nc;
   Line(Point p1, Point p2)
       if (p1.x == p2.x)
       {
           a = 1;
           b = 0;
           c = -p1.x;
       }
       else
           b = 1;
           a = -(p1.y-p2.y)/(p1.x-p2.x);
           c = -(a * p1.x) - (b * p1.y);
   bool operator == (Line const &ln)
```

```
return ( (a == ln.a) && (b == ln.b) && (c == ln.c) );
};
bool pointlessthan (Point left, Point right)
   if (left.x < right.x) return true;</pre>
   if ((left.x == right.x) && (left.y < right.y)) return true;</pre>
   return false;
bool ccw(Point p1, Point p2, Point p3)
    // true if p3 is to the left of the line made by p1 and p2
   double k = p1.x*p2.y + p3.x*p1.y + p2.x*p3.y - p3.x*p2.y - p2.x*p1.y - p1.x*p3.y;
   bool collinear(Point p1, Point p2, Point p3)
   return ( Line(p1, p2) == Line(p2, p3) );
double sqr(double a)
   return a * a;
double pt_distance(Point p1, Point p2)
   return sqrt(sqr(p1.x - p2.x) + sqr(p1.y - p2.y));
void printpoint(Point &pt)
   pt.print();
Polygon Area
double polygon_area(vector<Point> poly)
                                               int main()
   double total = 0.0, val = 0.0;
   int i, j;
                                                  vector<Point> poly;
   for (i=0; i < poly.size(); i++)</pre>
                                                  // Points must be in CCW order
                                                  poly.resize(3);
       j = (i+1) % (poly.size());
                                                  poly[0] = Point(2,2);
       val = (poly[i].x * poly[j].y) -
                                                  poly[1] = Point(5,2);
           (poly[j].x * poly[i].y);
                                                   poly[2] = Point(5,6);
       total += val;
                                                  cout << polygon_area(poly) << endl;</pre>
                                               }
   return total / 2;
Convex Hull
Point first_point;
bool smaller_angle(Point p1, Point p2)
   if (collinear(first_point, p1, p2))
       if (pt_distance(first_point, p1) <=</pre>
              pt_distance(first_point, p2))
           return true;
       else
           return false;
    if (ccw(first_point, p1, p2))
       return true;
   else
       return false;
void GSHull(list<Point> ps)
   vector<Point> hull;
   hull.resize(ps.size());
   int n = ps.size();
    if (n \le 3) // all points on hull
       for (list<Point>::iterator cur = ps.begin();
                      cur != ps.end(); cur++)
           hull.push_back(*cur);
```

Geometry else ps.unique(); ps.sort(pointlessthan); //for_each(ps.begin(), ps.end(), printpoint); //cout << endl; hull[0] = *ps.begin(); ps.pop_front(); first_point = hull[0]; ps.sort(smaller_angle); hull[1] = *ps.begin(); ps.pop_front(); ps.push_back(first_point); int top = 1;for (list<Point>::iterator cur = ps.begin(); cur != ps.end();) if (!ccw(hull[top-1], hull[top], *cur)) top--; else { top++; hull[top] = *cur; cur++; hull.resize(top); cout << "Hull:" << endl; for_each(hull.begin(), hull.end(), printpoint); **Triangulation** class Triangulation public: int n; int t[MAXPOLY][3]; Triangulation() void addTriangle(int a, int b, int c, vector<Point> &p) { t[n][0] = a; t[n][1] = b; t[n][2] = c;printpoint(p[a]); printpoint(p[b]); printpoint(p[c]); cout << endl;</pre> }; class Triangle public: Point a,b,c; Triangle(Point na = Point(), Point nb = Point(), Point nc = Point()) a = na; b = nb; c = nc;bool pointIn(Point p) if (ccw(a,b,p)) return false; if (ccw(b,c,p)) return false; if (ccw(c,a,p)) return false; return true; }; bool ear_Q(int i, int j, int k, vector<Point> &p) Triangle t(p[i], p[j], p[k]); if (ccw(p[i], p[j], p[k])) return false;

for (int m = 0; m < p.size(); m++)</pre>

if ((m != i) && (m != j) && (m != k))

```
int main()
   list<Point> ps;
   ps.push_back(Point(8,9));
   ps.push_back(Point(6,8));
   ps.push_back(Point(6,11));
   ps.push_back(Point(11,9));
   ps.push_back(Point(10,5));
   ps.push_back(Point(4,8));
   ps.push_back(Point(6,13));
   ps.push_back(Point(6,4));
   ps.push_back(Point(10,12));
   ps.push_back(Point(10,2));
   ps.push_back(Point(13,11));
   ps.push_back(Point(12,6));
   GSHull(ps);
   return 0;
}
```

```
if (t.pointIn(p[m])) return false;
   return true;
void triangulate(vector<Point> &p, Triangulation &t)
    vector<int> 1, r;
   l.resize(MAXPOLY);
   r.resize(MAXPOLY);
   int i;
   for (i = 0; i < p.size(); i++)
       l[i] = ((i-1) + p.size()) % p.size();
       r[i] = ((i+1) + p.size()) % p.size();
   t.n = 0;
   i = p.size() - 1;
    int numtriangles = p.size()-2;
   while (t.n < numtriangles)</pre>
       i = r[i];
       if (ear_Q(l[i],i,r[i],p))
           t.addTriangle(l[i], i, r[i], p);
           1[ r[i] ] = 1[i];
           r[ l[i] ] = r[i];
       else
           numtriangles--;
   }
}
```

```
int main()
{
    vector<Point> poly;
    Triangulation t;

    poly.resize(6);
    poly[0] = Point(1,1);
    poly[1] = Point(3,1);
    poly[2] = Point(3,4);
    poly[3] = Point(2,4);
    poly[4] = Point(1,3);
    poly[5] = Point(2,2);
    triangulate(poly,t);
    return 0;
}
```

Trigonometry

Sometimes it is necessary to convert a slope in rise/run to radians, which can be done with C's atan2(rise, run) function, or atan2(dy, dx). If you need to convert a radian measure to rise/run, you can use tan(angle), or sin(angle) / cos(angle).

```
Pythagorean Relations
Alpha / beta relations
sin(a+B) = (sin a)(cos B)+(cos a)(sin B)
                                                         \sin^2 a + \cos^2 a = 1
cos(a+B) = (cos a)(cos B)-(sin a)(sin B)
                                                         1 + \tan^2 a = \sec^2 a
                                                         \cot^2 a + 1 = \csc^2 a
tan(a+B) = (tan a+tan B)/(1-(tan a)(tan B))
sin(a-B) = (sin a)(cos B) - (cos a)(sin B)
cos(a-B) = (cos a)(cos B) + (sin a)(sin B)
tan(a-B) = (tan a-tan B)/(1+(tan a)(tan B))
                                                         Reciprocal Relations
Cofunction Relations
sin a = cos(90 - a)
                                                         \sin a = 1 / \csc a
                                                         \cos a = 1 / \sec a
cos a = sin(90 - a)
tan a = cot(90 - a)
                                                         \tan a = 1 / \cot a
\cot a = \tan(90 - a)
                                                        \cot a = 1 / \tan a
sec a = csc(90 - a)
                                                        sec a = 1 / cos a
csc\ a = sec(90 - a)
                                                         \csc a = 1 / \sin a
Double angle relations
                                                         Quotient Relations
\sin 2a = 2(\sin a)(\cos a)
                                                         tan a = (sin a)/(cos a)
cos 2a = cos^2 a - sin^2 a
                                                         \cot a = (\cos a)/(\sin a)
\cos 2a = 1 - 2\sin^2 a
cos 2a = 2cos^2 a - 1
tan 2a = (2tan a)/(1 - tan^2 a)
Half angle relations
                                                         Sum and differences to product relations
\sin x/2 = \pm ((1 - \cos x)/2)^{1/2}
                                                         \sin x + \sin y = 2(\sin(x+y/2)(\cos(x - y)/2)
\cos x/2 = \pm ((1 + \cos x)/2)^{1/2}
                                                         \sin x - \sin y = 2(\cos(x+y)/2)(\sin(x-y)/2)
\tan x/2 = \pm ((1 - \cos x)/(1 + \cos x))^{1/2}
                                                         \cos x + \cos y = 2(\cos(x + y)/2)(\cos(x-y)/2)
                                                         \cos x - \cos y = -2(\sin(x+y)/2)(\sin(x-y)/2)
                                                         \sin^2 x = (1 / 2)(1 - \cos 2x)
\cos^2 x = (1 / 2)(1 + \cos 2x)
Law of sines
                                                         Law of cosines
a / sin A = b / sin B = c / sin C
                                                         a^2 = b^2 + c^2 - 2bc(\cos A)
                                                        b^2 = a^2 + c^2 - 2ac(\cos B)
                                                         c^2 = a^2 + b^2 - 2ab(\cos C)
```

String Algorithms

Longest Common Sequence

This is a dynamic programming solution to finding the longest common (not necessarily consecutive) sequence in two strings. For example, if the two strings are abcbdab and bdcaba, the lcs is bcba. The c [i][j] is the length of the longest sequence using the first i chars of the first string and the first j chars of the second string. The b array is used to remember which part of the recurrence we used so that we can rebuild the lcs at the end.

```
#include <stdio.h>
#include <string.h>
#define MAX_CHARS 8
#define DIAG 0
#define UP 1
#define RIGHT 2
int c[MAX_CHARS][MAX_CHARS]; // length of lcs
int b[MAX_CHARS][MAX_CHARS]; // direction we used in the table
void lcs_build_table(char* s1, char *s2)
    int m = strlen(s1); int n = strlen(s2);
    for (int i=1; i<=m; i++)
       c[i][0] = 0;
    for (int j=0; j <= n; j++)
       c[0][j] = 0;
    for (int i=1; i<=m; i++)
        for (int j=0; j<=n; j++)
        {
            if (s1[i-1] == s2[j-1])
                c[i][j] = c[i-1][j-1]+1;
                b[i][j] = DIAG;
            else if (c[i-1][j] >= c[i][j-1])
                c[i][j] = c[i-1][j];
                b[i][j] = UP;
            }
            else
                c[i][j] = c[i][j-1];
                b[i][j] = RIGHT;
    }
int main()
    char s1[MAX_CHARS],s2[MAX_CHARS];
    while (gets(s1))
        qets(s2);
        lcs_build_table(s1,s2);
        print_lcs(strlen(s1),strlen(s2),s1);
        printf("\n");
}
```

```
c[i,j] = \begin{cases} 0 & or(i=0, j=0) \\ c[i-1, j-1] + 1 & x_i = y_j \\ \max(c[i, j-1], c[i-1, j] & x_i \neq y \end{cases}
```

```
void print_lcs(int i, int j, char *s)
{
    if ((i==0) || (j==0)) return;
    if (b[i][j] == DIAG)
    {
        print_lcs(i-1,j-1,s);
        printf("%c",s[i-1]);
    }
    else if (b[i][j] == UP)
        print_lcs(i-1,j,s);
    else
        print_lcs(i,j-1,s);
}
```

Pattern Matching

This is the Boyer-Moore algorithm for searching for a pattern (string) within another string. It gets its efficiency by looking at the pattern from the end instead of the beginning so that it can shift the pattern by multiply spaces at each mismatch (depending on the character that didn't match) #include <stdio.h>

```
#include <string.h>
char s[80];
char p[80];
int last[128];

void compute_last()
{
   for (int i=33; i<128; i++)
        last[i] = 0;
   for (int j = 0; j<strlen(p); j++)
        last[p[j]] = j+1;</pre>
```

```
int main()
    while (gets(s))
        gets(p);
       int n = strlen(s);
int m = strlen(p);
       compute_last();
       int t = 0;
while (t <= n-m)
            int j = m-1;
            while ((j>=0) \&\& (p[j] == s[t+j]))
               j--;
            if (j == -1)
                printf( "pattern match at %d\n",t);
                t++;
            }
            else
                int mismatch = (int)s[t+j];
                if (j < last[mismatch])</pre>
                    t++;
                else
                    t = t+j-last[mismatch]+1;
            }
       }
  }
}
```

Graph Algorithms

Storing in arrays

Rather than using a data structure, it can be much easier to store data in binary trees in a linear array. Place the head node in position 1. Now, the left child of any node can be placed in 2n and the right child can be placed in 2n+1 with no gaps or redundancy.

```
Left child: 2*n+1
Right child: 2*n+2
Parent: (n-1)/2 truncated
Level: log2(n+1) = log(n+1) / log(2) truncated
```

Storing a graph explicitly:

```
#define MAXV 100 // max number of vertices
#define MAXDEGREE 50 // max out degree of a vertex
typedef struct {
    int edges[MAXV+1][MAXDEGREE]; // storing edges
               // in adjacency lists
    int degree[MAXV+1]; // degree of each vertex
   int nvertices;
   int nedges;
} graph;
void insert_edge(graph *g, int x, int y, bool
   if (g->degree[x] > MAXDEGREE)
       printf("Error: degree: %d %",x,y);
    g->edges[x][g->degree[x]] = y;
   g->degree[x]++;
    if (directed == FALSE)
       insert_edge(g,y,x,TRUE);
    else
       g->nedges++;
void print_graph(graph *g)
    int i, j;
    for(i=1;i<=g->nvertices;i++)
       printf("%d: ",i);
       for (j=0;j<g->degree[i];j++)
           printf(" %d",g->edges[i][j]);
       printf("\n");
```

```
void init_graph(graph *g)
{
   int i;
   g->nvertices = 0;
   g->nedges = 0;
   for (i=1;i<=MAXV; i++) g->degree[i] = 0;
}
```

Breadth First Search

Remember:

- each vertex goes from undiscovered to discovered to processed
- parent[i] is the vertex that we discovered the ith vertex from (we traveled the edge from i to this node. You can use this to recreate the path backwards (see the **find path** method which takes the starting node, the ending node and the parents array as its parameters).
- this will always give a shortest path
- you may need to do something for each vertex or each edge. Use process_vertex or process_edge as appropriate

```
void bfs(graph *g, int start){
    deque<int> q;
    int v; // current vertex;
int i; // counter
    q.push_front(start);
    while (q.empty() == FALSE){
       v = q.front();
        q.pop_front();
       process_vertex(v); // do whatever
                   // you're supposed
                    // to do at each vertex
                                                      }
       processed[v] = TRUE;
        for (i=0;i<g->degree[v];i++) {
            if (discovered[g->edges[v][i]] == FALSE){
                q.push_front(g->edges[v][i]);
               discovered[g->edges[v][i]] = TRUE;
```

```
#include <deque>
using namespace std;
bool processed[MAXV];
bool discovered[MAXV];
bool parent[MAXV];
void init_search(graph *g){
   int i;
   for (i=1; i<=g->nvertices; i++) {
      processed[i] = discovered[i] = FALSE;
      parent[i] = -1;
   }
}
```

```
int main()
{
    graph realg;
    graph *g = &realg;
    init_graph(g);
    for (int i=1;i<5;i++)
    {
        insert_edge(g,i, i+1,TRUE);
    }
    insert_edge(g,3,5,FALSE);
    insert_edge(g,1,4,FALSE);
    bfs(g,1);
    find_path(1,4,parent);
}</pre>
```

Connected Components

This will print out the vertices in each component of a graph; if you make process_vertex just print out the number of that vertex: printf("%d",v);

```
connected_components(graph *g)
{
   int c,i;
   init_search(g);
   c = 0;
   for (i=1;i<=g->nvertices;i++){
      if (discovered[i] == FALSE){
            c++;
            printf("Component %d:",c);
            dfs(g,i);
            printf("\n");
      }
}
```

Cycle Detection

```
To find cycles in a graph, use dfs with this process_edge method (and no need for process_vertex) process_edge(int x, int y) 
{

// if we've been here from a different parent, we found a back 
// edge which means we found a cycle 
if (parent[x] != y) {

printf("Cycle from %d to %d:",y,x); 
find_path(y,x,parent); 
finished = TRUE; 
}
```

Depth First Search on a Graph

Remember

- Uses the same graph structure and initialization as bfs
- the boolean "finished" can be used to stop the search before traversing the entire graph if necessary. Just set it "true" based on the appropriate condition

```
dfs(graph *g, int v){
   int i;
   int y;
   if (finished) return;
   discovered[v] = TRUE;
   process_vertex(v);//do what you need to do as you see each vertex
   for(i=0; i<g->degree[v]; i++)
         = g->edges[v][i];
       if (discovered[y] == FALSE){
           parent[y] = v;
           dfs(v);
       } else{
           if (processed[y] == FALSE)
               process\_edge(v,y); // do what you need to as
                   // you see each edge
       if (finished) return;
   processed[v] = TRUE;
}
```

Graph Algorithms

Network Flows

```
#include <deque>
#include <iostream>
using namespace std;
#define MAXV 100 // max number of vertices
#define MAXDEGREE 50
                      // max out degree of a vertex
typedef struct {
   int v;
    int capacity;
    int flow;
    int residual;
} edge;
typedef struct {
   edge edges[MAXV+1][MAXDEGREE]; // storing edges
           // in adjacency lists
    int degree[MAXV+1]; // degree of each vertex
    int nvertices;
    int nedges;
} graph;
void init_graph(graph *g)
    int i;
    g->nvertices = 0;
   g->nedges = 0;
    for (i=1;i<=MAXV; i++) g->degree[i] = 0;
void insert_edge(graph *g, int x, int y, bool directed, int capacity)
    if (g->degree[x] > MAXDEGREE)
       printf("Error: bigger than max degree: %d %d",x,y);
    g->edges[x][g->degree[x]].v = y;
    g->edges[x][g->degree[x]].capacity = capacity;
   g->edges[x][g->degree[x]].flow = 0;
   g->edges[x][g->degree[x]].residual = capacity;
   g->degree[x]++;
    if (directed == false)
       insert_edge(g,y,x,true,capacity);
    else
       g->nedges++;
}
void print_graph(graph *g)
    int i,j;
    for(i=1;i<=g->nvertices;i++)
       printf("%d: ",i);
       for (j=0;j<g->degree[i];j++)
printf(" %d (cp: %d, fl: %d, rs: %d)",
                   g->edges[i][j].v,
                   g->edges[i][j].capacity,
                   g->edges[i][j].flow,
                   g->edges[i][j].residual);
       printf("\n");
bool processed[MAXV];
bool discovered[MAXV];
int parent[MAXV];
void init_search(graph *g){
    for (i=1; i<=g->nvertices; i++) {
       processed[i] = discovered[i] = false;
       parent[i] = -1;
}
```

```
bool valid_edge(edge e)
   if (e.residual > 0) return true; else return false;
void bfs(graph *g, int start){
   deque<int> q;
   int v; // current vertex;
int i; // counter
   q.push_front(start);
    while (q.empty() == false){
       v = q.front();
       q.pop_front();
       processed[v] = true;
       for (i=0;i<g->degree[v];i++) {
           if ((discovered[g->edges[v][i].v] == false) &&
                   (valid_edge(g->edges[v][i]))){
               q.push_front(g->edges[v][i].v);
               discovered[g->edges[v][i].v] = true;
               parent[g->edges[v][i].v] = v;
           }
        }
   }
void find_path(int start, int end, int
                                                      edge * find_edge(graph *g, int x, int y)
parents[])
                                                         int i;
   if ((start == end) || (end = -1))
                                                         for (i = 0; i < g->degree[x]; i++)
       printf("\n%d",start);
                                                             if (g->edges[x][i].v == y)
    else{
                                                                 return (&g->edges[x][i]);
       find_path(start, parents[end], parents);
       printf(" %d",end);
                                                         return NULL;
}
int path_volume(graph *q, int start, int end, int parents[])
   edge *e;
   if (parents[end] == -1) return 0;
   e = find_edge(g,parents[end],end);
    if (start == parents[end])
       return (e->residual);
   else
       return(min(path_volume(g,start,parents[end],parents),
                   e->residual));
void augment_path(graph *g, int start, int end, int parents[],
       int volume)
    edge *e;
                                                           int main()
   edge *find_edge();
                                                               graph g;
   if (start == end) return;
                                                               init_graph(&g);
                                                               insert_edge(&g,1,2,false,30);
   e = find_edge(g, parents[end], end);
                                                               insert_edge(&g,1,3,false,100);
   e->flow += volume;
                                                               insert_edge(&g,1,4,false,25);
   e->residual -= volume;
                                                               insert_edge(&g,2,6,false,25);
   e = find_edge(g,end,parents[end]);
                                                               insert_edge(&g,3,6,false,30);
   e->residual += volume;
                                                               insert_edge(&g,3,5,false,15);
                                                               insert_edge(&g,4,5,false,20);
   augment_path(g,start,parents[end],parents,volume);
                                                               insert_edge(&g,5,6,false,45);
                                                               insert_edge(&g,6,7,false,50);
void netflow(graph *g, int source, int sink)
                                                               g.nvertices = 7;
                                                               netflow(&g, 1, 7);
    int volume, totalvolume;
                                                               return 0;
   init_search(g);
   bfs(g,source);
   volume = path_volume(g, source, sink, parent);
   totalvolume = volume;
   while (volume > 0)
```

augment_path(g,source,sink,parent,volume);

Strongly Connected Components

In dfs, compute finish time of each vertex by adding an int array f and a counter. After each vertex completes, increment the counter and store it at that vertices position in f.

Reverse all of the edges in the graph

DFS again, but visit the vertices in the order of their finish times. each component will be a strongly connected component

Topological Sorting

Construct an ordering of vertices so all edges go from left to right (edges represent precedence relations) Remember:

- The strategy here is to find all of the vertices with no incoming edges they can go at the left of the output. For each of those, delete all of its edges and you'll find more vertices with indegree of zero (who could then be output). If it is a directed, acyclic graph, every vertex will eventually have an indegree of zero and get processed
- this uses a queue to hold the vertices with no preceding edges (indegree = 0). In a true topological sort, it doesn't matter which of these vertices we process next. In a particular problem, if the ordering matters, replace the queue with an appropriate structure (sorted vector?)

```
#include <deque>
                                                        compute_indegrees(graph *g, int in[])
using namespace std;
topsort(graph *g, int sorted[])
                                                            int i, i;
                                                            for (i=1;i<=g->nvertices;i++)
   int indegree[MAXV];
                                                               in[i] = 0;
   deque<int> zeroin;
                                                            for (i=1;i<=q->nvertices;i++)
    int x,y,i,j;
                                                                for (j=0;j<degree[i];j++)</pre>
    compute_indegrees(g,indegree);
                                                                   in[g->edges[i][j]]++;
    for (i=1;i<=g->nvertices;i++)
       if (indegree[i] == 0) zeroin.push_front(i);
    j=0;
    while (zeroin.empty() == FALSE) {
       x = zeroin.pop_front()
       sorted[j] = x;
       for (i=0;i<g->degree[x];i++){}
           y = g->edges[x][i];
           indegree[y]--;
           if (indegree[y] == 0) zeroin.push_front(y);
   if (j != g->nvertices)
       printf ("Not a DAG\n");
```

Traversal

```
#define NULLCHILD -1
int tree[7] = {0, 1, 2, 3, NULLCHILD, 5, 6};

void dfPreOrder(int n)
{
   if(tree[n] == NULLCHILD) return;

   printf("%d\n", tree[n]); // Do us
   dfPreOrder(2*n+1); // Do left child
   dfPreOrder(2*n+2); // Do right child
}
```

```
dfInOrder(2*n+1); // Do left child
  printf("%d\n", tree[n]); // Do us
  dfInOrder(2*n+2); // Do right child
}

void dfPostOrder(int n)
{
  if(tree[n] == NULLCHILD) return;

  dfPostOrder(2*n+1); // Do left child
  dfPostOrder(2*n+2); // Do right child
  printf("%d\n", tree[n]); // Do us
```

if(tree[n] == NULLCHILD) return;

void dfInOrder(int n)

Breadth-first can be implemented by simply traversing the array linearly. Right to $\,$

left traversals are implemented as above, just switch left and right child calls.

String-Based Math Addition: Addition:

String-Based Math

Negative numbers are not supported, decimals are not supported, and division by zero is not checked for. The division used is integer division, therefore the decimal point and after are chopped off. The functions will work for arbitrarily large string integers, and will support any base 2 - 36. Leading zeros should be stripped off of all input.

Prototypes:

```
int Map(char);
char ReverseMap(int);
int Max(int, int);
int bigcmp(char*, char*);

char* Add(char*, char*, int);
char* Subtract(char*, char*, int);
char* Multiply(char*, char*, int);
char* Divide(char*, char*, int);
char* Modulus(char*, char*, int);
```

```
ReverseMap a value to its associated ASCII value.
Map a character to its associated value.
                                                         char ReverseMap(int input)
int Map(char input)
                                                         if ((input<10))</pre>
if ((input>='0')&&(input<='9'))</pre>
                                                             return '0' + input;
   return input - '0';
                                                             return 'A' + input - 10;
if ((input>='A')&&(input<='Z'))</pre>
    return input - 'A' + 10;
Max finds the larger of two values.
                                                         BigCmp is like strcmp, except for char* numbers of the type used
int Max(int a, int b)
                                                         in these functions.
                                                         int bigcmp(char* op1, char* op2)
if (a>b)
                                                         if (strlen(op1)>strlen(op2))
   return a;
                                                             return 1;
    return b;
                                                         if (strlen(op2)>strlen(op1))
                                                             return -1;
                                                         return strcmp(op1, op2);
```

Addition:

```
char* Add(char* op1, char* op2, int base)
unsigned int a;
int startfound = 0;
unsigned int c = 1;
int* sum;
char* response;
unsigned int size = Max(strlen(op1), strlen(op2)) + 2;
sum = new int[size];
response = new char[size];
for (a=0;a<size;a++)
    sum[a] = 0;
while (c<=size)
    if (c <= strlen(op1))</pre>
        sum[size - c] += Map(op1[strlen(op1) - c]);
    if (c <= strlen(op2))</pre>
   sum[size - c] += Map(op2[strlen(op2) - c]);
sum[size - c - 1] += sum[size - c] / base;
sum[size - c] %= base;
    C++;
}
c = 0;
for (a=0;a<size;a++)
    if (startfound==0)
        if ((sum[a] == 0)&&(a!=size-1))
            continue;
        else
            startfound=1;
    response[c] = ReverseMap(sum[a]);
response[c] = '\0';
return response;
```

Division

```
char* Divide(char* op1, char* op2, int base)
unsigned int a;
int b;
int startfound = 0;
unsigned int c = 1;
char* response;
unsigned int size = strlen(op1);
response = new char[size];
response[0] = ReverseMap(base-1);
response[1] = ' \ 0';
c = 1i
while (bigcmp(Multiply(response,op2,base),op1) < 0)</pre>
{
   response[c] = ReverseMap(base-1);
   response[c+1] = ' \0';
response[c] = '\0';
for (a=0;a<strlen(response);a++)</pre>
   response[a] = '0';
for(a=0;a<strlen(response);a++)</pre>
    for(b=0;b<base;b++)
       response[a] = ReverseMap(b);
       if (bigcmp(Multiply(response,op2,base),op1) > 0)
           break;
   response[a] = ReverseMap(b);
return response;
Modulus (requires Divide, Multiply, Subtract):
char* Modulus(char* op1, char* op2, int base)
return Subtract(op1, Multiply(Divide(op1, op2, base), op2,base),base);
Multiplication
char* Multiply(char* op1, char* op2, int base)
unsigned int a;
int startfound = 0;
unsigned int c = 1;
int* sum;
char* response;
unsigned int size = strlen(op1) + strlen(op2) + 1;
sum = new int[size];
response = new char[size];
for (a=0;a<size;a++)
    sum[a] = 0;
while (c<=size)
    for (a=1;a<=c;a++)
       if ((strlen(op2) >= c - a + 1)&&(strlen(op1) >= a))
           sum[size - c] += Map(op1[strlen(op1) - a]) *
   Map(op2[strlen(op2) - c + a - 1]);
           while (sum[size - c] >= base)
               sum[size - c -1] += 1;
               sum[size - c] -= base;
           }
    }
   C++;
c = 0;
```

String-Based Math Subtraction: Subtraction:

```
for (a=0;a<size;a++)
{
    if (startfound==0)
       if ((sum[a] == 0)&&(a!=size-1))
           continue;
       else
           startfound=1;
   response[c] = ReverseMap(sum[a]);
response[c] = '\0';
return response;
Subtraction:
char* Subtract(char* op1, char* op2, int base)
unsigned int a;
int startfound = 0;
unsigned int c = 1;
int* sum;
char* response;
unsigned int size = strlen(op1);
sum = new int[size];
response = new char[size];
for (a=0;a<size;a++)
   sum[a] = Map(op1[a]);
while (c<=size)
   if (c <= strlen(op2))</pre>
       sum[size - c] -= Map(op2[strlen(op2) - c]);
   while (sum[size-c]<0)
       sum[size - c - 1] -= 1;
sum[size - c] += base;
   C++;
}
c = 0;
for (a=0;a<size;a++)
    if (startfound==0)
       if ((sum[a] == 0)&&(a!=size-1))
           continue;
       else
          startfound=1;
   response[c] = ReverseMap(sum[a]);
response[c] = ' \ 0';
return response;
```

Dynamic Programming

Edit Distance

```
-Set of rules for finding the difference between two strings.
-Substitution: One character changes to another.
-Insertion: Add a character.
-Deletion: Remove a character.
#define MATCH 0
#define INSERT 1
#define DELETE 2
struct cell {
   int cost;
                   //Cost of reaching
   int parent; //Parent Cell
cell table[maxlength + 1][maxlength + 1];
//Change S into T
//Pad S and T with a character and start with 1 to make initialization easier.
int compare(String s, String t)
{int i, j, k, opt[3];
for (i=0;i<maxlength;i++)</pre>
   //Initialize the first row and column here. Typically, across
    // the row, INSERT and down the column DELETE, and give an
    // appropriate COST to each}
for (i=1;i<s.size();i++)</pre>
    for (j=1;j<t.size();j++) {</pre>
    opt[MATCH] = table[i-1][j-1].cost + match(s[i], t[j]);
    opt[INSERT] = table[i][j-1].cost + {cost of deleting t[j]};
    opt[DELETE] = table[i-1][j].cost + {cost of deleting s[i]};
     table[i][j].cost = opt[MATCH];
    table[i][j].parent = MATCH;
    for(k=INSERT;k<=DELETE;k++)</pre>
       if (opt[k] < table[i][j].cost) {
           table[i][j].cost = opt[k];
           table[i][j].parent = k;}
    //Identify the appropriate goal cell. This is typical.
   return tables[s.size()-1][t.size()-1].cost;
}
```

Memoization

Take any algorithm that might repeat a certain calculation multiple times.

Create an STL map that takes as input the critical parameter and saves the value of the result, and check to see if the memoized value exists in the map before evaluating the function. If the parameter is an int, you can use an array.

Greedy Activity Selection

In this problem, you are given a list of jobs that are specified by starting and finishing times. You have to select the largest set of jobs whose times don't overlap (one job can start exactly when another is finishing). The strategy is to sort the jobs by finish time and then be greedy by picking the job with the earliest finish time that is compatible with the set you've already chosen

```
#include <iostream>
#include <string.h>
#include <vector>
using namespace std;

typedef pair<int,int> job;
vector<job> joblist;
vector<job> picked;

bool compare_finish(const job &p1, const job &p2)
{
    printf("comparing %d, %d\n",p1.second, p2.second);
    return(p1.second < p2.second);
}
void printjob(const job &p1)
{</pre>
```

```
printf("(%d %d)\n",p1.first, p1.second);
int main()
    char s[80];
   while (gets(s))
       char *p = strtok(s," ");
       int s = atoi(p);
       p = strtok(NULL, "\n");
       int f = atoi(p);
       joblist.push_back(make_pair(s,f));
    sort(joblist.begin(), joblist.end(), compare_finish);
   picked.push_back(joblist[0]);
    for (int i=1;i<joblist.size();i++)</pre>
       if (joblist[i].first >= picked.back().second)
           picked.push_back(joblist[i]);
   printf("Selected\n");
   for_each(picked.begin(),picked.end(),printjob);
```

Maze Traversal Simple- Simple

Maze Traversal

Simple

```
#include <iostream>
#include <bitset>
#include <vector>
using namespace std;
#define MAX_SIZE 8
typedef struct row_struct
   bitset<MAX_SIZE+2> d;
} row;
vector<row> maze;
int log(int x)
    int l = 0;
   while (x >= 1)
       x = x/10;
       1++;
   return 1;
int main()
   int temp,n;
   maze.reserve(MAX_SIZE+2);
   cin >> n;
    // rows and columns are numbered from 1
    for (int i=1;i<=n;i++)
       maze[i].d.set(0);
       maze[i].d.set(n+1);
       maze[0].d.set(i);
       maze[n+1].d.set(i);
       for (int j=1; j <= n; j++)
           cin >> temp;
           maze[i].d[j] = temp;
    if (!search(1,1,n,n))
       cout << "No path found" << endl;</pre>
}
```

```
bool search(int sx, int sy, int fx, int fy)
   cout << "(" << sx << "," << sy << ")";
   if ((sx == fx) \&\& (sy == fy))
      return true;
   maze[sx].d.set(sy);
   if ((!maze[sx+1].d[sy]) &&
       search (sx+1, sy, fx, fy))
       return true;
   if ((!maze[sx-1].d[sy]) &&
       search (sx-1, sy, fx, fy))
       return true;
   if ((!maze[sx].d[sy+1]) \&\&
      search (sx, sy+1, fx, fy))
       return true;
   if ((!maze[sx].d[sy-1]) &&
      search (sx, sy-1, fx, fy))
      return true;
   for (int i=0;i<3+\log(sx)+\log(sy);i++)
      cout << (char)8;
   return false;
```

Min Spanning Trees and Shortest Path

The strategy for Prim's MST and Dijkstra's shortest path algorithms is exactly the same. The comments highlight the differences.

- Dijkstra's gives a shortest path tree, so giving shortest paths from starting vertex to ALL other vertices
- You can find a maximum spanning tree by negating the weights and then finding the min spanning tree
- If we want a min spanning tree with the smallest product of weights, we can use log(a*b) = log(a)+log(b) and replace each with with its logarithm and use the normal MST algorithm

```
void mst_sp(graph *g, int start){
                                             #define MAXV 50
   int i,j,v,w,weight,dist;
   bool intree[MAXV];
                                             #define MAXDEGREE 40
                                             #define FALSE 0
   int distance[MAXV];
   int parent[MAXV];
                                             typedef struct{
                                                 int v; // neighboring vertex
   for (i=1; i<=g->nvertices;i++){
       intree[i] = FALSE;
                                                 int weight; // edge weight
       distance[i] = MAXINT;
                                             } edge;
                                             typedef struct{
       parent[i] = -1;
                                                 edge edges[MAXV+1][MAXDEGREE];// adjacency list
   distance[start] = 0;
                                                 int degree[MAXV+1];
                                                                      // outdegree
   v = start;
                                                 int nvertices;
   while (intree[v] == FALSE) {
                                                 int nedges;
                                             } graph;
       intree[v] = true;
       for (i=0;i<q->degree[v];i++){
           w = g - sedges[v][i].v;
           weight = g->edges[v][i].weight;
           // use the following for MST
           if ((distance[w] > weight) && (intree[w] = FALSE)){
               distance[w] = weight;
               parent[w] = v;
           }
           // use the following for shortest path
           if (distance[w] >distance[v]+weight){
               distance[w] = distance[v] + weight;
               parent[w] = v;
           }
       v = 1; // make v will be closest vertex that not in tree
       dist = MAXINT;
       for (i=2; i<=g->nvertices; i++){
           if ((intree[i] == FALSE) && (dist > distance[i])){
               dist = distance[i];
               v = i;
       }
```

All Pairs Shortest Path

If you care about the path, the most efficient algorithm is dijkstra's from each vertex. This will give the distance, but won't let you recreate the path.

```
typedef struct {
   int weight[MAXV+1][MAXV+1];
   int nvertices;
} adjacency_matrix;
init_adj_matrix(adjency_matrix *g){
   int i,j;
   g->nvertices=0;
   for(i=1;i<MAXV;i++)
        for(j=1; j<=MAXV; j++)
        g->weight[i][j] = MAXINT;
}
```

Grids

Rectilinear

Much like the Cartesian plane, they are relatively simple to traverse in array form. Note that a Hexagonal grid is practically identical to two rectilinear grids, slightly offset.

Triangular Lattice

This lattice is practically identical to a Hexagonal grid. From any point, there are 6 paths one might take.

Triangular Cell-Wise

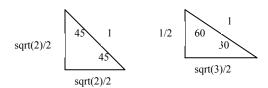
Similar to a Rectilinear grid, but do not allow passage between some cells.

Hexagonal

Choose one direction to be "x", and another to be "y". Moving in the other direction (the one that is a linear combination of the two) adds 1 to x and subtracts one from y or vice-versa.

Circle Packing

The densest possible placement of circles is with a placement analogous to the centers of hexagons in a hexagonal grid. To find the height and width of an array of packed circles, make use of the hexagonal grid and the properties of a 30-60-90 triangle. Both special triangles are below for reference.



Coding Tricks

STL algorithms

```
The examples are relative to vector<int> data; However, they can be applied to any of the containers.
```

adjacent_find: finedfirst element that has a value equal to its neighbor. Second form allows you to make the criteria of "equal" so you can make it anything. For instance, you could make a method "doubled" that return true if the first param has half of the second and find elements whose neighbors were doubles.

```
adjacent_find(data.begin(), data.end());
    adjacent_find(data.begin(), data.end(), compareOp);
count: count the number of elements with a particular value
    count(data.begin(), data.end(), value)
count_if: (operation is a boolean method with a single parameter that each element will be passed into)
    countif(data.begin(), data.end(), operation)
equal: write your own compare operation to make this very powerful. For example, compareOp could check if the elements in the second list
are doubles of the elements in the first list (in order).
    equal(data.begin(), data.end(), data2.begin())
    equal(data.begin(), data.end(), data2.begin(), compareOp)
find:
    find(data.begin(), data.end(), value)
find if: (operation is a boolean method with a single parameter that each element will be passed into)
    find_if(data.begin(), data.end(), operation)
find first of: return position of first element of first range that is also in second range
    find_first_of(data.begin(), data.end(),
                 data2.begin(), data2.end());
    find_first_of(data.begin(), data.end(),
                  data2.begin(), data2.end(), compareOp);
for_each: (operation is a method to be applied to every member of the vector. can be applied to any InputInterators)
    for_each(data.begin(), data.end(), operation)
min element:
    min_element(data.begin(), data.end())
    min_element(data.begin(), data.end(), compareOperation)
    max_element(data.begin(), data.end())
    max_element(data.begin(), data.end(), compareOperation)
next permutation: permutes the elements to give the next permutation. returns FALSE if elements have lexicographic order (see also:
prev_permutation)
    next_permutation(data.begin(), data.end();
prev_permutation: permutes the elements to give the previous permutation. returns FALSE if elements have lexicographic order (see also:
next permutation)
    prev_permutation(data.begin(), data.end();
remove:
    remove(data.begin(), data.end(), remVal);
remove if:
    remove_if(data.begin(), data.end(), booleanOp);
replace:
    replace(data.begin(), data.end(), old, new);
replace copy:
    replace_copy(data.begin(), data.end(), newdata.begin(),
    old, new);
replace_copy_if:
    replace(data.begin(), data.end(), newdata.begin(), booleanOp,
                 old, new);
    replace(data.begin(), data.end(), booleanOp, newValue);
search: return an iterator pointing at the first occurrence of one container in another.
    vector<int>:: iterator pos;
    pos = search(data.begin(), data.end(),
                 data2.begin(), data2.end())
    pos = search(data.begin(), data.end(),
                 data2.begin(), data2.end(),compareOp)
search_n: returns position of the first of size consecutive elements in the range whose value match. Note that the criteria operation in the second
for MUST be a binary operation.
    search_n(data.begin(), data.end(), size, value)
    search_n(data.begin(), data.end(), size, greater<int>())
unique: remove duplicate values (with "equal" defined by the boolean binary operator compareOp)
    unique(data.begin(), data.end());
unique(data.begin(), data.end(), compareOp);
```

C/C++ Tricks

ItoA: Need something like itoa(int x, string targ)? Use sprintf(targ,"%d",x);

STL - Reading Input

```
ifstream dataFile("ints.dat");
istream_iterator<int> dataBegin(datafile);
istream_iterator<int> dataEnd;
list<int> data(dataBegin,dataEnd);
Map Example
#include <iostream>
#include <map>
#include <string>
using namespace std;
void main()
   map<string,int> freq;
   string word;
   while (cin >> word)
       freq[word]++;
   }
   for (map<string,int>::iterator iter = freq.begin(); iter != freq.end(); iter++)
       cout << iter->second << " " << iter->first << endl;</pre>
Java Framework
       Be careful to name to class (and file) in the way the problem specifies (This one is Main)
import java.io.*;
import java.util.*;
public class Main
   static String readLn (int maxLg)
       byte lin[] = new byte [maxLg];
       int lg = 0, car = -1;
       String line = "";
       try
       {
           while (lg < maxLg)</pre>
               car = System.in.read();
               if ((car < 0) || (car == '\n')) break;
               lin [lg++] += car;
       catch (IOException e)
           return (null);
       if ((car < 0) && (lg == 0)) return (null); // eof
       return (new String (lin, 0, lg));
   public static void main (String args[]) // entry point from OS
       String input;
       StringTokenizer idata;
       int n,curr;
       while ((input = readLn (255)) != null)
           idata = new StringTokenizer (input);
           n = idata.countTokens();
           curr = Integer.parseInt (idata.nextToken());
    }
```