LAKSHYA BATCH

EM Waves
Maxwell Equations And Displacement Current

LECTURE - 1

GOALS OF THE DAY

- Maxwell Equations
- Displacement Current

The best time to plant a tree was 20 years ago.
The second best time is now.

Antar Pantar
Ve
Antar Pantariya.

Maxwell Equations

These are four Set of Equation which Can Completely Explain Electro-Magnetic theory

(EMT)

1. Gauss Law of Electrostats

Electric flux through a closed Surface $= \dfrac{q_i}{\varepsilon_0}$

$$\boxed{\phi_E = \oint_S \vec{E}\cdot\vec{A} = \frac{Q_i}{\varepsilon_0}}$$

2.

Gauss Law Of Magnetostatics

Magnetic flux through a Surface = 0

$$\phi_B = \int \bar{B}\,\bar{A} = 0 = \mu_0 (m_{in})$$

(Monopoles do not Exist)

$N = +m$

$S = -m$

3.

Faraday's Law

Whenever there is change in Magnetic flux, there is an induced EMF

$$EMF = \left|\frac{d\phi}{dt}\right|.$$

$$E = -\frac{dV}{dr}$$

$$\int -\vec{E}\,\vec{dr} = \int dV$$

$$-\int \vec{E}\cdot\vec{dr} = V.$$

$$\boxed{\int \vec{E}\cdot\vec{dr} = -\frac{d\phi}{dt}}$$

4. Ampere Circuital Law

Line Integral of $\vec{B}$ across a Closed loop = $\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_i)}$

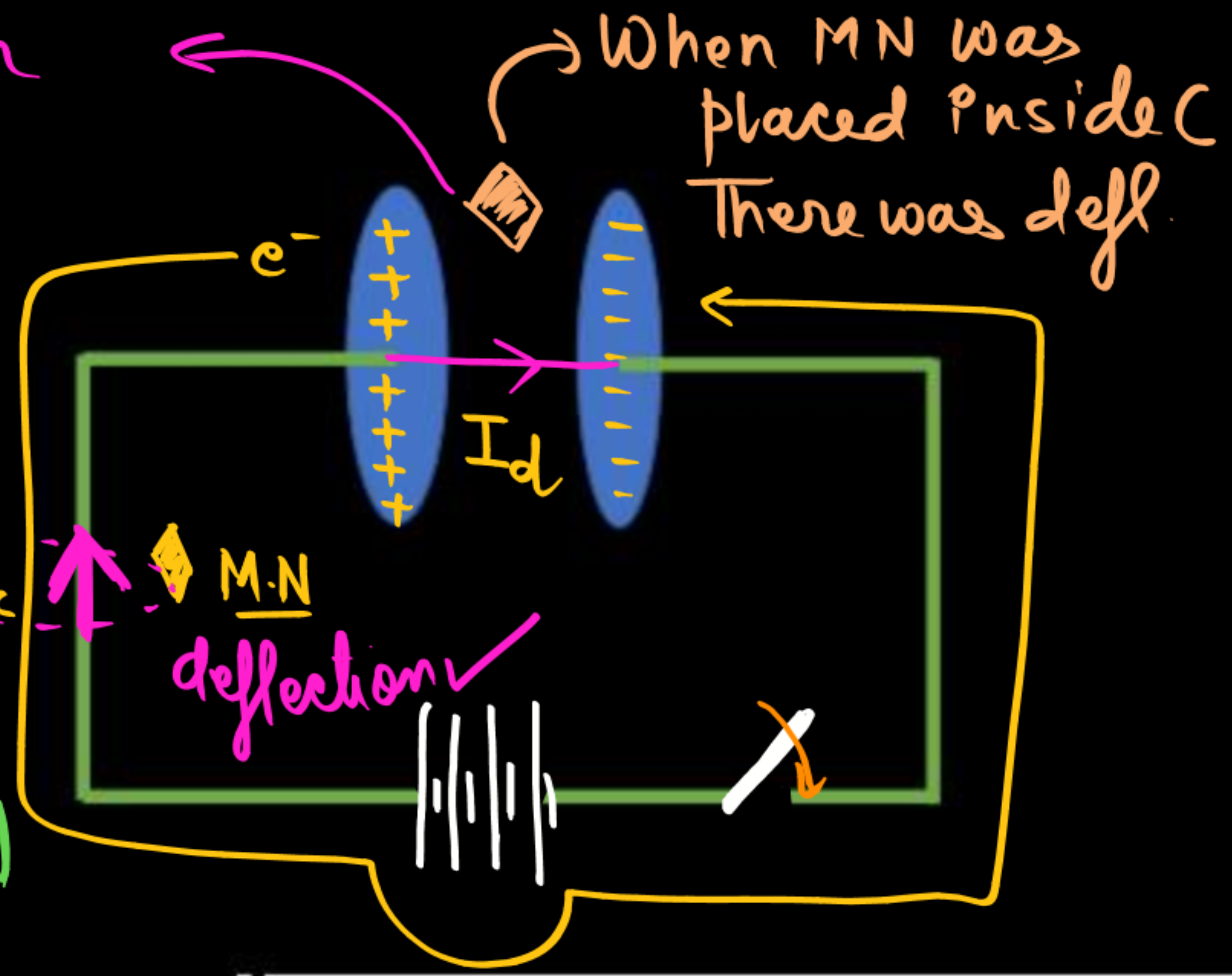
This Region also have MF

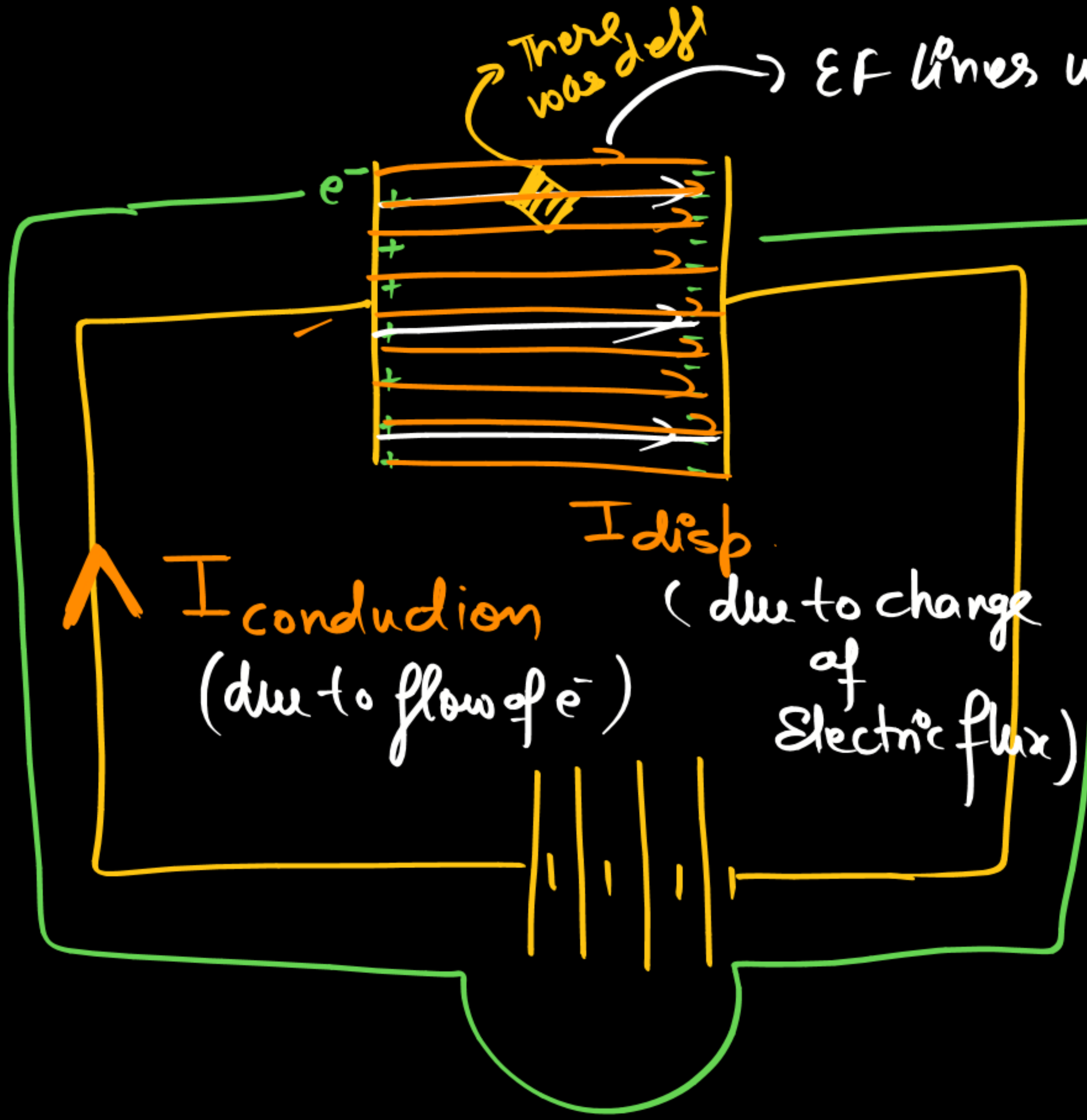
Experimental Issue which led to New Approach

When MN was placed between plates of C there was deflection.

There are two Current in CKT $I_{conduction}$

$I_{displacement}$





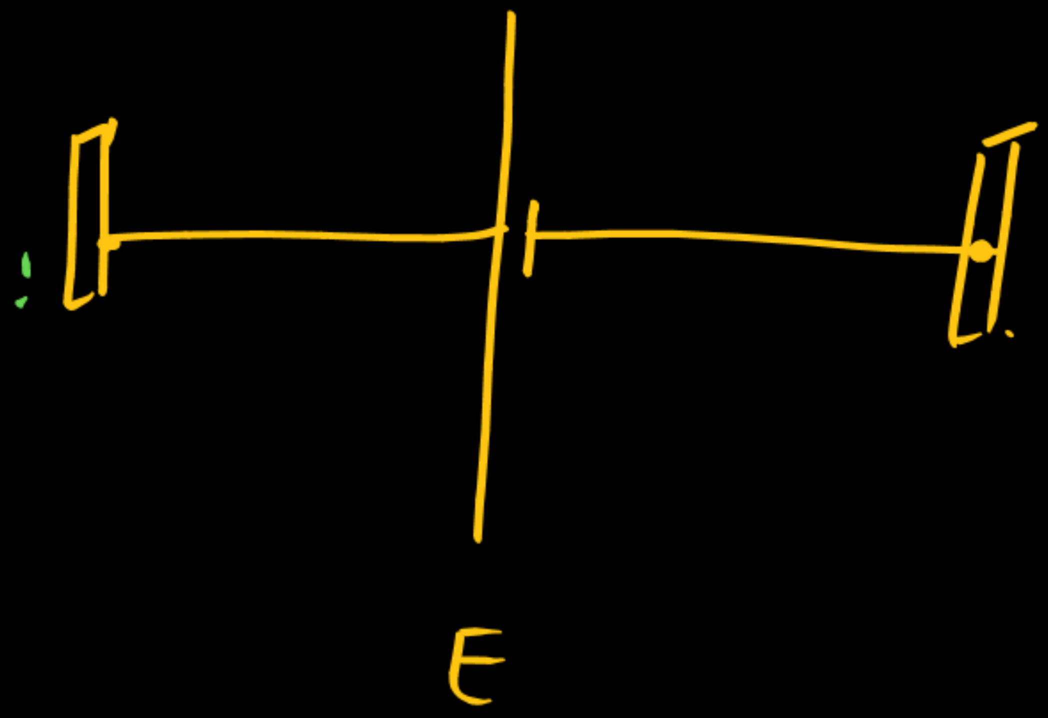
EF lines will increase with time.

In this Region EF is Variable (with time)

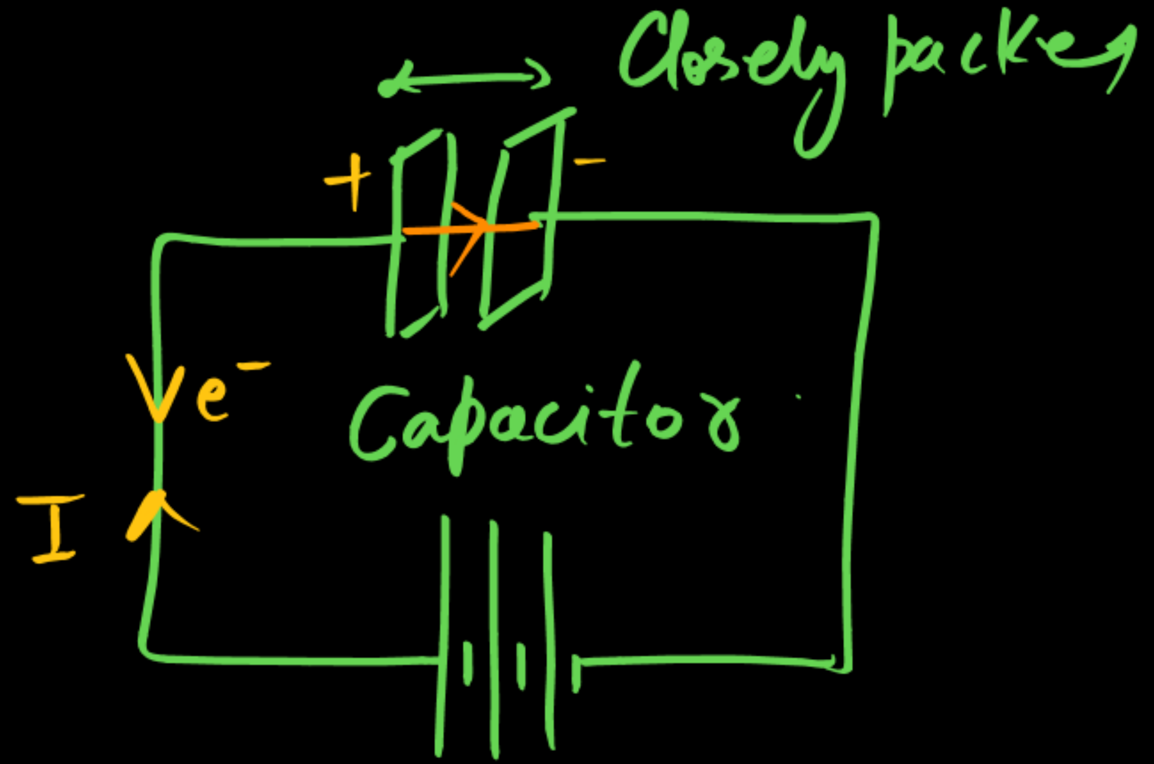
This is developing a Current between plates of Capacitor → $I_{displacement}$

due to Continuity

$$\boxed{I_{cond} = I_{disp}}$$



No



This is Not an open ckt.

Concept of Displacement Current

(*) Whenever there is change in Electric flux, there is an induced MF due to development of Current. Which is Called disp Current.

$$\phi_E = \frac{q}{\varepsilon_0}$$

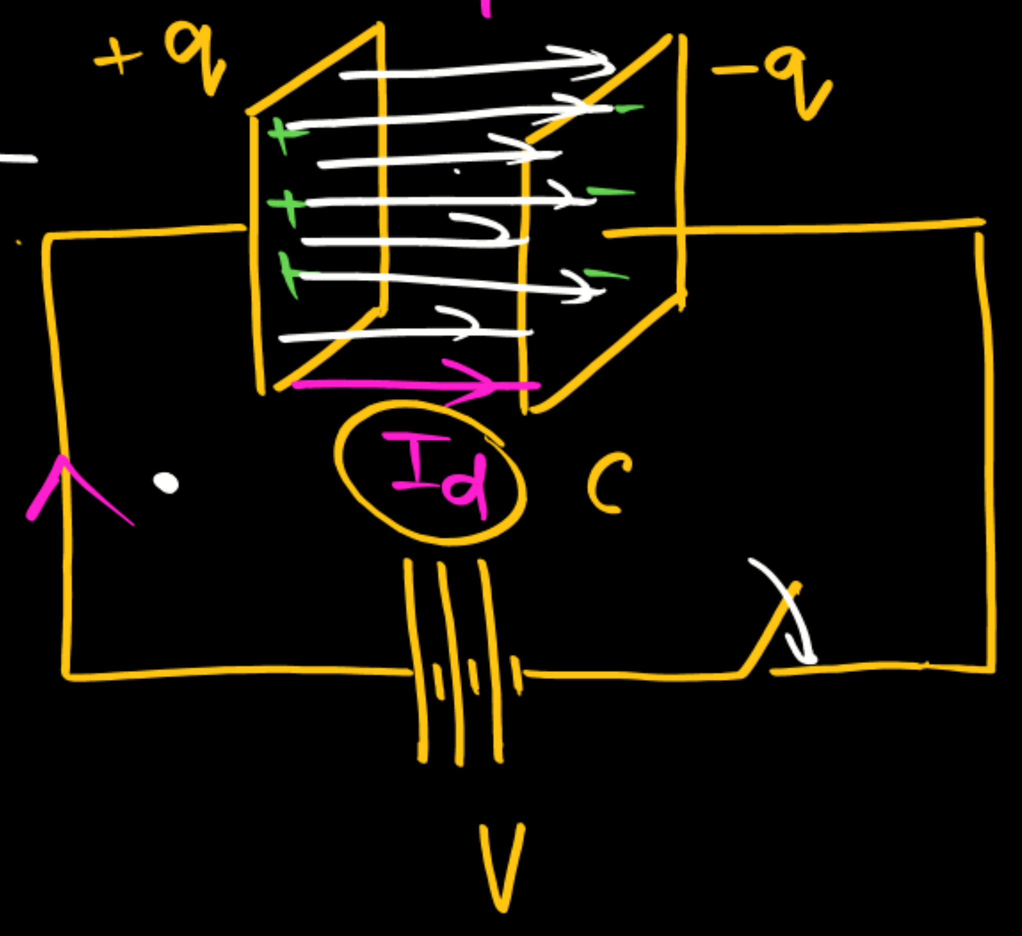
$$q = \varepsilon_0 \phi_E$$

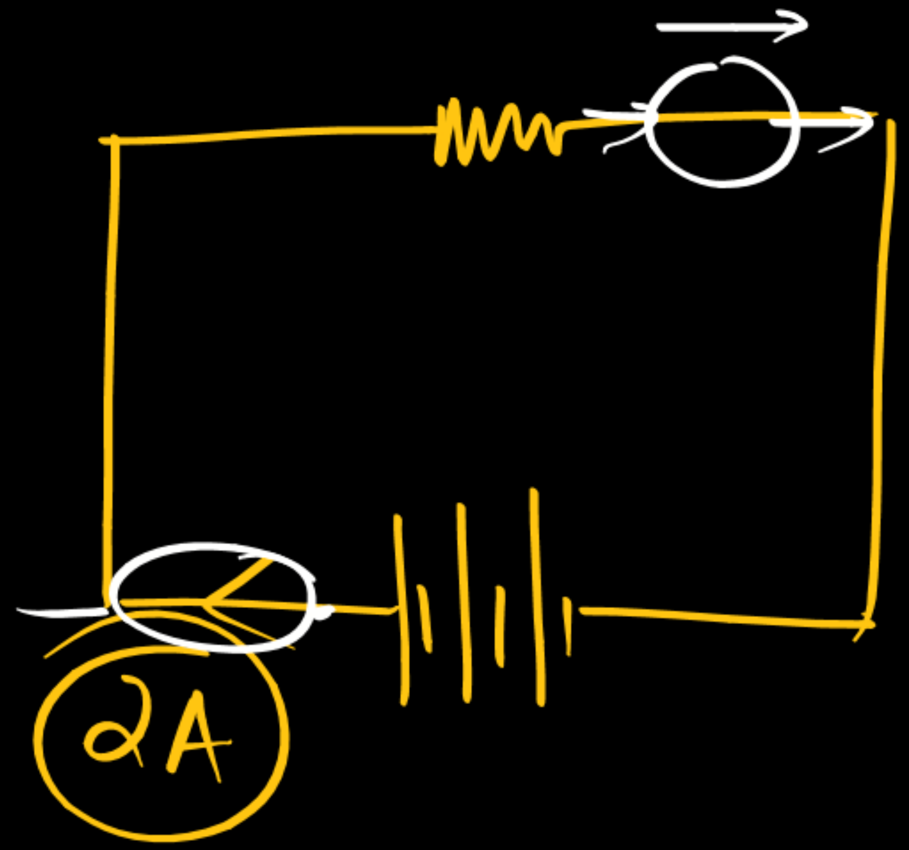
diff wrt time

$$\frac{dq}{dt} = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\boxed{I_c = I_d = \varepsilon_0 \frac{d\phi_E}{dt}}$$

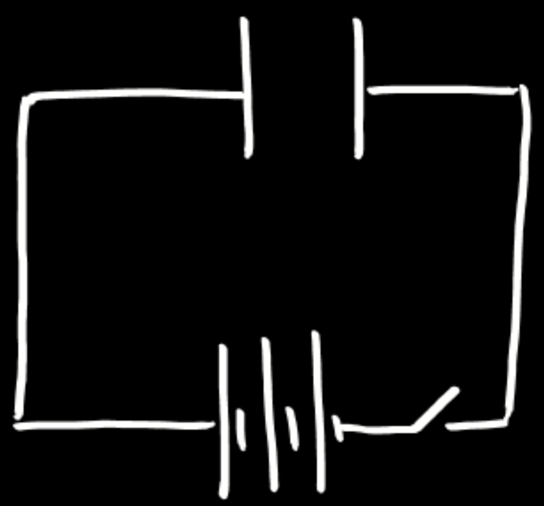
with time No of EFL ↑.



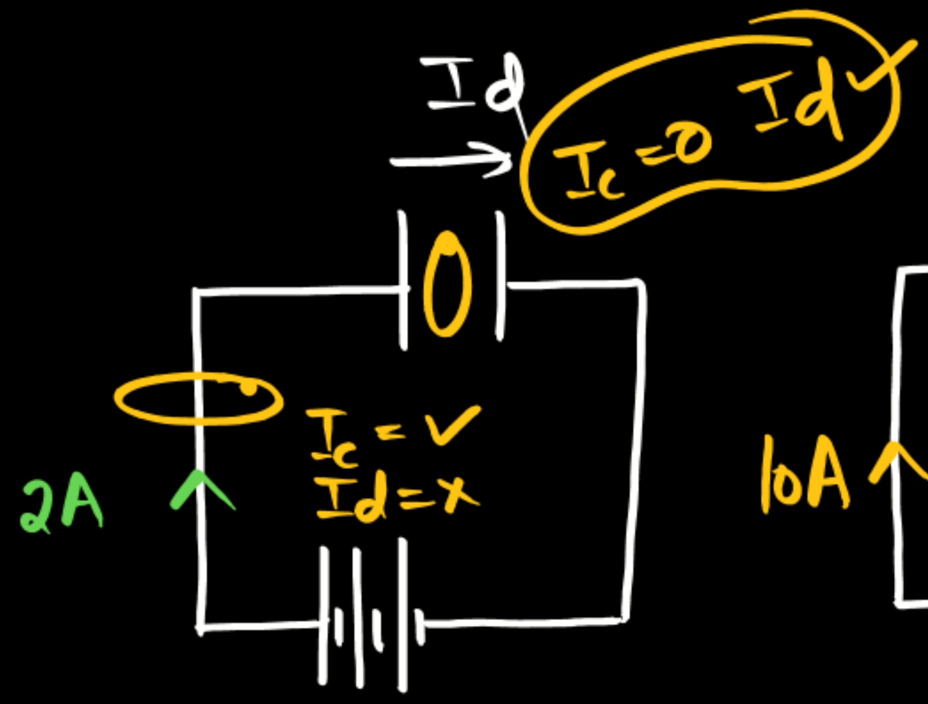


2A

(*) There is a Continuity of Current in a CKt.

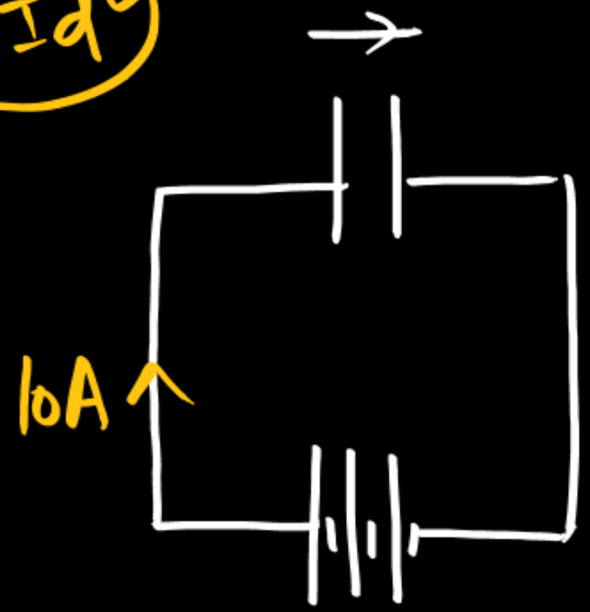


$I_d = I_c = 0$



$I_c = 2A$

$I_d = 2A$



$I_c = 10A$

$I_d = 10A$

So Modified Ampere Circuital Law.

Maxwell chachaji bole Apna bhi Kuch Karna Padega warna Amar nhi ho payenge!

$$\Rightarrow \int \vec{B} \cdot \vec{dl} = \mu_0 (I_c + I_d)$$

$$I_c = I_d$$

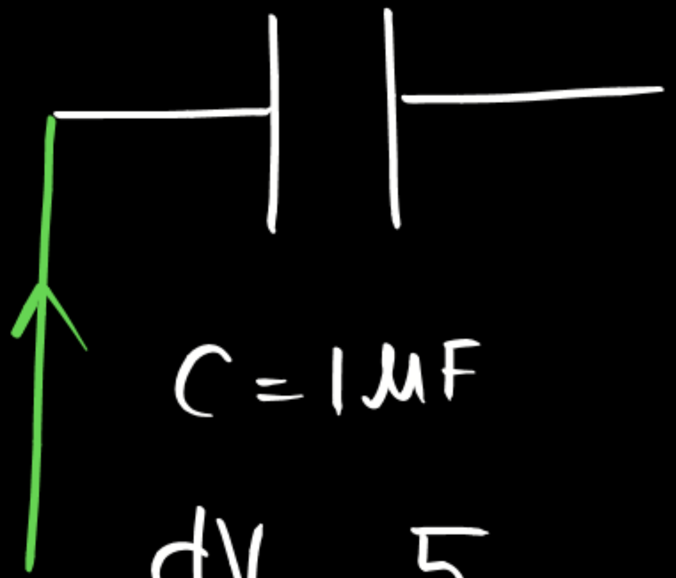
$$\Rightarrow \int \vec{B} \cdot \vec{dl} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Ex.: The Voltage between plates of Parallel plate Capacitor is 1μF is changing at rate 5 V/s find displacement Current.

a) 5μA b) 10μA c) 15μA d) Zero.

Ans

Sol:-



$\frac{dq}{dt} = I_c$

$C = 1\mu F$

$\frac{dV}{dt} = 5$

$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

I_c = आसानी से Calculate Karenge.

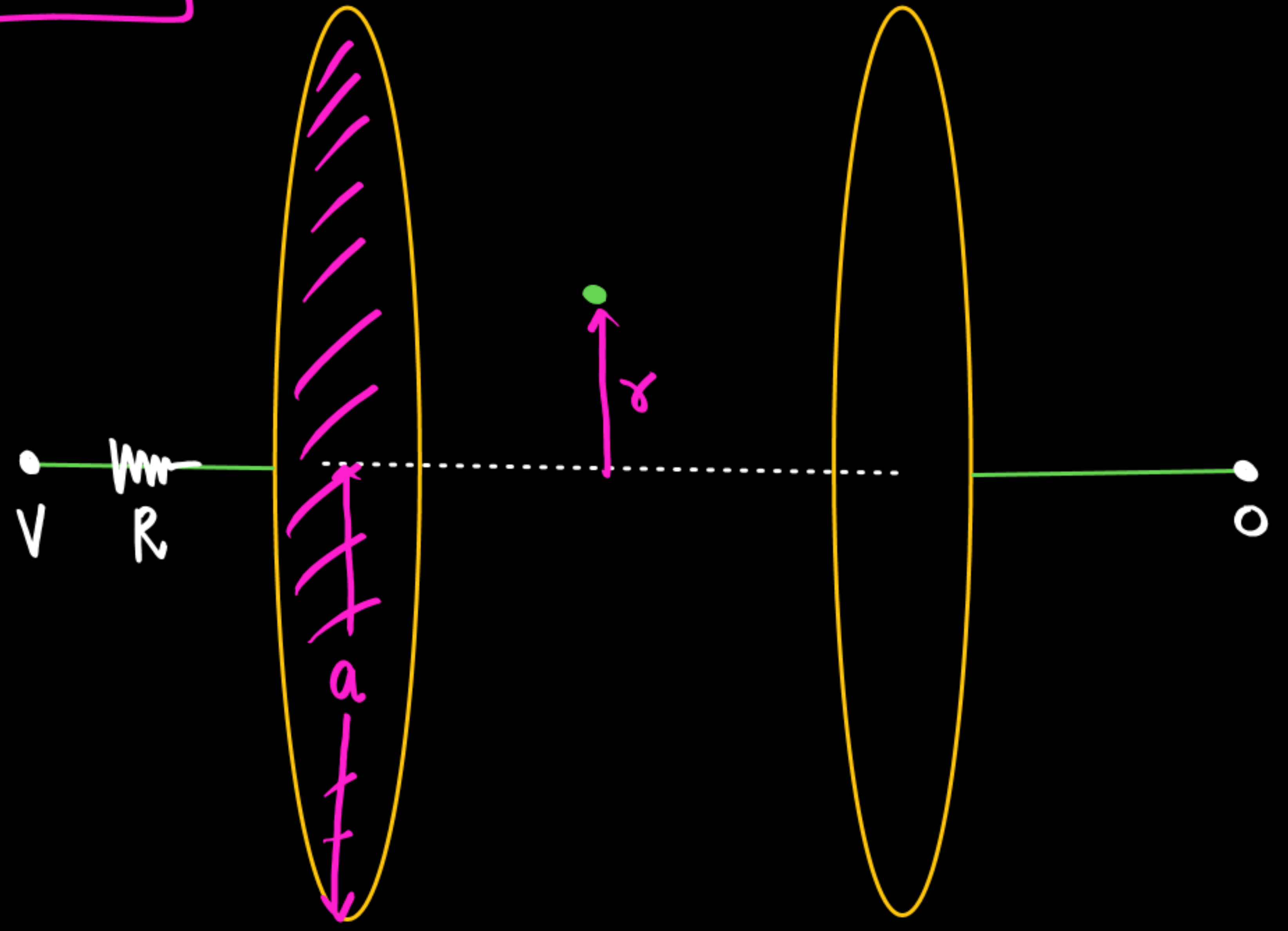
$= \frac{dq}{dt} = \frac{d}{dt} CV = C\frac{dV}{dt}$

$= 10^{-6} \times 5 \; A$

Que. A Capacitor C is Connected with battery of Voltage V. If Resistance of ckt is R. Find Magnetic field at distance r from axis of plate. take plate Area = $\boxed{A = \pi a^2}$

Adv

(*) Draw Variation of B with r.

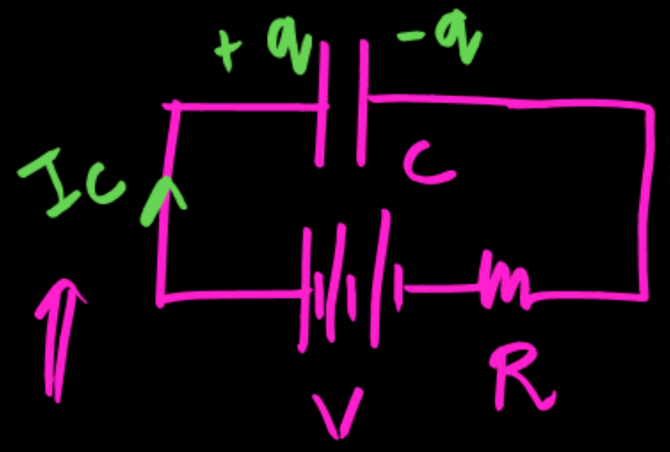


In RC Ckt

Charge on Capacitor = $q = CV(1-e^{-t/RC})$

$$\boxed{\tau = RC}$$

$$q = Q_{max}(1-e^{-t/\tau}).$$



$t = 0$

$q = 0$

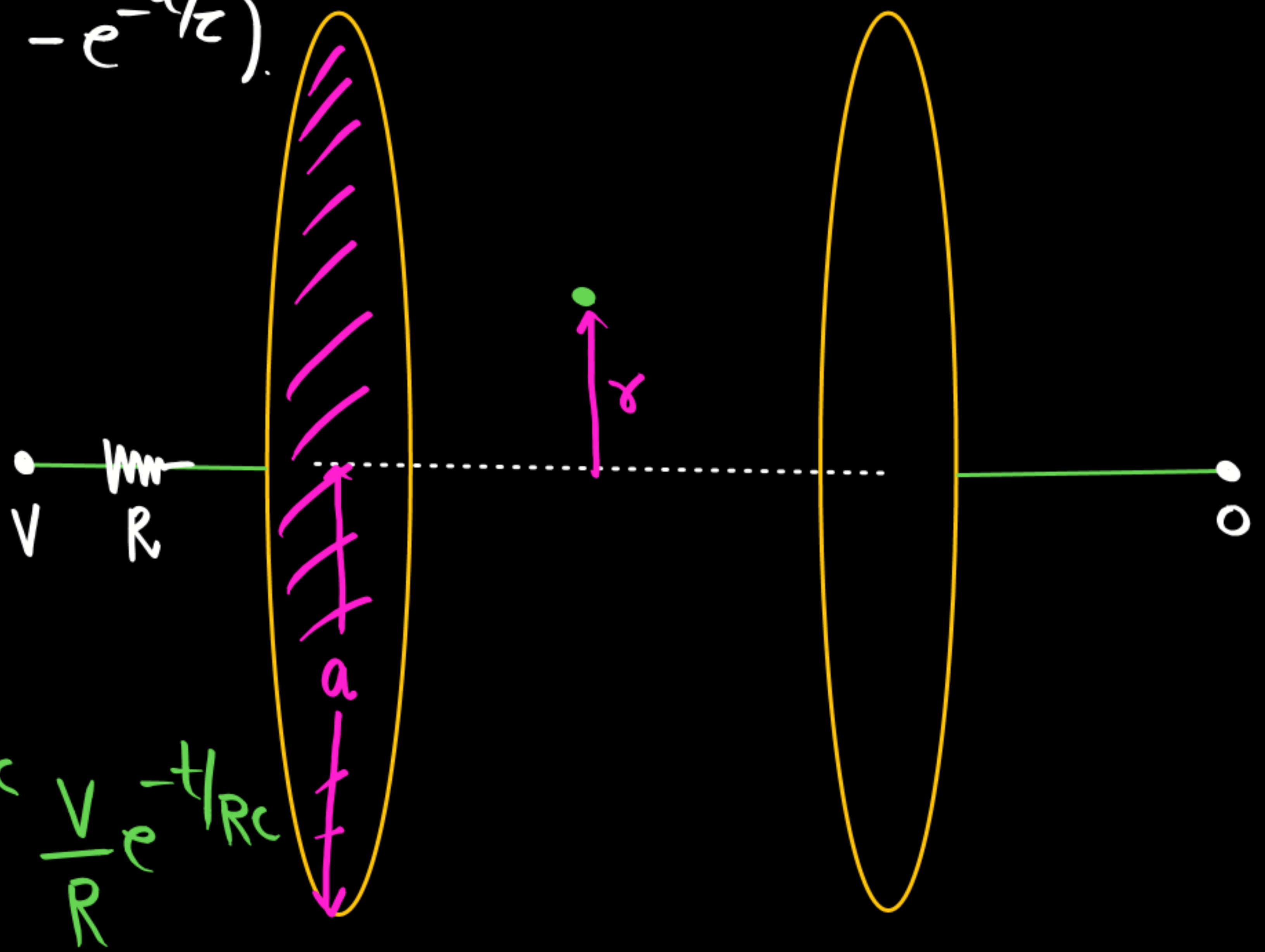
(5τ)

Steady State

$t = \infty$

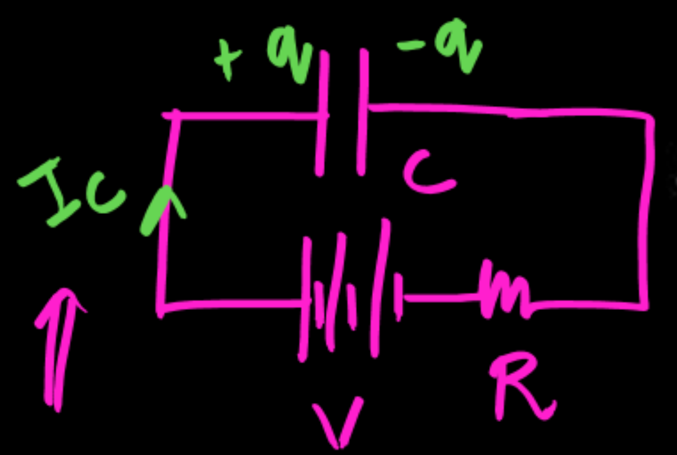
$q = Q_{max} = CV.$

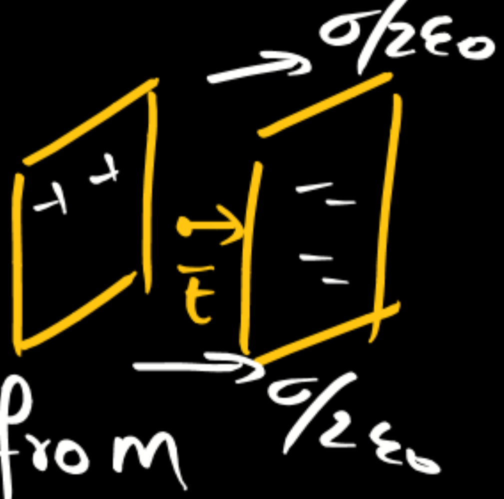
"With time charge on Capacitor is Varying" $I_c = \frac{dq}{dt} = \frac{CVe^{-t/RC}}{RC} = \frac{V}{R}e^{-t/RC}$



Charge on plate $q = CV(1 - e^{-t/RC})$.

due to this EF between plates will vary.



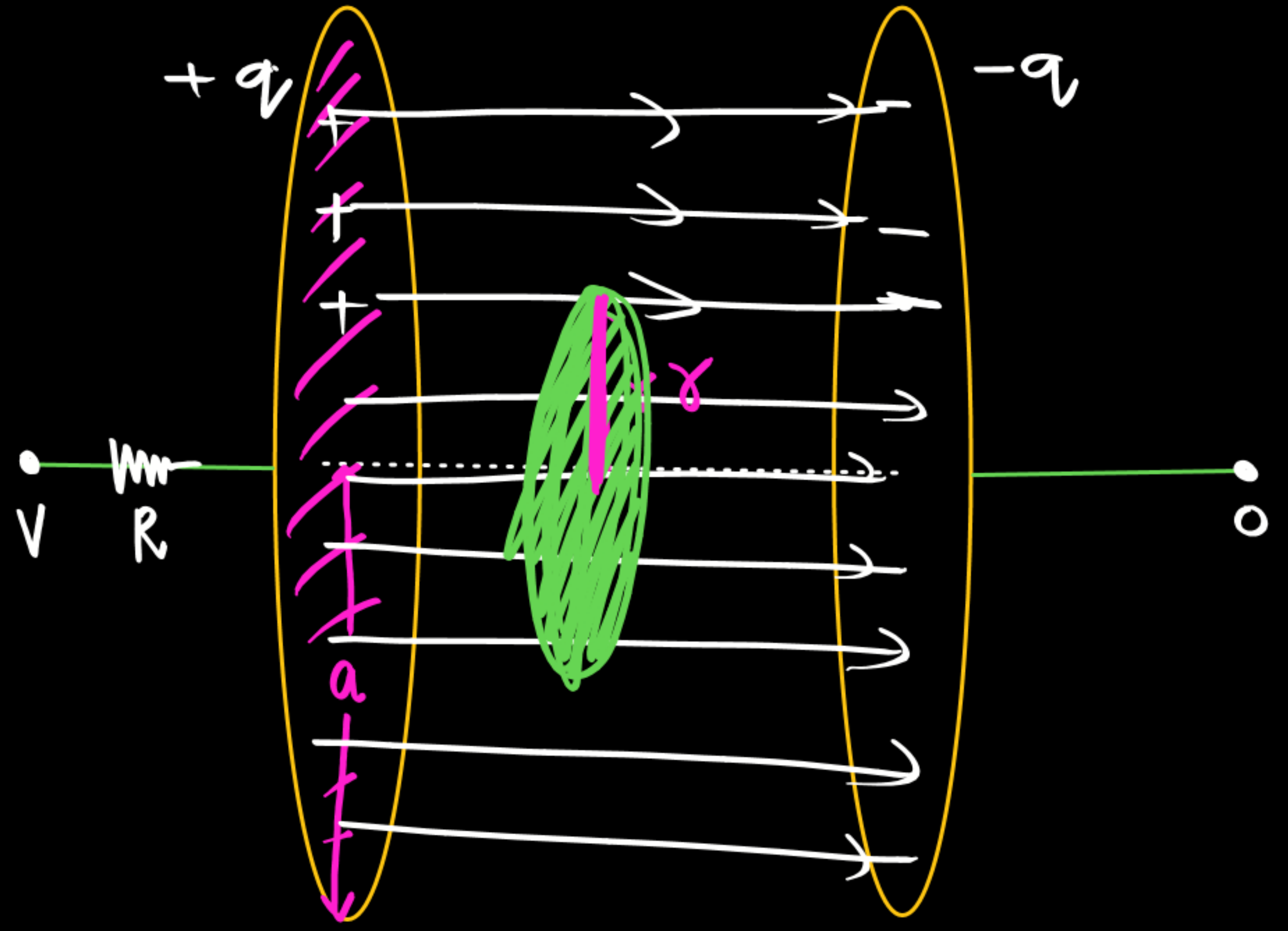


from Capacitor

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

$$\boxed{E = \frac{CV(1 - e^{-t/RC})}{\varepsilon_0 A}}$$

EF will vary with time



$$EF = \frac{CV}{\varepsilon_0 A}\left(1-e^{-t/RC}\right).$$

Flux through Green Area

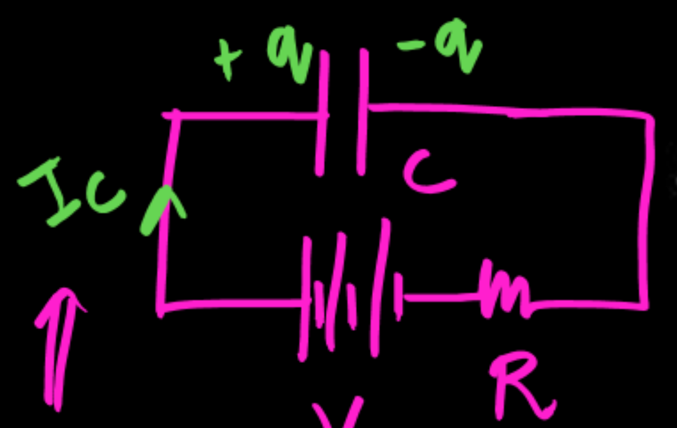
$$\phi_E = E \cdot A_{loop}$$

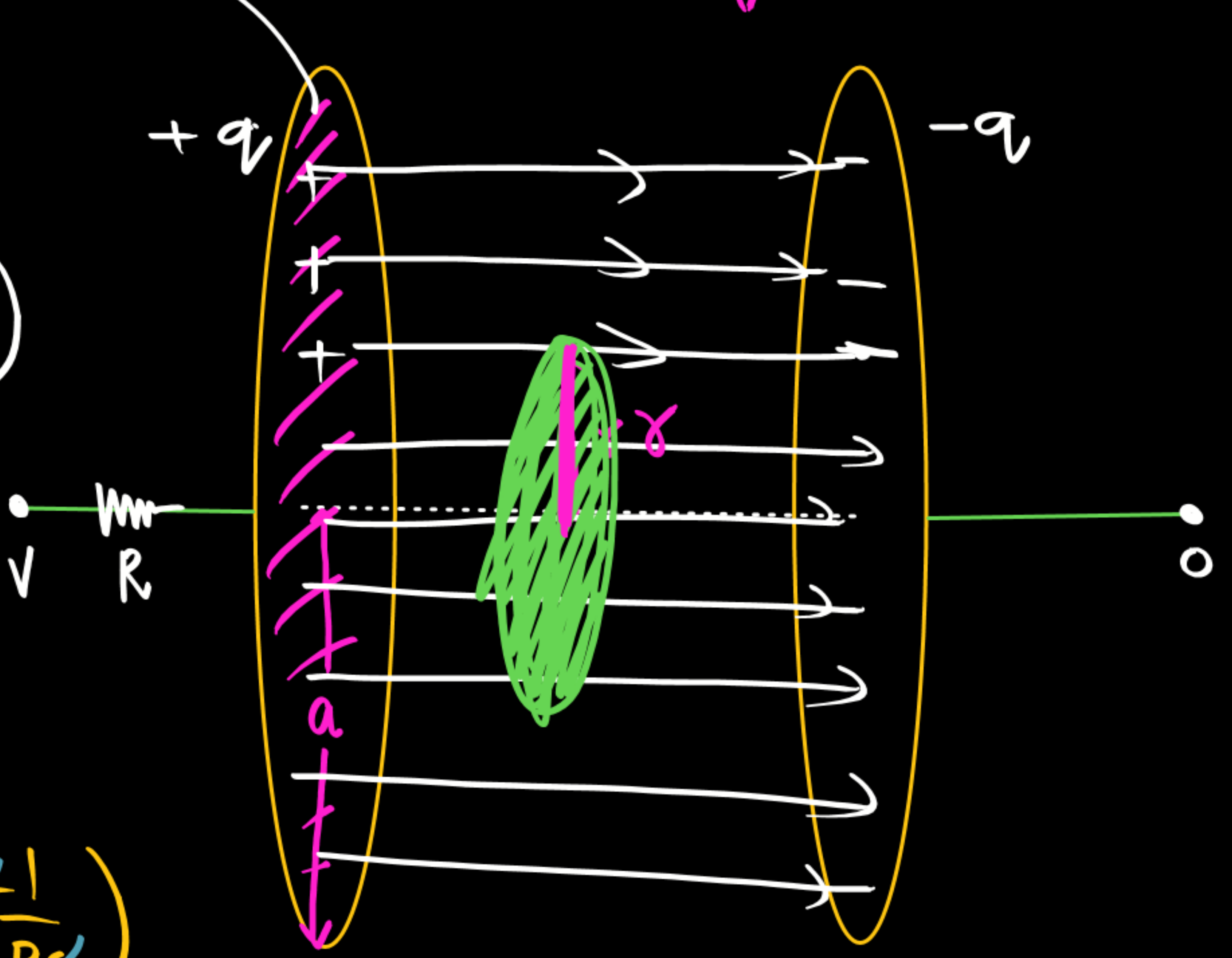
$$= \frac{CV}{\varepsilon_0 \cancel{\pi} a^2} \cdot \cancel{\pi} r^2 \left(1-e^{-t/RC}\right)$$

ε flux time flun

$$\rightarrow \phi_E = \frac{CV}{\varepsilon_0 a^2} r^2 \left(1-e^{-t/RC}\right).$$

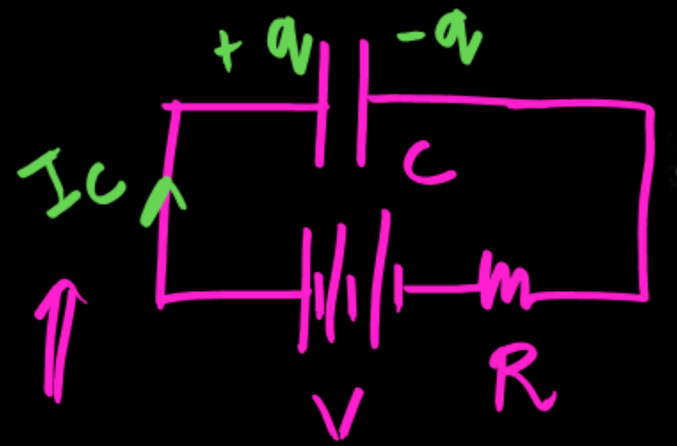
$$\boxed{I_d = \varepsilon_0 \frac{d\phi_E}{dt}} = \frac{\cancel{\varepsilon_0} \cancel{C} V r^2}{\cancel{\varepsilon_0} a^2}\left(\cancel{-}e^{-t/RC} \times \frac{\cancel{-}1}{R\cancel{C}}\right)$$

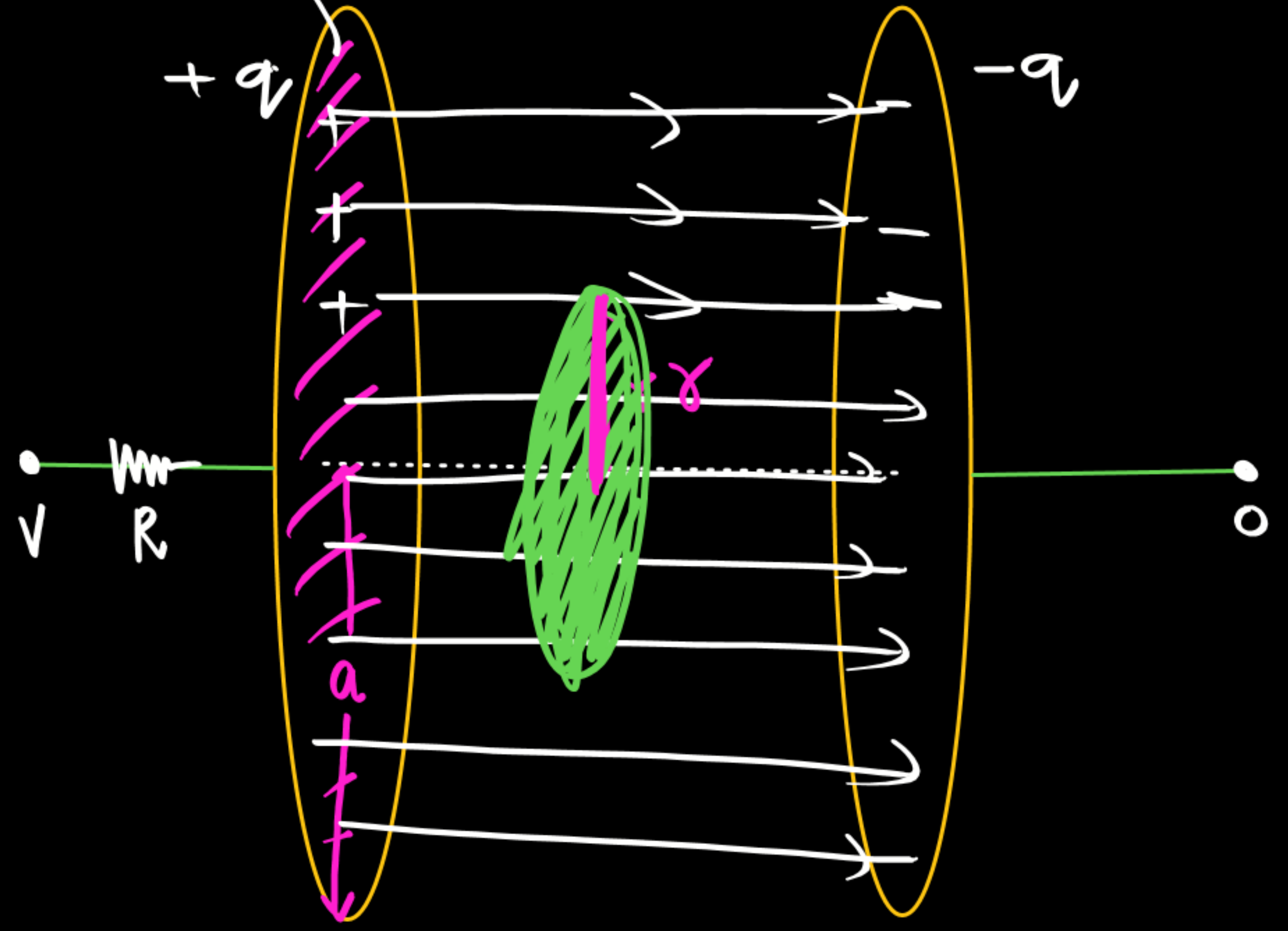




$$\boxed{I_d = \frac{V r^2}{R a^2} e^{-t/RC}} = \frac{I_c r^2}{a^2}$$

$A = \pi a^2$

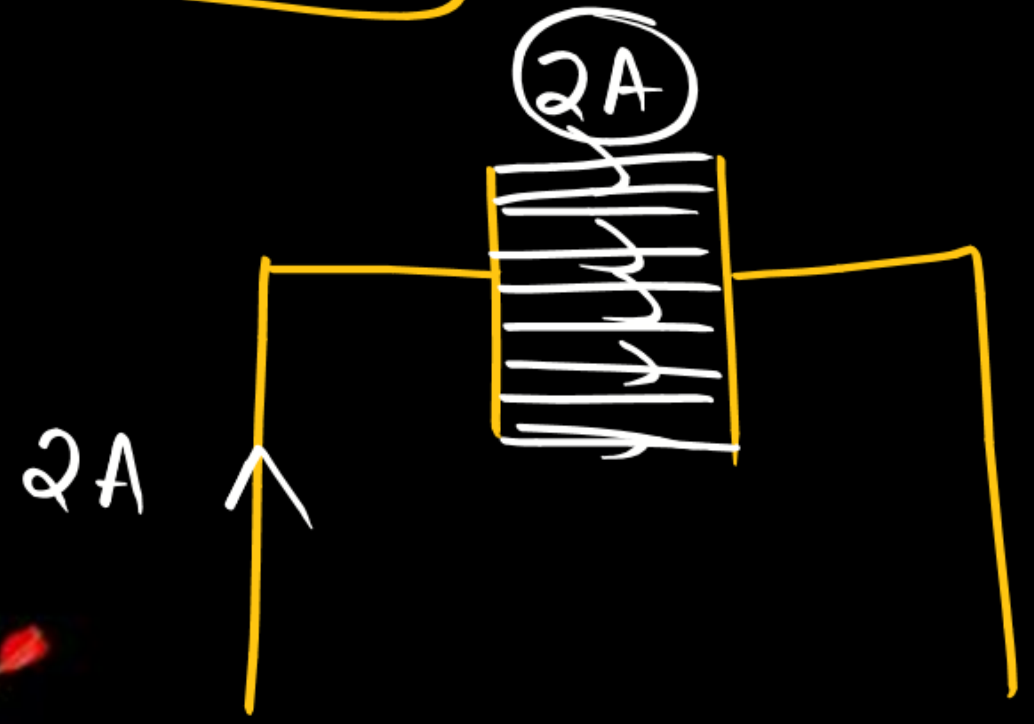




(*) $\boxed{r = a}$

$$\boxed{I_d = \frac{V}{R} e^{-t/RC}}$$

$$\boxed{I_c = \frac{V}{R} e^{-t/RC}}$$



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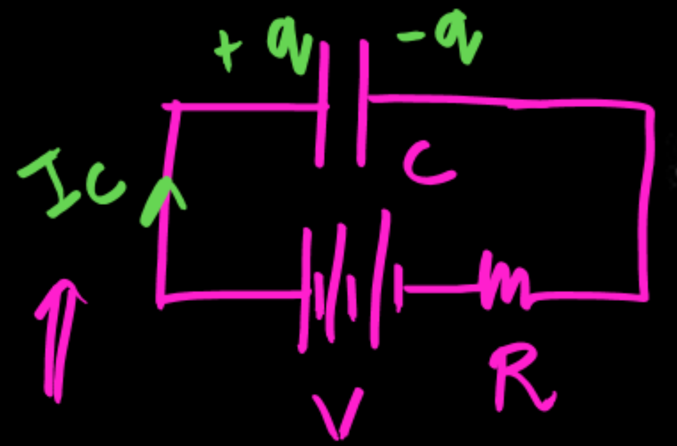
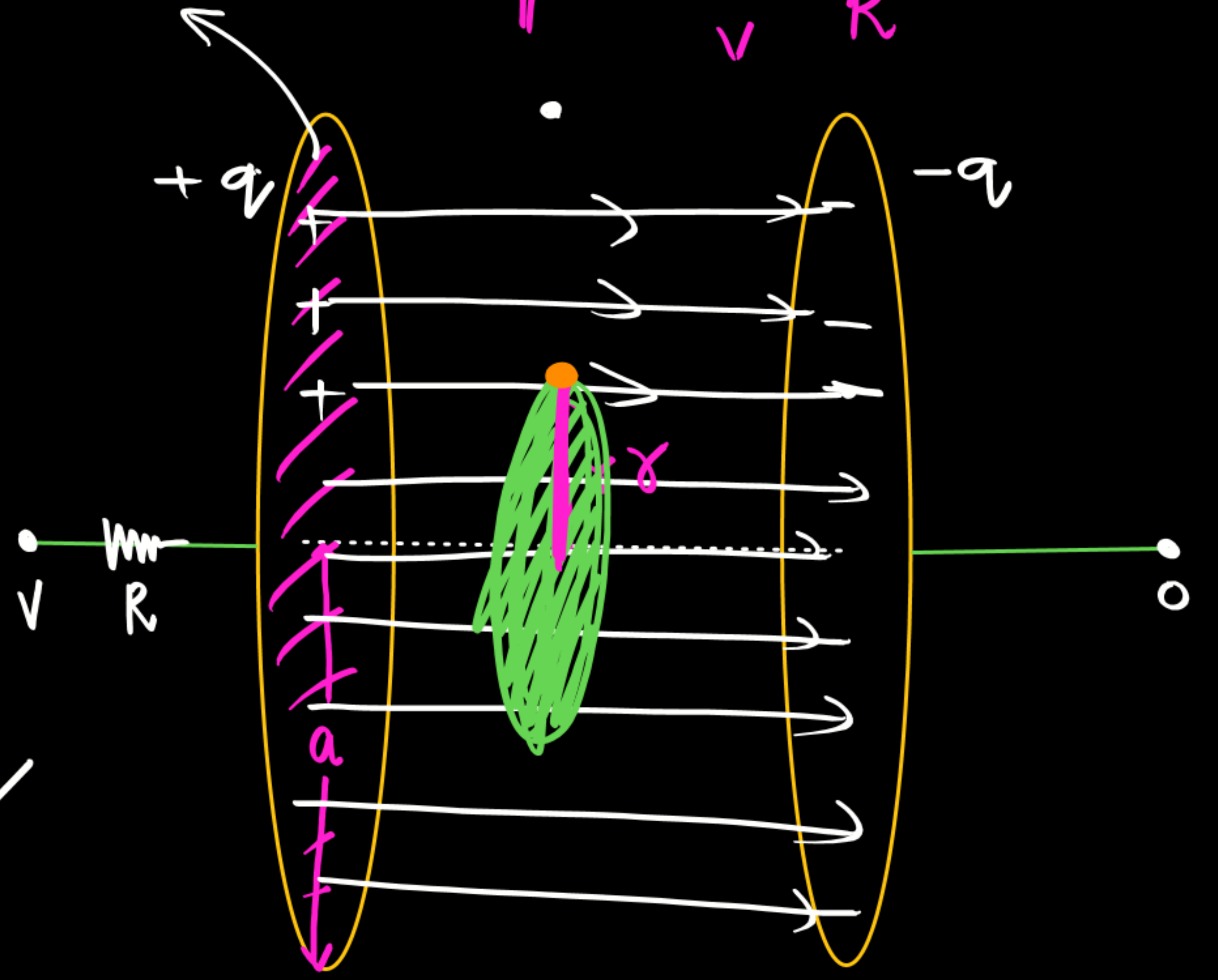


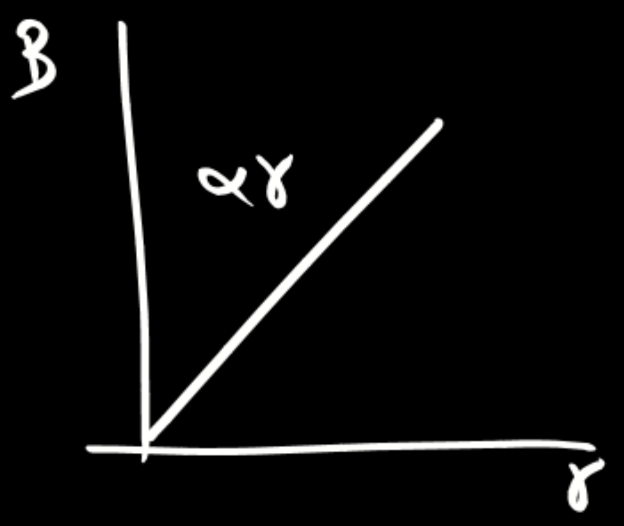


If we have to find B at r

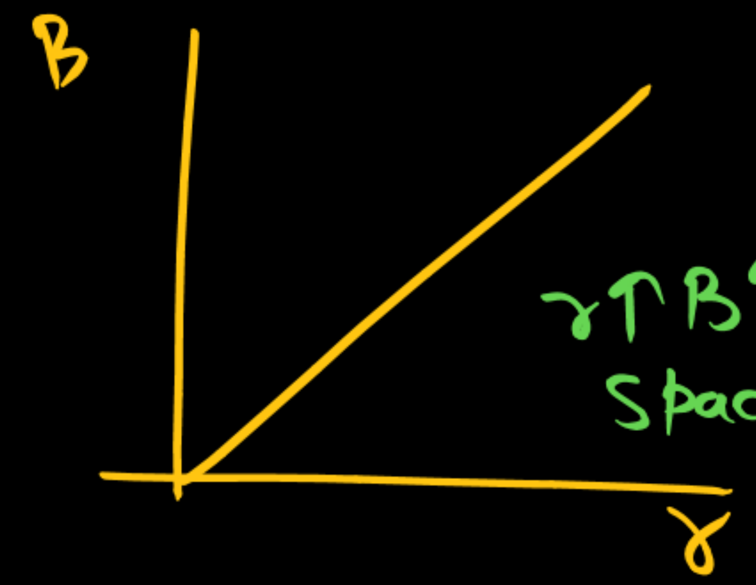
$$\oint B \cdot dl = \mu_0 (\cancelto{0}{I_c} + I_d)$$

$$B \cdot 2\pi r = \frac{\mu_0 V r^2}{R a^2} e^{-t/RC}$$

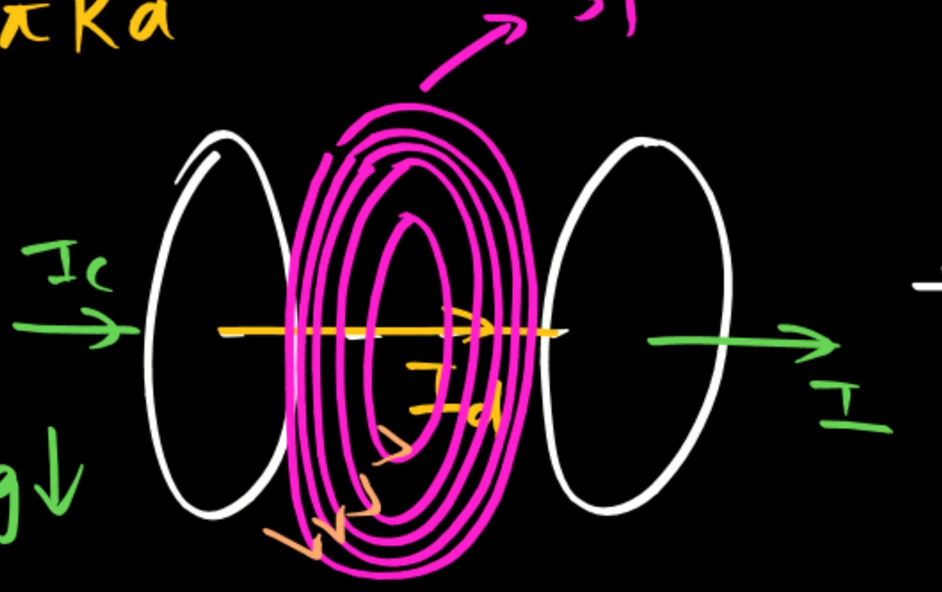
$$\boxed{B = \frac{\mu_0 V r}{2\pi R a^2} e^{-t/RC}}$$

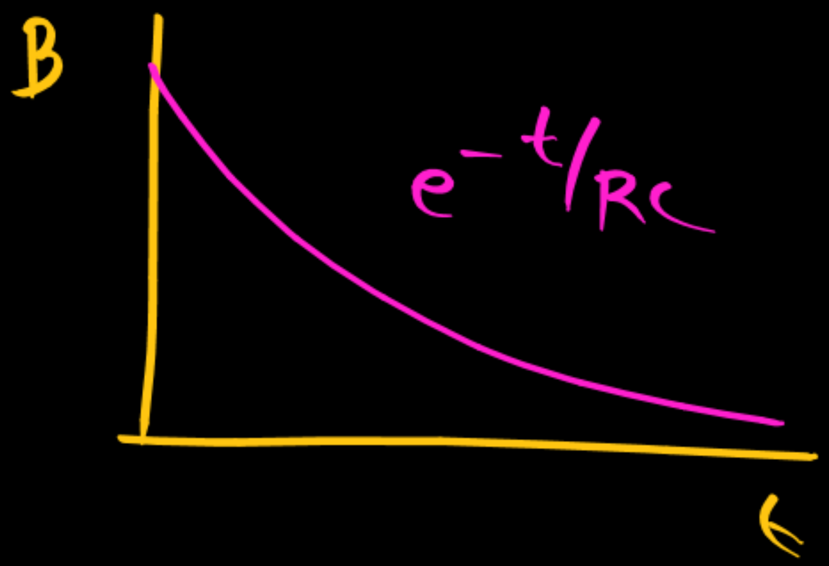


B: Capacitor $= \dfrac{\mu_0 V r}{2\pi R a^2} e^{-t/RC}$



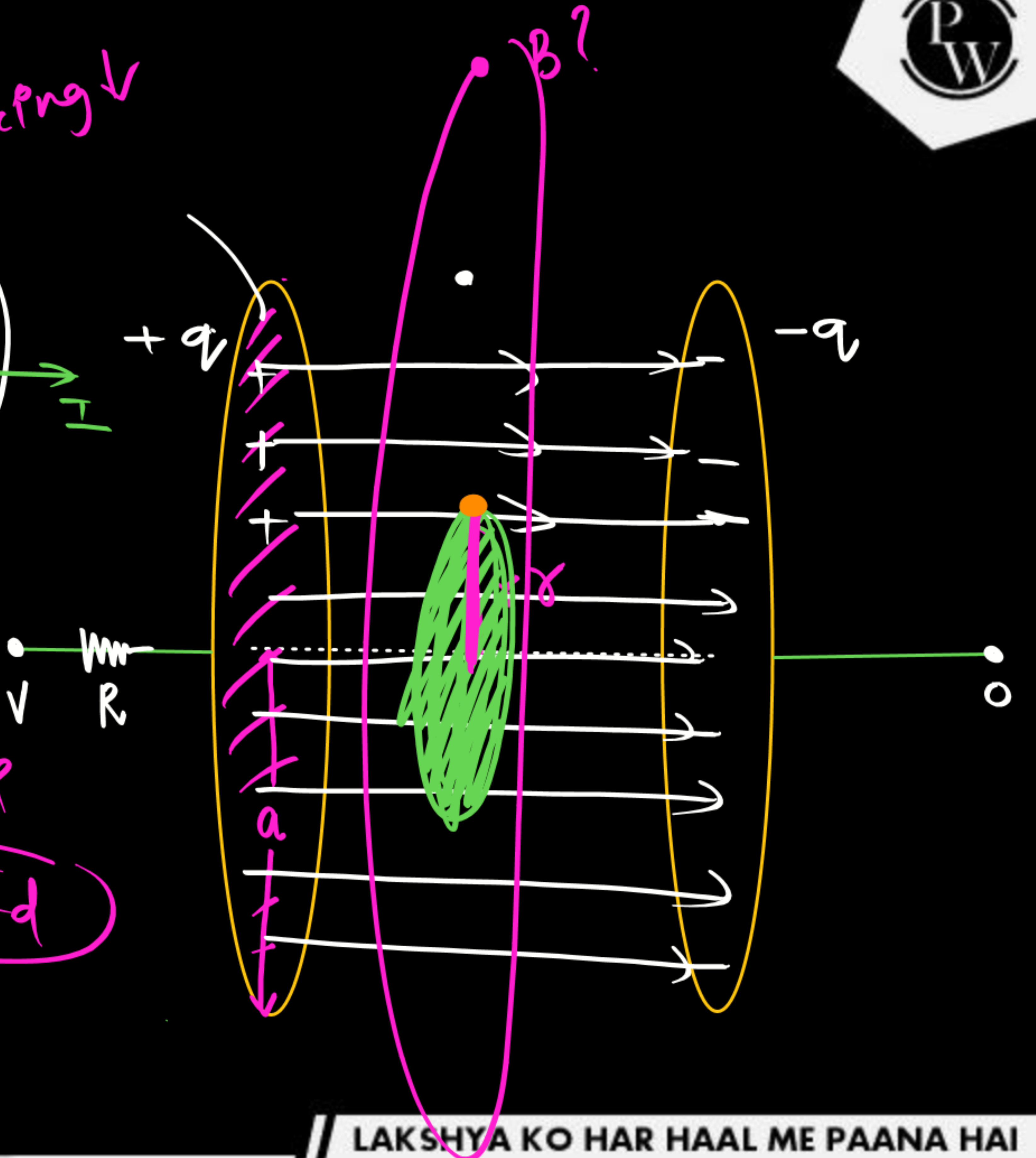
$r\uparrow B\uparrow$
spacing↓





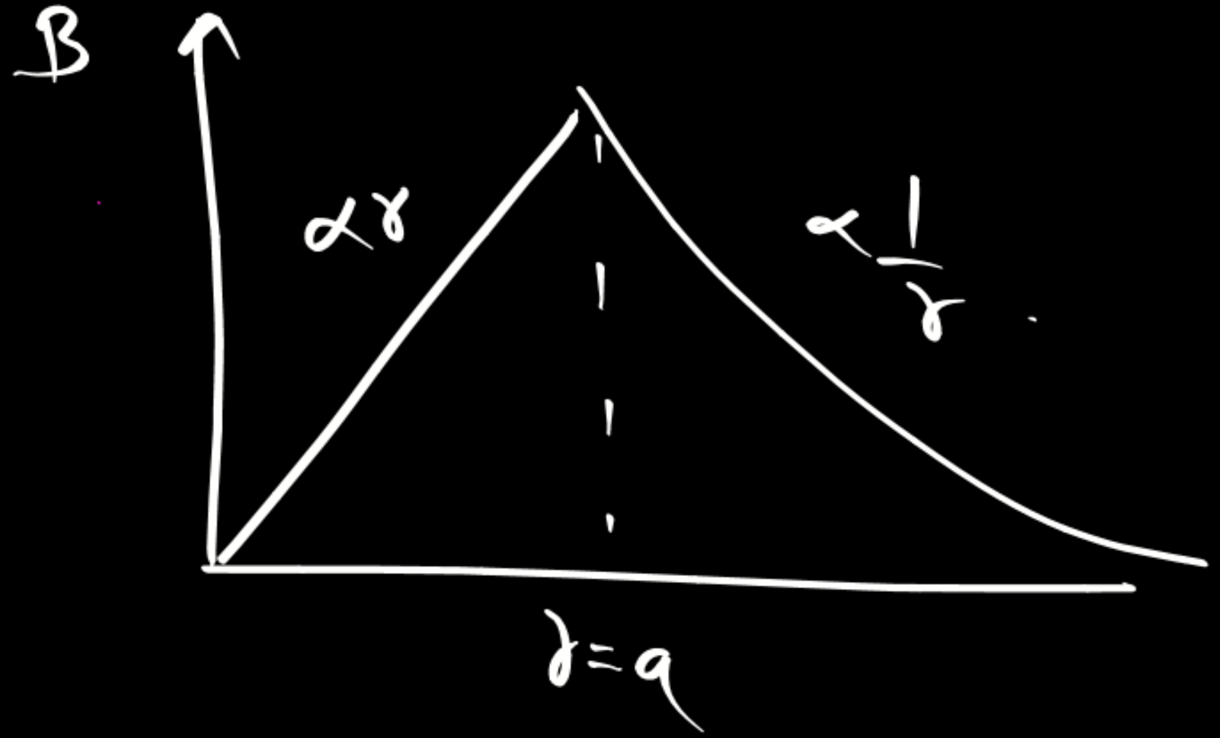
$t \to \infty$
Capacitor fully charge

$I_c = 0 = I_d$



Outside Region of Capacitor

$$\boxed{B \propto \frac{1}{r}}$$



Thank You Lakshyians