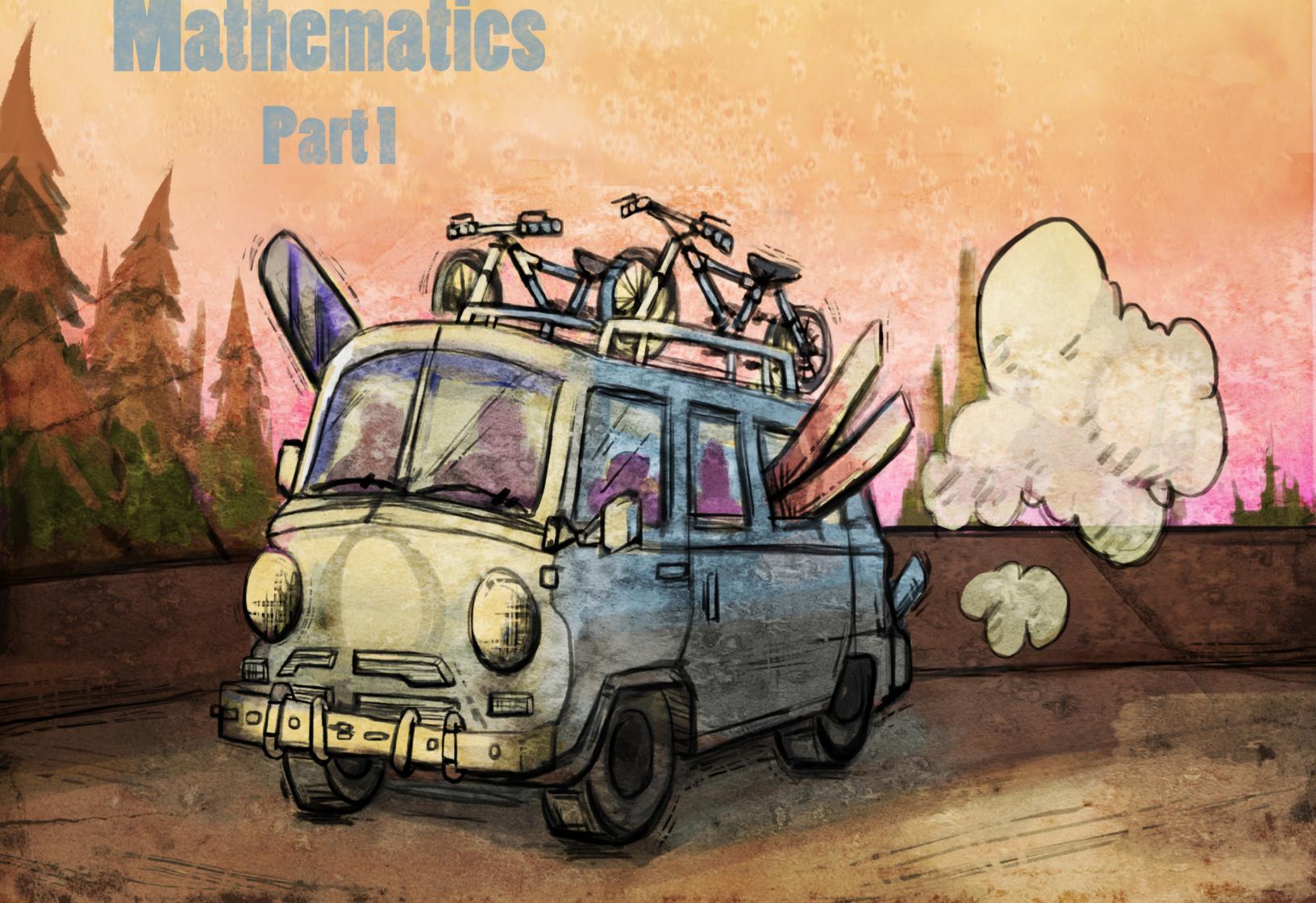


# MATH 035

## Eighth-Grade Mathematics

### Part 1



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**MATH 035—EIGHTH-GRADE MATHEMATICS, PART 1**

Section S001 / 0.5 Semester hours

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## Introduction to This Reading Packet

Thank you for purchasing the lesson packet for MATH 35: Eighth-Grade Mathematics, Part 1. These items are included in this packet:

- this introductory letter
- a copy of the readings from each lesson (the pages following this letter)

Begin your online course by reading the syllabus; it contains the information you need to successfully complete the course. As you begin, you will notice that each lesson includes a brief introduction, learning outcomes for the lesson, reading assignments, analysis of the importance of each reading, and lesson reading material. The reading material for each lesson is included in this packet, as well as in the online course.

This packet is designed to give you the best experience for reading the detailed lesson content and taking notes. To complete the course, you will need access to the online assignments and exams.

Best wishes for your success in this course!

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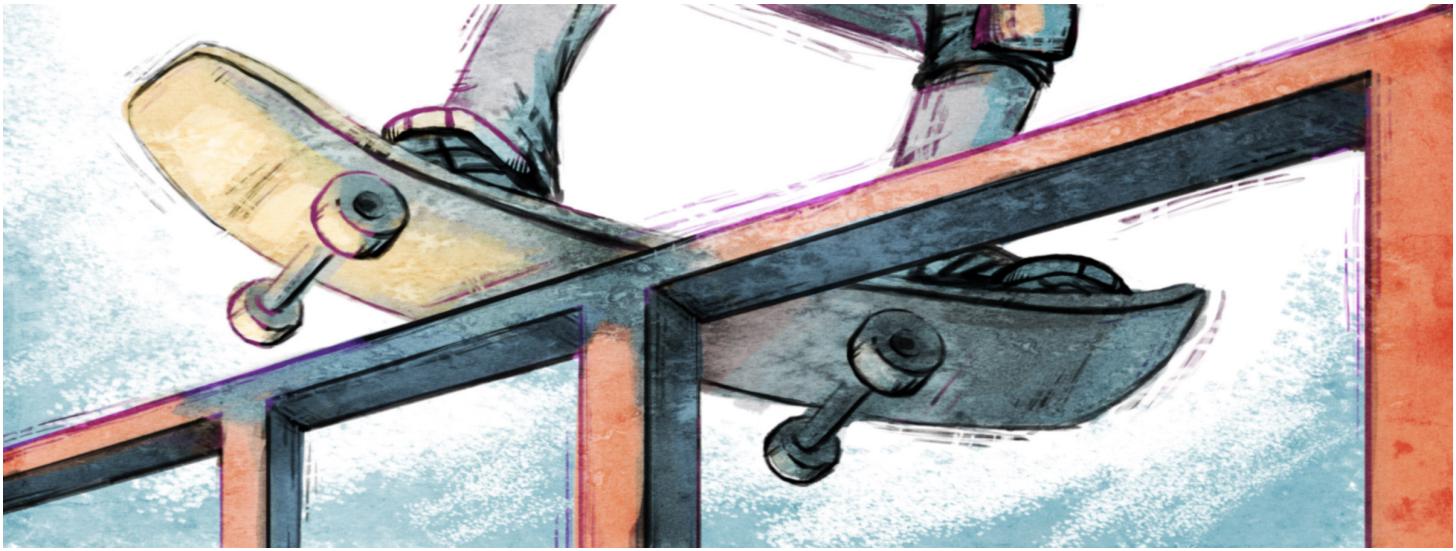
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## Unit 1

### Equation Concepts

**I**N THIS UNIT YOU WILL LEARN ABOUT SOLVING DIVERSE TYPES OF EQUATIONS USING MULTIPLE tools. These tools include properties of equality, the distributive property, and combining like terms. You will learn when to apply each tool (property) and how it helps you solve any given equation. You will also learn how to write equations with one variable to represent the world around us and then apply tools to solve each equation.

#### Learning Outcomes

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1. Solve basic one-step and two-step equations with integer coefficients.
2. Solve basic equations with rational coefficients.
3. Solve multi-step equations with variables on one side of the equal sign.
4. Solve multi-step equations with variables on both sides of the equal sign.
5. Solve complex multi-step equations.
6. Write and solve equations with one variable.



## 1.1: Basic Equations

*Solve one-step and two-step equations with integer coefficients.*

Many different equations are used in the world around us, but we will focus on basic equations that only use one or two steps to solve and that have integer coefficients. (Remember that an *integer* is a whole number, or a number that is not a fraction.)

The value we are solving for is a *variable*. We typically use  $x$  or  $y$  to represent the unknown value. However, any letter could be used to represent the unknown value, depending on the unknown.

A *term* is a group of numbers and variables that have been connected with multiplication or division. Examples of these would be  $2x$ , or  $213y$ . When terms are in their simplest form, the number with the variable is the *coefficient*. The coefficient of  $2x$  is 2, and the coefficient of  $213y$  is 213.

Remember

- Variables are unknown values.
- Terms are numbers and variables connected by multiplication or division.
- Coefficients are numbers with variables in a simplified term.

Equations are two expressions that are set equal to each other. We can use properties of equality to find the value of the variable in the equation. With properties of equality, what you do to one side of the equation must be done to the other side to keep it equal. You can think of a scale and keeping the weight the same.

These are equally weighted.



These are not equally weighted.



## Solving Equations

When solving equations, our goal is to get the variable isolated on one side of the equation. We do this by adding, subtracting, multiplying, or dividing both sides of the equation. There are two parts to solving equations: (1) undoing what has been done, and (2) doing the same operation to both sides. You will be able to answer these two examples mentally. The following examples show you how the properties of equality work.

### Example 1

Find the value of the variable.

$$4x = 8$$

Dividing by 4 undoes multiplying of  $x$  (the variable) by 4. We need to divide by 4 on both sides.

$$x = 2$$

### Example 2

Find the value of the variable.

$$y + 7 = 19$$

Subtracting 7 undoes the addition of 7 to  $y$  (the variable). We need to subtract 7 from both sides.

$$y = 12$$

Notice that we have done the opposite operation from what was in the equation. In example 1, we divided 4 because it was 4 times  $x$ . The second example added 7, so we subtracted by 7 to isolate the variable.

The number of operations in an equation determines the number of operations it will take to solve the equation. Recall that when we used order of operations, it meant that we did multiplication and division first, and then we did addition and subtraction. When solving

## **Math 35: Eighth-Grade Mathematics, Part 1**

equations using reverse order of operations, we will first add or subtract and then multiply or divide. Read through the next two examples on how to solve these types of equations.

### **Example 3**

$$2x + 7 = 3$$

What operations do you see? (The variable is being multiplied by 2, and 7 is being added.) Now we use reverse order of operations and undo the addition first. We subtract 7 to undo the addition, and we subtract from both sides to keep the equation equal.

$$2x = -4$$

The variable is being multiplied by 2; we undo the multiplication of 2 by dividing by 2 on both sides of the equation to keep it equal. Then we get our solution.

$$x = -2$$

### **Example 4**

$$9 + \frac{b}{4} = 17$$

The operations we see are addition and division. We undo addition with subtraction, so we subtract 9. We must keep the equation equal, so we subtract 9 from both sides:

$$\frac{b}{4} = 8$$

Now to undo the division by 4, we multiply by 4. Keeping the equation equal, we need to multiply both sides by 4:

$$b = 32$$

When solving equations it is a matter of undoing what has been done to the variable and doing the same thing to both sides to keep it equal.

If you need more examples or someone to help review how to solve equations, please watch the videos at the end of the unit by clicking on the links.

## **Videos**

- “Solving Equations 1”: TeacherTube Math” (running time 6:12)
- “Solving Equations #2 with Mr. Karaba” (running time 7:54)
- “Solving Equations #3 with Mr. Karaba” (running time 6:46)



**Video**

Go to unit 1 in your course online to watch these videos.

- “Solving Equations 1” (running time 3:03)

## **Practice**

---

Complete the practice problems in Worksheet 1. After you have completed the worksheet, check your answers in the Unit 1 Worksheet Key (found in the Resources folder of the online course).



## 1.2: Equations with Rational Coefficients

*Solve basic equations with rational coefficients.*

In unit 1.1, you learned that coefficients are the numbers with variables in a simplified term (the number in front of the variable). *Rational coefficient* means that the number in front of the variable is a rational number. To solve equations with rational coefficients, you need to undo the operations and keep both sides equal, the same as you did with integer coefficients.

### Reading Assignment

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Read pages 111–14 and 121–24 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

---

- “Solving Equations #3 with Mr. Karaba” (running time 6:46)

This video was also included in unit 1.1; view it again if you missed it or if you want a refresher.



**Video**

Go to unit 1 in your course online to watch this video.

### Practice

---

Work through the selected problems in the *Glencoe Math* textbook:

- pages 115–18, problems 2, 7, 16, 20, 21, 23, 26, 30
- pages 125–28, problems 3–6, 8–10, 13, 22–29

After you have completed these practice problems, check your answers in the Unit 1 Practice Key (found in the Resources folder of the online course).



## 1.3: Equations with Variables on One Side

*Solve multistep equations with variables on one side of the equal sign.*

When equations have multiple steps, we use more tools than we have previously. We are going to add one more part to the solving of equations. Previously, we had two parts: (1) undo what has been done to our variable, and (2) keep the equation equal by doing the same operation to both sides of the equation. There will be times that we need to simplify each side of the equation before we complete these parts.

Parts to solving equations:

1. Simplify each side of the equation.
2. Undo what is being done to the variable.
3. Do the same operation to both sides of the equation.

Simplifying each side of the equation is done with the tools for simplifying expressions, which you learned in the 7th-grade course. The two most common methods for simplifying are using the distributive property and combining like terms.

*Distributive property* is multiplying a sum or difference by a term.

*Combining like terms* means performing operations between terms with the same exponent(s) on one side of the equation. (Remember, *exponents* show the number of times to multiply.)

In the next few examples, notice how and when we use the distributive property and combine like terms.

### Example 1

$$49 = -7(t + 1)$$

Write the equation.

$$49 = -7t - 7$$

Use the distributive property.

$$49 + 7 = -7t - 7 + 7$$

Use the addition property of equality.

$$56 = -7t$$

Simplify.

$$\frac{56}{-7} = \frac{-7t}{-7}$$

Use the division property of equality.

$$-8 = t$$

Simplify.

**Example 2**

$$2(4a - 5) = 30$$

$$8a - 10 = 30$$

$$8a - 10 + 10 = 30 + 10$$

$$8a = 40$$

$$\frac{8a}{8} = \frac{40}{8}$$

$$a = 5$$

Write the equation.

Use the distributive property.

Use the addition property of equality.

Simplify.

Use the division property of equality.

Simplify.

**Example 3**

$$-5p - 10 + 6p + 2 = 15$$

$$p - 8 = 15$$

$$p - 8 + 8 = 15 + 8$$

$$p = 23$$

Remember to combine only terms with the same exponent(s).

Write the equation.

Combine like terms.

Undo subtraction.

Simplify.

**Example 4**

$$3p + 4(p + 11) = 79$$

$$3p + 4p + 44 = 79$$

$$7p + 44 = 79$$

$$7p + 44 + (-44) = 79 + (-44)$$

$$7p = 35$$

$$\frac{7p}{7} = \frac{35}{7}$$

$$p = 5$$

Write the equation.

Use the distributive property.

Simplify.

Use the addition property of equality.

Simplify.

Use the division property of equality.

Simplify.

**Videos**

- “Combining Like Terms and the Distributive Property” (running time 4:06)
- “Combining Like Terms 2” (running time 3:42)



Go to unit 1 in your course online to watch these videos.

**Practice**

Complete the practice problems in Worksheet 2. After you have completed the worksheet, check your answers in the Unit 1 Worksheet Key (found in the Resources folder of the online course).



## 1.4: Equations with Variables on Both Sides

*Solve multistep equations with variables on both sides of the equal sign.*

When equations have variables on both sides, you use the properties of equality to find an equivalent equation. Properties of equality were covered in unit 1.2 and on pages 121–24 in the textbook, if you want a refresher.

Keep these tips in mind:

- Remember to get all variables to one side of the equation.
- Only like terms (those with the same variables) can be combined.
- Undo addition and subtraction first and then multiplication and division to solve for the variable.
- Check for any incorrect signs. (Multiplying an even number of negative terms is positive; multiplying an odd number of negative terms is negative.)
- Ending with a false statement is “no solution.”
- Ending with a true statement is an “all real numbers” solution.

### Reading Assignment

Read pages 145–48 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Solving Equations with the Distributive Property 2” (running time 4:49)
- “Solving Equations 2” (running time 4:06)



**Video**  
Go to unit 1 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 149–52, problems 1–6, 10, 13, 15–17, 23, 25–32

After you have completed these practice problems, check your answers in the Unit 1 Practice Key (found in the Resources folder of the online course).



## 1.5: Complex Equations

*Solve complex multistep equations.*

Some equations contain expressions with grouping symbols (parentheses). To solve these complex equations, you first expand the expression using the distributive property.

You learned about using the distributive property for simpler equations in unit 1.3. The distributive property is used for expanding the expression by multiplying a sum or difference by a term. In this unit, you will work with equations that contain multiple sets of parentheses.

The basic tips listed in unit 1.4 continue to apply; here are some additional tips for complex equations:

- Simplify each side of the equation by using distributive property and combining like terms.
- Distribute on both sides of the equation before combining like terms.
- Distribute to both terms in each set of parentheses.
- Check for an incorrect use of the operations.
- To simplify fractions, you must multiply by the LCD on both sides of the equation.
- To clear fractions, you must multiply by the LCD of all fractions in the equation.
- Also, notice the third “solution” option in the following list (you learned the first two in unit 1.4):
  - Ending with a false statement is “no solution.”
  - Ending with a true statement is an “all real numbers” solution.
  - For an equation that has every value of the variable as its solution, the answer is “infinite number of solutions.”

### Reading Assignment

Read pages 153–56 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

- “Solving Equations with the Distributive Property”  
(running time 6:03)



**Video**

Go to unit 1 in your course online to watch this video.

## Practice

---

Work through the selected problems in the *Glencoe Math* textbook:

- pages 157–60, problems 1–8, 12, 17–24, 28–30

After you have completed these practice problems, check your answers in the Unit 1 Practice Key (found in the Resources folder of the online course).



## 1.6: Writing and Solving Equations

*Write and solve equations with one variable.*

In the previous units, you were given equations to solve. Now you will need to be able to write equations for a given word problem. To do this, you will translate sentences or statements into equations, so you will need to remember how to translate the words into the correct operations.

Here are some tips for writing equations:

- Use  $n$ ,  $x$ , or another logical letter to represent the number.
- “Is” means “equal to a number.”
- “Equal to” means that a number has to be by itself on one side of the equation.
- “And” means to use addition in that part of the equation.
- “More than” means to use addition.
- “Sum” means to use addition.
- “Difference” means to use subtraction.
- “Less than” means to use subtraction in that part of the equation.
- “Times” means to use multiplication.
- For fractions, use division in that part of the equation.

### Reading Assignment

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Read pages 129–32 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

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- “Linear Equation Word Problem Example” (running time 5:58)



**Video**

Go to unit 1 in your course online to watch this video.

## Practice

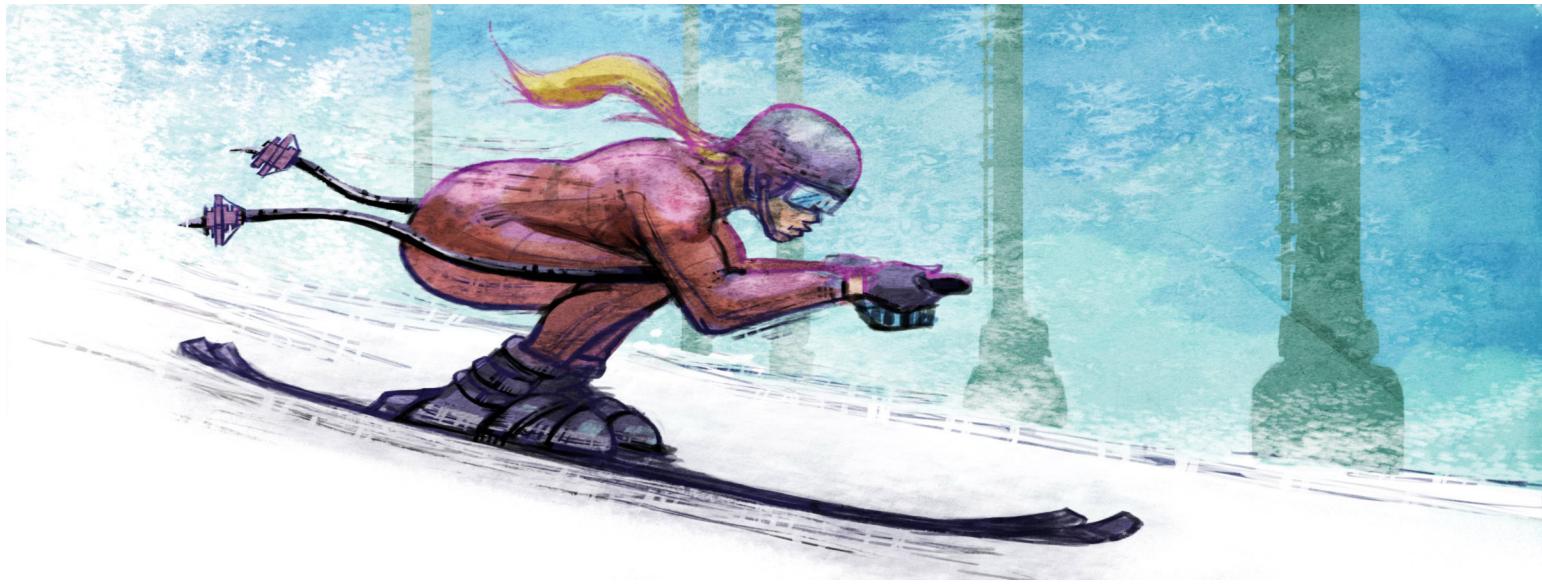
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Work through the selected problems in the *Glencoe Math* textbook:

- pages 133–36, problems 1–6, 13–28

After you have completed these practice problems, check your answers in the Unit 1 Practice Key (found in the Resources folder of the online course).





## Unit 2

### Linear Equations

**I**N THIS UNIT YOU WILL LEARN ABOUT LINEAR EQUATIONS AND HOW TO WRITE THEM GIVEN GRAPHS, tables, or points. *Linear equations* are equations with a graph that is a straight line. To create these linear equations, you will find the slope and constant rate of change and determine whether a linear equation is proportional. You will also find the simplest way to write the equations.

#### Learning Outcomes

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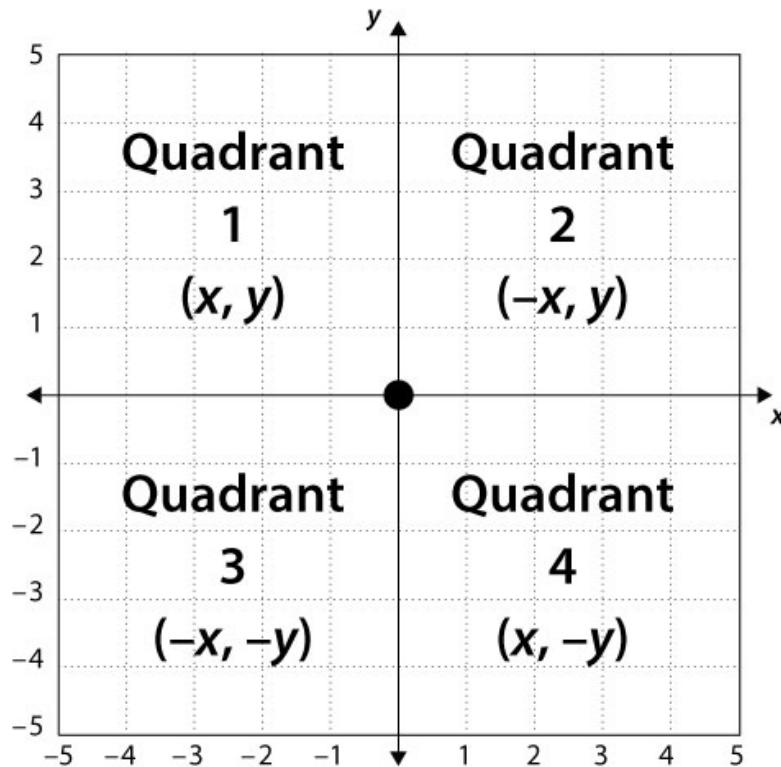
1. Identify parts of the coordinate plane, plot the points, and find the slope given a graph, table, or points.
2. Find the constant rate of change given a table, graph, or word problem.
3. Identify the slope and  $y$ -intercepts and graph the equations in tables.
4. Write equations using slope-intercept form in linear equations.
5. Graph and determine the slope of proportional relationships of linear equations.
6. Compare proportional relationships in linear equations.



## 2.1: Coordinate Plane and Slope

*Identify parts of the coordinate plane, plot the points, and find the slope given a graph, table, or points.*

The coordinate plane is a coordinate system in which a horizontal number line and a vertical number line intersect at their zero points. The coordinate plane is broken up by the y-axis and x-axis into four quadrants. The y-axis is the vertical line; the x-axis is the horizontal line. Notice the different parts of the coordinate plane in the following graph.



The center point is called the *origin*. The y-axis is a number line that has positive values above the origin and negative values below the origin. The x-axis is a number line that has positive values to the right of the origin and negative values to the left of the origin. This means

that each quadrant has a specific relationship with positive or negative values for both the  $x$  and  $y$  values.

The  $x$ - and  $y$ -values are both positive in quadrant 1. In quadrant 2, there is a negative  $x$ -value and positive  $y$ -value. In quadrant 3, the  $x$ - and  $y$ -values are negative. The  $x$ -values are positive and the  $y$ -values are negative in quadrant 4. Each point is represented as an *ordered pair* ( $x$ -value,  $y$ -value).

*Slope* is the rate of change, or the ratio of the vertical change (the *rise*) to the horizontal change (the *run*). Slope can be positive (slanting up or to the right) or negative (slanting down or to the left). The slope is written as change in  $y$  over change in  $x$ .

## Reading Assignment

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Read pages 181–84 and 226 in the *Glencoe Math* textbook. Carefully go through the examples.

## Videos

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- “Slope and Rate of Change” (running time 13:41)
- “Algebra: Slope” (running time 8:27)
- “Algebra: Slope 2” (running time 9:45)
- “Graphing Points in All Four Quadrants on the Coordinate Plane” (running time 5:01)



Video

Go to unit 2 in your course online to watch these videos.

## Practice

---

Work through the selected problems in the *Glencoe Math* textbook:

- page 266, problems 1–6
- pages 185–88, problems 1–10, 15–22

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).



## 2.2: Constant Rate of Change

*Find the constant rate of change given a table, graph, or word problem.*

A linear relationship has a *constant rate of change*, meaning that the rate of change between any two points is the same, or constant. Constant rate of change is calculated by the change in the  $y$ -value divided by the change in the  $x$ -value.

Two quantities ( $x$  and  $y$ ) are proportional if they have a constant ratio, meaning that the line connecting the points passes through the origin.

When you are finding the constant rate of change, you need to watch the direction (positive or negative) of the slope. If the line goes up, the slope is positive; if the line goes down, the slope is negative. Also remember that “ascend” means up and “descend” means down.

### Reading Assignment

---

Read pages 171–74 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

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- “1–7 Constant Rate of Change” (running time 1:53)



**Video**

Go to unit 2 in your course online to watch this video.

### Practice

---

Work through the selected problems in the *Glencoe Math* textbook:

- pages 175–78, problems 1–6, 10, 15, 17–21

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).



## 2.3: Equations in Tables

*Identify and graph equations in tables.*

Reviewing from unit 2.1, the slope of the line describes the steepness of the line or the rate of change. There are other ways that we refer to slope:

<u>rise</u>	<u>change in y</u>	<u>vertical change</u>
run	change in x	horizontal change

We can find the slope using the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The  $y$ -intercept is the ordered pair  $(0, b)$ , where  $b$  is the  $y$ -value where the line crosses the  $y$ -axis. The  $x$ -value is 0 in the ordered pair of the  $y$ -intercept. If you find zero for the  $x$ -value of an ordered pair, the corresponding  $y$ -value is the  $y$ -intercept.

Pay attention to the rules for operations with negative numbers:

- A negative number divided by a positive number is negative.
- A negative number divided by a negative number is positive.
- A positive number divided by a negative number is negative.
- When subtracting, double negatives change to a positive.

Watch the first video (“Linear Equations from XY Tables”) about how to graph and find the slope and  $y$ -intercept of a linear equation.

### Example 1

Find the slope and  $y$ -intercept, and then graph the equation.

<b>x</b>	-1	1	3	5
<b>y</b>	-2	0	2	4

Find the change in  $y$ -values over the change in  $x$ -values.

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change in $y$	$0 - -2 = 2$	2
change in $x$	$1 - -1 = 2$	2

The slope of the equation is 1.

There is no  $x$ -value of 0 in the table. Notice that 0 is halfway between  $-1$  and  $1$ , so the  $y$ -value would be halfway between  $-2$  and 0. The point where the line will cross the  $y$ -axis is  $(0, -1)$ . The  $y$ -intercept is  $-1$ .

Remember that the slope is the rise over the run, or change in  $y$  over change in  $x$ . When you solve equations, make sure you find the change the same way for the  $x$ - and  $y$ -value.

## Videos

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- “Linear Equations from XY Tables” (running time 5:01)
- “SAT & ACT Math: How to Find the Slope and Y-Intercept of a Line” (running time 5:48)



Video

Go to unit 2 in your course online to watch these videos.

## Practice

---

Work through the selected problems in the *Glencoe Math* textbook:

- Page 184: problem 3
- Pages 185–87, problems 4, 5, 18, 19

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).



## 2.4: Slope-Intercept Form

*Write equations using slope-intercept form in linear equations.*

Nonproportional linear relationships can be written in *slope-intercept form*, which is an equation written in the following form:

$$y = mx + b$$

In this form,  $m$  is the slope and  $b$  is the  $y$ -intercept. For example, in the following equation,  $y = 3x + 2$ , the slope ( $m$ ) is 3 and the  $y$ -intercept ( $b$ ) is 2.

The *slope* is the coefficient of  $x$ . The  *$y$ -intercept* is the other term, or the term added to the term with  $x$ . When solving these equations, solve for  $y$  so that you have it in slope-intercept form.

### Reading Assignment

Read pages 199–202 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “SAT & ACT Math: How to Find the Slope and  $Y$ -Intercept of a Line” (running time 5:48)
- “Graphing a Line in Slope-Intercept Form” (running time 3:01)



**Video**  
Go to unit 2 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 203–5, problems 1–3, 17, 18

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).



## 2.5: Proportional Relationships

*Graph and determine slope of proportional relationships of linear equations.*

Proportional graphs are linear, meaning they have straight lines, but they also go through the *origin*, which is the bottom left corner. “Proportional” means having the same or a constant ratio. Two quantities have a *proportional linear relationship* if they have a constant ratio and a constant rate of change.

The equation is as follows:

$$y = mx$$

In this equation,  $m$  represents the constant of variation, the constant of proportionality (constant ratio), the slope, and the *unit rate* (rate with a denominator of 1).

### Reading Assignment

Read pages 189–93 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Graphing, Interpreting, and Comparing Proportional Relationships 1” (running time 4:44)
- “Graphing, Interpreting, and Comparing Proportional Relationships 2” (running time 5:08)



**Video**

Go to unit 2 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 195–198, problems 1–2, 5–7, 13, 15–16, 18–20

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).



## 2.6: Comparisons

*Compare proportional relationships in linear equations.*

Now that you have learned to graph and determine the slope of proportional relationships of linear equations, you will compare two proportional relationships. Remember, the slope is written as change in  $y$  over change in  $x$ , or rise over run in simplest form. Also, the constant of variation is  $y$  over  $x$ .

### Reading Assignment

Read pages 189–93 in the Glencoe Math textbook. Carefully go through the examples.

### Videos

- “Graphing, Interpreting, and Comparing Proportional Relationships 4” (running time 5:12)
- “Graphing, Interpreting, and Comparing Proportional Relationships 5” (running time 4:42)



**Video**

Go to lesson 2 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- page 194, problems 2, 3
- pages 195–97, problems 3, 8–10, 17

After you have completed these practice problems, check your answers in the Unit 2 Practice Key (found in the Resources folder of the online course).





## Unit 3

### Systems of Equations

**T**WO OR MORE EQUATIONS WITH THE SAME SET OF VARIABLES ARE CALLED A *SYSTEM OF EQUATIONS*. In this unit you will learn about writing equations and solving systems of equations. You can solve a system of equations by graphing, by substitution, by elimination, or algebraically. You will learn when it is best to use each method of solving a system of equations.

#### **Learning Outcomes**

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1. Write linear equations given the slope and  $y$ -intercept of graphs.
2. Write equations given two points, and solve real-world problems.
3. Analyze and solve systems of equations by graphing.
4. Analyze and solve systems of equations by substitution.
5. Analyze and solve systems of equations by elimination.
6. Analyze and solve systems of equations algebraically.



## 3.1: Equations with Two Variables

*Write linear equations given the slope and y-intercept or graphs.*

### Review of Definitions

The following definitions were discussed in earlier units. They are listed here for easy reference:

- A *linear equation* is an equation with a graph that has a straight line.
- The *slope* is the ratio of the vertical change to the horizontal change.
- The *y-intercept* of a line is the *y*-coordinate of the point where the line crosses the *y*-axis.
- *Slope-intercept form* is used to write nonproportional linear relationships. The formula is as follows:
  - ◊  $y = mx + b$
  - ◊ In this formula, *m* is the slope and *b* is the *y*-intercept.

### Tips

The following tips might be helpful as you are working through the practice problems and the Self Checks:

- The slope should be the coefficient of *x* in the equation.
- The signs of the slope and the *y*-intercept do not change when placed in an equation.
- Pay attention to the direction of the slope (up is positive; down is negative).

### Reading Assignment

Read pages 199–202 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

- “Algebra I Help: Write an Equation of a Line Given Its Slope and Y-Intercept” (running time 2:44)



**Video**

Go to unit 3 in your course online to watch this video.

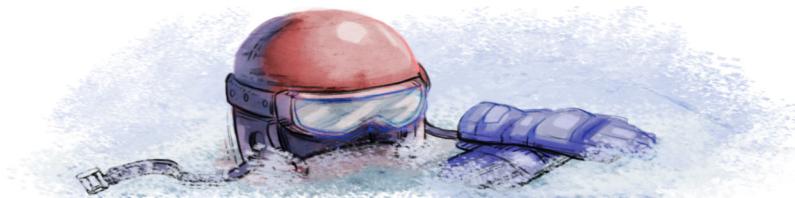
## Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 203–206, problems 4–7, 12, 15, 19–21, 24, 27

After you have completed these practice problems, check your answers in the Unit 3 Practice Key (found in the Resources folder of the online course).



## 3.2: Writing Equations

*Write equations given two points, and solve real-world problems.*

In this unit, you will learn to write equations when you are given two points. When you have two points, you can use the coordinates of the points to find the slope.

You already learned about slope-intercept form in unit 3.1, but now you need to know point-slope form as well. How do you tell the difference between slope-intercept form and point-slope form? They are just two different ways of writing the same thing; point-slope form can be used to find the information needed for slope-intercept form equations. View “Linear Equations in Point Slope Form” (see video links) for a detailed explanation.

You use *slope-intercept form* alone when you have the slope and  $y$ -intercept, or you use *point-slope form* first when you have the slope and the coordinates of a point that is not the  $y$ -intercept.

When an equation in slope-intercept form applies to a real-world situation, the slope is the rate of change and the  $y$ -intercept is the initial value.

### Formulas

As you learned in unit 3.1, the formula for slope-intercept form is  $y = mx + b$ . The formula for point-slope form is as follows:

$$y - y_1 = m(x - x_1)$$

For point-slope form, you subtract the  $x$  and  $y$  values from the points, and then you distribute the slope to both terms.

### Tips

Keep the following tips in mind when you are working on the practice problems and Self Checks:

- For equations in slope-intercept form given two points, find the slope of the line by finding the change in  $y$  over the change in  $x$ .
- Start with the same point when finding each change.
- Distribute the slope to both terms in parentheses.

## Reading Assignment

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Read pages 221–24 in the Glencoe Math textbook. Carefully go through the examples.

## Videos

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- “Linear Equations in Point Slope Form” (running time 9:38)
- “Linear Equations in Slope Intercept Form” (running time 14:57)



Video

Go to unit 3 in your course online to watch these videos.

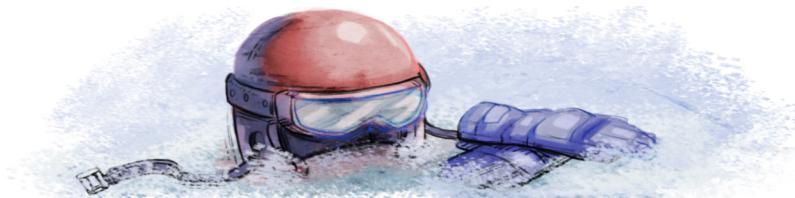
## Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 225–28, problems 1–8, 14, 17, 18

After you have completed these practice problems, check your answers in the Unit 3 Practice Key (found in the Resources folder of the online course).



### 3.3: Solving by Graphing

Analyze and solve systems of equations by graphing.

You can use graphing to estimate the solution of a system of equations. The *ordered pair* ( $x$ -coordinate and  $y$ -coordinate) for the point where the lines cross is the solution.

The graph also shows the number of solutions:

- If the lines cross (different slopes and  $y$ -intercepts), there is one solution.
- If the lines are parallel (same slopes, different  $y$ -intercepts), there is no solution.
- If the lines are the same (looks like one line), there will be an infinite number of solutions.

You will need to remember these ideas when you complete the Self Check questions.

#### Reading Assignment

Read pages 233–38 in the *Glencoe Math* textbook. Carefully go through the examples.

#### Videos

- “Solving Linear Systems by Graphing” (running time 8:29)
- “Graphing Systems of Equations” (running time 6:34)



**Video**

Go to unit 3 in your course online to watch these videos.

#### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 239–42, problems 1–6, 15–17, 22

After you have completed these practice problems, check your answers in the Unit 3 Practice Key (found in the Resources folder of the online course).



## 3.4: Solving by Substitution

Analyze and solve systems of equations by substitution.

Substitution can be used to find the exact solution of a system of equations. First, you write the equation and replace (substitute) a variable with an expression inside parentheses. From there, you simplify, collect like terms, and use properties of equality to find the solution.

### Number of Solutions

As you learned in earlier units, you need to pay attention to the number of solutions in a system of equations. When solving a system of equations, ask, “How many points do these equations have in common?”

1. **One point.** The solution is the point  $(x, y)$  that the two equations have in common.
2. **An infinite number of points.** If you get a true statement, then the two equations are the same equation and have every point in common. Answer: **Infinite number of solutions.**
3. **None.** If you get a false statement, then the two equations have the same slope but different  $y$ -intercepts. Their graphs will be parallel, so they will never have a point in common. Answer: **No solution.**

There is another possible answer when you get a true statement when solving by substitution: **All real numbers.**

### Tips

Keep the following tips in mind as you work on the Self Check questions:

- Make sure you copy each sign correctly from the equation you are replacing and the number you are substituting.
- Distribute the number to both terms that you substitute for  $x$ .
- Be sure of what variable you are solving for.
- Remember that subtracting a negative is simply addition.
- Combine only like terms, meaning terms with the same variables.
- Follow through on the signs in your work.

## **Reading Assignments**

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Read pages 243–46 in the *Glencoe Math* textbook. Carefully go through the examples.

## **Videos**

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- “Solving Linear Systems by Substitution” (running time 9:20)
- “Solving Systems of Equations Using Substitution 2” (running time 5:02)
- “Solving Systems of Equations Using Substitution 3” (running time 4:20)



**Video**

Go to unit 3 in your course online to watch these videos.

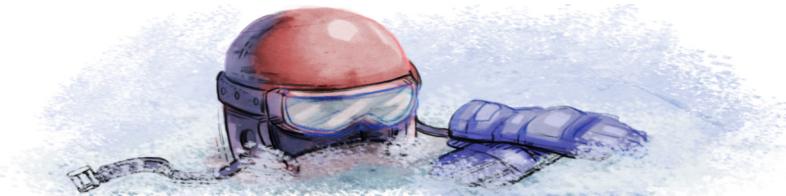
## **Practice**

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 247–250, problems 1–11, 16–22

After you have completed these practice problems, check your answers in the Unit 3 Practice Key (found in the Resources folder of the online course).



## 3.5: Solve by Elimination

Analyze and solve systems of equations by elimination.

In unit 3.4, we learned about solving systems of equations algebraically by substitution. Now we will look at elimination, which is a second way to solve systems of equations. *Elimination* is a method of adding or subtracting two equations in order to get an equation with one variable.

### Number of Solutions

As you learned in earlier units, you need to pay attention to the number of solutions in a system of equations. When solving a system of equations, ask, “How many points do these equations have in common?”

1. **One point.** The solution is the point  $(x, y)$  that the two equations have in common.
2. **An infinite number of points.** If you get a true statement, then the two equations are the same equation and have every point in common. Answer: **Infinite number of solutions.**
3. **None.** If you get a false statement, then the two equations have the same slope but different  $y$ -intercepts. Their graphs will be parallel, so they will never have a point in common. Answer: **No solution.**

When the coefficient of one variable is the opposite of the coefficient of the same variable in the equation, you add the two equations.

### Example 1

$$3y + 2x = 6$$

$$5y - 2x = 10$$

Notice that the coefficients of  $x$  are opposites, so we can add the equations and eliminate the  $x$ -variable.

**Math 35: Eighth-Grade Mathematics, Part 1**

$$(1) \quad \begin{array}{r} 3y + 2x = 6 \\ + 5y - 2x = 10 \\ \hline 8y + 0 = 16 \end{array}$$

$$(2) \quad 8y = 16$$

$$(3) \quad \frac{8y}{8} = \frac{16}{8}$$

$$(4) \quad y = 2$$

Now you substitute in your answer for  $y$  in one of the two equations and solve for  $x$ . Notice that it does not matter which equation you choose.

$$3y + 2x = 6 \qquad \qquad 5y - 2x = 10$$

$$3(2) + 2x = 6 \qquad \qquad 5(2) - 2x = 10$$

$$\begin{array}{rcl} 6 + 2x = 6 & & 10 - 2x = 10 \\ -6 \qquad -6 & & -10 \qquad -10 \end{array}$$

$$0 + 2x = 0 \qquad \qquad 0 - 2x = 0$$

$$2x = 0 \qquad \qquad -2x = 0$$

$$\frac{2x}{2} = \frac{0}{2} \qquad \qquad \frac{-2x}{-2} = \frac{0}{-2}$$

$$x = 0 \qquad \qquad x = 0$$

The solution of the systems of equations is  $(0, 2)$ .

We can check this solution by substituting in the values in our solution for  $x$  and  $y$  in each equation.

$$(1) \quad 3y + 2x = 6 \qquad \qquad 5y - 2x = 10$$

$$(2) \quad 3(2) + 2(0) \stackrel{?}{=} 6 \qquad \qquad 5(2) - 2(0) \stackrel{?}{=} 10$$

$$(3) \quad 6 + 0 \stackrel{?}{=} 6 \qquad \qquad 10 - 0 \stackrel{?}{=} 10$$

$$(4) \quad 6 = 6 \qquad \qquad 10 = 10$$

Since each equation resulted in a true statement when we substituted in the values of the ordered pair for  $x$  and  $y$ , we know that  $(0, 2)$  is the solution for the system of equations.

**Example 2**

$$-15x - 4y = -57$$

$$15x - 12y = 9$$

The coefficients of  $x$  in both equations are opposites, so we need to add the two equations.

$$\begin{array}{r} -15y - 4y = -57 \\ + 15x - 12y = 9 \\ \hline 0 - 16y = -48 \end{array}$$

$$-16y = -48$$

$$\frac{-16y}{-16} = \frac{-48}{-16}$$

$$y = 3$$

Now you substitute in your answer for  $y$  in one of the two equations and solve for  $x$ .

$$15x - 12y = 9$$

$$15x - 36 = 9$$

$$+36 +36$$

$$15x + 0 = 45$$

$$15x = 45$$

$$\frac{15x}{15} = \frac{45}{15}$$

$$x = 3$$

The solution for the system of equations is  $(3, 3)$ .

When the coefficient of one variable is the same as the coefficient of the same variable in the equation, you subtract the two equations. Subtraction is the distribution of a negative one to all terms in an equation and then addition of the equations.

**Example 3**

$$-3x + y = -21$$

$$-2x + y = -8$$

Notice that in both equations we have the term  $y$ , or positive  $1y$ . We multiply the second equation by  $-1$ .

**Math 35: Eighth-Grade Mathematics, Part 1**

$$-1(-2x + y = -8) \xrightarrow{-1} 2x - y = 8$$

Now we add the two equations to get an equation with one variable.

$$\begin{array}{r} -3x + y = -21 \\ + 2x - y = 8 \\ \hline -x + 0 = -13 \end{array}$$

We solve the new equation by subtracting the two equations.

$$\frac{-x}{-1} = \frac{-13}{-1}$$

$$x = 13$$

Now we substitute in the answer for  $x$  in one of the two original equations and solve for  $y$ . The original second equation has smaller numbers so we use the second equation.

$$-2x + y = -8$$

$$-2(13) + y = -8$$

$$\begin{array}{r} -26 + y = -8 \\ +26 \quad +26 \end{array}$$

$$0 + y = 18$$

$$y = 18$$

The solution is one point  $(13, 18)$ .

We check this solution by substituting in the values of  $x$  and  $y$  from our solution into each equation to see if we get a true statement. If we get a true statement, then the solution is correct.

$$-3x + y = -21 \qquad -2x + y = -8$$

$$-3(13) + 18 \stackrel{?}{=} -21 \qquad -2(13) + 18 \stackrel{?}{=} -8$$

$$-39 + 18 \stackrel{?}{=} -21 \qquad -26 + 18 \stackrel{?}{=} -8$$

$$-21 = -21 \qquad -8 = -8$$

Since both equations result in a true statement, the solution to the system of equations is one point  $(13, 18)$ .

**Example 4**

Twice a number added to another number is 25. Three times the first number minus the second number is 20. Find the two numbers.

First we write the equations:

$$2x + y = 25$$

$$3x - y = 20$$

Now we add the two equations to eliminate the  $y$ -variable.

$$\begin{array}{r} 2x + y = 25 \\ + \quad 3x - y = 20 \\ \hline 5x + 0 = 45 \end{array}$$

$$5x = 45$$

$$\frac{5x}{5} = \frac{45}{5}$$

$$x = 9$$

Now we substitute the answer for  $x$  in one of the two equations and solve for  $y$ .

$$2x + y = 25$$

$$2(9) + y = 25$$

$$\begin{array}{r} 18 + y = 25 \\ -18 \quad -18 \\ \hline 0 + y = 7 \end{array}$$

$$y = 7$$

The solution of the system of equations is  $(9, 7)$ .

## Tips

Keep the following tips in mind when you work on the Self Check questions:

- To get opposite coefficients, subtract one equation from the other by multiplying one equation by  $-1$  and then adding the two equations together.
- Substitute the value of the first variable into one of the equations and solve for the second variable.
- When the  $y$  terms have opposite coefficients, you add the equations.

## Videos

- “Solving Systems of Equations Using Elimination (2)” (running time 4:46)
- “Solving Systems of Equations by Elimination” (running time 12:43)



Go to unit 3 in your course online to watch these videos.

**Practice**

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Complete the practice problems in Worksheet 3. After you have completed the worksheet, check your answers in the Unit 3 Worksheet Key (found in the Resources folder of the online course).



## 3.6: Solving Algebraically

Analyze and solve systems of equations algebraically.

Solving equations algebraically is done by using the two methods taught in units 3.4 and 3.5: substitution and elimination. There are times when one method would be better than the other. The use of substitution is best when one variable is solved for in one of the two equations. Elimination is better when the equations are in standard form ( $ax + by + c$ ).

### Number of Solutions

As you learned in earlier units, you need to pay attention to the number of solutions in a system of equations. When solving a system of equations, ask, “How many points do these equations have in common?”

1. **One point.** The solution is the point  $(x, y)$  that the two equations have in common.
2. **An infinite number of points.** If you get a true statement, then the two equations are the same equation and have every point in common. Answer: **Infinite number of solutions.**
3. **None.** If you get a false statement, then the two equations have the same slope but different  $y$ -intercepts. Their graphs will be parallel, so they will never have a point in common. Answer: **No solution.**

### Substitution

For a refresher on substitution, review unit 3.4; reread pages 243–46 in the *Glencoe Math* textbook if you have questions.

### Elimination

There are situations when you cannot eliminate a variable by simply adding or subtracting the equations. To eliminate a variable from the system of equations and find an equation that you can use to solve for one variable, you must have opposite coefficients for the same variable. In order to get opposite coefficients, you multiply one of the equations by an integer and then add the equations to eliminate a variable and find the solution.

### Example 1

$$\begin{aligned} 6x + 3y &= 0 \\ -3x &= 2y = 9 \end{aligned}$$

We will multiply the second equation by 2 so that the coefficient of  $x$  will be -6. All terms will be multiplied by 2, so the equation will be  $-6x - 4y = 18$ . Now we can add the two equations to eliminate the  $x$  variable.

$$\begin{array}{rcl} 6x + 3y &=& 0 \\ + -6 -4y &=& 18 \\ \hline -y &=& \frac{18}{-1} \\ -1 &=& -1 \\ y &=& -18 \end{array}$$

Now substitute in -18 for the  $y$ -value in one of the two equations. The following has substitution in the first equation.

$$\begin{array}{rcl} 6x + 3y &=& 0 \\ 6x + 3(-18) &=& 0 \\ 6x - 54 &=& 0 \\ +54 &+& 54 \\ \hline \frac{6x}{6} &=& \frac{54}{6} \\ x &=& 9 \end{array}$$

The solution of the system of equations is one point  $(9, -18)$ .

### Example 2

$$\begin{aligned} 3x + y &= -2 \\ -9x - 3y &= 12 \end{aligned}$$

We will multiply the first equation by 3 so that the coefficient of  $x$  will be 9. All terms will be multiplied by 3, so the equation will be  $9x + 3y = -6$ . Now we can add the two equations to eliminate the  $x$  variable.

$$\begin{array}{rcl} 9x + 3y &=& -6 \\ -9x - 3y &=& 12 \\ \hline 0 &\neq& 6 \end{array}$$

The resulting equation is a false statement, so the answer is “no solution.”

**Example 3**

$$12x - 8y = -24$$

$$-3x + 2y = 6$$

We will multiply the second equation by 4 so that the coefficient of  $x$  will be  $-12$ . All terms will be multiplied by 4, so the equation will be  $-12x + 8y = 24$ . Now we can add the two equations to eliminate the  $x$  variable.

$$\begin{array}{r} 21x - 8y = -24 \\ + \quad -21x + 8y = \quad 24 \\ \hline 0 + \quad 0 = \quad 0 \\ 0 = \quad 0 \end{array}$$

Since the result of adding these two equations gives us a true statement of  $0 = 0$ , the lines are the same; therefore, the solution is “infinite number of solutions.”

**Videos**

- “Solve Systems of Equations Using Elimination (3)” (running time 4:51)
- “Solve Systems of Equations Using Elimination (5)” (running time 3:58)

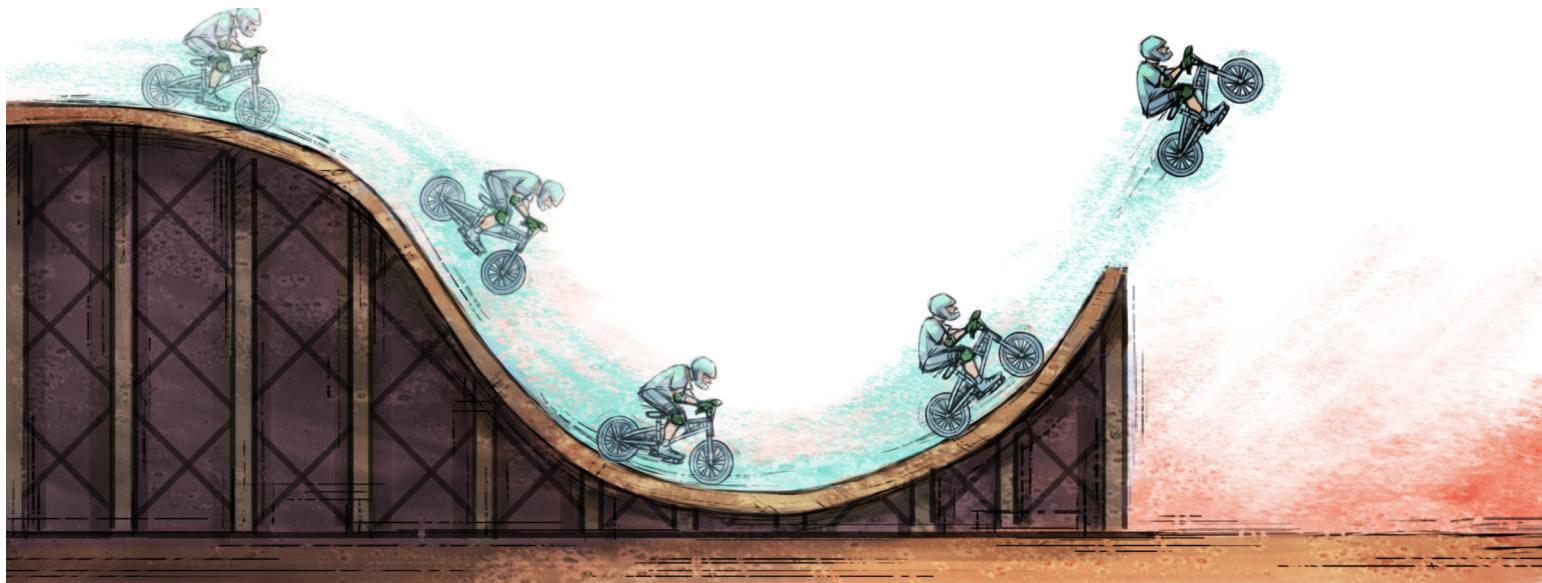
**Video**

Go to unit 3 in your course online to watch these videos.

**Practice**

Complete the practice problems in Worksheet 4. After you have completed the worksheet, check your answers in the Unit 3 Worksheet Key (found in the Resources folder of the online course).





## Unit 4

### Functions

**W**HAT IS A FUNCTION? WE WILL ANSWER THIS QUESTION AND TALK ABOUT OTHER ASPECTS OF functions in this unit. We will look at how we can represent functions, identify the types of functions, and construct functions in different representations. Once we have identified what functions look like in different forms (graphs, equations, numerically), we will identify different parts of a function and compare different functions with each other. This helps us to understand real-world situations such as which phone plan is better or which job offer will pay you more over time, as well as many other everyday situations. We will also interpret functions to give them meaning.

#### Learning Outcomes

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1. Identify the correct definition of a function.
2. Construct a function to model a linear relationship between two quantities.
3. Distinguish between linear and nonlinear functions; graph the functions correctly by type.
4. Compare properties of two functions when they are represented in different ways (algebraically, graphically, numerically, and verbally).
5. Interpret the functional relationship between two quantities by analyzing a graph.



## 4.1: Definition of Functions

*Identify the correct definition of a function.*

Functions are relations in which every member of the *domain* (input) is paired with exactly one member of the *range* (output). Functions can be represented by graphs, tables, ordered pairs, and equations. You will be writing equations at first, but this is just one way to express a function.

You will need to be able to determine whether a *relation* (set of ordered pairs) is a function. For example, if one person chooses two favorite colors, the relation is not a function because it is not a one-to-one relation.

Also, you will need to remember the difference between the dependent variable and the independent variable. The variable for the domain is the *independent variable* because it changes and can be any number; the variable for the range is the *dependent variable* because it depends on the other value.

### Reading Assignment

Read pages 267–272 and 285–290 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Understanding and Comparing Functions 1” (running time 5:15)
- “Understanding and Comparing Functions 2” (running time 5:09)
- “Understanding and Comparing Functions 3” (running time 3:43)



#### Video

Go to unit 4 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 273–276, problems 1–2, 7, 11, 12
- pages 285–286, problems 1–3

- pages 291–294, problems 1–3, 7, 9, 10, 12–16, 22–23

After you have completed these practice problems, check your answers in the Unit 4 Practice Key (found in the Resources folder of the online course).



## 4.2: Constructing Functions

*Construct a function to model a linear relationship between two quantities.*

Before you construct a function, you need to find the rate of change between two points. (Remember, the rate of change is  $y$  over  $x$ .) Then you find the *initial value* of the function, which is the corresponding  $y$ -value when  $x$  equals zero, or where the line crosses the  $y$ -axis. In other words, it is  $y$  minus the rate of change.

When you place ordered pairs in tables, the domain ( $x$ ) goes in the first column and the range ( $y$ ) goes in the second column. For graphing, the ordered pairs are placed on a coordinate plane;  $x$  is the horizontal line and  $y$  is the vertical line.

### Reading Assignment

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Read pages 277–286 and 319–322 in the *Glencoe Math* textbook. Carefully go through the examples.

### Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 281–284, problems 1, 2, 5, 9, 11, 13–15
- pages 323–326, problems 1–5, 9, 11, 13, 14

After you have completed these practice problems, check your answers in the Unit 4 Practice Key (found in the Resources folder of the online course).



## 4.3: Linear and Nonlinear Functions

*Distinguish between linear and nonlinear functions; graph the functions correctly by type.*

For a *linear function*, the graph forms a straight line. A linear function has an equation in slope-intercept form ( $y = mx + b$ ).

For a *nonlinear function*, the rate of change is not constant. That is, its graphs are not straight lines.

### Reading Assignment

Read pages 295–299 and 327–330 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Linear and Nonlinear Functions 1” (running time 3:34)
- “Linear and Nonlinear Functions 2” (running time 2:12)



**Video**

Go to unit 4 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 301–304, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21
- pages 331–334, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 4.4: Comparing Functions

Compare properties of two functions when they are represented in different ways (algebraically, graphically, numerically, and verbally).

Functions can be represented algebraically by an equation, graphically by a graph, numerically by a table, or verbally in words. You can compare two functions represented in different forms; the method you use will depend on the information you are given.

### Reading Assignment

Read pages 309–313 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Understanding and Comparing Functions 4” (running time 5:06)
- “Understanding and Comparing Functions 5” (running time 4:50)
- “Understanding and Comparing Functions 6” (running time 5:18)



Video

Go to unit 4 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 315–318, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 4.5: Interpreting Functions

*Interpret the functional relationship between two quantities by analyzing a graph.*

In this unit, you will analyze qualitative graphs to explain functional relationships. Sample graphs will be provided for you to interpret. *Qualitative graphs* are used to represent situations that may not show numerical values. That is, you must look at the lines to describe the situation or relationship. You will need to pay close attention to the shape and direction of the lines to interpret the graphs correctly.

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### Reading Assignment

Read pages 347–350 in the *Glencoe Math* textbook. Carefully go through the examples.

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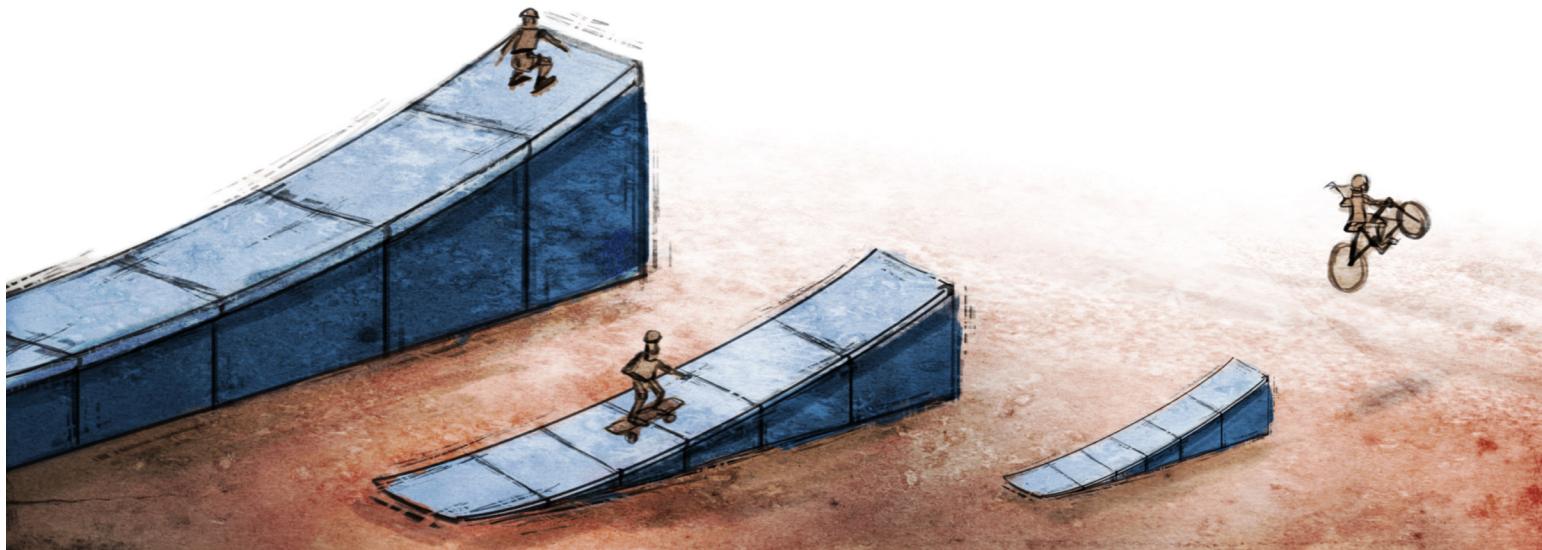
### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 351–354, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.





## Unit 5

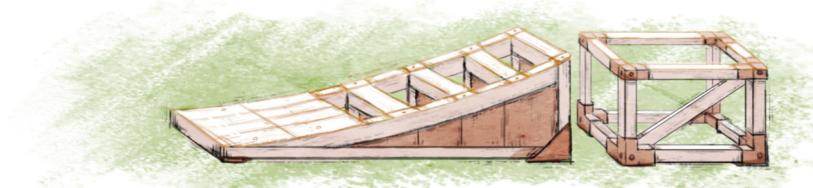
# Exponents

**I**N THIS UNIT YOU WILL LEARN ABOUT HOW TO WORK WITH EXPONENTS, INCLUDING WHAT THEY mean, how to use them, and how they can be helpful in expressing very large or small quantities.

### Learning Outcomes

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1. Identify the properties of integer exponents.
2. Multiply and divide mathematical expressions consisting of a single term (monomials).
3. Use powers of monomials to generate equivalent numerical expressions.
4. Use negative exponents to generate equivalent numerical expressions.
5. Use numbers expressed in scientific notation to estimate size and quantities of integers.
6. Perform operations with numbers expressed in scientific notation.



## 5.1: Powers and Exponents

*Identify the properties of integer exponents.*

A product of repeated factors can be shown as a *power*, which uses an exponent and a base. The *exponent* shows how many times the base number is multiplied by itself. A common mistake is to multiply the base number by the number of the exponent. The *base* is the common factor in a power. In  $10^3$ , the base is 10;  $10^3$  equals 10 times 10 times 10 (1,000), not 10 times 3 (30).

**Note:** Some Self Check questions will ask you to evaluate an expression, which means to find its value.

### Reading Assignment

Read pages 15–18 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Level 1 Exponents” (running time 9:46)
- “Understanding Exponents” (running time 1:54)



#### Video

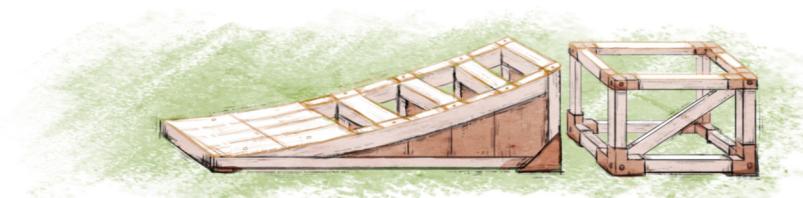
Go to unit 5 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 19–22, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 5.2: Multiplying and Dividing Monomials

*Multiply and divide mathematical expressions consisting of a single term (monomials).*

A *monomial* is a number, a variable, or a product of a number and one or more variables. To simplify monomials, you use the laws of exponents.

- to multiply powers with the same base, you add their exponents.
- to divide powers with the same base, you subtract their exponents.

A common mistake is multiplying the exponents instead of adding or dividing the exponents instead of subtracting. Also, remember to consider each base separately when multiplying or dividing monomials.

### Reading Assignment

Read pages 23–26 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Exponent Rules Part 1” (running time 9:42)
- “Perform Operations with Numbers Expressed in Scientific Notation, Including Decimals” (running time 4:06)



**Video**

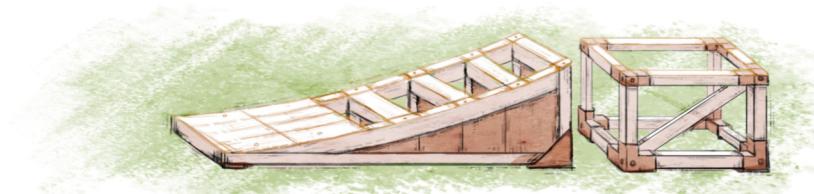
Go to unit 5 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 27–30, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 5.3: Powers of Monomials

*Use powers of monomials to generate equivalent numerical expressions.*

Another rule of exponents is used for finding the product of powers. Raising a power to a power means we multiply the exponents. Be careful not to confuse the rules for raising powers with those for multiplying monomials.

### Reading Assignment

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Read pages 31–34 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

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- “Exponent Rules 1” (running time 2:28)



**Video**

Go to unit 5 in your course online to watch this video.

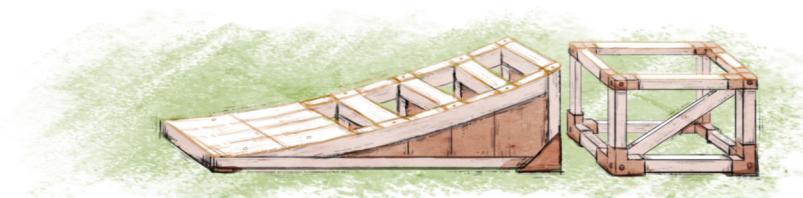
### Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 35–38, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 5.4: Negative Exponents

*Use negative exponents to generate equivalent numerical expressions.*

In this unit, you will work with negative exponents and simplify expressions with negative exponents. The rules for product of powers and quotient of powers can be used to multiply and divide powers with negative exponents.

### Tips

Here are a few things to remember about negative exponents:

- If you have a negative exponent in the numerator, you put that base in the denominator with a positive exponent.
- The sign of the base does not change when the exponent changes signs.

Remember this rule: any base raised to a negative power is actually 1 over that base raised to the positive power ( $3^{-2} = 1 \text{ over } 3^2 = 1 \text{ over } 9$ ).

### Reading Assignment

Read pages 43–46 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Negative Exponents” (running time 7:13)
- “Simplify Expressions Involving 0 and Negative Exponents (Part 1)” (running time 5:01)
- “Simplify Expressions Involving 0 and Negative Exponents (Part 2)” (running time 5:01)



Video

Go to unit 5 in your course online to watch these videos.

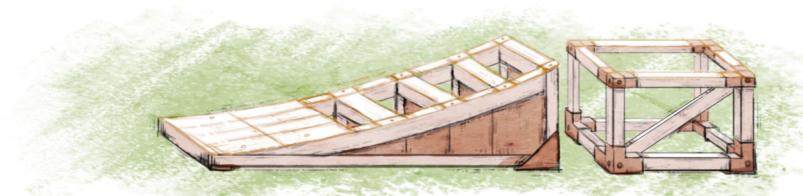
### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 47–50, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47

**Math 35: Eighth-Grade Mathematics, Part 1**

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 5.5: Scientific Notation

*Use numbers expressed in scientific notation to estimate size and quantities of integers.*

*Scientific notation* is a compact way of writing numbers with values that are very large or very small. For example, 5,500 is  $5.5 \times 10^3$  in scientific notation.

Use these rules to express a number in scientific notation:

- If the number is greater than or equal to 1, the power of 10 is positive.
- If the number is between 0 and 1, the power of 10 is negative.

You will also need to know how to convert scientific notation to standard form: multiplying by a positive power of 10 moves the decimal point to the right; multiplying by a negative power of 10 moves the decimal point to the left.

### Reading Assignment

Read pages 51–54 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Scientific Notation” (running time 11:25)
- “Convert from Standard to Scientific Notation” (running time 5:01)



**Video**

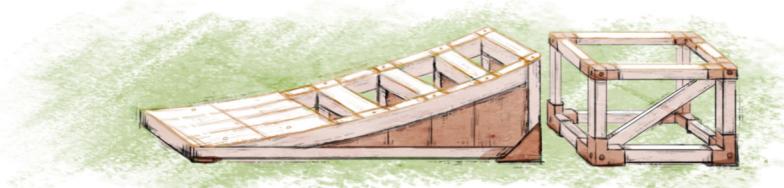
Go to unit 5 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 55–58, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 5.6: Computing

*Perform operations with numbers expressed in scientific notation.*

You need to line up the place values when you add or subtract decimals in standard form, but the exponent shows the place values in scientific notation. Both numbers must be expressed in the same form before you add or subtract.

**Note:** You will be asked to evaluate expressions in the Self Check, review, and unit questions. “Evaluating an expression” simply means finding its value.

### Reading Assignment

Read pages 59–62 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Perform Operations with Numbers Expressed in Scientific Notation, Including Decimals” (running time 3:34)
- “Scientific Notation 2” (running time 3:58)



**Video**

Go to unit 5 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 63–66, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## Unit 6

### The Number System

**I**N THIS UNIT YOU WILL LEARN ABOUT RATIONAL AND IRRATIONAL NUMBERS. WE WILL WORK WITH these numbers to determine their value, approximate irrational numbers, and compare rational and irrational numbers. We will work with roots to identify what they mean and what properties they contain.

#### **Learning Outcomes**

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1. Identify rational and irrational numbers.
2. Convert rational numbers to decimals and convert decimals to rational numbers.
3. Identify the definition and properties of roots.
4. Simplify square roots and cube roots to solve equations.
5. Use rational approximations of irrational numbers to estimate the value of expressions.
6. Order rational approximations of irrational numbers to compare the size of irrational numbers.



## 6.1: The Number System

*Identify rational and irrational numbers.*

To know how to work with rational and irrational numbers, we must first identify what they are. In this unit we will talk about what different number sets we deal with and how they are similar and different.

### Natural Numbers

The smallest set of numbers is what we call the *natural numbers*. These numbers are the first numbers we learned, and we often use them to count something like how many jelly beans we have in our hand: 1, 2, 3, and so on. This set is also called the *counting numbers*.

### Whole Numbers

The set of whole numbers includes the natural numbers, but it also includes 0.

### Integers

Notice that as we continue to add more sets, the sets previously mentioned are a part of the next group. The integer set includes all numbers from the natural and whole number sets. The integer group also adds negative natural numbers. This means that the numbers included in this set would be  $\dots -3, -2, -1, 0, 1, 2, 3, \dots$  and so on in both directions.

### Rational Numbers

This set includes all the other sets above plus fractions, decimals, percents, and mixed numbers. The best way to check for a rational number is to see if you can write it as a fraction. When you look at a number as a decimal and it repeats or terminates (ends), it is a rational number.

### Real Numbers

This set includes all the sets of numbers above plus a set called the irrational numbers. This set includes all numbers that we will deal with in this course.

### Irrational Numbers

This set of numbers includes those numbers that will not fit into any of the other number sets except real numbers. An irrational number expressed as a decimal will continue forever without repeating. The most common numbers in this set are pi and the roots of prime numbers.

In the following examples, we will determine which number sets each number belongs to.

### Example 1

$$\begin{array}{r} -43 \\ \hline -55 \end{array}$$

This number is a fraction. Numbers that can be written as a fraction are included in the rational numbers set and real numbers set. This number cannot be simplified, so it is not a natural number, whole number, or integer.

### Example 2

$$\frac{10}{5}$$

The number should be simplified if possible to find what number sets it fits in. This fraction can be simplified by dividing both the numerator and denominator by 5. The result would be 2. The resulting 2 is in all number sets except the irrational numbers.

### Example 3

$$4.5698202\dots$$

This number continues on and does not repeat, so it cannot be included in the rational numbers set. It will be included only in the real numbers and irrational numbers sets.

## Reading Assignment

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The diagram on page 90 in the *Glencoe Math* textbook will help in understanding how all the number sets relate.

## Videos

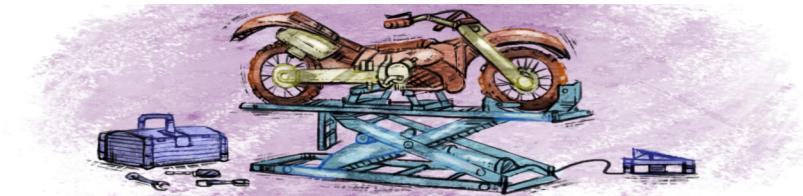
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- “Introduction to Rational and Irrational Numbers” (running time 5:54)
- “Identifying Rational Numbers” (running time 5:46)
- “Distinguish Between Rational and Irrational Numbers” (running time 5:00)



**Video**

Go to unit 6 in your course online to watch these videos.



## 6.2: Converting Rational Numbers

*Convert rational numbers to decimals and convert decimals to rational numbers.*

A *rational number* is a number that can be written as the ratio of two integers (that is, it can be expressed as a fraction) in which the denominator is not zero.

Every rational number can be expressed as a decimal by dividing the numerator by the denominator.

*Bar notation* can be used to indicate repeating digits. A bar is placed above the repeating part of the number.

### Reading Assignment

Read pages 7–10 in the *Glencoe Math* textbook. Carefully go through the examples.

### Video

- “Converting Fractions to Decimals Example” (running time 2:16)



**Video**

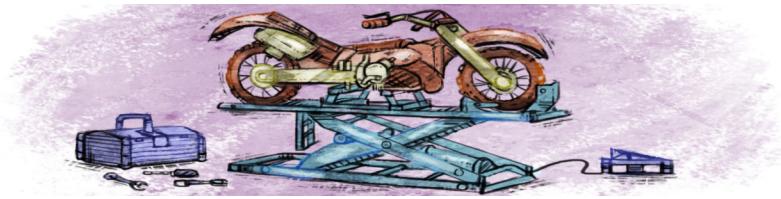
Go to unit 6 in your course online to watch this video.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 11–14, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 6.3: Introduction to Roots

*Identify the definition and properties of roots.*

A *square root* of a number is one of its two equal factors. Numbers such as 1, 4, and 9 are called perfect squares because they are squares of integers.

A *cube root* of a number is one of its three equal factors. Numbers such as 8, 27, and 64 are perfect cubes because they are the cubes of integers.

It is helpful to know all the perfect squares through 15 and perfect cubes through 10 to make it easier to solve problems dealing with *radicals* (square roots and cube roots.)

PERFECT SQUARES	PERFECT CUBES
$1^2 = 1$	$1^3 = 1$
$2^2 = 4$	$2^3 = 8$
$3^2 = 9$	$3^3 = 27$
$4^2 = 16$	$4^3 = 64$
$5^2 = 25$	$5^3 = 125$
$6^2 = 36$	$6^3 = 216$
$7^2 = 49$	$7^3 = 343$
$8^2 = 64$	$8^3 = 512$
$9^2 = 81$	$9^3 = 729$
$10^2 = 100$	$10^3 = 1000$
$11^2 = 121$	
$12^2 = 144$	
$13^2 = 169$	
$14^2 = 196$	
$15^2 = 225$	

### Reading Assignment

Read pages 71–74 in the *Glencoe Math* textbook. Carefully go through the examples.

## Video

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- “Understanding Square Roots” (running time 1:20)



**Video**

Go to unit 6 in your course online to watch this video.

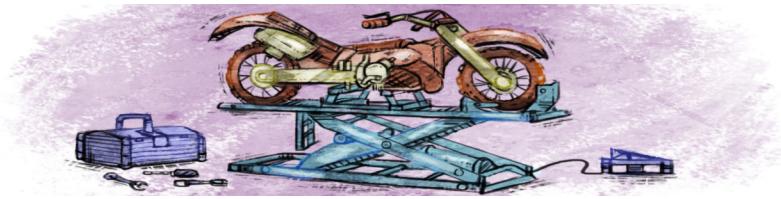
## Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 75–76, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 6.4: Simplifying Roots

*Simplify square roots and cube roots to solve equations.*

This unit continues from 6.3 on using square roots and cube roots. Simplifying is used to solve equations. Here are some tips for simplifying:

- To undo a variable squared, take the square root of both sides.
- To undo a variable cubed, take the cube root of both sides.
- Taking the cube root does not undo a square root.
- Cubing both sides does not undo a square root.

Also, you will need to know how to find the perimeter of a shape given its area. Remember that squaring an area does not provide the perimeter of a shape.

It is helpful to know all the perfect squares through 15 and perfect cubes through 10 to make it easier to solve problems dealing with square roots and cube roots.

PERFECT SQUARES	PERFECT CUBES
$1^2 = 1$	$1^3 = 1$
$2^2 = 4$	$2^3 = 8$
$3^2 = 9$	$3^3 = 27$
$4^2 = 16$	$4^3 = 64$
$5^2 = 25$	$5^3 = 125$
$6^2 = 36$	$6^3 = 216$
$7^2 = 49$	$7^3 = 343$
$8^2 = 64$	$8^3 = 512$
$9^2 = 81$	$9^3 = 729$
$10^2 = 100$	$10^3 = 1000$
$11^2 = 121$	
$12^2 = 144$	
$13^2 = 169$	
$14^2 = 196$	
$15^2 = 225$	

## **Math 35: Eighth-Grade Mathematics, Part 1**

### **Reading Assignment**

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Review pages 71–74 in the *Glencoe Math* textbook. Carefully go through the examples.

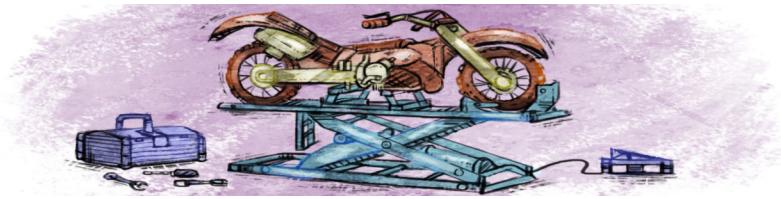
### **Practice**

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 77–78, problems 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 6.5: Estimating Value of Irrational Numbers

*Use rational approximations of irrational numbers to estimate the value of expressions.*

When numbers are not perfect squares, you must estimate the square root with approximations. You estimate the value of an irrational number by finding the best rational approximation.

Start by truncating, which is dropping the digits after the first decimal place, then dropping the digits after the second decimal place, and so on until the closest approximation is reached. Also, you can use the nearest perfect square to help find the best approximation.

### Reading Assignment

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Read pages 81–84 in the *Glencoe Math* textbook. Carefully go through the examples.

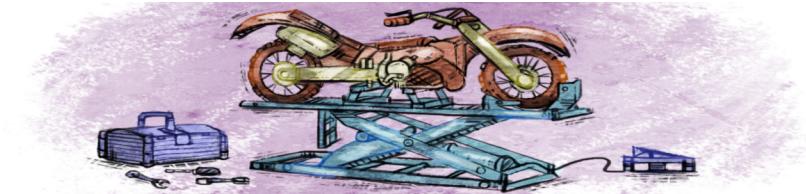
### Practice

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Work through the selected problems in the *Glencoe Math* textbook:

- pages 85–88, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.



## 6.6: Ordering Rational and Irrational Numbers

*Use approximations of irrational numbers to compare rational and irrational numbers.*

In review, numbers that are not rational are called irrational numbers. Rational numbers include integers, whole numbers, and natural numbers. Irrational numbers include pi, decimals that do not terminate or repeat, and some roots (when the square root is not the root of a perfect square). The set of *real numbers* includes the set of rational numbers and the set of irrational numbers.

You can compare and order rational and irrational numbers by writing them in decimal form. Approximations are used for irrational numbers.

### Reading Assignment

Read pages 89–92 in the *Glencoe Math* textbook. Carefully go through the examples.

### Videos

- “Comparing Rational Numbers” (running time 6:58)
- “Comparing Decimals 4” (running time 2:07)
- “Compare Irrational and Rational Numbers” (running time 5:00)



#### Video

Go to unit 6 in your course online to watch these videos.

### Practice

Work through the selected problems in the *Glencoe Math* textbook:

- pages 93–96, problems 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43

After you have completed these practice problems, check your answers in the key at the back of the *Glencoe Math* textbook.