Chapter 12 Neutron stars

12.1 Neutron degeneracy pressure

- ✓ For a dying star of core mass > 1.4 $M_\odot$ (Chandrasekhar limit), the electron degeneracy pressure (*Box 9.1*) cannot support the gravity, the core continues to contract rapidly, density goes up, hence its density increases.

- ✓ At an extremely high density ~10^{14} g cm^{-3}, electrons are squeezed into atomic nuclei, the electrons combine with protons and become neutrons: $\mathbf{p^+ + e^- \rightarrow n + \nu}$, most of the core becomes neutron matter. Furthermore, the resistance to the compression rises due to **neutron degeneracy pressure**.

- ✓ In the case of core mass < 2-3 $M_\odot$, neutron degeneracy pressure can stop further contraction of the core, and it becomes a stable, small, but extremely high density **neutron star**. On the other hand, how about the core in the case of core mass larger than 3 $M_\odot$?

12.2 Properties of neutron stars

- ✓ The typical density of a neutron star is about 10^{14}-10^{15} g cm^{-3} (the entire Sun is compressed into a city, or the entire Hong Kong into a tablespoon, it is then a neutron star Fig. 12-1).

Fig. 12-1: Neutron stars are not much larger than many of the Earth′s major cities. In this fanciful comparison, a typical neutron star sits atop of Manhattan.

- ✓ Why is the density of nuclei so high? The size of an atom is about 1 Å (~10^{-10} m), whereas the size of an atomic nucleus is about 10^{-15} m. But the mass of an atom concentrates at the nucleus (99.95%). If one compresses normal matter (made up of atoms) until the atomic nuclei ″touches″ each other, the linear dimensions of the matter will be reduced by a factor of 10^{-5}, the volume reduces by a factor of ~10^{-15}, so the density (1 g cm^{-3} for water) increases to ~10^{15} g cm^{-3}.

✓ The typical radius and mass of a neutron star are, respectively, about 10 km and 1.4 M⊙. The surface gravity is then extremely strong. For example, $g \sim GM/r^2 = 2\times10^{12}$ ms^{-2}, it is about 190 billion times larger than that on the Earth′s surface! Under such environment, a man would have a weight of about million tons there, there would have no ″mountains″ on its surface.

✓ The escape velocity is close to the speed of light: $v_{esc} = \sqrt{\dfrac{2GM}{R}} = 0.64c$. The gravitational energy of a mass m is then comparable to its rest mass energy: $\dfrac{GMm/R}{mc^2} = 0.21$. Newtonian mechanics is *inadequate*. The theory of General relativity has to be considered there.

✓ Most neutron stars are spinning very rapidly (period ~10^{-1} s - 10^{-3} s) [1]. By the conservation of angular momentum (L = $CMR^2\omega$, where CMR^2 is the momentum of inertia, and C is a constant depending on geometry), we have $CMR_f^2\omega_f = CMR_i^2\omega_i$; therefore, $\omega_f = \omega_i\left(\dfrac{R_i}{R_f}\right)^2$. The angular speed increases with $1/R^2$ as R decreases, a contracting core will spin faster and faster (Fig. 12-2).

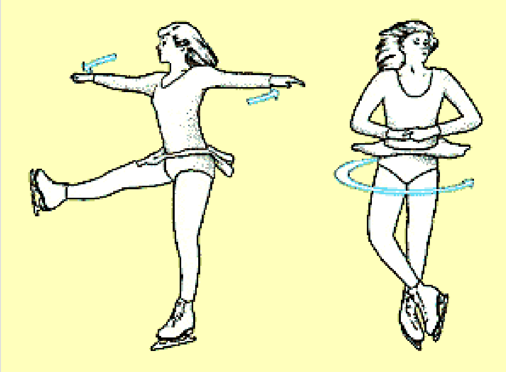

Fig. 12-2: An ice-skater spins faster as she contracts her body, illustrating the principle of conservation of angular momentum.

✓ Similarly, neutron stars have very strong and concentrated magnetic: B ~10^{12} G (Gauss) (c.f. 0.5 G on the surface of Earth). Since the magnetic flux $\Phi \sim B\,A$, where B is the magnetic field, and A is the area ~ R^2, so the magnetic field lines are ″freezing in″ during the contraction. i.e., $B_f R_f^2 = B_i R_i^2$; therefore, we have $\boxed{B_f = B_i\left(\dfrac{R_i}{R_f}\right)^2}$. The magnetic field increases with $1/R^2$ as R decreases also.

✓ The surface temperature of a neutron star is very high (~10^6 K), hence thermal radiation is primarily in X-rays. However, the luminosity (by thermal radiation) is very small

[1] There are some rare neutron stars with extremely slow rotation. The pulsar SXP 1062 spins once every 18 minutes. More details can be found here: http://chandra.harvard.edu/photo/2011/sxp1062/

because of its small size. In fact, we seldom find a neutron star, which is *not* a pulsar (details shown later).

✓ *Theoretical model of a neutron star*: It has a solid, crystal *crust* of heavy nuclei. Deeper in the crust are increasingly neutron-rich nuclei. The nuclei effectively dissolve, a mixture of neutrons, protons and electrons approaches a ratio of 8:1:1. A solid core may also exist. (Fig. 12-3). The actual structures are not very clear in fact!

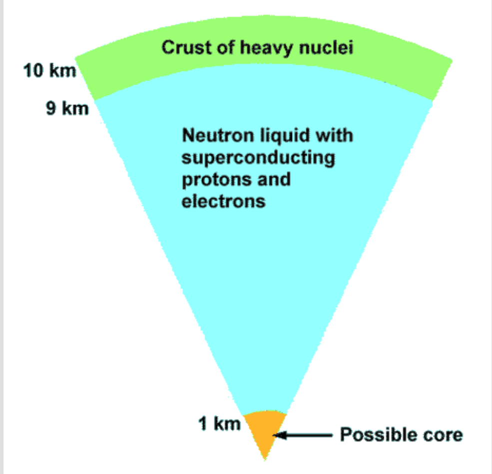

Fig. 12-3: The theoretical model of neutron star.

12.3 Pulsars

✓ Pulsars emit very regular pulses (period ~10^{-1} s -10^{-3} s) of radiation with wide rang of frequencies (from X-rays to radio waves) observed on the Earth (Fig. 12-4). The pulsation periods are precisely measured by atomic clocks to 17 digits!

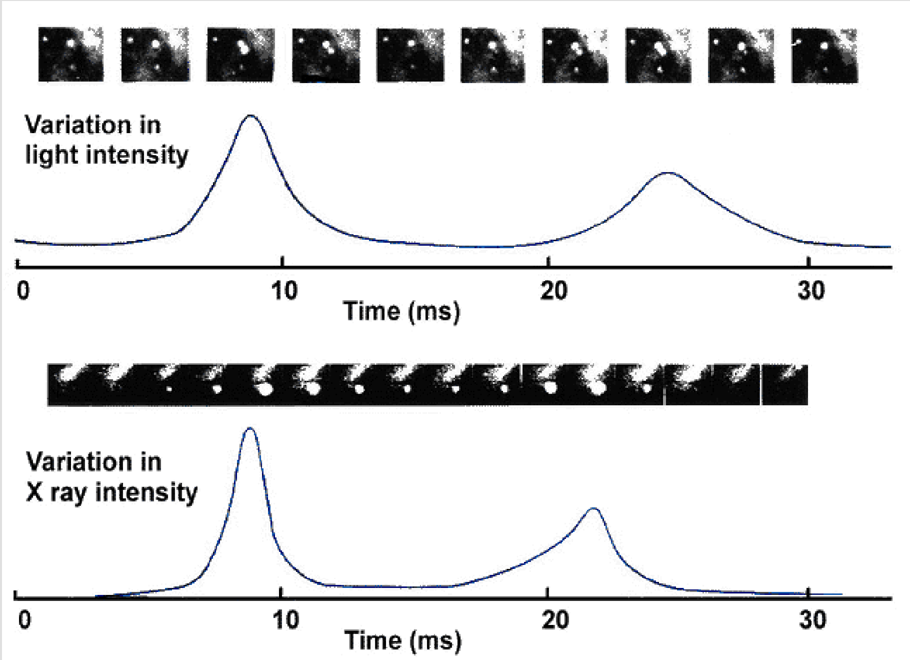

Fig. 12-4: High-speed images of the Crab Nebula pulsar show that it is pulsing at (a) visible wavelengths and (b) at X-ray wavelengths.

✓ **Lighthouse Model:** Strong magnetic fields are created along the rotation axis. High-energy charged particles flow along the magnetic field lines, producing radiation beam outward. Hence, the beams of radiations will sweep across in space like a lighthouse.

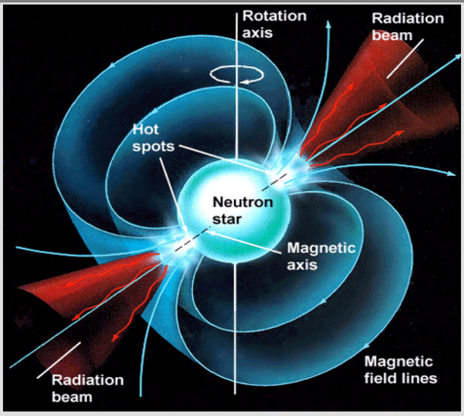

Fig. 12-5: The "lighthouse model" of neutron star emission accounts for many of the observed properties of pulsars.

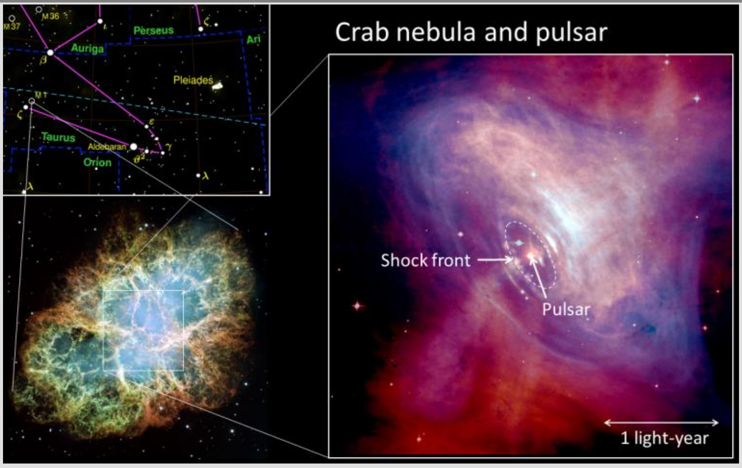

Fig. 12-6: The Crab Nebula (M1) is a supernova remnant in the Taurus constellation. The pulsar at the centre spins 30 times per second. (Graphics: MPIK, photo: NASA)

(Fig. 12-5) Regular and rapid pulses are observed on the Earth, hence it is called **pulsar.** For example, the Crab Nebula pulsar is the remnant of the supernova in 1054. (Figs. 12-4, 12-6)

- ✓ Even for white dwarfs such a rapid rotation is so fast that the centrifugal force would have torn the star apart. Only neutron stars can have such a strong gravity to resist the hug centrifugal force.

- ✓ Accurate measurement of the period P of a pulsar shows that the period is increasing very gradually, typical $dP/dt \sim 10^{-15}$, as the pulsar is radiating energy in rotation. Its **characteristic lifetime** (it is the time for pulses to cease, and cannot be observed on the Earth then) is very long, i.e., $P/(dP/dt) \sim 10^{7}$ years. Why? Where does the energy loss?

- ✓ As mentioned above, the rotation period is *slowing down* because **rotational energy** is being converted into **radiation energy**. For example, one records 60 pulses per second on the Earth from the neutron star in Crab Nebula, i.e., $P = 1/30\,\mathrm{s} = 0.033\,\mathrm{s}$. Accurate measurement shows that the period is increasing very slowly at a rate of $dP/dt = 1.3\times10^{-5}\ \mathrm{s/years}$. Taking $M = 1.4\mathrm{M}_\odot$ and $R = 10\,\mathrm{km}$, the rotational energy and the rate of energy being radiated away can be estimated as follows. The rotational kinetic energy of the pulsar is $K = \frac{1}{2} I\omega^2$, where I is the momentum of inertia. Assuming (for simplicity) that the star is *uniform* sphere, we have

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2$$
$$= 2.0\times10^{42}\text{ J}$$

The rate of decrease of K is $\frac{dK}{dt} = I\omega\frac{d\omega}{dt} = \frac{1}{2}I\omega^2 \times \frac{2}{\omega}\frac{d\omega}{dt} = \frac{2K}{\omega}\frac{d\omega}{dt}$. Since $P = 2\pi/\omega$ and $dP/P = -d\omega/\omega$, so $\frac{dK}{dt} = -\frac{2K}{P}\frac{dP}{dt} \approx -5\times10^{31}$ W. It is ~10^5 times the radiation power of the Sun. Remarkably, this is the energy required to power the Crab Nebula.

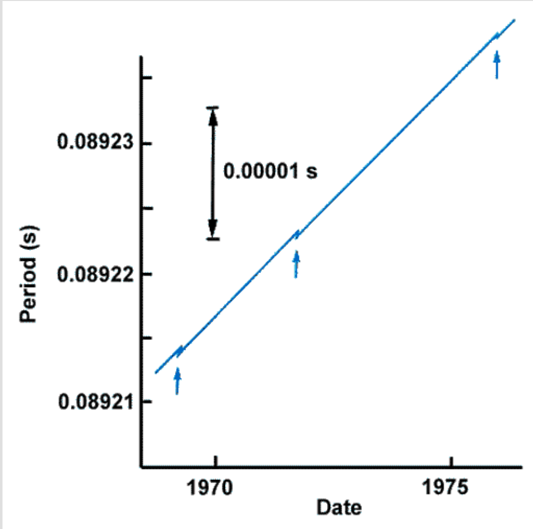

Fig. 12-7: Glitches (arrows) occur when the pulsar suddenly speeds up by a very small amount.

- ✓ **Glitches:** Sudden *decreases* in rotation periods (pulsation periods) of pulsars by very small amounts ($|\Delta P|/P$ ~10^{-6}-10^{-8}, Fig. 12-7). It might be due to ″star-quakes″ - crust broken by stress built up from rotation. It results in a more spherical pulsar, spinning faster a little bit afterward.

12.4 Pulsars in binary systems

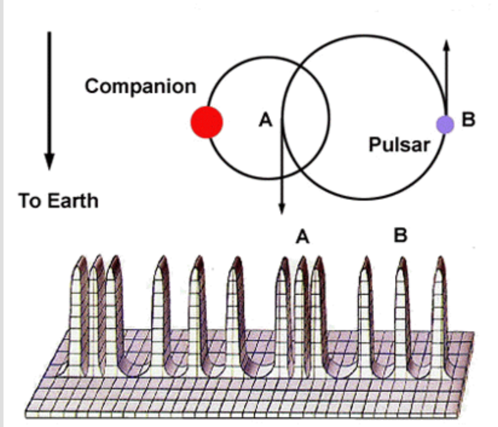

Fig. 12-8: A schematic diagram shows a pulsar′s orbital motion in a binary system.

- ✓ Pulsars in binary systems are usually discovered by observing periodic, tiny changes of pulsar period. When the pulsar is approaching the Earth, its pulsation period would appear shorter because of Doppler effect; whereas when the pulsar is receding, its pulsation period would appear longer. As the pulsar period measured to great precision, the details of the orbital motion are revealed. (Fig. 12-8)
- ✓ Unlike isolated pulsars, matter is transferred from the companion (a normal star or giant) to the neutron star in a binary system, so an accretion disk is formed in the pulsar. (Fig. 12-9)

- ✓ Since neutron stars always have *very strong* magnetic fields, the accreted matter follows the field lines and is funnelled onto one of the magnetic poles. Consequently, X-rays radiations are emitted over a large solid angle. **Binary X-ray pulsars** are seen on the Earth.
- ✓ X-rays pulses may be *eclipsed* periodically by its companion, e.g., Hercules X-1. (Fig. 12-10) It seems that X-rays pulses disappear as the pulsar is eclipsed behind its stellar companion.

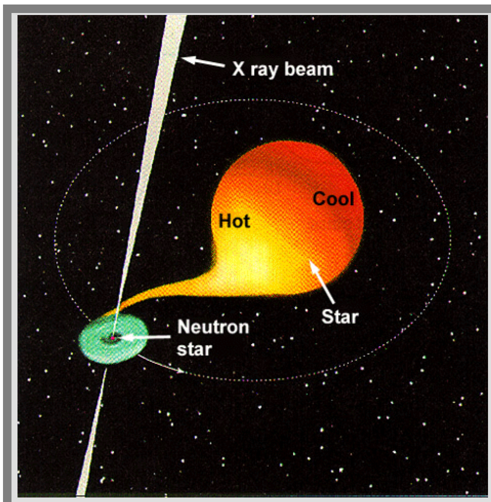

Fig. 12-9: A model of Hercules X-rays.

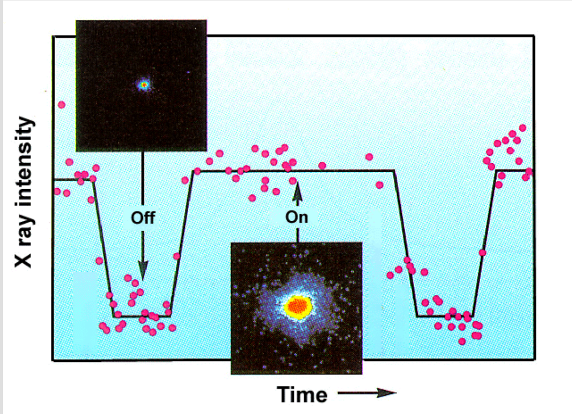

Fig. 13-10: X rays from Hercules X-1. Insets show the pulsar seems to be "turned off" during an eclipse and "on" between the eclipses.

12.5 Planets around a neutron star

- ✓ The rotation period of a pulsar can be measured very accurately (up to 17 digits). However, for some cases, e.g., PSR B1257+12, the measured period does not agree very well with the theoretical model unless one includes planets orbiting the neutron star. In 1992, it became the first confirmed case of exoplanet [2].
- ✓ Even such a violent environment of a neutron star have planets, one concludes planets should be common everywhere in our universe! Most known exoplanets are however orbiting main-sequence star.

[2] The first confirmed case of exoplanet around a main-sequence star is 51 Pegasi b, discovered in 1995.