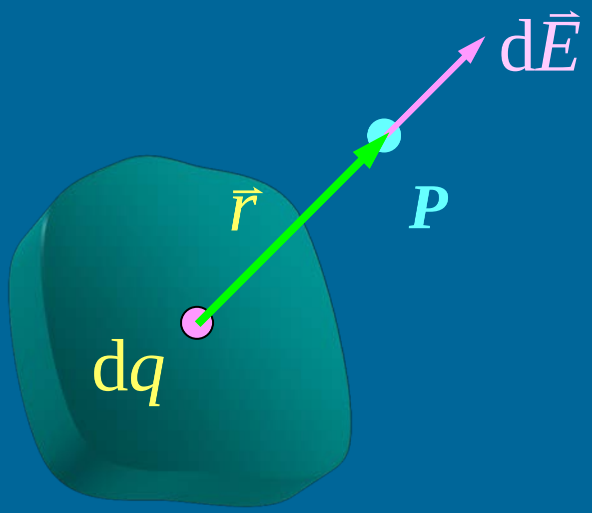
1. 库仑定律 $\boxed{\vec{F}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}\vec{r}^{\,0}}$ $\vec{r}^{\,0}$ 由施力电荷指向受力电荷

2. 场强的计算

点电荷的电场 $$\vec{E}=\frac{\vec{F}}{q_0}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{r}^{\,0}$$

点电荷系的电场 $$\vec{E}=\sum_k\frac{1}{4\pi\varepsilon_0}\frac{q_k}{r_k^2}\vec{r}_k^{\,0}$$

连续分布带电体 $$\vec{E}=\int\frac{\mathrm{d}q}{4\pi\varepsilon_0 r^2}\vec{r}^{\,0}$$

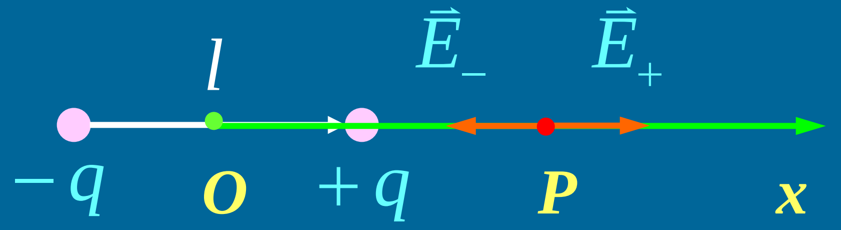


例 求电偶极子在延长线上和中垂线上一点产生的电场强度。

解

$$\vec{E}_{+}=\frac{q}{4\pi\varepsilon_0(x-l/2)^2}\vec{i}$$

$$\vec{E}_{-}=-\frac{q}{4\pi\varepsilon_0(x+l/2)^2}\vec{i}$$



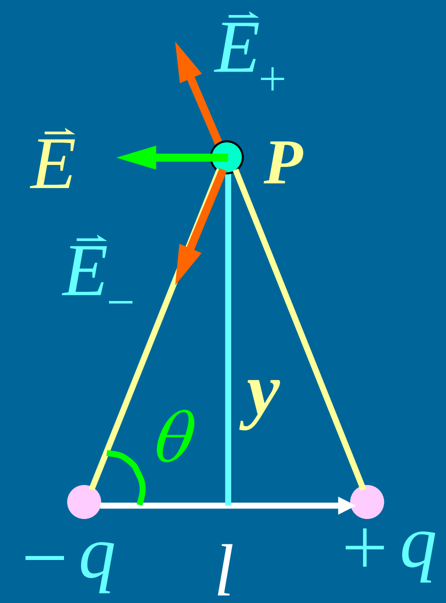
$$\vec{E}=\vec{E}_{+}+\vec{E}_{-}=\frac{q\cdot 2xl}{4\pi\varepsilon_0(x^2-l^2/4)^2}\vec{i}$$

令：电偶极矩 $\vec{p}=q\vec{l}$

$$=\frac{2x\vec{p}}{4\pi\varepsilon_0(x^2-l^2/4)^2}\approx\frac{1}{4\pi\varepsilon_0}\cdot\frac{2\vec{p}}{x^3}\qquad(\because x>>l)$$

在中垂线上

$$E_{+} = E_{-} = \frac{q}{4\pi\varepsilon_0 (y^2 + l^2/4)}$$



$$E = 2E_{+}\cos\theta \qquad \cos\theta = \frac{l/2}{(y^2 + l^2/4)^{1/2}}$$

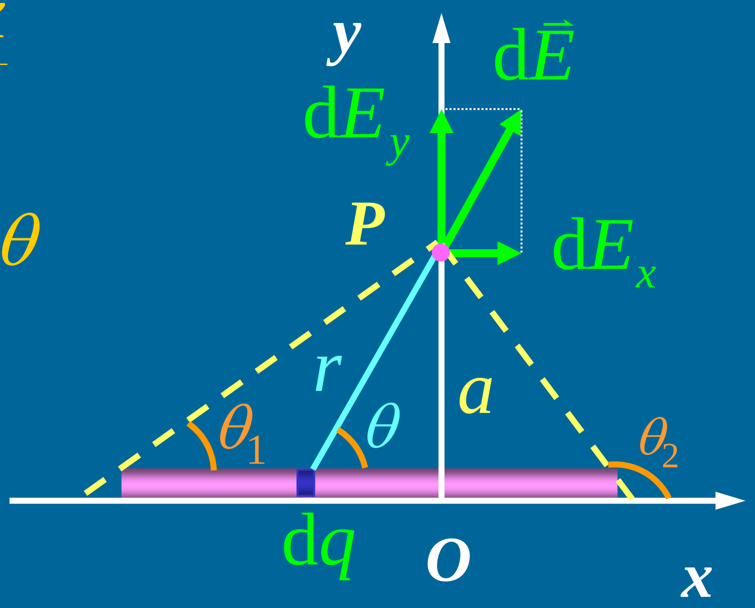
$$\Rightarrow \quad E = \frac{ql}{4\pi\varepsilon_0 (y^2 + l^2/4)^{3/2}} \qquad y >> l$$

$$\Rightarrow \quad E = \frac{p}{4\pi\varepsilon_0 y^3} \qquad \vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 y^3}$$

例 长为L的均匀带电直杆，电荷线密度为λ

求 它在空间一点P产生的电场强度（P点到杆的垂直距离为a)

解 $\mathrm{d}q=\lambda\mathrm{d}x$ $\qquad$ $\mathrm{d}E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\mathrm{d}x}{r^2}$



$\mathrm{d}E_x=\mathrm{d}E\cos\theta \qquad \mathrm{d}E_y=\mathrm{d}E\sin\theta$

由图上的几何关系

$$x=a\tan(\theta-\frac{\pi}{2})=-a\cot\theta$$

$$\mathrm{d}x=a\csc^2\theta\,\mathrm{d}\theta \qquad r^2=a^2+x^2=a^2\csc^2\theta$$

$$\mathrm{d}E_x=\frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta\mathrm{d}\theta \qquad \mathrm{d}E_y=\frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta\mathrm{d}\theta$$

$$E_x = \int \mathrm{d}E_x = \int_{\theta_1}^{\theta_2} \frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta \,\mathrm{d}\theta = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

$$E_y = \int \mathrm{d}E_y = \int_{\theta_1}^{\theta_2} \frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta \,\mathrm{d}\theta = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$

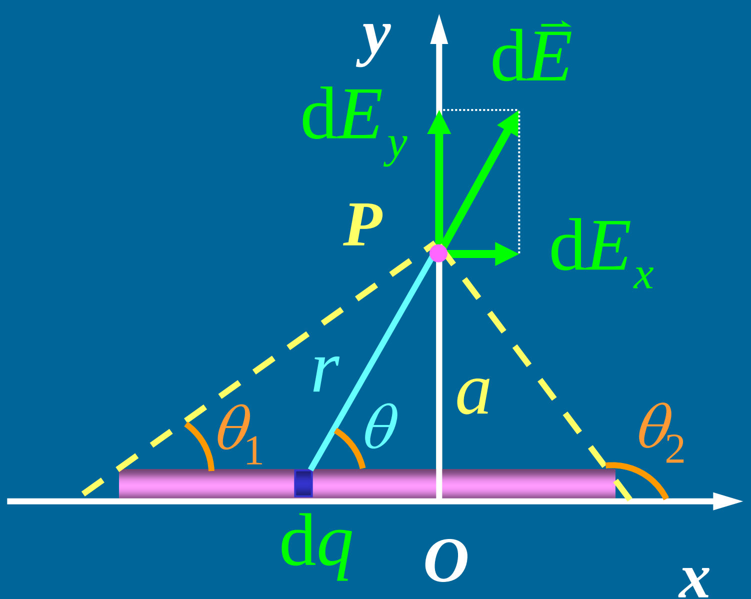
讨论

(1) $a >> L$ 杆可以看成点电荷

$$E_x = 0 \qquad E_y = \frac{\lambda L}{4\pi\varepsilon_0 a^2}$$

(2) 无限长直导线

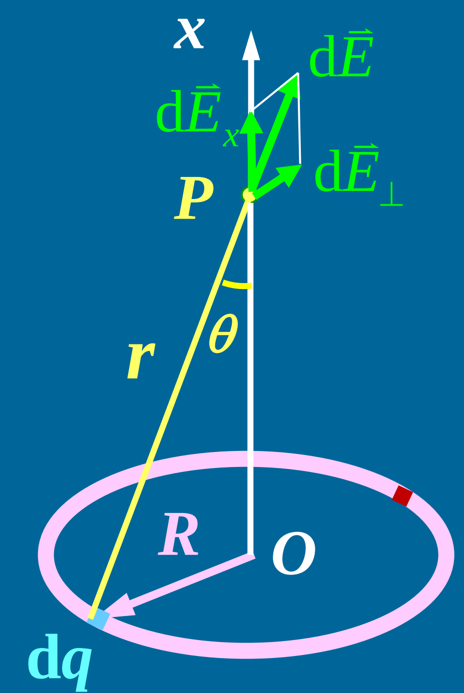
$$\left.\begin{array}{l}\theta_1 = 0 \\ \theta_2 = \pi\end{array}\right\} \Rightarrow \left\{\begin{array}{l} E_x = 0 \\ E_y = \dfrac{\lambda}{2\pi\varepsilon_0 a}\end{array}\right.$$



例 半径为R 的均匀带电细圆环，带电量为 q

求 圆环轴线上任一点P 的电场强度

解 $\mathrm{d}q=\lambda \mathrm{d}l$ $\qquad \mathrm{d}E=\dfrac{1}{4\pi\varepsilon_0}\dfrac{\mathrm{d}q}{r^2}$



$$\mathrm{d}E_\perp=\mathrm{d}E\sin\theta \qquad \mathrm{d}E_x=\mathrm{d}E\cos\theta$$

圆环上电荷分布关于x 轴对称 $\quad E_\perp=0$

$$E_x=\frac{1}{4\pi\varepsilon_0}\int\frac{\mathrm{d}q}{r^2}\cos\theta=\frac{1}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}\int\mathrm{d}q=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta$$

$$\cos\theta=\frac{x}{r} \qquad r=(R^2+x^2)^{1/2} \qquad E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}\vec{i}$$

讨论

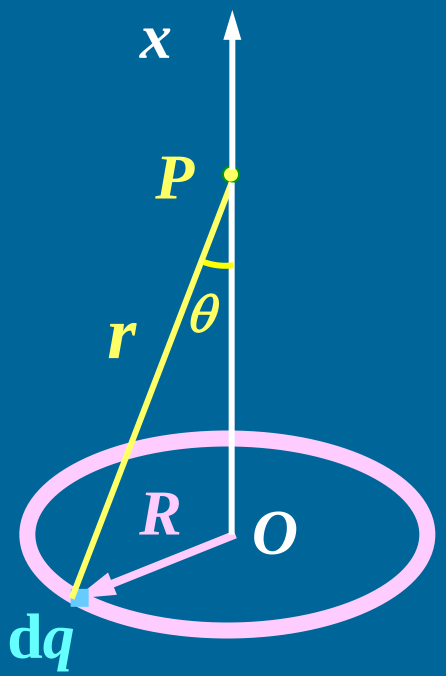
(1) 当 $x = 0$（即P点在圆环中心处）时，

$$E=0$$

(2) 当 $x>>R$ 时

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

可以把带电圆环视为一个点电荷



例 面密度为 σ 的圆板在轴线上任一点的电场强度

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}\vec{i}$$

解 $\mathrm{d}q = 2\pi r\mathrm{d}r\sigma$

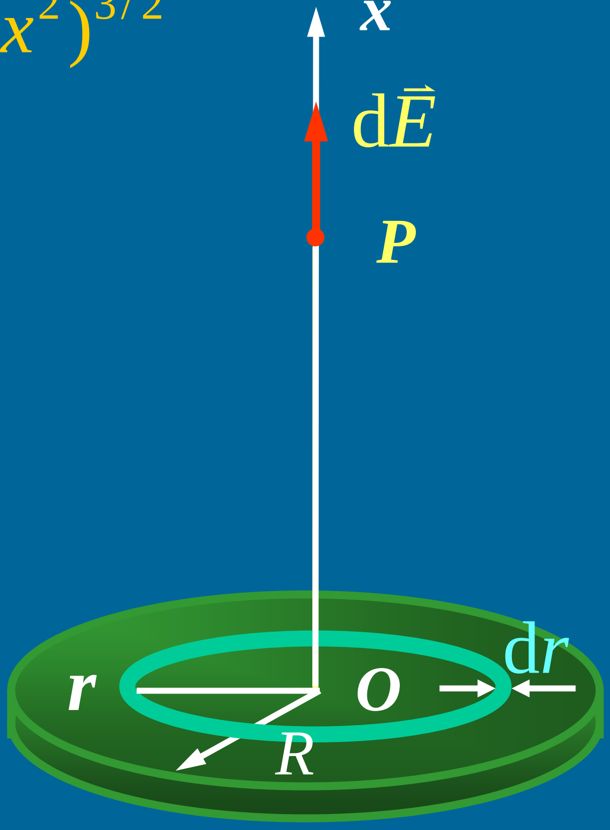
$$\mathrm{d}E = \frac{1}{4\pi\varepsilon_0}\frac{x\mathrm{d}q}{(r^2+x^2)^{3/2}} = \frac{x\sigma}{2\varepsilon_0}\frac{r\mathrm{d}r}{(r^2+x^2)^{3/2}}$$

$$E = \int \mathrm{d}E = \frac{x\sigma}{2\varepsilon_0}\int_0^R \frac{r\mathrm{d}r}{(r^2+x^2)^{3/2}}$$

$$= \frac{x\sigma}{2\varepsilon_0}\left[\frac{1}{|x|} - \frac{1}{(R^2+x^2)^{1/2}}\right]$$

若 $x>0$ $\quad \vec{E} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]\vec{i}$

若 $x<0$ $\quad \vec{E} = -\frac{\sigma}{2\varepsilon_0}\left[1 + \frac{x}{(R^2+x^2)^{1/2}}\right]\vec{i}$



若$x>0$ $\quad \vec{E}=\frac{\sigma}{2\varepsilon_0}[1-\frac{x}{(R^2+x^2)^{1/2}}]\vec{i}$

若$x<0$ $\quad \vec{E}=-\frac{\sigma}{2\varepsilon_0}[1+\frac{x}{(R^2+x^2)^{1/2}}]\vec{i}$

讨论

(1) 当$R>>x$ ，圆板可视为无限大薄板

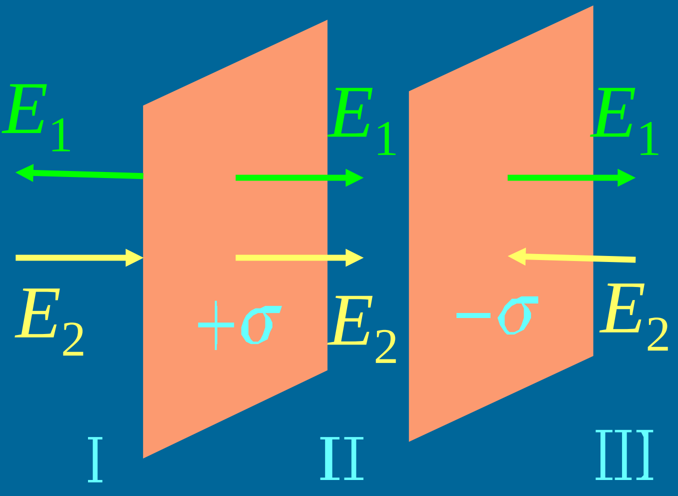
$$\boxed{E=\frac{\sigma}{2\varepsilon_0}}$$

(2)

$$E_{\mathrm{I}}=-E_1+E_2=0$$

$$E_{\mathrm{II}}=E_1+E_2=\frac{\sigma}{\varepsilon_0}$$

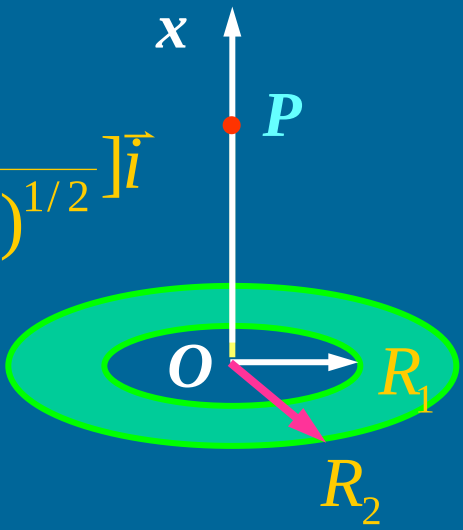
$$E_{\mathrm{III}}=E_1-E_2=0$$



(3) 当$x>>R$ 圆盘可视为点电荷

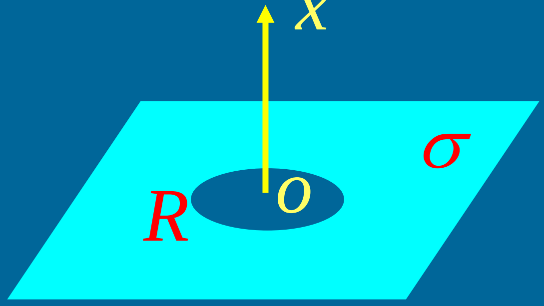
(4) 补偿法

$$\vec{E}=\vec{E}_{R_1}+\vec{E}_{R_2}=\frac{x\sigma}{2\varepsilon_0}\left[\frac{1}{(R_1^2+x^2)^{1/2}}-\frac{1}{(R_2^2+x^2)^{1/2}}\right]\vec{i}$$

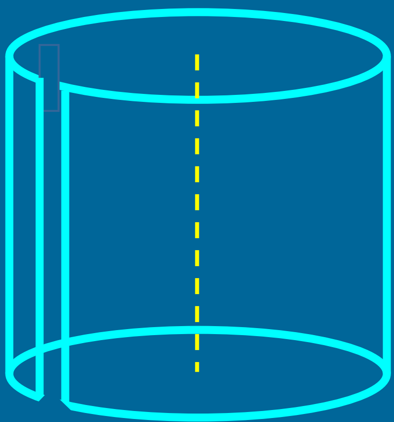


例1 无限大的带电平板，面密度σ，其上有半径为R的孔，求：孔轴线上的电场强度。

解：总场强可看作是面密度$-\sigma$的圆盘与面密度σ无限大平板的电场的叠加



例2 无限长圆柱面上有一平行于轴线的狭缝，缝宽为a，如图，求轴线上的场强。（面密度σ）

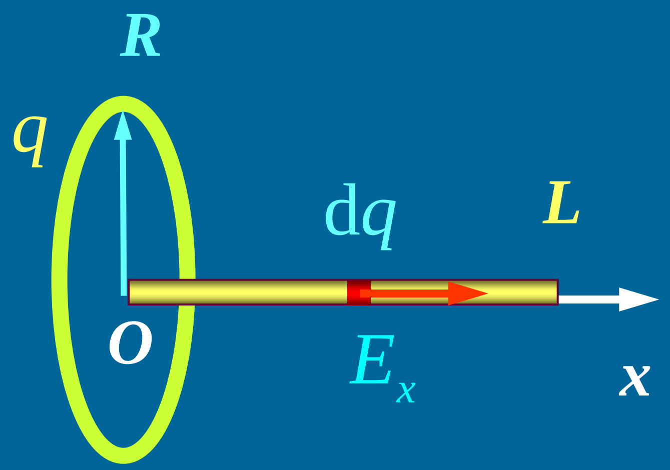


解：总场强可看作是无限长带正电圆柱面和带负电的无限长直线的电场的叠加

$$-\lambda=-a\sigma$$

例 已知圆环带电量为q，杆的线密度为λ，长为L

求 杆对圆环的作用力



解

$$\mathrm{d}q = \lambda \mathrm{d}x$$

圆环在 dq 处产生的电场

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2 + x^2)^{3/2}}$$

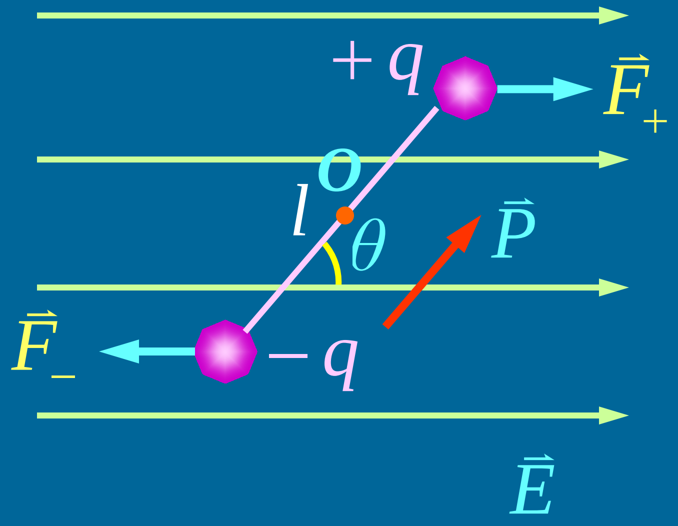
$$\mathrm{d}F = E_x \mathrm{d}q = E_x \lambda\ \mathrm{d}x$$

$$F = \int_0^L \frac{q\lambda\ x\mathrm{d}x}{4\pi\varepsilon_0 (R^2 + x^2)^{3/2}} = \frac{q\lambda}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + L^2}}\right)$$

例 求电偶极子在均匀电场中受到的力偶矩。

解 $\vec{F}_+ = q\vec{E}$ $\qquad \vec{F}_- = -q\vec{E}$

相对于O点的力矩



$$M = F_+ \cdot \frac{1}{2} l \sin\theta + F_- \cdot \frac{1}{2} l \sin\theta$$

$$= qlE\sin\theta$$

$$\vec{M} = q\vec{l} \times \vec{E} = \vec{p} \times \vec{E}$$

讨论

(1) $\theta = \dfrac{\pi}{2}$ 力偶矩最大

(2) $\theta = 0$ 力偶矩为零 (电偶极子处于稳定平衡)

(3) $\theta = \pi$ 力偶矩为零 (电偶极子处于非稳定平衡)

总结

1. 均匀带电直杆

$$\begin{cases} E_x = \dfrac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1) \\ E_y = \dfrac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2) \end{cases}$$

无限长直导线

$$\begin{cases} E_x = 0 \\ E_y = \dfrac{\lambda}{2\pi\varepsilon_0 a} \end{cases}$$

2. 均匀带电细圆环

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2 + x^2)^{3/2}}\vec{i}$$

3. 无限大带电薄板

$$E = \frac{\sigma}{2\varepsilon_0}$$

4. 补偿法