Chapter 13 Black holes

13.1 Newtonian picture

✓ In 1783, John Michell showed that if the size of a star is smaller than its critical radius, then light corpuscles (light was not known to be waves at that time) emitted from the star would fall back to its surface.

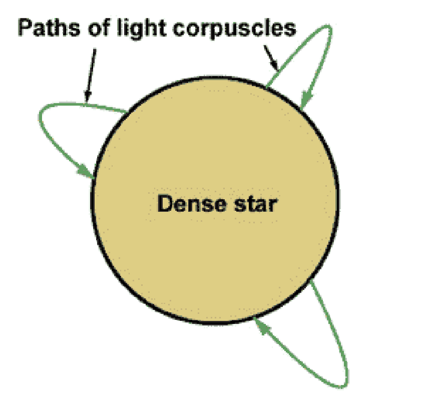

Fig. 13-1: An early picture of the behavior of light emitted from a high-density star.

✓ In Newtonian mechanics, one estimates the initial speed required to escape from a celestial body, namely e**scape velocity**, as $v_{esc} = \sqrt{2GM/R}$.

✓ For a massive (large M) and small (small R) celestial body, it has a larger escape velocity. When a celestial body collapses to a high density such that its escape velocity equals the speed of light c, the body has a radius of $R = 2GM/c^2$. Hence, not even light can escape from the body (Fig. 13-1) with a radius $R < R_S$, the body becomes a **black hole**!

13.2 General Relativistic picture

✓ In relativistic mechanics, massive body causes **curved space-time** (analogy: a heavy ball on a rubber sheet, Fig. 13-2). A *small* and *massive* celestial body causes the curvature of space-time so large that even light cannot escape from the massive body. One estimates that a star's core of mass >3 $M_\odot$ will continue to contract under gravity beyond nuclear matter density, *even* neutron degeneracy pressure cannot stop the contraction!

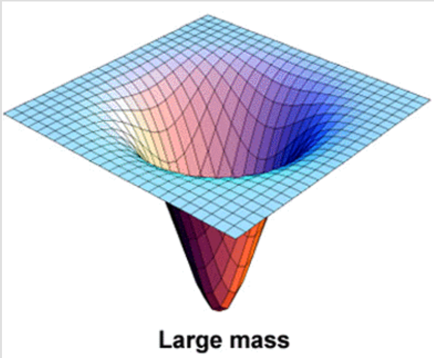

Fig. 13-2: The curvature of space-time around a massive object, as predicted by general relativity, can be loosely pictured as the curvature on a rubber sheet caused by a heavy ball.

Schwarzschild black holes

- ✓ Karl Schwarzschild first calculated the contraction of a *non-rotating*, massive star's core in 1915. He concluded that the core would contract to *a point of infinite density*, namely **Schwarzschild singularity** (Fig. 13-3). Not even light can escape from a position $R < R_S$, where R_s is called **Schwarzschild radius**. The object is called a **Schwarzschild black hole**.

- ✓ Interesting enough, after tedious calculation one shows that the Schwarzschild radius is the same as that obtained in Newtonian mechanics, i.e., $\boxed{R_s = 2GM/c^2}$.

- ✓ The boundary of the region inside which *nothing* can escape (it is the Schwarzschild radius for a Schwarzschild black hole) is called **event horizon**. For example, a black hole of 1 $M_\odot$ has only $R_S \approx 3$ km. Moreover, the curvature of space-time is described by the **Schwarzschild metric**, it is the only solution of Einstein equation for a spherically symmetric distribution of mass.

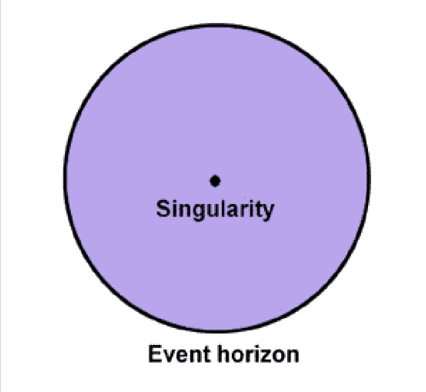

Fig. 13-3: Schwarzschild black hole.

- ✓ An observer views far away from the event horizon, there is nothing special – just like replacing the black hole by a star of the same mass. Newton′s laws (and hence Kepler′s laws) are also approximately true if the observer is *far* from the event horizon. The observer sees a black region with no light coming out.

- ✓ When a spaceship is approaching the black hole, it will be *stretched by an increasing tidal force* along the direction to the black hole. The spaceman in the spaceship sees distant starlight concentrated to a small cone overhead. (Fig. 13-4)

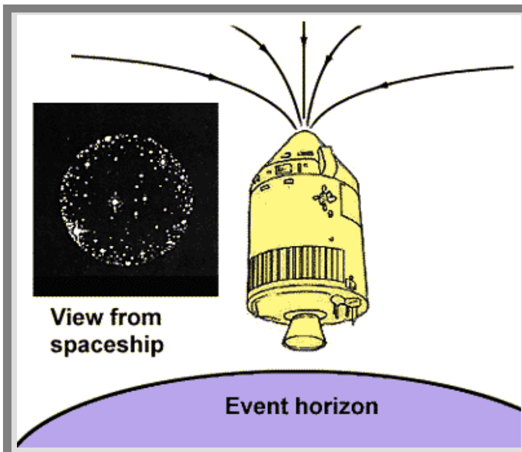

Fig. 13-4: A spaceship hovers above the black-hole horizon.

- ✓ In addition, by general relativity clocks in a spaceship falling into a black hole run *slower* compared to clocks far away. It is called **gravitational time dilation**. The effect increases to infinity when the spaceship is falling through the event horizon. Consequently, *as seen from an observer very far away*, an astronaut in a spaceship would takes an infinitely long time for him to fall through the event horizon.

- ✓ As a light pulse is emitted from a massive body (in a direction against a gravitational field) to far outside the body, its frequency will be decreased. This phenomenon is called the **gravitational red shift**. Exact calculation shows that the observed frequency becomes $\boxed{\nu' = \nu\sqrt{1-2GM/rc^2}}$.

- ✓ For $r = R_S$, a light beam takes infinite long time to escape from the black hole, so the red shift is infinite, that means *light loses all energy in climbing out from the event horizon*. If the pulse is emitted far from the *event horizon*, *weak* field limit is applied, i.e., $r >> R_S$, $\nu' = \nu\left(1-\frac{2GM}{rc^2}\right)^{1/2} \approx \nu\left(1-\frac{GM}{rc^2}\right)$, hence we have

$$\boxed{\frac{\Delta\nu}{\nu} = \frac{\nu'-\nu}{\nu} \approx -\frac{GM}{rc^2}}.$$

This result has been obtained when we discussed the general theory of relativity.

- ✓ In the case of $r >> R_S$, the effects of gravitational red shift are unimportant as $\frac{2GM}{rc^2} = \frac{R_S}{r} << 1$. The Newton′s laws are *approximately* true when $R_S << r$ - just like replacing the black hole with a star of the same mass.

Kerr black holes

- ✓ Since a star′s rotation speeds up in contraction, one should expect the most common black holes in our universe are rotating,
- ✓ **Kerr black holes** are solutions to general relativity describing a *rotating* black hole. It shows that this kind of black holes consists of two surfaces, namely:
  - ✧ **Event horizon**: a boundary inside of which nothing can escape, and
  - ✧ **Stationary limit**: a boundary inside of which nothing can resist being dragged around with the rotating mass (Fig. 13-5).

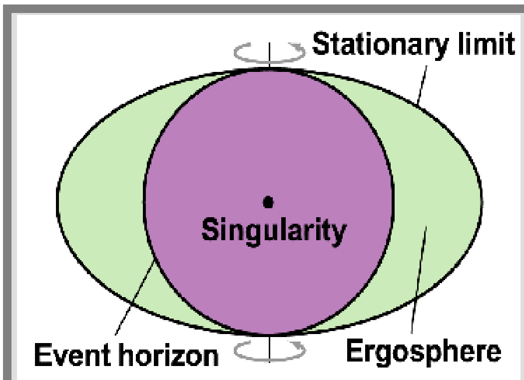

Fig. 13-5: The Kerr black hole is rotating black hole, consisting of two event horizons.

- ✧ The region between the event horizons and the stationary limit is called **ergosphere**. When a particle entering into the region will be split into two parts: one falls into the black hole, and the other ejected. Energy can then be extracted (Fig. 13-6).

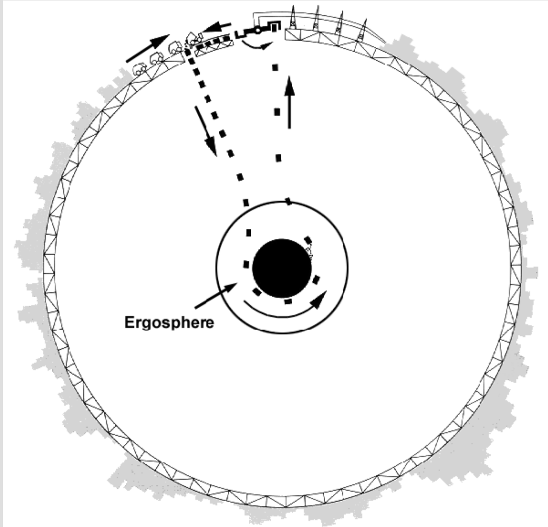

Fig. 13-6: Penrose suggests dropping garbage towards the black hole, in order to gain energy for electricity!

No hair theorem

- ✓ The theorem states that a black hole is characterized by 3 properties only, namely, the ***mass***, ***angular momentum***, and ***charge***. *Mass* is the *only* property of a Schwarzschild black hole, whereas a rotating black holes (Kerr black holes) are characterized by its *mass* and *angular momentum*. We don't care about charge because astronomical objects are *unlikely* to carry net charges (due to all the plasmas in the surroundings).
- ✓ In fact, the warping of space-time *inside* the event horizon is very complicated. Quantum effects become important in the space-time singularity at the centre. There is yet a successful theory to completely unify relativity and quantum mechanics. [1]

Hawking radiation

- ✓ Stephen Hawking predicted that black hole would actually radiate with a thermal spectrum due to quantum mechanical effects near to the event horizon. The so-called ***Hawking radiation*** from a black hole is so weak that it is impossible to measure directly. The *evaporation* of black hole is very long. For example, a one-solar-mass black hole will evaporate in 10^{67} years (for comparison, the age of the universe is 1.4×10^{10} years!).

[1] The so-called "Information Paradox" is one of the most famous examples about the problem of combining relativity with quantum mechanics. The problem led to a bet between Kip Thorne, Stephen Hawking and John Preskill.

13.3 The search for black holes

- ✓ By applying Kepler's law to the orbital data, one can deduce the mass of a central object provided that the velocity and mass of the matter rotating around the object are given.
- ✓ For example, detailed calculations of giant elliptical galaxy M87 (Fig. 13-7) reveal that there is about 3×10^9 M⊙ confined in a space no larger than the solar system! Takes NGC 4261 as a second example, there is about 1.2×10^9 M⊙ in a small region at the centre.

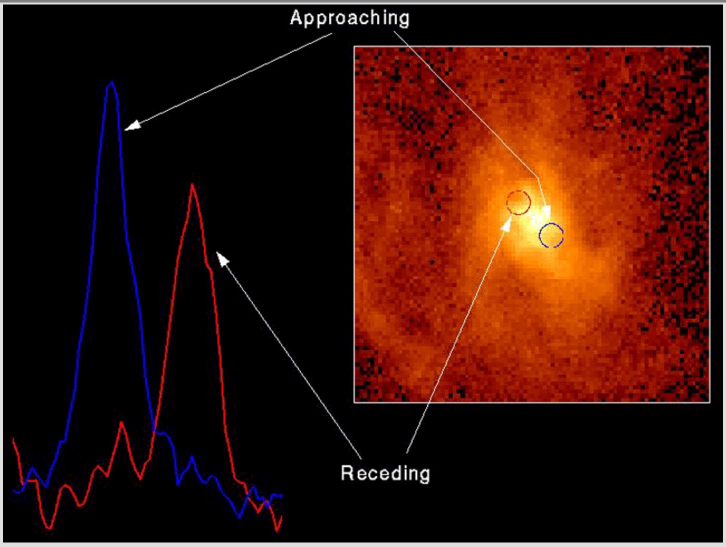

Fig. 13-7: Hubble Space Telescope imaged the centre of the giant elliptical galaxy M87 and revealed a small disk of gas surrounding the core. The blue and red lines show the Doppler shifts (blue shift when the source is approaching, red shift when it is receding) in the spectral lines caused by rapid rotation of the disk.

- ✓ The largest black hole, in terms of the angular size on the sky, is Sgr A* at the centre of our own Milky Way Galaxy. Even though its mass of over 3 million solar masses is not large compared to other supermassive black hole, its angular span is large due to its short distance from us.
- ✓ Over the last two decades, there have been continuing efforts to observe the stars around the galactic centre. (Fig. 13-8) By measuring the orbits of the so-called S stars, which are fast-moving young stars [2], it is

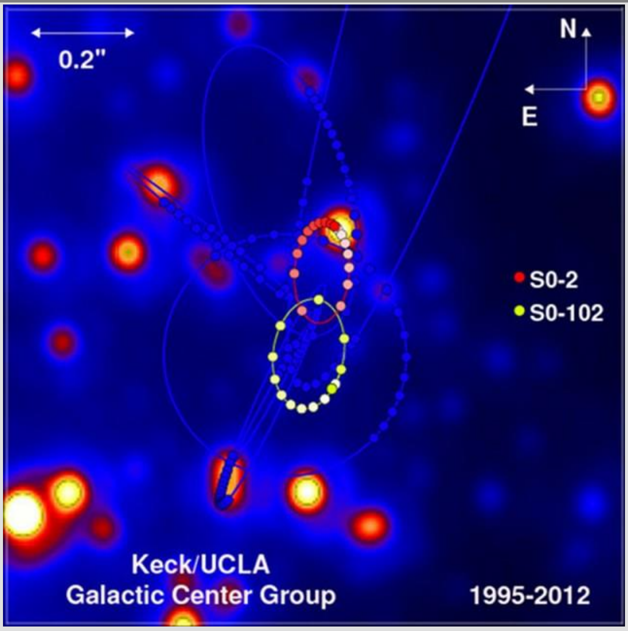

Fig. 13-8: Complete orbits of S0-102 and S0-2 within the central arcsecond of the Milky Way Galaxy. The IR observation was made possible with adaptive optics. (credit: Keck/UCLA, Andrea Ghez)

[2] The star that is closest to Sgr A* is S0-102, or simply S102, which has a period of merely 11.5 years. The observation of S0-102's orbit shows that the mass of Sgr A* is $4.1\pm0.4\times10^6$ solar masses. For details, see the

possible to prove the existence of a supermassive black hole at Sgr A*.

- ✓ Due to the advance in observation with Very Long Baseline Interferometry (VLBI), within ten years there would be direct observation that resolves the events horizons of supermassive black holes (Fig. 13-9) at Galactic Centre and M87. However, it is unclear whether the observation would really reveal the so-called silhouette or shadow of the black hole, because the image may be hidden behind the emission from the accreting plasma. [3]

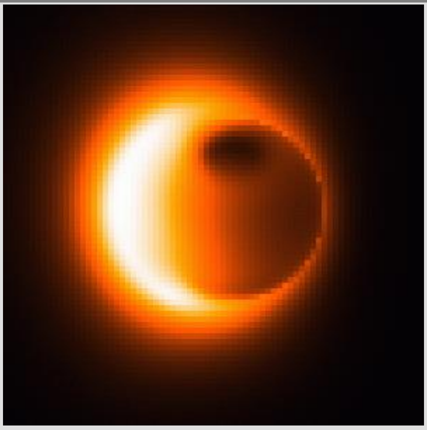

Fig. 13-9: Simulated observation (at 230 GHz) of the emission from the inner accretion flow in Sgr A* at the Galactic Centre. (Credit: the EHT collaboration)

- ✓ There is also effect to search for stellar black hole. Many binary systems have a dark massive member. The orbital data (Doppler shifts) of a visible star in a binary system can reveal the mass of the dark companion, hence black holes. Collision of black holes will generate relatively strong gravitational waves. Scientists have built detectors to detect gravitational waves.
- ✓ Other indirect method of detection involves observing high-energy radiation [4] from celestial objects, and modelling the accretion flow in the system. For example, the bright X-ray source Cygnus X-1 is believed to be a binary system of a supergiant and a black hole.

2012 article on Science Magazine (Meyer et al., 2012, Science, 338, 84, preprint at http://arxiv.org/abs/1210.1294)

[3] For details of the VLBI observation of supermassive black hole, see the white paper for Event Horizon Telescope (Doeleman et al., 2009, arXiv:0906.3899), or their website: http://www.eventhorizontelescope.org

[4] High-energy radiation could come from black holes of various masses. In particular, it is believed that some ultraluminous X-ray sources contains intermediate-mass black holes (with 100 to 10000 solar masses). However, the field is not as well established compared to stellar-mass and supermassive black holes. For details, see this news release: http://hubblesite.org/newscenter/archive/releases/2012/2012/11/full/ and also this article: http://www.nasa.gov/jpl/nustar/black-holes-20131126.html

13.4 Testing General Relativity in the strong field regime

- ✓ Since Albert Einstein published his General Theory of Relativity in 1915, there have been a lot of efforts to verify the theory (see Section 11.3). So far General Relativity has passed all the experimental and observational tests with flying colours.
- ✓ Most of the tests, however, are in the so-called weak-field limit.

Gravitational Wave

- ✓ The situation changed completely due to the discovery of the binary pulsar PSR B1913+16 in 1974 by Hulse and Taylor [5]. The system provides evidence that General Relativity still works when gravity is strong.
- ✓ According to General Relativity, the rapid movement of a pair of compact objects (black holes or neutron stars) will create outward going ripple, known as gravitational wave, in the space-time. (Fig. 13-10)

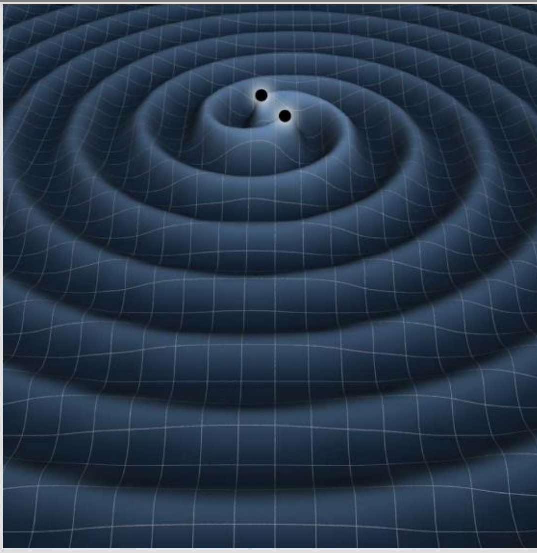

Fig. 13-10: Artist's conception of the gravitational wave produced by a binary black hole system. (credit: NASA)

- ✓ The pulsar signal of PSR B1913+16 allows us to measure the orbital period with high precision. After several years of its discovery, it was clear that the orbital period decreased slowly. The corresponding power loss in the system is in good agreement with the prediction of General Relativity.
- ✓ To directly measure gravitational wave, two LIGO (Laser Interferometer Gravitational-Wave Observatory) detectors were built in the United States. Each of the detectors consists of two 4-km arms in an L-shape configuration. Gravitational wave passing a LIGO detector

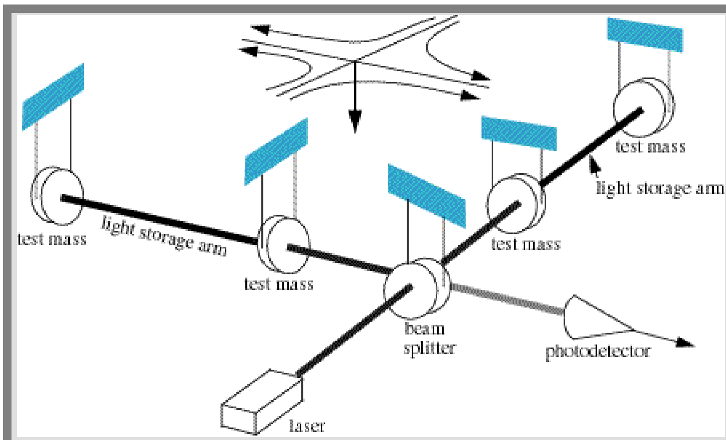

Fig. 13-11: Design of LIGO detector. (credit: LIGO)

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[5] Russell A. Hulse and Joseph H. Taylor Jr were awarded the Nobel Prize in Physics 1993, for "the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation." See the press release for details: http://www.nobelprize.org/nobel_prizes/physics/laureates/1993/press.html

would lead to small changes in the lengths of the arms, which could be measured with high precision by the interference of laser beam in the detector. (Fig. 13-11)

- ✓ Theories predict that LIGO is sensitive to the gravitational wave emitted from neutron star merger [6], black hole merger, and core collapse supernova.
- ✓ The first direction detection of gravitational wave was in 2015 (announced in 2016). The signal origin is the merging of two black holes, with ~35 and ~30 solar masses, at ~1.3 billion years ago.
- ✓ The first neutron star merger was detected in 2017, jointly by LIGO and Virgo. The corresponding EM signals were observed at multiple wavelengths.
- ✓ The LISA (Laser Interferometer Space Antenna) mission has been proposed to measure the gravitational radiation from space. It has gone through multiple redesigns and cancellations. The latest version of the design, led by ESA and joined by NASA, is expected to be launch in 2030s.

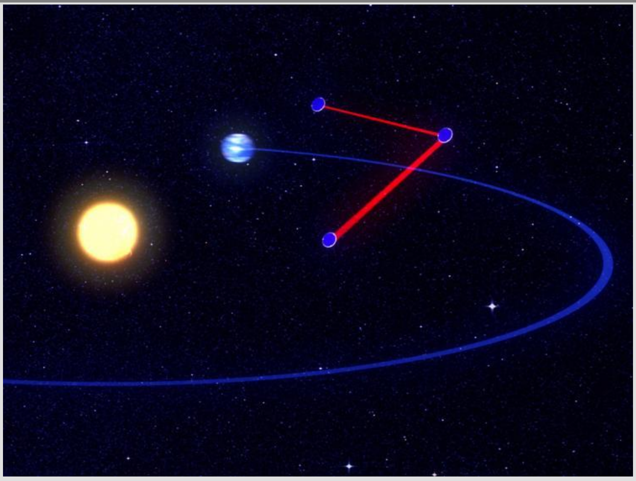

Fig. 13-12: The three eLISA spacecraft will be placed in orbits that form a triangular formation 20° behind the Earth and side length 1 million km. (The figure showing the formation is not to scale.) (Credit: AEI/MM/exozet)

Accreting matter

- ✓ Another way to test General Relativity in the strong field limit is by observing how matters are accreted into neutron stars and black holes.
- ✓ As the matters fall into the compact object, they form an accretion disk. Powerful outward flow, known as jet, could also be produced. (Fig. 13-13)
- ✓ By comparing the observation with simulation, it is possible to probe the innermost part of the disk to check whether General Relativity still works at that region.

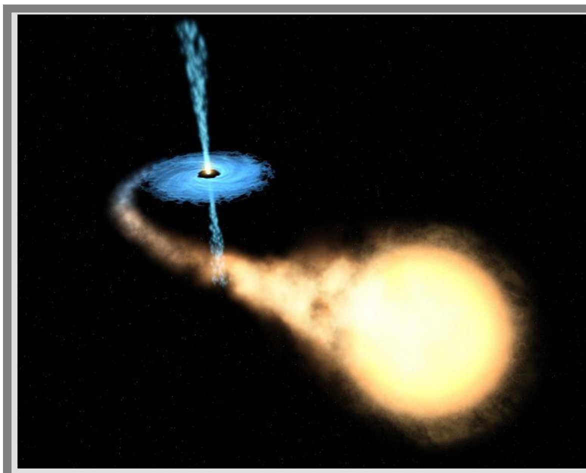

Fig. 13-13: Artist's conception of a black hole drawing matter from a nearby star. (Credit: NASA/STScl/ESA)

[6] As a pair of compact objects continues to lose energy, eventually they will merge. The final stage of their evolution produces significant perturbation in the space-time.