Chapter 3 Lunar phases, tides and eclipses

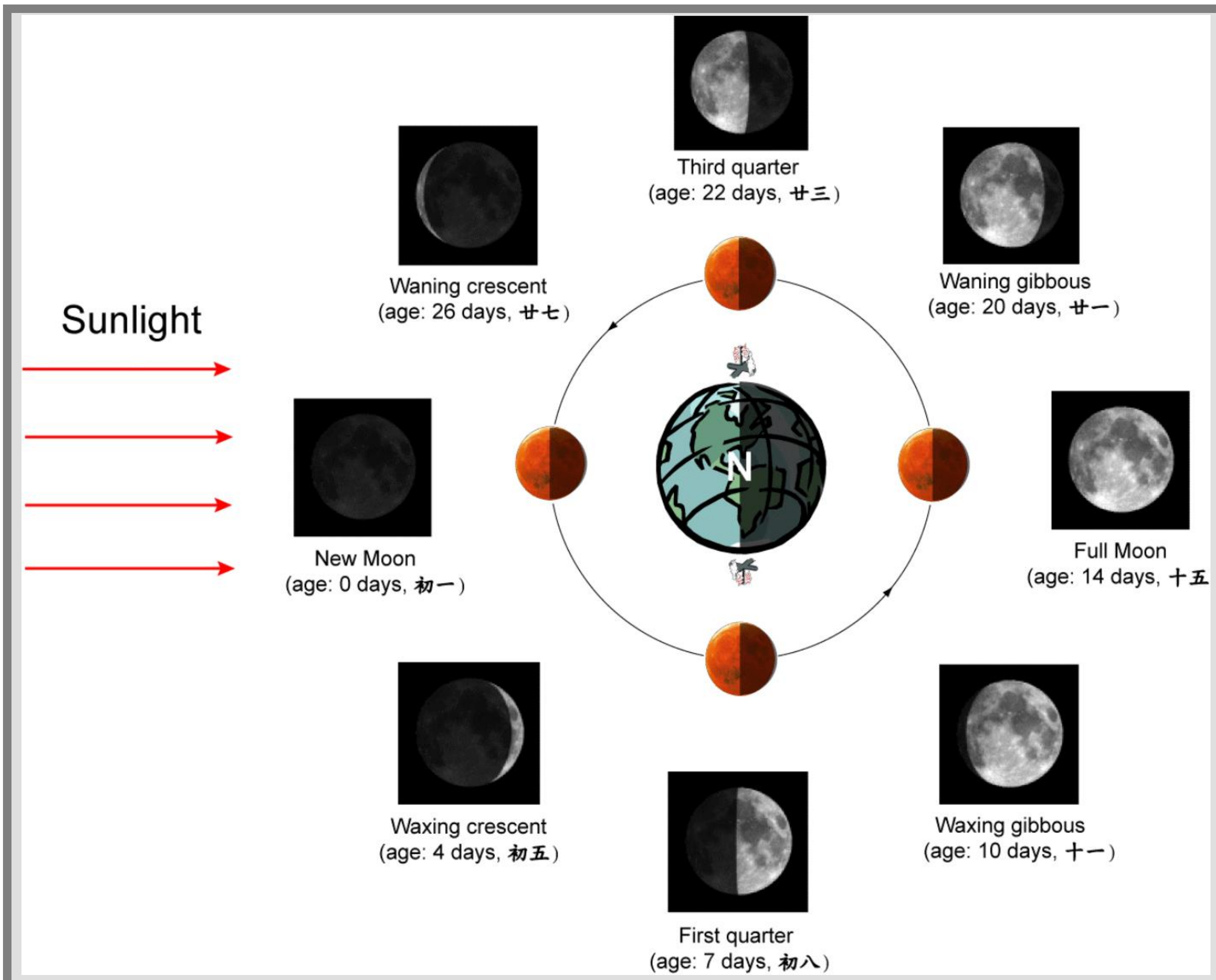

Fig. 3-1: The Moon's motion around the Earth produces phases. As viewed from above the North Pole, the Moon rotates in the counter-clockwise direction. The photos shown are approximately what an observer in the Northern Hemisphere would see.

3.1 Phase cycle

The Moon shines by reflecting sunlight. When the Moon travels to different positions (relative to the Sun and the Earth), different fractions of the lit hemisphere can be observed from Earth. (Fig. 3-1) The *apparent* shapes [1] of the sunlit part are called phases. The Moon goes through a complete set of phases in about 29.5 days.

Phase cycle : New Moon 朔(新月) → Waxing crescent 娥眉 → First quarter 上弦 → Waxing gibbous 盈凸 → Full Moon 望(滿) → Waning gibbous 虧凸 → Third quarter 下弦 → Waning crescent 殘月.

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[1] Another definition is the fraction of illuminated area on the Moon's apparent disk as visible from the Earth.

Question: When does the Third-quarter Moon rise and set?

Solution: By drawing a line (represents the observer's horizon) tangent to the Earth's surface on Fig. 3-2, the Moon rises at the eastern horizon at midnight, and it sets at the western horizon at noon. When does the Full Moon rise and set?

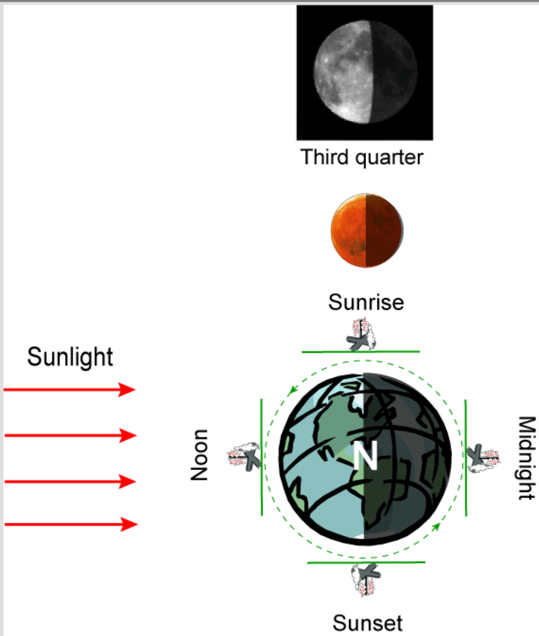

Fig. 3-2: The Moon rises at midnight and sets at noon.

- ✓ **Sidereal period:** The time to orbit around the Earth once *relative to* distant stars, about 27.3217 mean solar days.
- ✓ **Synodic period:** The lunar phase period (from one New Moon to another), about 29.5306 mean solar days.
- ✓ When the Moon finishes one cycle (one sidereal period), the Earth has already moved in its orbit with respect to the Sun. The Earth-Sun line has changed direction. (Fig. 3-3), the Moon needs two more days to catch up with the Earth-Sun line. As a result, *the synodic period is longer than the sidereal period*.

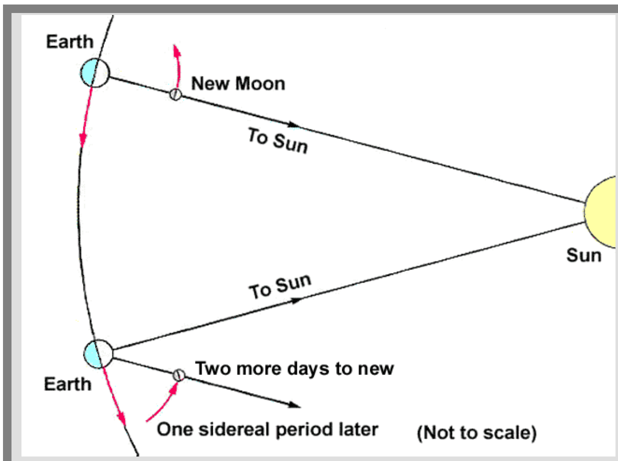

Fig. 3-3: One sidereal period after New Moon, the Moon returns to the same place relative to the background stars, but the Sun has moved eastward because of the Earth's orbital motion. Thus the Moon must travel a bit over 2 more days to catch up with the Sun and return to New Moon.

Question: Can you derive a relation between the synodic period and sidereal period of the Moon?

3.2 The Moon's orbit

✓ The orbital plane of the Moon is tilted by 5.1° to the orbital plane of the Earth. Therefore the declination of the Moon is always between -28.6° to 28.6° (5.1° beyond ±23.5°). Also, the Moon's path on the celestial sphere is close to the ecliptic.

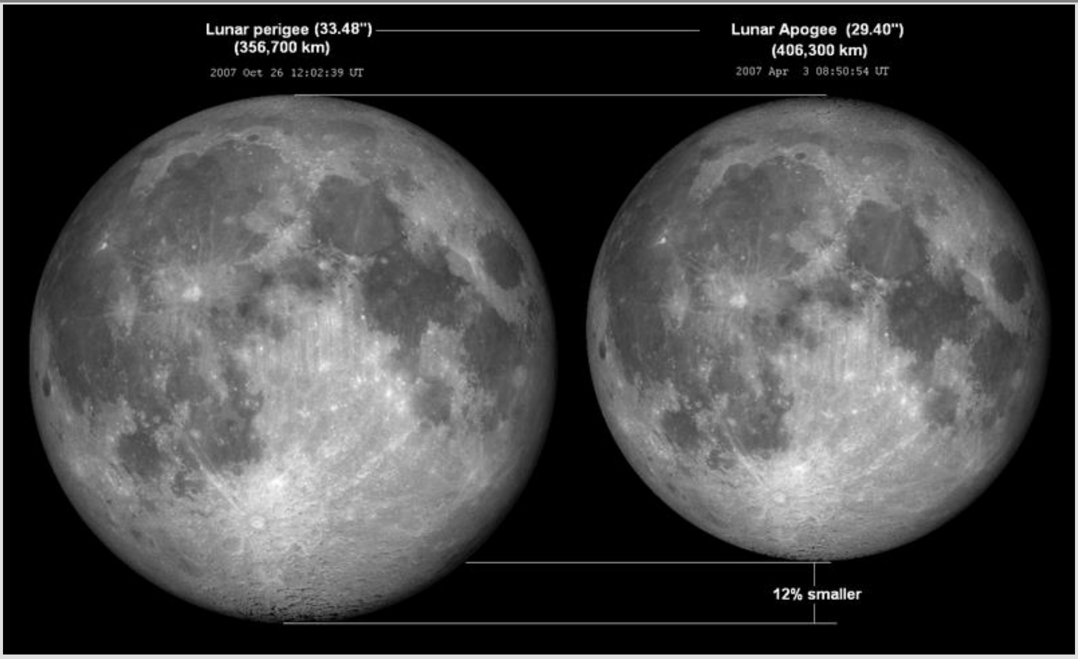

Fig. 3-4: The Perigee Moon alongside the Apogee Moon. (Image: Wikipedia Commons)

✓ The Moon's orbit is elliptical. Therefore the angular size of the Moon can vary by about 12% (Fig. 3-4). The so-called super Moon occurs when the Full Moon happens at the lunar perigee (nearest point).

3.3 Tides

Why are there two tides every day on Earth? [2]

✓ To understand tides, we consider the free fall of three objects under gravity as shown in Fig. 3-5. If the gravity is constant everywhere, since they fall with the same acceleration, their separations do not change. (Fig. 3-5 Left)

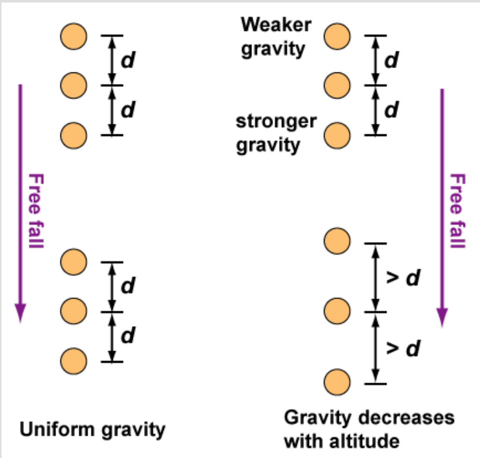

Fig. 3-5: (Left) If the gravity is uniform everywhere, the separations between the objects do not change upon falling. (Right) Actually, the gravity decreases with the altitude. As a result, the objects separate from one another.

[2] To be exact, there are two tidal bulges on Earth at any instant. Most locations have two tidal cycles per day (semidiurnal tides). Some areas, especially those far away from the equator, have only one tidal cycle per day (diurnal tides).

- ✓ However, Newton's gravitation law tells us that the gravitational attraction by any massive body decreases with increasing the distance from the body (goes as $1/r^2$). Details will be discussed in Chapter 4. Consequently, the upper/lower object will experience weaker/stronger gravity than the middle one. (Of course, it is insignificant in our daily life.) Consequently, the upper/lower object will fall slower/faster. The three objects will separate from one another. (Fig. 3-5 Right)

- ✓ Similar situation in the Earth-Moon system, regions on the Earth farther away from the Moon (e.g., region C) experience smaller gravitational pulls than the centre, whereas regions closer to the Moon (e.g., region A) experience larger pulls. As a result, both sides of the Earth along Earth-Moon line tend to separate from one another. (Fig. 3-6)

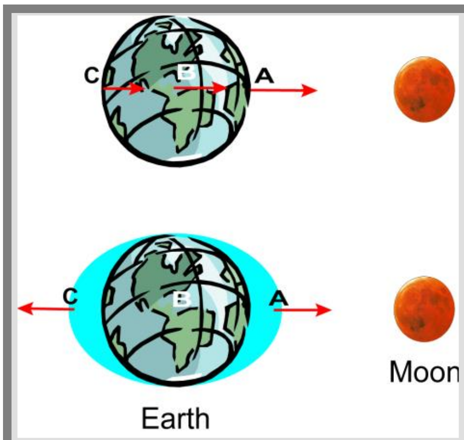

Fig. 3-6: (a) Positions A is closer to the Moon than B and C, so A experiences the largest gravitational force. (b) Because of the tidal force by the Moon, bulges on the both sides are observed on the Earth.

- ✓ From the perspective of the centre of the Earth, a force seems to have pushed C away from the Moon, and a force seems to have pulled A toward the Moon. It is called **tidal force**. One shows that the tidal force by the Moon on the unit mass of A or C is $F_{tidal} = \dfrac{2GMR}{r^3}$, where R, M and r are, respectively, the radius of the Earth, the mass of the Moon and the centre to centre separation between the Earth and the Moon. Derivation can be found in the later part of this section.

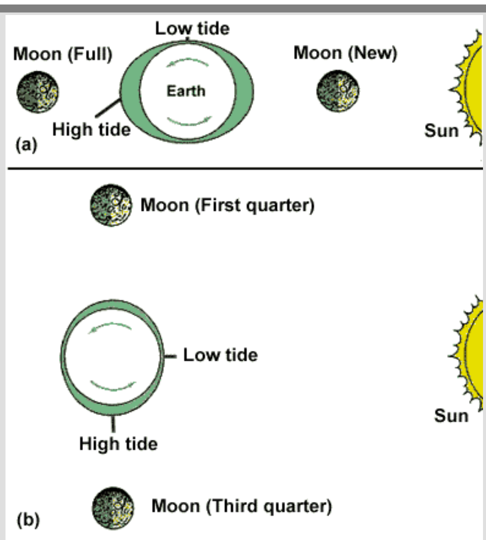

Fig. 3-7: (a) At New or Full Moon, the tidal forces by the Sun and the Moon are in the same direction, so the tidal bulges are larger. It is called spring tide. (b) At First or Third quarter Moon, the tidal forces are at right angles, and so the tidal bulges are smaller. It is called neap tide.

- ✓ Although the Moon's tidal force on Earth is small compared with the gravitational force between them (only about 3 %), the tidal force does pull water into bulges on *both* sides of the Earth along the direction of the Moon. Since the Earth self-rotates once a day, carrying every location on Earth through the two bulges, there are two high tides every day.

- ✓ There are similar tidal effects by the Sun on the Earth, but only half of the magnitude by the Moon, because the Sun is much farther away from the Earth. Note that the tidal force goes as $1/r^3$, decreasing very rapidly with the distance between them.
- ✓ **Spring tides:** At New and Full Moon, the Sun and the Moon pull in the same direction, causing *larger tides* (Fig. 3-7 (a)). [3]
- ✓ **Neap tides:** At First or Third quarters, their tidal effects by the Moon and the Sun are perpendicular to each other. Partial cancellation of their tidal effects leads to *smaller tides* (Fig. 3-7 (b)).
- ✓ In reality, the Earth's tidal bulges are not directly aligned with the Moon (Fig. 3-8), because the rotation period of Earth is shorter than the Moon's orbital period. The self-rotation period of the Earth decreases by 0.0016 second per century. Fossils of marine animals confirm that about 400 million years ago the Earth′s day was 22 hours only.

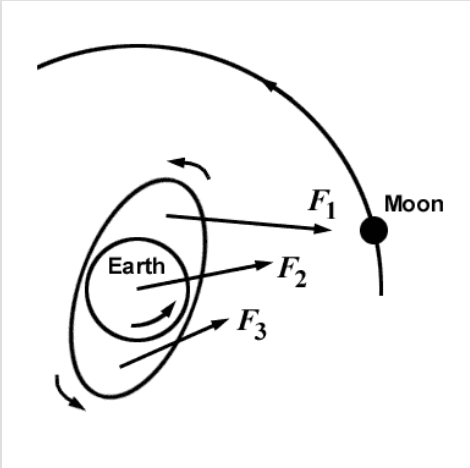

Fig. 3-8: The Earth's rotation drags the tidal bulges ahead of the Earth-Moon line. The gravitational forces of the bulges pull the forward in its orbit, forcing its orbit to grow in size.

- ✓ Meanwhile, the Moon is known to be drifting away from the Earth by 3 to 4 cm per year.[4] To see how the decrease in the Earth's rotation and the increase in the Earth-Moon distance are related, we need to consider the torque exerted on the Earth by the Moon's interaction. In Fig. 3-8, bulge F_1 leads the Moon and is closer to it than is bulge F_3. As a result, the force exerted on bulge F_1 by the Moon is greater, resulting in a net torque that is slowing the Earth's rotation. At the same time, bugle F_1 is pulling the Moon forward, increasing its rotational energy, and causing it move farther out (in other words, occupying a higher orbit). Because of tidal effects, given sufficient time the

[3] Note to be confused with *high* and *low* tides, which are related to the water level compared to other time on the same day. *Spring* and *neap* tides refer to the difference of the high and low tides. Specifically, spring (neap) tides refer to the case that the difference is large (small) compared to other days in the month.
[4] The distance can be determined with high precision by lunar laser ranging.

Earth will slow its rotation enough that the same side of the planet will always face the Moon.[5] It will happen when the length of the day is about 47 (current) days long.[6]

- ✓ In the past the Moon was much closer the Earth than it is today. It is also probable that the Moon's rotation period was once shorter than its orbital period. The Moon's present 1-to-1 synchronous rotation, called **tidal coupling** (潮汐耦合). It is due to the same tidal dissipation that is occurring on Earth today.

Now let's derive the tidal force.

- ✓ Since the gravitational force by the Moon on a unit mass on Earth is given by $F(r)=\frac{GM}{r^2}$, where G, M and r are, respectively, Gravitational constant, the mass of the Moon and the distance between the Moon and the unit mass.
- ✓ As position C is farther away, so the force acting on a unit mass of C (Fig. 3-6) is smaller, i.e., $F(r+R)=\frac{GM}{(r+R)^2}$ where R is the radius of the Earth. As observed on Earth, we consider the force relative to the centre of the Earth, i.e.,

$$\begin{aligned}F_{tidal} &= F(r+R)-F(r)\\ &= \frac{GM}{(r+R)^2}-\frac{GM}{r^2}\\ &= \frac{GM}{r^2}(1+\frac{R}{r})^{-2}-\frac{GM}{r^2}\\ &\approx \frac{GM}{r^2}(1-\frac{2R}{r})-\frac{GM}{r^2}\\ &= -\frac{2GMR}{r^3}\end{aligned}$$

$$\boxed{F_{tidal}=-\frac{2GMR}{r^3}}$$

The negative sign means the force pushes C away from the Moon. Similarly, one should show that the tidal force acting on the unit mass of A is the same except that the force is pulling A toward the Moon.

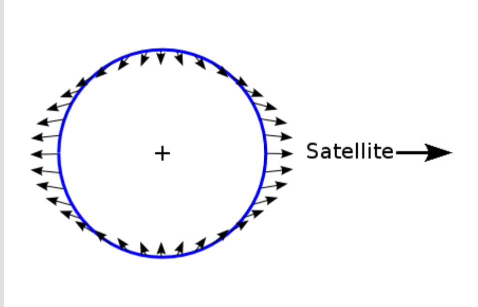

Fig. 3-9: The Moon's gravity differential field on the Earth's surface. (Image: Wikipedia Commons)

[5] Pluto and its largest satellite Charon are in such a *tidal lock*. Char
Pluto's surface!

[6] Carrell & Ostlie 2007, page 721-722.

- ✓ The real situation is more complicated. The gravity differential field is actually a function of the locations on the Earth. (Fig. 3-9)

3.4 Eclipses

- ✓ Since the Moon′s orbit tips by about 5° to the Earth′s orbit; therefore, on the celestial sphere, the Moon's orbit and the ecliptic intersect at two points called **nodes** (交點) (Figs. 3-10, 11). The Moon crosses each node once every month.
- ✓ In case where *both* the Sun and the Moon are close to the nodes, the Sun, the Earth and the Moon align almost on a straight line, an eclipse occurs (Figs. 3-10, 11).

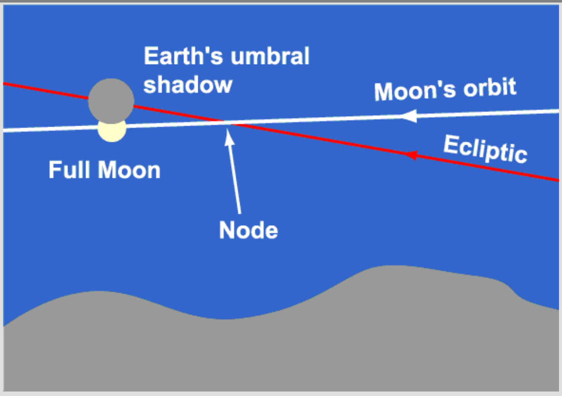

Fig. 3-10: Lunar eclipse occurs if both the Moon and the Earth's umbral shadow are near a node.

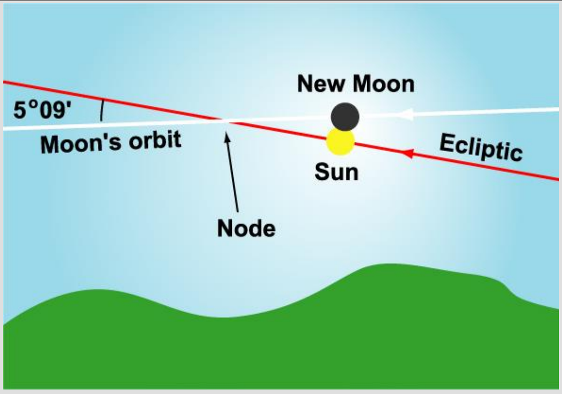

Fig. 3-11: A solar eclipse occurs if both the Sun and the Moon are close to a node.

- ✓ In the outer space, these nodes are just the intersections of the Moon's orbit and the plane of the Earth's orbit. The line joining the nodes is called the **line of nodes** (交線).
- ✓ If the line of nodes does not point at the Sun, i.e., the Sun, the Earth and the Moon are not on a straight line, there are no eclipses even at New moon and Full moon. (Fig. 3-12 (a))
- ✓ On the other hand, if the line of nodes points at or nearly at the Sun, eclipses occur at New moon and Full moon. (Fig. 3-12 (b))

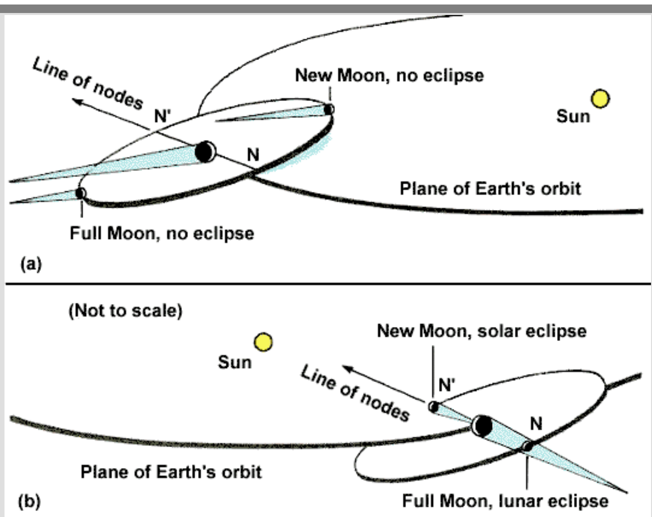

Fig. 3-12: Line of nodes. (a) The Sun, the Earth and the Moon are not on a straight line, eclipses do not occur. (b) The Sun, the Earth and the Moon are on the same straight line, eclipses occur.

Lunar eclipse

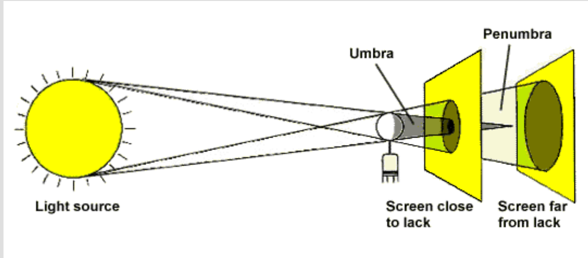

Fig. 3-13: The shadow cast by a map tack resembles those of the Earth and the Moon. The umbra is the region of total shadow. The penumbra is the region of partial shadow.

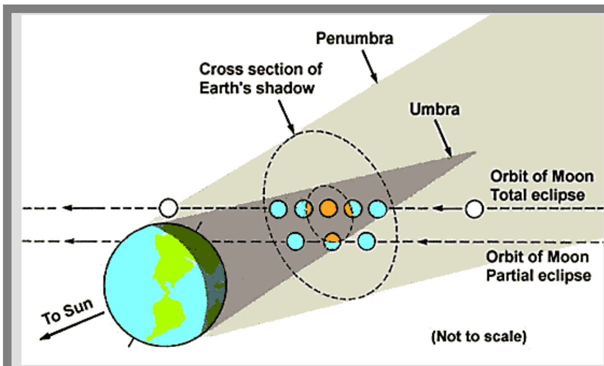

Fig. 3-14: During a total lunar eclipse, the Moon's orbit carries it through the penumbra and completely into the umbra. During a partial lunar eclipse, the Moon does not completely enter the umbra.

- ✓ **Lunar eclipses:** The Moon enters the Earth's shadow. The Earth's shadow has two parts. **Umbra**（本影）**:** No light from the Sun can reach this region (totally blocked by the Earth), it is *almost total darkness*. (Fig. 3-13). **Penumbra** （半影）**:** Light from parts of the Sun's surface is blocked by the Earth, it is *only partial darkness.* (Fig. 3-13)
- ✓ **Total eclipse:** The Moon enters the umbra completely (Fig. 3-14), only sunlight bend from the Earth's atmosphere can arrive to the Earth. It is known that blue light is scattered *more* than red light [7]; most blue light is scattered away, leaving only red light falling on the Moon, so the Moon appears red and dim.
- ✓ **Partial eclipse:** The Moon only enters the umbra partially, and only a part of the Moon is lit. (Fig. 3.14)
- ✓ **Penumbra eclipse:** The Moon only enters the penumbra, the Moon is a little bit dimmer than usual, but is fully lit.
- ✓ **Eclipse season:** A period during which the Sun is close enough to a node for an eclipse to occur.[8]
- ✓ The shadow of the Earth on the Moon is spherical: it shows the spherical shape of the Earth.

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[7] The scattering is called Rayleigh scattering. The scattering probability is proportional to the frequency to the power 4.

[8] Recall that the Sun completes a full cycle on the celestial sphere in a year, whereas the Moon completes a cycle in a month. The fast moving Moon is able to reach the other node on time after the first eclipse (either solar or lunar). Therefore, solar and lunar eclipses often occur two or three times in a row. Verify it with the info on NASA's eclipse website: http://eclipse.gsfc.nasa.gov/eclipse.html

Solar eclipse

- ✓ Due to a pure coincidence of nature, the angular sizes of both the Moon and the Sun are about 0.5°. Since the orbit of the Moon [9] is an ellipse, the angular size of the Moon could either be slightly larger than the Sun, or slightly smaller than the Sun.
- ✓ Another way to see the picture is to consider the Moon's shadow. In Fig.3-13, the screen in this case is the Earth and the object is the Moon. **Solar eclipses** occur when the Moon′s shadow falls on the Earth.
- ✓ If the Moon is relatively close, the Moon′s umbral shadow falls on the Earth (Fig. 3-15). The region is only about 270 km in diameter on Earth (the Earth′s is about 13,000 km), so only a small region on Earth can see total solar eclipse at the same time. As the Earth rotates, the shadow sweeps a path of total eclipse on Earth, so a **total eclipse** only lasts for at most a few minutes at a particular location on Earth. Meanwhile, a **partial eclipse** occurs at locations where the Moon′s penumbral shadow is falling on.

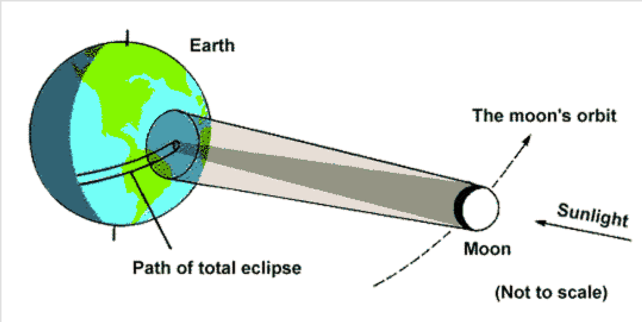


Fig. 3-15: Observers in the path of totality see a total solar eclipse when the umbral shadow sweeps over them. Those in the penumbra see a partial eclipse.

- ✓ **Annular eclipses:** The Moon is too far from the Earth to cover the Sun completely (Fig. 3-16). The umbra does *not* fall on the Earth, only the penumbra does. As a result, the Sun appears as a bright *ring* with the central part blocked by the Moon.

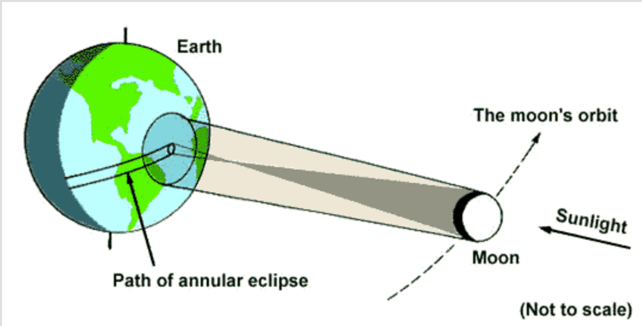


Fig. 3-16: If the Moon is near the farther part of its orbit, the umbral shadow does not reach the Earth, resulting in an annular eclipse.

- ✓ **Differential solar gravitational force:** Because of the solar gravitational force, the line of nodes (the line at which the Moon′s orbital plane crosses the ecliptic) precedes eastward with a period of about 8.85 years, so eclipses do not occur at the same time every year. (Fig. 3-17)

[9] and to a less extend, so does the orbit of the Earth

- ✓ **Eclipse seasons:** Considering the tilt of the Moon's orbit and the apparent sizes of the Moon and the Sun, every year there are two periods of about 34 days each, in which eclipse (either lunar or solar) can happen. The two eclipse seasons are separated by about half a year.
- ✓ **Saros cycle:** The Sun, Earth and Moon return to the same *relative* positions every 18 year 11.3 days. Two solar eclipses apart by one Saros cycle sweep similar paths on Earth.

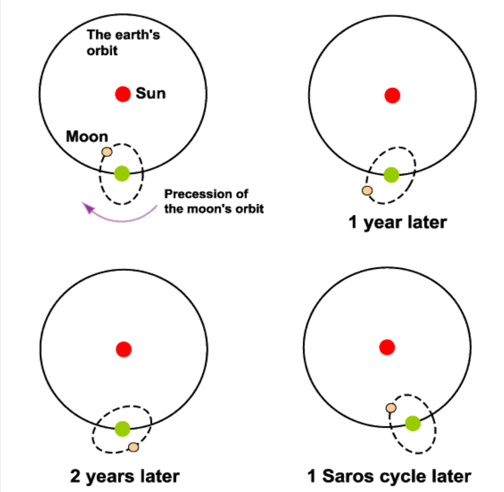
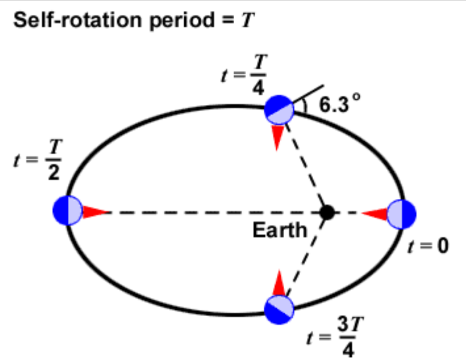


Fig. 3-17: Due to the precession of the Moon's orbit, the Sun, the Earth and the Moon can return to the same relative positions only after 1 Saros cycle.

Box 3.1 Lunar libration

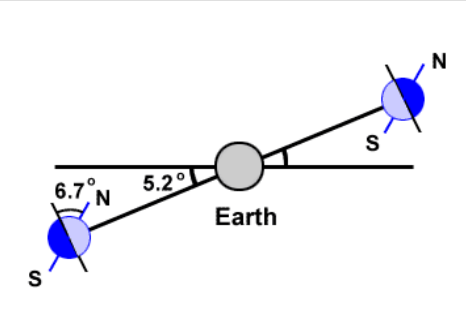
Because of the tidal coupling, the half of the Moon facing the Earth is always the same. Naively one would expect that we could only see 50% of the lunar surface.

As it turns out, the situation is a bit more complicated because the Moon's orbit is an eclipse instead of a circle. The Moon travels faster when it is near the *perigee* (nearest to the Earth), and slower near the *apogee* (farthest from the Earth). Because it self-rotates at a fairly steady rate, the overall effect is an east-west "rocking" of the Moon by about 7.9° as observed from the Earth. It is called **libration in longitude**.



Libration in longitude.

The lunar orbital plane tips by about 5.1° compared to the Earth's orbital plane. The inclination results in a north-south "nodding" motion over the month. It is called **libration in latitude**.



Libration in latitude.

A third effect, known as **diurnal libration**, is caused by the change in the observer's viewing point as the Earth self-rotates. Due to librations, one can see 59% of the total lunar surface from the Earth.