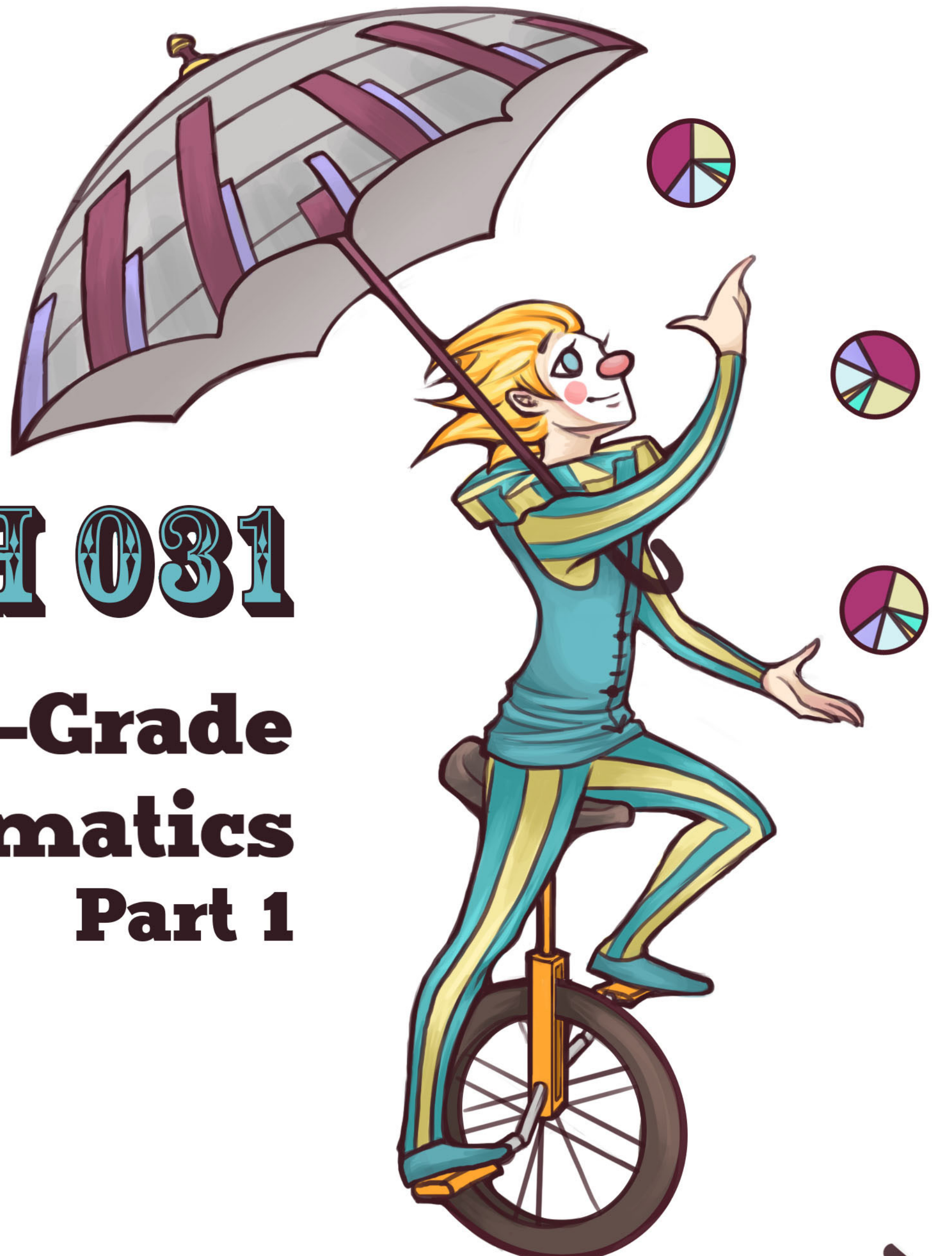
MATH 031

Seventh-Grade Mathematics
Part 1



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MATHEMATICS 031—SEVENTH-GRADE MATHEMATICS, PART 1
Section M002 / .5 Semester hours
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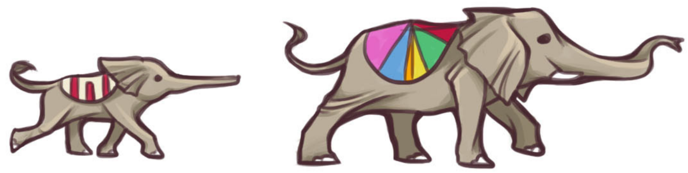

Introduction to This Reading Packet

Thank you for purchasing the lesson packet for MATH-031, Seventh-Grade Mathematics, Part 1. These items are included in this packet:

- This introductory letter
- A copy of the readings from each lesson (the pages following this letter)

Begin your online course by reading the syllabus; it contains the information you need to successfully complete the course. As you begin, you will notice that each lesson includes a brief introduction, learning outcomes for the lesson, and lesson reading material. The reading material for each lesson is included in this packet, as well as in the online course.

This packet is designed to give you the best experience for reading the detailed lesson content and taking notes. To complete the course, you will need access to the online assignments and exams.

Best wishes for your success in this course!

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Unit 1
Representing Data

In this lesson you will learn various ways to represent data. You will learn how to read and make graphs such as line graphs, bar graphs, and circle graphs. You will learn how to plot data values on a number line. You will also learn how to organize data in tables, spreadsheets, and frequency tables. Lastly, you will learn how to calculate data averages known as mean, median, and mode.

Learning Outcomes

1. Organize data in tables and spreadsheets.
2. Organize data in a line graph and a bar graph.
3. Organize data in a circle graph.
4. Organize data in frequency tables and line plots.
5. Find mean, median, and mode, and decide which average is most appropriate for a given situation.
6. Use random sampling to draw valid inferences about populations.

1.1: Data Tables and Spreadsheets

Organize data in tables and spreadsheets.

Suppose a manager of a music store wants to find out which type of compact disc is most popular. He decides to itemize his sales totals for several months by category. In the month of September, the music store sells 75 country, 45 rap, 100 rock, and 150 pop compact discs. In October, the store sells 100 country, 30 rap, 125 rock, and 200 pop compact discs. Lastly, in November the store sells 90 country, 35 rap, 115 rock, and 225 pop compact discs. The easiest way to organize the data is in a table.

Compact Disc Sales				
	Country	Rap	Rock	Pop
September	75	45	100	150
October	100	30	125	200
November	90	35	115	225

Notice the table is organized by month and by type of compact disc sold. This makes it very easy to read. You can quickly find the most popular type and least popular type of compact disc sold. The most popular type of compact disc is pop, whereas the least popular type is rap.

Another way to organize data is in a spreadsheet. A *spreadsheet* is a table in a computer application. Each individual entry in a spreadsheet is referred to as a *cell*. The cell is a box on the spreadsheet where the row and column meet. We identify cells by letter and number. Notice the previous table as shown in the following spreadsheet:

	A	B	C	D	E
1		Country	Rap	Rock	Pop
2	September	75	45	100	150
3	October	100	30	125	200
4	November	90	35	115	225

The most popular type of compact disc sold for the month of October is in cell E3. The least popular type of compact disc sold for the month of November is in cell C4. B2 contains the number of country compact discs sold in the month of September. These are examples of valuable information that can be represented on a table or a spreadsheet.

Example 1

Create a table to represent the scores of the BYU home football games: 45–28, 36–40, 32–15, 25–21, 42–35, and 21–14.

1. Decide how to organize the table. Figure out the number of rows and columns and their labels.

Game #	BYU	OTHER
1		
2		
3		
4		
5		
6		

2. Fill in the data values.

GAME #	BYU	OTHER
1	45	28
2	36	40
3	32	15
4	25	21
5	42	35
6	21	14

3. Add a title to the table.

BYU Home Game Football Scores		
Game #	BYU	OTHER
1	45	28
2	36	40
3	32	15
4	25	21
5	42	35
6	21	14

Example 2

Create a spreadsheet to represent the high and low temperatures for the month of December for the following cities: Salt Lake City 52–28, Provo 48–32, Ogden 45–25, Logan 42–21, Cedar City 55–35, and St. George 58–37.

1. Decide how to organize the spreadsheet. Figure out the number of rows and columns the table needs, as well as their labels. Remember to add the column of numbers and row of letters.

	A	B	C
1	City	High	Low
2	Salt Lake City		
3	Provo		
4	Ogden		
5	Logan		
6	Cedar City		
7	St. George		

2. Fill in the data values.

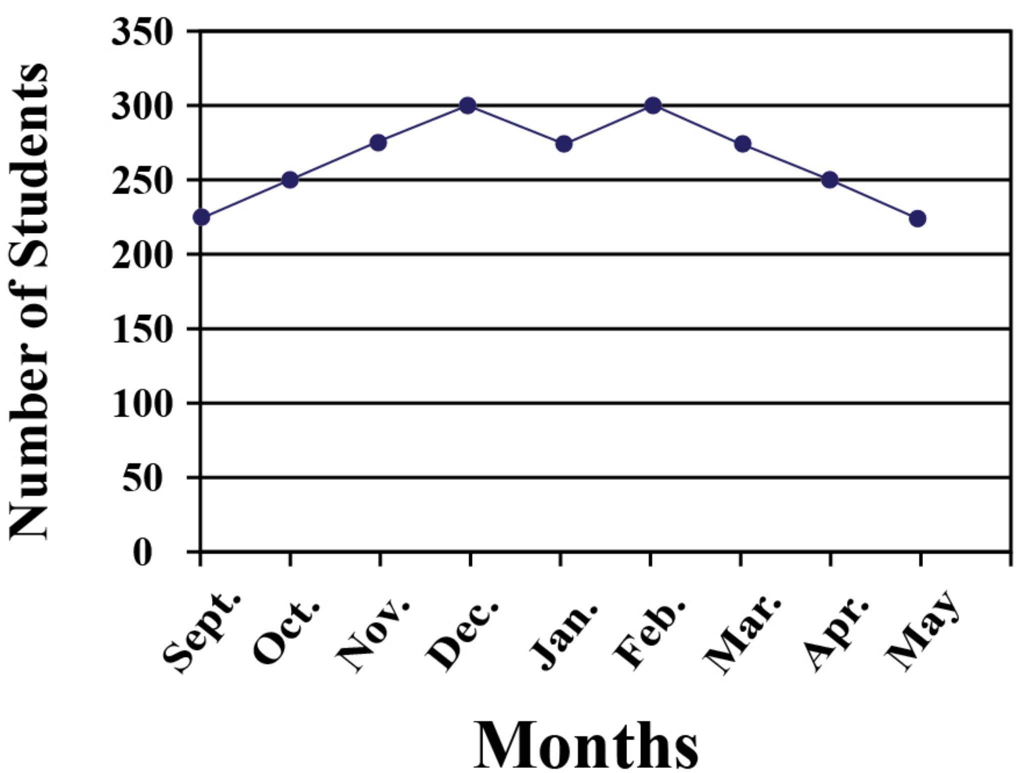
	A	B	C
1	City	High	Low
2	Salt Lake City	52	28
3	Provo	48	32
4	Ogden	45	25
5	Logan	42	21
6	Cedar City	55	35
7	St. George	58	37

1.2: Line and Bar Graphs

Organize data in a line graph and a bar graph.

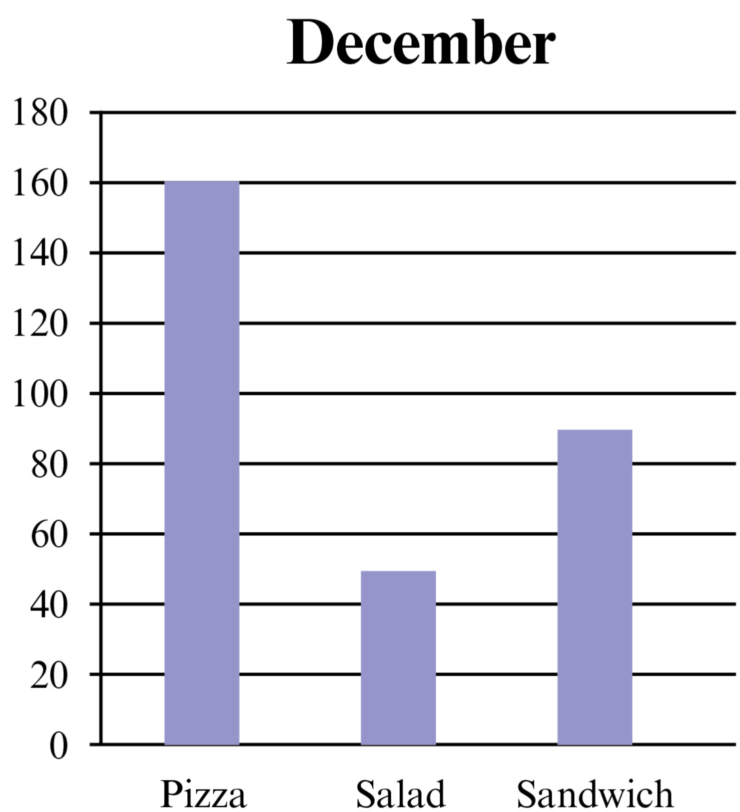
Suppose a school wanted to keep track of the number of students purchasing school lunch each month throughout the school year. We could represent this data on a line graph. A *line graph* is a graph used to show change over a period of time.

Students Purchasing School Lunches								
Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
225	250	275	300	275	300	275	250	225



Notice we can easily see the change in the number of students purchasing school lunch over the course of the school year. The number of students buying lunch increased as the year progressed and then decreased toward the year's end. We can see that the months when school lunch was the most popular were December and February, whereas September and May were unpopular months for school lunch.

Suppose the school wanted to keep track of the number of students purchasing pizza, salad, or a sandwich for a specific month. One way to compare the number of each lunch type is to create a bar graph. A bar graph is used to compare amounts. A bar graph is similar to a line graph because each data value is represented on the graph. However, the values on a bar graph are represented with a bar rather than a point. Another difference is that a bar graph compares amounts, whereas a line graph shows change over time. Let's look at the following bar graph.



Notice that the number of sandwiches purchased is almost double the number of salads purchased. Also, pizza was the most popular lunch choice in the month of December. The comparison is easy to see on this type of graph.

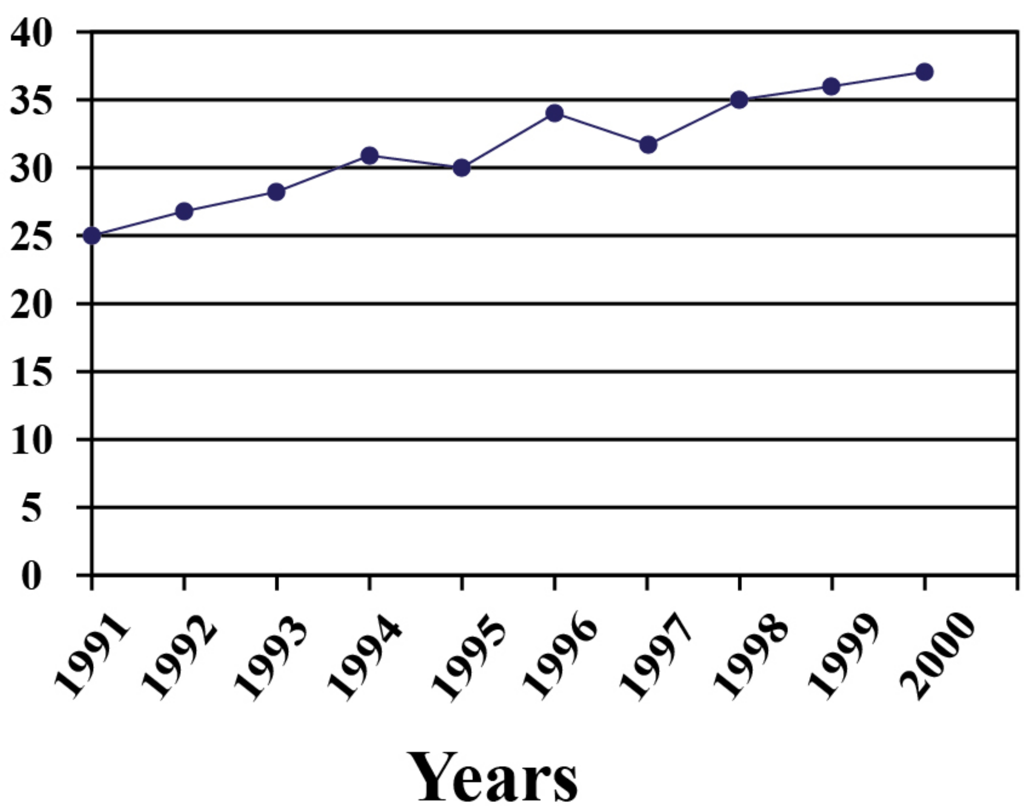
Example 1

Create a line graph for the number of registered voters in Utah County for the years from 1991 to 2000.

Registered Voters in Utah County (in thousands)									
1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
25	27	28	31	30	34	32	35	36	37

1. Draw the axes. Write the years on evenly spaced intervals on the horizontal axis.
2. Choose a scale for the vertical axis. The graph goes from 25 to 37. Draw and label a scale from 0 to 40, using intervals of 5.
3. Place a point on the graph for the registered voters for each year. Connect the points.
4. Label the vertical and horizontal axes. Put the title on the graph.

Registered Voters in Utah County (in thousands)



Notice we can see the trend for that decade on the line graph above. The increasing number of registered voters in Utah County is displayed by the increasing line. Let's suppose we wanted to further categorize this data. Maybe we wanted to compare the number of registered voters with the number of unregistered voters in Utah County for that same time period. We can do this by creating a *double line graph*. A double line graph has two data sets represented by two separate lines.

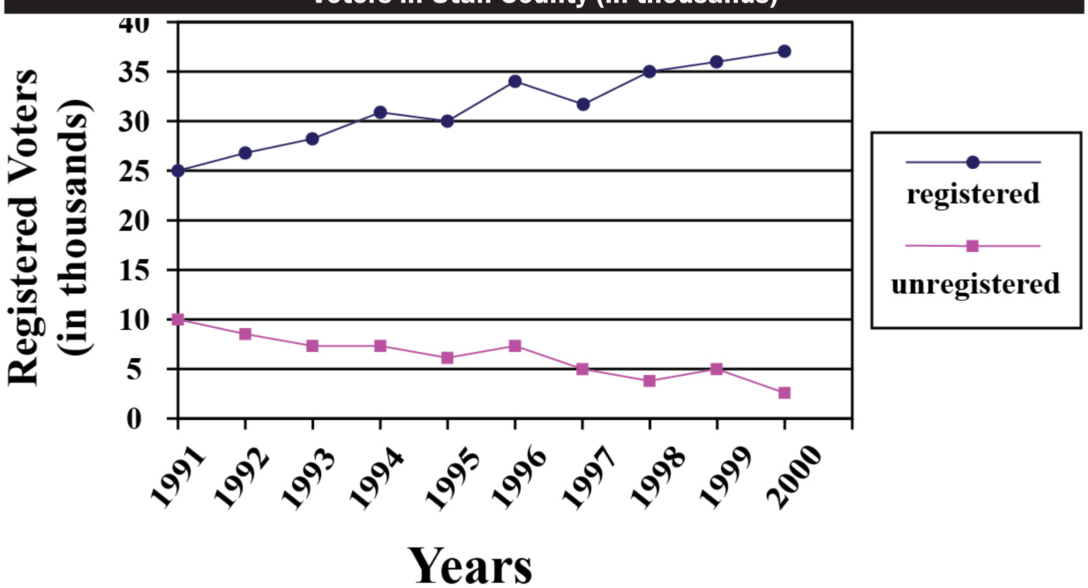
Example 2

Create a line graph for the number of registered voters in Utah County for the years from 1991 to 2000. (The second row of the table represents the number of registered voters, and the last row is the number of unregistered voters.)

Voters in Utah County (in thousands)									
1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
25	27	28	31	30	34	32	35	36	37
10	8	7	7	6	7	5	4	5	3

1. Draw the axes. Write the years on evenly spaced intervals on the horizontal axis.
2. Choose a scale for the vertical axis. The graph goes from 3 to 37. Draw and label a scale from 0 to 40, using intervals of 5.
3. Place a point on the graph for the registered voters for each year. Connect the points. Now place points on the graph for the unregistered voters and connect these dots.
4. Label the vertical and horizontal axes. Title the graph.

Voters in Utah County (in thousands)



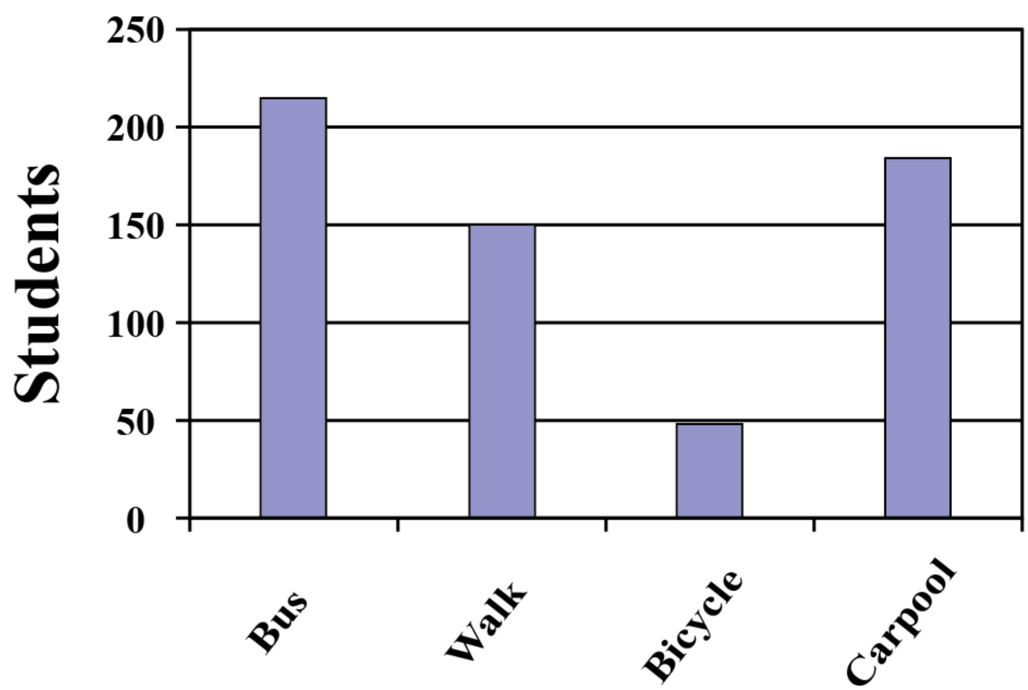
Example 3

Create a bar graph showing the transportation method students use to get to school.

Students' Method of Transportation			
Bus	Walk	Bicycle	Carpool
212	150	48	182

1. Draw the axes. Write the name of each transportation method on the horizontal axis.
2. Choose a scale for the vertical axis. The graph goes from 48 to 212. Draw and label a scale from 0 to 250, using intervals of 50.
3. Draw a bar to show the number of students using each transportation method.
4. Label the vertical and horizontal axes. Give your bar graph a title.

Students' Method of Transportation



Notice we can easily compare the number of students using each method of transportation by looking at the graph. Suppose we wanted to break the comparison down even further by comparing the elementary students' mode of transportation with the middle school students' transportation method. We can do this by creating a double bar graph. A *double bar graph* has two data sets represented by two separate bars.

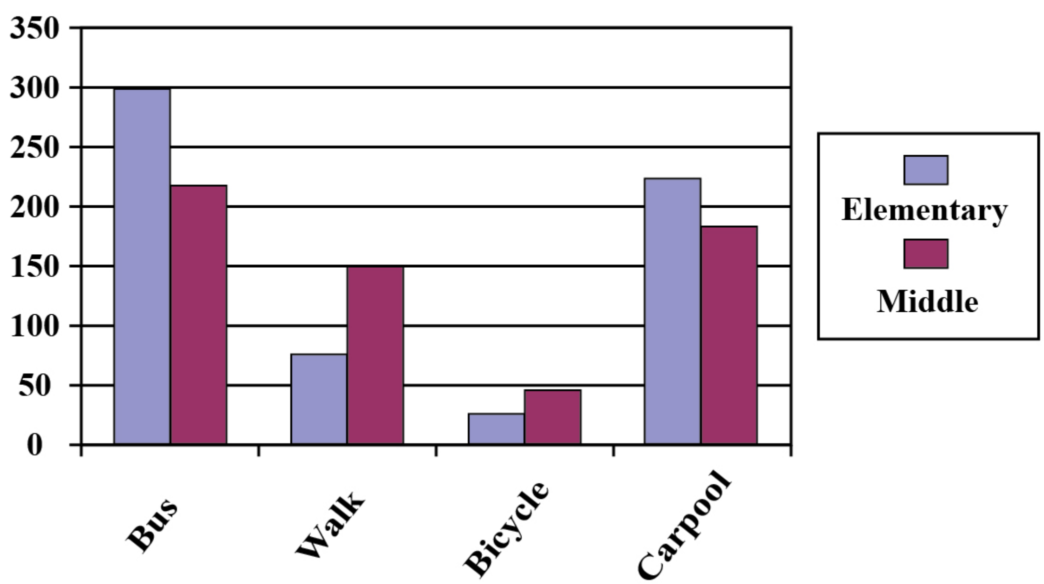
Example 4

Create a bar graph showing the transportation method middle school and elementary students use to get to school.

Students' Method of Transportation				
School	Bus	Walk	Bicycle	Carpool
Elementary	300	75	25	225
Middle	212	150	48	182

1. Draw the axes. Write the name of each transportation method on the horizontal axis.
2. Choose a scale for the vertical axis. The graph goes from 48 to 300. Draw and label a scale from 0 to 300, using intervals of 50.
3. Draw a bar to show the number of students using each transportation method.
4. Label the vertical and horizontal axes. Give your bar graph a title.

Students' Method of Transportation

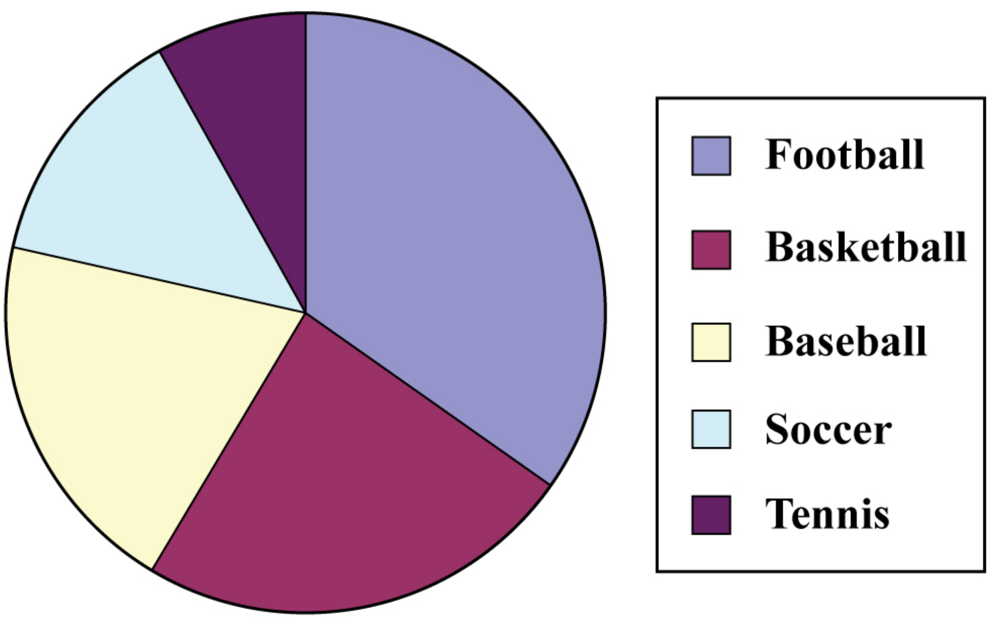


1.3: Circle Graphs

Organize data in a circle graph.

Suppose you conducted a survey of 50 students to find out their favorite sport. If you discovered 18 students prefer football, 12 students prefer basketball, 9 prefer baseball, 7 prefer soccer, and 4 prefer tennis, you could represent this data on a circle graph. A *circle graph* is a visual representation of the data which compares parts to the whole. Each pie piece in a circle graph represents a part of the whole circle. The circle graph below shows a visual representation of the survey results.

Favorite Sports



Notice that on the circle graph we can easily compare values. There are twice as many students who prefer football as baseball, and three times as many students who prefer basketball as there are students who prefer tennis. Let's try creating a circle graph.

Example 1

Create a circle graph showing the number of students who study a foreign language in school if 85 students study Spanish, 55 students study French, 35 students study German, and 25 students study Japanese.

Students Studying a Foreign Language			
Spanish	French	German	Japanese
85	55	35	25

1. Calculate the total number of students.

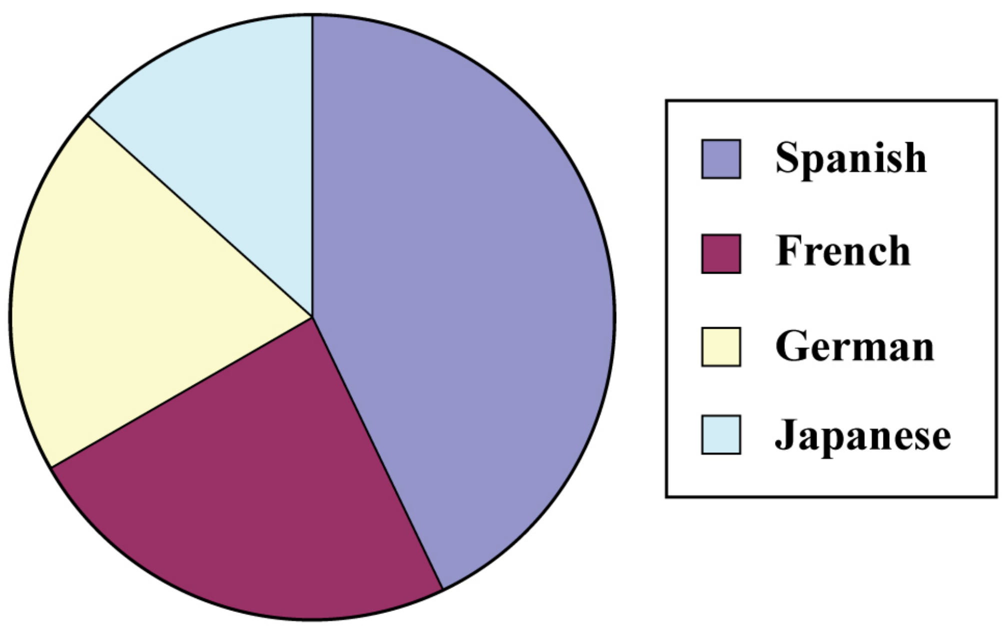
   85 + 55 + 35 + 25 = 200 students

2. Divide each category by the total number of students. Multiply the quotient by 360°. This is the number of degrees in the circle for each category.

85 ÷ 200 = .425	.425 × 360° = 153°
55 ÷ 200 = .275	.275 × 360° = 99°
35 ÷ 200 = .175	.175 × 360° = 63°
25 ÷ 200 = .125	.125 × 360° = 45°

3. Using a protractor, graph each category on the circle. Start with the largest category and end with the smallest one.
4. Label the categories or make a key for the graph. Give your circle graph a title.

Students Studying a Foreign Language



Example 2

Create a circle graph showing the number of students participating in extracurricular activities if 40 students are members of the drama club, 30 students are members of the math club, 20 students are members of the science club, and 10 students are members of the chess club.

Students in Extracurricular Activities			
Drama	Math	Science	Chess
40	30	20	10

1. Calculate the total number of students.

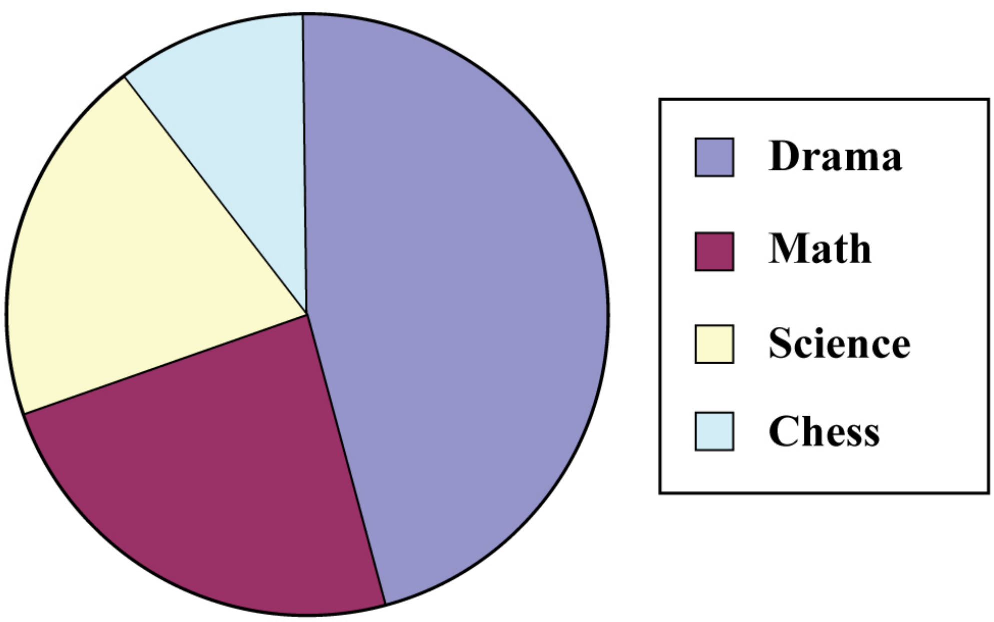
   40 + 30 + 20 + 10 = 100 students

2. Divide each category by the total number of students. Multiply the quotient by 360°. This is the number of degrees in the circle for each category.

40 ÷ 100 = .4	.4 × 360° = 144°
30 ÷ 100 = .3	.3 × 360° = 108°
20 ÷ 100 = .2	.2 × 360° = 72°
10 ÷ 100 = .1	.1 × 360° = 36°

3. Using a protractor, graph each category on the circle. Start with the largest category and end with the smallest one.
4. Label the categories or make a key for the graph. Give your circle graph a title.

Students in Extracurricular Activities



1.4: Frequency Tables and Line Plots

Organize data in frequency tables and line plots.

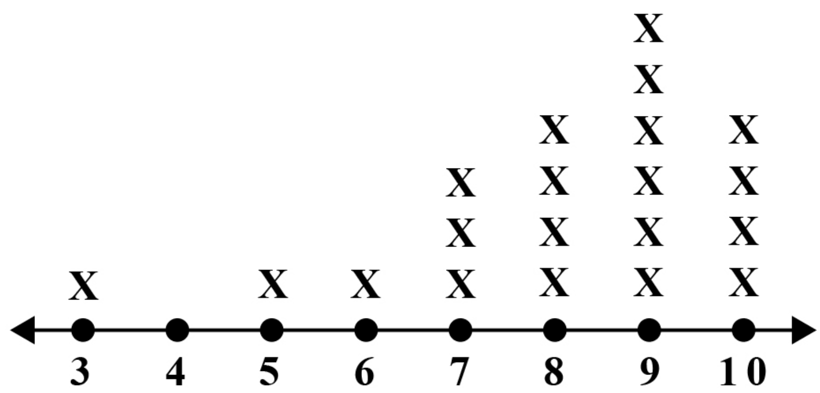
Suppose a teacher gave a 10-point quiz, and the students' scores were as follows: 10, 8, 6, 9, 10, 9, 9, 8, 8, 7, 8, 5, 10, 9, 3, 7, 9, 7, 9, and 10. There are many ways the teacher could represent the students' quiz scores. One way to represent this data is a frequency table. A *frequency table* shows how often a result occurs. Another way to represent data is a line plot. A *line plot* is a numerical plot where data is recorded on a number line. When making a frequency table, you first need to list each type of outcome. Next you need to tally the number of results for each outcome. Lastly, you find the sum of the tally marks and record it as your frequency for each type of outcome. Let's look at the frequency table for the example above, and notice how the teacher organized the data.

Quiz Scores

SCORE	TALLY	FREQUENCY
3	I	1
5	I	1
6	I	1
7	III	3
8	IIII	4
9	IIIIII	6
10	IIII	4

When making a line plot, the first step is to find the data *range*. The range is the difference in the high and low values of data. Notice that in the example above, the highest quiz score was 10, whereas the lowest quiz score was 3. The range of the data would be 7. We need to draw a number line between the highest and lowest value. Next, we put an X at each value on the number line. The frequency table above helps you to decide the number of X's for each value. Last, we put a title on the line plot. Let's look at the line plot for the example above.

Quiz Scores



A frequency table and a line plot help you to see the least-occurring data entry and the most-occurring data entry. In the example above, the least-occurring quiz scores were 3, 5, and 6, while the most-occurring quiz score was 9. Let's try an example together.

Example 1

Create a frequency table and line plot for following students' heights in inches: 65, 60, 63, 62, 67, 60, 65, 66, 65, 62, 66, 65, 62, 66, and 64.

Create a Frequency Table

1. Create a table with all the heights' outcomes.
2. Tally the number of people represented by each height.
3. Find the sum of each outcome to find the frequency of each height.
4. Label the frequency table.

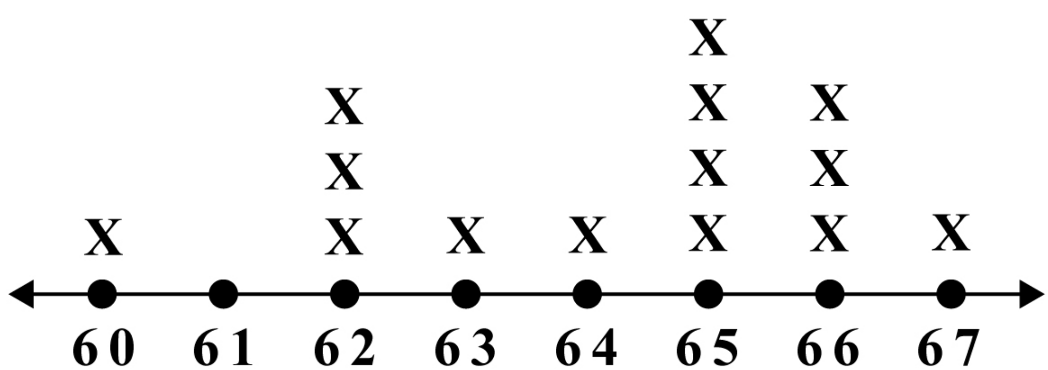
Students' Height (in inches)

HEIGHT	TALLY	FREQUENCY
60	II	2
62	III	3
63	I	1
64	I	1
65	IIII	4
66	III	3
67	I	1

Create a Line Plot

1. Find the data range. The students' heights were between 60 and 67 inches, so the range is 7 inches.
2. Draw a number line between the highest and lowest value. Since the shortest person is 60 inches, and the tallest person is 67 inches, you need a number line from 60 to 67.
3. Put an X at each height on the number line. Use the frequency table to determine the number of X's for each height.
4. Label the line plot.

Students' Height (in inches)



In the example above, the least occurring heights were 63, 64 and 67, while the most occurring height was 65. Let's try another example.

Example 2

Create a frequency table and line plot for the high temperatures in the month of January: 32°, 30°, 28°, 25°, 27°, 28°, 29°, 28°, 29°, 30°, 31°, 32°, 28°, 28°, 27°, 25°, 27°, 29°, 30°, 32°, 30°, 28°, 27°, 28°, 29°, 30°, 31°, 31°, 29°, 28°, and 29°.

Create a Frequency Table

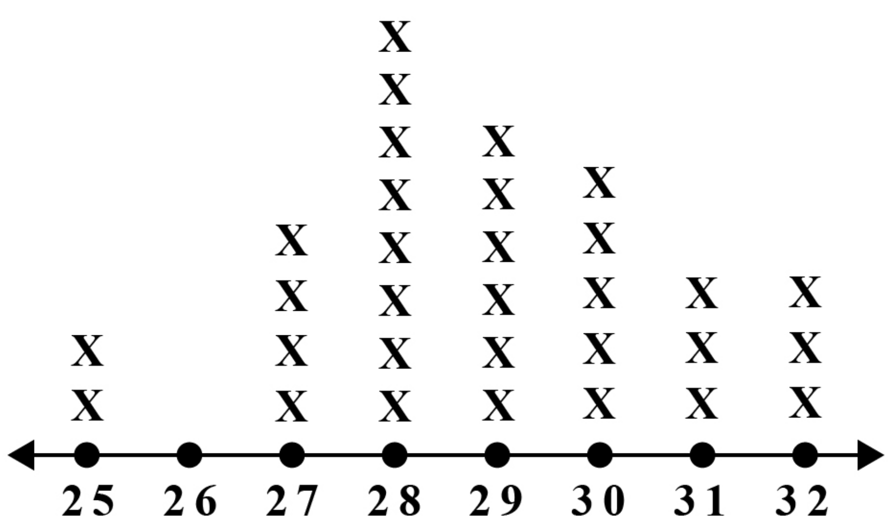
1. Create a table with all the temperatures that were outcomes.
2. Tally the occurrences of each temperature.
3. Find the sum of each outcome to find the frequency of each temperature.
4. Label the frequency table.

Temperature Highs (in degrees Fahrenheit)		
TEMPERATURE	TALLY	FREQUENCY
25	II	2
27	IIIII	4
28	IIIIIIII	8
29	IIIIII	6
30	IIIII	5
31	III	3
32	III	3

Create a Line Plot

1. Find the data range. The temperatures were between 25 and 32, so the range is 7 degrees.
2. Draw a number line between the highest and lowest values. Since the lowest temperature is 25 degrees and the highest temperature is 32 degrees, you need a number line from 25 to 32.
3. Put an X at each temperature on the number line. Use the frequency table to determine the number of X's for each temperature.
4. Label the line plot.

Temperature Highs (in degrees Fahrenheit)



In the previous example, the most infrequent temperature was 25°, while the most frequent temperature was 28°.

1.5: Mean, Median, and Mode

Find mean, median, and mode, and decide which average is most appropriate for a given situation.

Suppose a teacher asked her class to record the average number of minutes they spent reading each day. The results were 30, 35, 15, 20, 30, 25, 15, 10, 30, 45, 20, 30, 25, 30, and 15 minutes. We could find three measures of central tendency that are representative of this set of data. These measures are the *mean*, *median*, and *mode*. The *mean* is the average of the values; this can be calculated by dividing the sum of the values by the total number of values. The *median* is the middle value when the data is organized from least to greatest. Lastly, the *mode* is the most frequent value, or the value that occurs the most. Let's find the mean, median, and mode for the values above.

$$\textbf{Mean} = \frac{\textbf{sum of values}}{\textbf{number of values}}$$

$$= \frac{30+35+15+20+30+25+15+10+30+45+20+30+25+30+15}{15}$$

$$= \frac{375}{15}$$

$= 25$

Median = middle value (ranked least to greatest)

= 10, 15, 15, 15, 20, 20, 25, 25, 30, 30, 30, 30, 30, 35, 45

= 25

**Notice that the middle value could be between two other values. If this is the case, take the average or mean of the two values.

Mode = most frequent value

= 10, 15, 15, 15, 20, 20, 25, 25, 30, 30, 30, 30, 30, 35, 45

= 30

**Notice that there could be more than one mode. Also notice that if none of the values are more frequent than other values, then there is no mode.

The measures of central tendency indicate that the average number of minutes students spent reading each day and the middle value was 25 minutes. The most occurring number of minutes students spent reading each day was 30 minutes.

Example 1

Find the mean, median, and mode of the following values: 3, 9, 5, 7, 4, 3, 9, 10, 3, 1, 3, 5, and 3.

$$\textbf{Mean} = \frac{\textbf{sum of values}}{\textbf{number of values}}$$

$$= \frac{3 + 9 + 5 + 7 + 4 + 3 + 9 + 10 + 3 + 1 + 3 + 5 + 3}{13}$$

$$= \frac{65}{13}$$

= 5

Median = middle value (ranked least to greatest)

= 1, 3, 3, 3, 3, 3, 4, 5, 5, 7, 9, 9, 10

= 4

Mode = most frequent value

= 1, 3, 3, 3, 3, 3, 4, 5, 5, 7, 9, 9, 10

= 3

Example 2

Find the mean, median, and mode of the following values: 38, 54, 40, 32, 48, and 52.

$$\textbf{Mean} = \frac{\textbf{sum of values}}{\textbf{number of values}}$$

$$= \frac{38 + 54 + 40 + 32 + 48 + 52}{6}$$

$$= \frac{264}{6}$$

= 44

Median = middle value (ranked least to greatest)

= 32, 38, 40, 48, 52, 54

$$= \frac{40 + 48}{2}$$

= 44

Mode = most frequent value

= 32, 38, 40, 48, 52, 54

= no mode because each data value occurs once

1.6: Statistics

Use random sampling to draw valid inferences about populations.

Statistics is the science that deals with collecting, analyzing, and presenting numerical data; it is useful for drawing conclusions from a sample. A *random sample* contains objects or individuals chosen in a way that gives each one an equal chance of being selected; a *valid inference* is one in which you can correctly draw a conclusion; and a *population* is the specific group of objects or individuals being studied.

Assume that you want to know if males or females at your school have longer first names. You know you won't be able to ask all males and females their name, so you set up a survey to collect some data. You ask 50 males and 50 females, "How many letters are in your first name?" Your survey must be random for this sample data to represent the whole population, so you ask the students this question as they enter the lunchroom.

After collecting the data, you compute the mean for the male names and the mean for the female names. You want to know if your data is consistent, so you also compute the variability or mean absolute deviation. This number shows the average distance a data point is from the actual mean of all the data. The closer the variability is to 0, the more consistent the data is.

You find that the average length of the male names is 5 letters long with a variability of 1 letter, and the average length of the female names is 6 letters long with a variability of 1 letter. You conclude that, on average, female names have one more letter than male names. The variability of each type of name is 1, so the random sample is consistent and is a good representation of the population.

You make the following *inference*, or conclusion: At your school, females have longer names than males.

Example 1

To decide if teenagers or adults go swimming more often, use the question, "How many times per month do you go swimming?"

1. Decide where, how, and when you might ask this question to collect a random sample of the population of your city.

The most likely choice would be to do a survey at your city pool for a week. You could leave a short survey that asks the patrons if they are older or younger than 20 and how many times they went swimming in the past month.

2. Organize the data by age, with the mean and the variability.

AGE	19 OR YOUNGER	20 OR OLDER
Mean	12	5
Variability	2	4

3. Make comparisons about the data and list an inference about your original question.
   - ◇ The mean number of times going swimming is 7 more times for the younger group than the older group.
   - ◇ The variability of the older group is twice the variability of the younger group.
   - ◇ Inference: In your city, teenagers go swimming more often than adults.

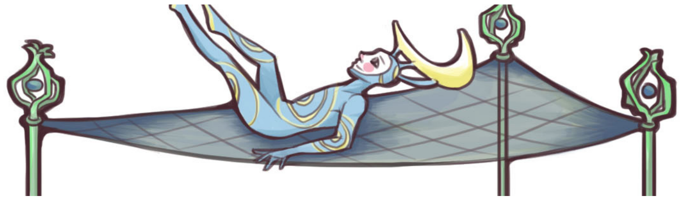
4. Questions to consider:
   - ◇ Would you get the same results in winter as you would in summer?
   - ◇ Would you get the same results in Alaska as you would in Hawaii?

Unit 2
Probability Models

In this lesson you will learn how to find the *probability*, or likelihood, that an event will occur. You will learn how to collect data from experiments and probability models to give the probability of a certain outcome and make predictions that an outcome will happen. You will also learn how to find the probability and make predictions when dealing with a compound event or more than one event at a time.

Learning Outcomes

1. Identify the probability of a chance event.
2. Collect data and predict the relative frequency, given the probability.
3. Use a probability model to find probabilities of events.
4. Identify probabilities of compound events.
5. Identify sample spaces for compound events and the outcome of the events.
6. Use simulation to identify frequencies for compound events.



2.1: Chance Processes

Identify the probability of a chance event.

Have you ever heard statement like the following?

- There is an 80% chance of rain tonight.
- There is a 1/6 chance that you will roll a 4 when rolling a six-sided die.
- There is a 0.5 chance you will get heads when flipping a coin.

All of these statements are explaining the probability or likelihood that the given event will occur. Probabilities are recorded as a number between 0 and 1 as a fraction, decimal, or a percent as shown in the statements. If an event has a probability of 0 then the event is impossible or will never occur. If an event has a probability of 1 then the event is certain or will always occur. All other probabilities are given a likelihood of occurring based on how close they are to 0, ½, or 1.

The first statement indicates that it is more likely that it will rain than not rain because 80% is closer to 100% than to 0%. The second statement indicates that rolling a 4 is more likely not to happen than to happen because 1/6 is closer to 0 than to 1. The third statement indicates that the likelihood of flipping a head is equally likely and unlikely because 0.5 is in the middle of 0 and 1.

Example 1

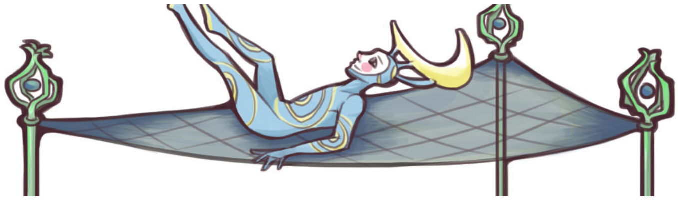
Using a six-sided number cube, think of an event that would be impossible, unlikely, equally unlikely as likely, likely, and an event that is certain.

- It would be impossible to roll the number 7 because the number cube doesn't contain a 7.
- It would be unlikely to roll the number 1 because there is only a 1 out of 6 chance that it will land on 1.
- It would be equally likely and unlikely to roll an odd because there are three odds and three evens on a number cube.
- It would be likely that you roll any number other than 1 because there is a 5 out of 6 chance it won't land on 1.
- It is certain that you will roll a number between 1 and 6 because those are the only six numbers on the number cube.

Example 2

Using a spinner that contains 26 sections with each of the letters in the English alphabet, think of an event that would be impossible, unlikely, equally unlikely as likely, likely, and an event that is certain.

- It would be impossible to spin "ñ" because this is part of the Spanish alphabet, not the English alphabet.
- It would be unlikely to spin a vowel because only 5 out of the 26 letters in the English alphabet are vowels.
- It would be equally unlikely and likely to spin a letter between A and M because this represents the first half of the English alphabet.
- It would be likely to spin a consonant because 21 out of the 26 letters in the English alphabet are consonants.
- It would be certain that you spin a letter in the English alphabet because that is all that the spinner contains.



2.2: Predictions

Collect data and predict the relative frequency, given the probability.

Suppose you want to find out if there really is a 50 percent chance of getting heads when flipping a coin. This is the *theoretical probability* because there should be an equally likely chance of getting a heads or a tails. To find the theoretical probability, you take each possible outcome and divide by the total number of possible outcomes.

You conduct an experiment by flipping a coin 25 times and keeping track of the outcomes.

Trial	1	2	3	4	5
Outcome	H	H	H	T	T

6	7	8	9	10	11	12	13	14	15
H	H	T	H	H	H	H	T	H	H

16	17	18	19	20	21	22	23	24	25
H	T	T	H	H	T	T	H	H	H

You use your experiment to find the *experimental probability* of getting a heads when flipping a coin. To find the experimental probability, you take the outcome, based on the experiment, and divide by the number of times you conducted the experiment. For this experiment you flipped a heads 17 out of 25 times or 68 percent of the time.

Because of the discrepancy, you decide to try the experiment again and flip the coin another 25 times and record the outcomes.

Trial	1	2	3	4	5
Outcome	T	H	T	T	T

6	7	8	9	10	11	12	13	14	15
H	T	T	T	T	T	T	T	H	H

16	17	18	19	20	21	22	23	24	25
H	T	T	T	H	H	T	H	T	H

The experimental probability of getting heads, based on this experiment, is 9 out of 25 or 36 percent. However, if you combine the outcomes of both experiments, you can find the *relative frequency*, or how often something happens when divided by all outcomes. To find the relative frequency, you take the total number of times the event occurs and divide by the total number of times you conducted the experiment. The relative frequency is 26 out of 50 or 52 percent.

As you continue doing the experiment, you are able to predict the long-term relative frequency. The longer you do the experiment, the closer the long-term relative frequency should get to the theoretical probability. If you flip the coin 1,000 times, you should get heads about 500 times.

Example 1

Jordan creates a deck of cards that has 40 cards, 4 cards each of the numbers 1 through 10. What is the theoretical probability that Jordan will draw a seven from the deck of cards?

There are 4 sevens in the deck of 40 cards, so the probability that he draws a seven is 4 out of 40 or 10 percent of the time.

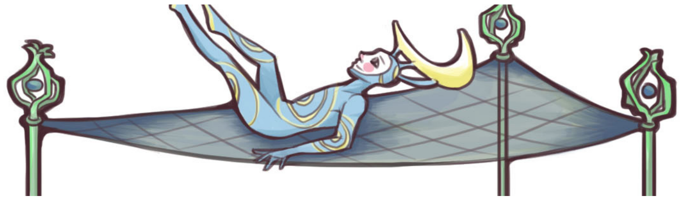
Jordan draws a card, records the outcome, puts the card back in the deck, draws another card, records the outcome, puts the card back in the deck, and continues this process 10 times. The table lists his outcomes. What is the experimental probability that he will draw a seven?

1	2	3	4	5	6	7	8	9	10
10	1	10	2	6	8	8	4	7	1

He got a seven once out of the 10 trials, so the experimental probability is 1 out of 10 or 10 percent of the time.

What would be the long-term relative frequency that Jordan would draw a seven if he continues the process 100 times?

Both the theoretical and experimental probabilities are 10 percent, so he should draw 10 sevens in 100 tries.



2.3: Probability of Events

Use a probability model to find probabilities of events.

Suppose you want to figure out if your basketball team will beat the top-ranked team tonight in the tournament. Based on the theoretical probability that there are two teams, your team has a 50% chance of winning. The problem is that it is hard to make a prediction based on this type of theoretical probability because there are other factors that need to be considered, such as skill level and teamwork of both teams.

Instead we will look at a probability model that will help make a more accurate prediction. For example, if you have played this team 10 times before and have only won twice, the relative frequency gives a probability of 2/10 or 20%. You can then make the prediction that it is unlikely that your team will win the game, but there is still a small chance that your team will win.

Example 1

You have been fishing several times at the local fishing pond over the last couple of months. You decide to make a probability model based on the last 12 times you went fishing at the pond. The table represents the results of your recent fishing outings. What is the probability that you will catch a fish? What is the probability that you will catch 4 or more fish?

Number of Times Fishing	1	2	3	4	5	6	7	8	9	10	11	12
Number of Fish Caught	2	0	5	3	3	1	4	0	2	3	5	1

The theoretical probability is that there is a 50-50 chance because there are only two options; you will catch a fish or you won't catch a fish.

Based on your model, the experimental probability shows that during your last 12 visits there were only 2 times that you didn't catch a fish, so the probability that you will catch a fish is 10/12 or about 83%. Of those same 12 visits, there were only 3 times that you caught 4 or more fish, so the probability of you catching 4 or more fish is 3/12 or 25%.

It is likely that you will catch a fish today, but it is unlikely that you will catch 4 or more fish today. Factors that could lead to the discrepancies between the theoretical and experimental probabilities are the time of day that you go fishing, how many people are fishing at the pond, and whether fish were recently added to the pond.

Example 2

You have 8 blue shirts (B), 3 white shirts (W), 5 red shirts (R), and 4 pink shirts (P) for a total of 20 shirts in your closet. Use the probability model that lists the color of shirt you wore each day for the last two weeks to predict what color of shirt you will wear tomorrow.

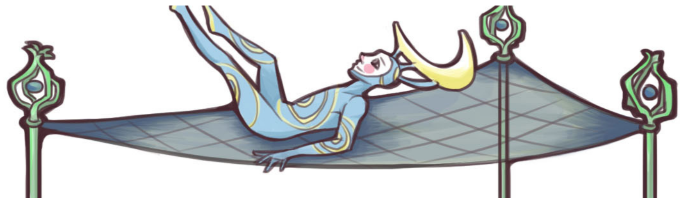
Day of the Week	S	M	T	W	T	F	S
Color of Shirt	R	B	R	B	B	R	W

Day of the Week	S	M	T	W	T	F	S
Color of Shirt	R	R	B	B	W	B	P

Based on the theoretical probability, there is an 8/20 or 40% chance you will wear a blue shirt, a 3/20 or 15% chance you will wear a white shirt, a 5/20 or 25% chance you will wear a red shirt, and a 4/20 or 20% chance you will wear a pink shirt.

Based on the experimental probability from your model, you wore a blue shirt 6 out of 14 times or about 43% of the time. You wore a white shirt 3 out of 14 times or about 14% of the time. You wore a red shirt 5 out of 14 times or about 36% of the time. You wore a pink shirt 1 out of 14 times or about 7% of the time.

It is most likely that you will wear a blue or red shirt tomorrow and least likely that you will wear a pink shirt tomorrow. Any discrepancies between the probabilities could be due to which shirts are clean and the mood you are in that day.



2.4: Compound Events

Identify probabilities of compound events.

Berkley learned a new game called "Flip and Roll." In the game you earn points by flipping a coin and rolling a number cube. Each combination earns a different amount of points, with the highest number of points being awarded to flipping tails and rolling a five.

What is the probability that Berkley will flip tails on the coin and roll a five on the number cube? To answer this question, we must find the probability of a *compound event* because there are two things involved at the same time. Each part of the game, flipping a coin and rolling a number cube, are simple events if done individually. However, combining both parts of the game to get one score makes it a compound event.

Example 1

What is the probability of flipping tails on the coin and roll a five on the number cube?

To find the compound probability, you find the simple probability of each event and then multiply those probabilities together.

1. The probability of flipping tails on the coin is ½.
2. The probability of rolling a five on the number cube is 1/6.
3. The probability of flipping tails on the coin and rolling a five on the number cube is (1/2)(1/6) = 1/12.

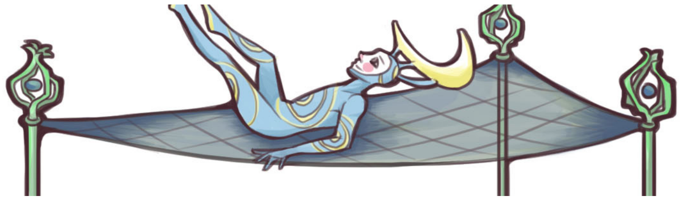
There is a 1/12 chance or about an 8% chance that Berkley will get the high points on her turn.

Example 2

Brandon needs to buy a new car, but he is unsure what type of car he wants to buy. He knows that his choices are between a 2-door car and a 4-door car. Also, the color of car has to be red, black, or blue. What is the probability that Brandon will buy a 2-door, black-colored car?

1. The probability that Brandon chooses a 2-door car is ½.
2. The probability that Brandon chooses a black-colored car is 1/3.
3. The probability that Brandon buys a 2-door, black-colored car is (1/2)(1/3) = 1/6.

There is a 1/6 chance or about a 17% chance that Brandon will buy a 2-door, black-colored car.



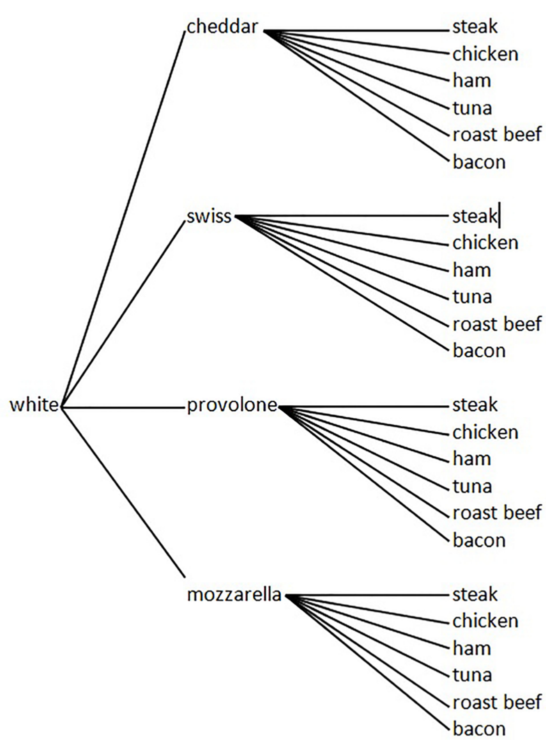
2.5: Sample Space

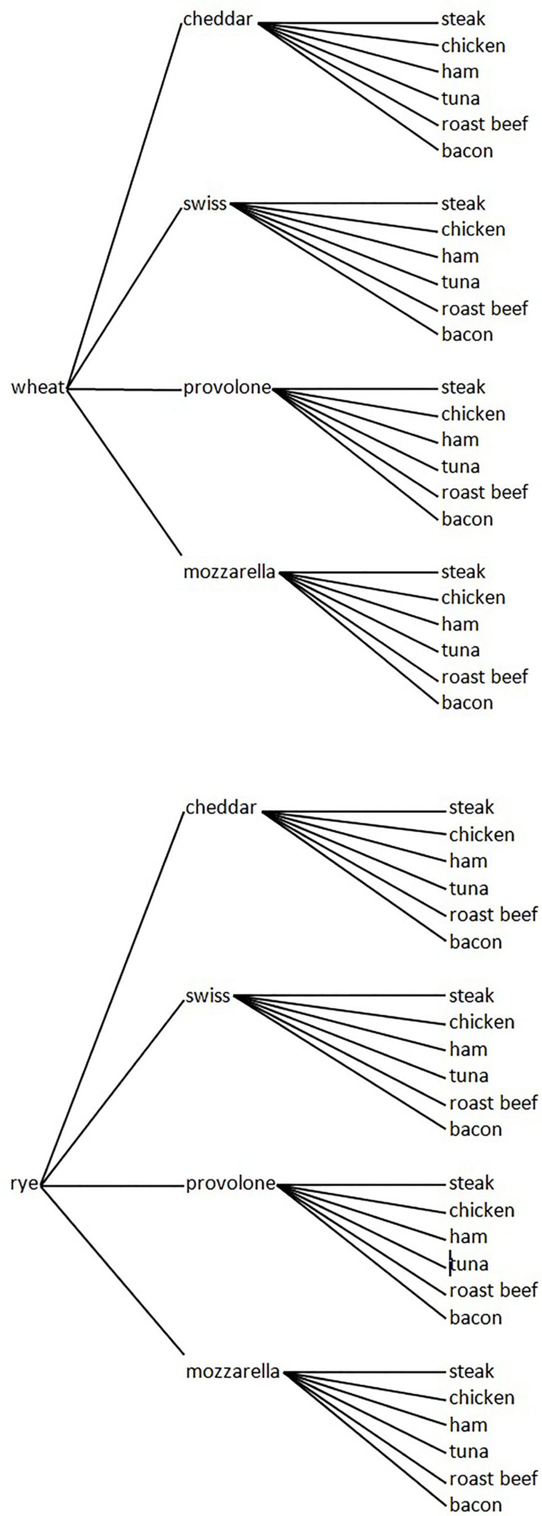
Identify sample spaces for compound events and the outcomes of the events.

The *sample space* of an experiment is the set of all possible outcomes of that experiment. Small numbers of outcomes are often grouped in parentheses (1, 2, 3, and 4) or shown in a simple table; large numbers of outcomes with compound events are usually shown in a tree diagram.

Suppose you want to know how many different types of sandwiches can be made at your local deli shop. You have 3 choices of bread: white, wheat, or rye. You have 4 choices of cheese: cheddar, swiss, provolone, or mozzarella. You have 6 choices of meat: steak, chicken, ham, tuna, roast beef, or bacon.

You can use a tree diagram to show the different types of sandwiches you can make.




wheat
cheddar
steak
chicken
ham
tuna
roast beef
bacon
swiss
steak
chicken
ham
tuna
roast beef
bacon
provolone
steak
chicken
ham
tuna
roast beef
bacon
mozzarella
steak
chicken
ham
tuna
roast beef
bacon
rye
cheddar
steak
chicken
ham
tuna
roast beef
bacon
swiss
steak
chicken
ham
tuna
roast beef
bacon
provolone
steak
chicken
ham
tuna
roast beef
bacon
mozzarella
steak
chicken
ham
tuna
roast beef
bacon

If you choose one type of bread, one type of cheese, and one type of meat, there are 72 different types of sandwiches.

What is the probability of someone ordering a ham and cheddar on wheat bread sandwich? This is a compound event because there are three choices to make in order to make the sandwich.

The probability would be (1/3)(1/4)(1/6) = 1/72, or about 1%.

Example 1

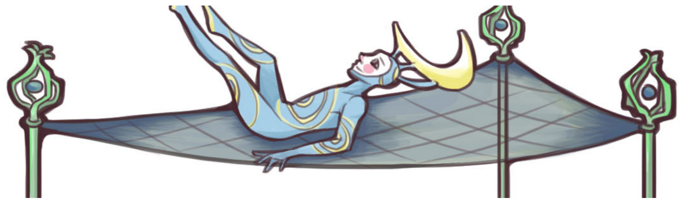
You are signed up to take math and English next year at school. There are 6 teachers that teach English: Miss Anderson, Mr. Croft, Miss King, Mrs. Lindy, Mr. Solomon, and Mr. White. There are 8 teachers that teach math: Miss Butler, Mrs. Cannon, Mr. Fairbanks, Mr. Lopez, Mr. Matthews, Miss Salisbury, Mr. Trudy, and Mr. Waters. What is the probability that your two teachers will have last names that start with the same initials?

Another way to organize the data is in a table that lists each of the possibilities.

	English					
Math	A & B	C & B	K & B	L & B	S & B	W & B
	A & C	**C & C**	K & C	L & C	S & C	W & C
	A & F	C & F	K & F	L & F	S & F	W & F
	A & L	C & L	K & L	**L & L**	S & L	W & L
	A & M	C & M	K & M	L & M	S & M	W & M
	A & S	C & S	K & S	L & S	**S & S**	W & S
	A & T	C & T	K & T	L & T	S & T	W & T
	A & W	C & W	K & W	L & W	S & W	**W & W**

There are 48 combinations of English and math teachers. There are 4 combinations that have the same initials for their last name: Mr. Croft and Mrs. Cannon, Mrs. Lindy and Mr. Lopez, Mr. Solomon and Miss Salisbury, and Mr. White and Mr. Waters.

The probability of having teachers with the same initial for the last name is 4/48, or about 8%.



2.6: Frequencies

Use a simulation to identify frequencies for compound events.

The cafeteria at your dad's work just got a new soda machine, called "Random Cola." When you insert your money into the machine it will randomly give you 1 of 6 different variations of cola: cola, diet cola, vanilla cola, cherry cola, lime cola, or lemon cola. You want to know how many times your dad will have to purchase a cola (the frequency) before he gets one of each type of cola.

One way to figure out the frequencies without actually putting all your money in the cola machine is by setting up a simulation. A *simulation* is a way to model real-life events using random samples.

Because there are 6 different types of cola, you decide to roll a six-sided number cube to simulate buying the colas. You assign 1 as cola, 2 as diet cola, 3 as vanilla cola, 4 as cherry cola, 5 as lime cola, and 6 as lemon cola. You roll the number cube and tally each cola until you have at least one of each type. You decide to do the simulation four times to look at the different frequencies. Here is the data for the simulations.

Simulation	1	2	3	4
1 – Cola	3	3	1	1
2 – Diet cola	3	1	1	7
3 – Vanilla cola	1	2	2	2
4 – Cherry cola	2	5	1	5
5 – Lime cola	1	2	3	4
6 – Lemon cola	3	1	1	6
Number of rolls to get each type of soda	13	14	9	25

The four frequencies range from 9 to 25, meaning your dad would have to buy at least 9 colas and at most 25 colas before he has tried each variation. On average, he would have to buy 15 colas before he has tried each variation.

Example 1

Your dad prefers the flavored colas over the non-flavored colas. If there are 4 out of 6 colas that are flavored, or 2/3, what is the probability that your dad would only have to buy 2 colas in order to get a flavored cola?

You do another simulation, with the number cube, but this time you keep track of the colas he receives from the machine.

Simulation 1: 3 and 5, vanilla and lime cola
Simulation 2: 4 and 5, cherry and lime cola
Simulation 3: 3 and 2, vanilla and diet cola
Simulation 4: 1 and 2, cola and diet cola
Simulation 5: 1 and 1, cola and cola
Simulation 6: 6 and 5, lemon and lime cola
Simulation 7: 6 and 4, lemon and cherry cola
Simulation 8: 2 and 1, diet and cola
Simulation 9: 4 and 1, cherry and cola
Simulation 10: 2 and 2, diet and diet cola

Six of the ten simulations had at least one cola that was flavored. Based on this simulation, the experimental probability that he only has to buy two sodas in order to get a flavored soda is 6/10 or 60% of the time.

Example 2

Sabina did some research and found that 40% of the population uses Mobile Nation as the cell phone provider instead of any other provider. What is the probability that she would only have to ask four people in order to find one that has Mobile Nation as the provider?

Sabina creates a simulation using 10 colored cards; she has 4 red cards to represent those who use Mobile Nation (R) and 6 black cards to represent those who use another provider (B). She will shuffle the cards and randomly draw out cards, looking to see if she has at least one red card out of the four.

Simulation	1	2	3	4	5	6	7	8	9	10	11	12
	B	B	B	B	R	R	B	B	B	R	R	B
	B	R	B	R	R	B	R	B	R	R	B	B
	B	B	B	R	B	B	B	B	B	B	B	B
	B	R	B	R	R	R	B	B	R	R	B	B

Eight out of the twelve simulations had at least one red card in the four cards chosen. Based on this simulation, the experimental probability is 8/12, or about 67%. It is likely that she will only have to ask four people in order to find at least one person who uses Mobile Nation.

Unit 3

Geometric Concepts

In this lesson you will learn about geometric figures such as points, lines, and planes. You will also learn about segments, rays, and types of angles. You will learn how to determine if a figure is a polygon. You will be able to name different polygons and further explore triangles and special quadrilaterals. You will also study different parts of circles.

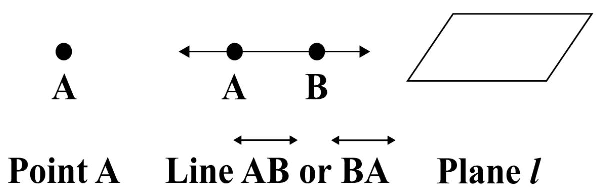
Learning Outcomes

1. Identify points, lines, planes, segments, and rays.
2. Classify angles by their measures; use facts about angles to write and solve simple equations for an unknown angle.
3. Classify triangles and quadrilaterals by their angles and sides.
4. Describe the two-dimensional figures that result from slicing three-dimensional solids.
5. Determine if a figure is a polygon and identify types of polygons.
6. Identify and work with parts of a circle.

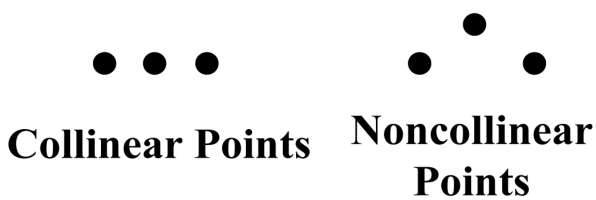
3.1: Points, Lines, Planes, Segments, and Rays

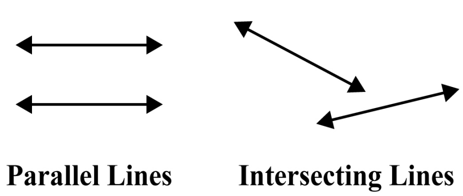
Identify points, lines, planes, segments, and rays.

There are three basic geometric terms that are defined. These are points, lines, and planes. A *point* is a dot that is generally labeled with a capital letter. A point has no length, width, or depth. On the other hand, a *line* is made up of infinite points and is generally labeled with two capital letters. A line has length but no width or depth. A *plane* is made up of infinite lines and is generally labeled with a single lower case letter. A plane has length and width but no depth.

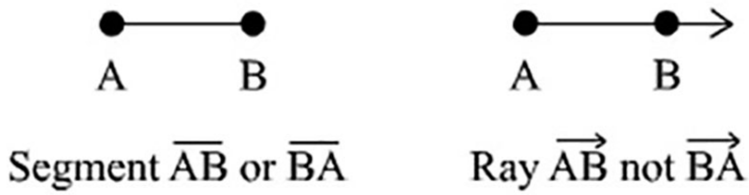


Points can be classified as *collinear* or *noncollinear*. Points are collinear if they lie on the same line, and they are noncollinear if they do not lie on the same line. Lines in the same plane can be classified as intersecting or parallel. *Intersecting lines* are lines that intersect at a point, and *parallel lines* are lines that never intersect.





Another geometric figure is a segment. A *segment* is part of a line. It consists of two endpoints and all the points in between the endpoints. A segment is finite because it does not go on forever. Another geometric figure is a ray. A *ray* is also part of a line. It consists of one endpoint and all the other points on one side of the endpoint. Notice these geometric figures below.

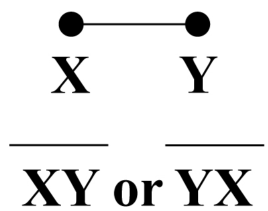


Notice that lines, segments, and rays are named with any two capital letters on the line or segment. These letters can be switched around when naming lines and segments; however, the letters cannot be switched when naming a ray. To name a ray you use the endpoint first and then any other point on the ray.

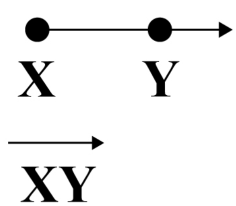
Example 1

Name each geometric figure.

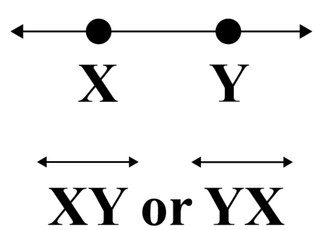
1.



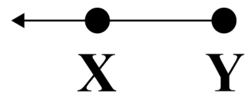
2.



3.



4.



$$\overrightarrow{YX}$$

Example 2

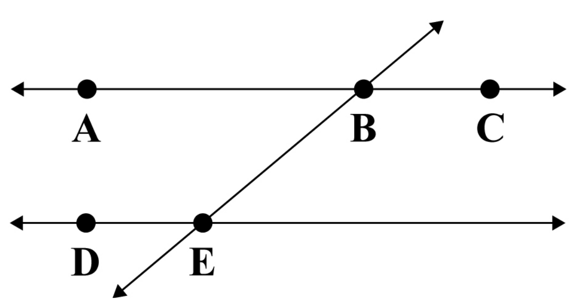
Name the line below in different ways.

$$\overleftrightarrow{AB}, \overleftrightarrow{BC}, \overleftrightarrow{CD}, \overleftrightarrow{AC}, \overleftrightarrow{AD}, \overleftrightarrow{BD} \text{ or}$$
$$\overleftrightarrow{BA}, \overleftrightarrow{CB}, \overleftrightarrow{DC}, \overleftrightarrow{CA}, \overleftrightarrow{DA}, \overleftrightarrow{DB}$$

Example 3

From the diagram below, name each of the following:

1. three collinear points
2. three noncollinear points
3. three segments
4. three rays
5. two lines that look like parallel lines
6. two lines that look like intersecting lines



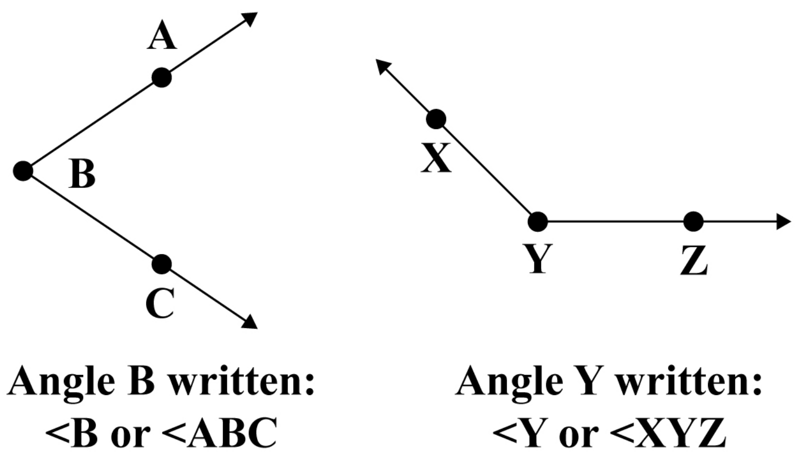
1. three collinear points
   a. A, B, C
2. three noncollinear points

   a. A, C, E or B, D, E or A, B, E or A, D, E or A, C, D or A, B, D (These are some of the possible answers.)
3. three segments
   a. AC, AB, BC, BE, or DE
4. three rays
   a. DE, DE, AB, BC, BA, CA, AC, BE, or EB
5. two lines which look like parallel lines
   a. AC and DE
6. two lines which look like intersecting lines
   a. AC and BE, DE and BE

3.2: Classify Angles

Classify angles by their measures; use facts about angles to write and solve simple equations for an unknown angle.

The hands on a clock form an angle. An *angle* is made up of two rays with a common endpoint, known as the *vertex*. The rays are called the sides of the angle. An angle is named using a capital letter for the vertex or three capital letters where the first and last letters are on different sides of the angle and the middle letter is the vertex of the angle. Let’s observe different angles below.



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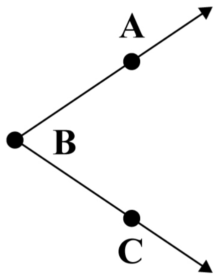
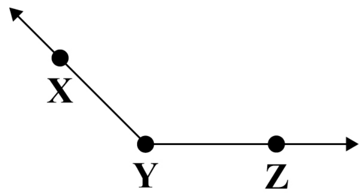
We can use a tool called a protractor to measure the size of an angle in degrees. To find the measure of an angle, you need to place the center of the protractor on the vertex of the angle. Next, line up one side with the bottom of the protractor. Lastly, you find the number of degrees where the second side of the angle intersects the protractor. If you are having difficulty deciding which scale on the protractor to use to measure the angle, you use the scale containing zero through which one the side of the angle passes.

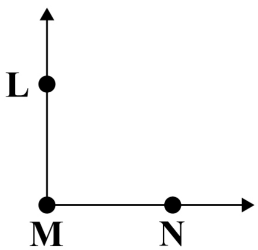
Angles are classified by their angle measures. An angle whose measure is less than 90° is known as an acute angle. An angle whose measure is between 90° and 180° is known as an obtuse angle. An angle whose measure is 180° is known as a straight angle. Another type of angle is a right angle. A right angle is an angle whose measure is 90°. Right angles are formed by intersecting lines referred to as perpendicular lines. Notice each of the types of angles below:



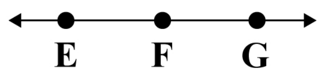
Acute Angle



Obtuse Angle

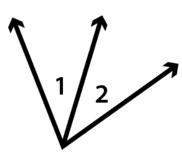


Right Angle

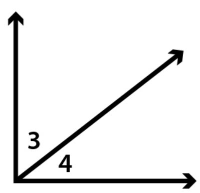


Straight Angle

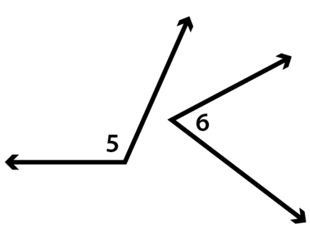
You can combine 2 angles at a time to form angles with specific properties. Adjacent angles are two angles that share a common ray. Complimentary angles are two angles that have a sum of 90°. Supplementary angles are two angles that have a sum of 180°. Both complementary and supplementary angles may be adjacent or non-adjacent. Vertical angles are formed by two intersecting lines and are nonadjacent, but congruent. Notice each of the types of angles below.



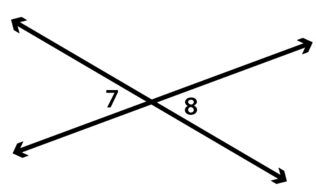
<1 & <2 are adjacent angles



<3 & <4 are adjacent, complementary angles



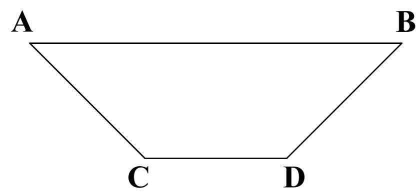
<5 & <6 are non-adjacent, supplementary angles



<7 & <8 are vertical angles

Example 1

Identify the type of each angle in the geometric figure.



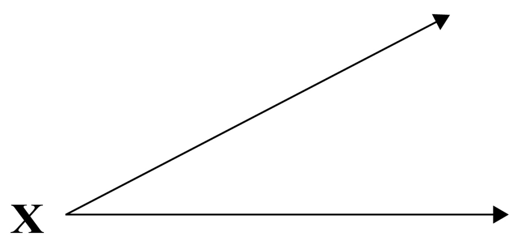
Angle	Type
$\angle A$ or $\angle BAC$	Acute angle
$\angle B$ or $\angle ABD$	Acute angle

Angle	Type
$\angle C$ or $\angle ACD$	Obtuse angle
$\angle D$ or $\angle CDB$	Obtuse angle

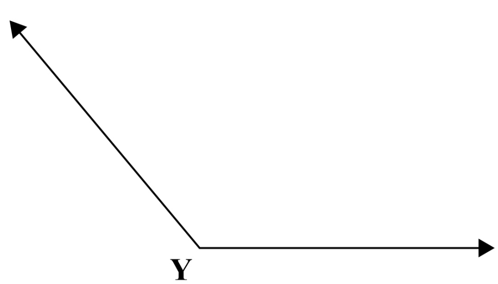
Example 2

Use a protractor to find the measure of $\angle X$ and $\angle Y$. Then classify the angles by their measures.

1.



2.

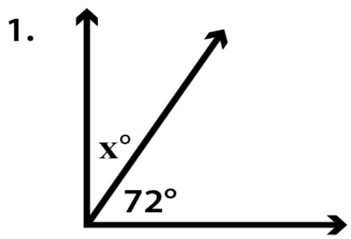


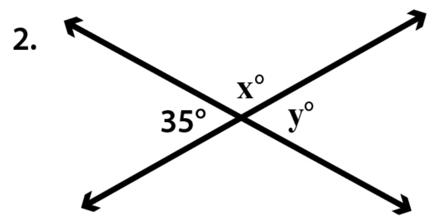
1. The measure of $\angle X$ is about 27°. This is an acute angle.

2. The measure of $\angle Y$ is about 136°. This is an obtuse angle. For more information, visit this page on the Math League website.

Example 3

1. Use the properties of angles to set up an equation and solve for the unknown angle.





The two angles are complementary or have a sum of 90°.

$x + 72 = 90$
$-72 \quad -72$
$x = 18°$

2. The two intersecting lines create both supplementary angles and vertical angles. Recall that supplementary angles have a sum of 180° and vertical angles are congruent.

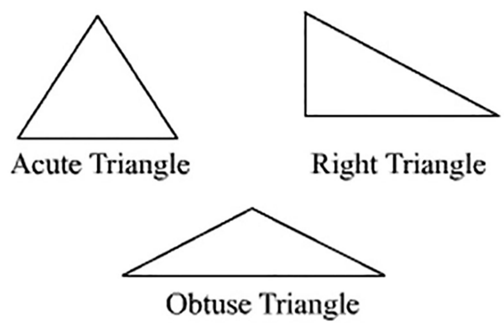
$x + 35 = 180$ and $y = 35°$
$-35 \quad -35$
$x = 145°$

3.3: Classify Triangles and Quadrilaterals

Classify triangles and quadrilaterals by their angles and sides.

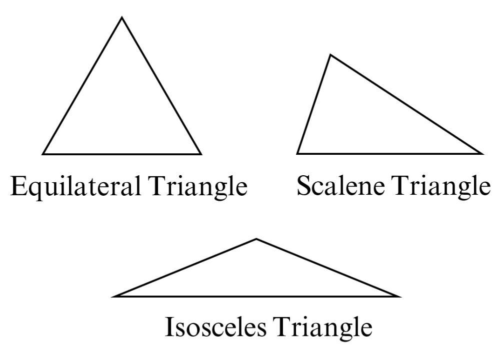
Classifying Triangles by Angles

A three-sided geometric figure is known as a *triangle*. Triangles are classified by their angle measures and the lengths of their sides. First, let's discuss how to classify triangles by their angles. A triangle with three acute angles is known as an *acute triangle*. A triangle with one obtuse angle is known as an *obtuse triangle*. Lastly, a triangle with one right angle is known as a *right triangle*. Notice each type of triangle below.



Classifying Triangles by Sides

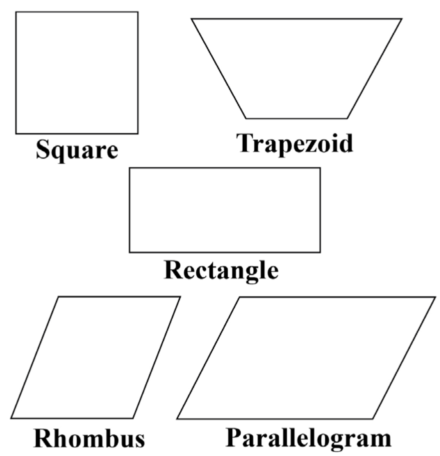
Next, let's discuss how to classify triangles by their sides. A triangle with three different length sides is known as a *scalene* triangle. A triangle with two sides the same length is known as an *isosceles* triangle. Lastly, a triangle with all three sides the same length is known as an *equilateral* triangle. Notice each type of triangle below.



When you identify a triangle, you classify the triangle by its angles and sides. For example, triangles could be acute equilateral, but not obtuse equilateral or right equilateral. There are also acute isosceles, obtuse isosceles, and right isosceles triangles. Lastly, there are acute scalene, obtuse scalene, and right scalene triangles. Let's identify each type of triangle in the following examples.

Classifying Quadrilaterals

A *quadrilateral* is a four-sided geometric figure. There are many special types of quadrilaterals. These special types of quadrilaterals are rectangles, squares, parallelograms, rhombuses, and trapezoids. A *rectangle* is a quadrilateral with four right angles and opposite sides the same length. A *square* is a quadrilateral with four right angles and all sides the same length. A *parallelogram* is a quadrilateral with opposite parallel sides. A *rhombus* is a quadrilateral with opposite parallel sides and all sides the same length. Finally, a *trapezoid* is a quadrilateral with one pair of parallel sides. Notice each of the quadrilaterals below:

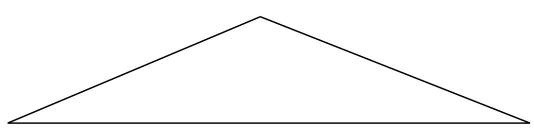


When you are identifying quadrilaterals, you need to use the lengths of the sides and the relationship of opposite sides. You also need to use the angle measures.

Example 1

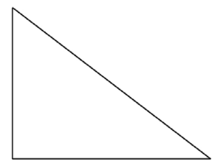
Identify each of the triangles.

1.



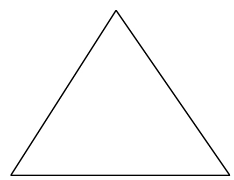
Notice the triangle has sides that are the same length and one angle greater than 90°; therefore, the triangle is an obtuse isosceles triangle.

2.



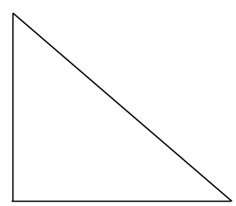
Notice the triangle has sides that are not the same length and one angle that is 90°; therefore, the triangle is a right scalene triangle.

3.



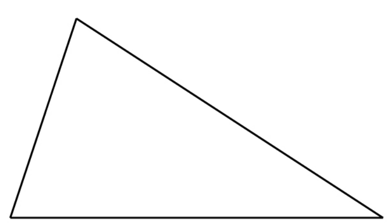
Notice the triangle has sides that are all the same length and all angles less than 90°; therefore, the triangle is an acute equilateral triangle.

4.



Notice the triangle has two sides that are the same length and an angle that is 90°; therefore, the triangle is a right isosceles triangle.

5.

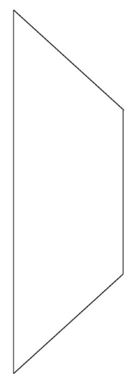


Notice the triangle has sides that are not the same length and all angles less than 90°; therefore, the triangle is an acute scalene triangle.

Example 2

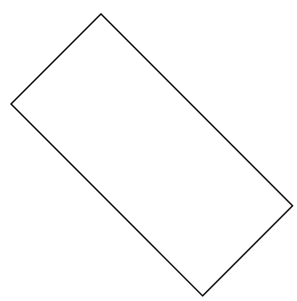
Identify each of the quadrilaterals.

1.



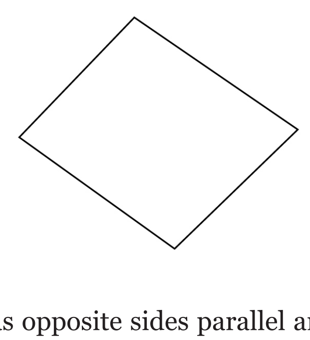
Notice this quadrilateral has one set of parallel sides; therefore, it is a trapezoid.

2.



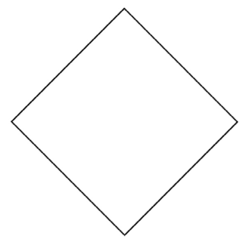
Notice this quadrilateral has four right angles and opposite sides are the same length; therefore, it is a rectangle.

3.



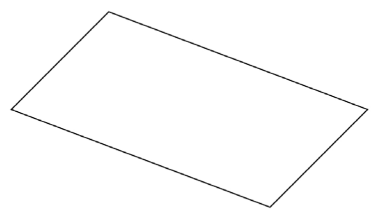
Notice this quadrilateral has opposite sides parallel and all sides are the same length; therefore, it is a rhombus.

4.



Notice this quadrilateral has four right angles and all sides are the same length; therefore, it is a square.

5.



Notice this quadrilateral has opposite parallel sides; therefore, it is a parallelogram.

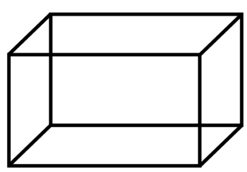
3.4: Plane Sections

Describe the two-dimensional figures that result from slicing three-dimensional solids.

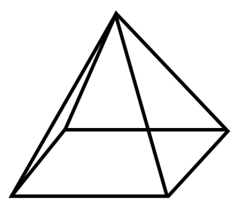
A *plane* has two dimensions (is a flat surface) and is often called 2D. A two-dimensional figure has width and height as the dimensions, such as a rectangle. A three-dimensional (3D) solid has width, height, and depth as the dimensions, such as a cube.

When you slice a 3D solid, the cross-section that results is a 2D figure. The slicing can be done horizontally, vertically, or diagonally. You can visualize the resulting 2D figure by thinking about the number of faces that were intersected on the 3D solid.

A right rectangular prism is a 3D solid that has six faces: a front face, a back face, a top face, a bottom face, a left face, and a right face. A right rectangular pyramid is a 3D solid that has a rectangular base and four triangular faces. Notice the 3D solids below.



Right Rectangular Prism

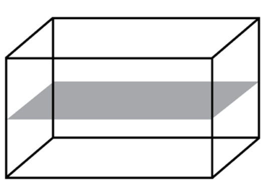


Right Rectangular Pyramid

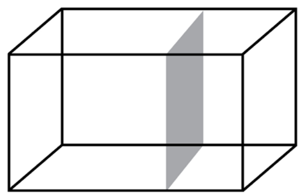
Example 1

What 2D figure would result if you sliced the right rectangular prism horizontally? Vertically? Diagonally?

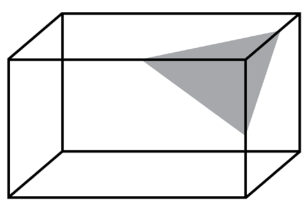
When you slice a right rectangular prism horizontally, it will slice through four faces: the front face, the back face, the left face, and the right face. The result will be a four-sided figure that is parallel to the top face and the bottom face, or in this case, a rectangle.



When you slice a right rectangular prism vertically, it will slice through four faces: the front face, the back face, the top face, and the bottom face. The result will be a four-sided figure that is parallel to the left face and the right face, or in this case, a square.



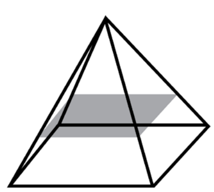
When you slice a right rectangular prism diagonally, the result will be determined by how many faces you slice through. If you slice through three faces, the result will be a triangle, as shown below.



Example 2

What 2D figure would result if you sliced the right rectangular pyramid horizontally? Vertically? Diagonally?

When you slice a right rectangular pyramid horizontally, it will slice through the four triangular faces. The result will be a four-sided figure that is parallel and similar to the base.



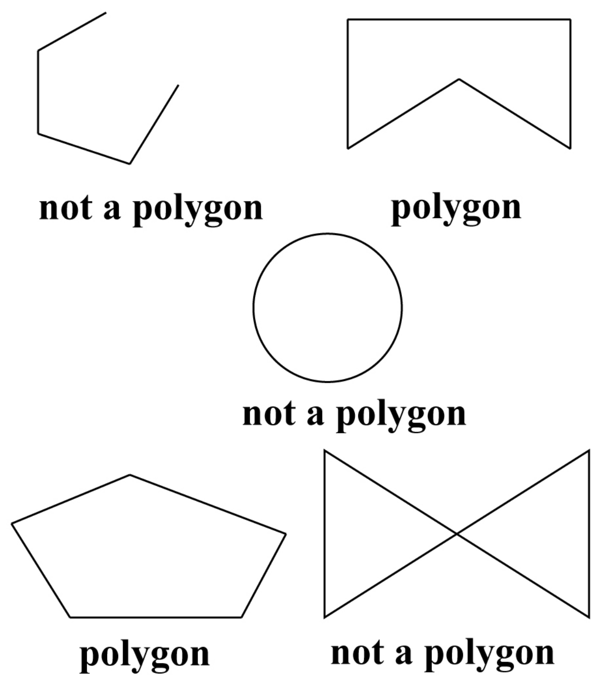
When you slice a right rectangular pyramid vertically through the *apex*, or the top point when all triangular faces meet, it will slice through two of the triangular faces and the base, thus creating a triangle. When you slice vertically and not through the apex, it will slice through three of the triangular faces and the base, creating a trapezoid.

When you slice a right rectangular pyramid diagonally, the result will be determined by how many faces you slice through. If you slice through three faces, the result is a triangle; if you slice through four faces, the result is a quadrilateral.

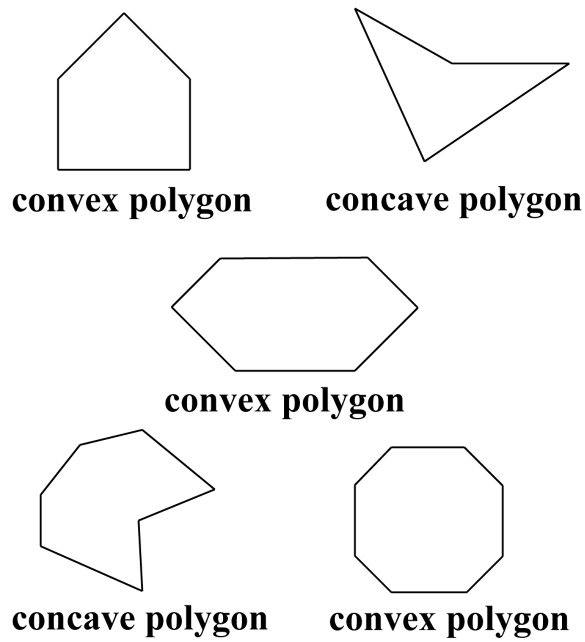
3.5: Identify Polygons

Determine if a figure is a polygon and identify types of polygons.

A *polygon* is a closed geometric plane figure made up of three or more segments. These noncollinear segments intersect only at their endpoints. Each endpoint is referred to as a vertex, and all the endpoints are vertices. Let's notice each of the polygons shown below.



There are two types of polygons: concave and convex. A *concave polygon* is a polygon around which a rubber band cannot fit tightly. That is, you can draw a straight line through a concave polygon that crosses more than two sides. (Think of a concave polygon as one with some sides that are "caved in.") On the other hand, a *convex polygon* is a polygon that a rubber band can fit tightly around; a straight line drawn through a convex polygon crosses two sides at most.



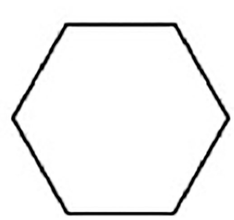
Polygons are named for the number of their sides. Here is a chart that summarizes each type of polygon:

Polygon	Number of Sides	Polygon	Number of Sides
Triangle	3	Octagon	8
Quadrilateral	4	Nonagon	9
Pentagon	5	Decagon	10
Hexagon	6	Dodecagon	12
Heptagon	7		

Example 1

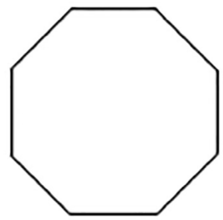
Identify each polygon.

1.



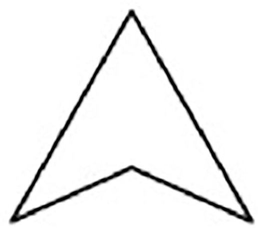
Notice this polygon has six sides and is not caved in anywhere; therefore, it is a convex hexagon.

2.



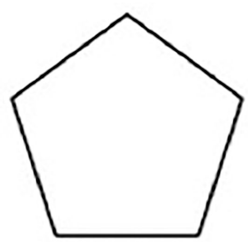
Notice this polygon has eight sides and is not caved in anywhere; therefore, it is a convex octagon.

3.



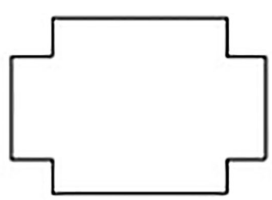
Notice this polygon has four sides and is caved in; therefore, it is a concave quadrilateral.

4.

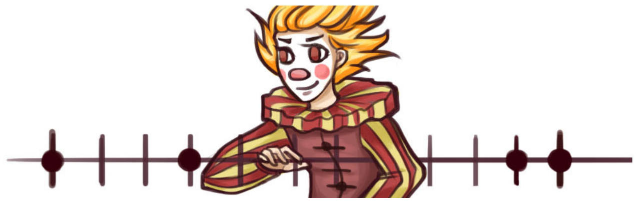


Notice this polygon has five sides and is not caved in anywhere; therefore, it is a convex pentagon.

5.



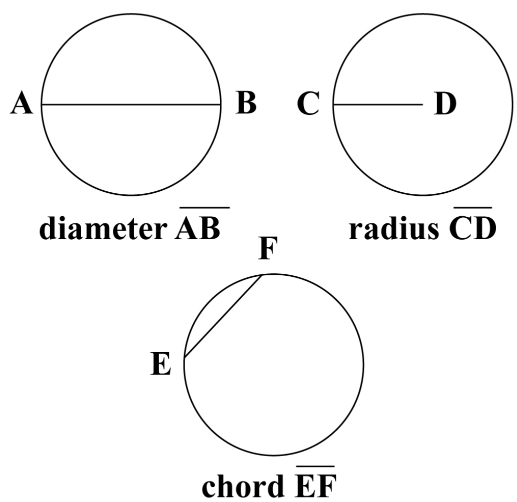
Notice this polygon has twelve sides and is caved in; therefore, it is a concave dodecagon. For more information, visit this page on the Ask Dr. Math website.



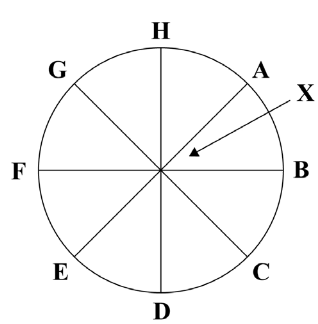
3.6: Parts of a Circle

Identify and work with parts of a circle.

A *circle* is the set of all points equidistant from a given point in a plane. The point in the middle of a circle is called the *center*. Circles are named by the center point. For example, a circle with the center at point O would be named circle O. A *radius* is a segment that has one endpoint on the circle and the other endpoint at the center. A *chord* is a segment whose endpoints are on the circle. Lastly, a *diameter* is a segment that passes through the center of the circle and whose endpoints are on the circle. Notice that the length of a diameter is twice the length of a radius.



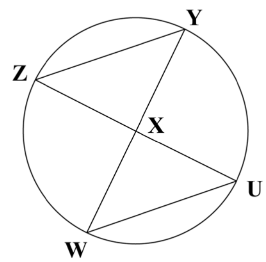
There are angles formed by radii of a circle; these angles are called *central angles*. The sum of all the angles in a circle is 360°. Notice the many central angles in the circle below.



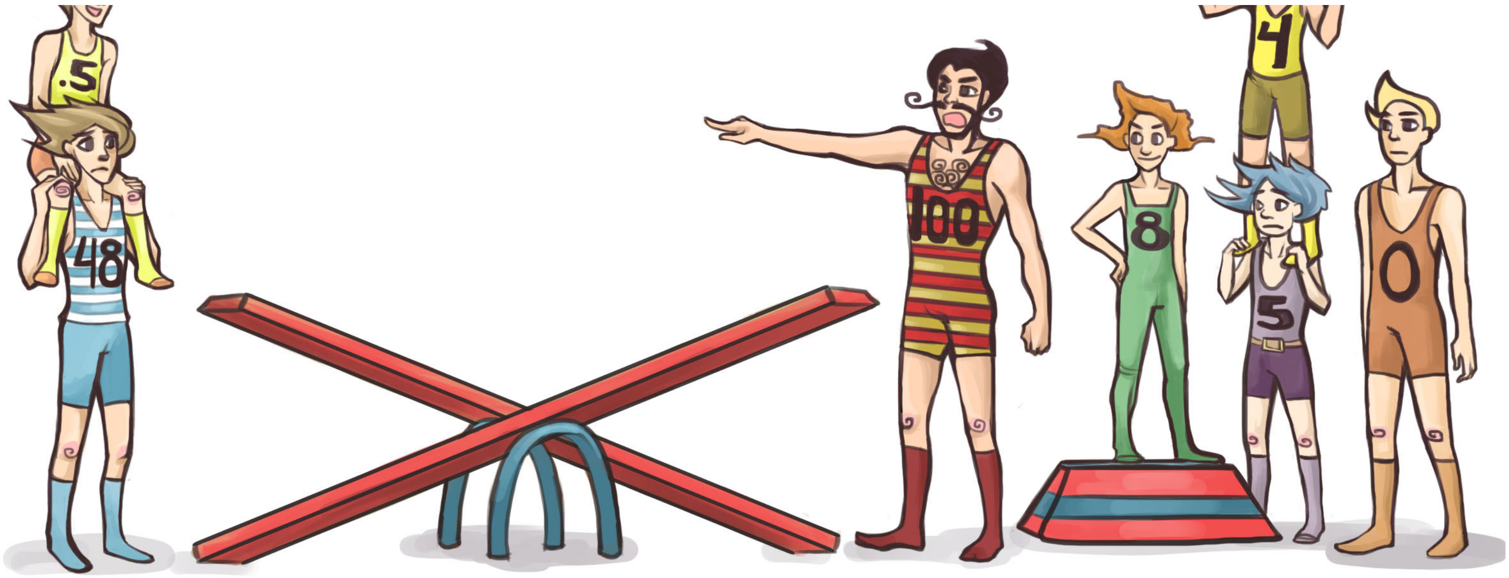
Some of the central angles are: ∠AXB, ∠BXC, ∠CXD, ∠DXE, ∠EXF, ∠FXG, ∠GXH, ∠AXH, ∠HXD, ∠HXF, ∠GXA, ∠CXE,∠FXB, and so on. Let’s try some examples.

Example 1

Name each of the diameters, radii, chords, and central angles in the diagram below.



The diameters are UZ, WY.
The radii are XZ, XU, XW, XY.
The chords are UZ, WY, UW, YZ.
The central angles are ∠UXW, ∠UXY, ∠YXZ, ∠ZXW.

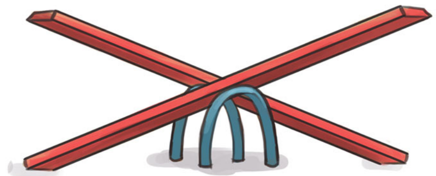


Unit 4
Decimals

In this lesson you will learn how to use and apply decimal concepts. First, you will learn how to estimate products and quotients of decimals. Then you will learn how to estimate sums and differences. Then you will learn how to multiply decimals and how to divide by decimals and whole numbers. You will also learn how to use metric units and how to choose appropriate units for measurements. Finally, you will learn how to multiply and divide by powers of ten.

Learning Outcomes

1. Estimate decimal products using rounding and clustering.
2. Estimate decimal quotients using rounding and clustering.
3. Estimate sums and differences of decimals.
4. Multiply and divide a decimal by a whole number and a decimal.
5. Measure using metric units and choose appropriate units of measurement.
6. Multiply and divide numbers by powers of ten.



4.1: Estimate Decimal Products

Estimate decimal products using rounding and clustering.

Suppose you wanted to plan a surprise birthday party for your friend. First, you would need to estimate the cost of the party supplies. When you go to the party supply store, you get the following price list. After you decide how much you need of each item, you can get an idea how much the party will cost.

Party Supplies	Unit Cost
Streamers	$0.99
Table Cloths	$2.85
Napkins	$1.29
Balloons	$2.15
Games	$4.75
Invitations	$3.29

Suppose you want to purchase three of each item. You could estimate your party cost quickly by rounding to the nearest dollar and multiplying by three. This process is known as estimating products. A *product* is an answer to a multiplication problem.

Party Supplies	Rounded Price	Product
STREAMERS	$0.99 ≈ $1	$1 × 3 = $3
TABLE CLOTHS	$2.85 ≈ $3	$3 × 3 = $9
NAPKINS	$1.29 $1	$1 × 3 = $3
3BALLOONS	$2.15 ≈ $2	$2 × 3 = $6
GAMES	$4.75 ≈ $5	$5 × 3 = $15
INVITATIONS	$3.29 ≈l $3	$3 × 3 = $9
		TOTAL: $45

Notice it was easy to estimate the cost of the party because we could quickly multiply each rounded party supply by three. Another method of estimating products is to use compatible numbers. *Compatible numbers* are numbers that are easy to multiply mentally. For example, if you wanted to estimate the product of 78.3 and 4.3, we could consider the product of the compatible numbers 75 and 4, therefore 75 × 4 = 300. Let's look at some examples of each method.

Example 1

Estimate the product of 11.6 × 3.23 using rounded whole numbers.

11.6 × 3.23	Write the problem
12 × 3 = 36	Use rounded numbers.

The estimated product is 36.

Example 2

Estimate the product of 2.25 × 16.91 using rounded whole numbers.

2.25 × 16.91	Write the problem
2 × 17 = 34	Use rounded numbers.

The estimated product is 34.

Example 3

Estimate the product of 39.26 × 1.98 using compatible numbers.

39.26 × 1.98	Write the problem
40 × 2 = 80	Use rounded numbers.

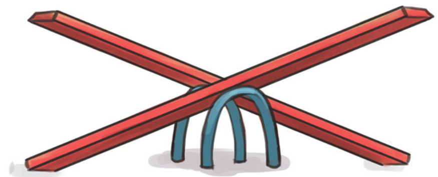
The estimated product is 80.

Example 4

Estimate the product of 18.75 × 41.5 using compatible numbers.

18.75 × 41.5	Write the problem.
20 × 40 = 36	Use rounded numbers.

The estimated product is 800.



4.2: Decimal Quotients

Estimate decimal quotients using rounding and clustering.

Suppose you had a summer job mowing lawns. If you earned \$74.25 in a month and charged \$8.25 per lawn, we could estimate how many lawns you mowed in the month by rounding the decimals to the nearest dollar and dividing them. This process is known as estimating a quotient. A *quotient* is an answer to a division problem.

\$74.25 ÷ \$8.25 = ???

\$74 ÷ \$8 ≈ 9

Therefore, you mowed about 9 lawns in the month. Notice it was simple to estimate the number of lawns mowed because we could quickly divide each decimal. Another method of estimating quotients is to use compatible numbers. Remember compatible numbers are numbers that are easily divided. For example, the computation 76.3 ÷ 15.7 would be much easier calculated if we used the compatible numbers 75 ÷ 15, which would result in a quotient of 5.

Example 1

Estimate the quotient of 99.8 ÷ 5.12 using rounded whole numbers.

99.8 ÷ 5.12	Write the problem.
100 ÷ 5 = 20	Use rounded numbers.

The estimated quotient is 20.

Example 2

Estimate the quotient of 88.4 ÷ 10.5 using rounded whole numbers.

88.4 ÷ 10.5	Write the problem.
88 ÷ 11 = 20	Use rounded numbers.

The estimated quotient is 8.

Example 3

Estimate the quotient of 15.76 ÷ 2.51 using compatible numbers.

15.79 ÷ 2.51	Write the problem.
15 ÷ 3 = 5	Use rounded numbers.

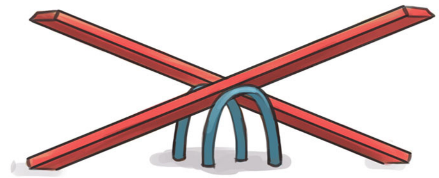
The estimated quotient is 5.

Example 4

Estimate the quotient of 39.3 ÷ 8.7 using compatible numbers.

39.3 ÷ 8.7	Write the problem.
40 ÷ 8 = 5	Use rounded numbers.

The estimated quotient is 5.



4.3: Sums and Differences

Estimate sums and differences of decimals.

Suppose you were calculating an estimate of a company's profits over the past year. You could find the sum profits by rounding each quarterly profit to the same place value before finding their sum.

Yearly Profits			
1st Quarter	*2nd Quarter*	*3rd Quarter*	*4th Quarter*
$1,978,591	$3,126,973	$2,278,256	$2,735,865

If we round each of the quarterly profits to the nearest hundred thousands, then we can add these estimates together to find their sum.

Given Place	Original Number	Up or Down?	Rounded Number
Hundred Thousands	$1,978,591	Up	2,000,000
Hundred Thousands	$3,126,973	Down	3,100,000
Hundred Thousands	$2,278,256	Up	2,300,000
Hundred Thousands	$2,735,865	Down	2,700,000

Now we can add the rounded quarterly profits together. This would result in $2,000,000 + $3,100,000 + $2,300,000 + $2,700,000 = $10,100,000. The company's yearly profits were about $10.1 million.

A *sum* is an answer to an addition problem, whereas a *difference* is an answer to a subtraction problem. Often, it is helpful to estimate a sum or difference. This is especially true when the exact amount is not needed. Let's try some examples.

Example 1

Chelsea went shopping for new school clothes. Suppose she bought jeans for $44.35, a sweater for $29.95, a skirt for $24.55, and shoes for $36.25. Estimate to the nearest dollar how much Chelsea spent on her school clothes.

Given Place	Original Number	Up or Down?	Rounded Number
Ones	\$44.35	Down	\$44.00
Ones	\$29.95	Up	\$30.00
Ones	\$24.55	Up	\$25.00
Ones	\$36.25	Down	\$36.00

Chelsea spent \$44.00 + \$30.00 + \$25.00 + \$36.00 = \$135.00 on her school clothes.

Example 2

Josh went shopping for new rollerblades. Suppose Oshman's Sports had the rollerblades he wanted on sale for \$64.15, while Big 5 Sports had the same ones on sale for \$79.65. To the nearest dollar, find the difference in the price of the rollerblades.

Given Place	Original Number	Up or Down?	Rounded Number
Ones	\$64.15	Down	\$64.00
Ones	\$79.65	Up	\$80.00

Josh would save \$16.00 (\$80.00 – \$64.00 = \$16.00) if he bought the less expensive roller blades.

Notice we used rounding to estimate these sums and differences. We can also use another method to estimate sums and differences. This method is called clustering. Clustering can be used when numbers are close to the same number. Let's look at an example using this method.

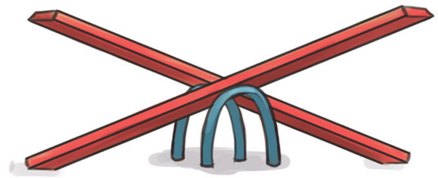
Example 3

Chandler bought some candy priced \$0.24, \$0.26, \$0.27, \$0.23 and \$0.25. Estimate the amount of money he spent on candy.

Chandler spent \$0.24, \$0.26, \$0.27, \$0.23, and \$0.25. Notice these amounts are all close to \$0.25, so we will add this amount five times.

\$0.25 + \$0.25 + \$0.25 + \$0.25 + \$0.25 = \$1.25

Chandler spent about \$1.25 on candy.



4.4: Multiplying and Dividing

Multiply and divide a decimal by a whole number and a decimal

Suppose you wanted to start a new exercise program. If you wanted to calculate the number of calories burned while exercising each day, you could multiply the number of hours spent exercising by the amount of calories burned per hour. Notice the following chart that summarizes the number of calories burned for each type of exercise.

Exercise Activity	Calories Burned/Hour
Jogging	600
Swimming	300
Bicycling	240
Walking	180
Aerobics	300
Rollerblading	300

You can calculate the number of calories you burn each time you exercise. For example, if you went jogging for 45 minutes, which is .75 hours, you could calculate the number of calories burned by multiplying 600 by .75. Notice we need to multiply a whole number by a decimal. The rules are summarized in the chart below.

Multiplying a Decimal by a Whole Number
1) Count the number of digits to the right of the decimal.
2) Move the decimal to the left that many spaces.

600
× .75 — 2 digits after the decimal
3000
42000
450.00 — move the decimal 2 places ←

Therefore, you would burn 450 calories by jogging for 45 minutes.

Now let's suppose you spent $7.50 on 5 orders of french fries. If you wanted to calculate the price of each order of french fries, you could divide the amount you spent by the number of orders of fries you purchased, or $7.50 divided by 5. Notice we need to divide a decimal by a whole number. The rules are summarized in the chart below.

Dividing by Whole Numbers
1) Divide by the divisor.
2) Place the decimal in the quotient by bringing it straight up from the dividend.

Let's review some basic division concepts in order to make the explanation clearer. A divisor is the number you are dividing by, while the dividend is the number being divided. The answer is known as the quotient, and the left-over amount is called the remainder.

```
        QUOTIENT + REMAINDER
DIVISOR ⟌DIVIDEND
```

Let's complete the example above.

1. Rewrite using the division box.
2. Divide by the divisor.
3. Bring the decimal straight up into the quotient.

$7.50 ÷ 5

```
    1 50
5 ⟌ 7. 50
    7 5
      0
    1  50
5 ⟌ 7.↑50
```

We can check the answer by multiplying

1.50
× 5 2 digits after the decimal
7.50 move the decimal 2 places

Therefore, the cost of each order of french fries is $1.50.

Let's try some multiplication examples.

Example 1

Find the product of 523 × .5.

1. Count the number of digits to the right of the decimal.
2. Move the decimal to the left that many spaces.

523
× .5 1 digit after the decimal
261.5 move the decimal 1 place
←

The product is 261.5.

We could also multiply a decimal by another decimal. The rules are similar to multiplying a decimal by a whole number. The rules are summarized in the chart below.

Multiplying a Decimal by a Decimal Number
1) Count the number of digits to the right of each decimal.
2) Move the decimal to the left the total amount of spaces.
3) If there are digits missing, fill in with zeros.

Example 2

Find the product of .31 × .04.

1. Count the number of digits to the right of each decimal.
2. Move the decimal to the left the total number of spaces.

.31	2 digits after the decimal
× .04	2 digits after the decimal
.0124	← move the decimal 4 places add one zero to the product

Let's try some division examples.

Example 3

Find the quotient of 1.44 ÷ 12.

1. Rewrite using the division box.
2. Divide by the divisor.
3. Bring the decimal straight up into the quotient.

1.44 ÷ 12

$$\begin{array}{r} 12 \\ 12\,\overline{)\,1.\;44} \\ -\underline{1\;\;2}\downarrow \\ 24 \\ -\underline{24} \\ 0 \end{array}$$

$$\begin{array}{r} .\;12 \\ 12\,\overline{)\,1.\,44} \end{array}$$

We can check the answer by multiplying

$$\begin{array}{r} .12 \\ \underline{\times\;12} \\ 24 \\ \underline{+\;120} \\ 1.44 \end{array} \leftarrow \text{ move the decimal 2 places}$$

The quotient is .12.

Example 4

Find the quotient of 19.5 ÷ .15.

Notice that the divisor of this problem is a decimal with two numbers behind the decimal. Before you can divide, you need to change the divisor to a whole number. However, you can't change just the divisor. You must change both the divisor and the dividend. Because the divisor is a decimal with the hundredths decimal place, you will multiply by 100 to make it a whole number.

Multiply the divisor and the dividend by 100 to create a new, equivalent problem.

19.5 × 100 = 1950 and .15 × 100 = 15

So, the new problem will be 1950 ÷ 15

1. Rewrite using the division box.
2. Divide by the divisor.
3. Bring the decimal straight up into the quotient.

1950 ÷ 15

```
       1  3
15 | 19.  5
    -15
       4  5
     - 4  5
          0
```

We can check the answer by multiplying

130

× .15

650

+ 130

19.50 ← move the decimal 2 places

The quotient is 130.

Example 5

Find the quotient of .099 ÷ .3.

The divisor for this problem is a decimal with a tenths decimal place. So, you must multiply both divisor and dividend by 10 to make a new, equivalent problem.

.099 × 10 = .99 and .3 × 10 = 3

So, the new problem will be .99 ÷ 3

1. Rewrite using the division box.
2. Divide by the divisor.
3. Bring the decimal straight up into the quotient.

$$
\begin{array}{r|l}
 & 33 \\
\hline
3 & .099 \\
 & -9 \\
\hline
 & 09 \\
 & -9 \\
\hline
 & 0
\end{array}
$$

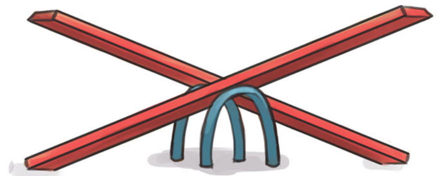
We can check the answer by multiplying

.33

<u>× .3</u>

.099 ← move the decimal 3 places and add a zero

The quotient is .33.



4.5: Metric Units

Measure using metric units, and choose appropriate units of measurement.

Suppose you wanted to find the length of your pencil. You could find this by measuring your pencil with a standard ruler. Most rulers are marked in inches as well as centimeters. A centimeter is one of the units of measurement in the metric system. The metric system is about 200 years old, and its basic unit of measurement is the meter. All other units of measurement in the metric system are powers of 10. Decimals are also used in the decimal system. Notice the following chart with the common metric units of measurement.

Unit	Abbreviation	Place Value	Size	Model
kilometer	km	thousands	1000m	length of five city blocks
hectometer	hm	hundreds	100 m	width of two football fields
decameter	dam	tens	10 m	width of a classroom
meter	m	ones	1 m	length of a baseball bat
decimeter	dm	tenths	.1 m	width of a hand
centimeter	cm	hundredths	.01 m	width of a finger
millimeter	mm	thousandths	.001 m	thickness of a penny

The most common metric units are kilometers, meters, centimeters, and millimeters. We will focus mainly on these units of measurement. Let's look at some examples where we estimate length by finding the appropriate metric unit of measurement.

Example 1

Choose the appropriate metric unit of measurement for each of the following:

1. height of a kitchen counter
2. width of a large paper clip
3. thickness of a dime
4. length of a main street

1. *height of a kitchen counter*: The height of a kitchen counter would be closest to the length of a baseball bat. Therefore, the appropriate metric unit of measurement is a meter.
2. *width of a large paper clip*: The width of a large paper clip would be closest to the width of a finger. Therefore, the appropriate metric unit of measurement is a centimeter.
3. *thickness of a dime*: The thickness of a dime would be closest to the thickness of a penny. Therefore, the appropriate metric unit of measurement is a millimeter.
4. *length of a main street*: The length of a main street would be closest to the length of six city blocks. Therefore, the appropriate metric unit of measurement is a kilometer.

We can also change units in the metric system. Because the metric system is based on powers of ten, this is an easy process. Notice the following metric conversion chart.

Metric Conversion Chart						
millimeters		centimeters		meters		kilometers
1 mm	=	.1 cm	=	.001 m	=	.000001 km
10 mm	=	1 cm	=	.01 m	=	.00001 km
1,000 mm	=	100 cm	=	1 m	=	.001 km
1,000,000 mm	=	100,000 cm	=	1,000 m	=	1 km

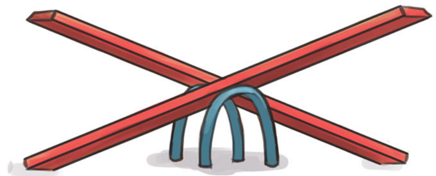
Example 2

Complete each metric conversion.

1. 1 mm =_____cm
2. _____ km = 1 m
3. _____ cm = 1 m
4. 1 km =______m
5. _____ mm = 1 cm
6. 1 m =_______cm

Let's use the metric conversion chart to fill in the blanks.

1. 1 mm = .1 cm
2. .001 km = 1 m
3. 100 cm = 1 m
4. 1 km = 1000 m
5. 10 mm = 1 cm
6. 1 m = 100 cm



4.6: Powers of Ten

Multiply and divide numbers by powers of ten.

Suppose you needed to complete an inventory of office supplies for the front office of your school. If there were 48.5 boxes of paperclips, and each box contains 100 paperclips, we could calculate the total number of paper clips by multiplying the number of boxes by the number of paperclips in each box, or 48.5 by 100. This is an easy process because we are multiplying by a power of 10, which means all we have to do is move the decimal. Notice the rules for multiplying by a power of 10 below.

Multiplying and Dividing by Powers of 10
1) Count the number of zeros in the number by which you are multiplying or dividing.
2) If you are multiplying, multiply by 1 and move the decimal in the product that number to the right.
3) If you are dividing, divide by 1 and move the decimal in the quotient that number to the left.
4) Add any needed zeros.

1. There are 2 zeros in 100.
2. Because this is a multiplication problem, move the decimal 2 places to the right in the product.
3. Add a needed zero.

48.5×100
$48.5 \times 100 = 4850$
decimal: →
Therefore, the school has 4,850 paperclips in stock.

Example 1

Multiply 3.42×10.

1. There is 1 zero in 10.
2. Because this is a multiplication problem, move the decimal 1 place to the right in the product.

3.42×10
$3.42 \times 10 = 34.2$

decimal: →
The product is 34.2.

Example 2
Multiply .65 × 1000.

1. There are 3 zeros in 1000.
2. Because this is a multiplication problem, move the decimal 3 places to the right in the product.
3. Add a necessary zero.

.65 × 1000
.65 × 1000 = 650
decimal: →
The product is 650.

Example 3
Multiply 75.445 × 100.

1. There are 2 zeros in 100.
2. Because this is a multiplication problem, move the decimal 2 places to the right in the product.

75.445 × 100
75.445 × 100 = 7544.5
decimal: →
The product is 7,544.5.

Example 4
Divide 332.6 ÷ 1000.

1. There are 3 zeros in 1000.
2. Because this is a division problem, move the decimal 3 places to the left in the quotient.

332.6 ÷ 1000
332.6 ÷ 1000 = .3326
decimal: ←
The quotient is .3326.

Example 5
Divide 2.45 ÷ 10.

1. There is 1 zero in 10.

2. Because this is a division problem, move the decimal 1 place to the left in the quotient.

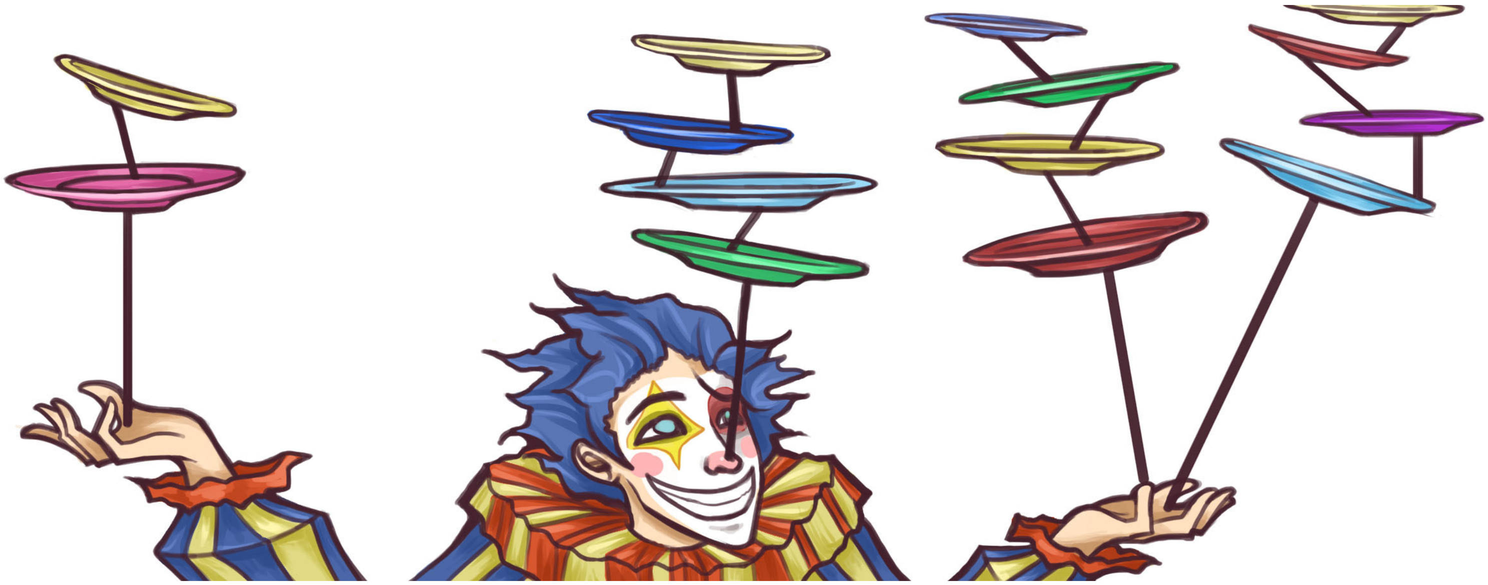
2.45 ÷ 10
2.45 ÷ 10 = .245
decimal: ←
The quotient is .245.

Example 6

Divide .85 ÷ 100.

1. There are 2 zeros in 100.
2. Because this is a division problem, move the decimal 2 places to the left in the quotient.
3. Add 2 necessary zeros.

85 ÷ 100
.85 ÷ 100 = .0085
decimal: ←
The quotient is .0085.

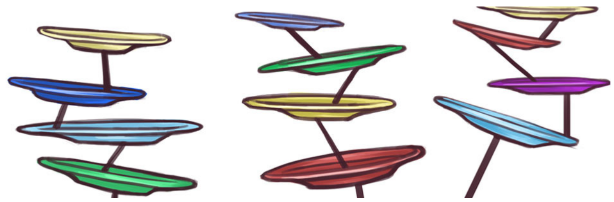


Unit 5
Number Theory

In this lesson you will learn about number patterns and number theory. First, you will explore divisibility using mental math. Then you will learn how to find the prime factorization of composite numbers. You will also learn how to find factors, as well as multiples of numbers. Finally, you will learn how to find the greatest common factor and the least common multiple of a set of numbers.

Learning Outcomes

1. Decide if one number is evenly divisible by another number and use divisibility rules to divide numbers.
2. Recognize prime and composite numbers and write a composite number as a product of prime numbers.
3. Find the factors of a given number.
4. Find multiples of a given number.
5. Find the greatest common factor of two whole numbers.
6. Find the least common multiple of two whole numbers.



5.1: Divisibility Rules

Decide if one number is evenly divisible by another number and use divisibility rules to divide numbers.

Suppose you are helping coach a soccer clinic and there are 128 kids signed up. You need to divide the kids into groups of 4 in order to run some soccer drills. Is there a way to know if 128 can be divided by 4 evenly without using a calculator? We can use divisibility tests to decide if a number can be divided evenly by another number. Let's look at the divisibility rules outlined below.

Divisibility Rules	
A NUMBER IS DIVISIBLE BY	
2	if the ones digit is an even number.
3	if the sum of the digits is divisible by 3.
4	if the last two digits are divisible by 4.
5	if the ones digit is a 5 or a 0.
6	if the last digit is an even number and the sum of the digits is divisible by 3.
8	if the last three digits are divisible by 8.
9	if the sum of the digits is divisible by 9.
10	if the ones digit is a 0.

Therefore, to decide if 128 can be divided by 4 evenly, you need to follow the divisibility test described above.

Is 128 divisible by 4?
Test: If the last two digits are divisible by 4.
128
28 ÷ 4 = 7, so the last two digits are divisible by 4.
Yes, 128 is divisible by 4.

The rules for divisibility can help you quickly decide if a number is divisible by another number. This will be a very helpful tool when we begin working with fractions. Let's go through some examples and try each divisibility test.

Example 1

Is 321 divisible by 2?
Test: If the ones digit is an even number.
321
1 is an odd number. Even digits are 0, 2, 4, 6, 8.
No, 321 is not divisible by 2.

Example 2

Is 213 divisible by 3?
Test: If the sum of the digits is divisible by 3.
213
2 + 1 + 3 = 6.
6 ÷ 3 = 2, so the sum of the digits is divisible by 3.
Yes, 213 is divisible by 3.

Example 3

Is 112 divisible by 4?
Test: If the last two digits are divisible by 4.
112
12 ÷ 4 = 3, so the last two digits are divisible by 4.
Yes, 112 is divisible by 4.

Example 4

Is 315 divisible by 5?
Test: If the ones digit is a 5 or a 0.
315
5 is a 5 or a 0.
Yes, 315 is divisble by 5.

Example 5

Is 134 divisible by 6?
Test: If the last digit is an even number, and the sum of the digits is divisible by 3.
134
4 is an even number. Even digits are 0, 2, 4, 6, 8.
134
1 + 3 + 4 = 8
8 ÷ 3 = 2 2/3, so the sum of the digits is not divisible by 3.
No, 134 is not divisible by 6.

Example 6

Is 2,088 divisible by 8?
Test: If the last three digits are divisible by 8.
2,088
88 ÷ 8 = 11, so the last three digits are divisible by 8.
Yes, 2,088 is divisible by 8.

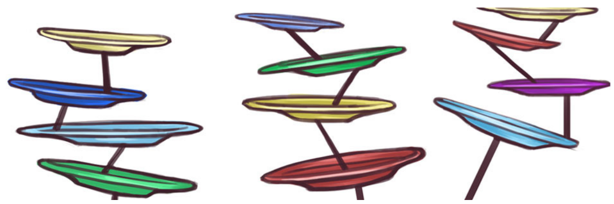
Example 7

Is 405 divisible by 9?
Test: If the sum of the digits is divisible by 9.
405
4 + 0 + 5 = 9.
9 ÷ 9 = 1, so the sum of the digits is divisible by 9.
Yes, 405 is divisible by 9.

Example 8

Is 2105 divisible by 10?
Test: If the ones digit is a 0.
2105
5 is not a 0.
No, 2,105 is not divisible by 10

.



5.2: Prime and Composite Numbers

Recognize prime and composite numbers and write a composite number as a product of prime numbers.

Suppose you are going to ship an item in the mail, and you are not sure if it will fit into the box. If you knew the volume of a box was 385 in3, you could find the dimensions of the box by using a process called prime factorization. A *prime number* is a number that can only be divided by 1 and itself. On the other hand, a *composite number* is a number that can be divided into factors. *Factors* are numbers that divide into whole numbers with a remainder of 0. *Prime factorization* is when we take a composite number and express it as a product of prime numbers. There are two methods we can use to find the prime factorization of a number. The first method is to use a factor tree and the second method is to use long division. A *factor tree* is a diagram where the factors branch out from previous factors until all the factors are prime numbers. Let's try the example above using the factor tree method .

1. Use the divisibility rules to divide the number.
2. When a number can no longer be divided, it is one of the prime factors.
3. Write the number as a product of prime factors from smallest to greatest.

```
  385
  / \
 5   77
     / \
    7   11
```

385 = 5 × 7 × 11

Therefore, the dimensions of the box are 5 by 7 by 11 inches. Let's try this example again using the long division method.

1. Use the divisibility rules to divide the number by prime numbers.
2. Repeat until the quotient is prime.
3. Write the number as a product of prime factors from smallest to greatest.

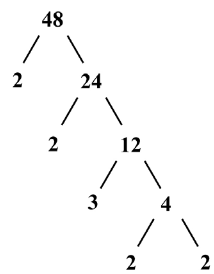
$$\begin{array}{r|l} & \mathbf{11} \\ \hline \mathbf{7} & \mathbf{77} \\ \hline \mathbf{5} & \mathbf{385} \end{array}$$

385 = 5 × 7 × 11

Example 1

Find the prime factorization of 48 using the factor tree method.

1. Use the divisibility rules to divide the number.
2. When a number can no longer be divided, it is one of the prime factors.
3. Write the number as a product of prime factors from smallest to greatest.

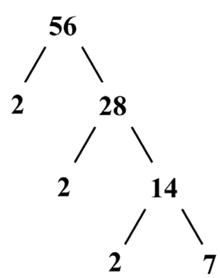


48 = 2 × 2 × 2 × 2 × 3

Example 2

Find the prime factorization of 56 using the factor tree method.

1. Use the divisibility rules to divide the number.
2. When a number can no longer be divided, it is one of the prime factors.
3. Write the number as a product of prime factors from smallest to greatest.



56 = 2 × 2 × 2 × 7

Example 3

Find the prime factorization of 45 using the long division method.

1. Use the divisibility rules to divide the number by prime numbers.
2. Repeat until the quotient is prime.
3. Write the number as a product of prime factors from smallest to greatest.

$$\begin{array}{r|r} & 5 \\ \hline 3 & 15 \\ \hline 3 & 45 \end{array}$$

45 = 3 × 3 × 5

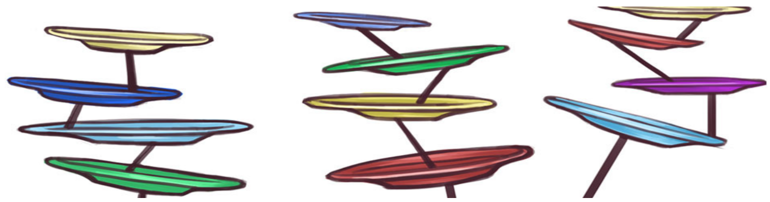
Example 4

Find the prime factorization of 72 using the long division method.

1. Use the divisibility rules to divide the number by prime numbers.
2. Repeat until the quotient is prime.
3. Write the number as a product of prime factors from smallest to greatest.

$$\begin{array}{r|r} & 3 \\ \hline 3 & 9 \\ \hline 2 & 18 \\ \hline 2 & 36 \\ \hline 2 & 72 \end{array}$$

72 = 2 × 2 × 2 × 3 × 3



5.3: Factors

Find the factors of a given number.

Suppose you and some friends earn money doing yard work. You are not sure how to divide the money in a manner that will be fair to everyone. You could find all the different ways the money could be divided by finding all the factors of the amount. If you earned $36, find all the different ways the money could be divided.

36 = **1** × **36**

36 = **2** × **18**

36 = **3** × **12**

36 = **4** × **9**

36 = **6** × **6**

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

Therefore, we could divide the money earned doing yard work in many different ways. We can find factors of a number by using the divisibility tests. This makes it easier to decide by which numbers a whole number can be divided. Let's try some further examples where we find the factors of a number.

Example 1

Find the factors of 48.

48 = **1** × **48**

48 = **2** × **24**

48 = **3** × **16**

48 = **4** × **12**

48 = **6** × **8**

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

Example 2

Find the factors of 60.

60 = **1** × **60**

60 = **2** × **30**

60 = **3** × **20**

60 = **4** × **15**

60 = **5** × **12**

60 = **6** × **10**

The factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

Example 3

Find the factors of 72.

72 = **1** × **72**

72 = **2** × **36**

72 = **3** × **24**

72 = **4** × **18**

72 = **6** × **12**

72 = **8** × **9**

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

Example 4

If you have a rectangle with an area of 80 cm2, find all its possible lengths and widths.

Area = Length × **Width**

80 = **1** × **80**

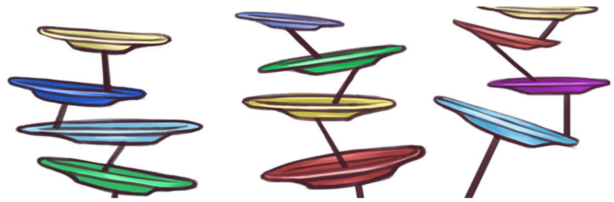
80 = **2** × **40**

80 = **4** × **20**

80 = **5** × **16**

80 = **8** × **10**

The factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80. These are also all the possible lengths and widths of the rectangle.



5.4: Greatest Common Factor

Find the greatest common factor of two whole numbers

Suppose you wanted to make fruit baskets for your neighbors. You have 12 apples, 20 oranges, and 32 pears, and you want to put the same number of each type of fruit in each basket. You could find the greatest number of each type of fruit you could put in each basket by finding the greatest common factor of 12, 20, and 32.

Factors of 12: 1, 2, 3, **4**, 6, 12

Factors of 20: 1, 2, **4**, 5, 10, 20

Factors of 32: 1, 2, **4**, 8, 16, 32

The greatest common factor of 12, 20, and 32 is 4.

Notice there are two factors common to all three numbers. They all have 2 and 4 as common factors. However, 4 is the greatest common factor of all three numbers. This means 4 is the greatest number of pieces of fruit you could put in each basket. The *greatest common factor* (GCF) is the largest number that will divide into two or more numbers.

There are two methods of finding the greatest common factor. One method is to list the factors of each number, and then find the greatest one they have in common. This is the method we used in the example above. Another method is to use the prime factorization of each of the numbers and find the product of the prime factors they have in common. Let's look at some examples of both methods.

Example 1

Find the GCF of 42 and 56 by making a list.

Factors of 42: 1, 2, 3, 6, 7, **14**, 21, 42

Factors of 56: 1, 2, 4, 7, 8, **14**, 28, 56

The GCF of 42 and 56 is 14.

Example 2

Find the GCF of 36 and 64 by making a list.

Factors of 36: 1, 2, 3, **4**, 6, 9, 12, 18, 36

Factors of 64: 1, 2, **4**, 8, 16, 32, 64

The GCF of 36 and 64 is 4.

Example 3

Find the GCF of 22, 33, and 55 by making a list.

Factors of 22: 1, 2, **11**, 22

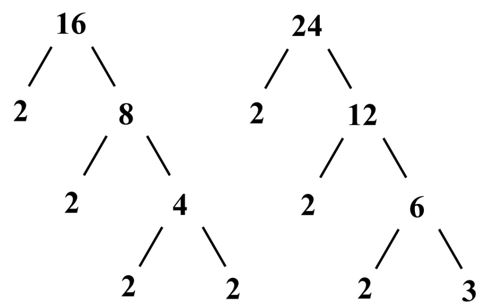
Factors of 33: 1, 3, **11**, 33

Factors of 55: 1, 5, **11**, 55

The GCF of 22, 33 and 55 is 11.

Example 4

Find the GCF of 16 and 24 using prime factorization.



$16 = 2 \times 2 \times 2 \times 2$

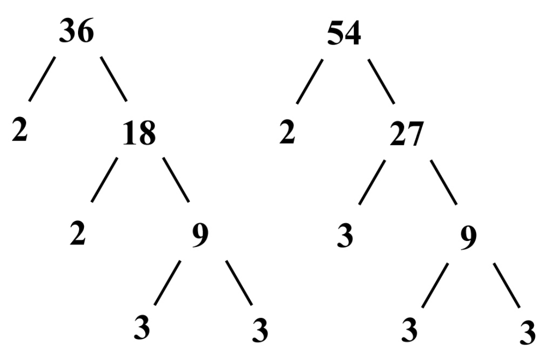
$24 = 2 \times 2 \times 2 \times 3$

GCF = $\mathbf{2} \times \mathbf{2} \times \mathbf{2} = 8$

The GCF of 16 and 24 is 8.

Example 5

Find the GCF of 36 and 54 using prime factorization.



$36 = 2 \times 2 \times 3 \times 3$

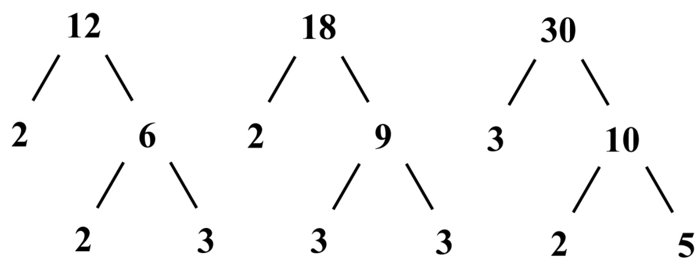
$54 = 2 \times 3 \times 3 \times 3$

GCF = $\mathbf{2} \times \mathbf{3} \times \mathbf{3} = 18$

The GCF of 36 and 54 is 18.

Example 6

Find the GCF of 12, 18, and 30 using prime factorization.



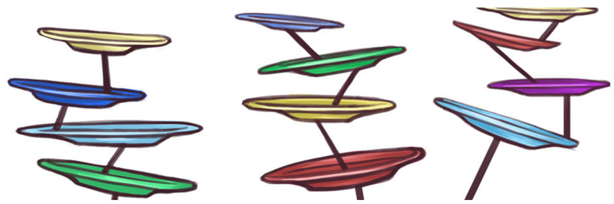
$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$30 = 2 \times 3 \times 5$

GCF = $\mathbf{2} \times \mathbf{3} = 6$

The GCF of 12, 24, and 30 is 6.



5.5: Multiples

Find multiples of a given number.

Suppose you go to get your hair cut every six weeks. You could create a list for your haircut schedule for the next eight haircuts. In order to do this you would have to list the first eight multiples of the number 6. A multiple of a number is the product of the number and a whole number other than 0.

Haircut Schedule:								
Haircut	1	2	3	4	5	6	7	8
Weeks	6	12	18	24	30	36	42	48

Therefore, the first eight multiples of 6 are 6, 12, 18, 24, 30, 36, 42, and 48.

Example 1

List the first six multiples of 12.

Number	1	2	3	4	5	6
Multiple	12	24	36	48	60	72

Therefore, the first six multiples of 12 are 12, 24, 36, 48, 60, and 72.

Example 2

List the first five multiples of 15.

Number	1	2	3	4	5
Multiple	15	30	45	60	75

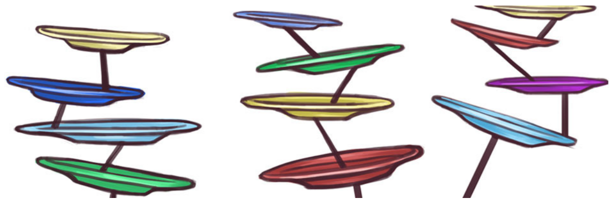
Therefore, the first five multiples of 15 are 15, 30, 45, 60, and 75.

Example 3

List the first seven multiples of 6.

Number	1	2	3	4	5	6	7
Multiple	6	12	18	24	30	36	42

Therefore, the first seven multiples of 6 are 6, 12, 18, 24, 30, 36, and 42.



5.6: Least Common Multiple

Find the least common multiple of two whole numbers.

Suppose you and a friend are walking laps on the local track. You are walking at a pace such that each lap takes 6 minutes, whereas your friend is walking at a pace where each lap takes 8 minutes. We can find the time it will take you and your friend to walk across the starting line together by finding the least common multiple of the two lap times. The *least common multiple* (LCM) is the smallest number that two other numbers will divide into. Let's find the LCM of 6 and 8.

LAP	1	2	3	4	5	6
YOUR TIME (IN MINUTES)	6	12	18	24	30	36
FRIEND'S TIME (IN MINUTES)	8	16	24	32	40	48

The LCM of 6 and 8 is 24.

Therefore, you and your friend will both cross the starting line together at 24 minutes. There are two methods to find the least common multiple. One method is to list the multiples of each number and then find the lowest one they have in common. This is the method we used in the example above. Another method is to use the prime factorization of each of the numbers and find the product of the different prime factors that appear the greatest number of times. Let's try some examples of both methods.

Example 1

Find the LCM of 75 and 100 by making a list of multiples.

Multiples of 75: 75, 150, 225, **300**, 375, 450

Multiples of 100: 100, 200, **300**, 400, 500, 600

The LCM of 75 and 100 is 300.

Example 2

Find the LCM of 12 and 20 by making a list of multiples.

Multiples of 12: 12, 24, 36, 48, **60**, 72

Multiples of 20: 20, 40, **60**, 80, 100, 120

The LCM of 12 and 20 is 60.

Example 3

Find the LCM of 5, 6 and 8 by making a list of multiples.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, **120**

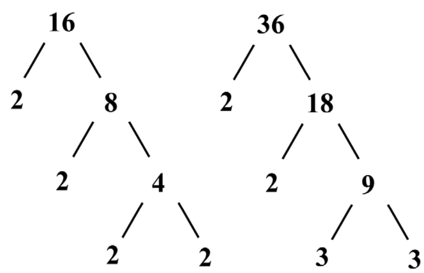
Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, **120**

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, **120**

The LCM of 5, 6 and 8 is 120.

Example 4

Find the LCM of 16 and 36 using the prime factorization.



16 = 2 × 2 × 2 × 2

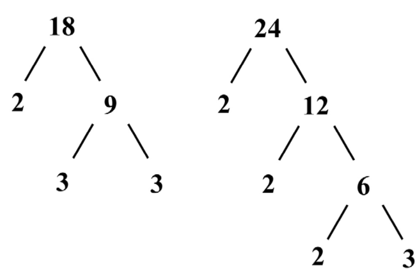
36 = 2 × 2 × 3 × 3

LCM = **2 × 2 × 2 × 2 × 3 × 3 = 144**

The LCM of 16 and 36 is 144.

Example 5

Find the LCM of 18 and 24 using the prime factorization.



18 = 2 × 3 × 3

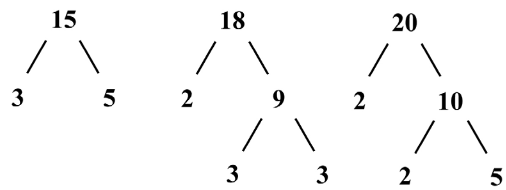
24 = 2 × 2 × 2 × 3

LCM = **2 × 2 × 2 × 3 × 3 = 72**

The LCM of 18 and 24 is 72.

Example 6

Find the LCM of 15, 18 and 20 using the prime factorization.



$15 = 3 \times 5$

$18 = 2 \times 3 \times 3$

$20 = 2 \times 2 \times 5$

LCM = $\mathbf{2 \times 2 \times 3 \times 3 \times 5 = 180}$

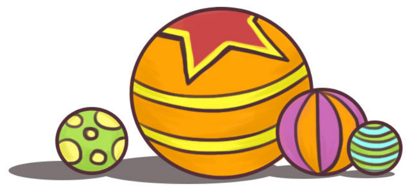
The LCM of 15, 18 and 20 is 180.

Unit 6
Measurement

In this lesson you will learn how to use the order of operations to solve problems. You will also use formulas to find the area of geometric figures and the surface area of geometric solids. You will also learn how to use and apply measurement concepts to find the perimeter of geometric figures. Finally, you will learn how to apply these concepts to solve real-life problems.

Learning Outcomes

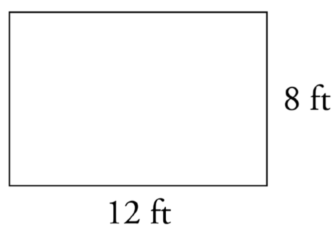
1. Use the order of operations to solve problems and use formulas. Also, find the area and perimeter of rectangles and squares.
2. Find the area and perimeter of parallelograms.
3. Find the area and perimeter of triangles.
4. Find the area and perimeter of trapezoids.
5. Find the area and circumference of circles.
6. Find the surface area of rectangular prisms.



6.1: Area and Perimeter of Rectangles

Use the order of operations to solve problems and use formulas. Also, find the area and perimeter of rectangles and squares.

Suppose you wanted to plant a rectangular garden with the dimensions 12 feet by 8 feet. You are planning on putting a wood border around the edge of the garden. First, you would need to find the area of the garden so you could estimate how much soil to order. Next, you would have to find the perimeter of the garden so you could calculate the amount of wood to buy for the border. Since the dimensions of the garden are 12 feet by 8 feet, the length is 12 feet, and the width is 8 feet. Notice the diagram of the garden below.



The *area* of a geometric figure is the number of square units inside the figure, whereas the *perimeter* of a geometric figure is the distance around the figure. There are different formulas to find the area and perimeter of various types of geometric figures. The formulas for the area and perimeter of rectangles are given below.

Area and Perimeter of Rectangles:
Area = length × width
A = l × w
Perimeter = 2 × (length + width)
P = 2 × (l + w)

In solving a problem such as this one, the first step is to define the variables. A *variable* is a letter that represents a number. The variables in the area and perimeter formulas are l and w. Therefore, $l = 12$ and $w = 8$.We can take these dimensions and substitute them into each formula to find the area and perimeter of the garden. Another important step to solving this problem is to understand that the order in which we do the problem matters. There is a set of rules defined so that people will do problems in the same order. These rules are called the *order of operations*. Notice the rules below.

Order of Operations
1) Operations in parentheses
2) Multiplication and division
3) Addition and subtraction

1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

$\boldsymbol{l}$ = 12 ft.
$\boldsymbol{w}$ = 8 ft.
$\boldsymbol{A = l \times w}$
$A = 12 \times 8$
A = 96 square ft.

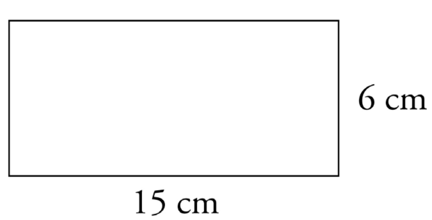
1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Remember to add inside the parentheses first.
4. Solve.

$\boldsymbol{P = 2 \times (l + w)}$
$P = 2 \times (12 + 8)$
$P = 2 \times 20$
P = 40 ft.

The area of the garden is 96 square feet, and the perimeter is 40 feet. Notice that the units for the area of the garden are in square feet, because area measures the number of square units in a geometric figure. On the other hand, the units for the perimeter of the garden are in feet, because perimeter measures the distance around a geometric figure. Let's try some further examples.

Example 1

Find the area and perimeter of the rectangle.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

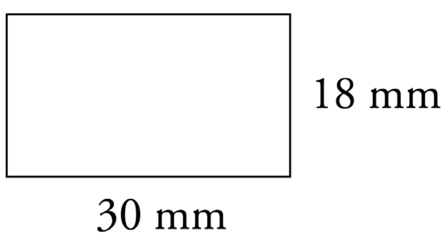
l = 15 cm
w = 6 cm
A* = *l* × *w
A = 15 × 6
A = 90 square cm

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Remember to add inside the parentheses first.
4. Solve.

P* = *2* × *(l* + *w)
P = 2 × (15 + 6)
P = 2 × 21
P = 42 cm
The area is 90 square cm, and the perimeter is 42 cm.

Example 2

Find the area and perimeter of the rectangle.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

l = 30 mm
w = 18 mm
A* = *l* × *w
A = 30 × 18
A = 540 square mm

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Remember to add inside the parentheses first.

4. Solve.

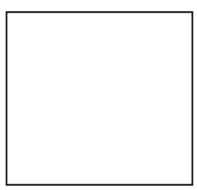
***P* = 2 × (*l* + *w*)**
$P = 2 \times (30 + 18)$
$P = 2 \times 48$
$P = 96$ mm
The area is 540 square mm, and the perimeter is 96 mm.

A square is a rectangle in which the length and width are the same. You can find the area and perimeter of a square using the formulas for rectangles. However, it is much easier to use area and perimeter formulas for a square. These formulas are given below.

Area and Perimeter of Squares:
Area = side × side
A = s × s
Perimeter = 4 × side
P = 4 × s

Example 3

Find the area and perimeter of the square.



5 in

1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

s = 5 in.
A* = *s* × *s
$A = 5 \times 5$
$A = 25$ square in.

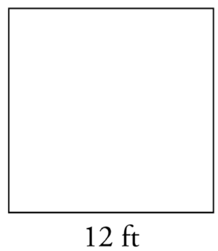
1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

P* = 4 × *s
$P = 4 \times 5$
$P = 20$ in.

The area is 25 square in., and the perimeter is 20 in.

Example 4

Find the area and perimeter of the square.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

s = 12 ft.

A* = *s* × *s

A = 12 × 12

A = 144 square ft.

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

P* = *4* × *s

P = 4 × 12

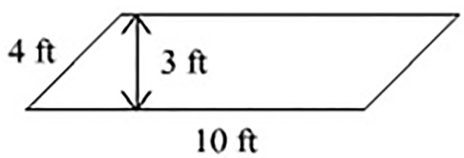
P = 48 ft.

The area is 144 square ft., and the perimeter is 48 ft.

6.2: Area and Perimeter of Parallelograms

Find the area and perimeter of parallelograms.

Suppose your school wanted to put in a diagonal walkway in front of the building. The principal decided to let your math class decide how to plan the project and calculate the estimates needed. The walkway would need a form made of wood around the edge so that you could pour cement. First, you would need to find the area of the walkway so you could estimate how much cement to order. Next, you would have to find the perimeter of the walkway so you could calculate the amount of wood to buy for the forms. You notice that the shape of the walkway is a parallelogram with dimensions 10 feet by 4 feet. However, the width of the walkway is 3 feet. Notice the diagram of the walkway below.



The area formula for a parallelogram uses the base length and the height. The base length of a parallelogram can be any side; however, it needs to be perpendicular to the height of the figure. Notice in the diagram above, the base length of 10 ft. is perpendicular to the height, which would be 3 ft. Be careful not to confuse the height with the side of the parallelogram, which is 4 ft. The area and perimeter formulas for a parallelogram are below.

Area and Perimeter of Parallelograms:
Area = base length × height
A = b × h
Perimeter = 2 × (base length + side)
P = 2 × (b + s)

1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

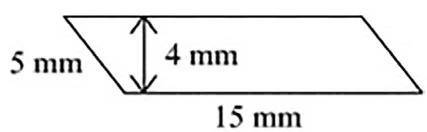
b = 10 ft.
h = 3 ft.
s = 4 ft.
A* = *b* × *h
A = 10 × 3
A = 30 square ft.

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Remember to add inside the parentheses first.
4. Solve.

P* = *2* × *(b* + *s)
P = 2 × (10 + 4)
P = 2 × 14
P = 28 ft.
The area of the walkway is 30 square ft., and the perimeter is 28 ft.
Let's try some more examples.

Example 1

Find the area and perimeter of the parallelogram.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

b = 15 mm
h = 4 mm
s = 5 mm
A* = *b* × *h
A = 15 × 4
A = 60 square mm

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Remember to add inside the parentheses first.
4. Solve.

$\boldsymbol{P = 2 \times (b + s)}$

$P = 2 \times (15 + 5)$

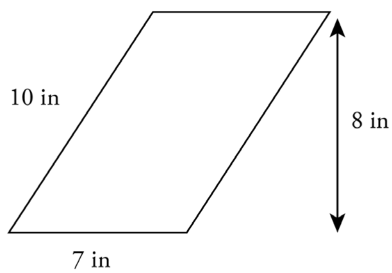
$P = 2 \times 20$

$P = 40$ mm

The area is 60 square mm, and the perimeter is 40 mm.

Example 2

Find the area and perimeter of the parallelogram.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Solve.

$\boldsymbol{b}$ = 7 in.

$\boldsymbol{h}$ = 8 in.

$\boldsymbol{s}$ = 10 in.

$\boldsymbol{A = b \times h}$

$A = 7 \times 8$

$A = 56$ square in.

1. Define the variables.
2. Substitute values into the **perimeter formula.**
3. Remember to add inside the parentheses first.
4. Solve.

$\boldsymbol{P = 2 \times (b + s)}$

$P = 2 \times (7 + 10)$

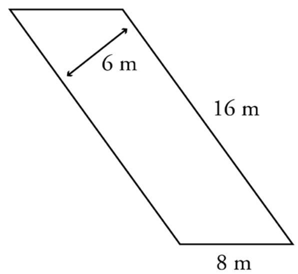
$P = 2 \times 17$

$P = 34$ in.

The area is 56 square in., and the perimeter is 34 in.

Example 3

Find the area and perimeter of the parallelogram.



1. Define the variables.
2. Substitute values into the area formula.
3. Solve.

b = 16 m
h = 6 m
s = 8 m
A* = *b* × *h
A = 16 × 6
A = 96 square m

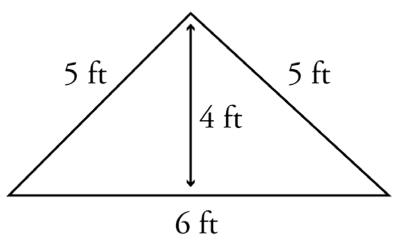
1. Define the variables.
2. Substitute values into the perimeter formula.
3. Remember to add inside the parentheses first.
4. Solve.

***P* = 2 × (*b* + *s*)**
P = 2 × (16 + 8)
P = 2 × 24
P = 48 m
The area is 96 square m, and the perimeter is 48 m.
For more information, visit the MathWizz website.

6.3: Area and Perimeter of Triangles

Find the area and perimeter of triangles.

Suppose you want to build a triangular planter with a brick border. First, you would need to find the area of the planter so you could estimate how many flowers you can plant. Next, you would have to find the perimeter of the planter so you could calculate the amount of bricks to buy for the border. The sides of the planter are 5 feet, 5 feet, and 6 feet, while the distance across the planter is 4 feet. Notice the diagram of the planter below.



The area formula of triangles is similar to that for parallelograms. The formula uses the base length and the height. The base length of a triangle can be any side; however, it needs to be perpendicular to the height of the figure. Notice in the diagram above, the base length of 6 ft. is perpendicular to the height, which is 4 ft. Be careful not to confuse the height with the sides of a triangle, which are 5 ft. The area and perimeter formulas for a triangle are below.

Area and Perimeter of Triangles:
Area = (base length × height) ÷ 2
A = (b × h) ÷ 2
Perimeter = sum of sides
P = s1 + s2 + s3

1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to multiply inside the parentheses first.
4. Solve.

b = 6 ft.

$\boldsymbol{h}$ = 4 ft.
$\boldsymbol{s_1}$ = 5 ft.
$\boldsymbol{s_2}$ = 5 ft.
$\boldsymbol{s_3}$ = 6 ft.
$\boldsymbol{A = (b \times h) \div 2}$
$A = (6 \times 4) \div 2$
$A = 24 \div 2$
$A = 12$ square ft.

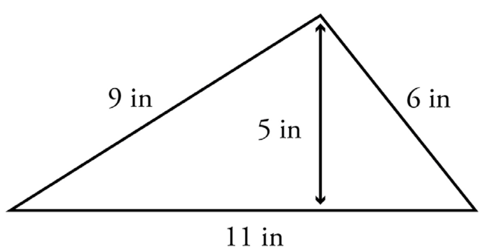
1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

$\boldsymbol{P = s_1 + s_2 + s_3}$
$P = 5 + 5 + 6$
$P = 16$ ft.

The area of the planter is 12 square ft., and the perimeter is 16 ft. Let's try some more examples.

Example 1

Find the area and perimeter of the triangle.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to multiply inside the parentheses first.
4. Solve.

$\boldsymbol{b}$ = 11 in.
$\boldsymbol{h}$ = 5 in.
$\boldsymbol{s_1}$ = 11 in.
$\boldsymbol{s_2}$ = 9 in.
$\boldsymbol{s_3}$ = 6 in.
$\boldsymbol{A = (b \times h) \div 2}$
$A = (11 \times 5) \div 2$
$A = 55 \div 2$
$A = 27.5$ square in

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

$\boldsymbol{P = s_1 + s_2 + s_3}$

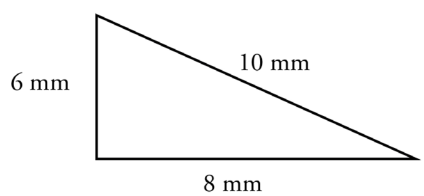
$P = 11 + 9 + 6$

$P = 26$ in

The area is 27.5 square in., and the perimeter is 26 in.

Example 2

Find the area and perimeter of the triangle.



1. Define the variables.
2. Substitute values into the area formula.
3. Remember to multiply inside the parentheses first.
4. Solve.

$\boldsymbol{b}$ = 8 mm

$\boldsymbol{h}$ = 6 mm

$\boldsymbol{s_1}$ = 6 mm

$\boldsymbol{s_2}$ = 8 mm

$\boldsymbol{s_3}$ = 10 mm

$\boldsymbol{A = (b \times h) \div 2}$

$A = (8 \times 6) \div 2$

$A = 48 \div 2$

$A = 24$ square mm

1. Define the variables.
2. Substitute values into the perimeter formula.
3. Solve.

$\boldsymbol{P = s_1 + s_2 + s_3}$

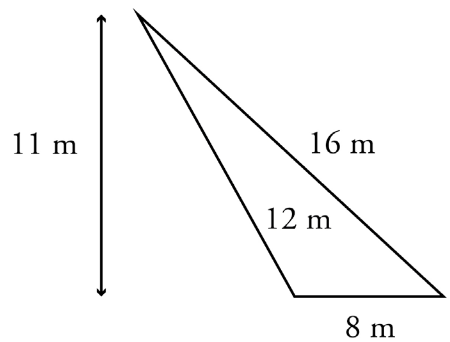
$P = 6 + 8 + 10$

$P = 24$ mm

The area is 24 square mm, and the perimeter is 24 mm.

Example 3

Find the area and perimeter of the triangle.



1. Define the variables.
2. Substitute values into the area formula.
3. Remember to multiply inside the parentheses first.
4. Solve.

$\boldsymbol{b} = 8$ m
$\boldsymbol{h} = 11$ m
$\boldsymbol{s_1} = 8$ m
$\boldsymbol{s_2} = 12$ m
$\boldsymbol{s_3} = 16$ m
$\boldsymbol{A = (b \times h) \div 2}$
$A = (8 \times 11) \div 2$
$A = 88 \div 2$
$A = 44$ square m

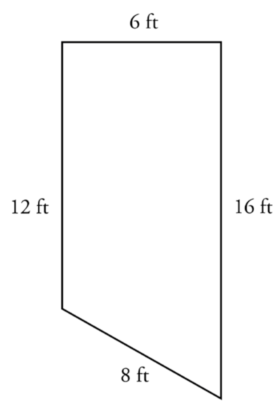
1. Define the variables.
2. Substitute values into the perimeter formula.
3. Solve.

$\boldsymbol{P = s_1 + s_2 + s_3}$
$P = 8 + 12 + 16$
$P = 36$ m
The area is 44 square m, and the perimeter is 36 m.
For more information, visit the Ask Dr. Math website.

6.4: Area and Perimeter of Trapezoids

Find the area and perimeter of trapezoids.

Suppose you wanted to add a cement pad to your existing driveway in order to have a place to park your truck. The cement pad would need a form made of wood around the edge so that you could pour cement. First, you would need to find the area of the cement pad, so you could estimate how much cement to order. Next, you would have to find the perimeter of the cement pad so you could calculate the amount of wood to buy for the forms. You notice that the shape of the cement pad is a trapezoid with the given dimensions. Notice the diagram of the cement pad below.



The area formula for a trapezoid uses both the base lengths and the height. The base lengths of a trapezoid are the parallel sides and are perpendicular to the height of the figure. Notice that in the diagram above, the base lengths are 12 ft. and 16 ft. and are perpendicular to the height, which is 6 ft. Be careful not to confuse the height with the side of the trapezoid, which is 8 ft. The area and perimeter formulas for a trapezoid are below.

Area and Perimeter of Trapezoids:
Area = height × (base length 1 + base length 2) ÷ 2
A = h × (b1 + b2) ÷ 2
Perimeter = sum of sides
P = s1 + s2 + s3 + s4

1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to add inside the parentheses first.
4. Solve.

$\boldsymbol{b_1}$ = 12 ft.
$\boldsymbol{b_2}$ = 16 ft.
$\boldsymbol{h}$ = 6 ft.
$\boldsymbol{s_1}$ = 16 ft.
$\boldsymbol{s_2}$ = 12 ft.
$\boldsymbol{s_3}$ = 8 ft.
$\boldsymbol{s_4}$ = 6 ft.
$\boldsymbol{A = h \times (b1 + b2) \div 2}$
$A = 6 \times (16 + 12) \div 2$
$A = 6 \times 28 \div 2$
$A = 168 \div 2$
A = 84 square ft.

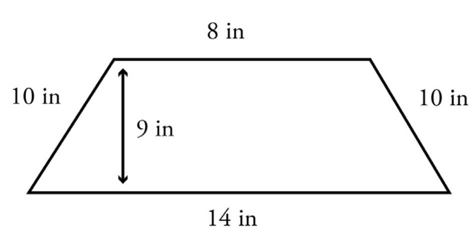
1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

$\boldsymbol{P = s1 + s2 + s3 + s4}$
$P = 16 + 12 + 8 + 6$
P = 42 ft.

The area of the cement pad is 84 square ft., and the perimeter is 42 ft. Let's try some more examples.

Example 1

Find the area and perimeter of the trapezoid.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to add inside the parentheses first.
4. Solve.

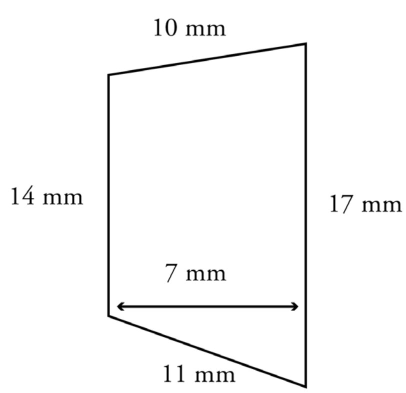
$\mathbf{b}_1$ = 8 in.
$\mathbf{b}_2$ = 14 in.
$\mathbf{h}$= 9 in.
$\mathbf{s}_1$ = 14 in.
$\mathbf{s}_2$ = 10 in.
$\mathbf{s}_3$ = 10 in.
$\mathbf{s}_4$ = 8 in.
A = h × (b1 + b2) ÷ 2
$A = 9 \times (8 + 14) \div 2$
$A = 9 \times 22 \div 2$
$A = 198 \div 2$
A = 99 square in.

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

P = s1 + s2 + s3 + s4
$P = 14 + 10 + 10 + 8$
P = 42 in.
The area is 99 square in., and the perimeter is 42 in.

Example 2

Find the area and perimeter of the trapezoid.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to add inside the parentheses first.
4. Solve.

b_1 = 14 mm
b_2 = 17 mm

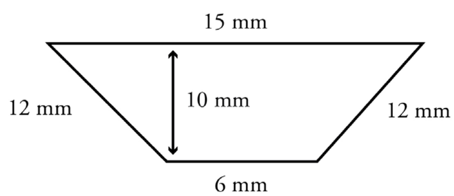
$\boldsymbol{h}$ = 7 mm
$\boldsymbol{s_1}$ = 17mm
$\boldsymbol{s_2}$ = 14 mm
$\boldsymbol{s_3}$ = 11 mm
$\boldsymbol{s_4}$ = 10 mm
$\boldsymbol{A = h \times (b1 + b2) \div 2}$
$A = 7 \times (14 + 17) \div 2$
$A = 7 \times 31 \div 2$
$A = 217 \div 2$
A = 108.5 square mm

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

$\boldsymbol{P = s1 + s2 + s3 + s4}$
$P = 17 + 14 + 11 + 10$
P = 52 mm.
The area is 108.5 square mm, and the perimeter is 52 mm.

Example 3

Find the area and perimeter of the trapezoid.



1. Define the variables.
2. Substitute values into the **area formula**.
3. Remember to add inside the parentheses first.
4. Solve.

$\boldsymbol{b_1}$ = 6 mm
$\boldsymbol{b_2}$ = 15 mm
$\boldsymbol{h}$ = 10 mm
$\boldsymbol{s_1}$ = 15 mm
$\boldsymbol{s_2}$ = 12 mm
$\boldsymbol{s_3}$ = 12 mm
$\boldsymbol{s_4}$ = 6 mm
$\boldsymbol{A = h \times (b1 + b2) \div 2}$
$A = 10 \times (6 + 15) \div 2$

$A = 10 \times 21 \div 2$

$A = 210 \div 2$

$A = 105$ square mm

1. Define the variables.
2. Substitute values into the **perimeter formula**.
3. Solve.

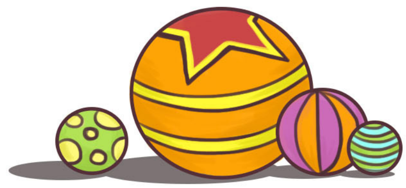
P = s1 + s2 + s3 + s4

$P = 15 + 12 + 12 + 6$

$P = 45$ mm

The area is 105 square mm, and the perimeter is 45 mm.

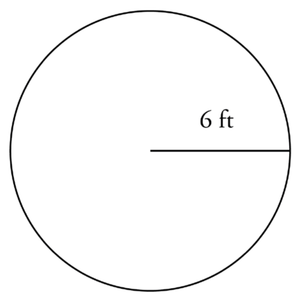
For more information, visit the Math2.org website.



6.5: Area and Circumference of Circles

Find the area and circumference of circles.

Suppose you tie your dog's 6-foot leash to a stake. We could find the area of the circle to determine how much space your dog can roam. We could find the perimeter of the circle, as well. The perimeter is called the circumference of the circle. Let's consider the diagram below.



Area and Circumference of Circles:
Area = radius × radius × π
$A = (r \times r) \times \pi$
Circumference = 2 × radius × π
$C = (2 \times r) \times \pi$
Circumference = diameter × π
$C = d \times \pi$

Remember from unit 2, a *radius* is a segment that has one endpoint on the circle and the other endpoint at the center. Also, a *diameter* is a segment that passes through the center of the circle and whose endpoints are on the circle. Notice these are in the area and circumference formulas for circles. The formulas contain the irrational number called pi.

Pi is a ratio of every circle's circumference to its diameter. It is approximately 3.14, and its symbol is π.

1. Define the variables.
2. Substitute values into the area formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{r} = 6$ ft.

$\boldsymbol{A = (r \times r) \times \pi}$

$A = 6 \times 6 \times \pi$

$A = 36\pi$ or 113 square ft.

1. Define the variables.
2. Substitute values into the circumference formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{C = (2 \times r) \times \pi}$

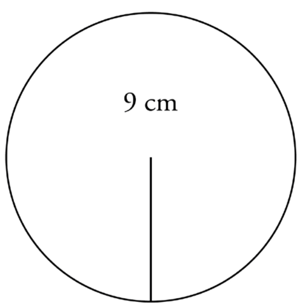
$C = 2 \times 6 \times \pi$

$C = 12\pi$ or 38 ft.

The area of land the dog can roam is approximately 113 square ft., and the perimeter is around 38 ft. Let's try some more examples.

Example 1

Find the area and circumference of the circle.



1. Define the variables.
2. Substitute values into the area formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{r} = 9$ cm.

$\boldsymbol{A = (r \times r) \times \pi}$

$A = 9 \times 9 \times \pi$

$A = 81\pi$ square cm.

1. Define the variables.
2. Substitute values into the circumference formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{C = (2 \times r) \times \pi}$

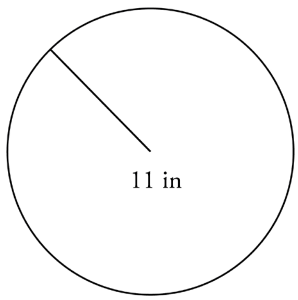
$C = 2 \times 9 \times \pi$

$C = 18\pi$ cm.

The area is 81π square cm, and the perimeter is 18π cm.

Example 2

Find the area and circumference of the circle.



1. Define the variables.
2. Substitute values into the area formula.
3. Multiply inside the parentheses first.
4. Solve.

r = 11 in.

$\boldsymbol{A = (r \times r) \times \pi}$

$A = 11 \times 11 \times \pi$

$A = 121\pi$ square cm.

1. Define the variables.
2. Substitute values into the circumference formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{C = (2 \times r) \times \pi}$

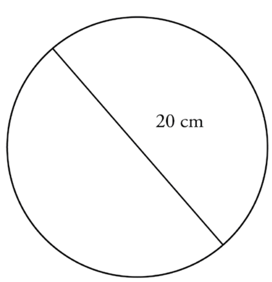
$C = 2 \times 11 \times \pi$

$C = 22\pi$ cm.

The area is 121π square in., and the circumference is 22π in.

Example 3

Find the area and circumference of the circle.



1. Define the variables.
2. Substitute values into the area formula.
3. Multiply inside the parentheses first.
4. Solve.

d = 20 cm

$\boldsymbol{r}$ = 10 cm

$\boldsymbol{A = (r \times r) \times \pi}$

$A = 10 \times 10 \times \pi$

$A = 100\pi$ square cm.

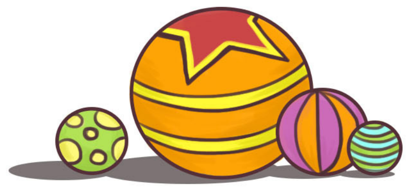
1. Define the variables.
2. Substitute values into the circumference formula.
3. Multiply inside the parentheses first.
4. Solve.

$\boldsymbol{C = (2 \times r) \times \pi}$

$C = 2 \times 10 \times \pi$

$C = 20\pi$ cm.

The area is 100π square cm, and the perimeter is 20π cm.



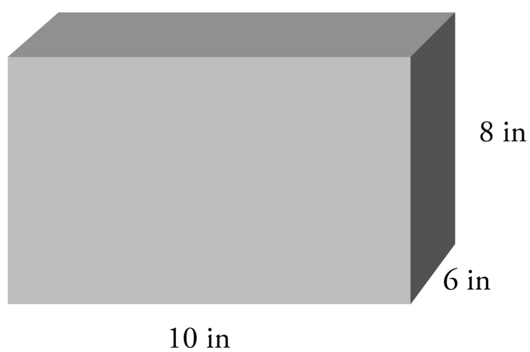
6.6: Surface Area of Prisms

Find the surface area of rectangular prisms.

Suppose you were wrapping a gift box and you weren't sure how much wrapping paper you needed. You could calculate this by finding the surface area of the gift box. Another name for the gift box is a rectangular prism. Notice the rectangular prism has 6 sides. These are known as faces. The surface area is the sum of the area of the faces of the rectangular prism. The formula for the surface area of a rectangular prism is below.

Surface Area of Rectangular Prisms:
SA = (2 × l × w) + (2 × w × h) + (2 × l × h)
where l = length, w = width, and h = height

You measure the gift box and find out the dimensions are 10 by 6 by 8. The length is 10 in., the width is 6 in., and the height is 8 in. Notice the diagram below.



1. Define the variables.
2. Remember to multiply before you add.

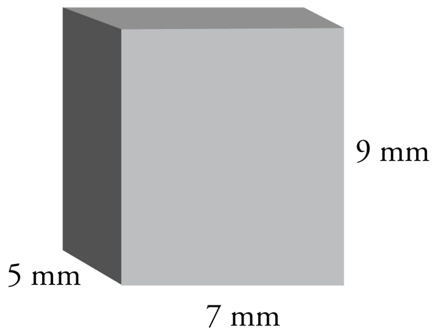
l = 10 in.
w = 6 in.
h = 8 in.
$\boldsymbol{SA = (2 \times l \times w) + (2 \times w \times h) + (2 \times l \times h)}$
$SA = (2 \times 10 \times 6) + (2 \times 6 \times 8) + (2 \times 10 \times 8)$
$SA = 120 + 96 + 160$

SA = 376 square in.

The surface area of the gift box is 376 square inches. Let's try some further examples.

Example 1

Find the surface area of the rectangular prism.



1. Define the variables.
2. Remember to multiply before you add.

l = 7 mm
w = 5 mm
h = 9 mm
$\boldsymbol{SA = (2 \times l \times w) + (2 \times w \times h) + (2 \times l \times h)}$
$SA = (2 \times 7 \times 5) + (2 \times 5 \times 9) + (2 \times 7 \times 9)$
$SA = 70 + 90 + 126$
SA = 286 square mm
The surface area is 286 square mm.

Example 2

Find the surface area of the rectangular prism with length = 6 cm, width = 10 cm, and height = 15 cm.

1. Define the variables.
2. Remember to multiply before you add.

l = 6 cm
w = 10 cm
h = 15 cm
$\boldsymbol{SA = (2 \times l \times w) + (2 \times w \times h) + (2 \times l \times h)}$
$SA = (2 \times 6 \times 10) + (2 \times 10 \times 15) + (2 \times 6 \times 15)$
$SA = 120 + 300 + 180$
SA = 600 square cm.
The surface area is 600 square cm.