Science in Classics
经典中的科学

Isaac Newton, *The Principia*

Content

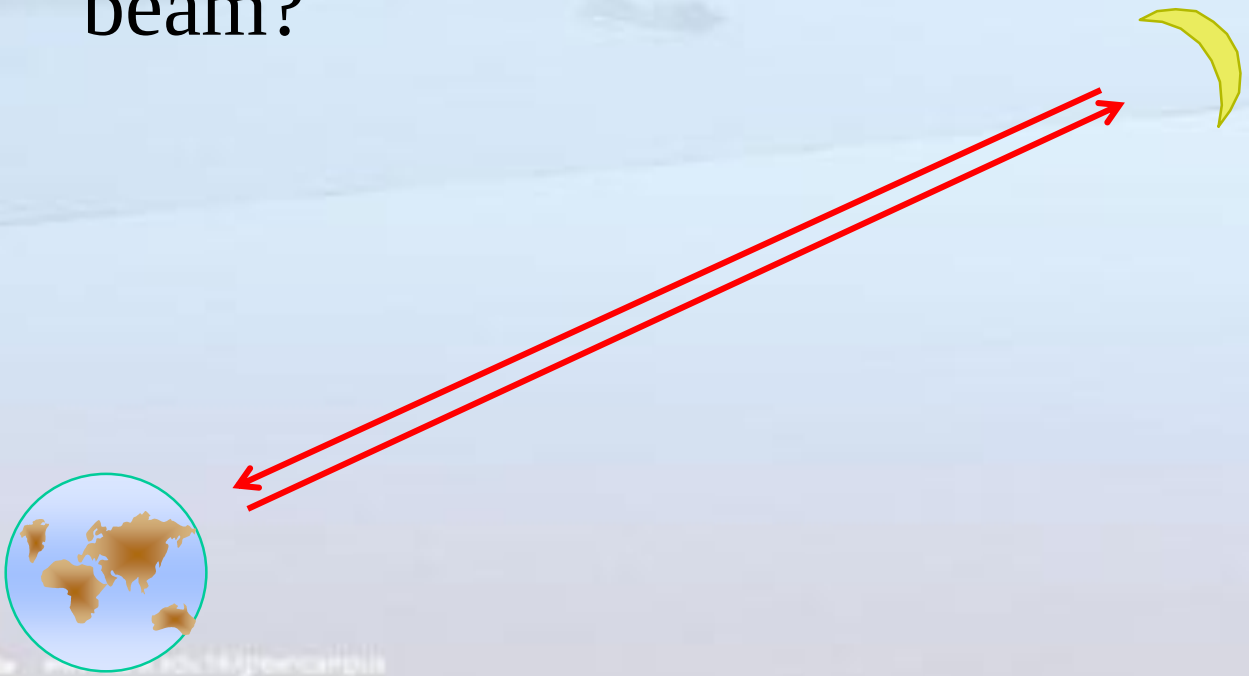
1. The terrestrial vs. the celestial
2. Galileo, Kepler and Hooke
3. Newton's worldview
   - unified universe
   - reductionist view
4. Newton's epistemology
5. Why did Newton write *The Principia*?

Question

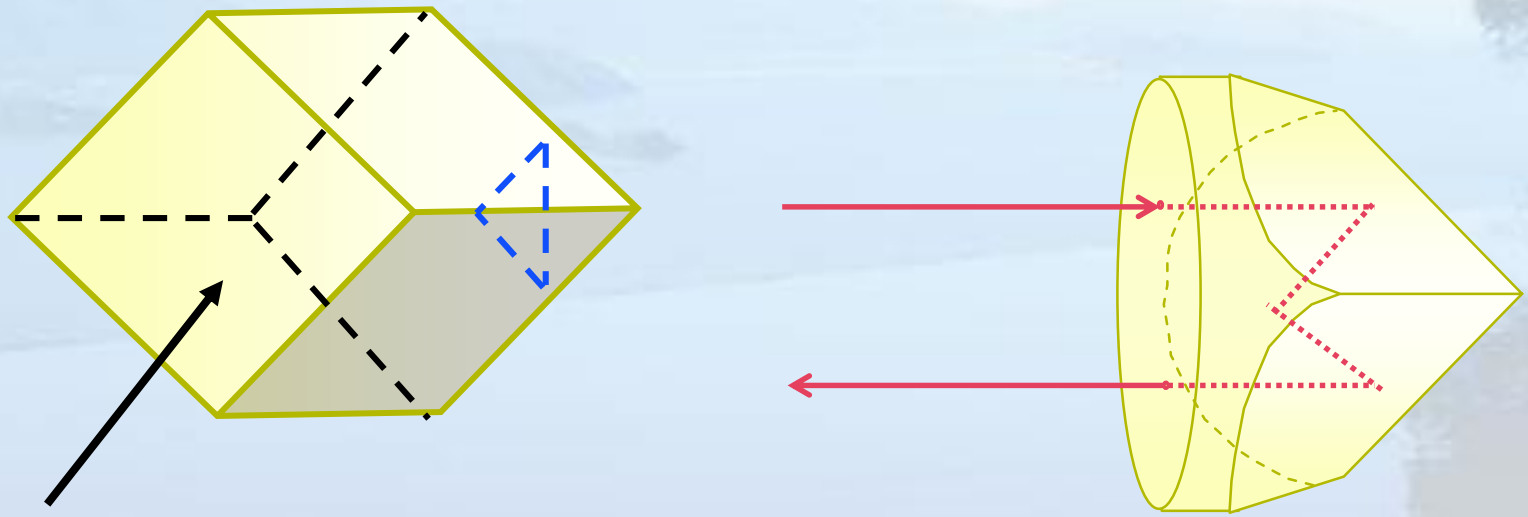
- How should we measure the distance between Earth and the moon?
  - A. By a ruler.
  - B. By reflection of light.
  - C. By reflection of ultrasound.

Question

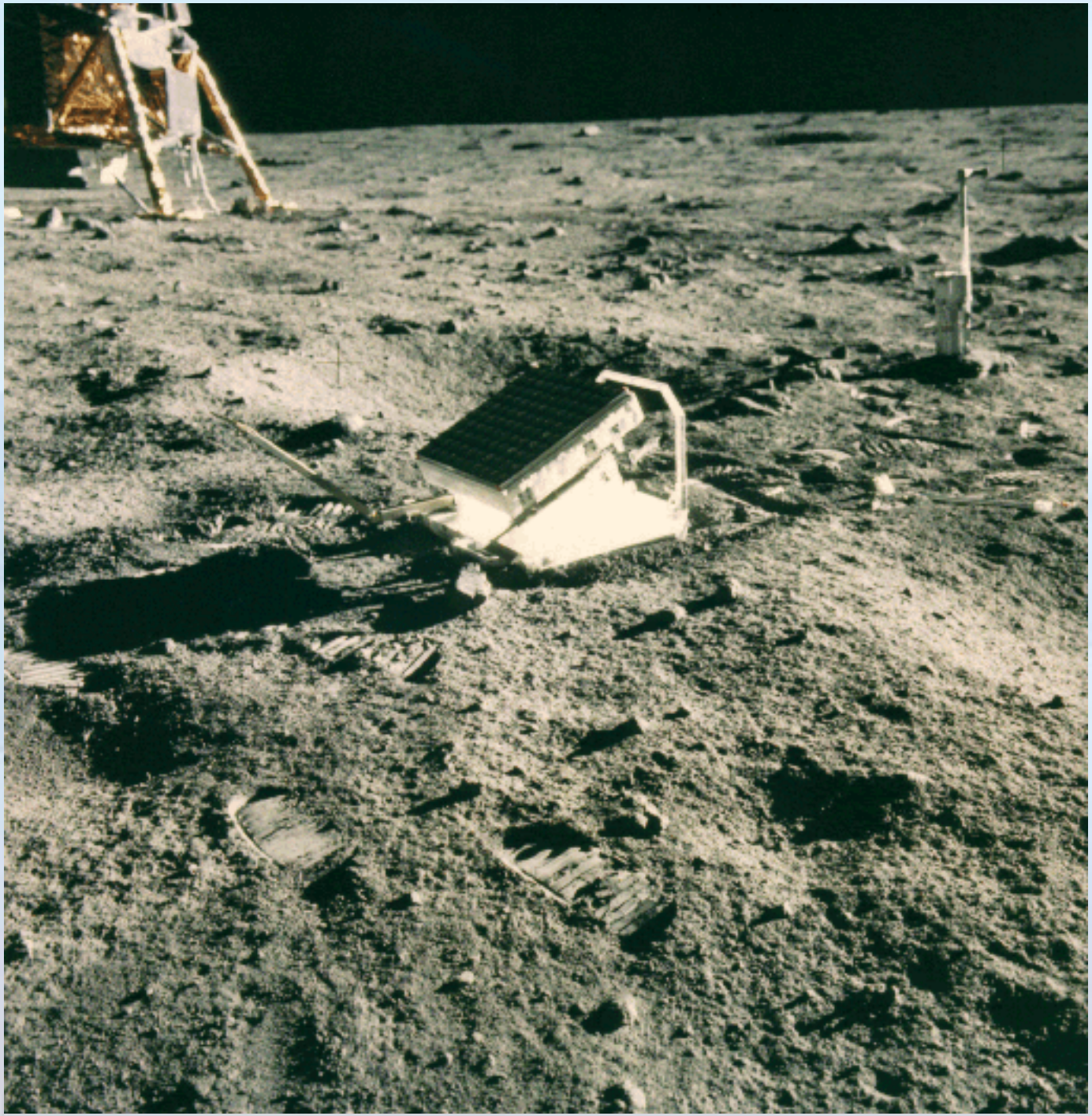
- But how can we make sure that the mirror on the moon is perpendicular to the laser beam?



Corner cube



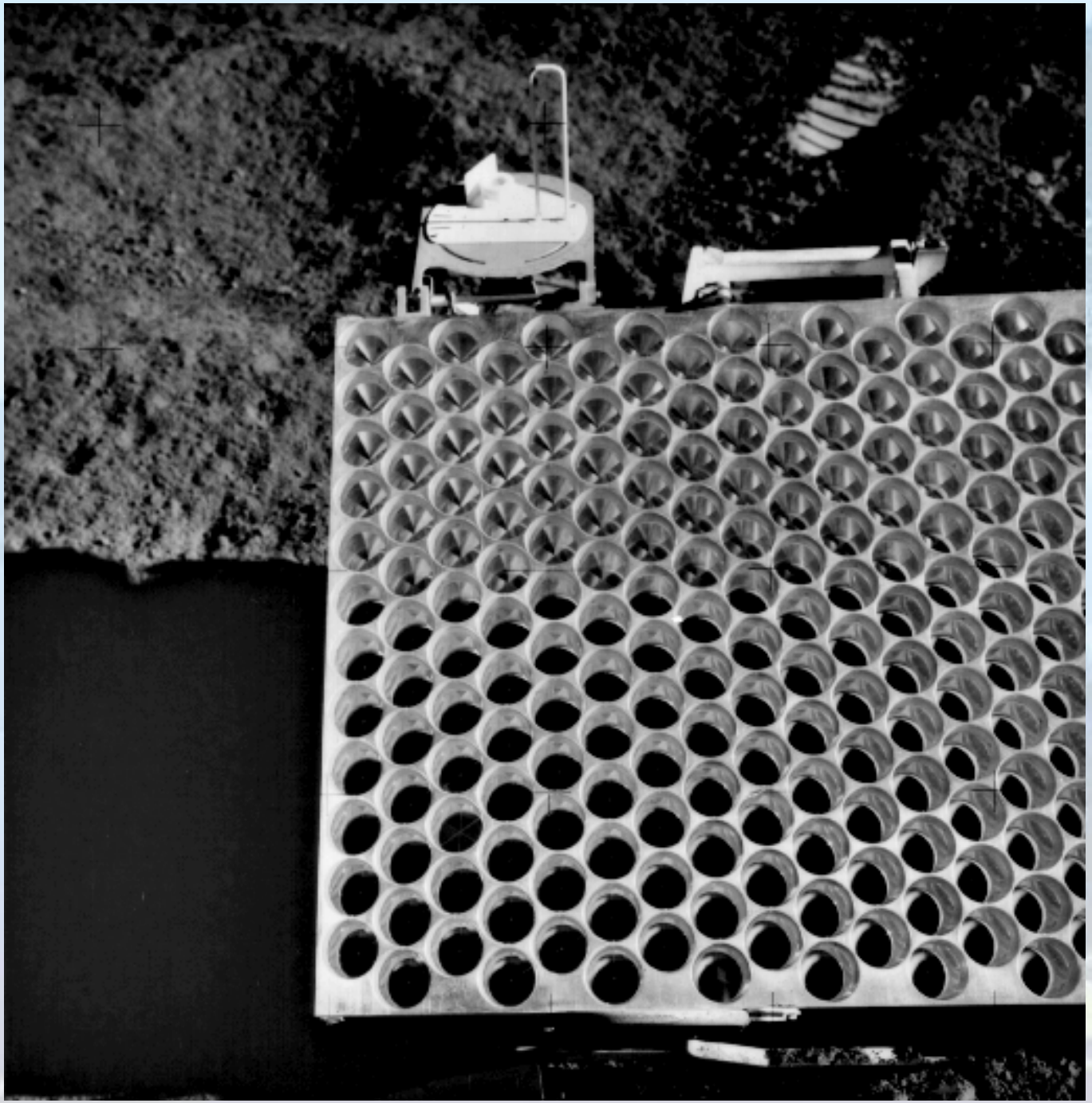
Glass



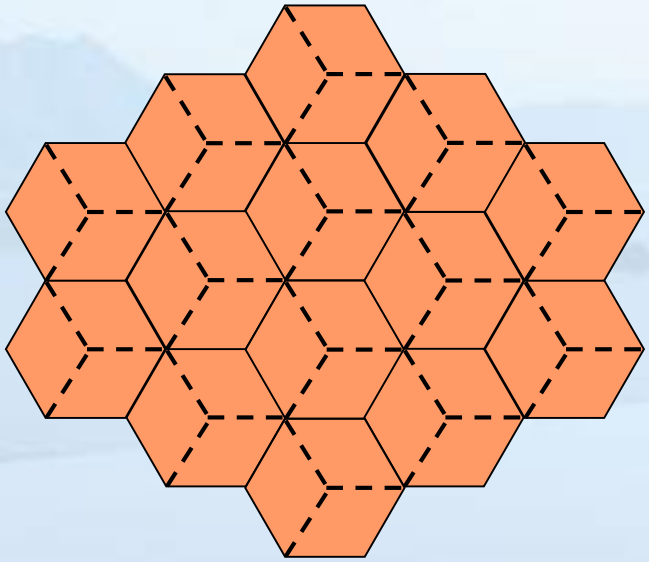
4 sets

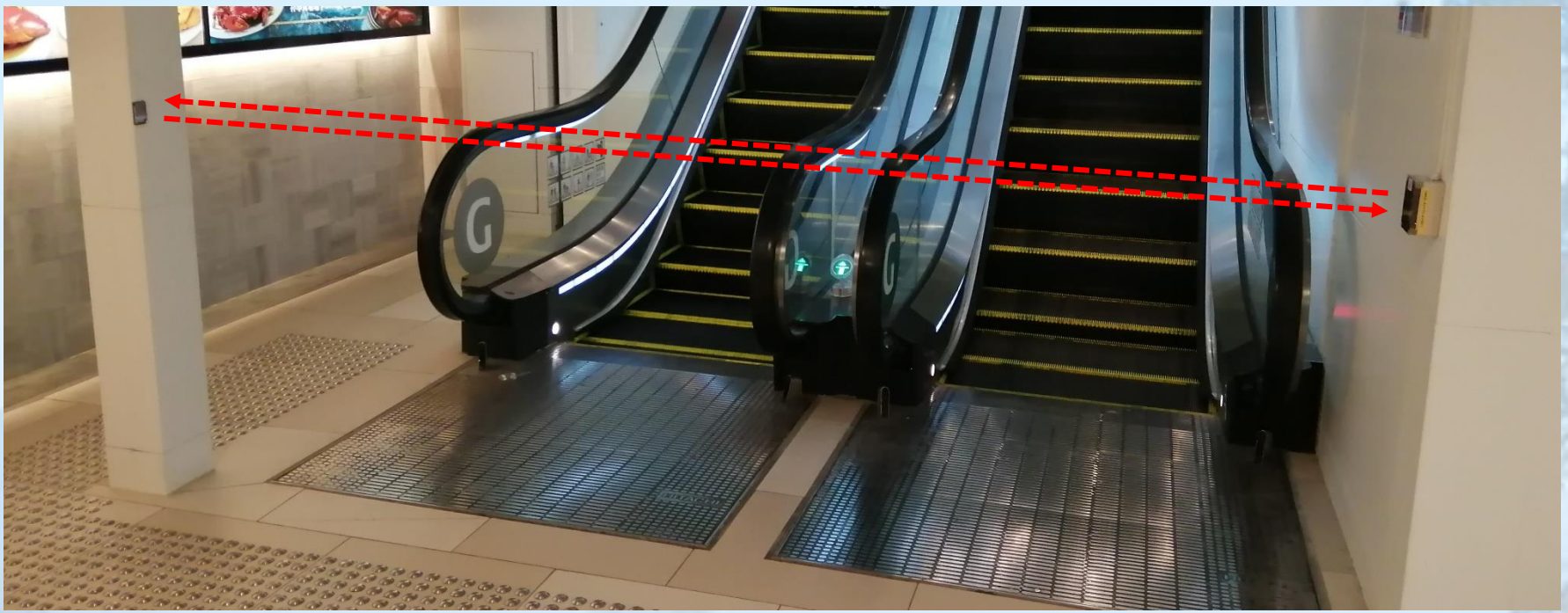
1969 Apollo 11

1971 Apollo 14, 15



Apollo 15



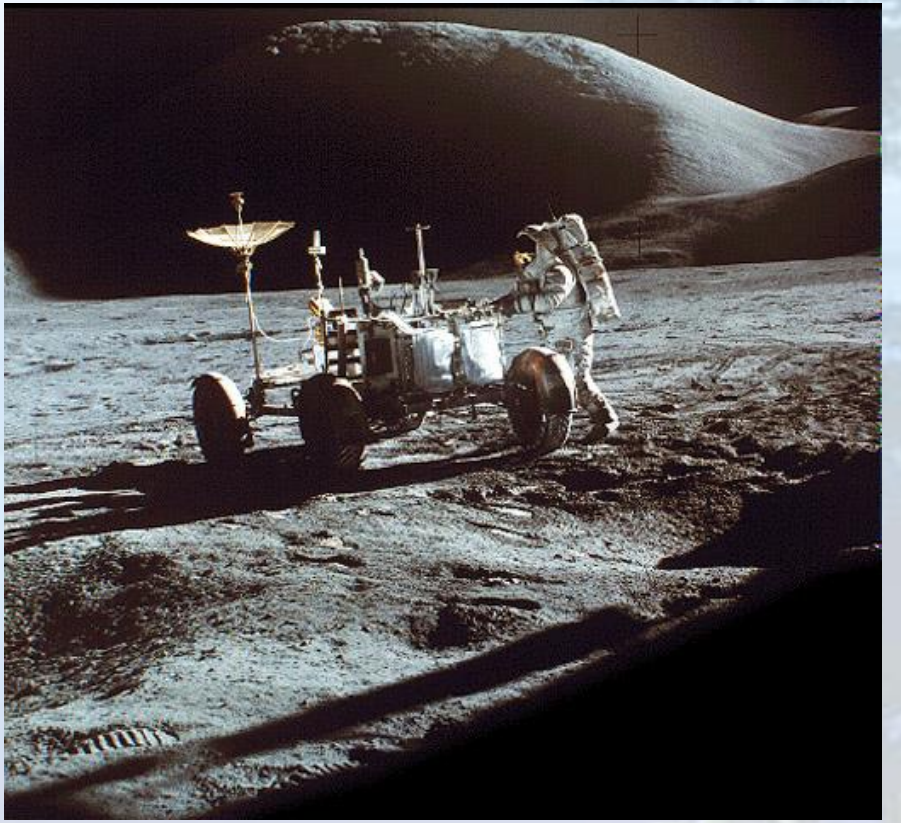


The moon is leaving

- The distance increases by ~ 3 cm/year.

The same laws

- The same law of light reflection.
- The same law of mechanics.

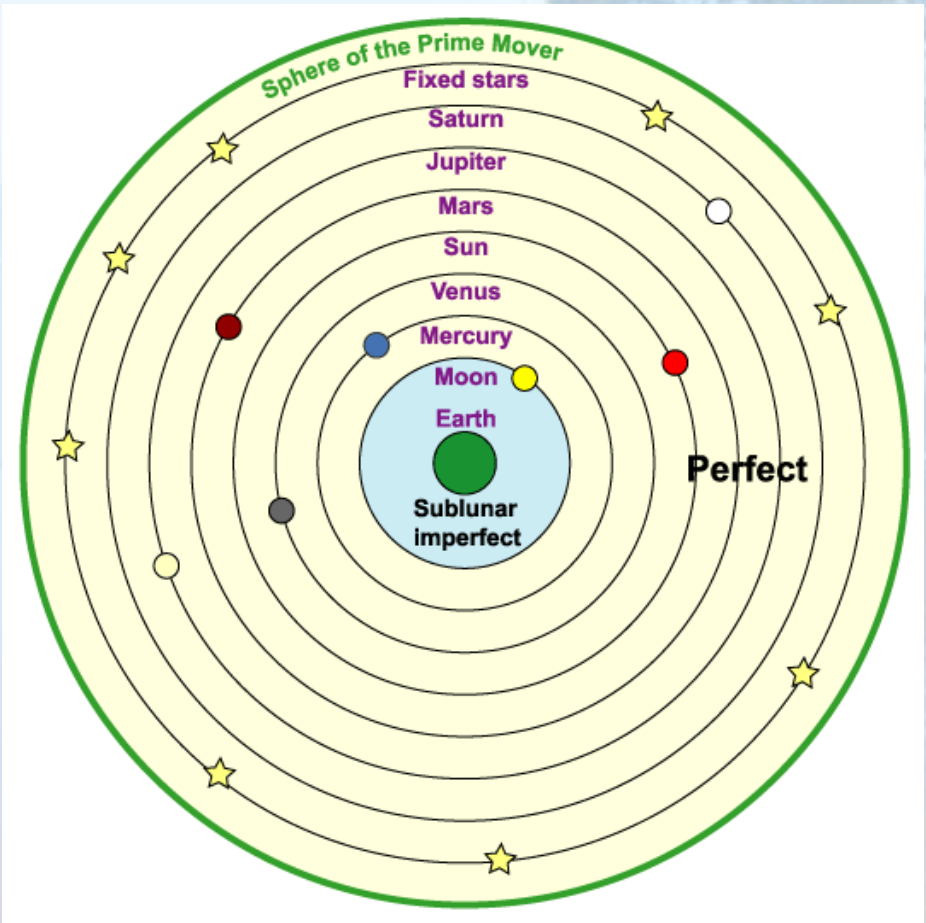


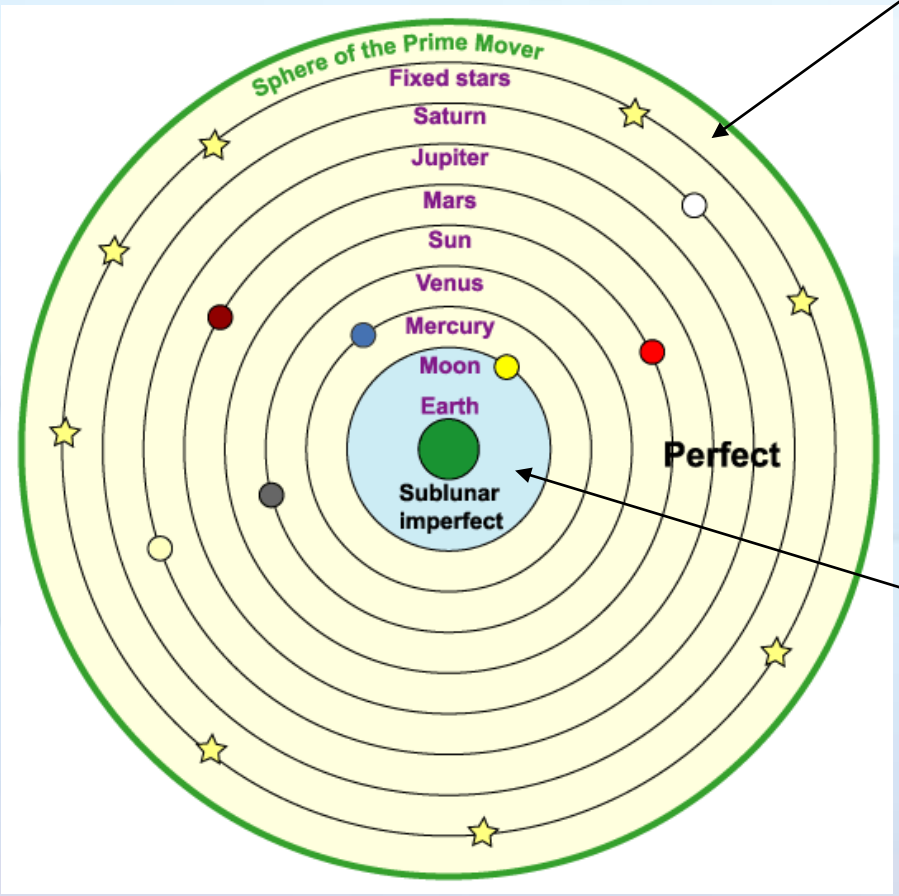
Core question

What is scientific understanding?

Aristotelian cosmology

- A geocentric universe.
- 55 planetary spheres plus the sphere of the fixed stars.
- On the right is a simplified version.





The celestial region

- Perfect (changeless, stars are constantly moving in circles)
- Made of aether (or ether 以太)
- No empty space.

The terrestrial region

- Imperfect (a changing world)
- Made of four elements (fire, air, water, earth / 火、风、水、土)
- No empty space.

Unification of the universe?

- Plato: two worlds
  - intelligible realm + material realm
- Aristotle: one world
  - but two regions: the terrestrial + the celestial
  - different rules in different regions.
- Question: Can the two regions be unified?
  - Galileo Galilei (1564-1642)
  - Isaac Newton (1643-1727)

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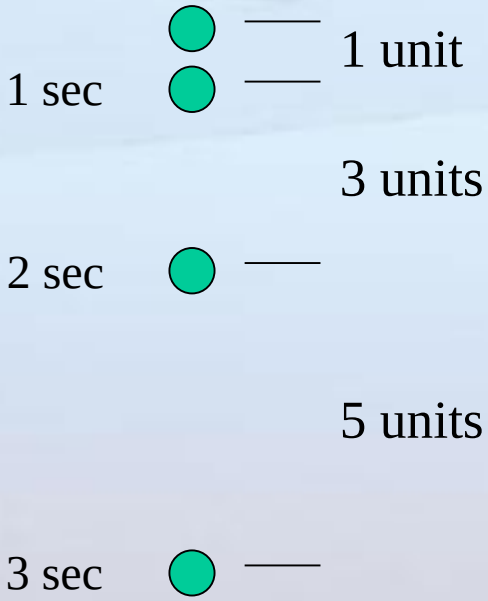
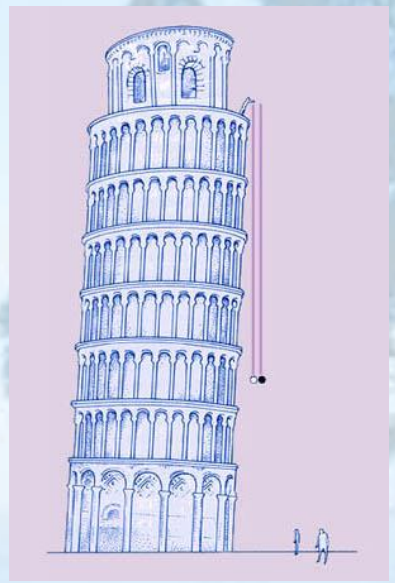
Galileo Galilei (伽利略)

- Falling speed in vacuum: independent of mass
- Velocity $\propto$ time



A true story?

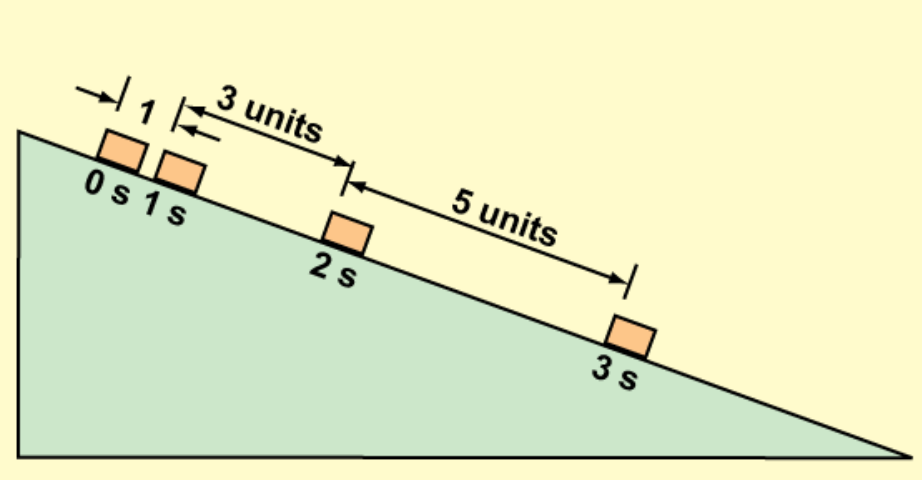


- Half and half.
- Then how did he know velocity $\propto$ time?

(12 June 2011)

Mass-independence of falling

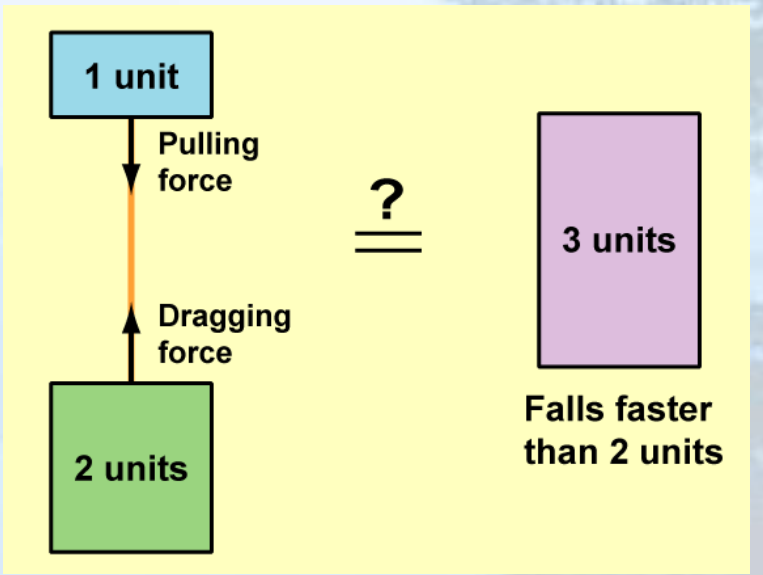
- Falling is too fast. He let an object slide on a smooth inclined plane and discovered:
  - velocity $\propto$ time elapsed
  - distances in the ratios 1:3:5:…
  - slope steeper, distances longer, the ratios keep unchanged.
  - independent of mass (e.g. 1 kg and 10 kg always move together)



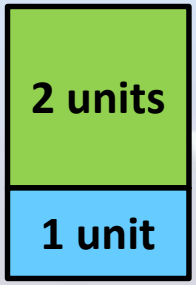
- Vertically falling = sliding along a vertical plane and hence also independent of mass.
- Velocity $\propto$ time
  - For both sliding and falling
  - Independent of mass

A thought experiment

- Compare
  - Aristotle: Falling speed $\propto$ weight (mass $\propto$ weight).
  - Galileo: Falling speed independent of weight
- Galileo designed a thought-experiment to disprove Aristotle's assertion.

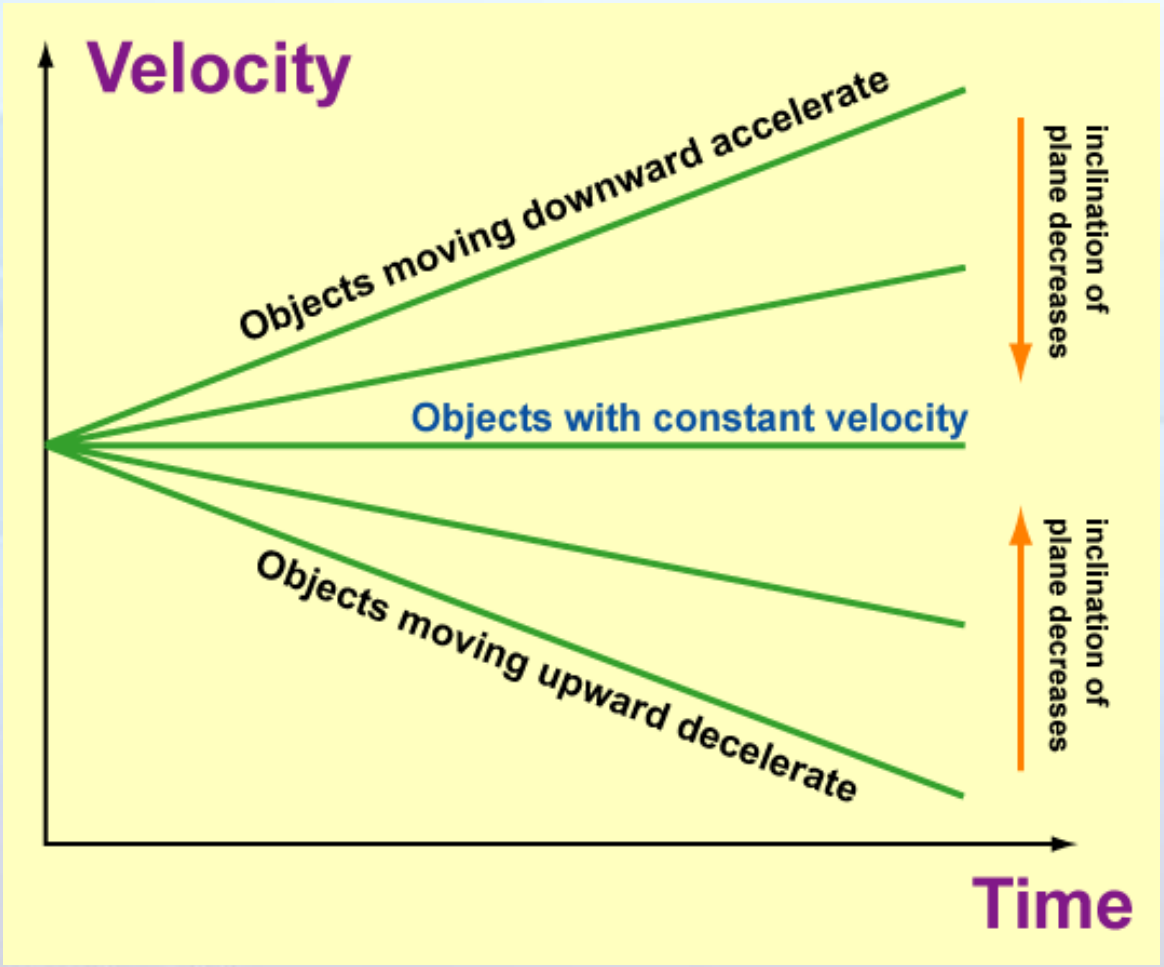


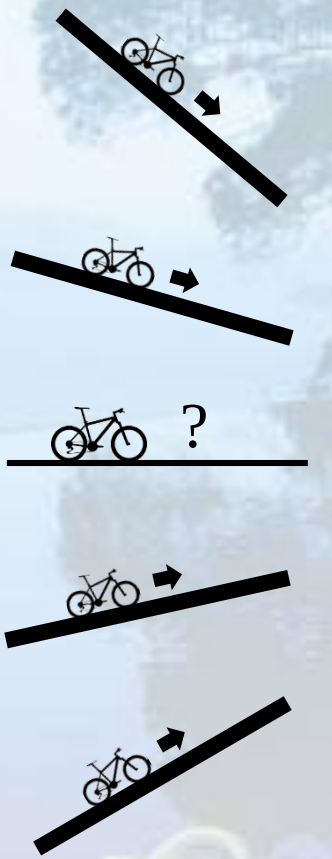
Original setting:



If the plane becomes horizontal …

the straight lines approach horizontal as the inclined plane approaches horizontal



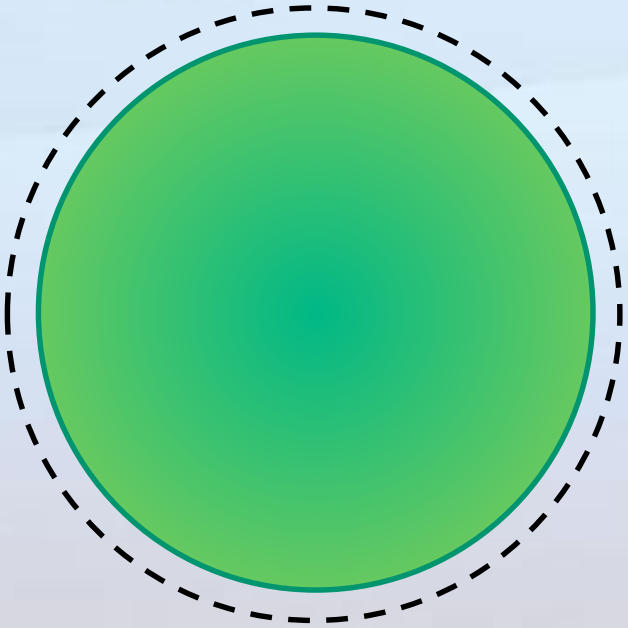


Horizontal motions are natural

- Galileo's conclusion: horizontal motion is natural.
- We need not explain why horizontal motion happens (as Aristotle did) but why it stops.
- The inertia (惯性) of the object keeps the object moving.
  - This later "became" Newton's first law.

Galileo's "horizontal"

- In fact, Galileo's "horizontal" direction is circular. Inertial motion (惯性运动) is along the spherical surface of Earth.



I. Bernard Cohen, *The Birth of a New Physics* (New York: W. W. Norton & Company, 1985). Chapter 7.

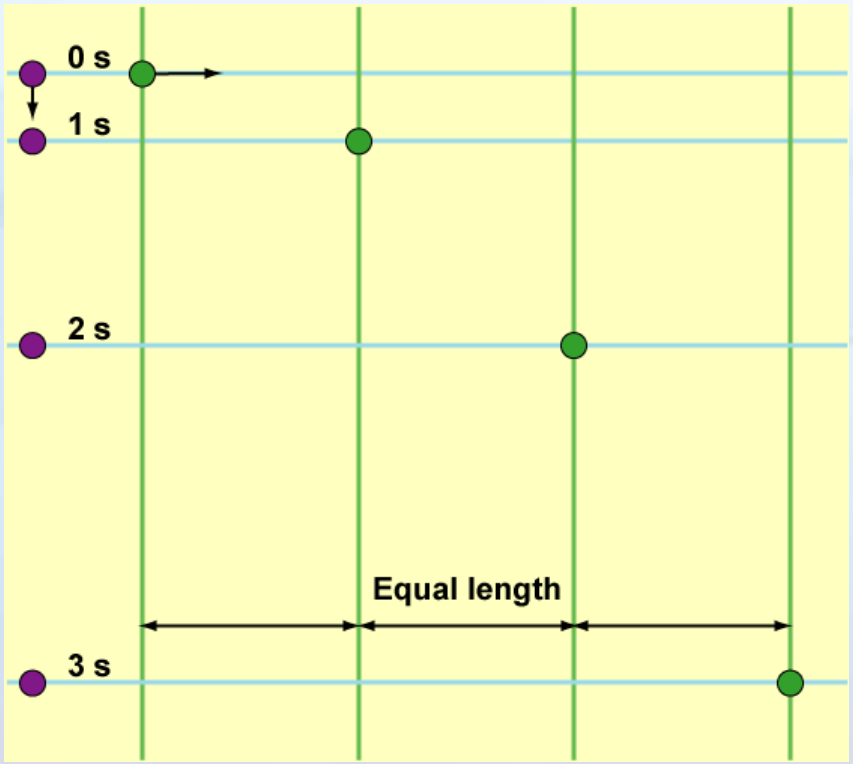
experiments. By the first two Laws and the first two Corollaries, Galileo discovered that the descent of bodies varies as the square of the time and that the motion of projectiles is in the curve of a parabola, experience agreeing with both, unless so far as these motions are a little retarded by the resistance of the air." The "two Corollaries" deal with methods used by Galileo and many of his predecessors to combine two different forces or two independent motions. Fifty years after the publication of Galileo's *Two New Sciences* it was difficult for Newton, who had already established an inertial physics, to conceive that Galileo could have come as close as he had to the concept of inertia without having taken full leave of circularity and having known the true principle of linear inertia.

I. Bernard Cohen, *The Birth of a New Physics*
(New York: W. W. Norton & Company, 1985).
Chapter 7.

A precise description of projectile

- The two independent motions adding together.

Falling vertically



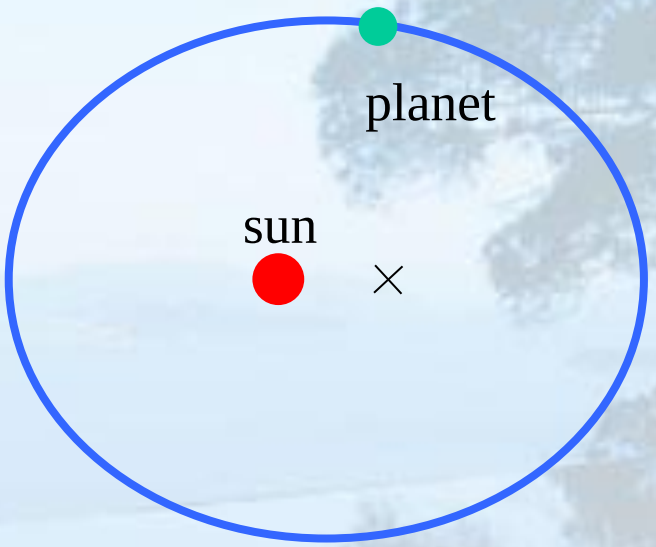
Horizontal natural motion

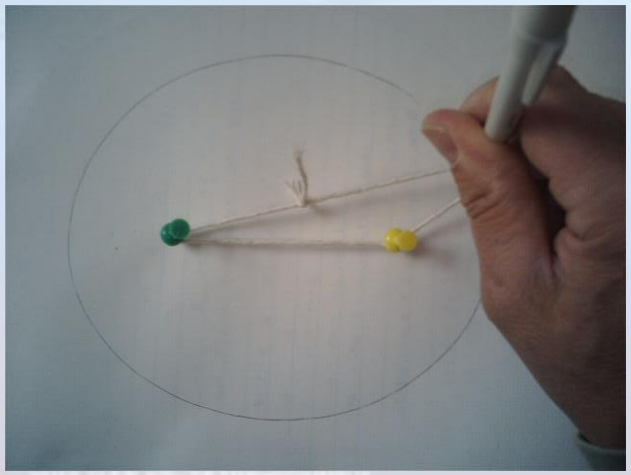
Comment on Galileo

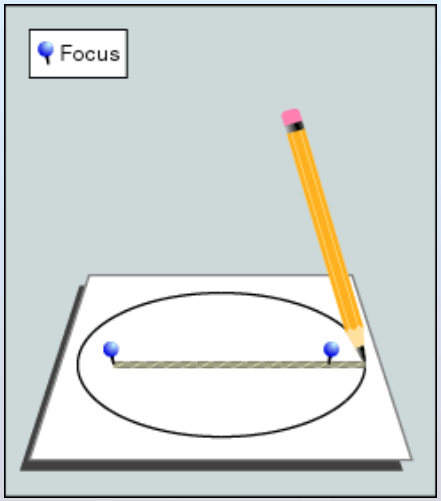
- proposed the law of inertia which later advanced to become Newton's first law.
- could not leave circularity.
- could NOT unify the celestial and the terrestrial regions successfully.

Kepler's First Law

- Kepler (1571-1630) discovered three laws of planetary motion.
- First law: the planetary orbits are ellipses.







It is NOT easy to discover …

- Does Earth's orbit really look elliptical?

: Earth's orbit

: a circle

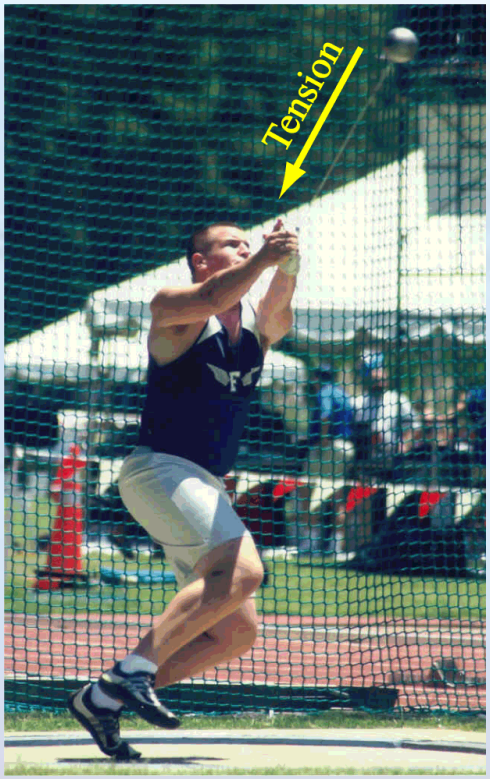
focus: 焦点
(plural: foci/ focuses)

Hooke, Halley and Wren

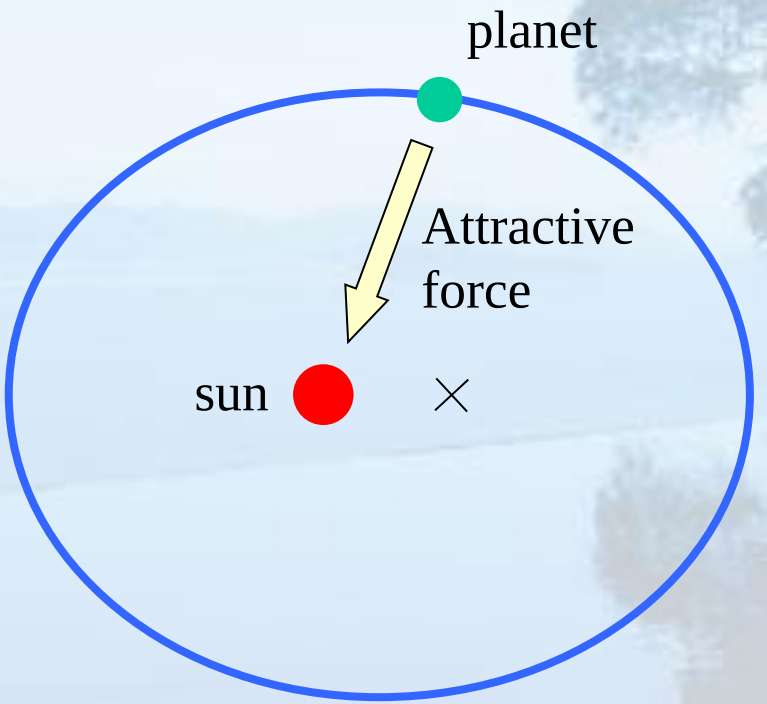
- 17th century
- They asked: Under what law of force would a planet follow an elliptical orbit?

(Hooke: 胡克; Halley: 哈雷; Wren: 雷恩, architect)

There is a force!

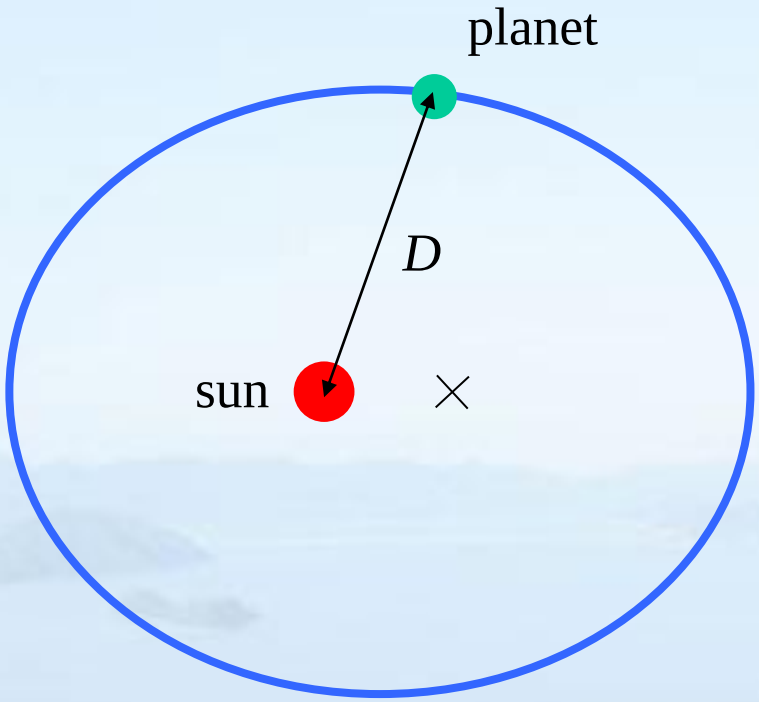


Hammer throw



Gravitation is an invisible string.

The solar system



- The problem is: How are the strength of the attractive force and the distance *D* related?

$$F \propto 1/D^2$$

- Who proposed first?
- Was he Newton?
- No.
- Perhaps Robert Hooke, a contemporary of Newton, or another person.

Robert Hooke
(1635-1703)

He [Robert Hooke] was clearly much closer to a solution by 6 January 1680, when he told Newton that gravitational attraction 'is always in a duplicate proportion to the Distance from the Center Reciprocall', … Hooke does not tell us exactly how he had come to these all-important conclusions,…

Allan Chapman, "England's Leonardo: Robert Hooke (1635-1703) and the art of experiment in Restoration England," *Proceedings of the Royal Institution of Great Britain*, **67,** 239 - 275 (1996).

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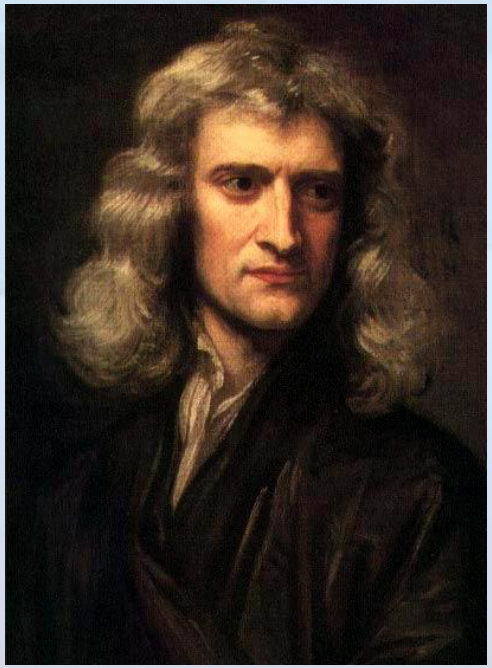
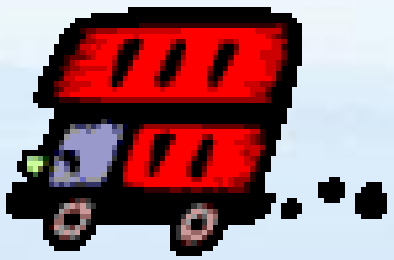
Could Newton unify the two regions?

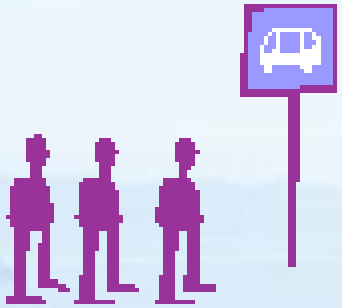


Question 1



- Will you hit the bus rear if you jump while the bus is in motion?

Question 2



- Why are you “pushed” forward when the bus is braked to a stop?

Inertia

- Now we know the answer …
- An internal tendency (倾向) to maintain the state of motion (either moving or at rest).
- Newton called it an “innate (固有的) force of matter”. (Definition III, *The Principia*)

The First Law

general terms

- Every body perseveres (坚持) in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.
- Also called “The law of inertia” (惯性定律).
- Galileo discovered this.

Let’s play the snooker …

- Newton’s 3 laws of motion answer 3 questions:
  - If the cue does not hit the ball, what will happen to the ball?
    - Newton’s first law
  - If the cue hits the ball, what will happen to the ball?
    - Newton’s second law
  - If the cue hits the ball, what will happen to the cue?
    - Newton’s third law

Second law

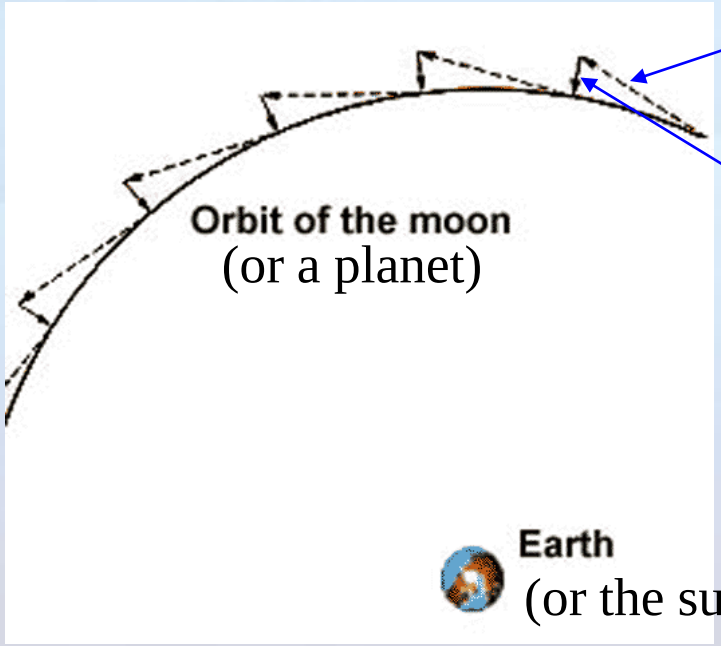
- The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
- Or simply: Force ⇨ acceleration

DEFINITION V.

A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.

Of this sort is gravity, by which bodies tend to the centre of the earth magnetism, by which iron tends to the loadstone; and that force, what ever it is, by which the planets are perpetually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits. A stone, whirled about in a sling, endeavours to recede from the hand that

First law



Gravity causes acceleration: Second law

Planetary motion is not natural. It is a combination of two kinds of motion.

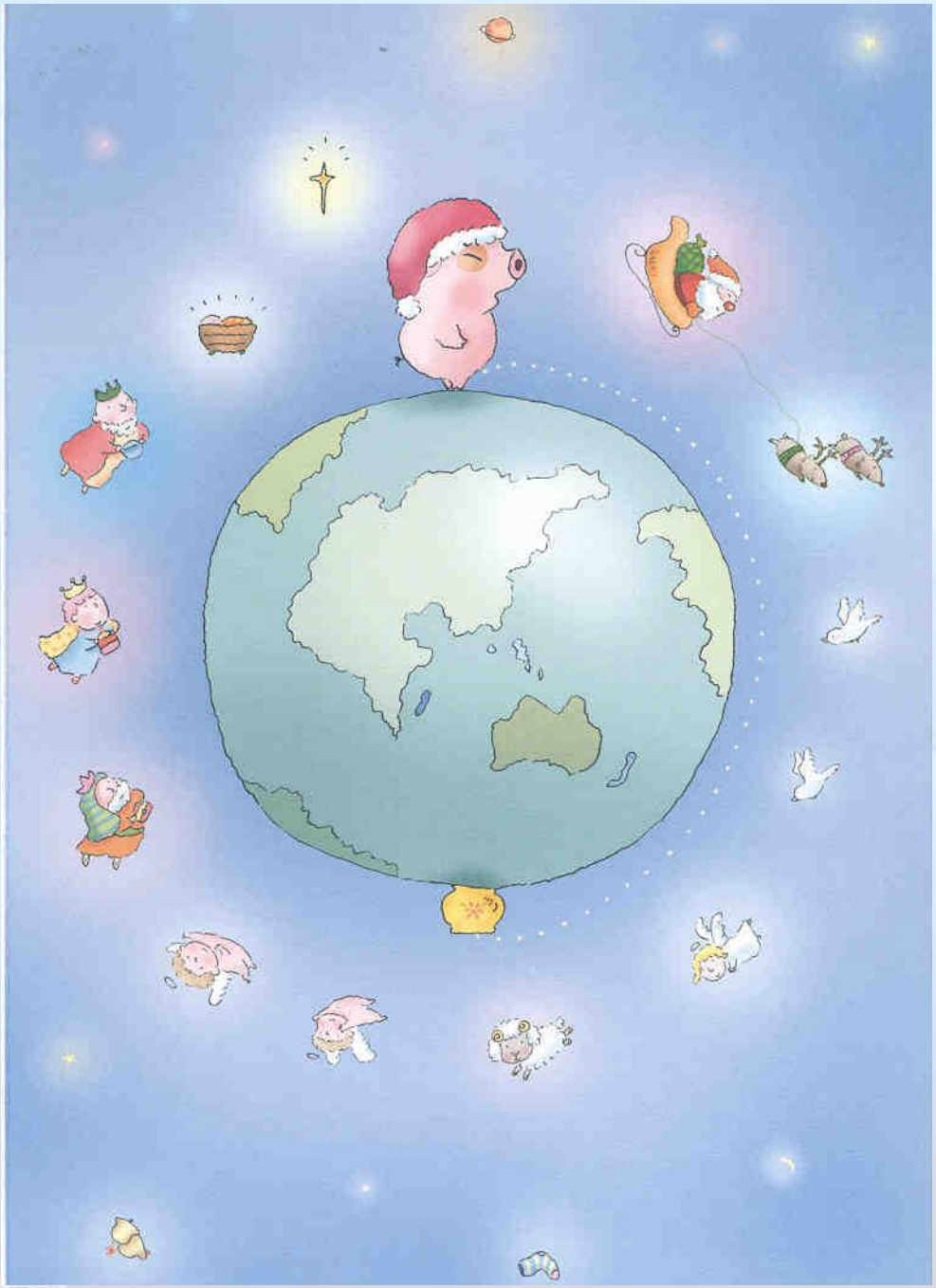
(Definition V, *The Principia*)

farther it will go. If a leaden ball, projected from the top of a mountain by the force of gunpowder with a given velocity, and in a direction parallel to the horizon, is carried in a curve line to the distance of two miles before it falls to the ground; the same, if the resistance of the air were taken away, with a double or decuple velocity, would fly twice or ten times as far. And by increasing the velocity, we may at pleasure increase the distance to which it might be projected, and diminish the curvature of the line, which it might describe, till at last it should fall at the distance of 10, 30, or 90 degrees, or even might go quite round the whole earth before it falls; or lastly, so that it might never fall to the earth, but go forward into the celestial spaces, and proceed in its motion *in infinitum*. And after the same manner that a projectile, by the force of gravity, may be made to revolve in an orbit, and go round the whole earth, the moon also, either by the force of gravity, if it is endued with gravity, or by any other force, that impels it towards the earth, may be perpetually drawn aside towards the earth, out of the rectilinear way, which by its innate force it would pursue; and would be made to revolve in the orbit which it now describes; nor could the moon with out some such force, be retained in its orbit. If this force was

See movie ☺

unification of the terrestrial and celestial regions

Orbit of McUrine



Unification of the universe

- Newton could unify the terrestrial and celestial regions ⇨ the universe is one.
- What understanding is Newton's understanding?

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Two questions

- Can all phenomena be reduced to several basic principles (axioms 公理)?
  - reductionism
- If yes, what is the language of these principles?
  - mathematics

Definitions

- Definitions
  - I. Quantity of matter
  - II. Quantity of motion
  - III. Inertia
  - IV. Impressed force
  - V. Centripetal force
  - …

Axioms (公理)

- Propositions that cannot be proved but can be accepted.
- In *The Principia*, there are three:
  - First Law of Motion
  - Second Law of Motion
  - Third Law of Motion

Disclaimer?

I likewise call attractions and impulses, in the same sense, accelerative, and motive; and use the words attraction, impulse or propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not physically, but mathematically: wherefore, the reader is not to imagine, that by those words, I anywhere take upon me to define the kind, or the manner of any action, the causes or the physical reason thereof, or that I attribute forces, in a true and physical sense, to certain centres (which are only mathematical points); when at any time I happen to speak of centres as attracting, or as endued with attractive powers.

(The last paragraph before Axioms)

A ~~Philosophical~~ Mathematical foundation

- *Principia mathematica philosophiae naturalis = The Mathematical Principles of Natural Philosophy*.
- Newton tried to establish a mathematical foundation of mechanics, but never a philosophical foundation.

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Idealization

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Of this sort is gravity, by which bodies tend to the centre of the earth magnetism, by which iron tends to the loadstone; and that force, what ever it is, by which the planets are perpetually drawn aside from the rectilinear motions, which otherwise they would pursue, and made to revolve in curvilinear orbits. A stone, whirled about in a sling, endeavours to recede from the hand that turns it; and by that endeavour, distends the sling, and that with so much the greater force, as it is revolved with the greater velocity, and as soon as ever it is let go, flies away. That force which opposes itself to this endeavour, and by which the sling perpetually draws back the stone towards the hand, and retains it in its orbit, because it is directed to the hand as the centre of the orbit, I call the centripetal force. And the same thing is to be understood of all bodies, revolved in any orbits. They all endeavour to recede from the centres of their orbits; and were it not for the opposition of a contrary force which restrains them to, and detains them in their orbits,

limitless in time

limitless in time

centre, orbit: mathematical concepts

- all are idealistic concepts.

right line: a mathematical object

uniform motion: limitless in time and space

which I therefore call centripetal, would fly off in right lines, with an uniform motion. A projectile, if it was not for the force of gravity, would not deviate towards the earth, but would go off from it in a right line, and that with a uniform motion, if the resistance of the air was taken away. It is by its gravity that it is drawn aside perpetually from its rectilinear course, and made to deviate towards the earth, more or less, according to the force of its gravity, and the velocity of its motion. The less its gravity is, for the quantity of its matter, or the greater the velocity with which it is projected, the less will it deviate from a rectilinear course, and the farther it will go. If a leaden ball, projected from the top of a mountain by the force of gunpowder with a given velocity, and in a direction parallel to the horizon, is carried in a curve line to the distance of two miles before it falls to the ground; the same, if the resistance of the air were taken away, with a double or decuple velocity, would fly twice or ten times as far. And by increasing the velocity, we may at pleasure increase the distance to which it might be projected, and diminish the curvature of the line, which it might describe, till at last it should fall at the distance of 10, 30, or 90 degrees, or even might go quite round the whole earth before it falls; or lastly, so that it might never fall to the earth, but go forward into the celestial spaces, and proceed in its motion *in infinitum*. And after the same manner that a projectile, by the force of

no resistance: an ideal situation.

limitless in space and time.

- None of these can be found in the real world.

Newton's method of science

Book III.

[…]

RULES OF REASONING IN PHILOSOPHY.

RULE I.

We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

To this purpose the philosophers say that Nature does nothing in vain, and more is in vain when less will serve; for Nature is pleased with simplicity, and affects not the pomp of superfluous causes.

RULE II.

Therefore to the same natural effects we must, as far as possible, assign the same causes.

As to respiration in a man and in a beast; the descent of stones in *Europe* and in *America*; the light of our culinary fire and of the sun; the reflection of light in the earth, and in the planets.

- There are 4 rules.

Rule I: Simplicity

RULE I.

We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

- If 3 causes can explain a phenomenon, we don't need the 4th.

Rule IV: Inductive reasoning

RULE IV.

In experimental philosophy we are to look upon propositions collected by general induction from phænomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phænomena occur, by which they may either be made more accurate, or liable to exceptions.

- Induction.
- New phenomena: either
  - More accurate theory.
  - Theory abandoned.

Questions to ponder on …

However, Newton only talked about the mathematical relation between physical quantities. He did not go into their meanings.

- Did Newton really understand the universe?
- What is scientific understanding?

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5. Why did Newton write *The Principia*?

General Scholium

idea of the substance of God. We know him only by his most wise and excellent contrivances of things, and final causes: we admire him for his perfections; but we reverence and adore him on account of his dominion: for we adore him as his servants; and a god without dominion, providence, and final causes, is nothing else but Fate and Nature. Blind metaphysical necessity, which is certainly the same always and every where, could produce no variety of things. All that diversity of natural things which we find suited to different times and places could arise from nothing but the ideas and will of a Being necessarily existing. But, by way of allegory, God is said to see, to speak, to laugh, to love, to hate, to desire, to give, to receive, to rejoice, to be angry, to fight, to frame, to work, to build; for all our notions of God are taken from the ways of mankind by a certain similitude, which, though not perfect, has some likeness, however. And thus much concerning God; to discourse of whom from the appearances of things, does certainly belong to Natural Philosophy.

(Para. 33)

- God's attributes (屬性) …. from natural phenomena

General Scholium (cont'd)

Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and planets, without suffering the least diminution of its force; that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes use to do), but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always in the duplicate proportion of the distances. Gravitation towards the sun is

[…]

from phænomena, and I frame no hypotheses; for whatever is not deduced from the phænomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of

(Para. 34)

- The origin of the natural laws.

End