第四章 根轨迹法

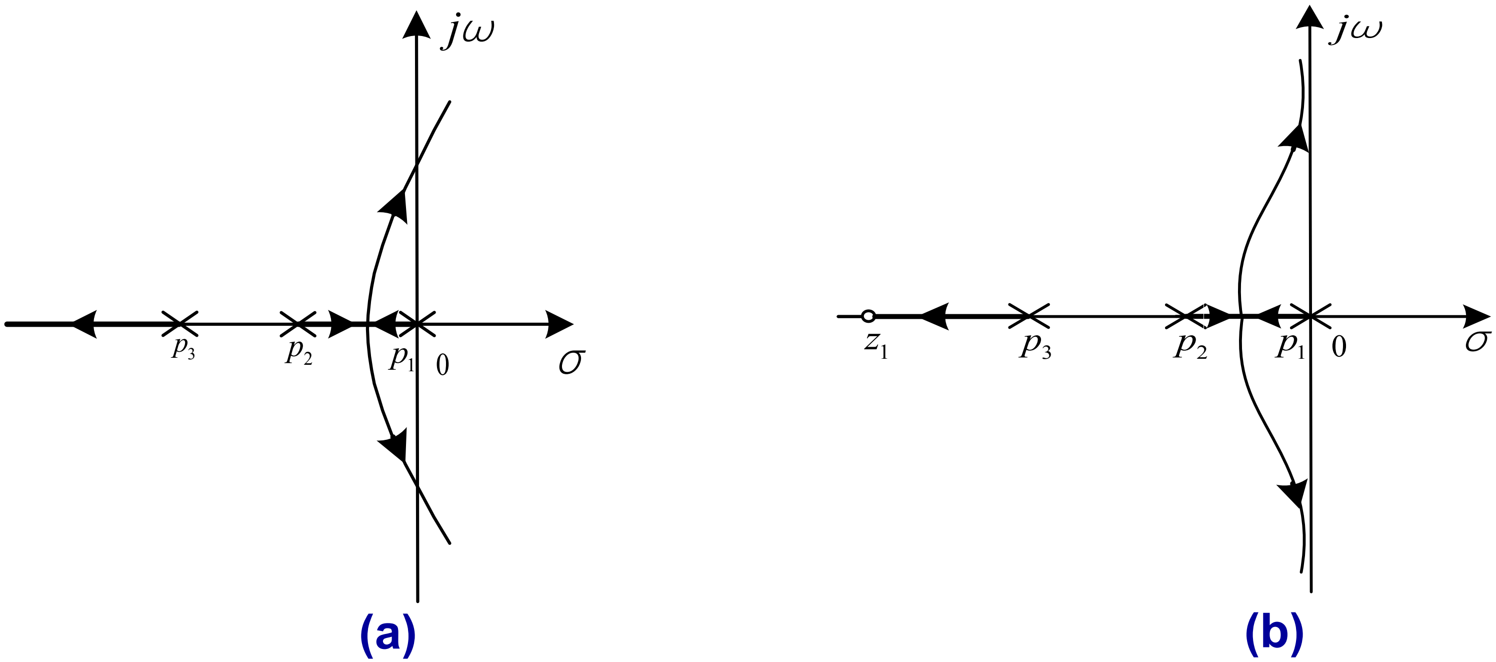
4.5 根轨迹校正

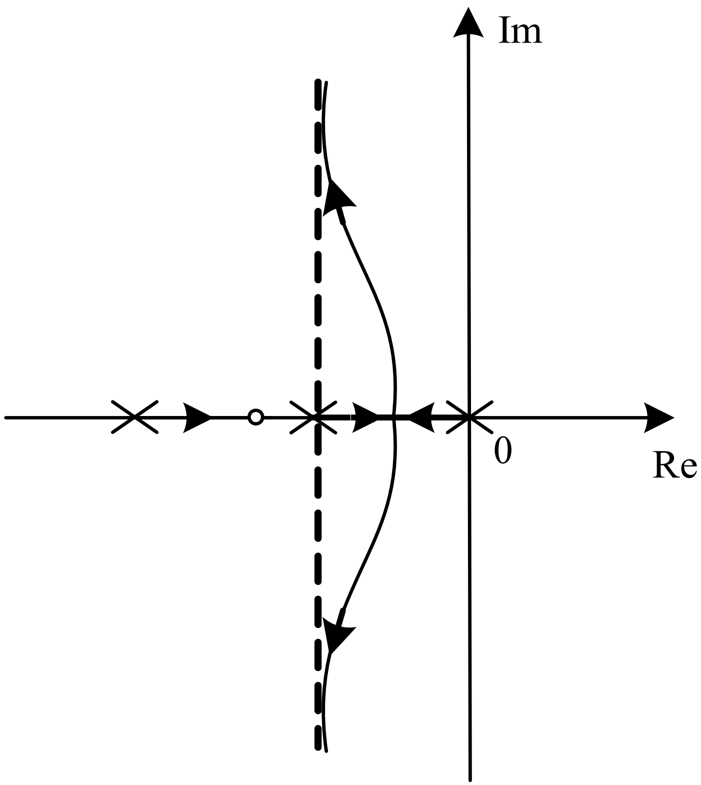
校正目的：选择合适的校正装置$K(s)$，使根轨迹在合适的增益K值下具有所希望的主导极点，从而获得所希望的性能。

1. 附加开环零点的作用

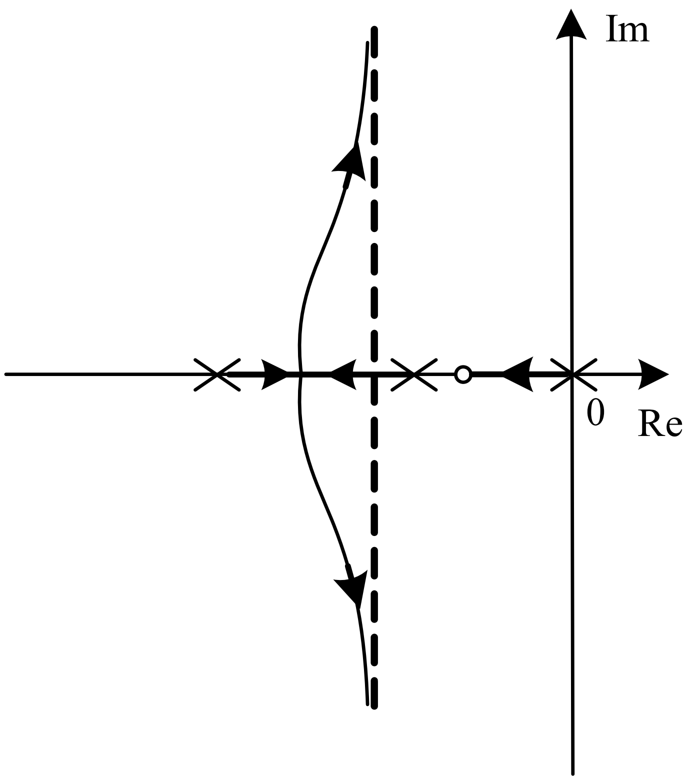
(1) 附加适当的开环零点**可以改善系统的稳定性。**

(2) 可以改善系统的动态性能。





(c)



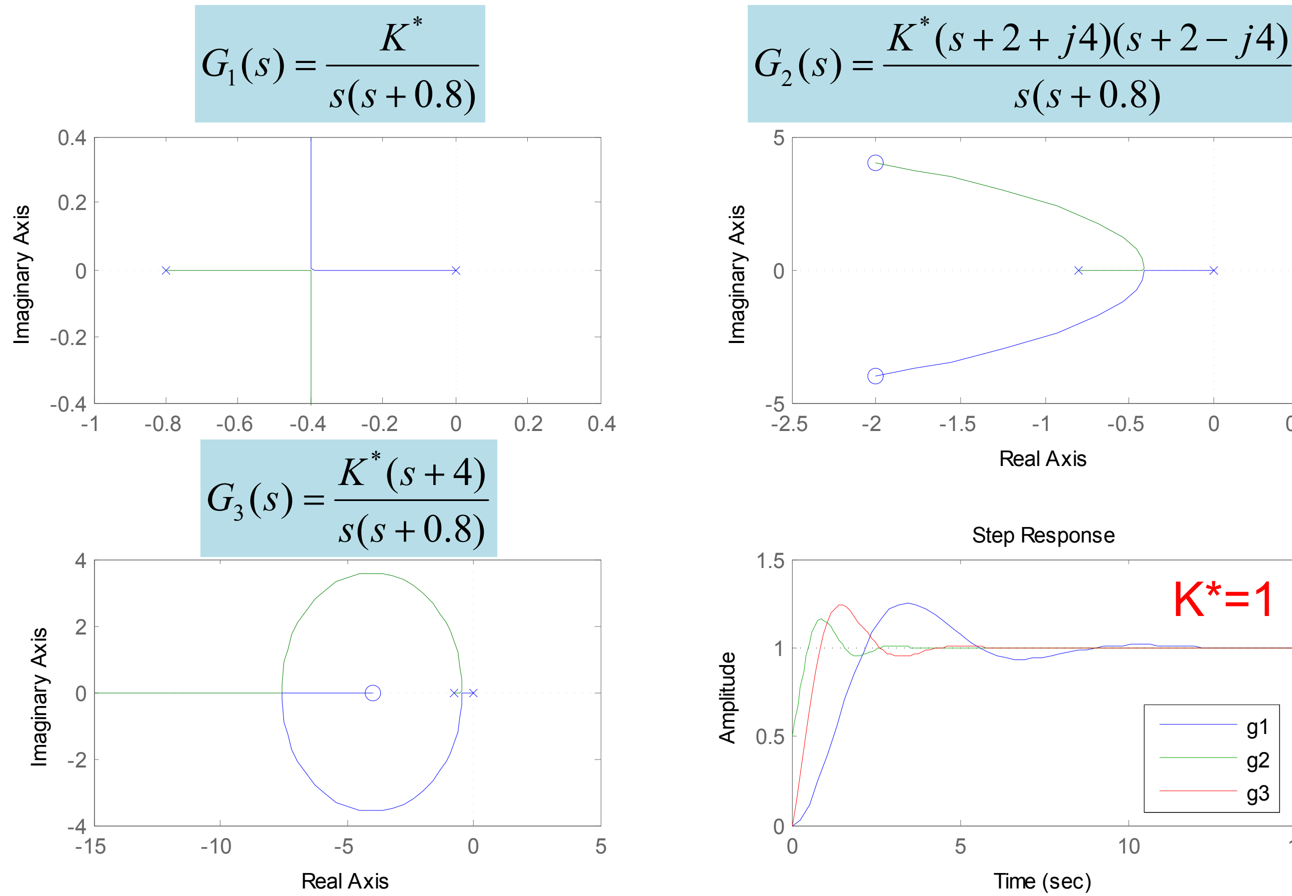
(d)

结论：只有当附加零点相对原有系统开环极点的位置选配适当，才有可能使系统的稳定性和动态性能同时得到明显的改善。

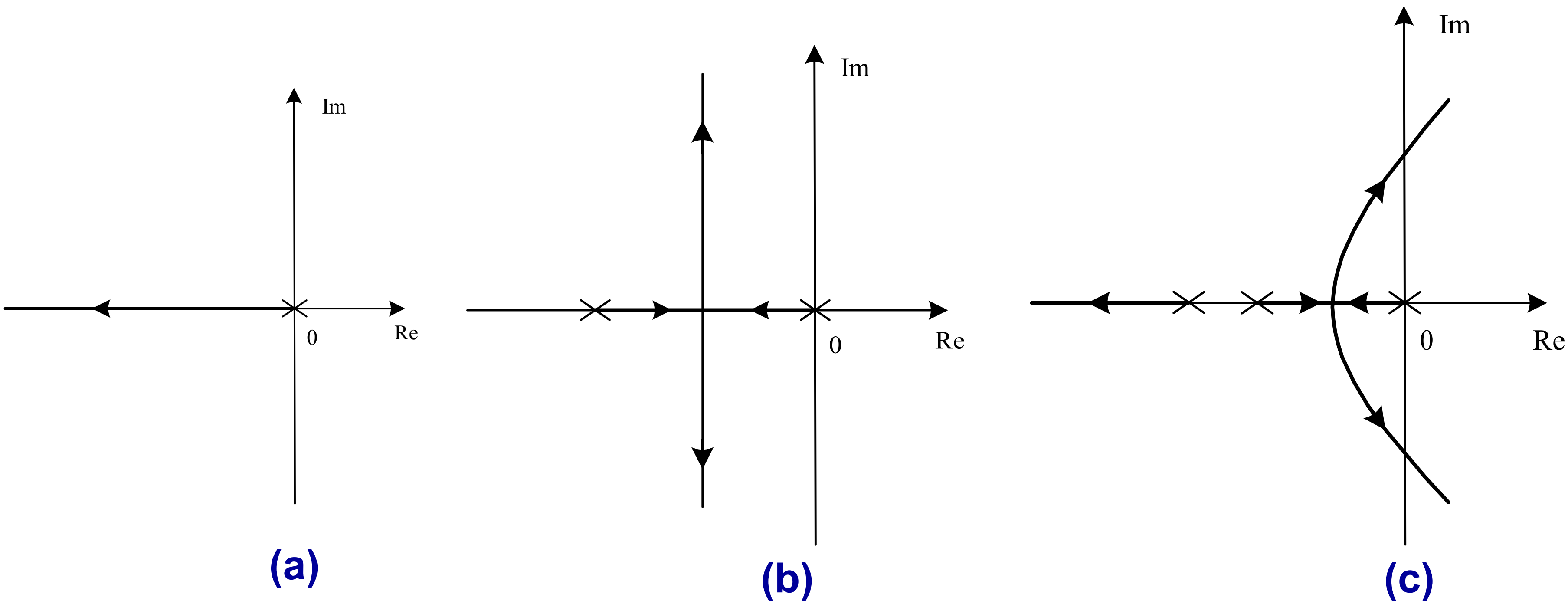
4.5 根轨迹校正

增加开环零点对根轨迹的影响

```
%增加开环零点的影响K*=1
den=[1 0.8 0];
g1=tf(1,den);
g2=tf([1 4 20],den);
g3=tf([1 4],den);

subplot(2,2,1)
rlocus(g1,'b-.');
subplot(2,2,2)
rlocus(g2);
subplot(2,2,3)
rlocus(g3);

subplot(2,2,4)
g1=tf(1,den+[0 0 1]);
g2=tf([1 4 20],den+[1 4 20]);
g3=tf([1 4],den+[0 1 4]);
step(g1,g2, g3);
```



2.附加开环极点的作用

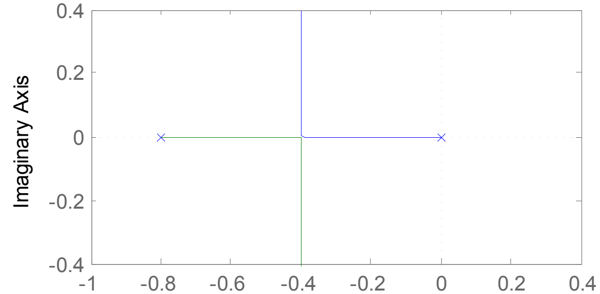


引入开环极点，可使根轨迹向右弯曲或移动，它增加了系统的滞后，相当于积分作用。引入的开环极点越接近原点，系统的稳定性被破坏的程度越大，甚至导致不稳定。

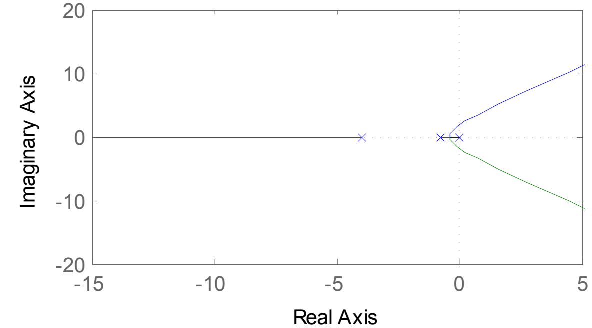
增加开环极点对根轨迹的影响

```
%增加开环极点的影响K*=1
den=[1 0.8 0];
g1=tf(1,conv(den,[1]));
g2=tf(1,conv(den,[1 4]));
g3=tf(1,conv(den,[1 4 20]));
subplot(2,2,1)
rlocus(g1);

subplot(2,2,2)
rlocus(g2);

subplot(2,2,3)
rlocus(g3);

subplot(2,2,4)
g1=tf(1,conv(den,[1])+[0 0 1])
g2=tf(1,conv(den,[1 4])+[0 0 0 1])
g3=tf(1,conv(den,[1 4 20])+[0 0 0 0 1])
step(g1,g2,g3);
```

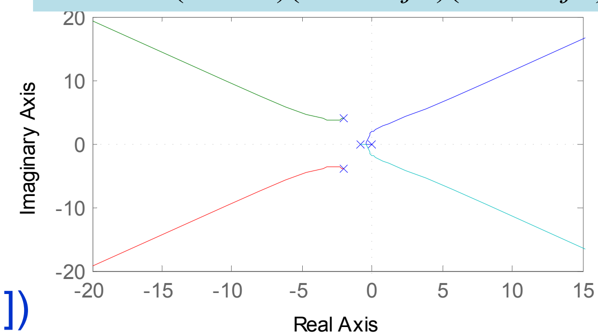
$$G_1(s)=\frac{K^*}{s(s+0.8)}$$

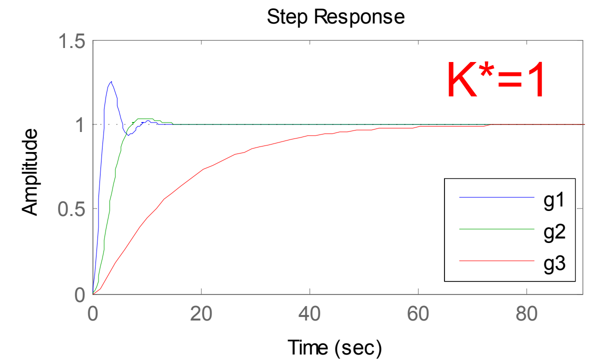


$$G_2(s)=\frac{K^*}{s(s+0.8)(s+4)}$$

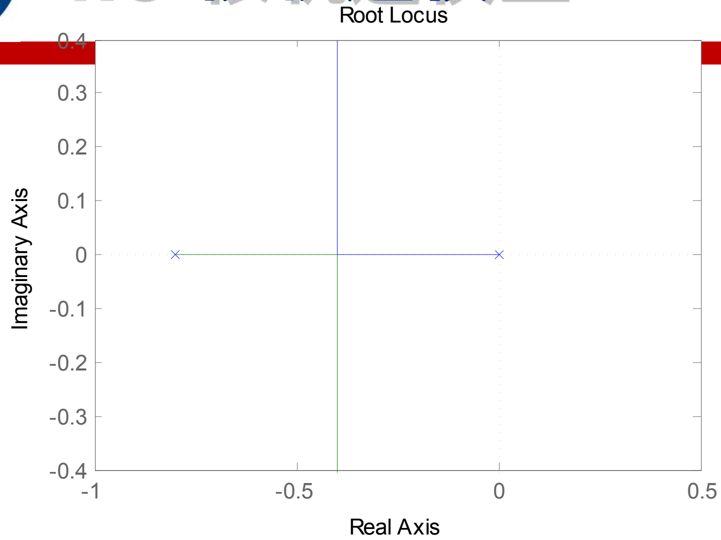


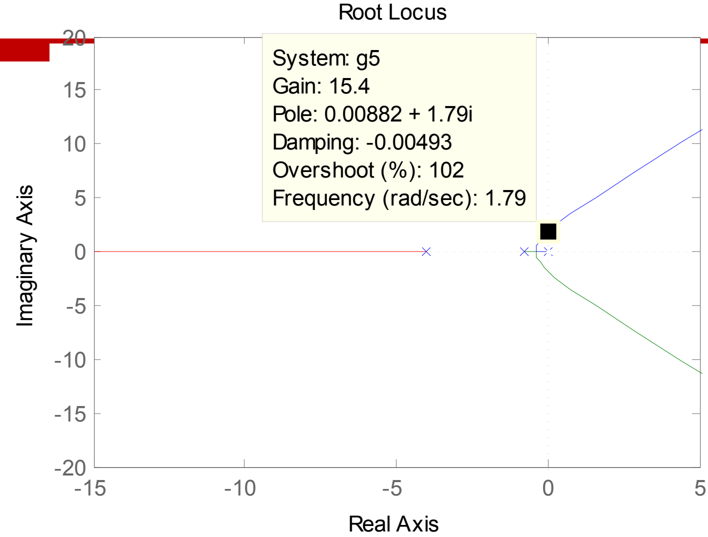
$$G_3(s)=\frac{K^*}{s(s+0.8)(s+2+j4)(s+2-j4)}$$

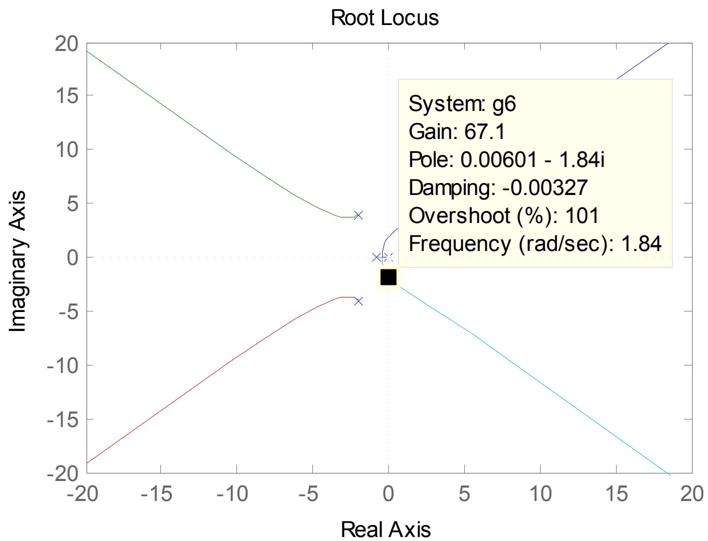


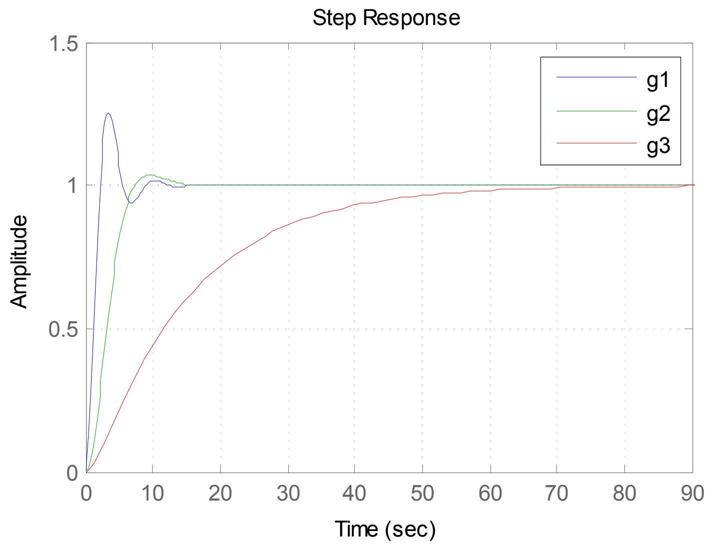


开环增益对系统响应的影响

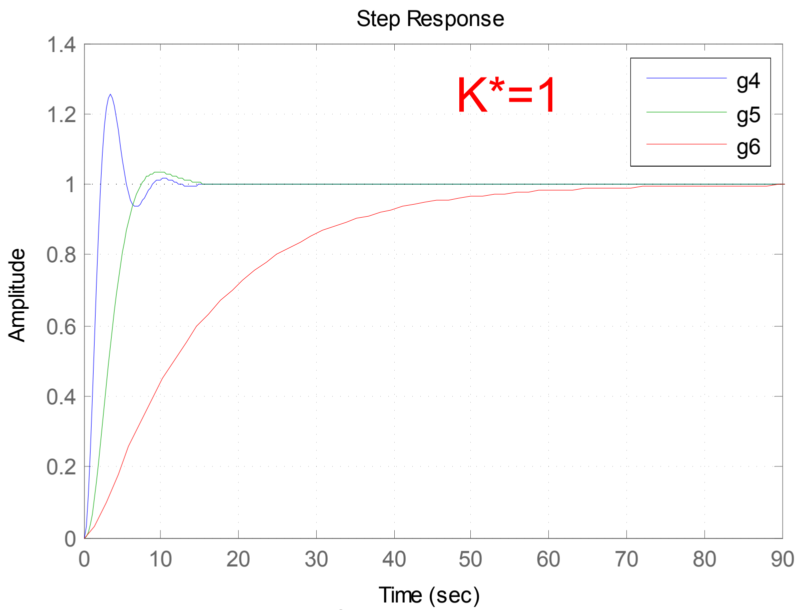


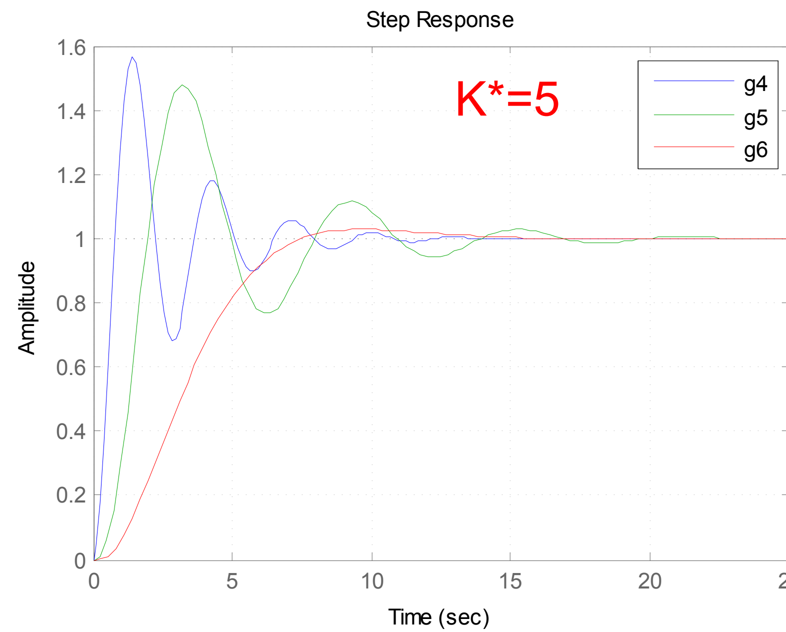


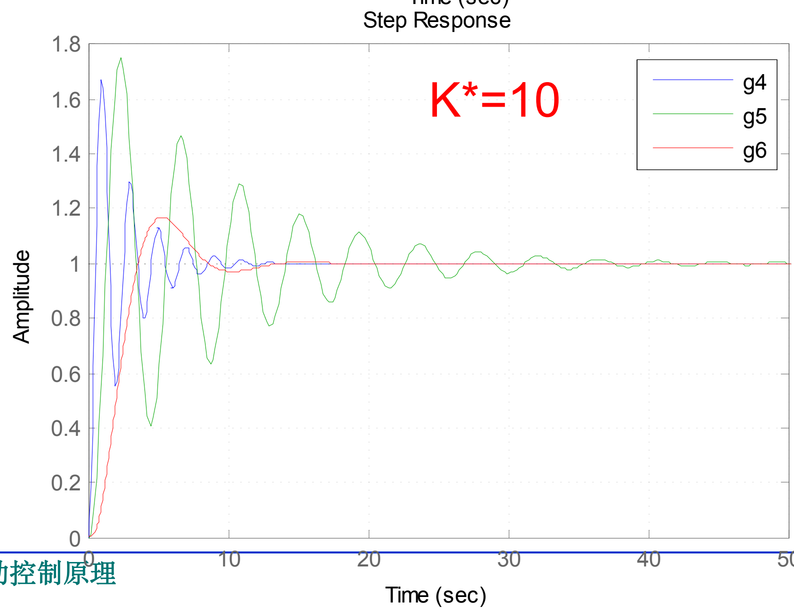


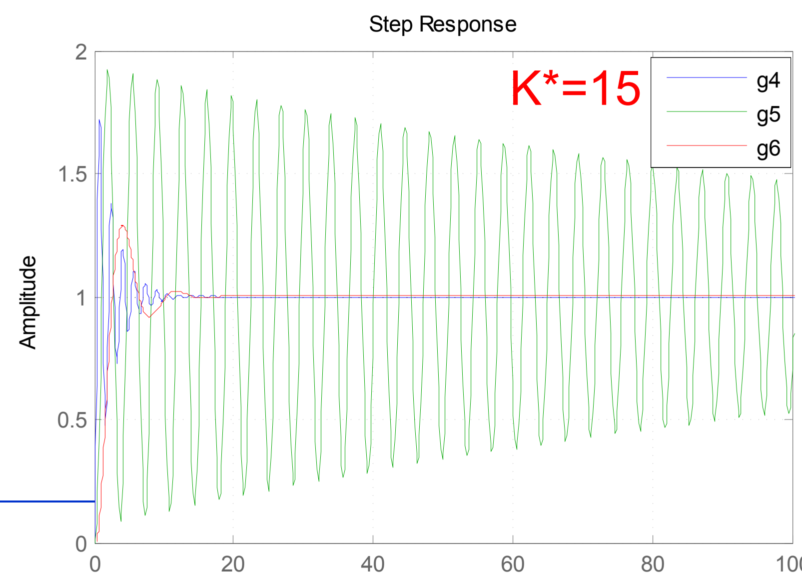


开环增益对系统响应的影响

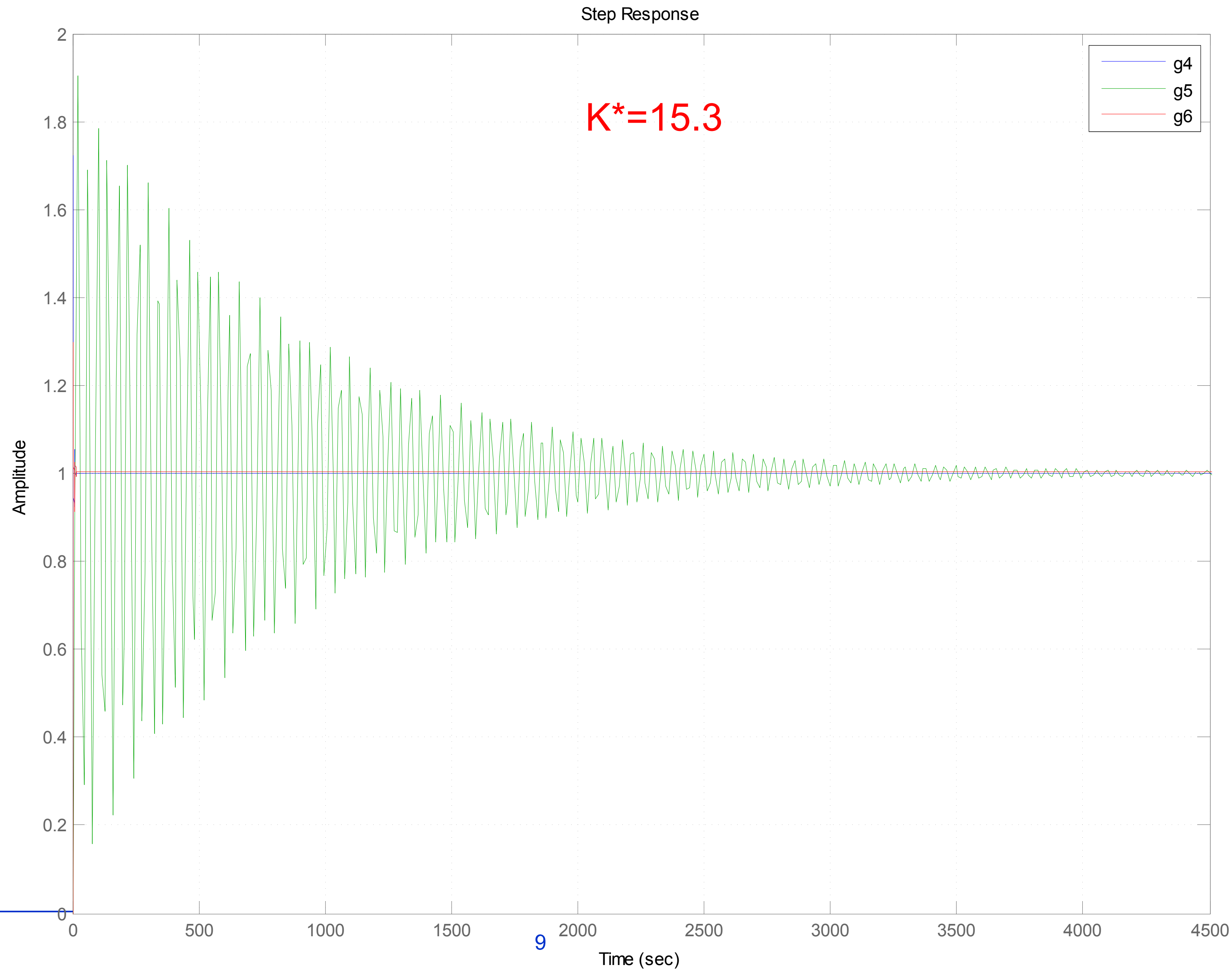






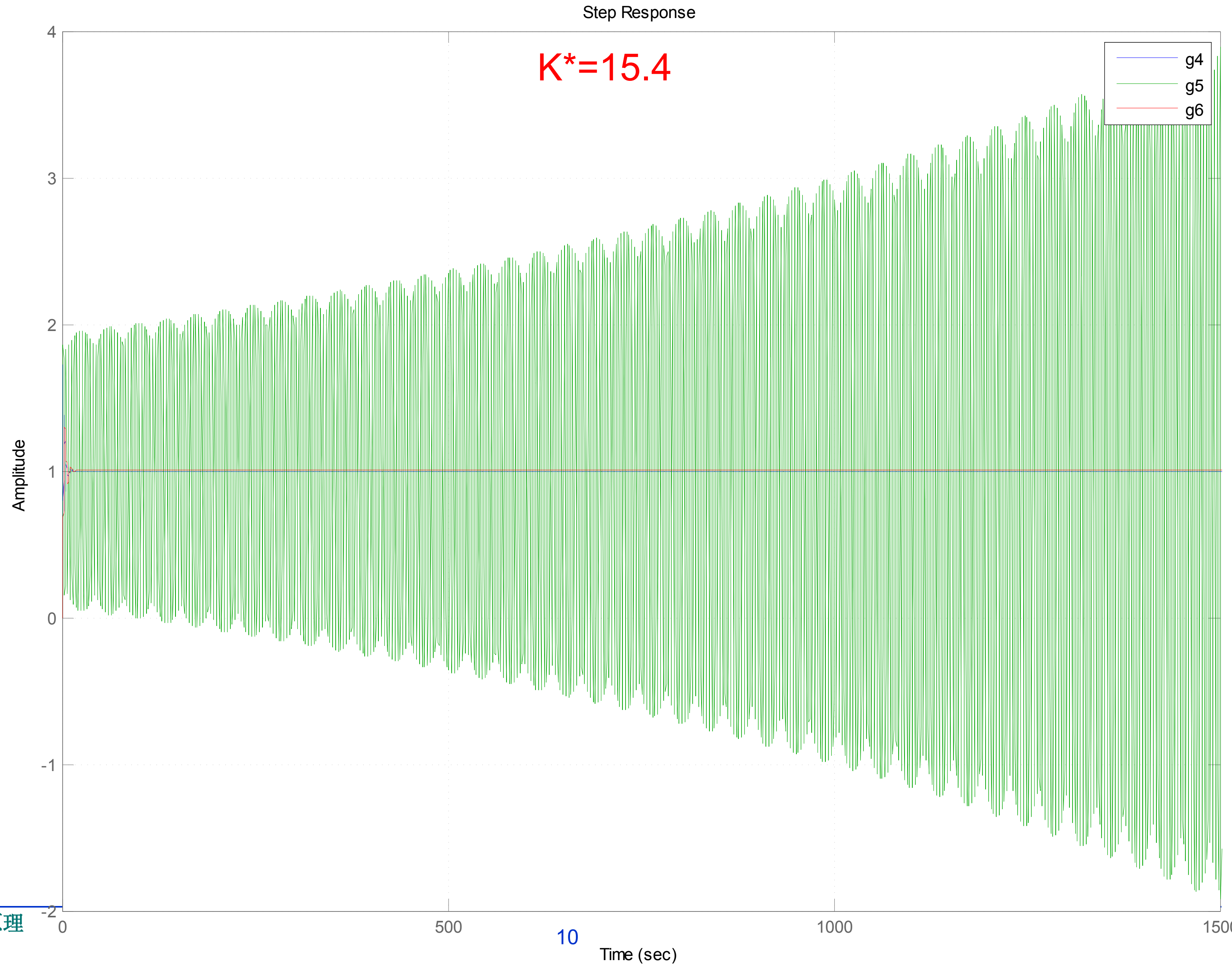


4.5 根轨迹校正



Step Response
K*=15.3
g4
g5
g6
Amplitude
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
500
1000
1500
2000
2500
3000
3500
4000
4500

4.5 根轨迹校正



Step Response
K*=15.4
g4
g5
g6
Amplitude
Time (sec)
4
3
2
1
0
-1
-2
0
500
1000
1500

4.5 根轨迹校正

1、增加开环零点对根轨迹的影响

1）改变了根轨迹在实轴上的分布；
2）改变了渐近线的条数、与实轴的交点、夹角；
3）可以抵消对系统不利的闭环极点（构成偶极子）；
4）根轨迹左移（零点要适当），对稳定性有利；
5）系统阻尼增加，过渡过程时间缩短；

2、增加开环极点对根轨迹的影响

1）改变了根轨迹的条数；
2）改变了根轨迹在实轴上的分布；
3）改变了渐近线的条数、与实轴的交点、夹角；
4）根轨迹右移（极点要适当），对稳定性不利；
5）系统阻尼减小，过渡过程时间加长；

3、增加开环偶极子对根轨迹的影响

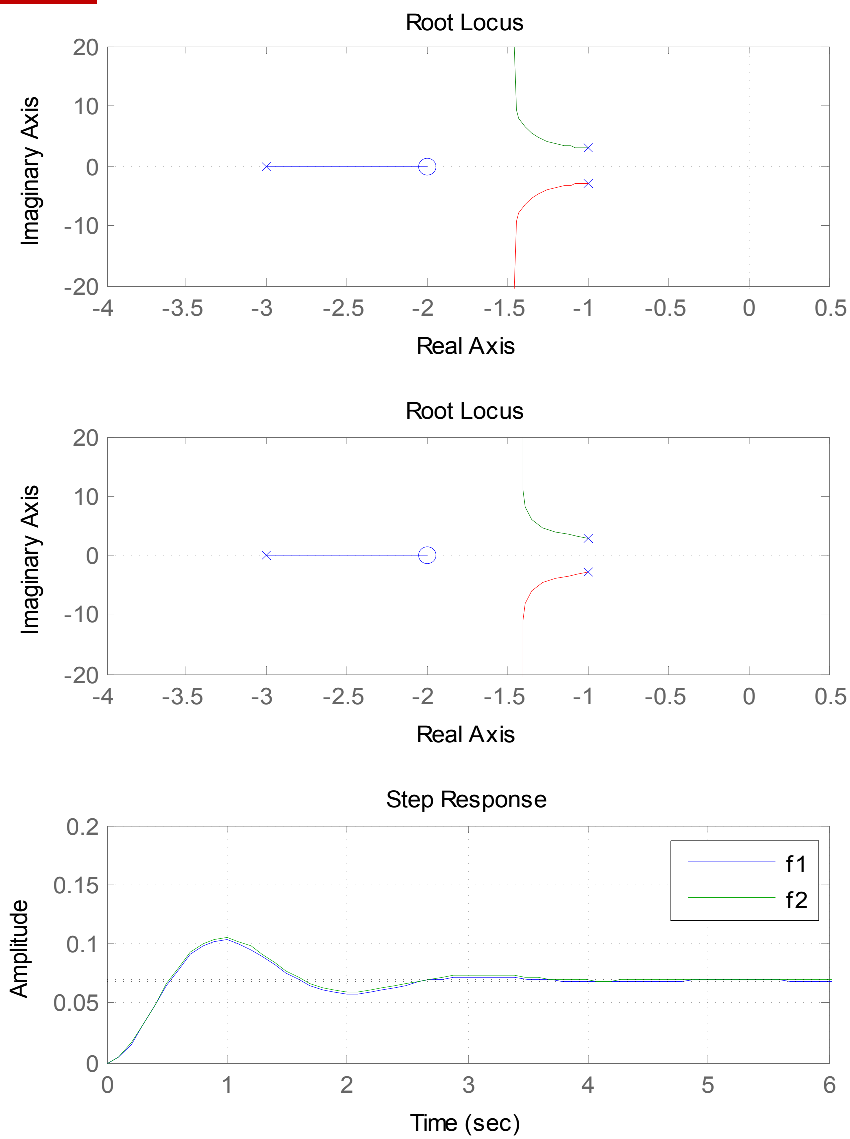
如果系统引入一对相距很近的开环零、极点(实数或复数)，且它们之间的距离比它们的模值小一个数量级时，则这一对开环零、极点称为开环偶极子。

$$\begin{cases} \angle\left(s-z_1^*\right) \approx \angle\left(s-p_1^*\right) \\ \left|s-z_1^*\right| \approx \left|s-p_1^*\right| \end{cases}$$

因此远离坐标原点的开环偶极子对系统性能的影响可以忽略不计。

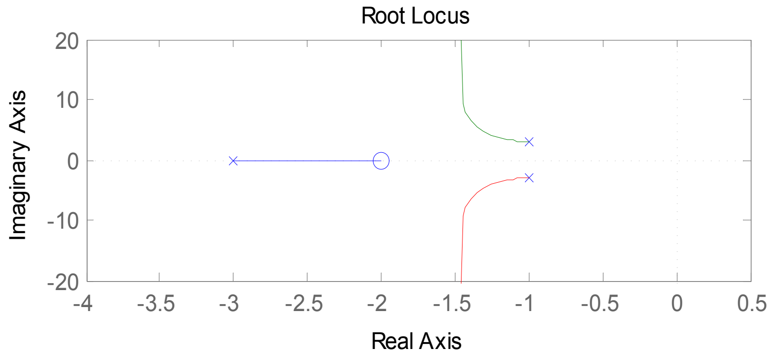
4.5 根轨迹校正

```
%增加开环偶极子对根轨迹的影响(远离原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,1)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -10];
pole=[-3 -1+2.83i -1-2.83i -9.8];
g2=zpk(zero,pole,1)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```

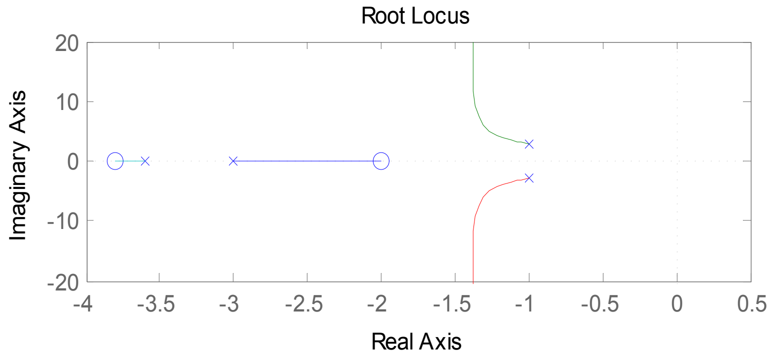


增加开环偶极子对根轨迹的影响(远离原点)

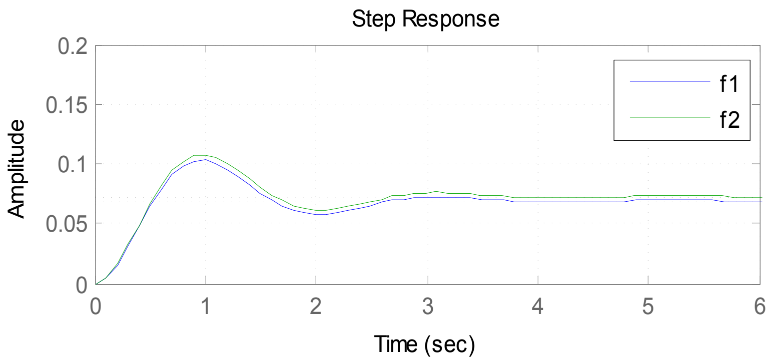
```
Zero/pole/gain:
           (s+2)
------------------------------
(s+3) (s^2 + 2s + 9.009)
```




```
Zero/pole/gain:
           (s+2) (s+3.8)
----------------------------------------
(s+3) (s+3.6) (s^2 + 2s + 9.009)
```




```
Zero/pole/gain:
                (s+2)
----------------------------------------
(s+2.921) (s^2 + 2.079s + 9.936)
```




```
Zero/pole/gain:
                (s+2) (s+3.8)
----------------------------------------------
(s+3.63) (s+2.9) (s^2 + 2.07s + 9.964)
```

4.5 根轨迹校正

靠近坐标原点的开环偶极子不能忽略，它对根轨迹的作用可以概括为：

（1）开环偶极子不影响稍远离开偶极子位置的根轨迹形状。

（2）开环偶极子不影响根轨迹上各点的根轨迹增益，但可影响根轨迹上各点的开环增益。合理地配置偶极子中的开环零、极点，可以在不影响动态特性的基础上，改善系统的稳态性能。

(3) 原点附近增加一对开环偶极子：提高系统开环增益，有利于改善静态性能，同时改变根轨迹的条数。

$$\begin{cases} |G_0(s)| = \dfrac{K^*\prod\limits_{i=1}^{m}|s-z_i||s-z_1^*|}{\prod\limits_{j=1}^{n}|s-p_j||s-p_1^*|} = 1 \\ ArgG_0(s) = \underbrace{\sum\limits_{i=1}^{m}Arg(s-z_i)}_{+Arg\left(s-z_1^*\right)} - \underbrace{\sum\limits_{j=1}^{n}Arg(s-p_j)}_{+Arg\left(s-p_1^*\right)} = (2k+1)\pi, k = 0,\pm1,\pm2\ldots \end{cases}$$

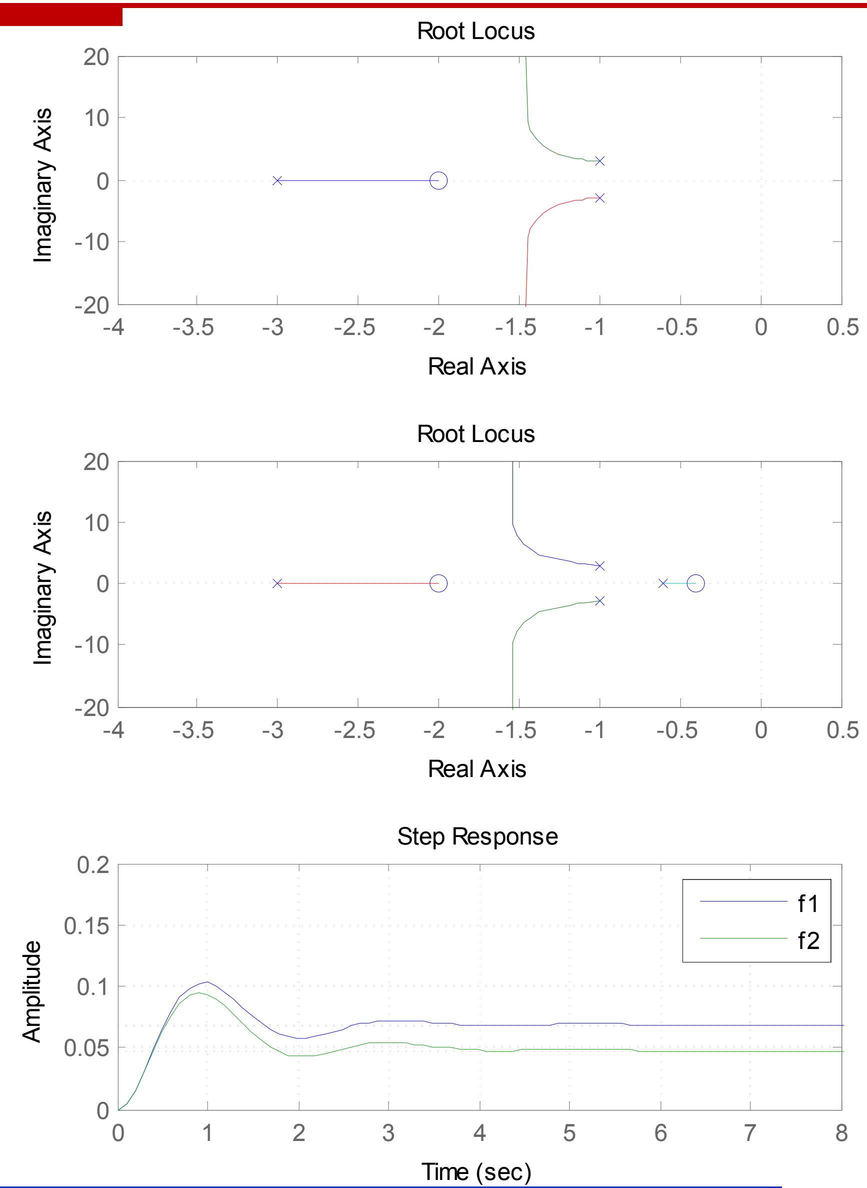
开环偶极子对远处根轨迹形状及根轨迹增益K*值影响不大，但开环增益：

$$K = \left.\frac{K^*\prod\limits_{i=1}^{m+1}|s-z_i|}{\prod\limits_{j=1}^{n+1}|s-p_j|}\right|_{s=0} = \frac{K^*\prod\limits_{i=1}^{m}|-z_i|}{\prod\limits_{j=1}^{n}|-p_j|}\frac{|-z_1^*|}{|-p_1^*|}$$

将提高$\dfrac{|-z_1^*|}{|-p_1^*|}$倍，$e_{ss}$降低!

4.5 根轨迹校正

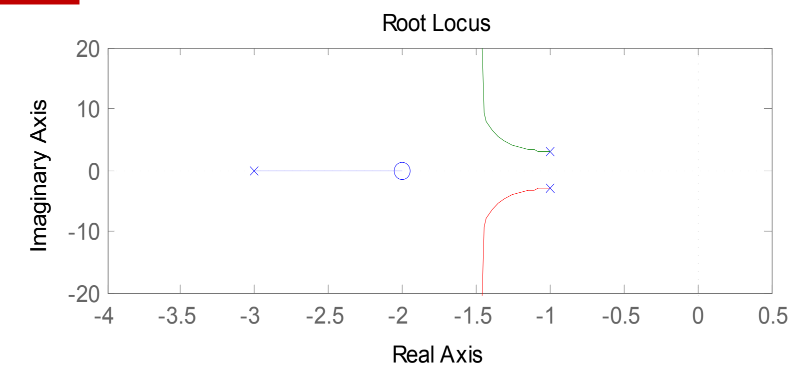
```
%增加开环偶极子对根轨迹的影响(接近原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,1)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -0.4];
pole=[-3 -1+2.83i -1-2.83i -0.6];
g2=zpk(zero,pole,1)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```



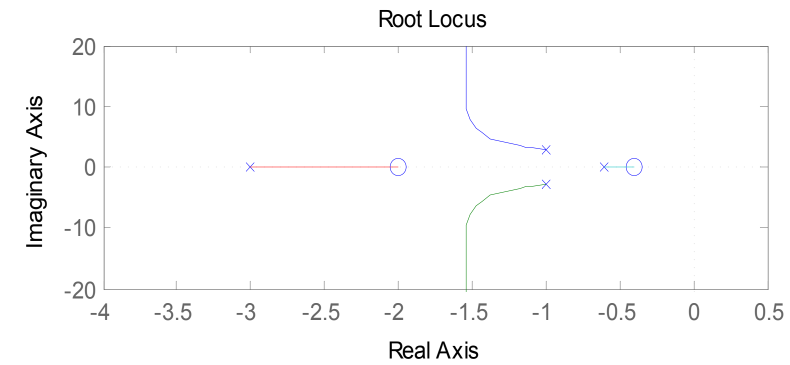
4.5 根轨迹校正

增加开环偶极子对根轨迹的影响(接近原点)

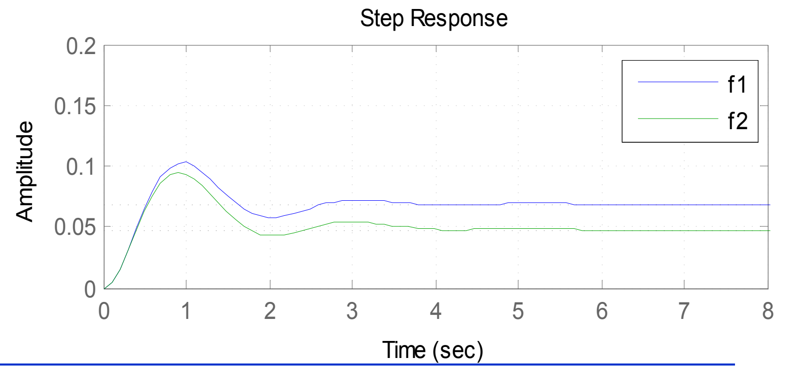
```
Zero/pole/gain:
           (s+2)
------------------------
(s+3) (s^2 + 2s + 9.009)
```




```
Zero/pole/gain:
           (s+2) (s+0.4)
--------------------------------
(s+3) (s+0.6) (s^2 + 2s + 9.009)
```



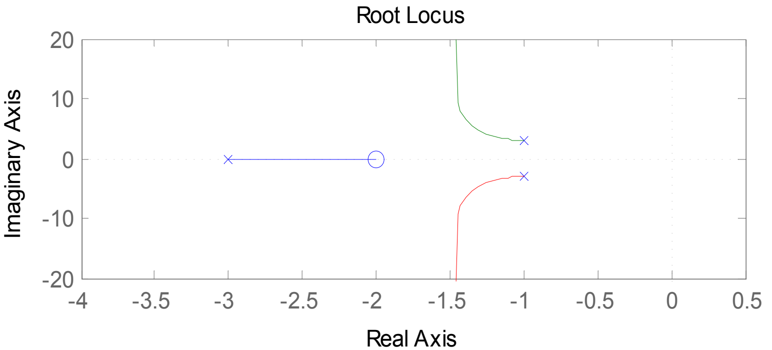

```
Zero/pole/gain:
               (s+2)
--------------------------------
(s+2.921) (s^2 + 2.079s + 9.936)
```

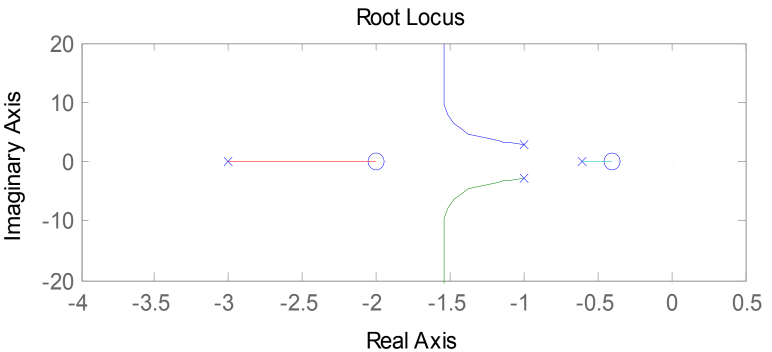


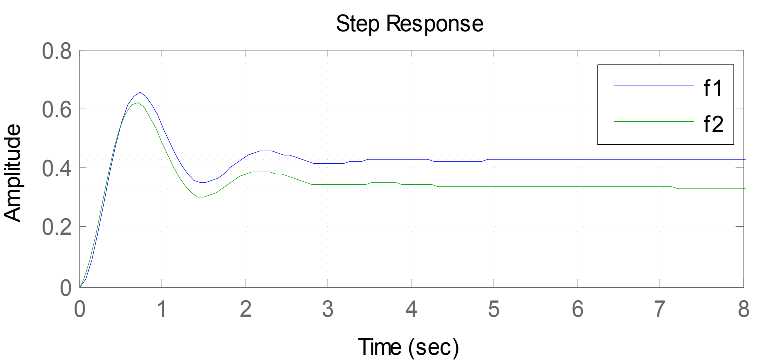

```
Zero/pole/gain:
                 (s+2) (s+0.4)
-------------------------------------------
(s+0.5866) (s+2.915) (s^2 + 2.098s + 9.951)
```

4.5 根轨迹校正

```
%增加开环偶极子对根轨迹的影响(接近原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,10)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -0.4];
pole=[-3 -1+2.83i -1-2.83i -0.6];
g2=zpk(zero,pole,10)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```



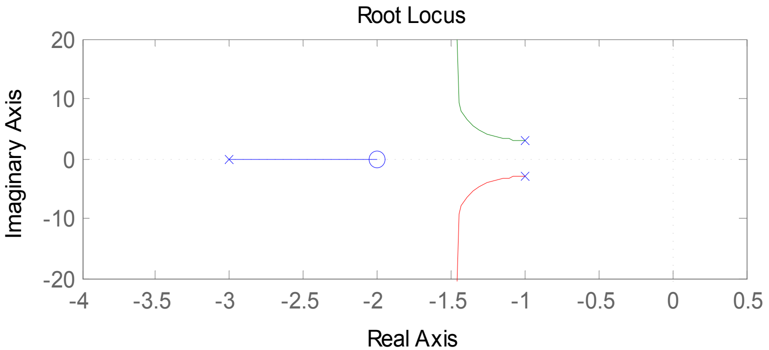




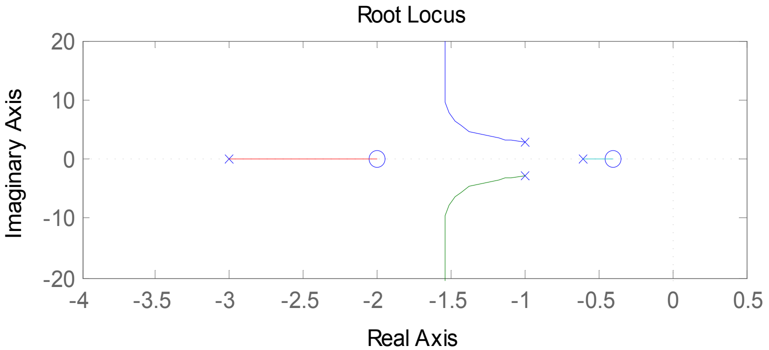
4.5 根轨迹校正

增加开环偶极子对根轨迹的影响(接近原点)

```
Zero/pole/gain:
        10 (s+2)
------------------------
(s+3) (s^2 + 2s + 9.009)
```

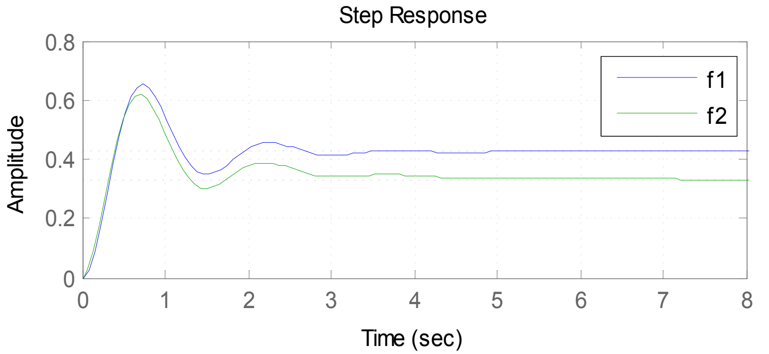



```
Zero/pole/gain:
        10 (s+2) (s+0.4)
--------------------------------
(s+3) (s+0.6) (s^2 + 2s + 9.009)
```



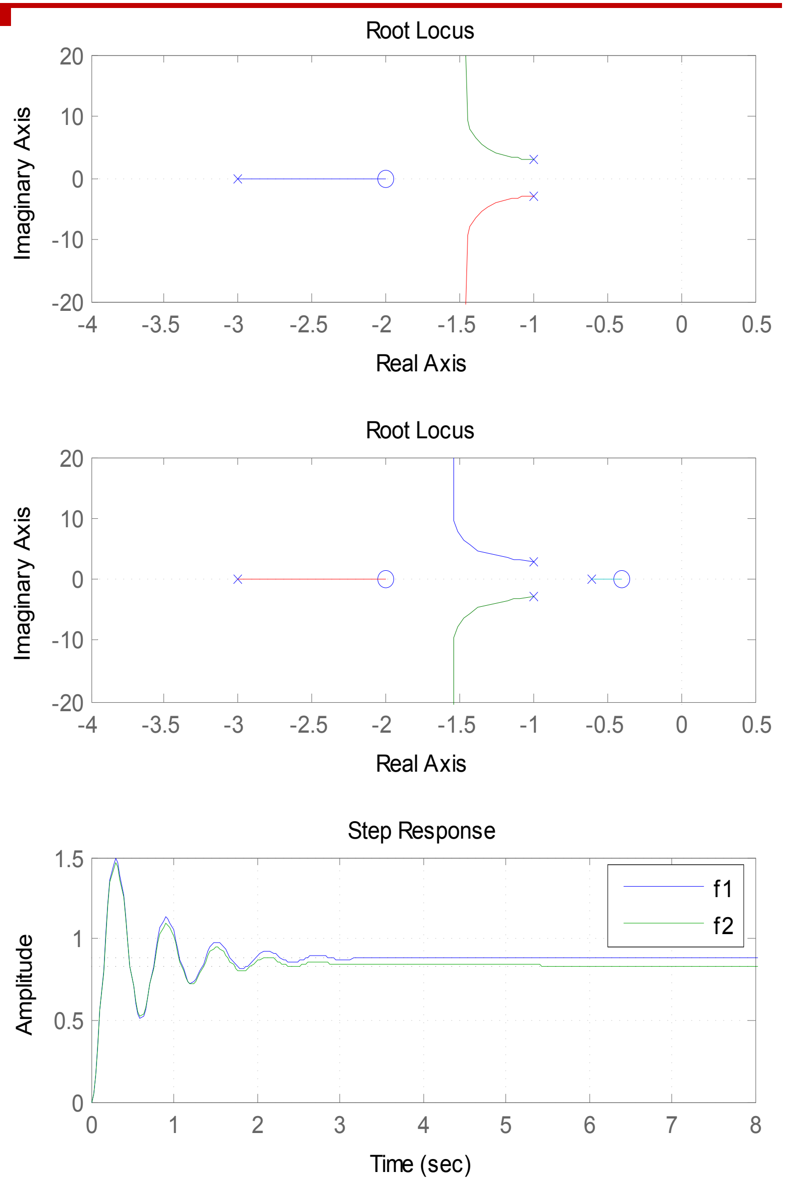

```
Zero/pole/gain:
            10 (s+2)
--------------------------------
(s+2.507) (s^2 + 2.493s + 18.76)
```

```
Zero/pole/gain:
            10 (s+2) (s+0.4)
-----------------------------------------
(s+2.48) (s+0.516) (s^2 + 2.604s + 18.93)
```



4.5 根轨迹校正

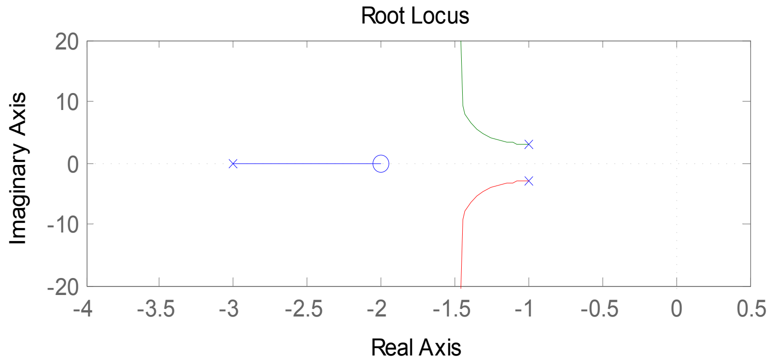
```
%增加开环偶极子对根轨迹的影响(接近原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,100)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -0.4];
pole=[-3 -1+2.83i -1-2.83i -0.6];
g2=zpk(zero,pole,100)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```

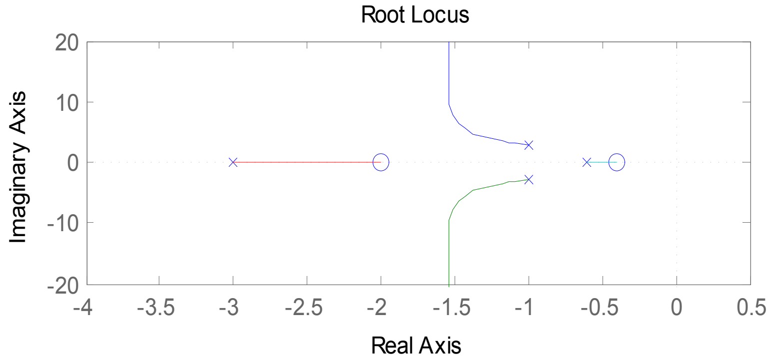


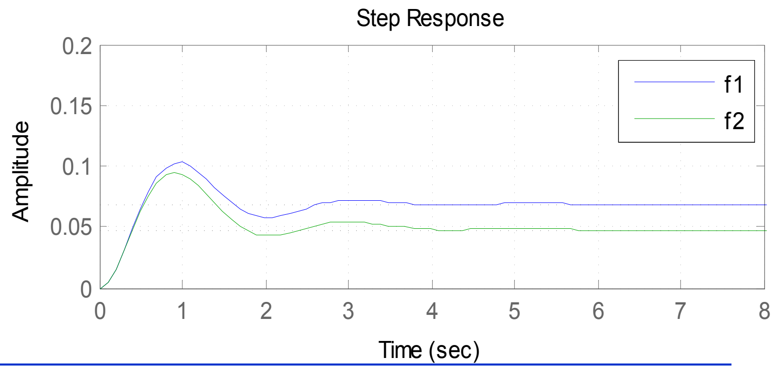
4.5 根轨迹校正

增加开环偶极子对根轨迹的影响(接近原点)

```
Zero/pole/gain:
      100 (s+2)
-------------------------
(s+3) (s^2 + 2s + 9.009)


Zero/pole/gain:
      100 (s+2) (s+0.4)
---------------------------------
(s+3) (s+0.6) (s^2 + 2s + 9.009)


Zero/pole/gain:
          100 (s+2)
---------------------------------
(s+2.084) (s^2 + 2.916s + 108.9)


Zero/pole/gain:
           100 (s+2) (s+0.4)
-------------------------------------------
(s+2.075) (s+0.424) (s^2 + 3.101s + 109.4)
```

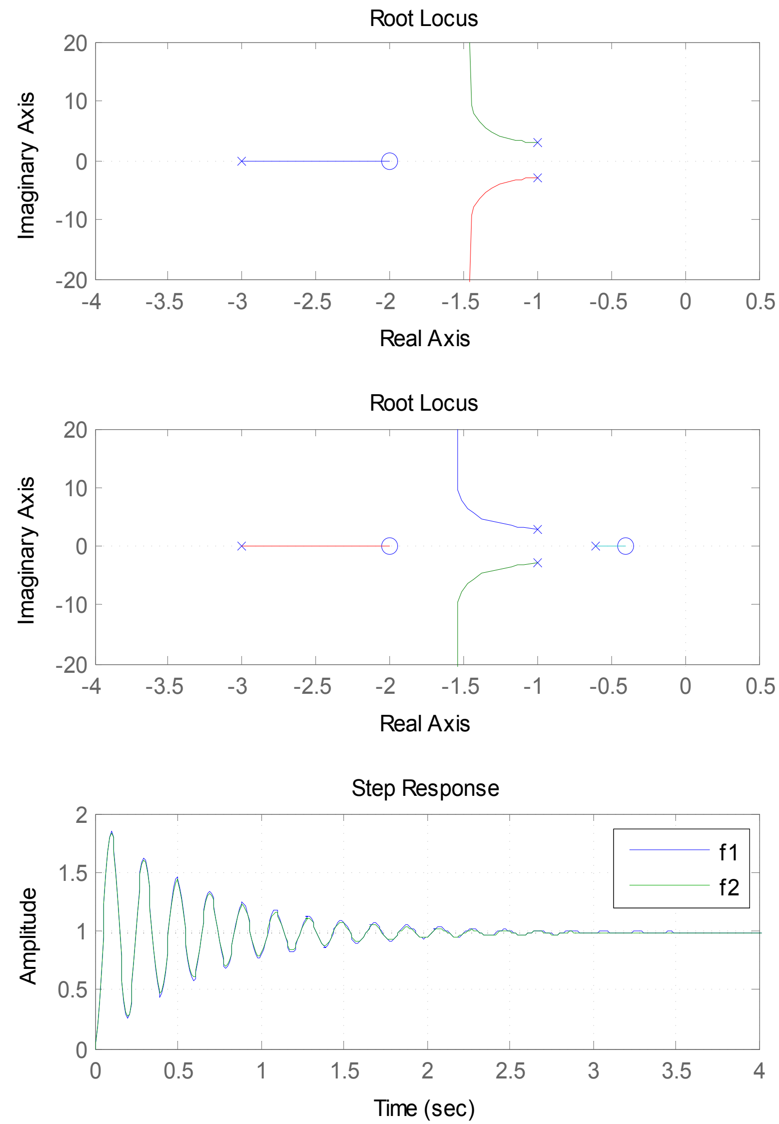






4.5 根轨迹校正

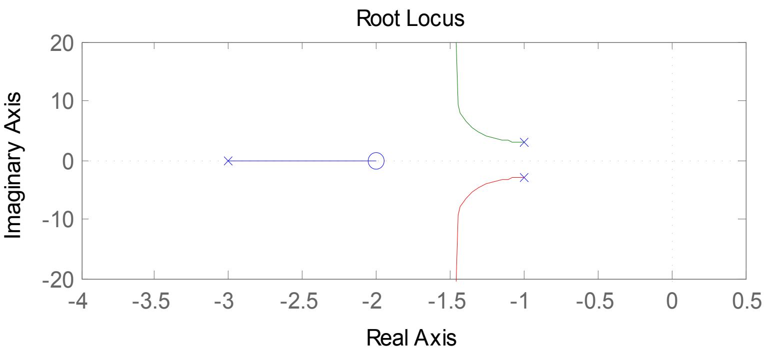
```
%增加开环偶极子对根轨迹的影响(接近原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,1000)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -0.4];
pole=[-3 -1+2.83i -1-2.83i -0.6];
g2=zpk(zero,pole,1000)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```



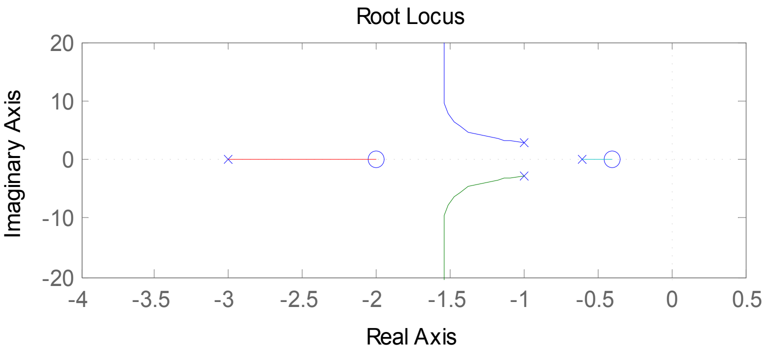
4.5 根轨迹校正

增加开环偶极子对根轨迹的影响(接近原点)

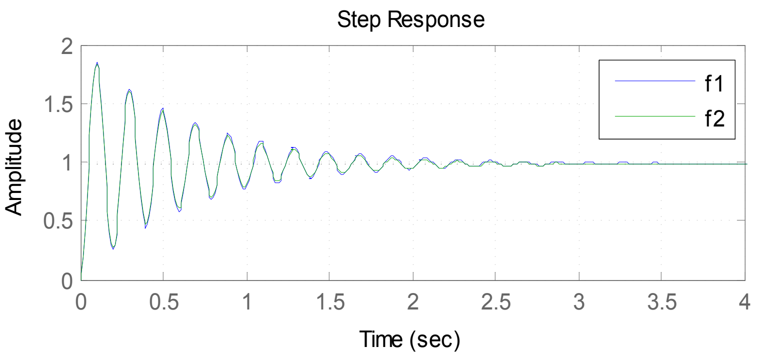
```
Zero/pole/gain:
       1000 (s+2)
------------------------
(s+3) (s^2 + 2s + 9.009)
```




```
Zero/pole/gain:
       1000 (s+2) (s+0.4)
--------------------------------
(s+3) (s+0.6) (s^2 + 2s + 9.009)
```



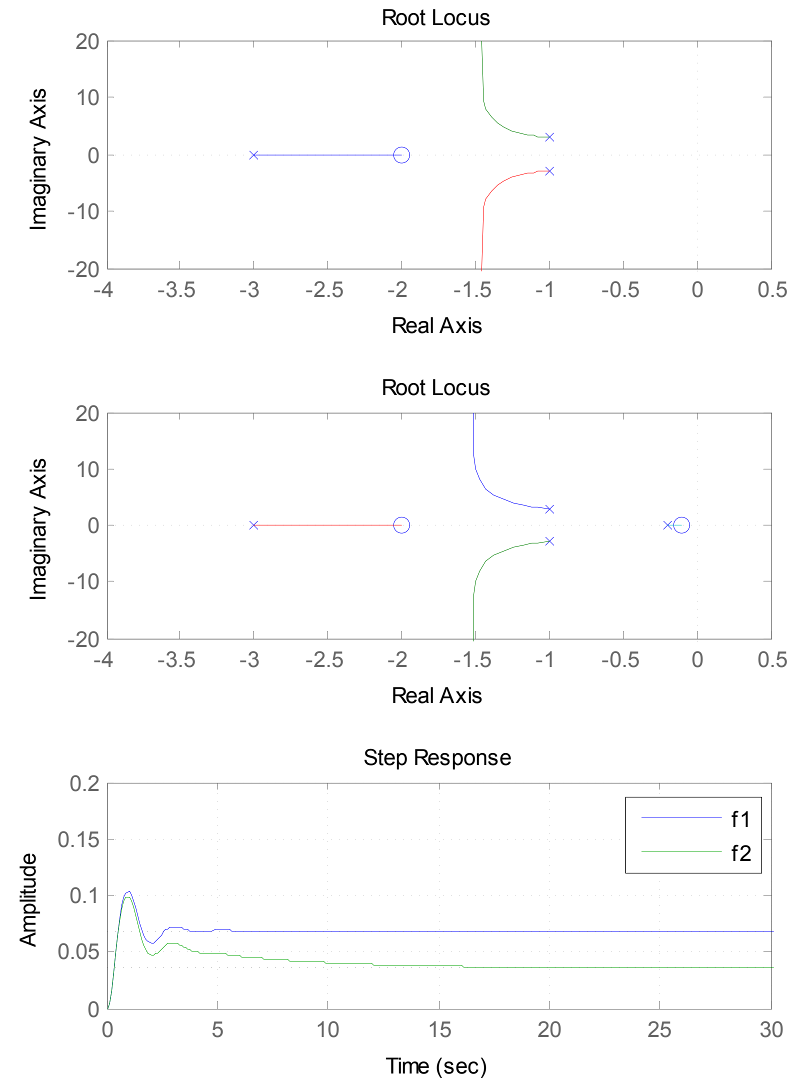

```
Zero/pole/gain:
          1000 (s+2)
-------------------------------
(s+2.009) (s^2 + 2.991s + 1009)
```




```
Zero/pole/gain:
             1000 (s+2) (s+0.4)
------------------------------------------
(s+2.008) (s+0.4027) (s^2 + 3.189s + 1010)
```

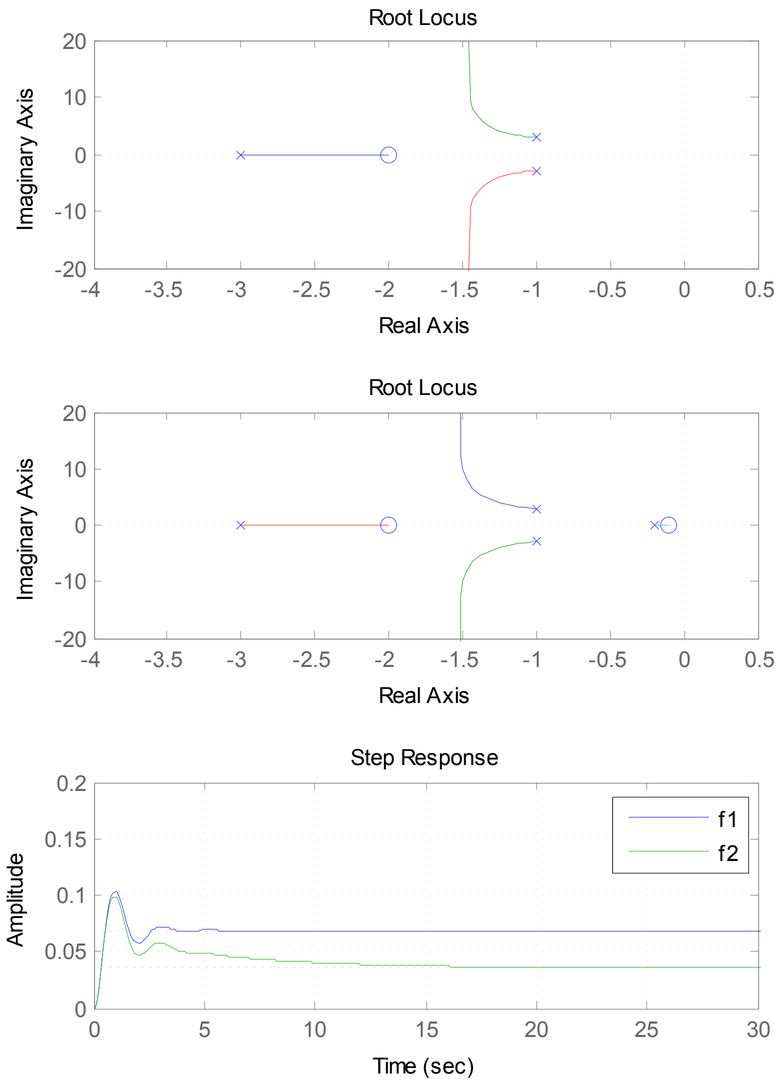
4.5 根轨迹校正

```
%增加开环偶极子对根轨迹的影响(接近原点)
zero=[-2];
pole=[-3 -1+2.83i -1-2.83i];
g1=zpk(zero,pole,1)
subplot(3,1,1)
rlocus(g1);
axis([-4 0.5 -20 20]);

zero=[-2 -0.1];
pole=[-3 -1+2.83i -1-2.83i -0.2];
g2=zpk(zero,pole,1)
subplot(3,1,2)
rlocus(g2);
axis([-4 0.5 -20 20]);

f1=feedback(g1,1)
f2=feedback(g2,1)
subplot(3,1,3)
step(f1,f2);
```



增加开环偶极子对根轨迹的影响(接近原点)

```
Zero/pole/gain:
          (s+2)
-------------------------
(s+3) (s^2 + 2s + 9.009)


Zero/pole/gain:
          (s+2) (s+0.1)
--------------------------------
(s+3) (s+0.2) (s^2 + 2s + 9.009)


Zero/pole/gain:
              (s+2)
--------------------------------
(s+2.921) (s^2 + 2.079s + 9.936)


Zero/pole/gain:
                    (s+2) (s+0.1)
---------------------------------------------
(s+0.1931) (s+2.919) (s^2 + 2.088s + 9.947)
```



4、根轨迹校正反馈系统

step: 1）暂态性能指标→期望的闭环主导极点;

2）绘制原系统的根轨迹，确定是否增加校正装置;

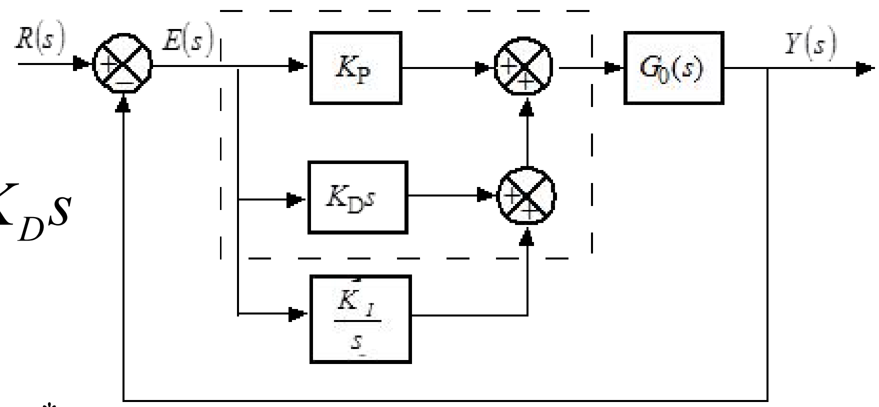
3）校正后系统根轨迹通过期望闭环主导极点，检验开环比例系数是否满足静态指标（否则，增加开环偶极子以改善之）；

4）检验暂、静态指标。

4.5 根轨迹校正

例： PID控制 （比例proportional，积分integral，微分derivative）

已知：$K_P = 1, K_D = 0.25, K_I = 1.5$



$$G_o(s) = \frac{K^*}{s(s+2)} \quad G_c(s) = K_P + \frac{K_I}{s} + K_D s$$

(1)P控制

$$G_P(s) = \frac{K_P K^*}{s(s+2)}$$

$$\begin{cases} K = \dfrac{K_P K^*}{2} = \dfrac{K^*}{2} \\ \upsilon = 1 \end{cases}$$

(2)PD控制

$$G_{PD}(s) = \frac{K^*(0.25s+1)}{s(s+2)} = \frac{\frac{K^*}{4}(s+4)}{s(s+2)}$$

$$\begin{cases} K = \dfrac{K^*}{2} \\ \upsilon = 1 \end{cases}$$

根轨迹向左偏移，系统动态性能得以有效改善。

4.5 根轨迹校正

(3)PI控制

$$G_{PI}(s)=\frac{K^*\left(1+\frac{1.5}{s}\right)}{s(s+2)}=\frac{K^*(s+1.5)}{s^2(s+2)} \qquad \begin{cases} K=\frac{3}{4}K^* \\ \upsilon=2 \end{cases}$$

系统由1型变为2型，稳态性能明显改善。但由于引入积分，系统动态性能变差。

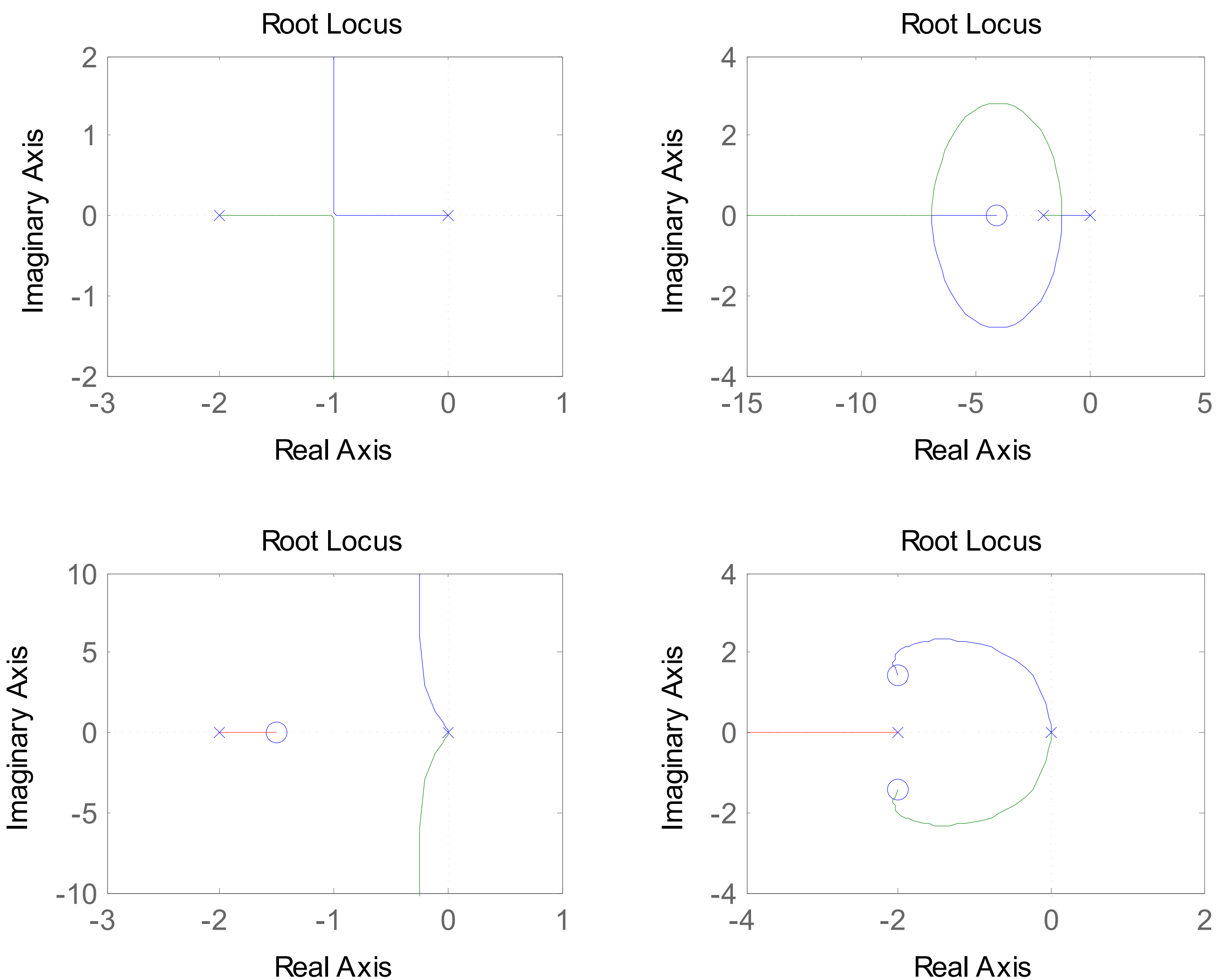
(4)PID控制

$$\mathrm{G_{PID}}(s)=\frac{\mathrm{K}^*\left(1+0.25\mathrm{s}+\frac{1.5}{\mathrm{s}}\right)}{\mathrm{s(s+2)}}=\frac{\frac{\mathrm{K}^*}{4}(\mathrm{s}+2+\mathrm{j}\sqrt{2})(\mathrm{s}+2-\mathrm{j}\sqrt{2})}{\mathrm{s}^2(\mathrm{s}+2)} \qquad \begin{cases} K=\frac{3K^*}{4} \\ \upsilon=2 \end{cases}$$

PID控制综合了微分控制和积分控制的优点，既能改善系统的动态性能，又保留了2型系统的稳态性能。

4.5 根轨迹校正

```
N1=[0 0 1];
D1=[1 2 0];
subplot(2,2,1)
Rlocus(N1,D1);
% grid on;

N2=[0 1 4];
D2=[1 2 0];
subplot(2,2,2)
Rlocus(N2,D2);
% grid on;

N3=[0 0 1 1.5];
D3=[1 2 0 0];
subplot(2,2,3)
Rlocus(N3,D3);
% grid on;

N4=[0 0.25 1 1.5];
D4=[1 2 0 0];
subplot(2,2,4)
Rlocus(N4,D4);
% grid on;
```



Thank You !