Chapter 13

Black holes

Black holes

13.1 Newtonian picture

13.2 General relativistic picture

13.3 The search for black holes

13.4 Testing General Relativity in the strong-field regime

13.1 Newtonian picture

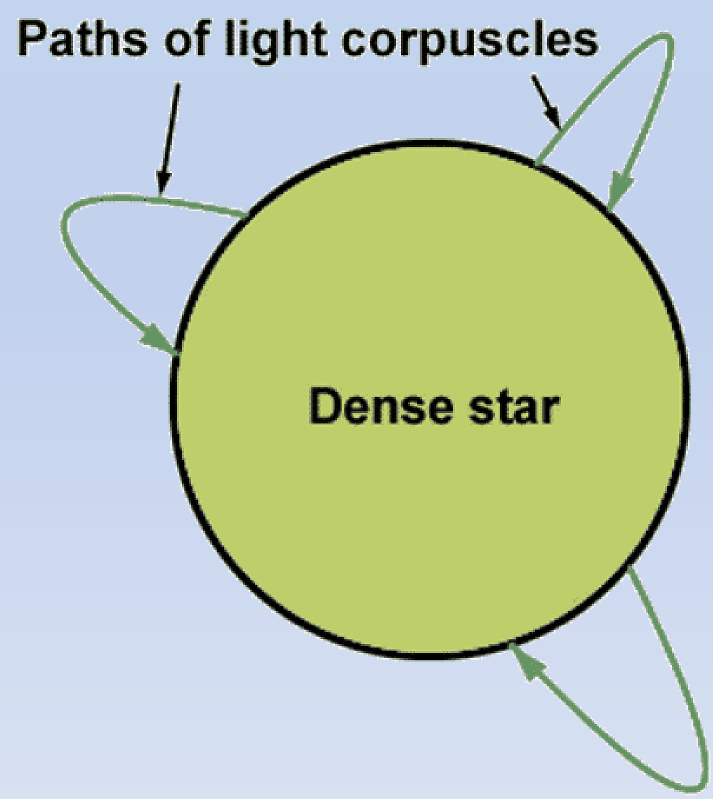
- ✓ Escape velocity

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

- ✓ For a massive and small star, v_{esc} is very large.
- ✓ If $v_{\mathrm{esc}} = c$

$$R = \frac{2GM}{c^2}$$

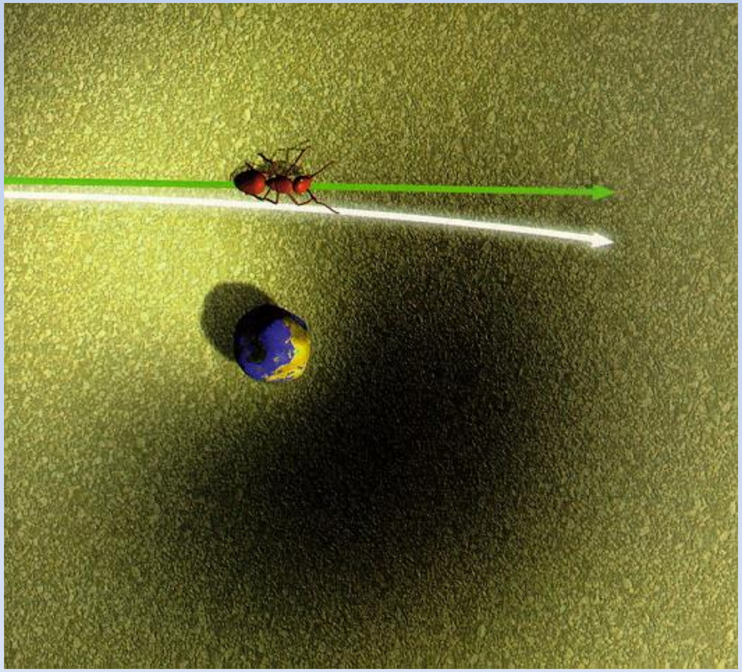
- ✓ Not even light can escape.

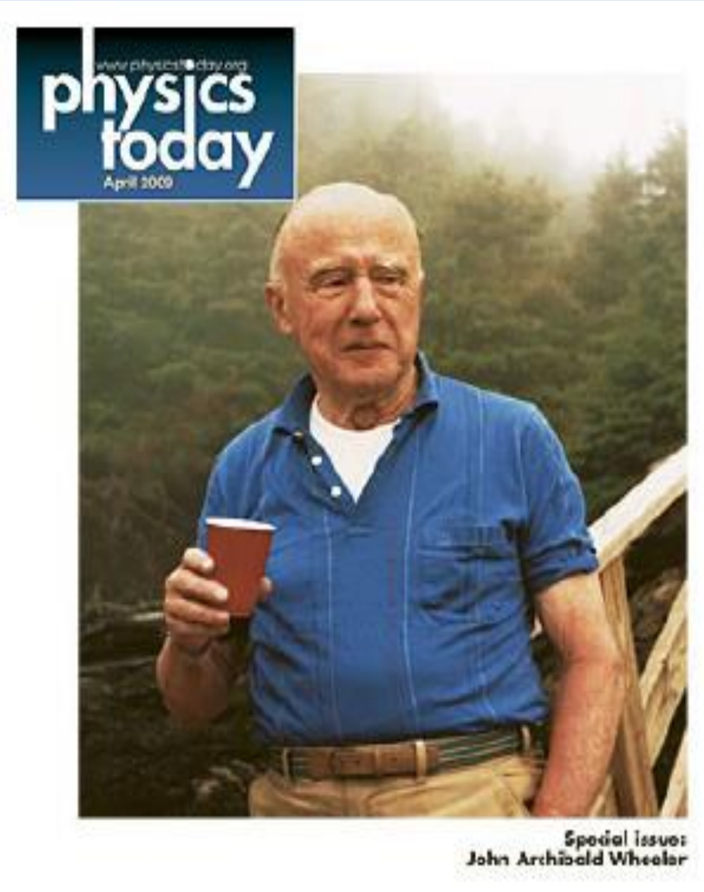


13.2 General relativistic picture

John Wheeler (1911-2008):

"Spacetime tells matter how to move; matter tells spacetime how to curve."



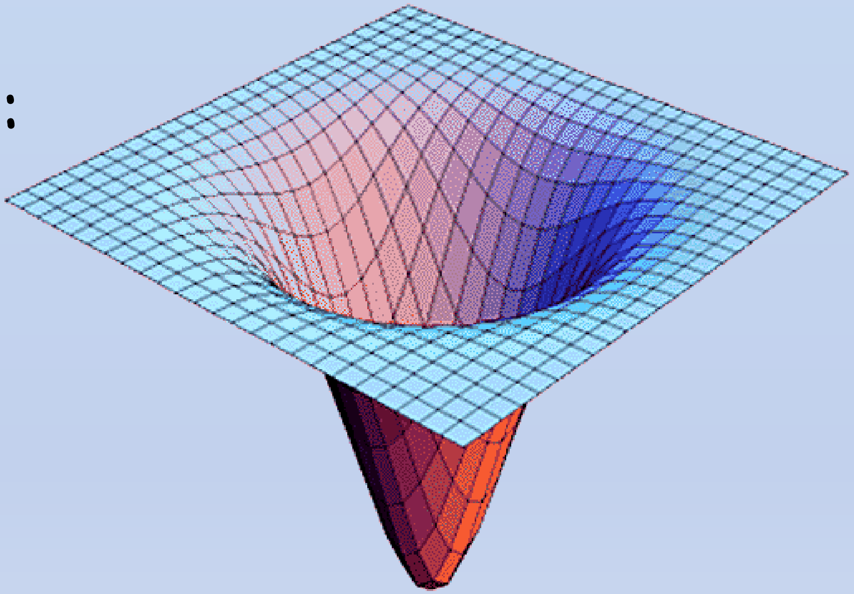


Wheeler coined the term "black hole"

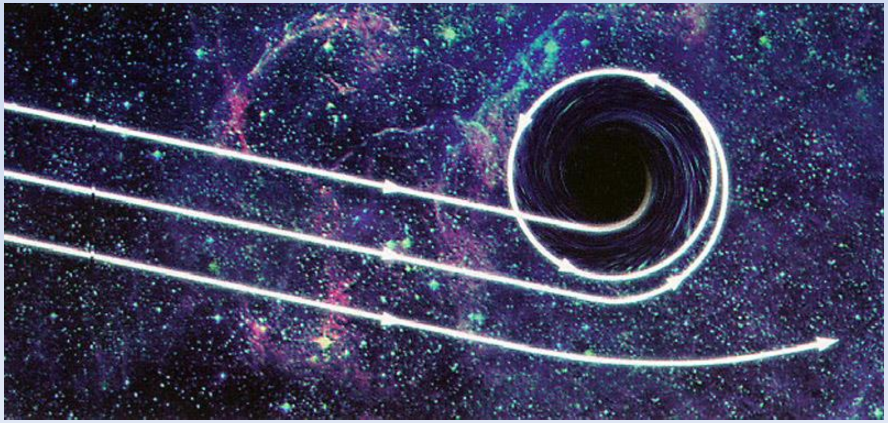
13.2 General relativistic picture

- ✓ Remaining core mass $> 3\, M_\odot$: contract beyond nuclear density, i.e., not even neutron degenerate pressure can withstand gravity.
- ✓ Severe curvature of space-time
- ✓ Even light cannot escape.

➔ black hole



Large mass



13.2 General relativistic picture

- ✓ Karl Schwarzschild was the first to calculate the radius of non-rotating black hole in vacuum.
- ✓ Core contracts to

$$R < R_S \equiv 2GM/c^2$$

same as the Newtonian result!

- ✓ From inside the Schwarzschild radius R_S, *not* even light can escape.

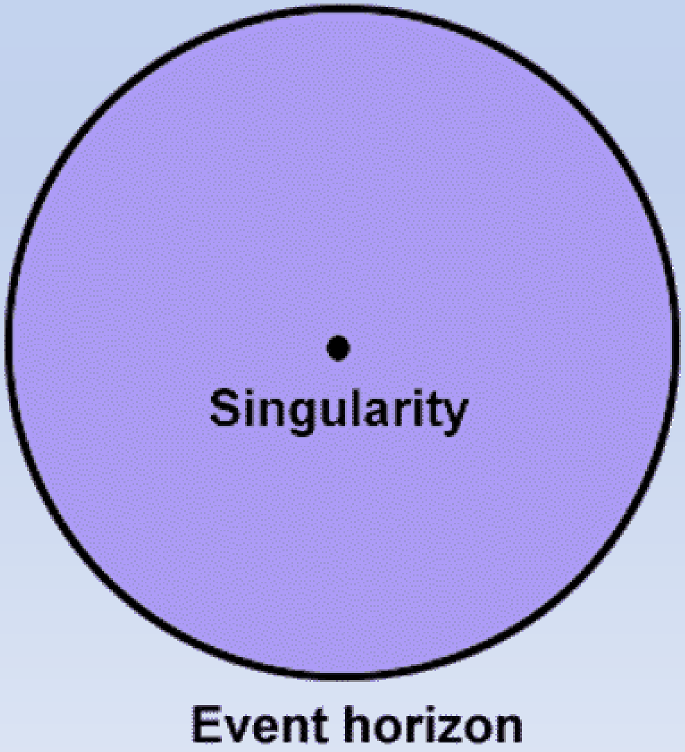
13.2 General relativistic picture

Schwarzschild black hole

- ✓ The surface with $r = R_S$ is called the event horizon.
- ✓ R_S marks the boundary inside which *nothing* (even light) can escape.
- ✓ For example,

1-solar-mass star: $R_S = 3$ km

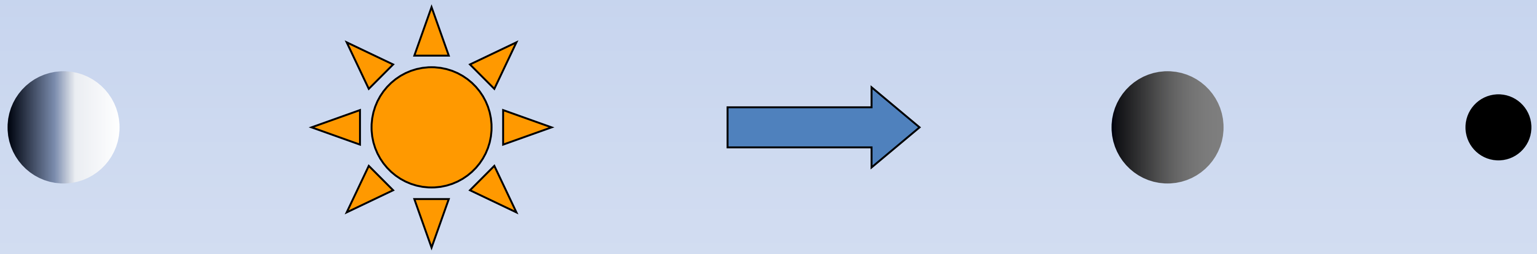
10-solar-mass star: $R_S = 30$ km



13.2 General relativistic picture

What will a distant observer see?

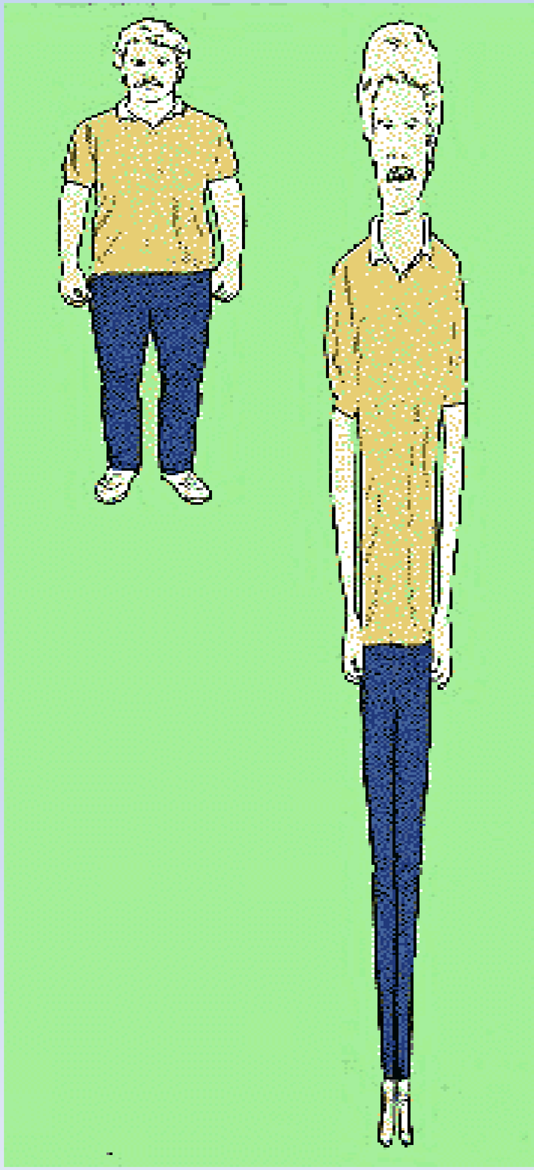
- ✓ No difference in gravitational force: just like the gravity of a star with the same mass
- ✓ A dark region



Don't worry! We won't fall into a black hole!

13.2 General relativistic picture

Tidal force



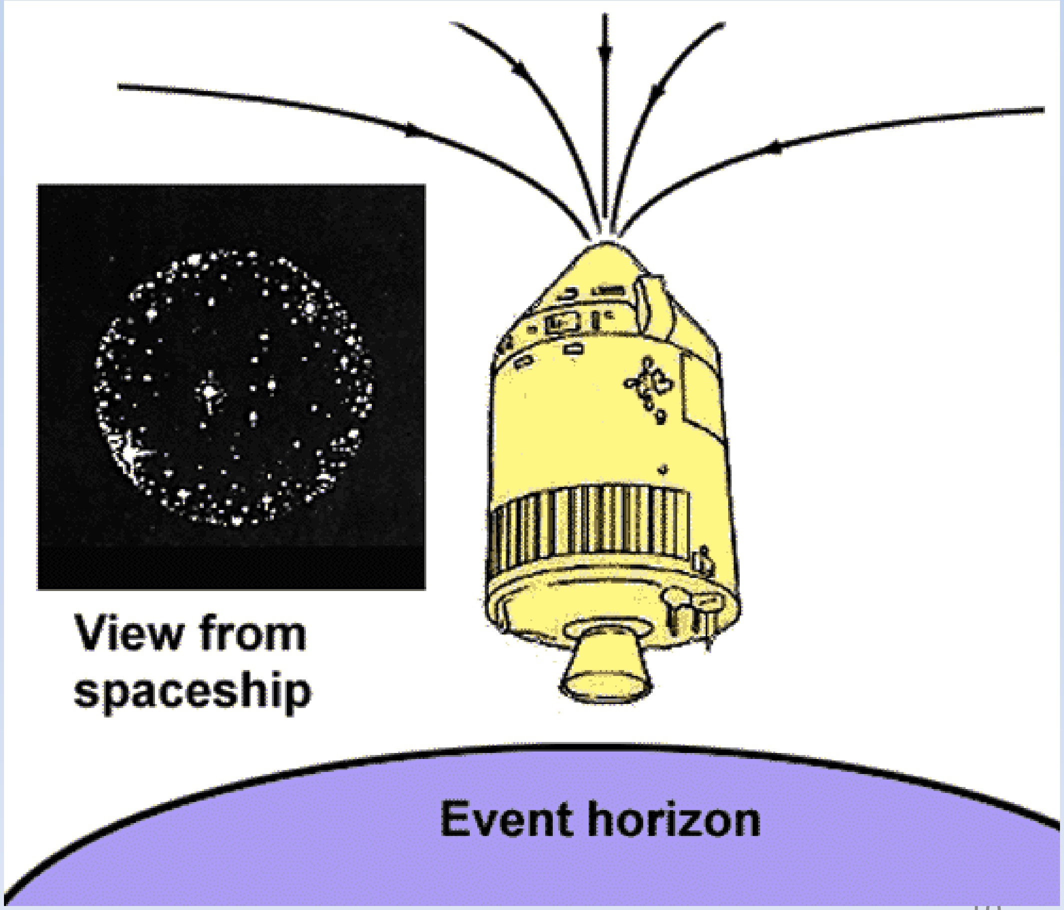
- ✓ Stretched in the direction towards the black hole (i.e., gravity stronger at the feet than the head), compressed in the perpendicular direction.
- ✓ may destroyed by the extreme tidal force !

(imagine being turned to spaghetti, and then torn apart…)

13.2 General relativistic picture

Bending of the path of light

- Light is also affected by the gravity. Their path is bent.
- The unfortunate astronaut sees distant starlight converges to a small cone overhead



13.2 General relativistic picture

Gravitational time dilation

- Because of curvature of space-time, the clocks falling into a black hole run slower

$$\Delta t = \Delta t_0 \left(1 - \frac{2GM}{rc^2}\right)^{-1/2}$$

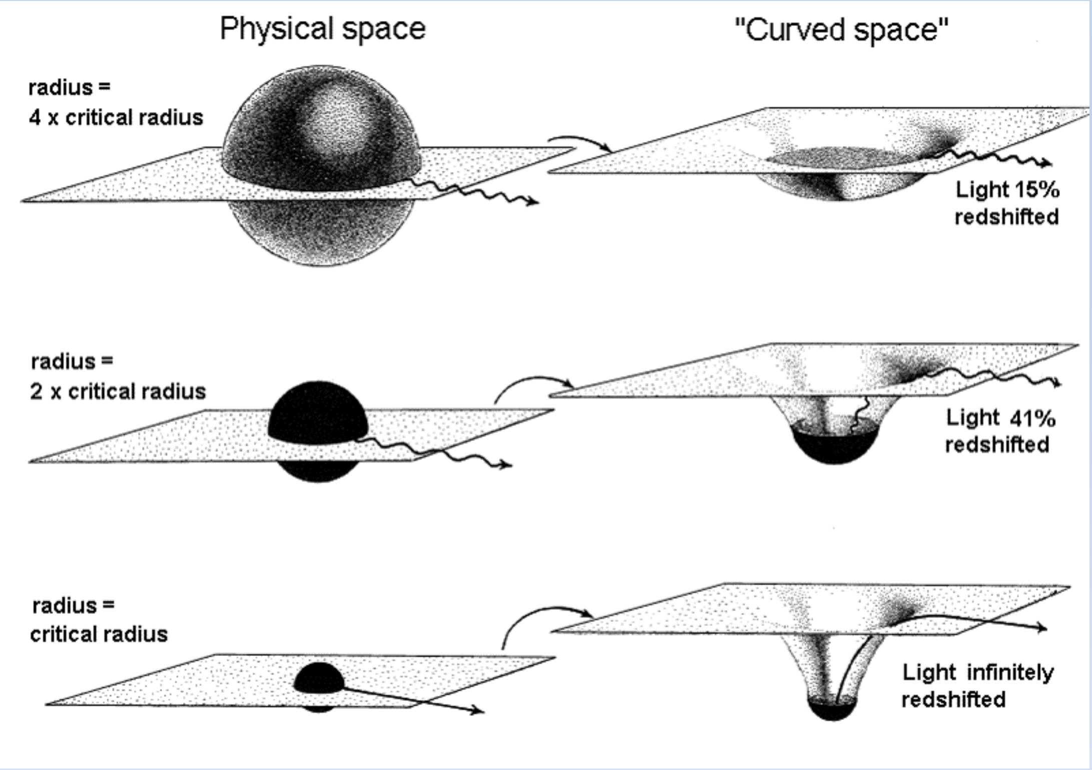
what happens if $r = R_s = 2GM/c^2$?

- The effect increases to infinity when the space shuttle falls through the event horizon.
- For $r < R_S$, i.e., $2GM/(rc^2) > 1$, no event inside the event horizon could be observed by a distant observer!

13.2 General relativistic picture

Gravitational redshift

- ✓ Light escape from a black hole is redshifted
- ✓ If $r \le R_S$: **infinitely** redshifted!
- ✓ A light pulse would lose all its energy when it is climbing out of the potential well.



13.2 General relativistic picture

Gravitational redshift

- ✓ Frequency is $1/\Delta t$, the gravitational redshift:

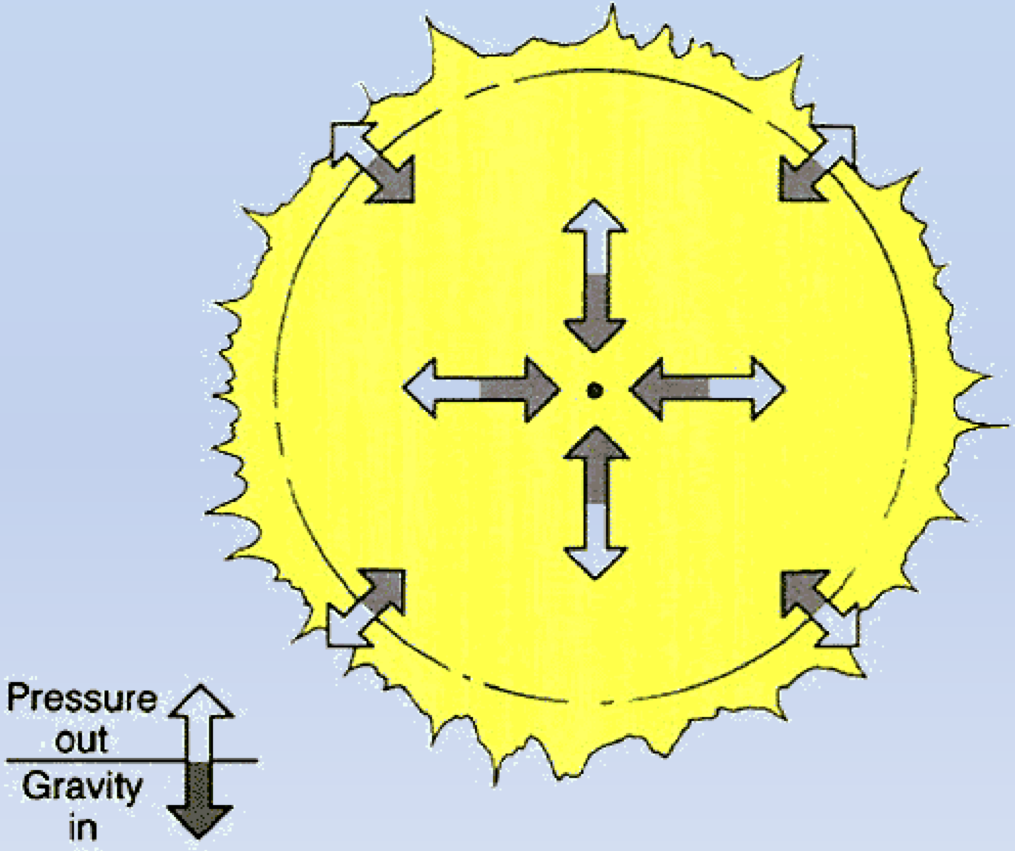
$$f = f_0 \left(1 - \frac{2GM}{rc^2}\right)^{1/2}$$

what happens if
$r = R_s = 2GM/c^2$?

- ✓ Note if the pulse is emitted far from the event horizon, i.e., $r >> R_s$, weak-field limit applies, i.e.,

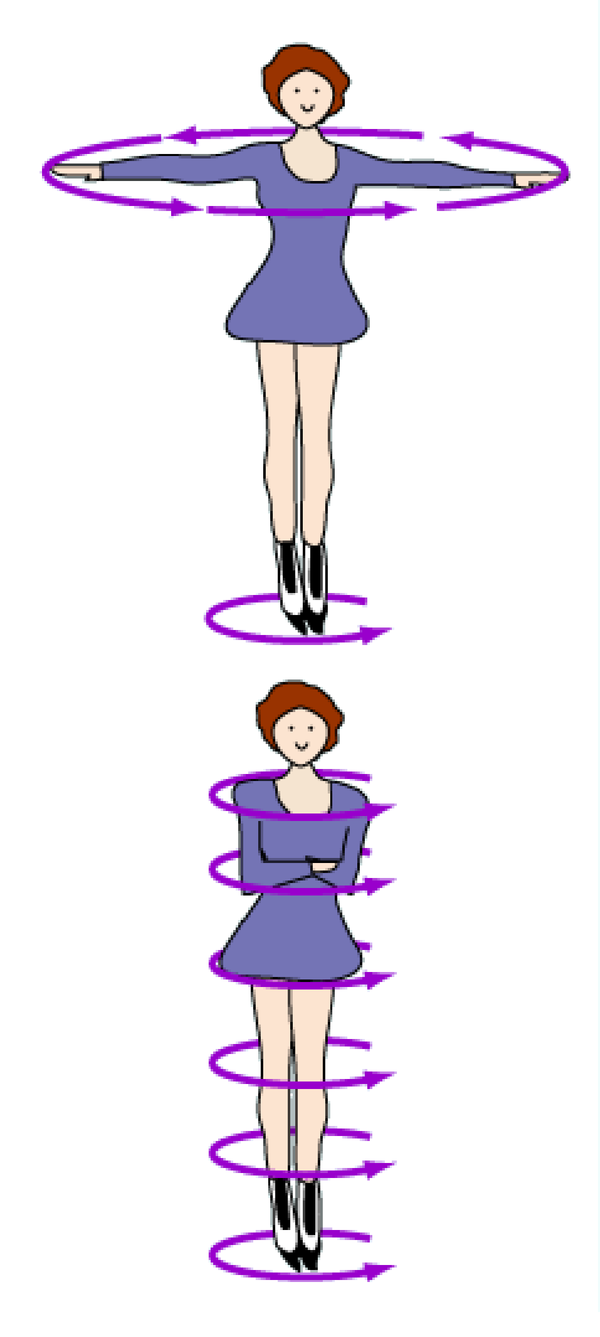
$$f = f_0 \left(1 - \frac{GM}{rc^2}\right) = f_0 \left(1 - \frac{\Delta\phi}{c^2}\right)$$

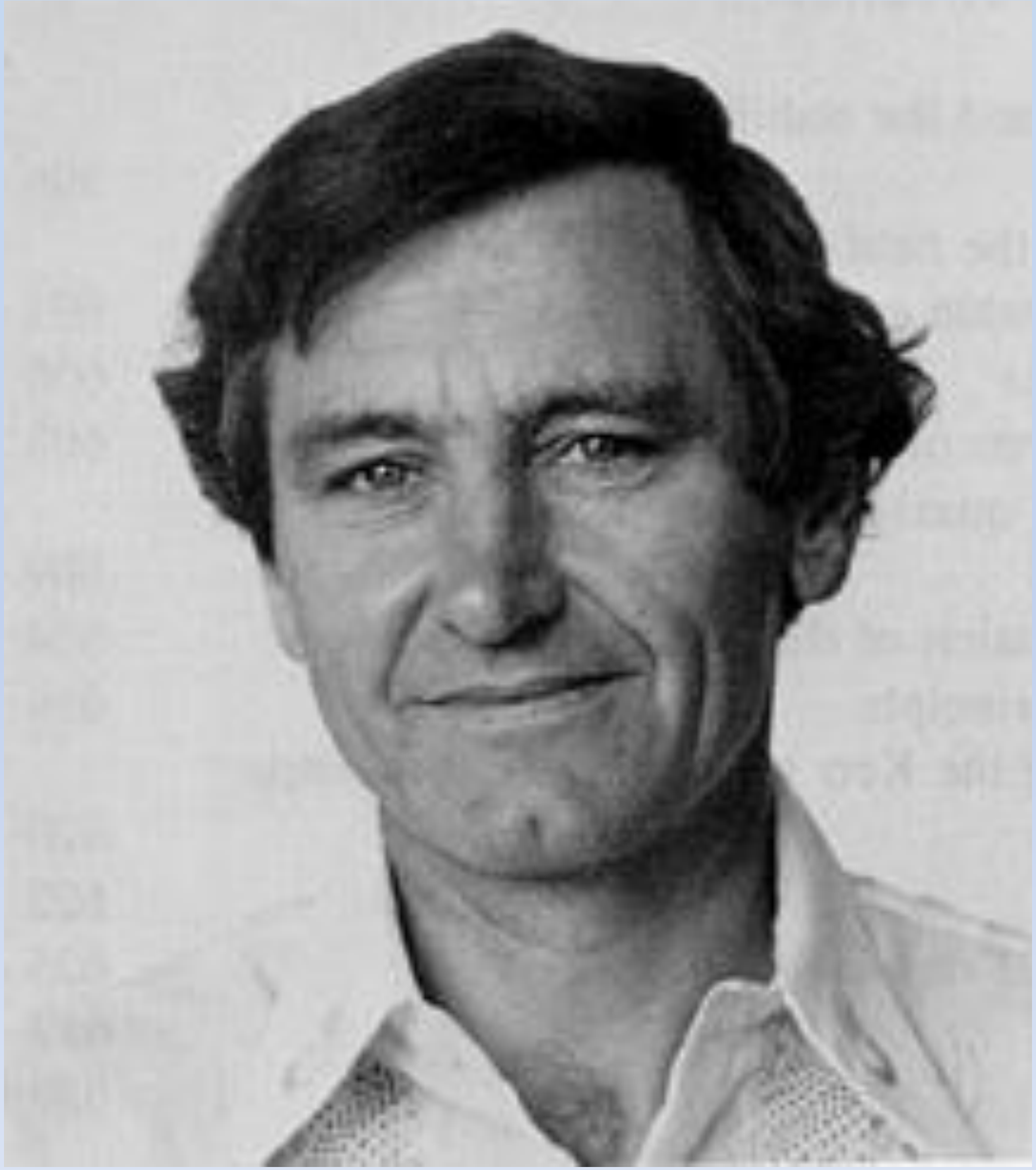
Before further discussion, remind me again how a star could form a **black hole**?



- Towards the end of life of a massive star
- remaining core of 2-3 solar masses:
- core contracted and becomes black hole

- Now recall that stars are rotating





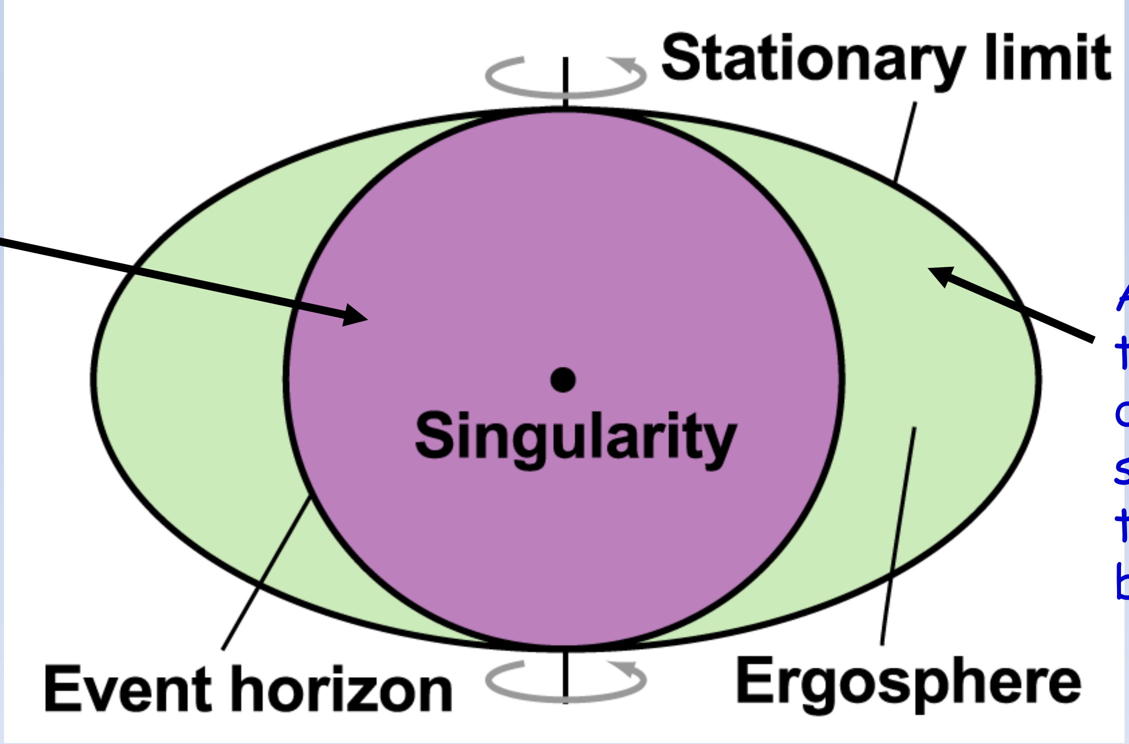
Roy Kerr (1934 -)

13.2 General relativistic picture

Kerr black hole

✓black holes with *angular momentum*

Even light cannot escape

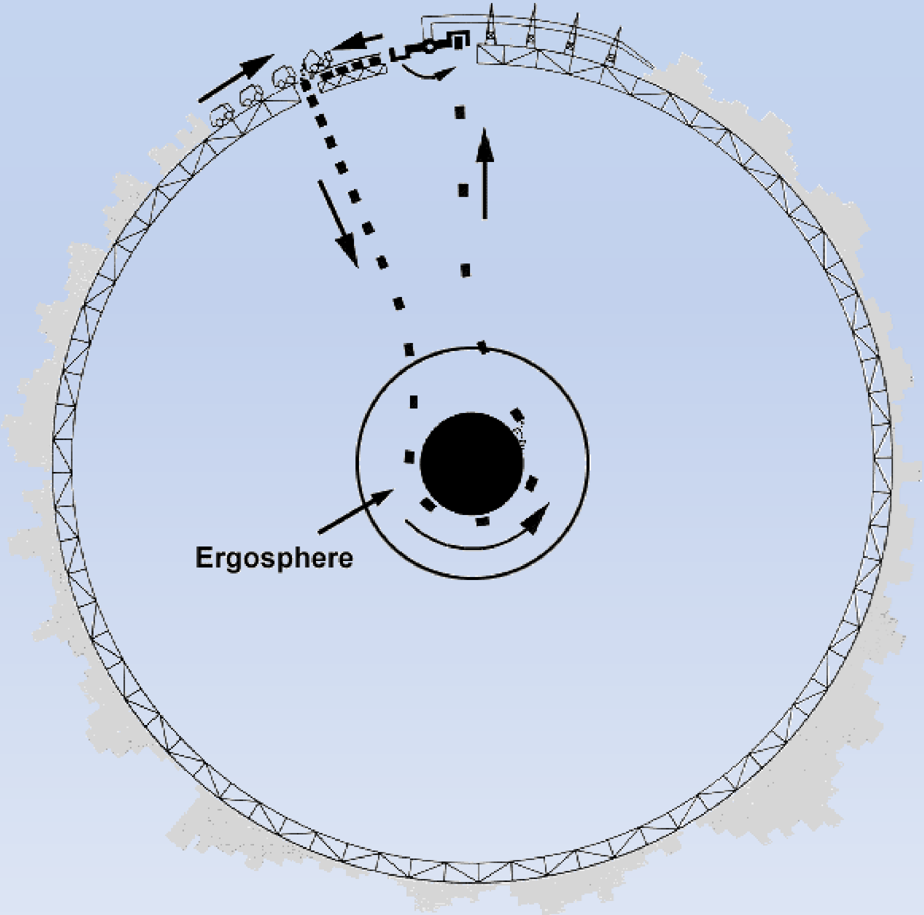


A region in which the spacetime is dragged significantly by the rotating black hole.

13.2 General relativistic picture

The Penrose process

- ✓ Roger Penrose: let's throw some garbage to a rotating black hole! (paraphased)
- ✓ It is possible for the infalling matter to be split into two parts. The outgoing part gets some energy from the rotation of the BH.



13.2 General relativistic picture

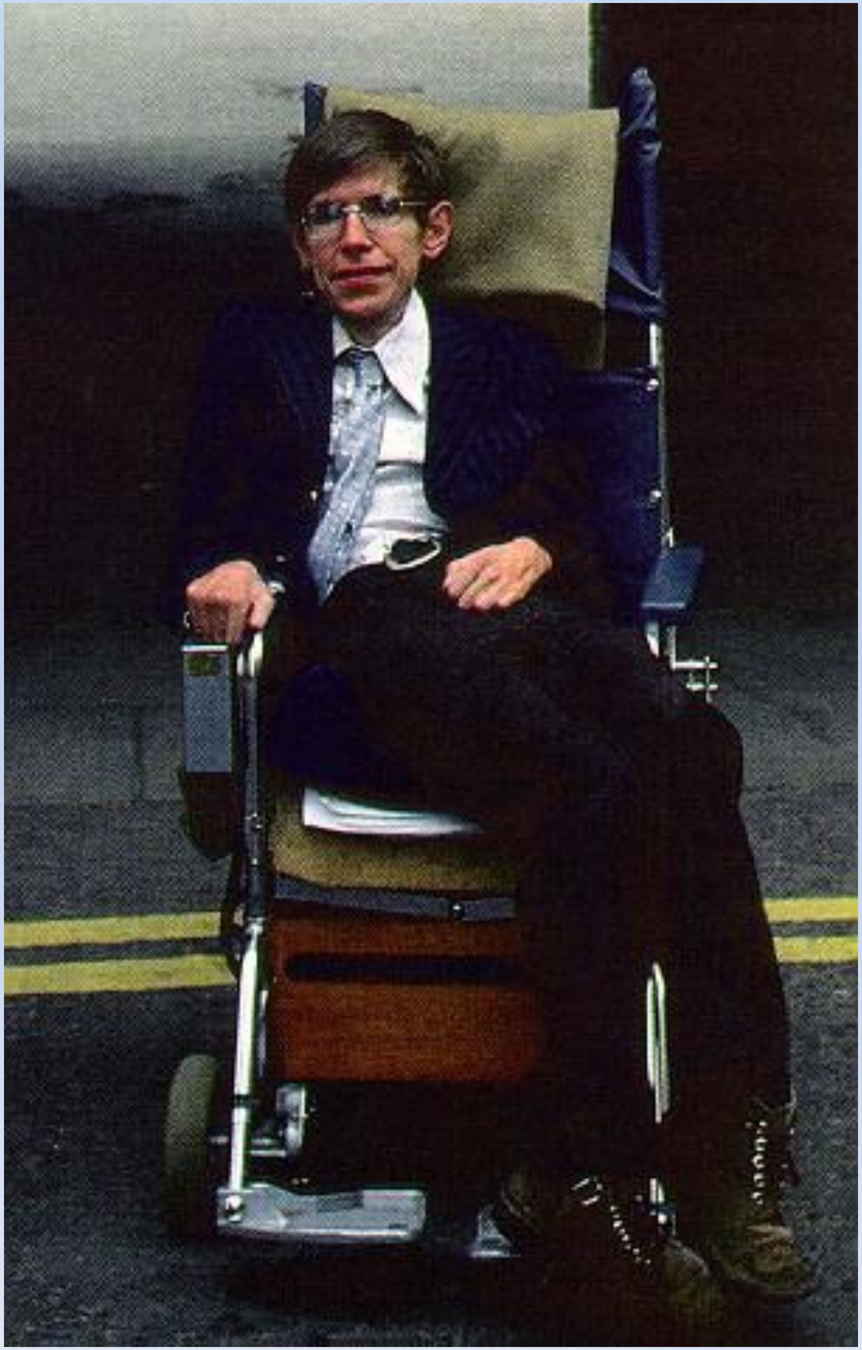
No hair theorem

- ✓ A black hole is completely characterized by 3 parameters: the *mass*, *angular momentum*, and *charge*.
- ✓ Mass is the *only* property of a Schwarzschild black hole
- ✓ Rotating black hole (Kerr black hole) with angular momentum
- ✓ a black hole with charge should NOT be common

13.2 General relativistic picture

- ✓ Significant warping of space-time
- ✓ everything swallowed by the singularity
- ✓ NO route from black holes to other parts of the universe
- ✓ Quantum effect becomes important at the singularity

Stephen Hawking
(1942 - 2018)



Whereas Stephen Hawking
has such a large investment in
General Relativity and Black
Holes and desires an insurance
policy, and whereas Kip Thorne likes
to live dangerously without an
insurance policy,

Therefore be it resolved that
Stephen Hawking bets 1 year's
subscription to "Penthouse" as against
Kip Thorne's wager of a 4-year
subscription to "Private Eye", that
Cygnus X1 does not contain a
black hole of mass above the
Chandrasekhar limit

Conceded
Stephen Hawking
June 1990

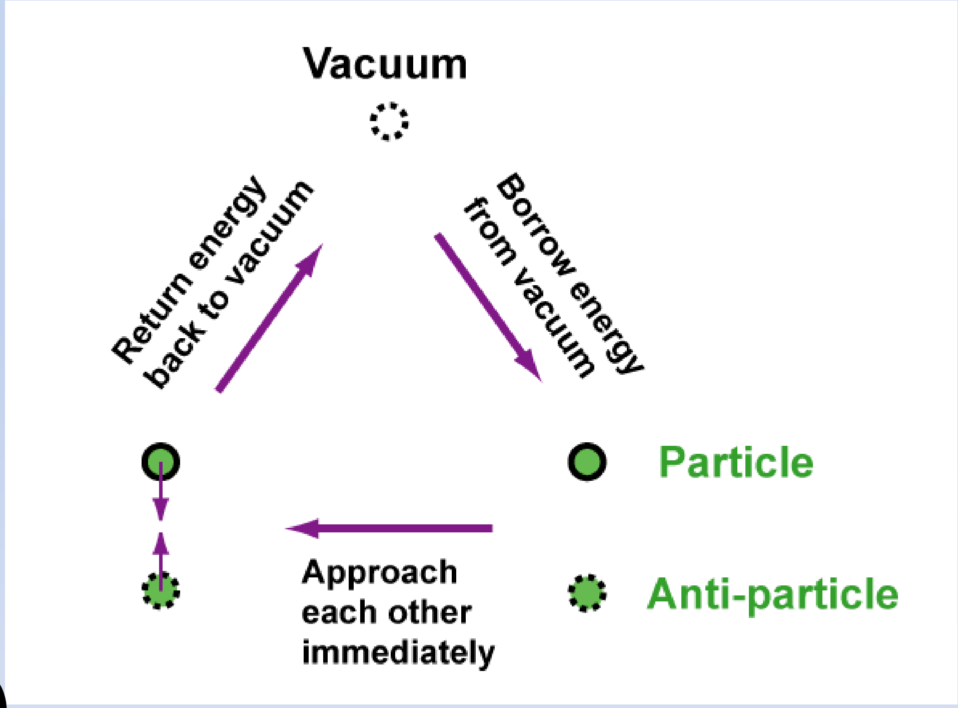
Stephen Hawking Kip S. Thorne

Witnessed this tenth
day of December 1974
Hrandman Anna Zytkow Werner Is

13.2 General relativistic picture

Hawking radiation

- ✓ Virtual particle pairs: Quantum mechanics allows energy to be "borrowed" from the vacuum, creating a pair of particles: "Particle-antiparticle pair"
- ✓ Under usual conditions, the pair can exist only temporarily (hence virtual). They will be together again and annihilate each other nearly immediately.

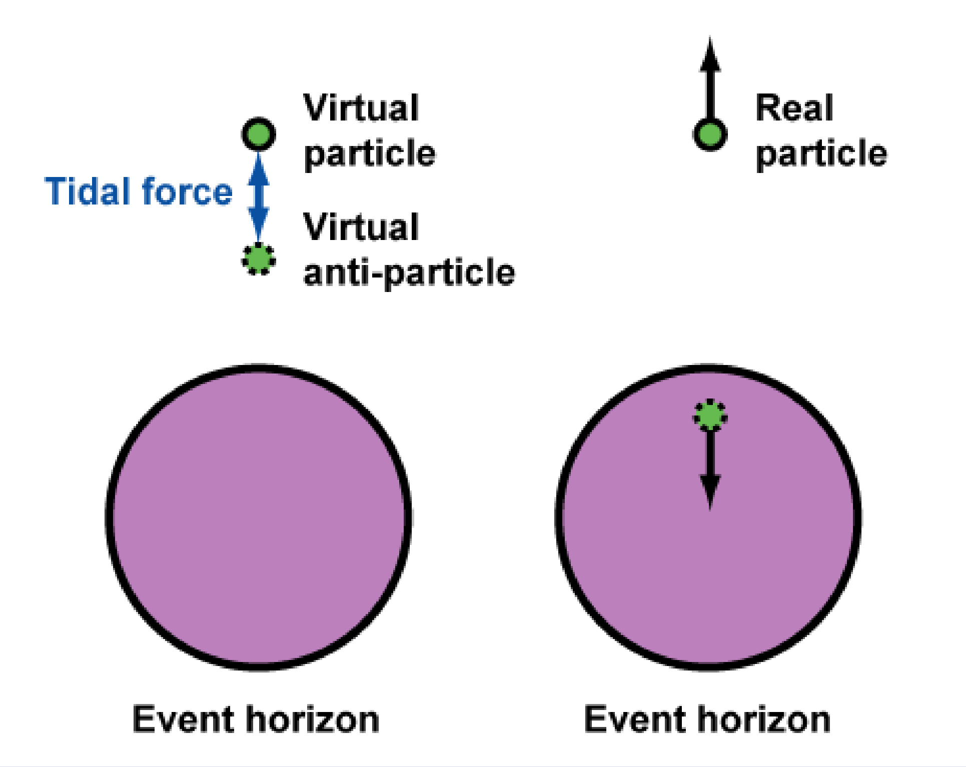


Example: Positron (正电子), which carries positive charge, is the antiparticle of electron.

13.2 General relativistic picture

Hawking radiation

- ✓ Strong tidal force → a virtual pair becomes a real pair
- ✓ One particle falls into the black hole.
- ✓ The other escapes
- ✓ Hawking radiation: emitting all kinds of particles



13.3 The search for black holes

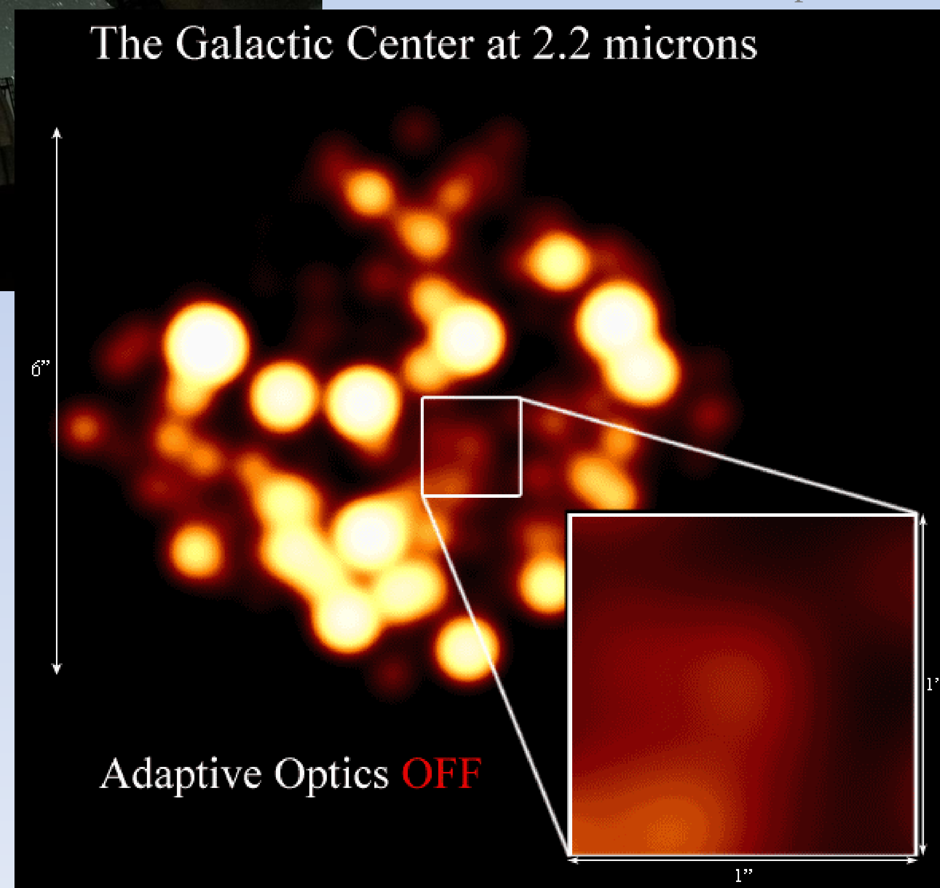
- ✓ Given the velocity and mass of the matter rotating around a region
- ✓ the mass of central object can be deduced
- ✓ Large mass in small volume, may be a black hole

13.3 The search for black holes

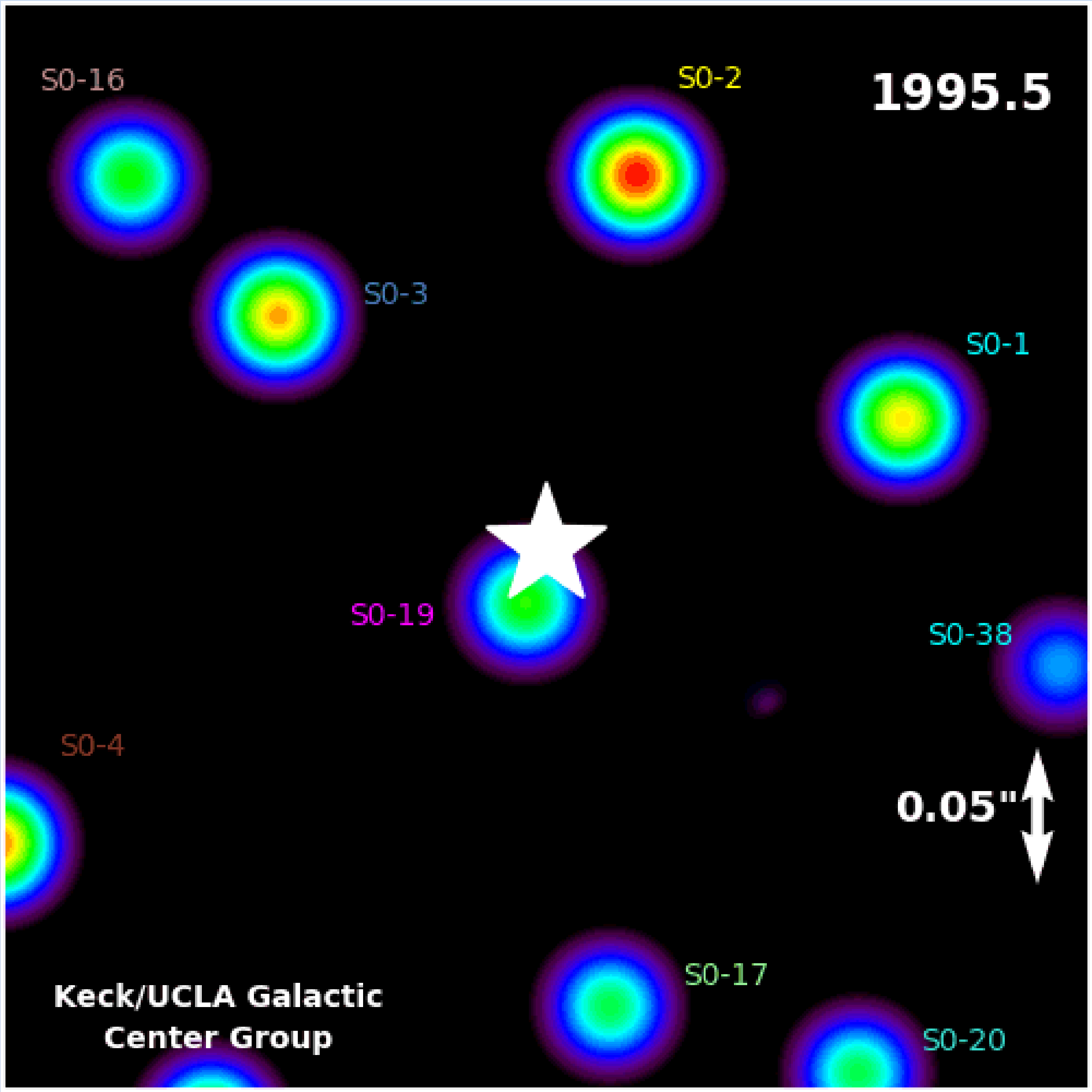
- ✓ Sgr A*: a compact radio source at the centre of the Milky Way Galaxy
- ✓ Over 3 million solar masses in a small region
- ✓ Believe to host a supermassive black hole
- ✓ How do we know?

Image: Ethan Tweedie

Image: UCLA Galactic Center Group



observing the galactic centre by using adaptive optics



Two stars have completed at least 1 orbit in the last 2 decades

13.3 The search for black holes

- ✓ Within the next few years, the region is expected to be resolved in sub-mm wavelength by using VLBI (Very Long Baseline Interferometry)

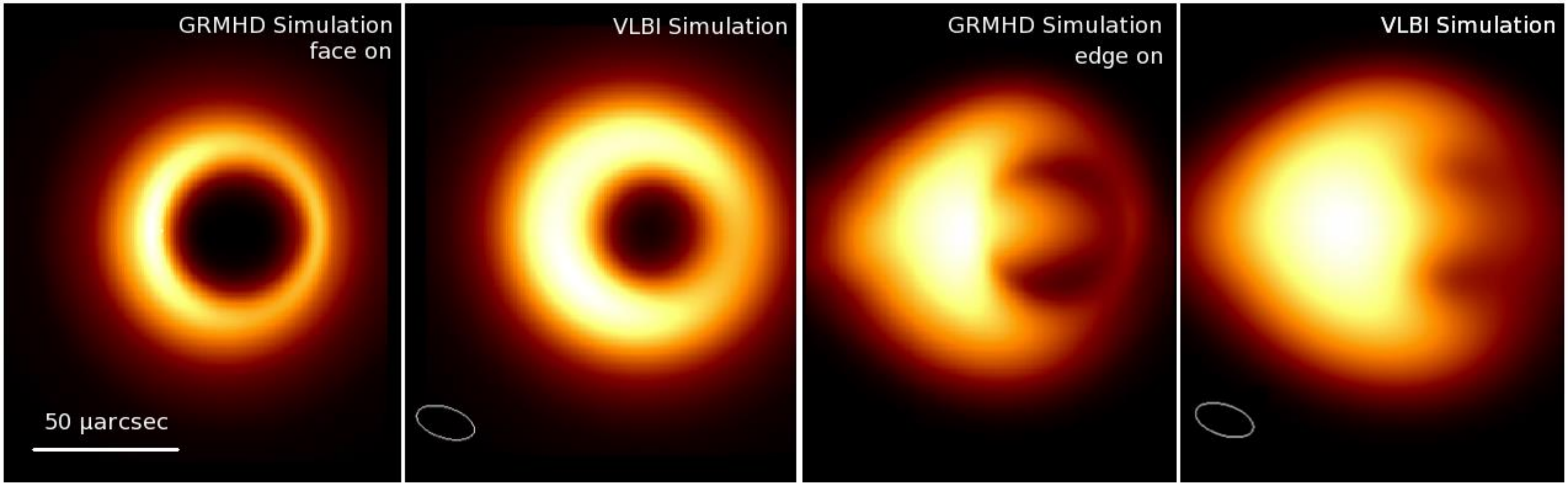
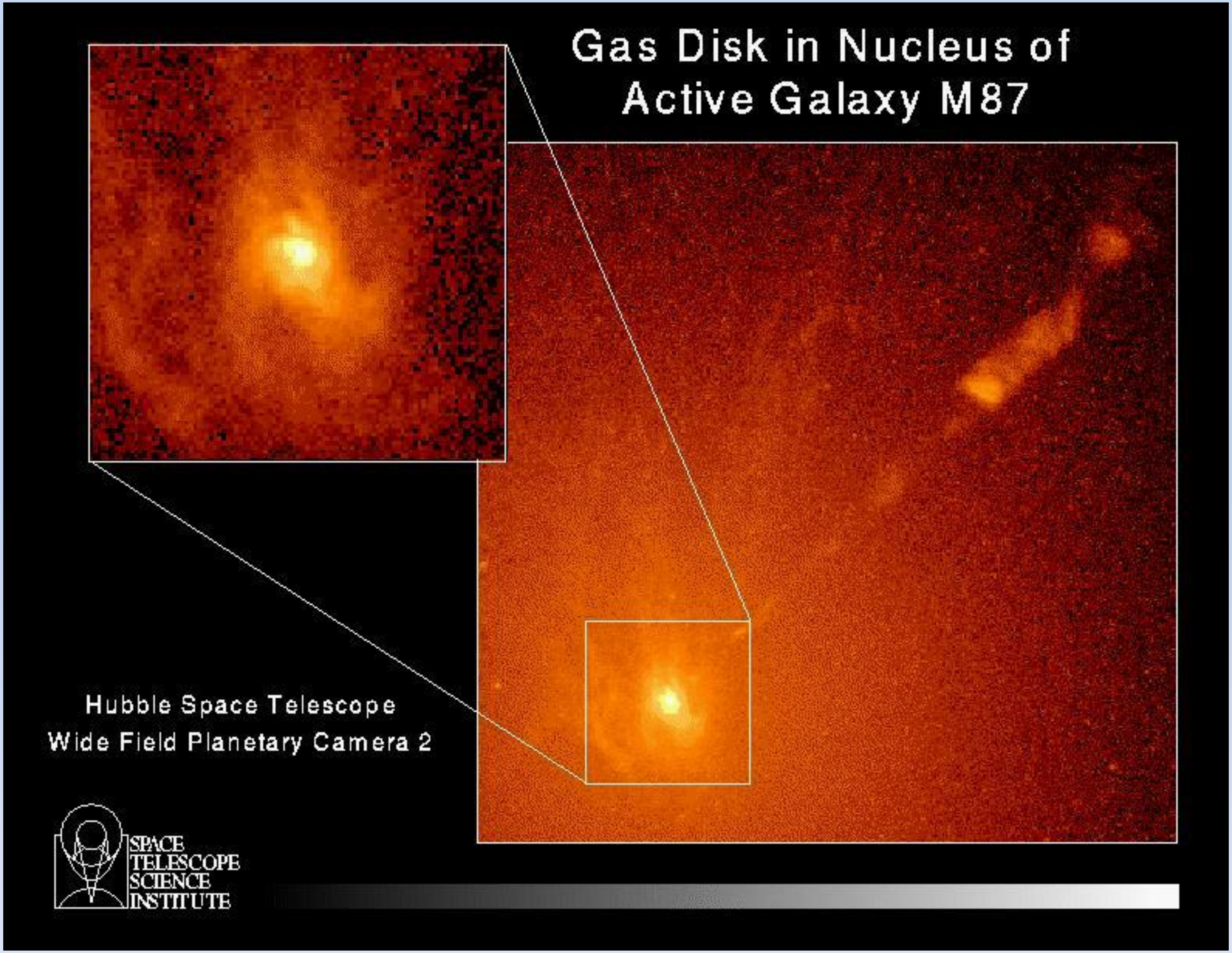
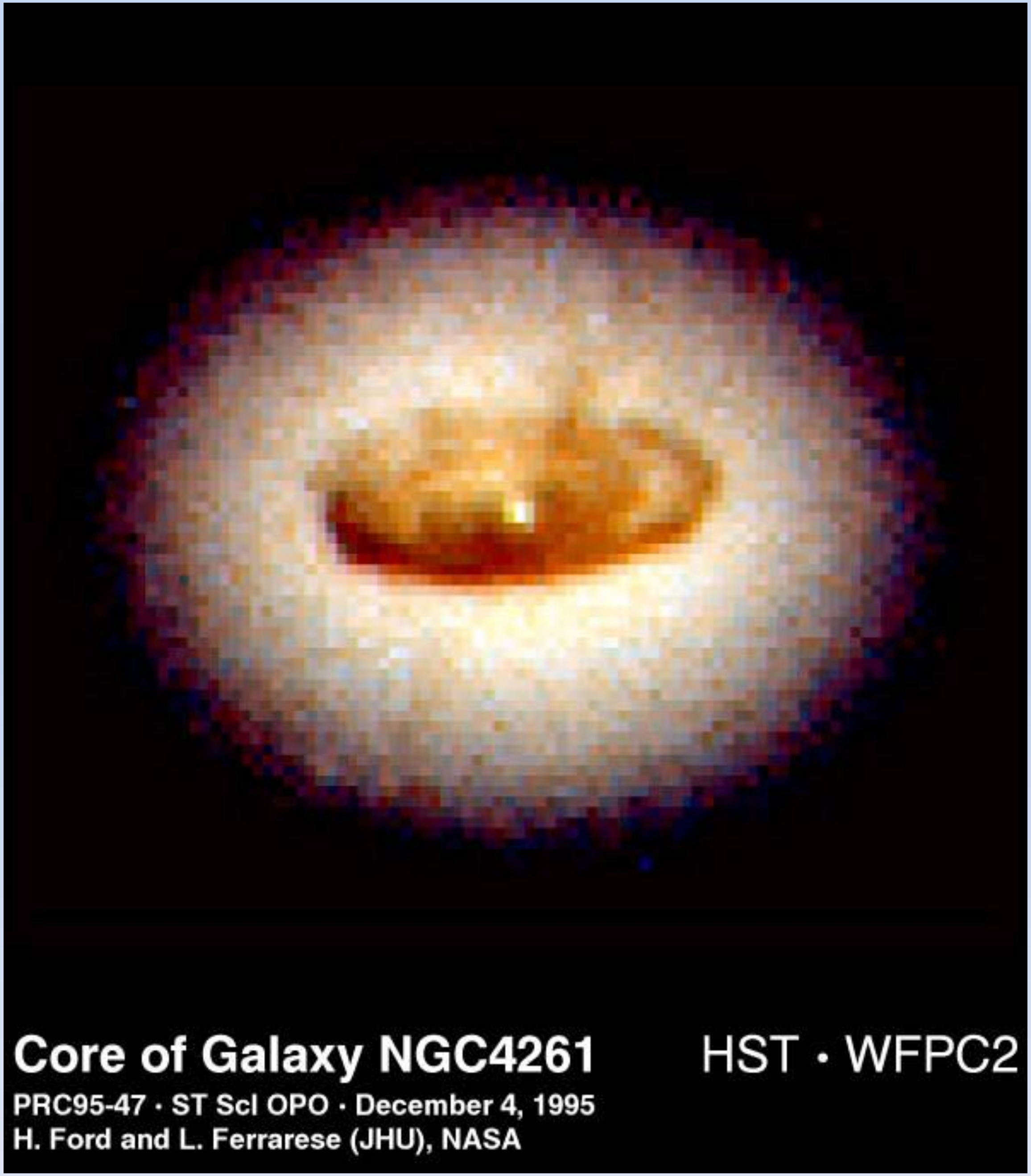


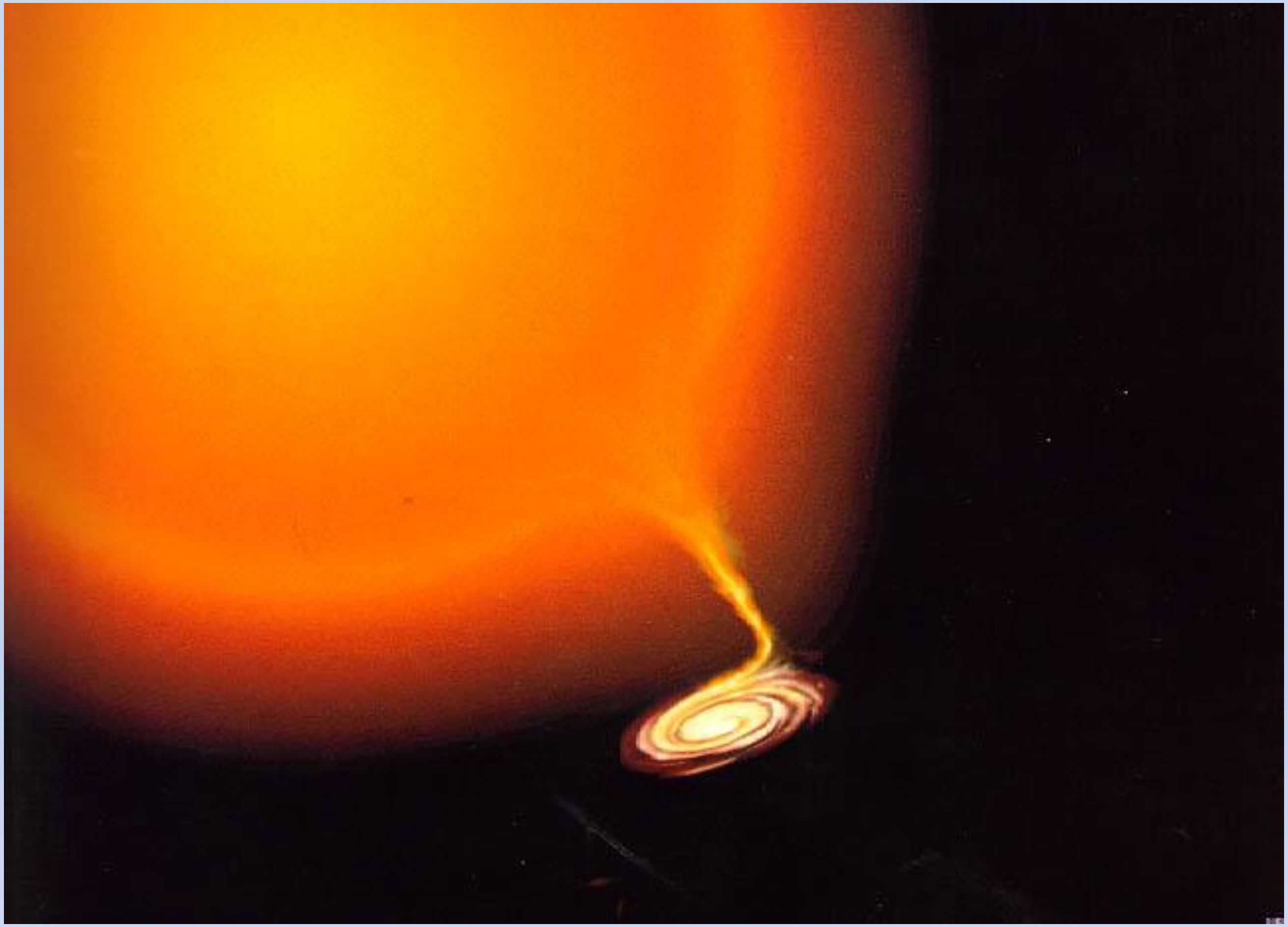
Image: Moschibrodzka et al 2009

13.3 The search for black holes

- M87 at Virgo cluster: Super-massive black holes may be at the centre of galaxies
- Gas rotating at ~ 500 km/s
- 3 billions $M_{\odot}$ confined in a space no larger than our solar system
- 2nd largest black hole in terms of angular size on the sky
- may be black hole power source at center?

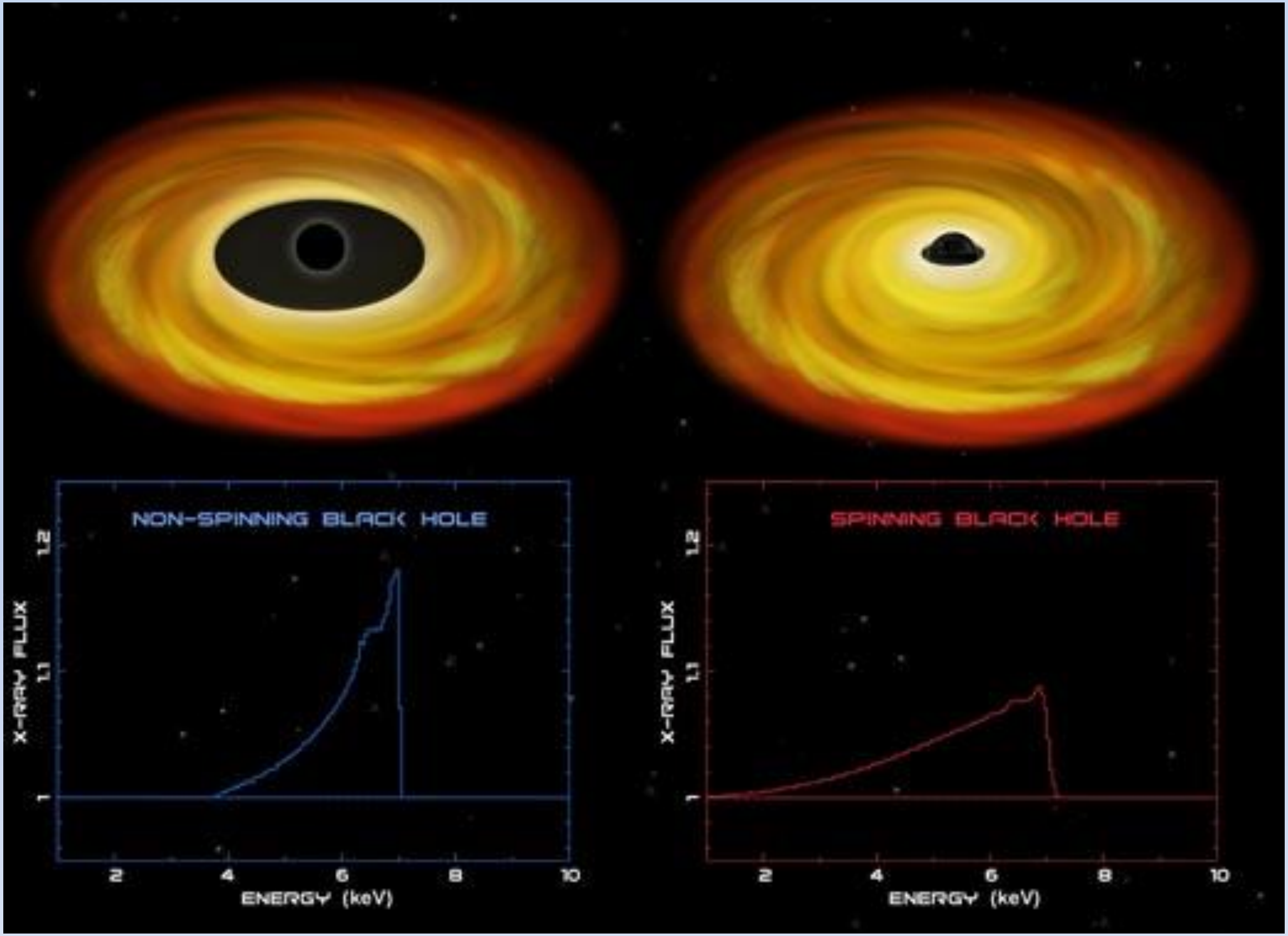

Gas Disk in Nucleus of
Active Galaxy M87
Hubble Space Telescope
Wide Field Planetary Camera 2
SPACE
TELESCOPE
SCIENCE
INSTITUTE


Core of Galaxy NGC4261
HST · WFPC2
PRC95-47 · ST ScI OPO · December 4, 1995
H. Ford and L. Ferrarese (JHU), NASA



- ✓ For stellar-mass black hole, we can observe close binary systems,
- ✓ X-rays emitted from the accreted matter
- ✓ Mass of the dark companion can be estimated

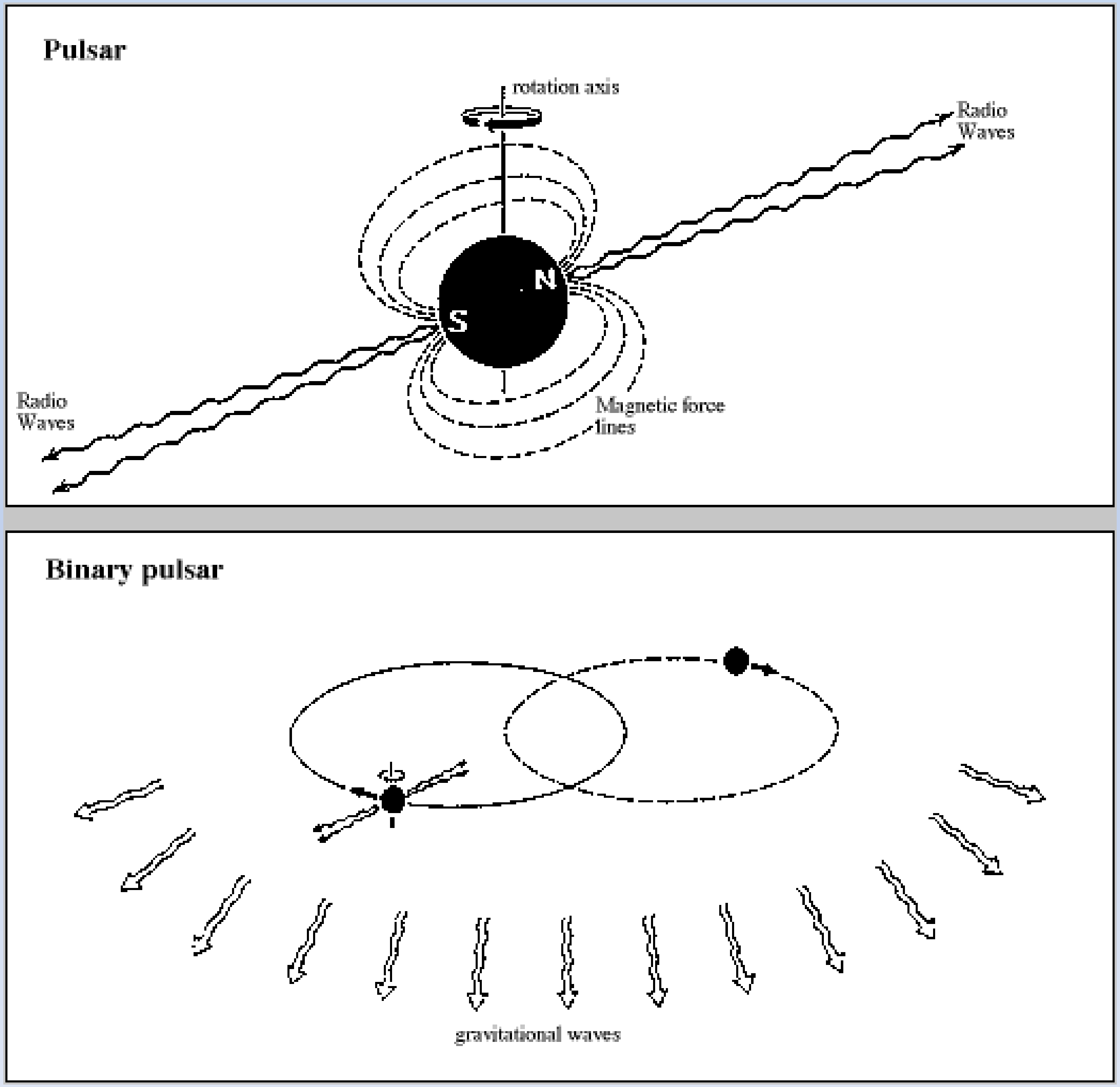
Image: NASA/CXC/M.Weiss/SAO/J.Miller et al.



The X-ray spectrum could also allow us to constrains the parameters of the black hole

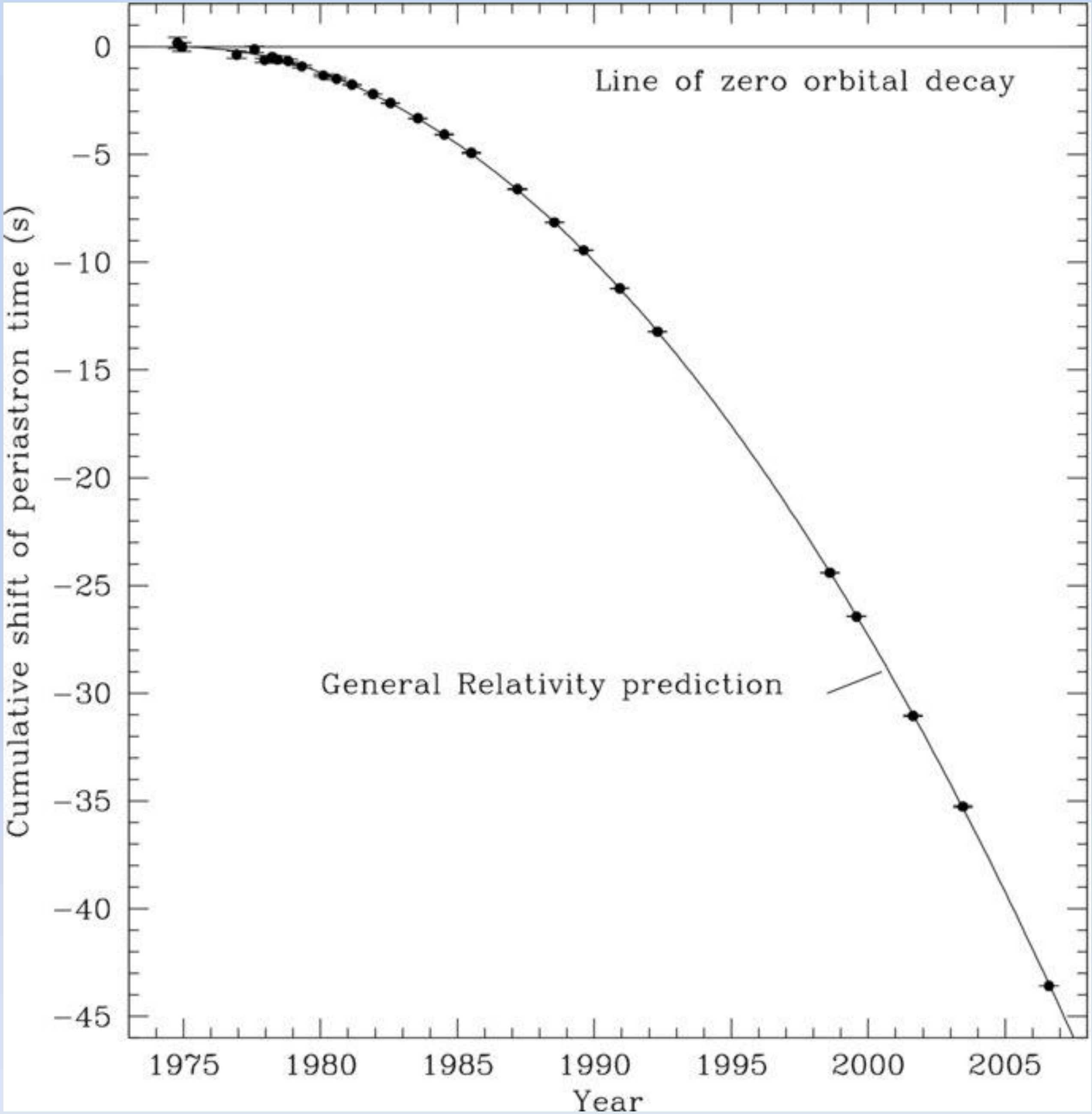
13.4 Testing General Relativity in the strong-field regime

- ✓ Almost all tests of GR are in the weak-field limit.
- ✓ Is GR correct in location of extremely strong gravity?
- ✓ Space-time distortion is significant around black holes, and to a lower extent, neutron stars. They allow us to test whether GR still works in the strong field limit.
- ✓ It all changes due to the discovery of a binary pulsar system B1913+16.



The orbit shrinks slowly over time.

Image: nobelprize.org

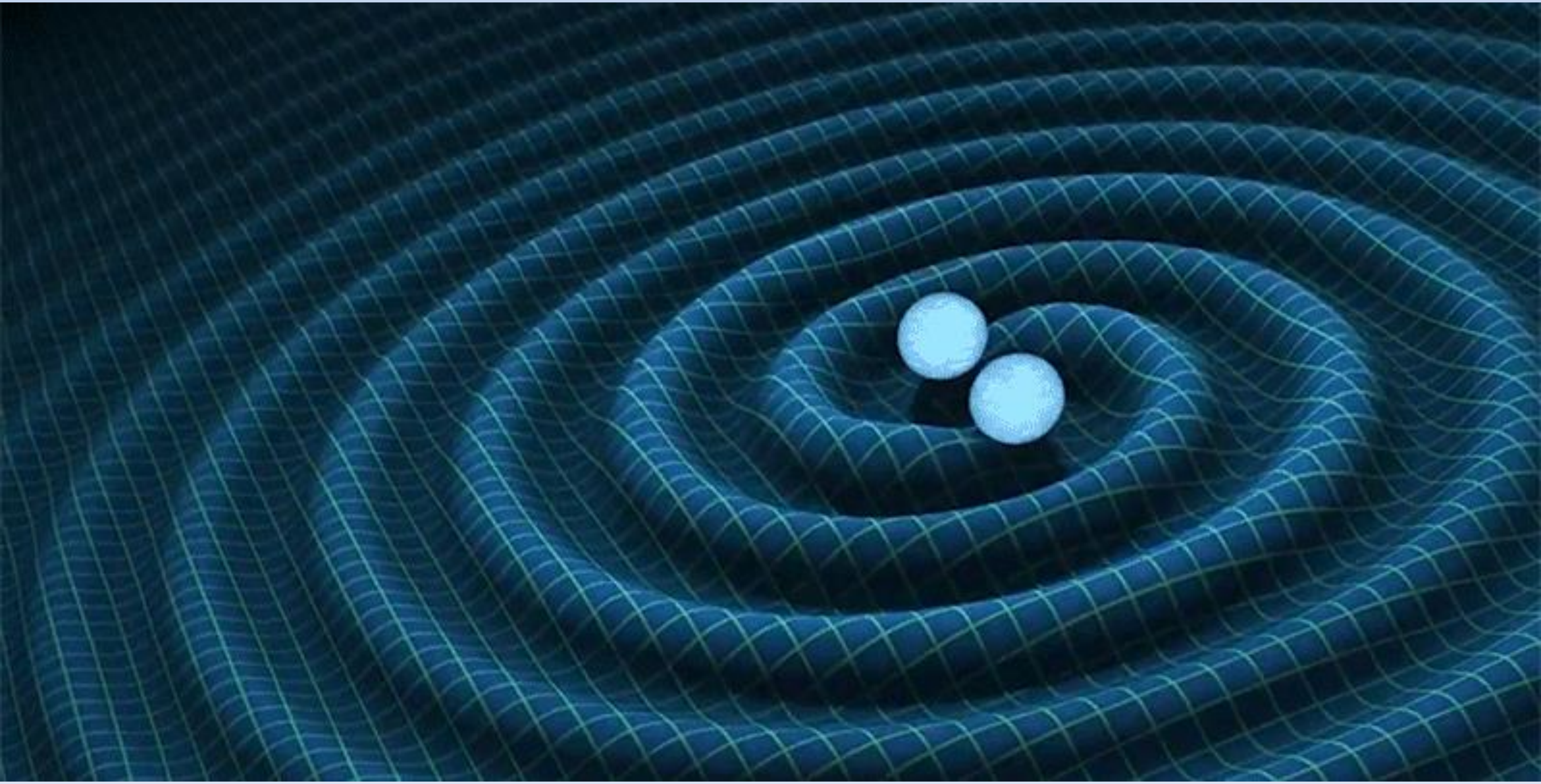


Observation of the orbital decay of the binary pulsar provides strong evidence for the existence of gravitational wave.

Credit: Weisberg, et al., 2010

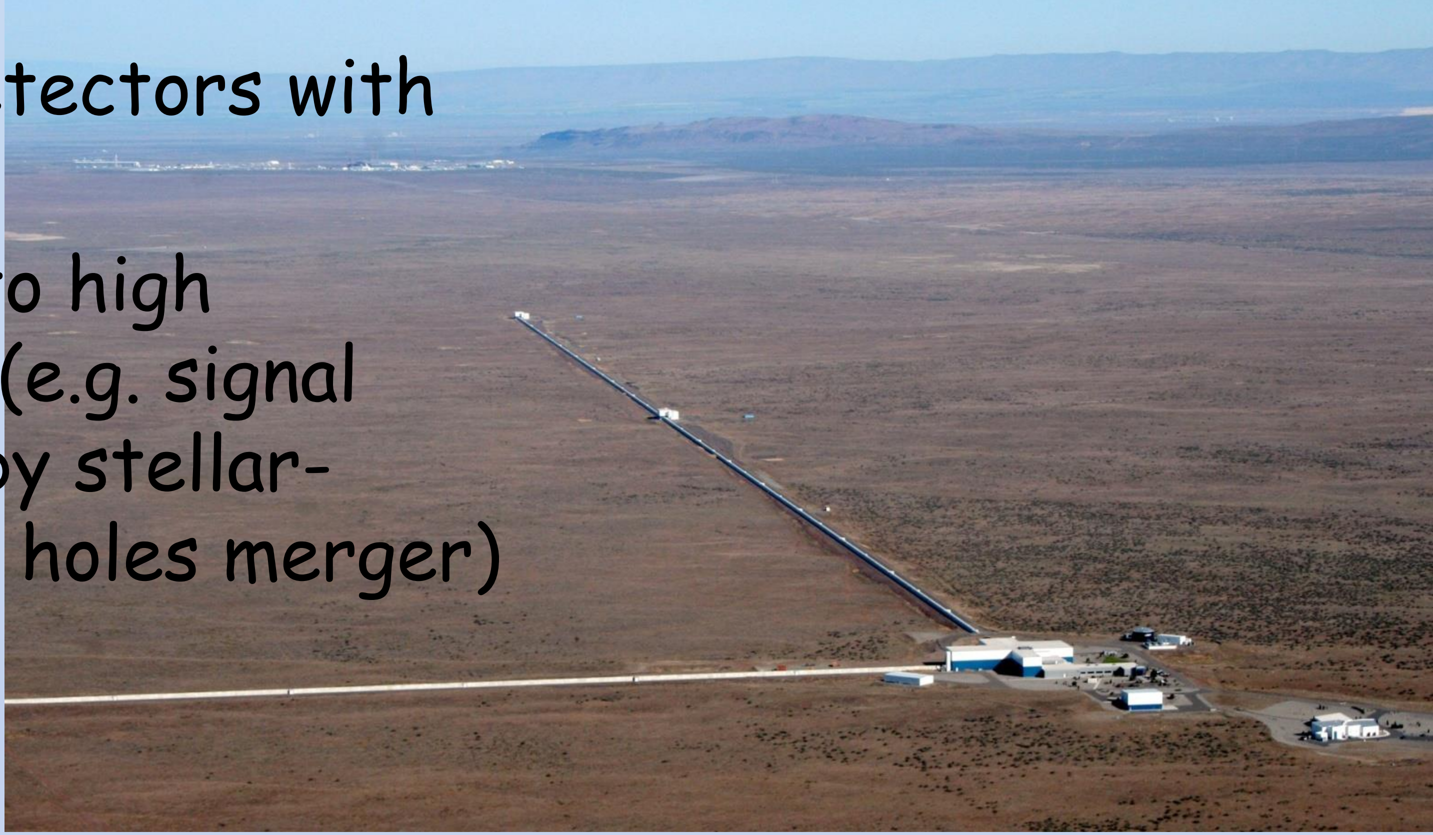
13.4 Testing General Relativity in the strong-field regime

Image credit: LIGO



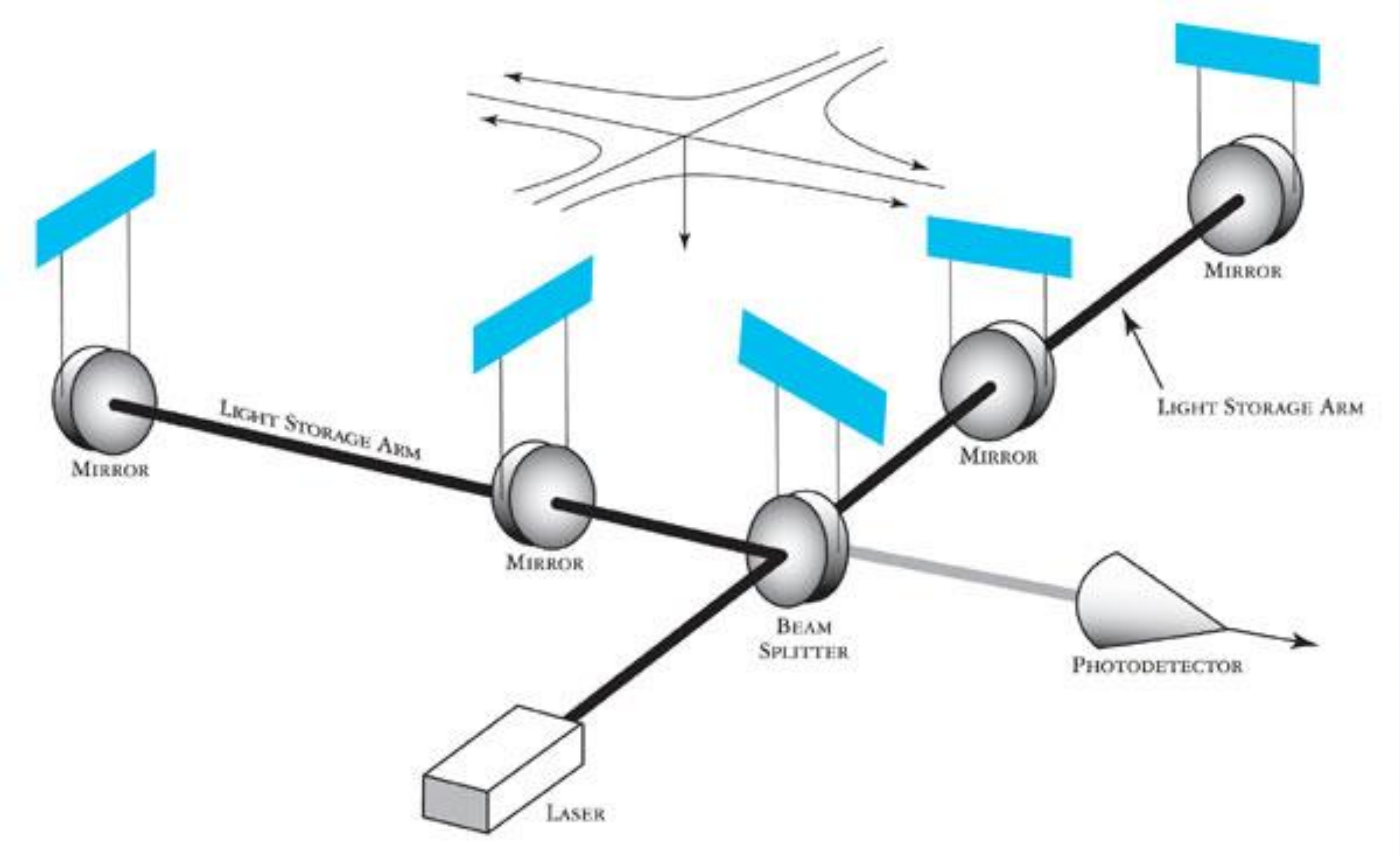
Gravitational wave: the ripple of the space-time

- ✓ L-shape detectors with 4-km arms
- ✓ Sensitive to high frequency (e.g. signal produced by stellar-mass black holes merger)



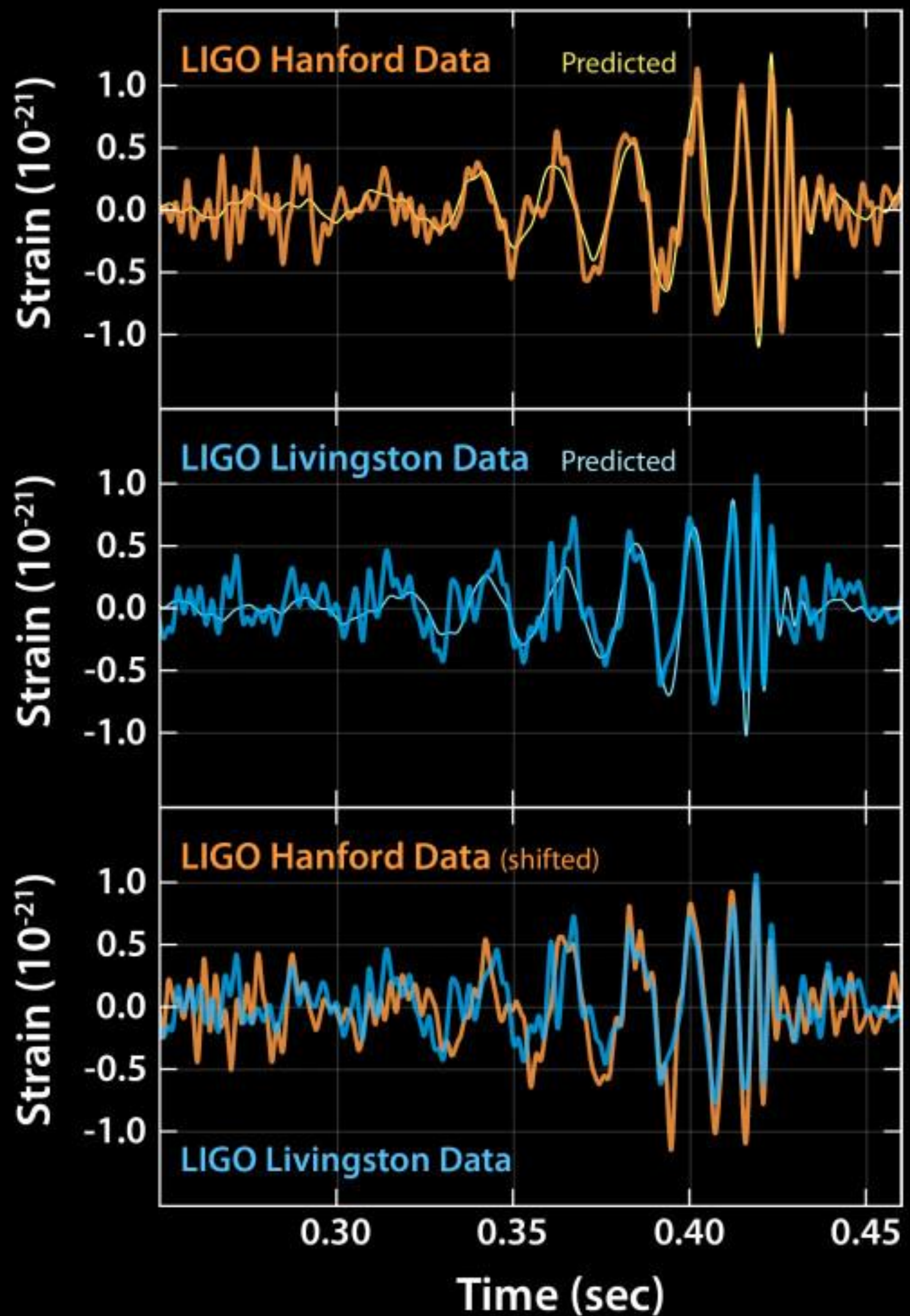
LIGO at Hanford

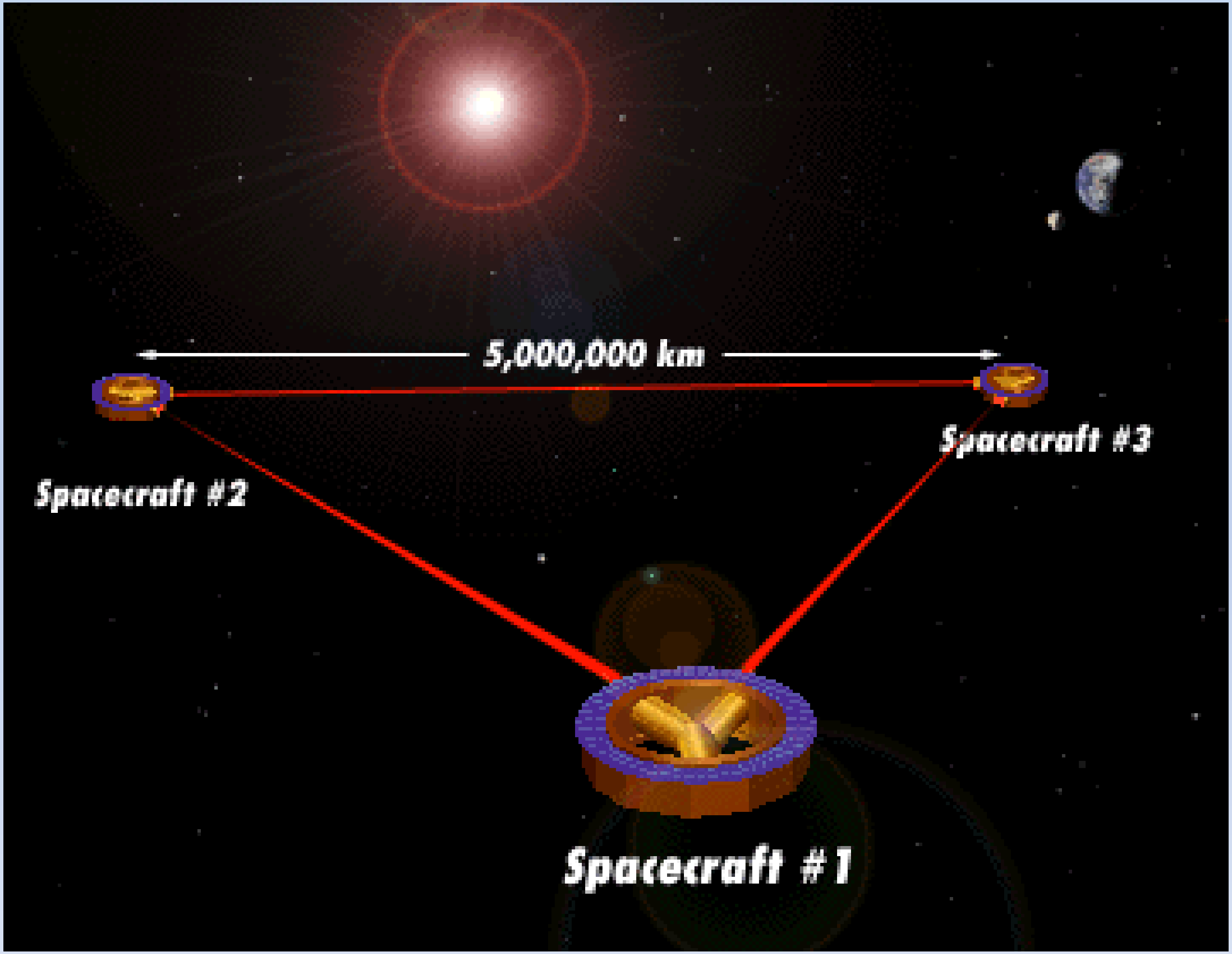
LIGO at Livingston



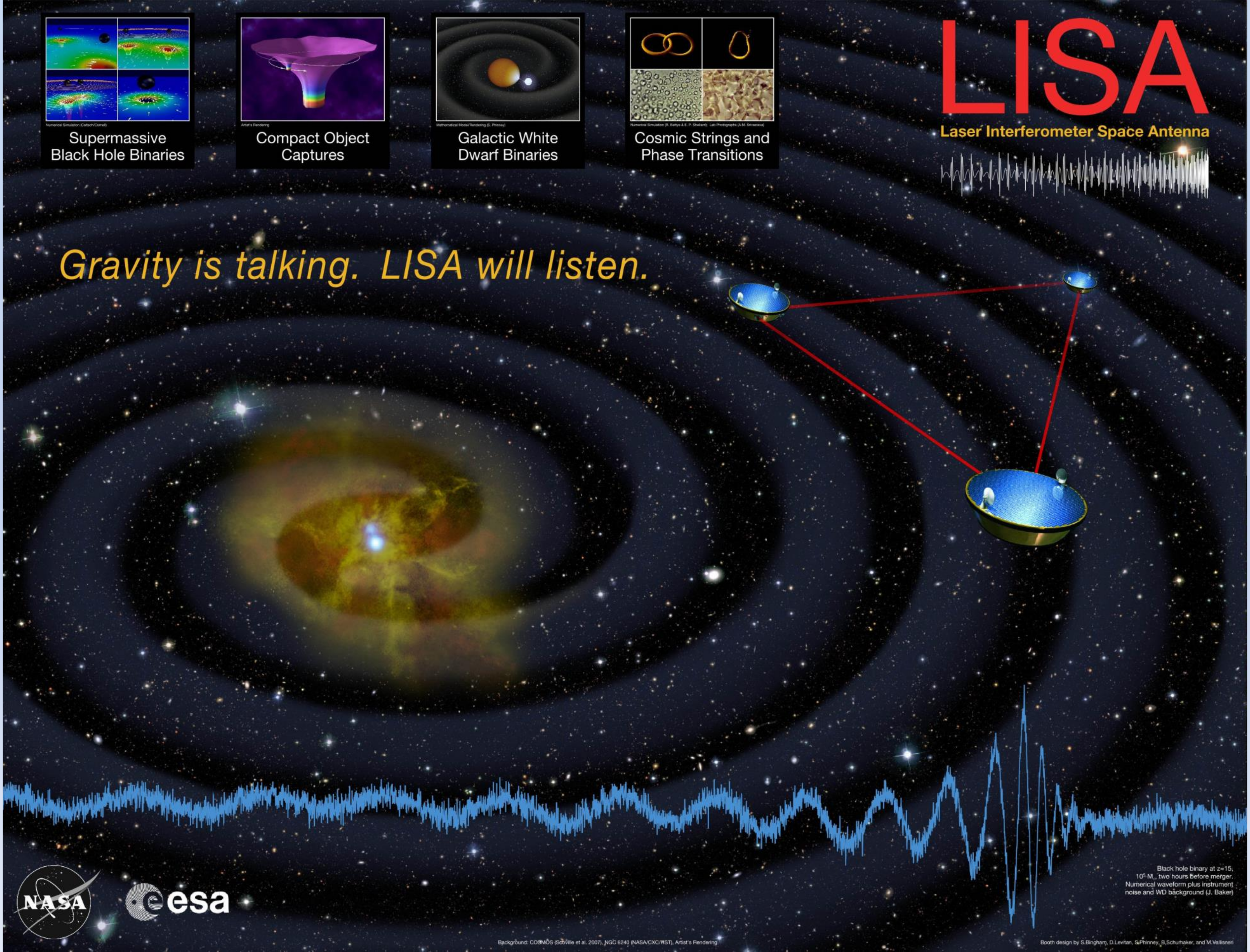
LIGO's design

- ✓ First direct detection by LIGO (the Laser Interferometer Gravitational-Wave Observatory) in 2015, announced in 2016/2.
- ✓ The GW was emitted when two black holes merged 1.3 billion years ago.
- ✓ GW from neutron stars merger has since been detected.

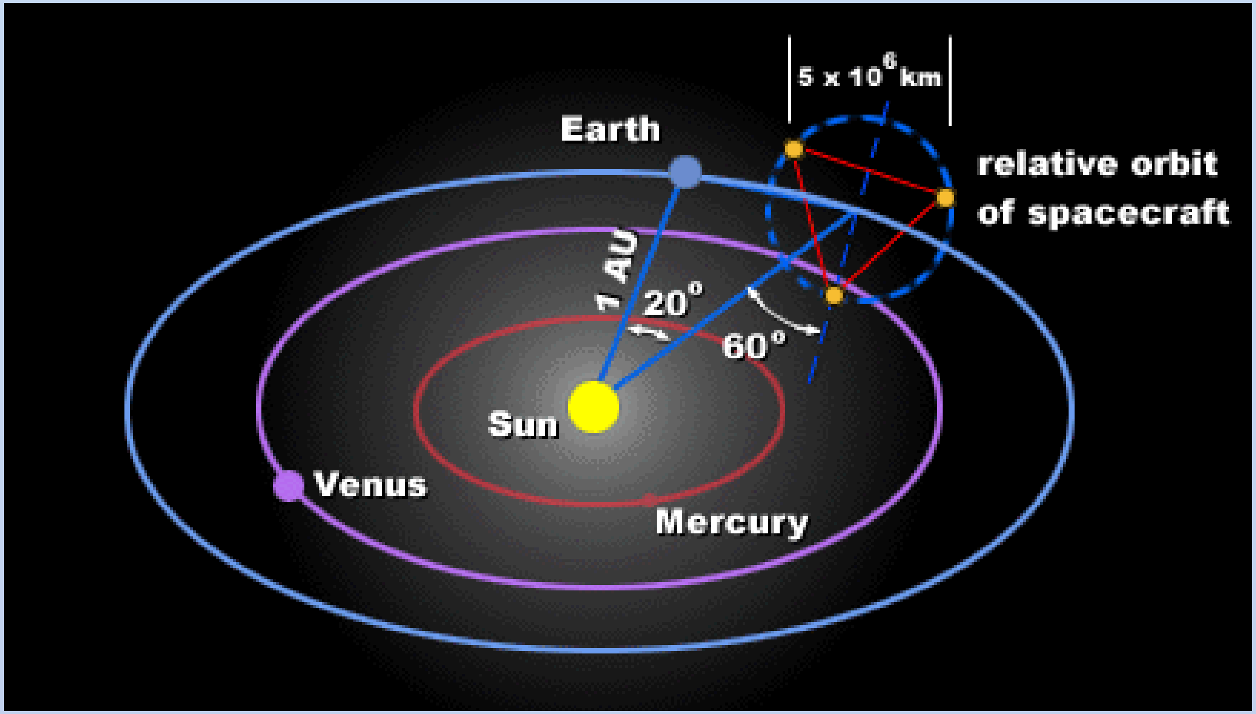


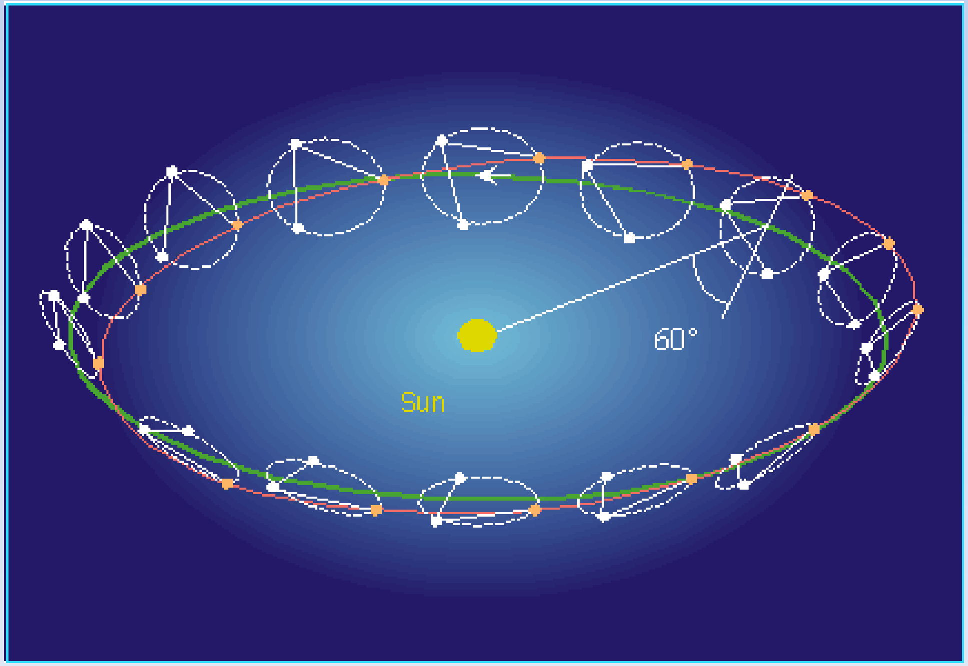


LISA: the Laser Interferometer Space Antenna



Supermassive Black Hole Binaries
Compact Object Captures
Galactic White Dwarf Binaries
Cosmic Strings and Phase Transitions
LISA
Laser Interferometer Space Antenna
Gravity is talking. LISA will listen.
Black hole binary at z=15,
10⁵ M, two hours before merger.
Numerical waveform plus instrument
noise and WD background (J. Baker)
NASA
esa
Background: COSMOS (Scoville et al. 2007), NGC 6240 (NASA/CXC/HST), Artist's Rendering
Booth design by S.Bingham, D.Levitan, S.Phinney, B.Schumaker, and M.Vallisneri


5 x 10^6 km
Earth
relative orbit
of spacecraft
1 AU
20°
60°
Sun
Venus
Mercury


60°
Sun

13.4 Testing General Relativity in the strong-field regime

- ✓ Another way to test GR at strong gravity region is to observe the accretion of matter
- ✓ The source of matter could be a nearby star, …

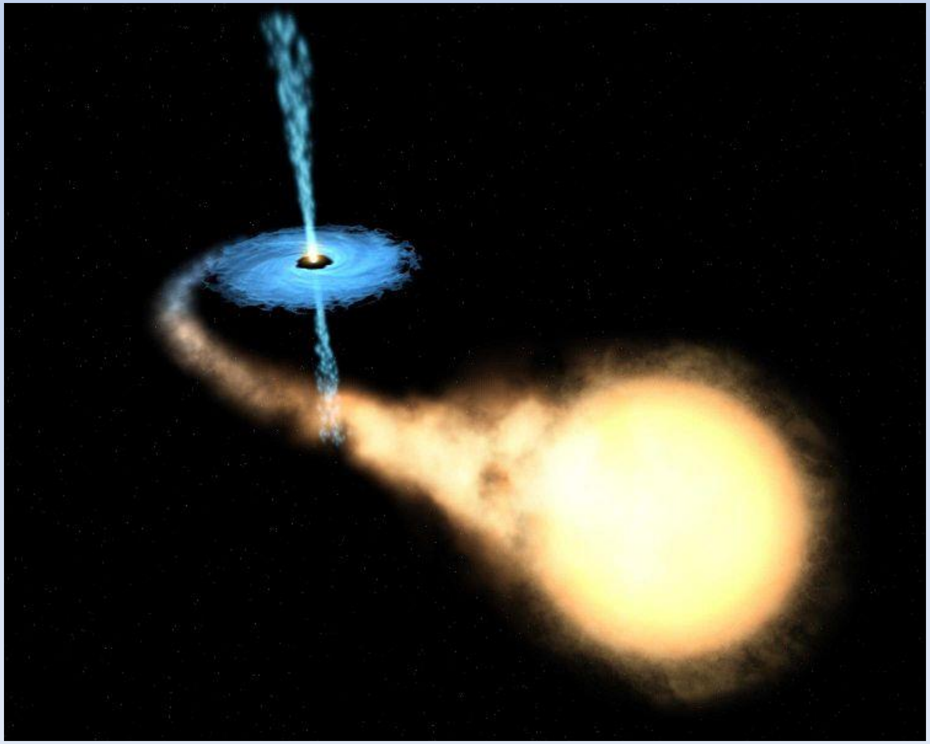
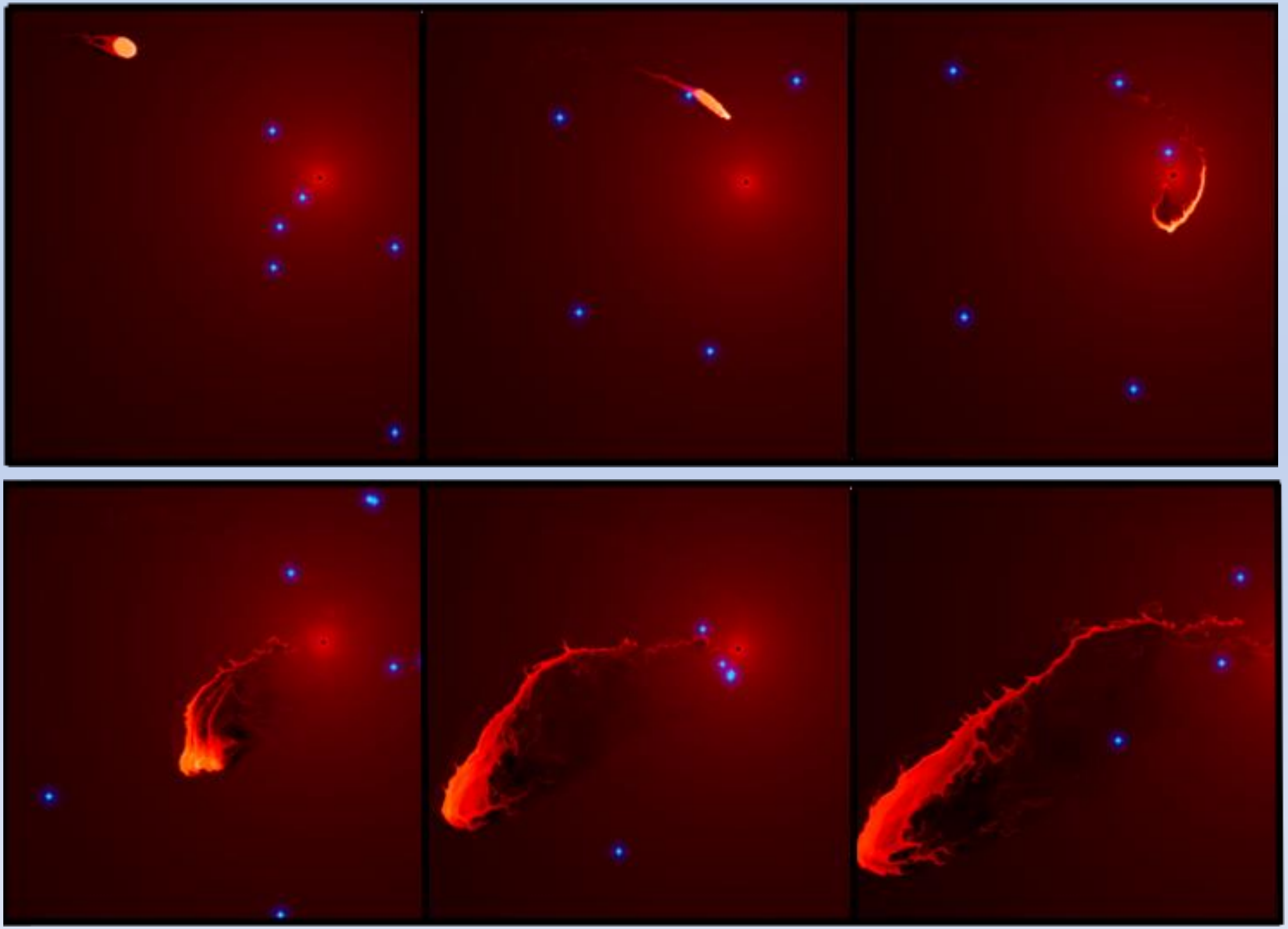


Image: NASA/STScl/ESA

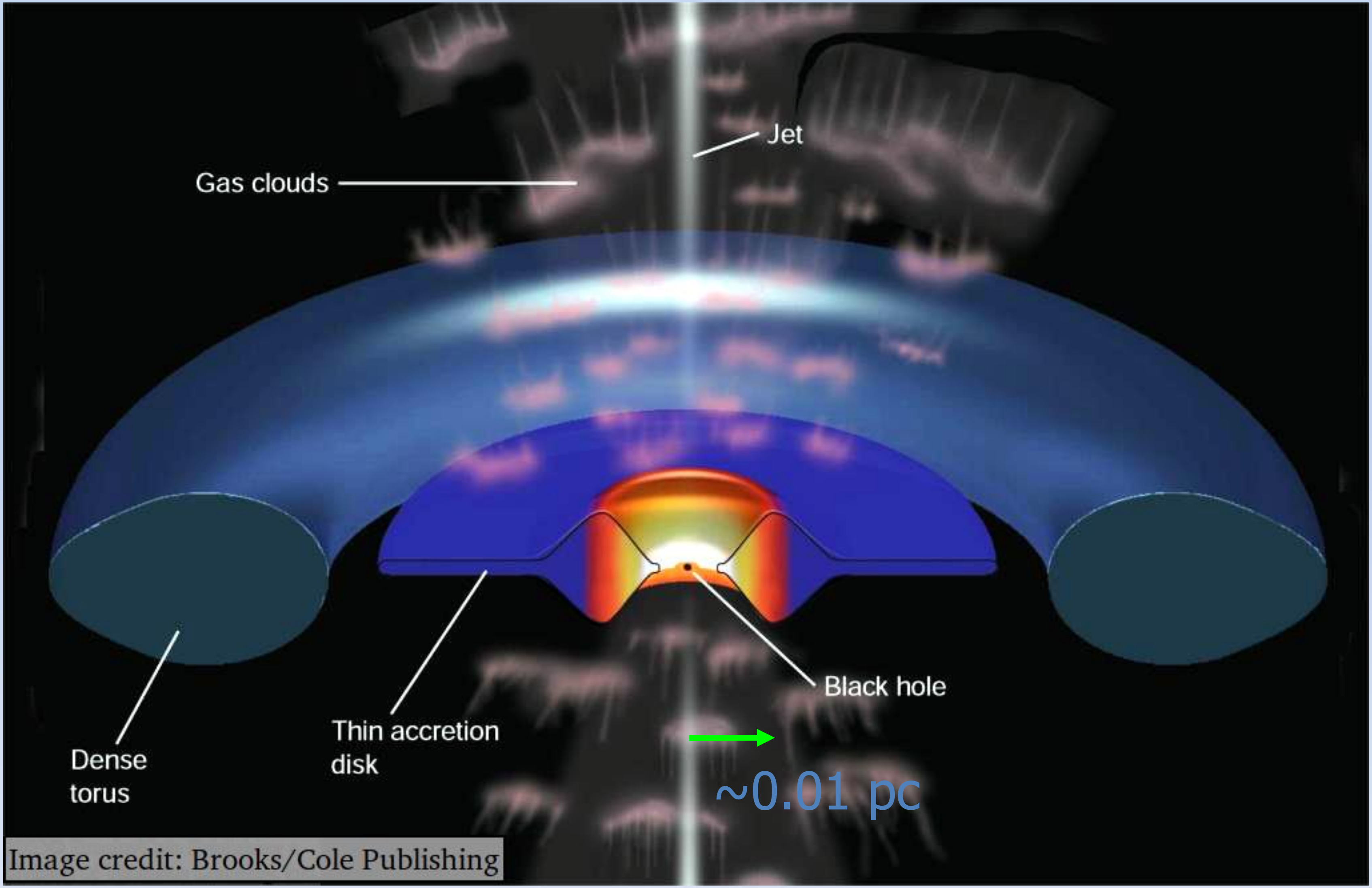
✓ … or an unfortunate clump of gas

Credit: ESO/MPE/M.Schartmann/L. Calçada



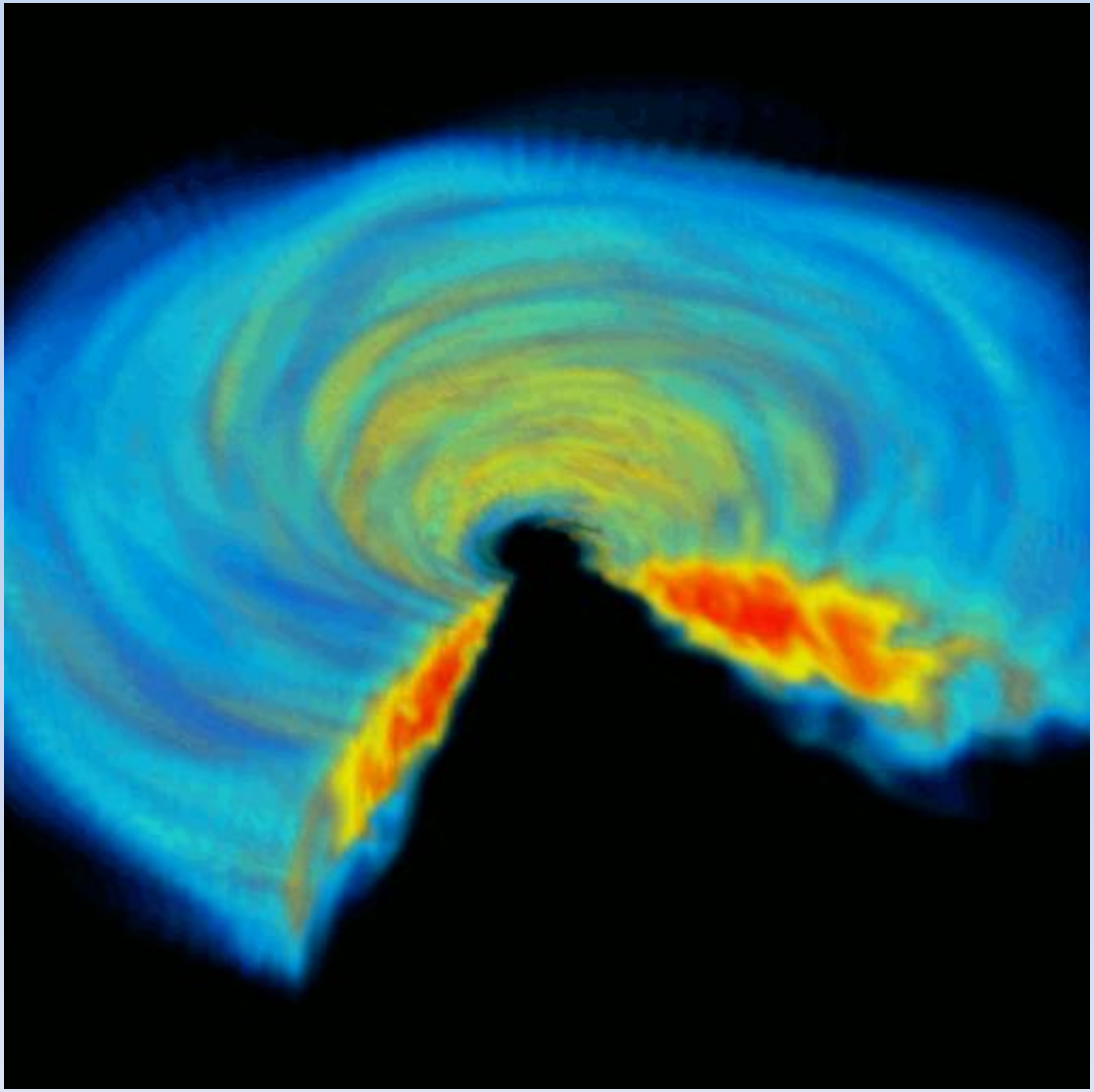
Simulation of the disruption of G2 gas cloud near to Sgr A*

✓ Accretion disk is formed around the central object. (e.g. a supermassive black hole)



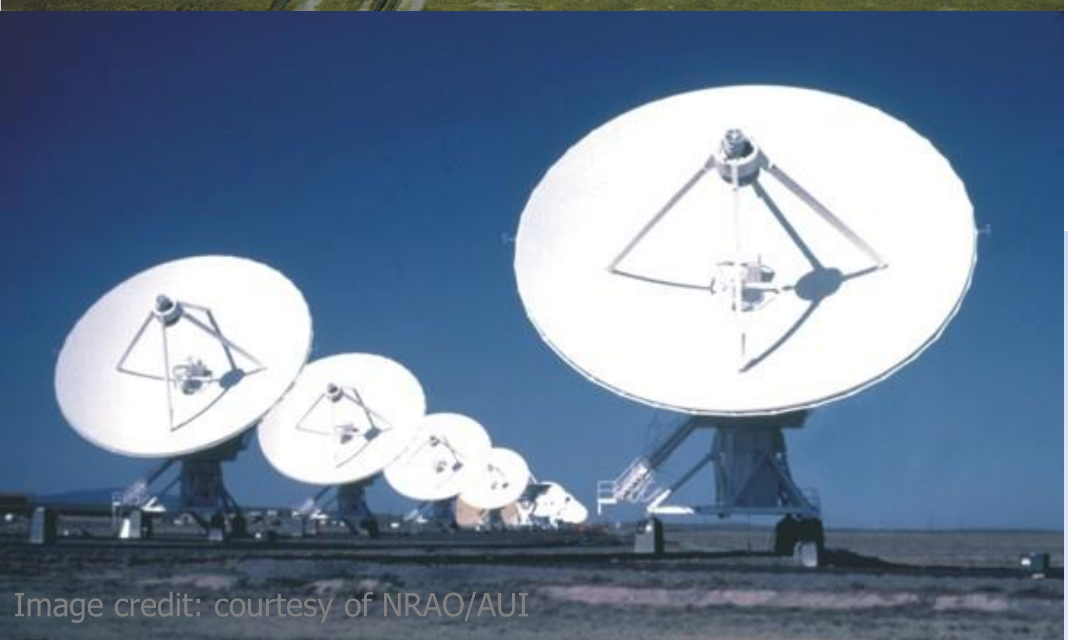
(not to scale) ~1000 pc

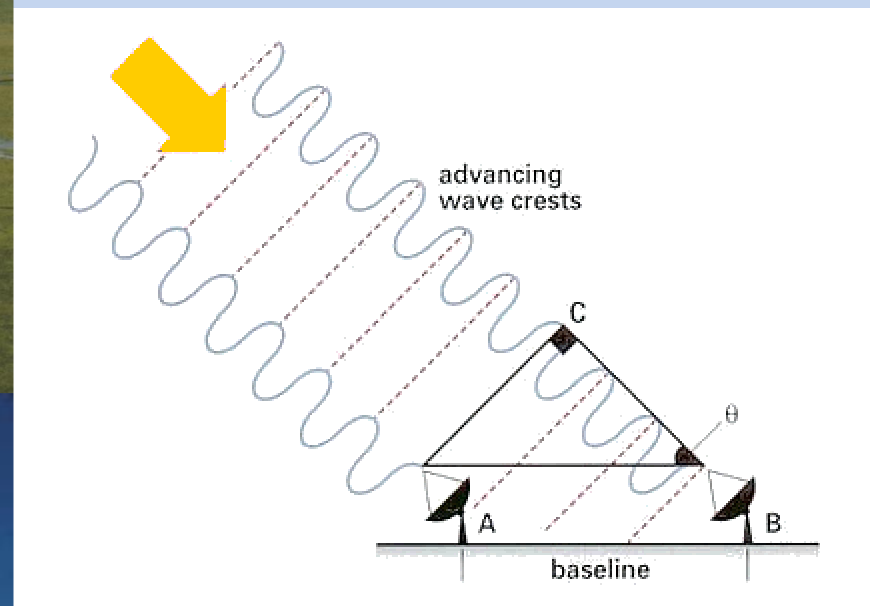
- ✓ We can exam GR at strong gravity region by comparing observation with simulation.



Credit: John Hawley

Interferometry (干涉测量)





Greatly enhance the angular resolution

Very Long Baseline Interferometry (VLBI)

The Event Horizon Telescope (EHT) is expected to resolve the event horizon of the galactic centre black hole.

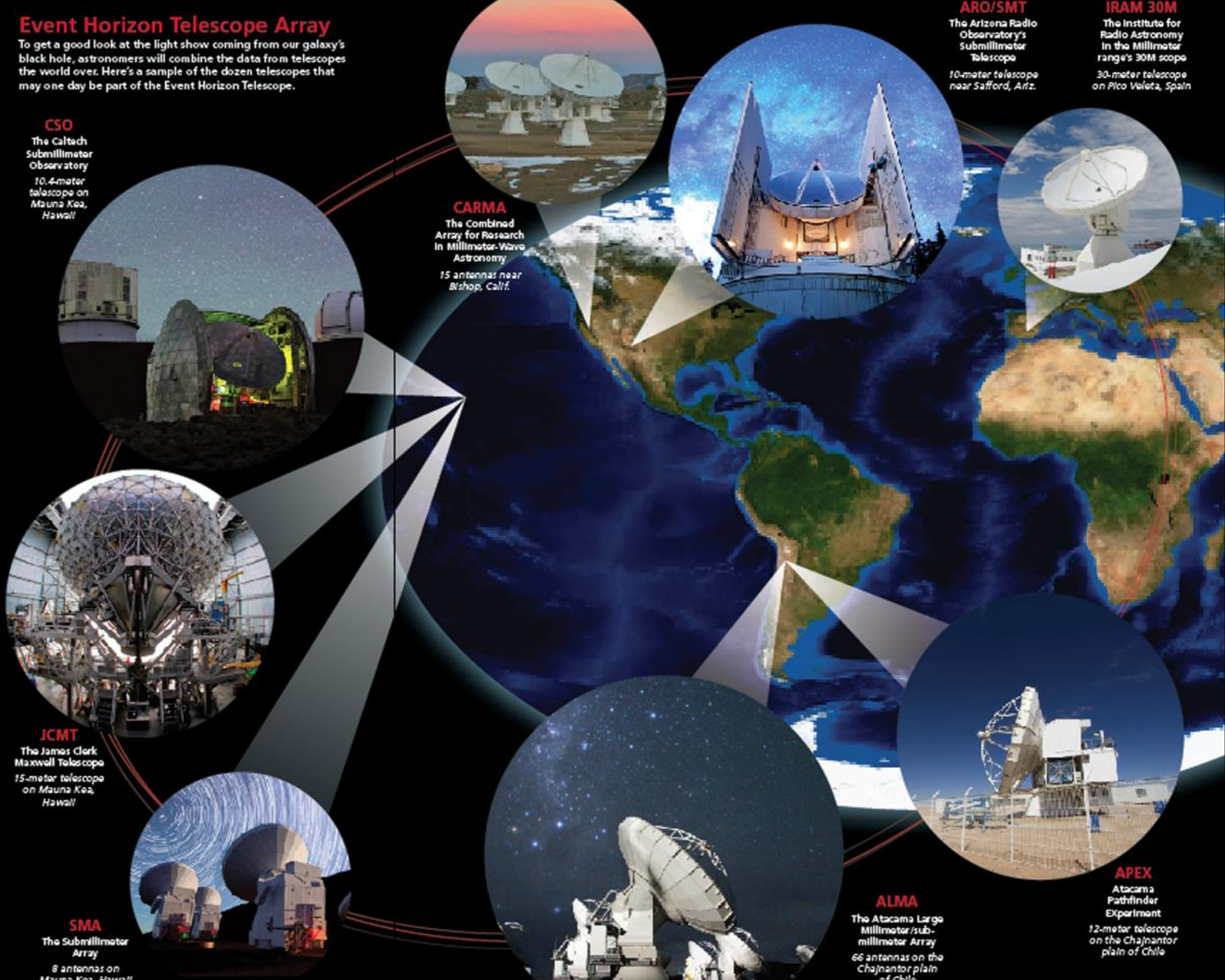


Image credit: Clockwise from top: Andrew A. West, UC Berkley/Radio Astronomy Lab; University of Arizona/David Harvey. Ewan O'Sullivan; Iztok Boncina/ESO; ALMA (ESO/NAOJ/NRAO), C. Padilla; Sean Goebel; Joint Astronomy Centre; Sean Goebel.
Via http://discovermagazine.com

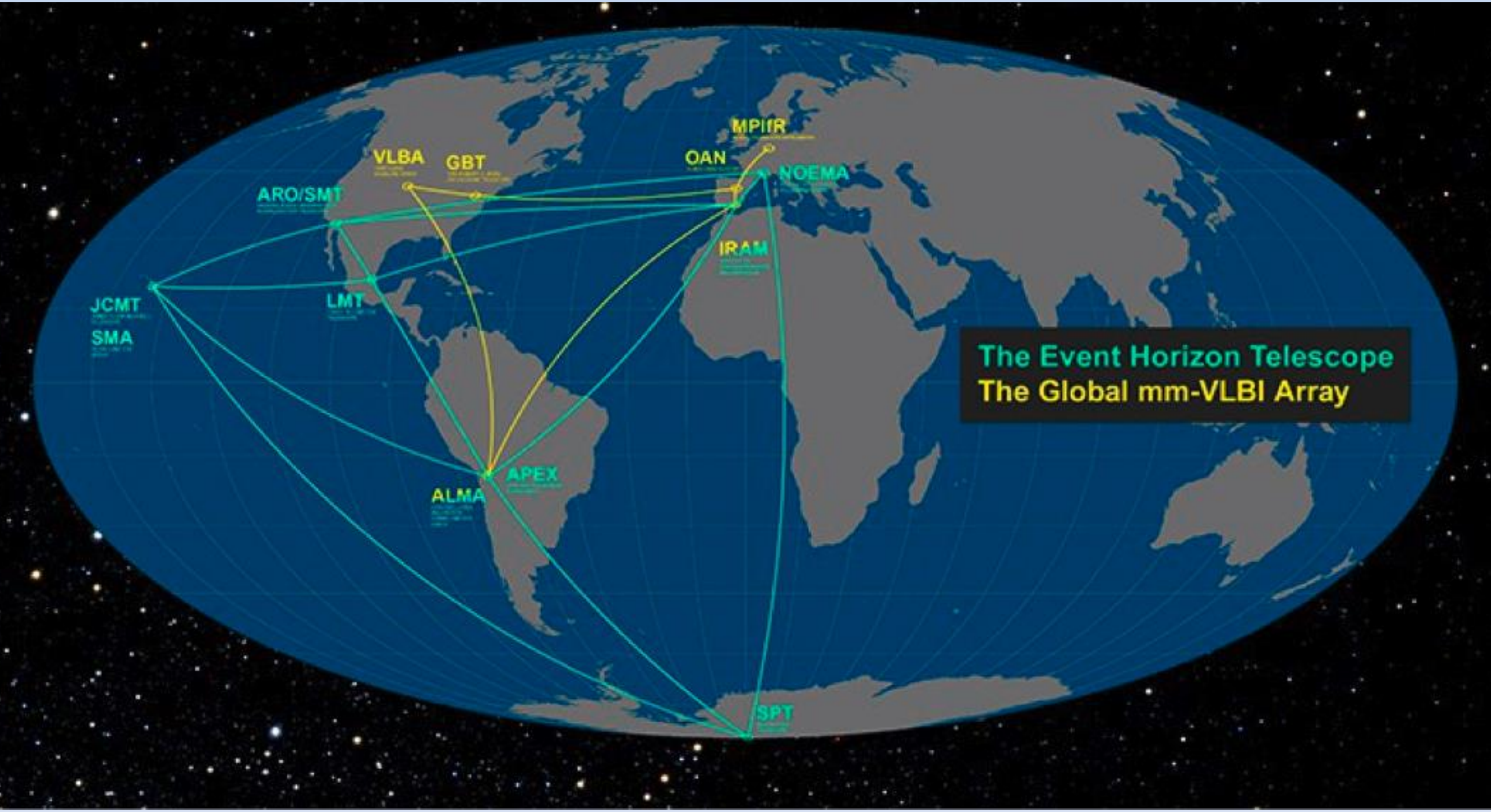
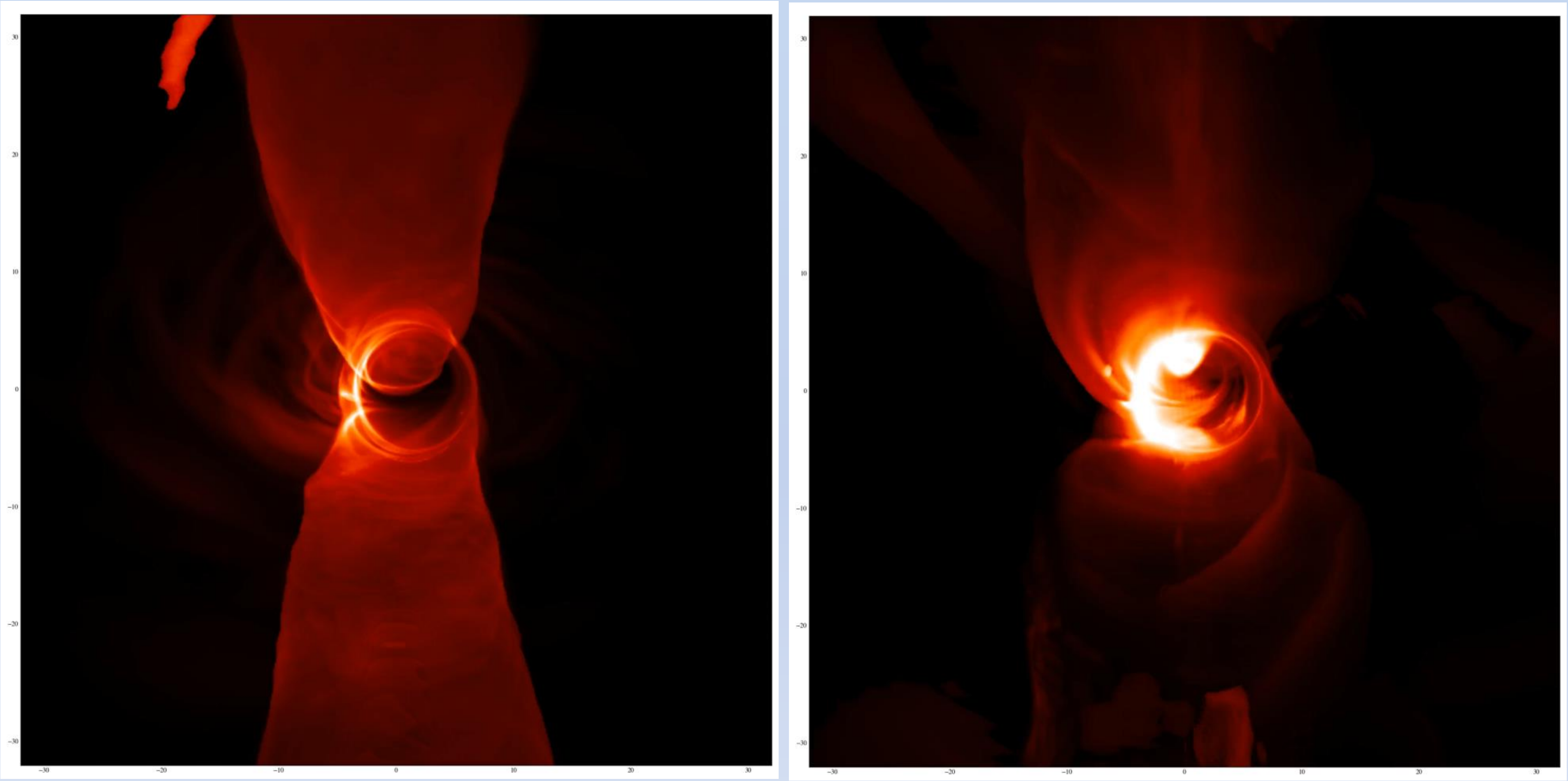


Image credit: ESO/O. Furtak

Image credit: Lia Medeiros, Chi-Kwan Chan, Feryal Özel, Dimitrios Psaltis



Mocked observation of what we might see.

Some final words

There are still lots to discover about black hole. We only covered the classical theory, and a little bit about the effect of quantum mechanics.

In the coming decade, gravitational wave measurements, and also VLBI observations of Sgr A* and M87, will hopefully allow us to know more about these strange objects!