Chapter 6 Measuring stars

Astronomers cannot perform experiments directly in the laboratory. If the objects of interest locate beyond the Solar System, we cannot even send probe to them. Instead, we have to *observe* what is going on and try to make sense of our observation. Most of the information that we obtain is in the form of electromagnetic (EM) radiation.

In this chapter, we will explain what information¹ we can decipher from the EM radiation from stars. It prepares us for the birth of stars (Ch.7), structure of stars (Ch.8), evolution and death of stars (Ch.9, 10). Our most familiar star, the Sun, will be discussed in Ch.8.

6.1 Telescopes

Ever since Galileo pointed his telescope to the sky, human has been building bigger and better telescope for astronomical observation. Normally the term "telescope" is referred to optical telescope, but it is also used for other wavelength. For millimetre wave to radio wave, normally the devices are called arrays or dishes.

- The **light-gathering power** of a telescope is directly proportional to the area of the objective lens/mirror, which is normally fixed for a particular telescope. The **magnification**, defined as the ratio of the apparent angular size of object (seen via the telescope) to the real angular size, normally can be adjusted by changed the eyepiece. Higher magnification comes with the expense of less radiation power per unit area, and thus dimmer image.
- Another important function of a telescope is to produce sharp image. A sharp image has high **angular resolution**, which is the angular distance between the finer details. Resolving power of a telescope can be limited by the conditions (as known as "seeing") of the atmosphere. Fluctuations and turbulence in the atmosphere lead to blurring of image and twinkling of star light. Light pollution "washes out" the sky and makes it impossible to see the dim objects. (Fig. 6-1)

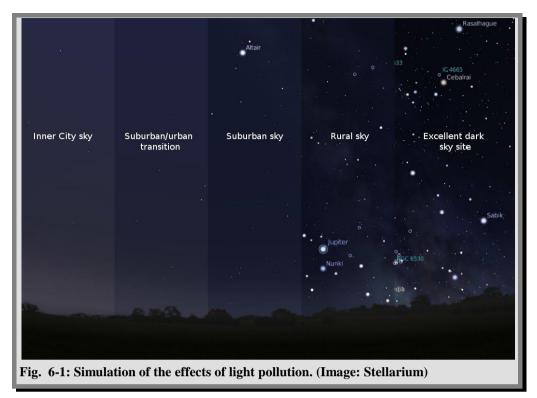
Astrometry: measuring the positions of objects in the sky *Photometry*: measuring the amount of energy from objects

Spectrometry: measuring the distribution of radiation with wavelength

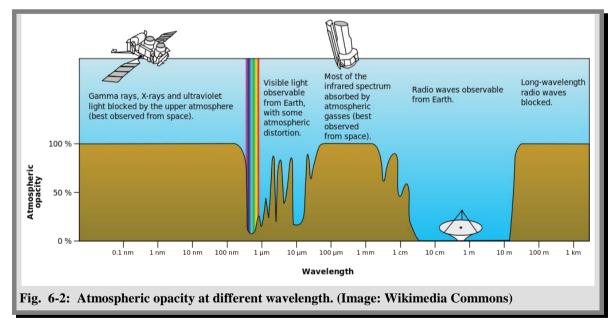
Polarimetry: measuring the polarization of light We will touch on the first four types of observation.

¹ In case you are into technical details, you may want to know that the basic EM observations include

Imaging: taking images/photos of celestial objects



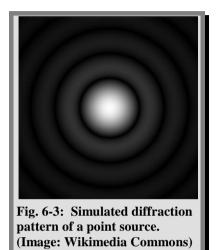
✓ To reduce the effects of atmosphere, some telescopes are built at high-altitude site or the pole ². At those locations, the amount of water vapour in the air is low and so the seeing is much better. For some wavelengths, the Earth's atmosphere is actually opaque. (Fig. 6-2) Space-based observatory is used for observing in those wavelengths.

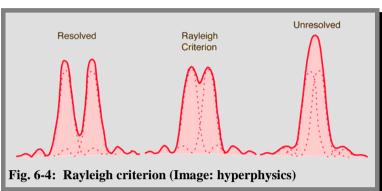


✓ Even without the atmosphere, and even if a telescope is free of aberration (optical defect), it is still not possible to find infinite resolution. The finite size of a telescope's aperture leads to diffraction, meaning that the image of even a point source will be a

² Another ground-based method to reduce atmospheric effect is adaptive optics, which involve the measurement and compensation of atmospheric turbulence by adjusting the shape of the mirror.

circle (known as Airy disk)³ surrounded by concentric rings (Fig. 6-3). Two light sources cannot be resolved if their Airy disks come too close and join one another. (Fig. 6-4) The resolution condition is known as the Rayleigh criterion, which is $\theta = 1.22 \frac{\lambda}{D}$, with θ being the diffraction-limited angular resolution in radian, λ the wavelength in metre, and D the diameter of the telescope's aperture. ⁴



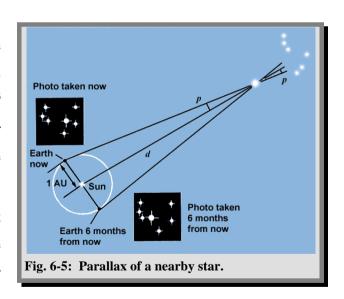


6.2 Distances and motions of stars

The method of parallax 5

As the Earth moves around the Sun, the apparent position of a *nearby star* changes relative to other distant stars. (Fig. 6-5) ⁶ The change in the apparent position of a star depends on its distance from the Earth. The effect is known as stellar parallax. ⁷

Parallax p is the half of the total shift in the angular position of a star. If the distance d to the star is much larger



³ The shape of a point source (or a star), as viewed by using a telescope, is the Point Spread Function (PSF). The Airy disc is the PSF of an ideal telescope.

⁴ Interferometry is a way to improve angular resolution by "interfering" the signal of multiple telescopes/dishes to imitate a giant telescope/dish. The technique has to applied to radio to sub-mm bands (e.g. VLA, VLBI, SMA), and optical and IR bands (e.g. CHARA, Keck, VLT).

⁵ The method of parallax is also known as the method of stellar parallax. Do not confuse it with "Spectroscopic parallax", which is an entirely different method of distance measurement.

⁶ More accurately, the nearby star moves in an ellipse.

⁷ Not to be confused with spectroscopic parallax described in **Box 6.3**.

where d is in parsec and p in second of arc.

than the average distance between the Earth and the Sun (~ 1 AU) ⁸, we have $d = \frac{1}{p}$

✓ Therefore, **1 parsec** (**pc**) is defined as the distance of a star with a parallax of 1 second of

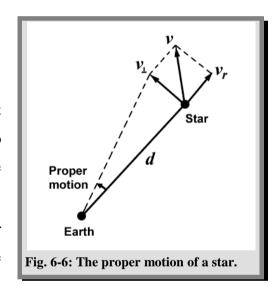
arc. So 1 pc =
$$\frac{1 \text{ AU}}{1"} = \frac{1 \text{ AU}}{\frac{1}{3600} \times \frac{\pi}{180} \text{ rad}}$$
, or $\boxed{1 \text{ pc} \approx 206265 \text{ AU} \approx 3.26 \text{ ly.}}$

✓ Since the Earth's atmosphere limits the angular resolution ⁹, the farthest distance measured by this method is ~100 pc. Stars farther than 100 pc from us should use other methods to measure the distance.

Stellar Motions

Stars can move in any direction. In general, a star's velocity v can be broken into components parallel and perpendicular to our line of sight (Fig. 6-6).

- The component perpendicular to our line of sight is called the star's **tangential velocity** (v_{\perp}) . To determine it, one must know the distance to the star (d) and its *proper motion*.
- **Proper motion** (μ), is the change in angular position of a star in one year. Therefore, the tangential velocity (in km/s) is given by



 $v_{\perp} = 4.74 \ \mu \times d$, where μ is in arcseconds per year and d is in parsecs.

The component of a star's motion parallel to our line of sight is its radial velocity (v_r) . It can be determined from measurements of the Doppler shifts (see **Box 6.1**) of the star's spectral lines.

⁸ The nearest star, Proxima Centauri, is 4.2 ly away from us. Therefore the assumption is always true for stars.

⁹ Space-based satellites can do much better. Hipparcos satellite, launched in 1989, can measure distance up to

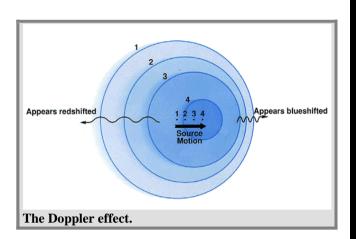
^{~1600} pc. Another space observatory Gaia, launched in Dec 2013, can measure distance up to ~ 8×10^6 pc to

 $[\]sim$ 2.5 × 10⁷ pc, depending on the magnitude of the source! Both missions are (were) done by ESA, which is the European counterpart of NASA.



Box 6.1 Doppler effect

In the direction of motion of a wave source, wave fronts get "compressed", the wavelength becomes *shorter*, the frequency becomes *higher*. The wave is said to be **blue shifted**. In the opposite direction, the wavelength becomes *longer*, frequency becomes *lower*; hence the wave is **red shifted**. As a whole, the phenomenon is called the *Doppler effect*.



More generally, both the source and receiver are moving. The classical Doppler effect states that observed wavelength λ is given by $\lambda = \left(\frac{c + v_s}{c - v_r}\right)\lambda_0$, where c is the

wave speed, v_r is the receiver's velocity relative to the medium, v_s is the source's velocity. The velocity is defined as +ve when the source/receiver is move away from one another.

Unlike other waves, EM radiation does not need a medium of propagation. Also, for relative velocity ν (+ve if moving away) comparable to the speed of light, relativistic effect (time dilation) must be taken into account. The relativistic Doppler effect is

described by
$$\lambda = \lambda_0 \sqrt{\frac{1+v/c}{1-v/c}}$$
, which reduces to the classical equation for $v << c$.

A predication by relativity is that there is Doppler shift even when the source's relative velocity is perpendicular to the light of sight. In the classical theory, there is no transverse Doppler effect.

6.3 Magnitude scales

Besides the positions and motions, another important piece of knowledge about a star is how bright it is. Here are several related concepts:

- **Luminosity** (光度,L) is the total radiation energy (including all wavelengths) emitted by a star per unit time (unit: W). For the Sun, $L_{\odot} = 3.90 \times 10^{26}$ W.
- **Apparent Brightness** (亮度, B): The amount of light energy <u>received</u> per unit time per unit area measured at a distance r from a star (unit: W m⁻²). Also known as **flux**.

Inverse-Square Law: The apparent brightness of light that an observer can see is inversely proportional to the square of the observer's distance from the source, i.e., $B = \frac{L}{4\pi r^2}$. If you double your distance from a source, its radiation

your distance from a source, its radiation is spread over an area four times larger, so the brightness drops by a factor of 4. (Fig. 8-7) Therefore, the luminosity of a

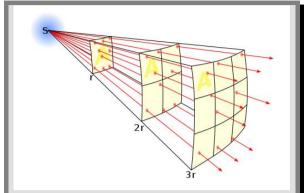


Fig. 6-7: The Inverse-Square Law. Farther away from the source, the radiation spreads out more. (Image: Wikimedia Commons)

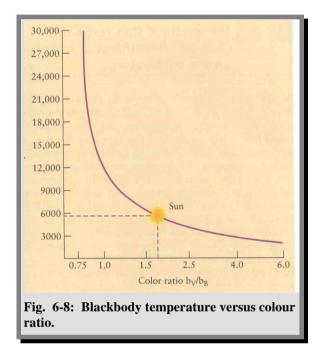
star is determined once the apparent brightness and the distance from the star are known.

- ✓ Magnitude scale is frequently used to denote brightness by astronomers.
 - ♦ Apparent magnitude (視星等,m): It is convenient to measure a star's brightness compared with a reference star. Vega is chosen as the reference star, and the apparent magnitude is defined as $m = -2.5\log(B/B_{\text{Vega}})$. The apparent magnitude of Vega is $m_{\text{Vega}} \equiv 0$. The magnitude difference is then defined as $m_1 m_2 = -2.5\log(B_1/B_2)$. Two stars with magnitude difference of 5 differ by 100 times in brightness.
 - **Absolute magnitude** (絶對星等,M): It is the magnitude of a star measured as if the star were at a distance of $10 \ pc$ from the Earth. Thus, the absolute magnitude measures the luminosity of a star. In addition, by the Inverse-Square Law, the difference between the apparent and absolute magnitude, named **distance modulus**, is equal to $M M = 5\log r 5$. The distance r is measured in pc.
- ✓ Strictly speaking, the values that we have defined are the bolometric magnitudes, which take into account the radiation power in all wavelengths.

6.4 Surface temperatures of stars

You may notice that the stars that you see in the nighttime sky have different colours. For example, you can easily distinguish between the colours of reddish Betelgeuse and bluish Bellatrix. In fact, the colour of a star does reveal its surface temperature.

- High-energy radiation, e.g., γ -ray, is produced by nuclear fusion at the core of a star. Before reaching the star's surface the energy is absorbed and re-emitted many times. Deep into a star, the temperature is so high that atoms are fully ionized. The state is called **plasma** (ions and electrons in random thermal motions).
- ✓ Random collisions of charged particles and photons with a wide range of energies, EM waves of *all* wavelengths are then produced. The radiation emitted by a star can be decomposed into a continuous spectrum.
- Like blackbody radiations (see **Box** 5.1), the shape of the star's spectrum depends on its surface temperature. Hence, the surface temperature is revealed by comparing the relative intensity in neighbouring wavelength bands. Astronomers must accurately measure its colours in order to obtain a star's surface temperature.
- ✓ **UBV photometric system**: Sometimes apparent brightness within a certain passband ¹⁰ is measured. Most commonly used passbands include U-band



(ultraviolet), B-band (blue), and V-band ("visual", which is actually yellow) 11 . By finding the apparent brightness in each band, one can find the colour ratios $b_{\rm V}/b_{\rm B}$ and $b_{\rm B}/b_{\rm U}$, which are indicators of the stellar surface temperature (Fig. 6-8). Similarly, one can specify the stellar temperature using the colour indices B-V and U-B, which are the difference in the apparent magnitudes in corresponding passbands.

6.5 Physics of Stellar Spectra

Because stars are so distant, the only information that we can hope to obtain about their physical nature – apart from temperature, structure, chemical composition, etc. – comes from analysing the light that they emit, i.e., *the spectra of stars*.

¹⁰ Light within a certain well-defined range of wavelength (band) is allowed to pass through the filter, so the range is called passband.

¹¹ U, B and V are wide-band filters centred at around 365nm, 440nm and 550nm.

- Since a hot, glowing object produces a continuous blackbody spectrum (see **Box 5.1**), or *continuum*. For the case of a star, e.g., the Sun, the continuum is produced at low-lying levels of the star's atmosphere where the gases are relatively hot and dense.
- The absorption lines are created when the continuum radiation flows outward through the cooler, less dense, upper layers of star's atmosphere (see **Box 6.2**). Atoms in the upper layers absorb radiation at specific wavelengths. Therefore, the details of stars' spectra depend on the star's temperature and elements present in the stars.

Harvard spectral classification

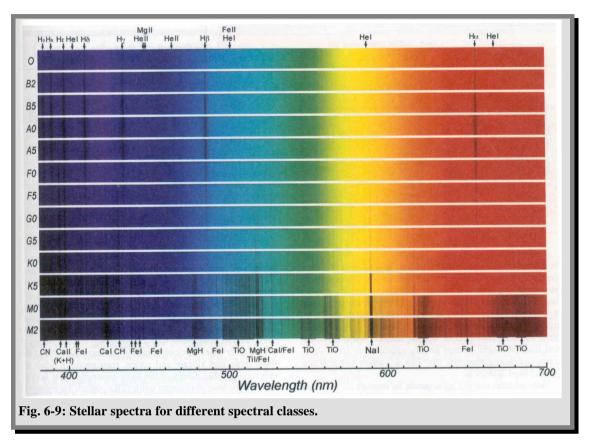
According to star's surface temperature, astronomers classify stars' spectra into seven spectral classes, namely Oh! Be A Fine Girl (Guy) Kiss Me. 12 The hottest stars are O stars; whereas the coolest stars are M stars. Each spectral class is divided into finer subclass with number below 10 (with 0 being the hottest subclass). For example, the spectral class F includes spectral types F0, F1, ..., up to F9.9, and then followed by G0, G1, Our Sun is a G2 star.

Spectral Class	Spectral Characteristics	Surface Temperature (K) ¹³
О	Hottest blue star with few lines. Strong He II absorption lines.	> 30,000
В	Hot blue-white stars. He I absorption strongest at B2. H I (Balmer) absorption lines becoming stronger.	20,000
A	White stars. Balmer absorption lines strongest at A0. Ca II absorption lines becoming stronger.	8,500
F	Yellow-white stars. Ca II lines continue to strengthen as Balmer lines continue to weaken.	6,500
G	Yellow stars. Ca II lines continue becoming stronger. Fe I, other neutral lines becoming stronger.	5,500
K	Cool orange star. Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.	4,500
M	Coolest red stars. Spectra dominated by molecular absorption bands, especially titanium oxide. Neutral metal absorption lines remain strong.	3,000

¹² There are three other recently added spectral classes below class M. The objects are known as brown dwarfs. Class L spectrum has strong metal hydride molecules, and neutral Na, K, Cs lines. It refers to surface temperature from ~1500K to ~2100K. Class T spectrum has methane bands, neutral K, and weak water lines. It refers to surface temperature below ~1500K. Objects with temperature below 500K is classified in class Y.

¹³ There is a temperature range for each class. Here I just provide a typical value around the middle of the class.

Question: Hydrogen is the most abundant element in the universe. Why do Balmer lines not show up in all stars' spectra?



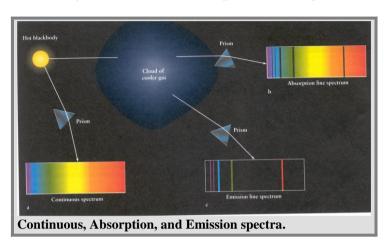
Answer: Balmer lines are produced when an electron transits between n = 2 and higher energy orbit (see $\Re ax$ 6.2). If the star is very hot enough, high-energy photons knock electrons out of the hydrogen atoms in the star's outer layers easily. When hydrogen's only electron is torn away, no hydrogen spectral lines can be produced. Hence, the Balmer lines will be relatively weak in such hot stars, such as O stars. (Fig. 6-9) Conversely, if the star's atmosphere is very cool, almost all of the hydrogen atoms are in the lowest (n = 1) energy state. Therefore, these unexcited atoms cannot absorb the photons characteristic of the Balmer lines. For the Balmer lines to be prominent the star must be hot enough to excite electrons out of the ground state but not so hot that hydrogen atoms are ionized.



Box 6.2 Types of Spectra

A hot, glowing object such as a blackbody emits a continuous spectrum of light.

If this light is passed through a cloud of a cooler gas, the cloud selectively absorbs light of certain specific wavelengths, depending on the kinds of specific elements of the cooler gas. As a result, the spectrum of this light has dark absorption lines, called absorption spectrum.

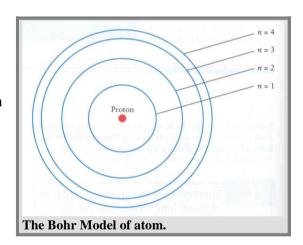


The cloud does not retain all the light energy that it absorbs but radiates it outward in all directions. If the light radiated from the gas is viewed against a cold, dark background, its spectrum contains bright emission lines, called **emission spectrum**.

According to the Bohr Model of atom (as shown in the figure), a photon is absorbed when an electron jumps from a lower energy orbit to another higher energy orbit; whereas a photon is emitted when an electron jumps from a higher energy orbit to lower energy one.

Bohr proved mathematically that the wavelength (in meter) of the (emitted or absorbed) photon in a *hydrogen atom* is

given by
$$\sqrt{\frac{1}{\lambda} = R\left(\frac{1}{N^2} - \frac{1}{n^2}\right)}$$
, where the



Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$, N is the number of inner orbit, and n is the number of outer orbit.

When an electron transits between the second Bohr orbit (N = 2) and a higher energy orbit, it is **Balmer series**. For examples, n = 3 (H_{α} , $\lambda \sim 656.3$ nm), n = 4 (H_{β} , $\lambda \sim 486.2$ nm). One the other hand, the transitions between (N = 1) and a higher energy orbit, it is **Lyman series**, which is entirely in the ultraviolet. For the transitions between (N = 3) and a higher energy orbit, it is **Paschen series**, which is in the infrared.

Structure of spectral lines

The detailed shape of a spectral line, called the **line profile**, contains important information about a star. (Fig. 6-10)

✓ The area of the line profile indicates how much energy is absorbed from continuum. It is in turn proportional to the amount of

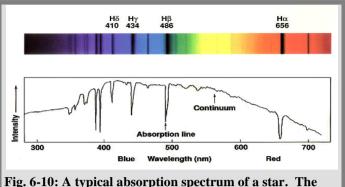
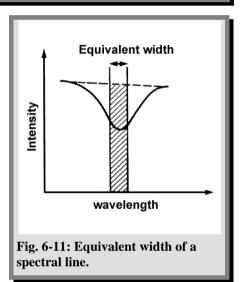


Fig. 6-10: A typical absorption spectrum of a star. The detailed shape of a line provides important information about a star.

atoms in the star's atmosphere to absorb the photons with the frequency in question. In a broad and deep spectral line more energy is absorbed at that wavelength, hence more atoms there are.

✓ Strength of an absorption line is usually described by its **equivalent width** *w*, which is the width that a rectangular spectral line would have if it had the same area as the observed line, but with the depth of the continuum (Fig. 6-11).



Doppler broadening: A spectral line may be broadened (Fig. 6-12) by Doppler shifts due to random thermal motions of particles. The shift in wavelength $\Delta\lambda$ is, for $v \ll c$, $\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$, where v_r is the component of the atom's velocity in the direction of observation. Following the Maxwell-Boltzmann distribution (see $\Re x$ 7.2), the root mean square speed of

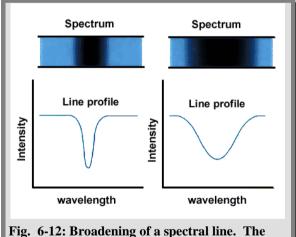


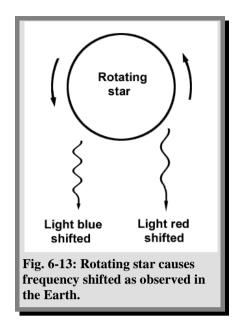
Fig. 6-12: Broadening of a spectral line. The area of the spectral line remains unchanged.

the particles is $v_{\text{rms}} = \sqrt{3kT/m}$. Therefore, the total width of a spectral line is about

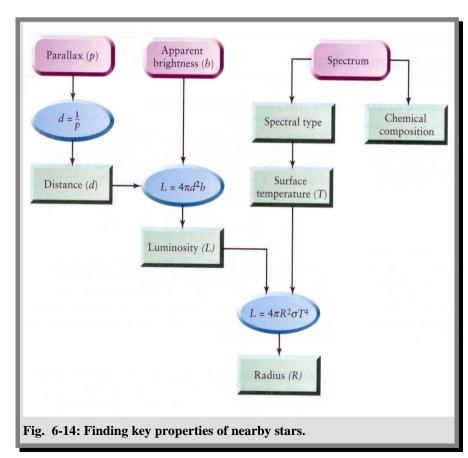
$$\frac{\Delta\lambda}{\lambda} \approx \frac{2}{c} \sqrt{\frac{3kT}{m}}$$
. For example, the Doppler broadening of H_{α} line in the Sun is about

$$\Delta \lambda \approx 0.51 \stackrel{\circ}{\text{A}}$$
 for $T = 5770 \,\text{K}$, $m = 1.67 \times 10^{-27} \,\text{kg}$, and $\lambda = 6563 \stackrel{\circ}{\text{A}}$.

- ✓ **Rotational broadening**: Light coming from different part of a rotating star is Doppler shifted in different ways (Fig. 6-13). By measuring the shape of the spectral lines, astronomers can deduce how fast the star is rotating.
- ✓ There are other processes responsible for line broadening of spectral lines. In general, the signature in the line profile allows one to determine the physical condition related to the particular process.

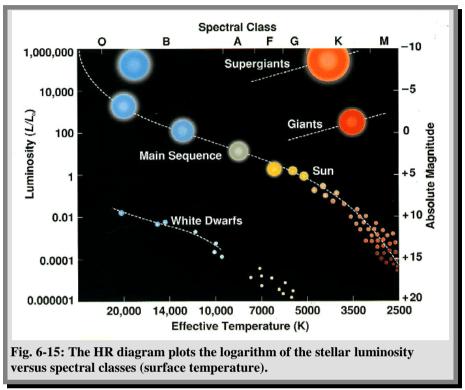


Summary: Fig. 6-14 shows how astronomers deduce the properties of relatively nearby stars (those close enough to measure it distance by method of parallax). Given the star's distance to the Earth, measuring the apparent brightness gives the star's luminosity from the inverse-square law. The star's spectrum reveals their surface temperature, as well as chemical compositions. And from luminosity and surface temperature, the Stefan-Boltzmann law allows the radius to be calculated. The line profile further reveals the quantity and motion of atoms in question.



6.6 Hertzsprung-Russell (H-R) diagram

✓ A graph of **luminosity** (or absolute magnitude) versus **spectral classes** (or surface temperatures). Each star is represented by a point on the H-R diagram (Fig. 6-15).



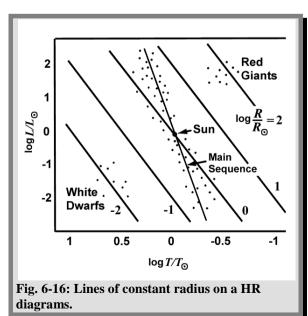
- ✓ Points at the *top* are *luminous* stars and at the *bottom* are *faint* stars; whereas points at the *left* are *hot* stars and at the *right* are *cool* stars.
- ✓ As a star *evolves*, its surface temperature, as well as luminosity, changes, so the position of the star on H-R diagram also changes.
- Stars of the same surface temperature can have very different luminosities. By Stefan-Boltzmann's law, one obtains $\ln L = 4 \ln T + 2 \ln R + \ln(4\pi\sigma)$; hence,

$$\ln\left(\frac{L}{L_{\odot}}\right) = 4\ln\left(\frac{T}{T_{\odot}}\right) + 2\ln\left(\frac{R}{R_{\odot}}\right). \text{ On the}$$

H-R diagram, stars with constant radii lie on straight lines (Fig. 6-16).

Different kinds of stars locate at different positions on the H-R diagram (Fig. 6-15).

Main sequence is a belt from upper left



to lower right, including ~90% of all stars in the sky. *Cool* stars are *faint* and *small*, *hot* stars are *luminous* and *large*. The Sun is a medium-temperature G2 main-sequence star.

- ✓ **Giants** lie in the upper right of the H-R diagram, and they are cool *but* luminous stars. Hence, they must have large surface area ($R/R_{\odot} \sim 10$ -100). The radii of **supergiants** are even larger, with $R/R_{\odot} \sim 100$ -1000.
- ✓ White dwarfs lie in the lower left of the diagram, and they are hot *but* faint. Hence, they must be very small (~ size of the Earth).
- Classifying stars by *spectral class* is essentially the same as categorizing them according to surface temperature. Stars of the same surface temperature can have very different luminosities. For example, a star with surface temperature 5800 K could be either a white dwarf, a main-sequence star, or a giant, depending on the star's luminosity. Nevertheless, one can distinguish them by examining features in a star's spectral lines.
- ✓ Hydrogen lines are good indicators of luminosity. The higher density and pressure of the gas in the star's atmosphere, the more frequently hydrogen atoms collide and interact with other atoms and ions in the atmosphere. These collisions shift the energy levels in the hydrogen atoms and thus broaden the hydrogen spectral lines ¹⁴. In the denser atmosphere of a main-sequence star, frequent interatomic collisions perturb the energy levels in hydrogen atoms, producing broader Balmer lines. In the low-density and low-pressure atmosphere of a luminous giant star are quite low, collisions between atoms and ions are so infrequent that hydrogen atoms produce narrow Balmer lines.
- ✓ **Luminosity classes** (Fig. 6-17) classify the stars according to the subtle differences in spectral lines. Hence, different luminosity classes represent different stages in the evolution of a star, so they provide a useful subdivision of the star types.

Ia: Luminous supergiant;

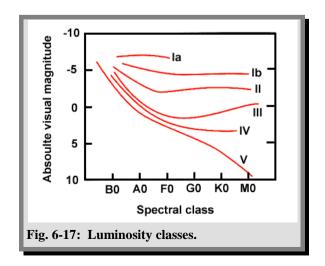
Ib: Supergiant;

II: Bright giant;

III: Giant;

IV: Subgiant;

V: Main-sequence star

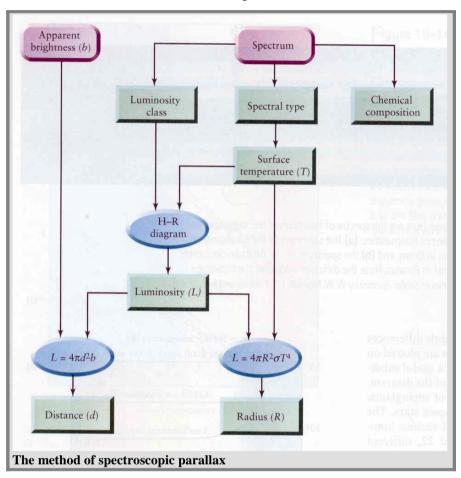


¹⁴ Known as pressure broadening, or collisional broadening.



Box 6.3 Spectroscopic parallax

With information about a star's location on the H-R diagram, and its spectrum, it is possible to estimate the distance of the star. The method is known as spectroscopic parallax (which has never to do with stellar parallax).

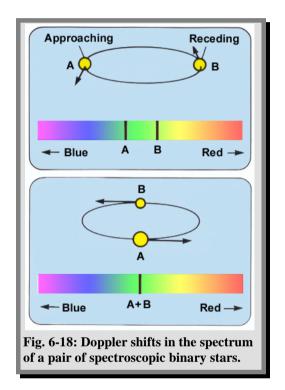


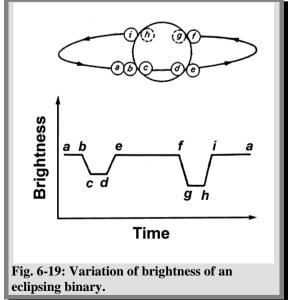
From the spectrum, one can learn the star's spectral type and luminosity class, from which the surface temperature and luminosity can be inferred using the H-R diagram. Given the star's luminosity and apparent brightness, the distance from Earth can be calculated. This technique can be used for estimating the distance for stars much further away (compared to stellar parallax). Also, the star's chemical composition can be determined from its spectrum, and the star's radius can be calculated from the luminosity and surface temperature by using the Stefan-Boltzmann law.

Since the luminosity classes in Fig. 6-17 actually locate on broad bands instead of thin lines, there are uncertainties in reading the figure. Distance estimated by using spectroscopic parallax is therefore only accurate to at best 10%. Distance measurement by stellar parallax, on the other hand, is a lot more accurate (provided that the star is close enough).

6.7 Binary stars

- ✓ It is believed that most stars belong to multiple-star systems. The most common type is the binary system, which consists of two stars.
- ✓ A pair of stars appears at nearly the same position in the night sky is called **double stars**. Some of them are **optical double stars**, which lie along nearly the same line of sight but are actually at very different distances from us. Many double stars are **binary stars**, or **binaries** pairs of stars that actually orbit each other.
- ✓ **Visual binaries** are gravitationally connected and can be *resolved* by optical telescopes.
- ✓ With the information of the period, distance, and angular size of the orbit of visual binaries, the mass of each star can be determined.
- ✓ **Spectroscopic binaries** are also gravitationally connected binaries, but they are too close (or too far) to be resolved by optical telescopes. It is discovered by observing periodic Doppler shifts in a spectrum. (6-18)
- ✓ It is possible to have more than two stars in a system. It has long been known that the second last star of the Big Dipper is a visual binary. However, it is actually a sextuplet system (i.e. a 6-star system) ¹⁵.
- Sometimes the stars are too close to be resolved by optical telescopes, but the system shows periodic variation in brightness by eclipses. By analysing the light curves (Fig. 6-19), it is possible to determine the ratio of luminosity and radii of the component stars.





¹⁵ The visual binary constains Mizar and Alcor. Mizar is a double binaries. Alcor is a binary. For details, see http://www.rochester.edu/news/show.php?id=3515