空气动力学基础

航天学院·空气动力学教学组

Northwestern Polytechnical University, XI’AN

课程内容安排

第一章 流体力学基础知识

第二章 流体力学基本原理与方程

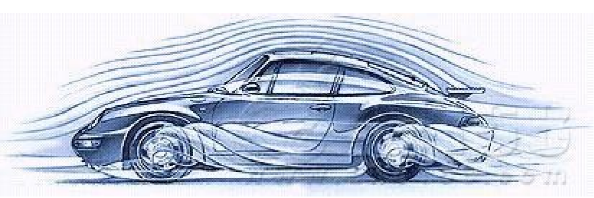
第三章 不可压理想流体绕物体的流动

第四章 高速可压缩流基础知识

第五章 一维定常可压缩管内流动

第六章 附面层和黏性流动

第七章绕翼型的低速流动



第八章绕翼型的可压缩流动

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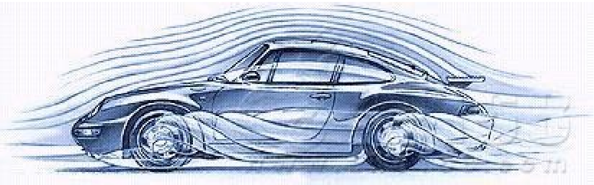
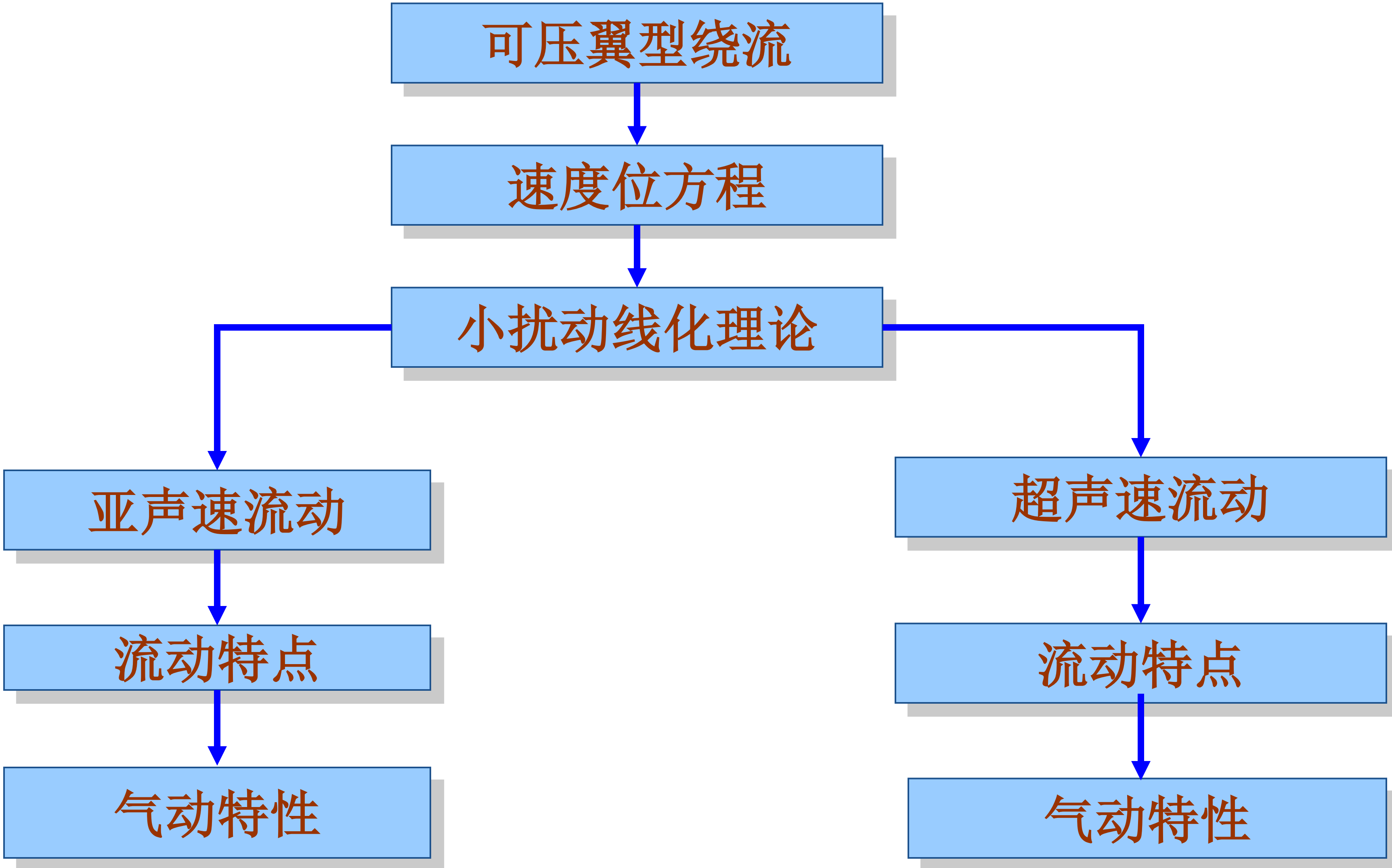


§8-1速度位方程

§8-2小扰动线化理论

§8-3亚声速流中薄翼型气动特性

§8-4超声速流中的翼型


可压翼型绕流
速度位方程
小扰动线化理论
亚声速流动
流动特点
气动特性
超声速流动
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◆ 速度位方程

1）低速不可压

拉普拉斯方程

$$\boxed{\nabla^2\phi = 0}$$

$$\boxed{\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0}$$

2）高速可压

连续方程

$$\boxed{\cancel{\frac{\partial\rho}{\partial t}}+\frac{\partial(\rho v_x)}{\partial x}+\frac{\partial(\rho v_y)}{\partial y}+\frac{\partial(\rho v_z)}{\partial z}=0}$$

定常流

$$\boxed{v_x\frac{\partial\rho}{\partial x}+v_y\frac{\partial\rho}{\partial y}+v_z\frac{\partial\rho}{\partial z}+\rho\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)=0}$$

等熵流

$$\boxed{\frac{p}{\rho^\gamma}=C}$$

$$\boxed{\frac{\partial\rho}{\partial x}=\frac{1}{a^2}\frac{\partial p}{\partial x},\frac{\partial\rho}{\partial y}=\frac{1}{a^2}\frac{\partial p}{\partial y},\frac{\partial\rho}{\partial z}=\frac{1}{a^2}\frac{\partial p}{\partial z}}$$

$$\boxed{\frac{1}{a^2}\left(\frac{v_x}{\rho}\frac{\partial p}{\partial x}+\frac{v_y}{\rho}\frac{\partial p}{\partial y}+\frac{v_z}{\rho}\frac{\partial p}{\partial z}\right)+\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)=0}$$

◆ 速度位方程

等熵流

$$\frac{1}{a^2}\left(\frac{v_x}{\rho}\frac{\partial p}{\partial x}+\frac{v_y}{\rho}\frac{\partial p}{\partial y}+\frac{v_z}{\rho}\frac{\partial p}{\partial z}\right)+\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)=0$$

欧拉方程

$$\frac{\partial p}{\partial x}=-\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)$$

$$\frac{\partial p}{\partial y}=-\rho\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)$$

$$\frac{\partial p}{\partial x}=-\rho\left(v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)$$

$$\left(1-\frac{v_x^2}{a^2}\right)\frac{\partial v_x}{\partial x}+\left(1-\frac{v_y^2}{a^2}\right)\frac{\partial v_y}{\partial y}+\left(1-\frac{v_z^2}{a^2}\right)\frac{\partial v_z}{\partial z}$$
$$-\frac{v_xv_y}{a^2}\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)-\frac{v_yv_z}{a^2}\left(\frac{\partial v_y}{\partial z}+\frac{\partial v_z}{\partial y}\right)-\frac{v_zv_x}{a^2}\left(\frac{\partial v_z}{\partial x}+\frac{\partial v_x}{\partial z}\right)=0$$

◆ 速度位方程

等熵流

$$\frac{1}{a^2}\left(\frac{v_x}{\rho}\frac{\partial p}{\partial x}+\frac{v_y}{\rho}\frac{\partial p}{\partial y}+\frac{v_z}{\rho}\frac{\partial p}{\partial z}\right)+\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)=0$$

欧拉方程

$$\frac{\partial p}{\partial x}=-\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)$$

$$\frac{\partial p}{\partial y}=-\rho\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)$$

$$\frac{\partial p}{\partial x}=-\rho\left(v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)$$

有位流动

声速可根据能量方程，写成速度的形式

$$\left(1-\frac{v_x^2}{a^2}\right)\frac{\partial^2\phi}{\partial x^2}+\left(1-\frac{v_y^2}{a^2}\right)\frac{\partial^2\phi}{\partial y^2}+\left(1-\frac{v_z^2}{a^2}\right)\frac{\partial^2\phi}{\partial z^2}$$
$$-2\frac{v_xv_y}{a^2}\frac{\partial^2\phi}{\partial x\partial y}-2\frac{v_yv_z}{a^2}\frac{\partial^2\phi}{\partial y\partial z}-2\frac{v_zv_x}{a^2}\frac{\partial^2\phi}{\partial z\partial x}=0$$

速度位的二阶非线性偏微分方程

§8-1速度位方程

§8-2小扰动线化理论

§8-3亚声速流中薄翼型气动特性

§8-4超声速流中的翼型

◆ 方程的线化

小扰动（小迎角、薄翼，小弯度）

$$\frac{v_x'}{v_\infty}、\frac{v_y'}{v_\infty}、\frac{v_z'}{v_\infty} << 1$$

合速度

$$v_x = v_\infty + v_x' \qquad v_y = v_y' \qquad v_z = v_z'$$

$$a^2 = a_\infty^2 - \frac{\gamma-1}{2}\left(2\,v_\infty v_x' + v_x'^2 + v_y'^2 + v_z'^2\right)$$

$$\begin{aligned}
&\left(1-Ma_\infty^2\right)\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\\
&= Ma_\infty^2\left[(\gamma+1)\frac{v_x}{v_\infty}+\cancel{\frac{\gamma+1}{2}\left(\frac{v_x}{v_\infty}\right)^2}+\cancel{\frac{\gamma-1}{2}\frac{v_y^2+v_z^2}{v_\infty^2}}\right]\frac{\partial v_x}{\partial x}\\
&+Ma_\infty^2\left[(\gamma-1)\frac{v_x}{v_\infty}+\cancel{\frac{\gamma+1}{2}\left(\frac{v_y}{v_\infty}\right)^2}+\cancel{\frac{\gamma-1}{2}\frac{v_x^2+v_z^2}{v_\infty^2}}\right]\frac{\partial v_y}{\partial y}\\
&+Ma_\infty^2\left[(\gamma-1)\frac{v_x}{v_\infty}+\cancel{\frac{\gamma+1}{2}\left(\frac{v_z}{v_\infty}\right)^2}+\cancel{\frac{\gamma-1}{2}\frac{v_y^2+v_x^2}{v_\infty^2}}\right]\frac{\partial v_z}{\partial z}\\
&+Ma_\infty^2\left[\frac{v_y}{v_\infty}\left(1+\cancel{\frac{v_x}{v_\infty}}\right)\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)+\frac{v_z}{v_\infty}\left(1+\cancel{\frac{v_x}{v_\infty}}\right)\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)+\cancel{\frac{v_y v_z}{v_\infty^2}}\left(\frac{\partial v_z}{\partial y}+\frac{\partial v_y}{\partial z}\right)\right]
\end{aligned}$$

省略'号

忽略高于一次微量的项 => ?

◆ 方程的线化

小扰动（小迎角、薄翼）

$$\frac{v_x'}{v_\infty}、\frac{v_y'}{v_\infty}、\frac{v_z'}{v_\infty} << 1$$

合速度

$$v_x = v_\infty + v_x' \qquad v_y = v_y' \qquad v_z = v_z'$$

忽略高阶项

$$\left(1 - Ma_\infty^2\right)\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$
$$= \cancel{Ma_\infty^2 \left(\gamma + 1\right)\frac{v_x}{v_\infty}\frac{\partial v_x}{\partial x}} + \cancel{Ma_\infty^2 \left(\gamma - 1\right)\frac{v_x}{v_\infty}\left(\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)}$$
$$\cancel{+ Ma_\infty^2 \frac{v_y}{v_\infty}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)} + \cancel{Ma_\infty^2 \frac{v_z}{v_\infty}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)}$$

马赫数不是很大，也不太接近1时，左侧各项相同数量级，故右侧多乘一个微量，可忽略右侧的微量 => ？

◆ 压强系数的线化

忽略高阶项，写成

$$\boxed{\left(1-Ma_\infty^2\right)\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0}$$

写成扰动速度位线化方程形式：

$$\Phi = v_\infty x + \phi \qquad v_x = \partial\phi/\partial x$$

$$\boxed{\left(1-Ma_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0}$$

二维流动（亚声速）

$$\boxed{\beta^2 = 1-Ma_\infty^2}$$

$$\boxed{\beta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0} \qquad Ma_\infty < 1 \qquad \text{椭圆型方程}$$

二维流动（超声速）

$$\boxed{B^2 = Ma_\infty^2-1}$$

$$\boxed{B^2\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}=0} \qquad Ma_\infty > 1 \qquad \text{双曲型方程}$$

不适用于：跨声速流动、马赫数较高的超声速流动

◆ 压强系数的线化

压强系数

$$\boxed{C_p=\frac{p-p_\infty}{\frac{1}{2}\rho_\infty v_\infty^2}},\quad \boxed{a^2=\gamma\frac{p_\infty}{\rho_\infty}} \Rightarrow C_p=\frac{2}{\gamma Ma_\infty^2}\left(\frac{p}{p_\infty}-1\right)$$

$$C_p=\frac{2}{\gamma Ma_\infty^2}\left\{\left[1+\frac{\gamma-1}{2}Ma_\infty^2\left(1-\frac{v^2}{v_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}}-1\right\}$$

薄翼

$$\boxed{C_p=-\frac{2v_x}{v_\infty}=-\frac{2}{v_\infty}\frac{\partial\phi}{\partial x}}$$

旋成体

$$\boxed{C_p=-\left(\frac{2v_x}{v_\infty}+\frac{v_r^2}{v_\infty^2}\right)}$$

压强系数只跟X扰动速度有关

◆ 边界条件的线化

物面条件（无穿透）

$$\boxed{v\cdot n=0}$$

$$(v_\infty+v_x)\cos(n,x)+v_y\cos(n,y)+v_z\cos(n,z)=0$$

小扰动下，厚度、弯度很小

$$\boxed{(v_y)_{y=0}=\left(\frac{\partial\phi}{\partial y}\right)_{y=0}=v_\infty\frac{\partial f}{\partial x}}$$

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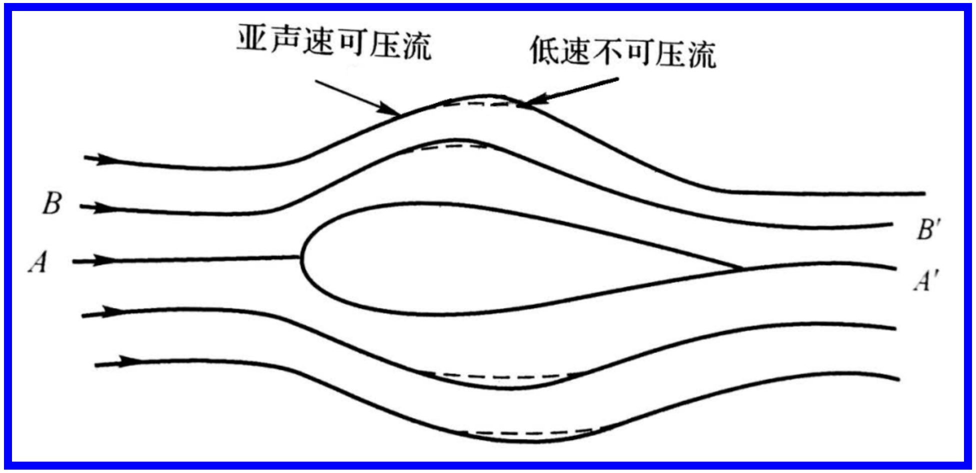
◆ 流动特点

在翼型上下流管收缩处，亚声速的流线在竖向受到的扰动的扩张，要比低速不可压流的流线为大



$$\frac{dA}{A}=-\left(1-Ma^2\right)\frac{dv}{v}$$

$$\frac{d\rho}{\rho}=-Ma^2\frac{dv}{v}$$

亚声速流动的压缩性影响，将使翼型在竖向所产生的扰动，要比低速不可压流的为强

◆ 气动特性

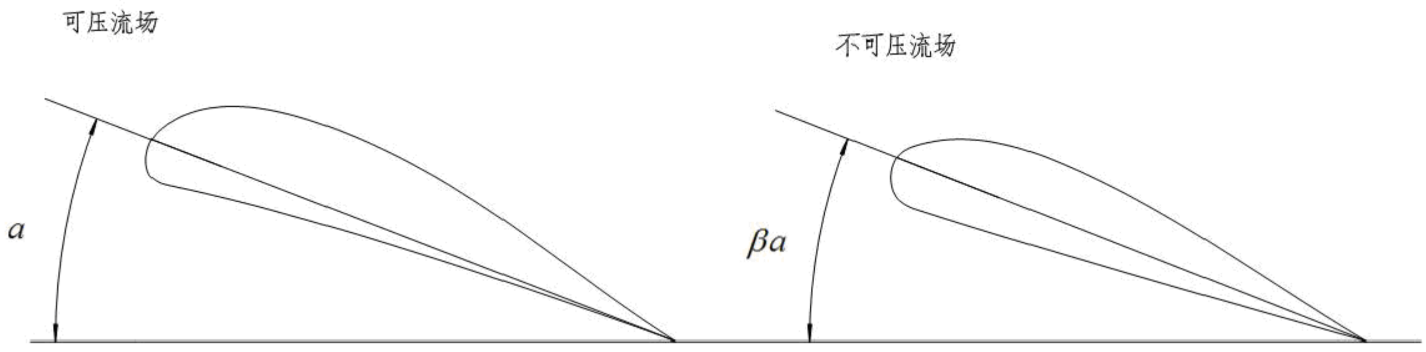
引入变换（仿射变换） $x'=x, y'=\beta y, \phi'=k\phi, v_\infty'=v_\infty$

亚声速线化方程 $\boxed{\beta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0} \rightarrow \boxed{\frac{\partial^2\phi'}{\partial x'^2}+\frac{\partial^2\phi'}{\partial y'^2}=0}$

边界条件 $\left(\frac{\partial\phi'}{\partial y'}\right)_{y'=0}=v_\infty'\frac{dy'}{dx'}$

对应翼型（仿射相似）

$$\left.\begin{aligned}&\text{相对厚度}\ \overline{c}'=\beta\overline{c}\\&\text{相对弯度}\ \overline{f}'=\beta\overline{f}\\&\text{迎角}\ \alpha'=\beta\alpha\\&k=\beta^2\end{aligned}\right\}$$



◆ 气动特性

压强系数只跟X扰动速度有关

压强系数

$$C_p = -\frac{2}{v_\infty}\frac{\partial \phi}{\partial x} = -\frac{1}{\beta^2}\frac{2}{v'_\infty}\frac{\partial \phi'}{\partial x'}$$

$$(C_p)_{Ma_\infty,\alpha,\bar{c},\bar{f}} = \frac{1}{\beta^2}(C_p)_{0,\beta\alpha,\beta\bar{c},\beta\bar{f}}$$

需要进行翼型仿射变换，不方便！？

$$(C_p)_{0,\alpha,\bar{c},\bar{f}} = \frac{1}{\beta}(C_p)_{0,\beta\alpha,\beta\bar{c},\beta\bar{f}}$$

$$(C_p)_{Ma_\infty,\alpha,\bar{c},\bar{f}} = \frac{1}{\beta}(C_p)_{0,\alpha,\bar{c},\bar{f}}$$

普朗特-格劳沃法则

$\frac{1}{\beta}$亚声速压缩修正因子

气动参数

$$(C_y)_{Ma_\infty,\alpha,\bar{c},\bar{f}} = \frac{1}{\beta}(C_y)_{0,\alpha,\bar{c},\bar{f}}$$

$$(m_z)_{Ma_\infty,\alpha,\bar{c},\bar{f}} = \frac{1}{\beta}(m_z)_{0,\alpha,\bar{c},\bar{f}}$$

$$(C_y^a)_{Ma_\infty} = \frac{1}{\beta}(C_y^\alpha)_0$$

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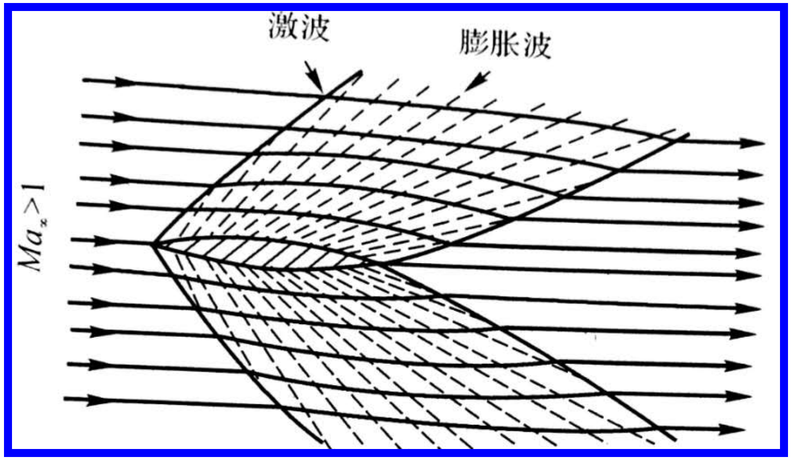
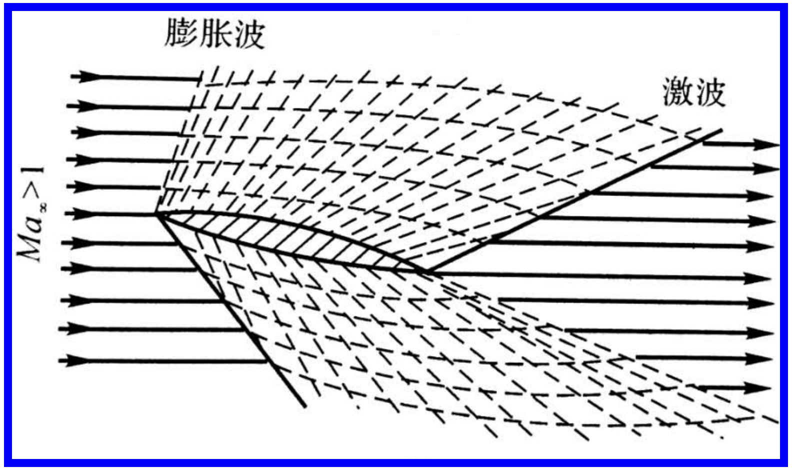
◆ 流动特点

$\alpha < \theta$ 气流在前缘上下表面内折产生激波;
沿前缘切线方向流动;
上下表面斜率减小，形成膨胀波;
后缘各产生一道斜激波。



$\alpha > \theta$ 气流在前缘上表面外折产生膨胀波，后缘上表面产生激波;
气流在前缘下表面内折产生激波，后缘下表面产生膨胀波



◆ 线化理论

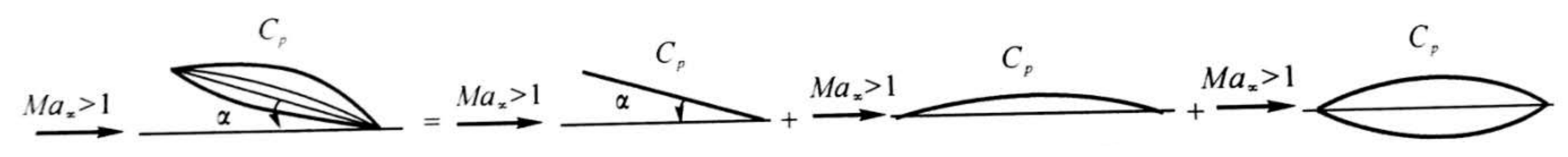
一级近似，将激波、膨胀波看成马赫波

$$C_p = \pm\frac{2\theta}{\sqrt{Ma_\infty^2-1}}$$

$$\left.\begin{aligned} C_{p_u}(x,\ +0) &= \frac{2\left(\dfrac{dy}{dx}\right)_u}{\sqrt{Ma_\infty^2-1}} \\ C_{p_l}(x,\ -0) &= \frac{-2\left(\dfrac{dy}{dx}\right)_l}{\sqrt{Ma_\infty^2-1}} \end{aligned}\right\}$$

薄翼条件下

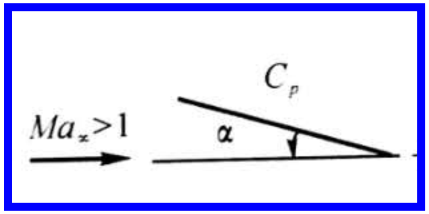
$$C_p = C_{p\alpha} + C_{pf} + C_{pc}$$



◆ 气动特性

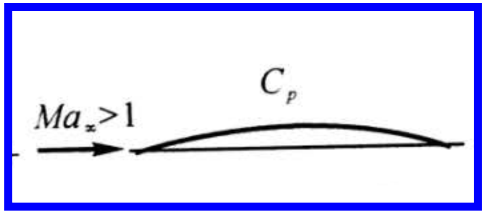
迎角问题

$$\Delta(C_p)_\alpha = (C_{p_l} - C_{p_u})_\alpha = \frac{4\alpha}{\sqrt{Ma_\infty^2 - 1}}$$



弯度问题

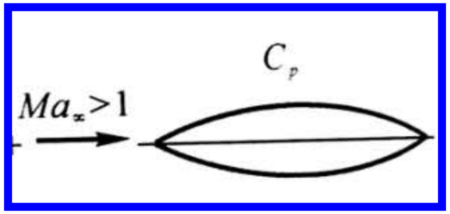
$$\left.\begin{aligned}(C_{p_u})_f &= 2\left(\frac{dy}{dx}\right)_f \Big/ \sqrt{Ma_\infty^2 - 1} \\ (C_{p_l})_f &= -2\left(\frac{dy}{dx}\right)_f \Big/ \sqrt{Ma_\infty^2 - 1}\end{aligned}\right\}$$



$$\Delta(C_p)_f = (C_{p_l} - C_{p_u})_f = -4\left(\frac{dy}{dx}\right)_f \Big/ \sqrt{Ma_\infty^2 - 1}$$

厚度问题

$$\left.\begin{aligned}(C_{p_u})_f &= 2\left(\frac{dy_u}{dx}\right)_c \Big/ \sqrt{Ma_\infty^2 - 1} \\ (C_{p_l})_f &= -2\left(\frac{dy_l}{dx}\right)_c \Big/ \sqrt{Ma_\infty^2 - 1}\end{aligned}\right\}$$



$$\Delta(C_p)_c = (C_{p_l} - C_{p_u})_c = 0$$

◆ 气动特性

升力贡献为零

升力系数

$$\boxed{C_y = (C_y)_\alpha + (C_y)_f + (C_y)_c}$$

$$\boxed{C_y = (C_y)_\alpha = \frac{4\alpha}{\sqrt{Ma_\infty^2 - 1}}}$$

波阻系数

$$\boxed{C_{x_b} = \frac{4}{\sqrt{Ma_\infty^2 - 1}} \left\{ \alpha^2 + \underline{\frac{1}{b}\int_0^b \left[\left(\frac{dy}{dx}\right)_f^2 + \left[\frac{dy}{dx}\right]_c^2 \right] dx} \right\}}$$

俯仰力矩系数（绕前缘）

$$\boxed{m_z = -\frac{C_y}{2} - \frac{4}{b^2\sqrt{Ma_\infty^2 - 1}} \int_0^b y_f \, dx}$$

$$\boxed{\bar{x}_p = \frac{x_p}{b} = -\frac{m_z}{C_y}} \qquad \boxed{\bar{x}_F = -\frac{\partial m_z}{\partial C_y} = \frac{1}{2}}$$

（1）亚声速时绕翼型的流动特点；

（2）亚声速时绕翼型的气动参数；

（3）超声速时绕翼型的流动特点；

（4）超声速时绕翼型的气动参数。

Thanks for your attention!

MERCI!

谢谢！