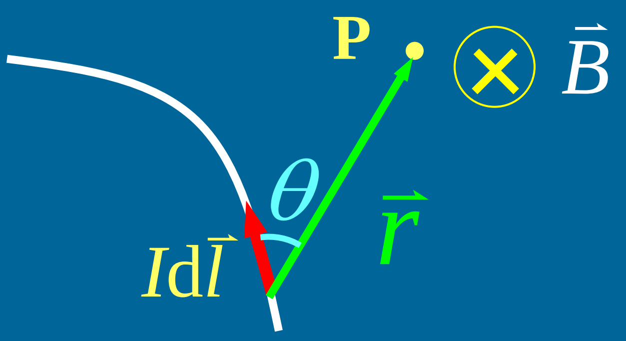
§ 11.2 毕奥－萨伐尔定律（续）

磁　场：取 $Id\vec{l} \longrightarrow d\vec{B}$

毕－萨定律：$\boxed{d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \vec{r}_0}{r^2}}$　　$\vec{r}_0$ —— 单位矢量

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$　　真空中的磁导率

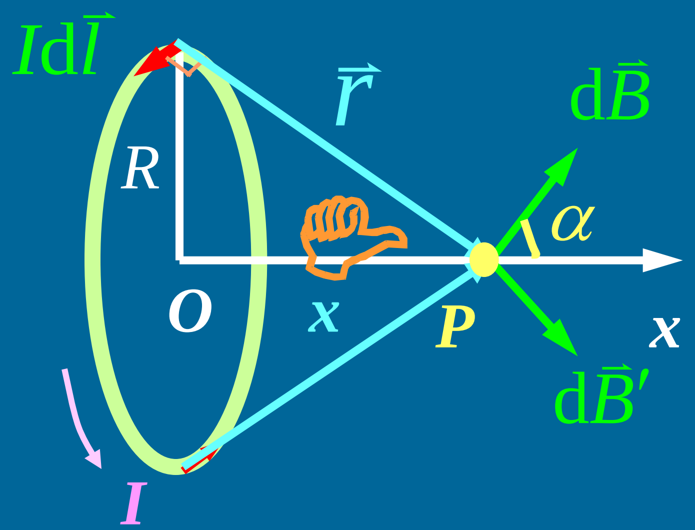
大小：$dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\theta}{r^2}$



方向：右螺旋法则

2. 载流圆线圈的磁场

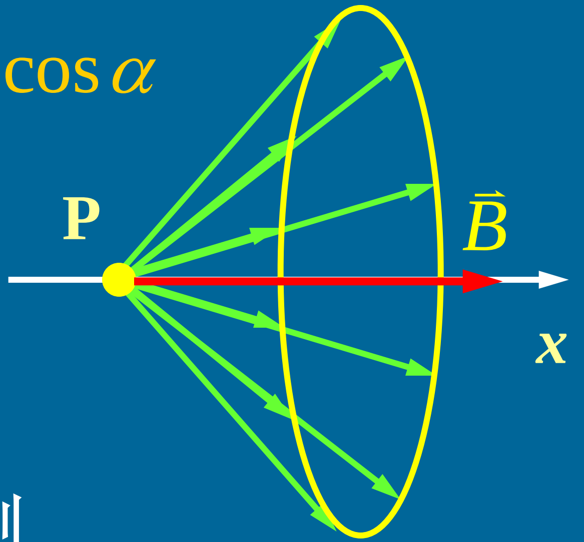
求轴线上一点 P 的磁感应强度



$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{(R^2 + x^2)}$$

根据对称性 $B_\perp = 0$

$$B = \int dB_x = \int dB\cos\alpha = \int \frac{\mu_0}{4\pi}\frac{Idl}{(x^2 + R^2)}\cos\alpha$$



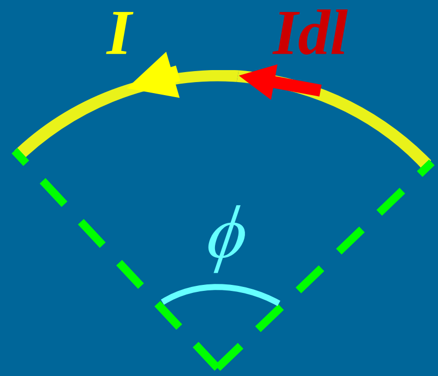
$$\cos\alpha = \frac{R}{r} = \frac{R}{(R^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}$$ 方向满足右手定则

讨论 $B=\dfrac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$

(1) $x=0$ 载流圆线圈的圆心处 $B=\dfrac{\mu_0 I}{2R}$

如果由N 匝圆线圈组成 $B=\dfrac{\mu_0 NI}{2R}$



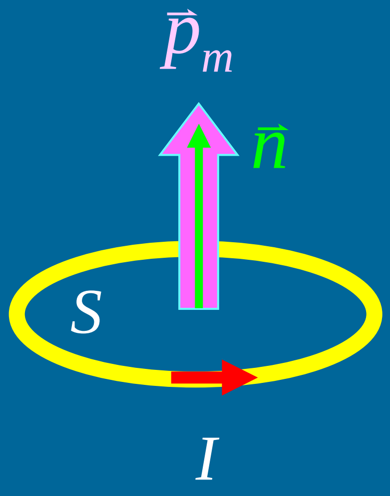
(2) 一段圆弧在圆心处产生的磁场

法一:
(看作圆的一部分) $B=\dfrac{\mu_0 I}{2R}\cdot\dfrac{\phi}{2\pi}=\dfrac{\mu_0 I\phi}{4\pi R}$

法二: $\mathrm{d}B=\dfrac{\mu_0}{4\pi}\dfrac{I\mathrm{d}l}{R^2}$ $B=\int\dfrac{\mu_0}{4\pi}\dfrac{I}{R^2}\mathrm{d}l=\dfrac{\mu_0 I}{4\pi R^2}\int\mathrm{d}l=\dfrac{\mu_0 I\varphi}{4\pi R}$

(3) $x >> R$ $\Rightarrow$ $B = \dfrac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

$$B \approx \frac{\mu_0 I R^2}{2x^3} \cdot \frac{\pi}{\pi} = \frac{\mu_0 I S}{2\pi x^3}$$

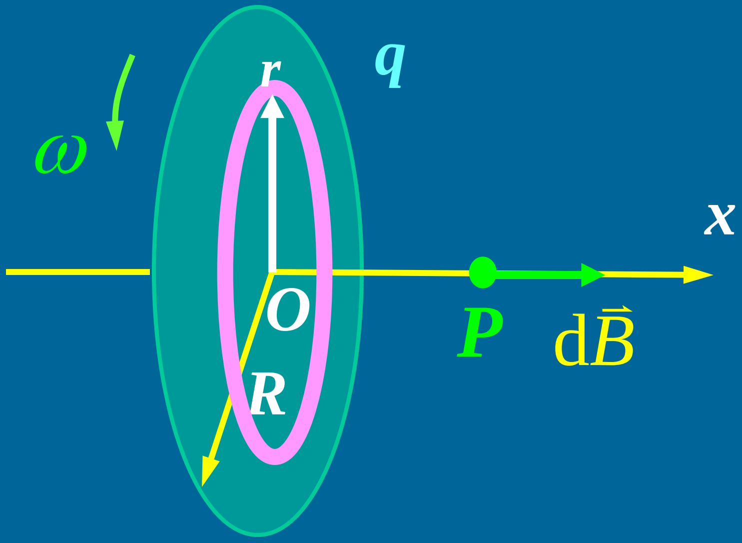


定义： $\boxed{\vec{p}_m = IS\vec{n}}$ ——载流圆线圈磁矩

若为N 匝，则：

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{p}_m}{x^3} \qquad \vec{p}_m = NIS\vec{n}$$

例 求绕轴旋转的带电圆盘轴线上的磁场和圆盘的磁矩



解 $\sigma = q/\pi R^2$

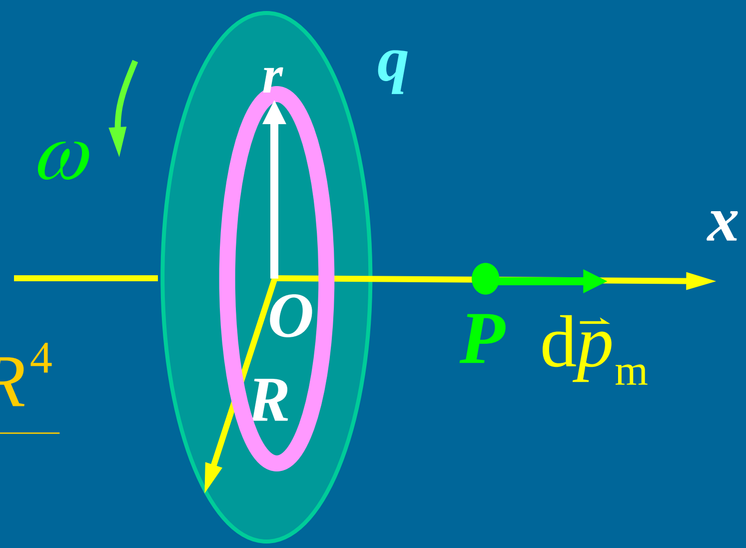
$$\mathrm{d}q = \sigma \cdot 2\pi r \mathrm{d}r$$

$$\mathrm{d}I = \frac{\mathrm{d}q}{T} = \frac{\sigma \cdot 2\pi r \mathrm{d}r}{2\pi/\omega} = \omega\sigma r \mathrm{d}r$$

$$\mathrm{d}B = \frac{\mu_0 r^2 \mathrm{d}I}{2(r^2 + x^2)^{3/2}} = \frac{\mu_0 \sigma\omega r^3 \mathrm{d}r}{2(r^2 + x^2)^{3/2}}$$

$$B = \int \mathrm{d}B = \frac{\mu_0 \sigma\omega}{2}\left[\frac{R^2 + 2x^2}{\sqrt{x^2 + R^2}} - 2|x|\right]$$

$$\mathrm{d}\vec{p}_m = \pi r^2 \mathrm{d}I\vec{n} = \pi r^3 \omega\sigma \mathrm{d}r\vec{n}$$



$$p_m = \int \mathrm{d}p_m = \int_0^R \pi r^3 \sigma\omega \mathrm{d}r = \frac{\pi\omega\sigma R^4}{4}$$

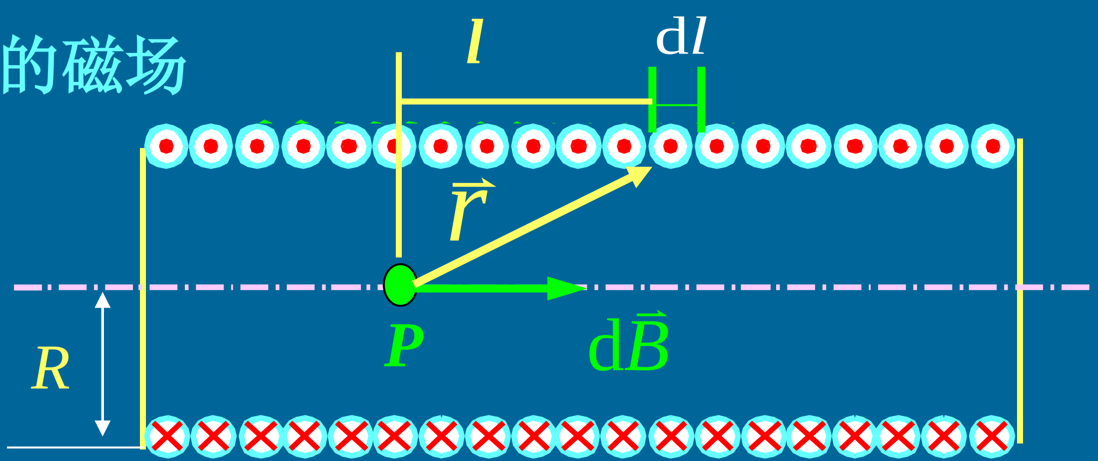
方向沿 x 轴正向

3. 载流直螺线管轴线上的磁场

已知螺线管半径为R

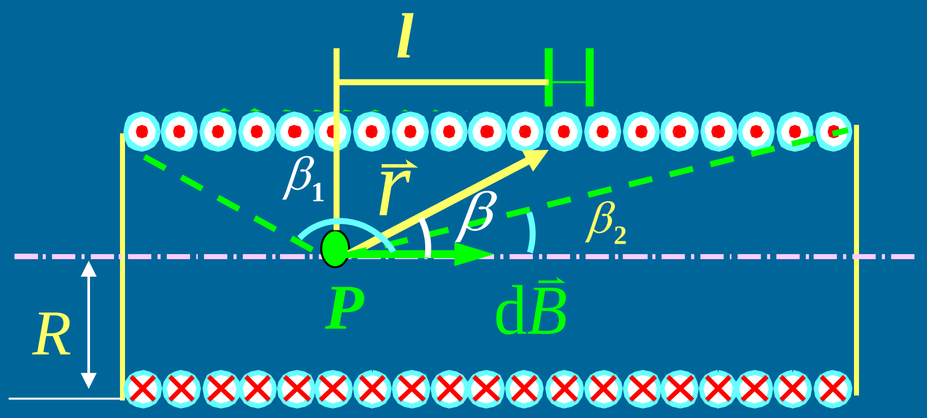
单位长度上有n 匝

$$\mathrm{d}I' = In\mathrm{d}l$$



$$\mathrm{d}B = \frac{\mu_0 R^2 \mathrm{d}I'}{2(R^2+l^2)^{3/2}} = \frac{\mu_0 R^2 I n \mathrm{d}l}{2(R^2+l^2)^{3/2}}$$

$$\begin{cases} l = R\cot\beta \\ R^2 + l^2 = R^2\csc^2\beta \end{cases}$$



$$\mathrm{d}B = -\frac{\mu_0}{2} nI \sin\beta \mathrm{d}\beta$$

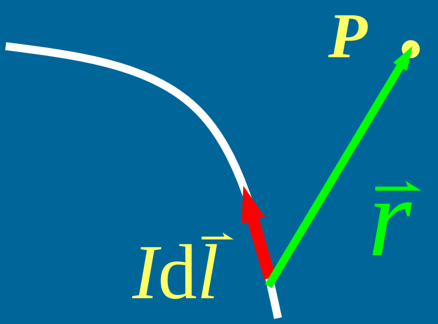
$$B = \int_{\beta_1}^{\beta_2} -\frac{\mu_0}{2} nI \sin\beta \mathrm{d}\beta = \frac{\mu_0 nI}{2}(\cos\beta_2 - \cos\beta_1)$$

讨 论

(1) 无限长载流螺线管 $\beta_1 \to \pi,\ \beta_2 \to 0$ $\Rightarrow$ $B = \mu_0 nI$

(2) 半无限长载流螺线管轴线上的端点处 $\beta_1 \to \pi/2, \beta_2 \to 0$ $\Rightarrow$ $B = \mu_0 n \frac{I}{2}$

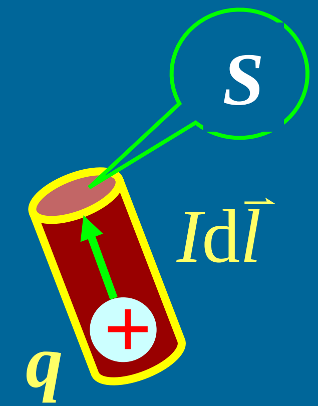
三. 运动电荷的磁场（下面从毕－萨定律出发导出运动电荷的磁场表达式）



$$\mathrm{d}\vec{B}=\frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{l}\times\vec{r}_0}{r^2}$$

载流子密度

$$I=\frac{\mathrm{d}Q}{\mathrm{d}t}=\frac{nq\mathrm{d}V}{\mathrm{d}t}=\frac{nqS\mathrm{d}l}{\mathrm{d}t}=nqS\upsilon$$



$$\mathrm{d}\vec{B}=\frac{\mu_0}{4\pi}\frac{(nqS\upsilon)\mathrm{d}\vec{l}\times\vec{r}_0}{r^2}=\frac{\mu_0}{4\pi}\frac{(nqS\vec{\upsilon})\mathrm{d}l\times\vec{r}_0}{r^2}$$

电流元内总载流子数 $\mathrm{d}N=nS\mathrm{d}l$

$\vec{\upsilon}$ 与 $\mathrm{d}\vec{l}$ 方向一致

$$\mathrm{d}\vec{B}=\frac{\mu_0}{4\pi}\frac{\mathrm{d}N\cdot q\vec{\upsilon}\times\vec{r}_0}{r^2}$$

一个运动电荷产生的磁场 $\boxed{\vec{B}=\frac{\mathrm{d}\vec{B}}{\mathrm{d}N}=\frac{\mu_0}{4\pi}\frac{q\vec{\upsilon}\times\vec{r}_0}{r^2}}$

例 如图的导线，已知电荷线密度为 λ，当绕 O 点以 ω 转动时

求 O 点的磁感应强度

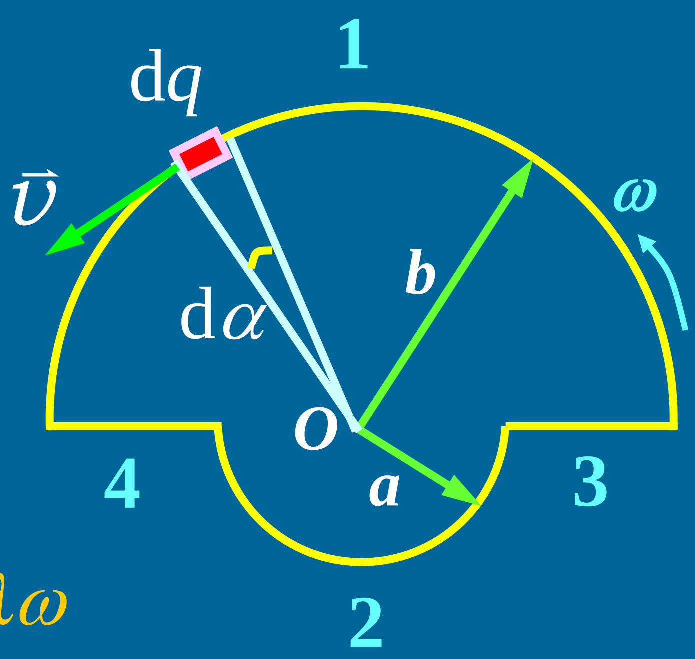
解 线段1：

$$\mathrm{d}q = \lambda \mathrm{d}l = \lambda b \mathrm{d}\alpha$$

$$\mathrm{d}B_1 = \frac{\mu_0}{4\pi}\frac{\mathrm{d}q \cdot \omega b}{b^2}$$

$$= \frac{\mu_0 \lambda \omega}{4\pi}\mathrm{d}\alpha$$

$$B_1 = \int_0^{\pi} \frac{\mu_0 \lambda \omega}{4\pi}\mathrm{d}\alpha = \frac{1}{4}\mu_0 \lambda \omega$$

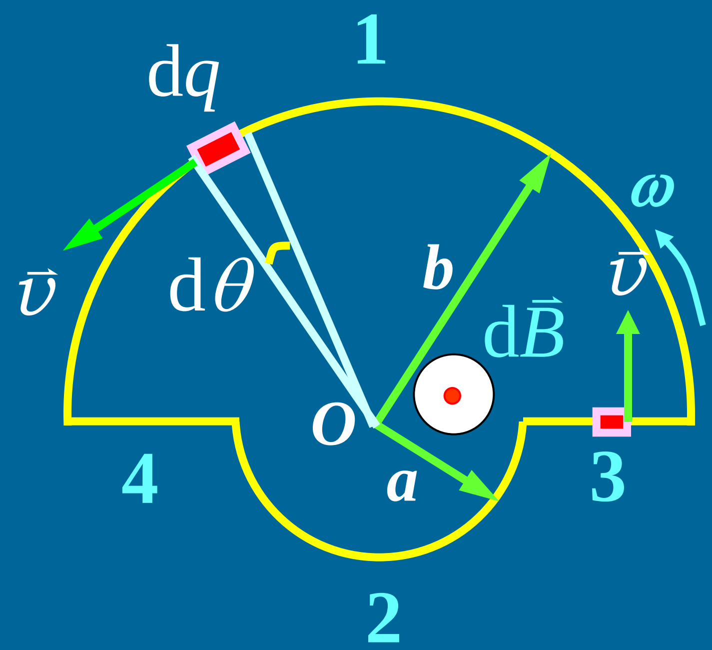


线段2：同理 $B_2 = \frac{1}{4}\mu_0 \lambda \omega$

线段3： $\mathrm{d}q = \lambda \mathrm{d}r$

$$\mathrm{d}B_3 = \frac{\mu_0}{4\pi}\frac{\lambda \mathrm{d}r \cdot \omega r}{r^2} = \frac{\mu_0 \lambda \omega}{4\pi r}\mathrm{d}r$$

$$B_3 = \int_a^b \frac{\mu_0 \lambda \omega}{4\pi r}\mathrm{d}r = \frac{\mu_0 \lambda \omega}{4\pi}\ln\frac{b}{a}$$



线段4： 同理 $B_4 = \dfrac{\mu_0 \lambda \omega}{4\pi}\ln\dfrac{b}{a}$

$$B = B_1 + B_2 + B_3 + B_4 = \frac{1}{2}\left(1 + \frac{1}{\pi}\ln\frac{b}{a}\right)\mu_0 \lambda \omega$$

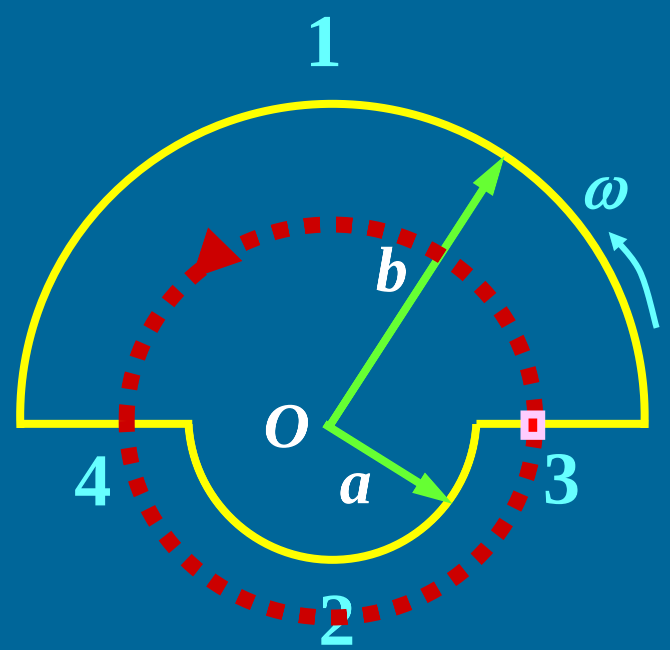
解法二：

线段1：（旋转时等效为载流圆线圈）

$$I_1=\frac{\mathrm{d}q}{\mathrm{d}t}=\frac{\lambda\pi b}{2\pi/\omega}=\frac{\lambda\omega b}{2}$$

$$B_1=\frac{\mu_0 I_1}{2b}=\frac{\mu_0}{2b}\frac{\lambda\omega b}{2}=\frac{\mu_0\lambda\omega}{4}\qquad\odot$$

线段2：同理 $B_2=\frac{1}{4}\mu_0\lambda\omega$ $\odot$



线段3：（旋转时等效为无数载流圆线圈，半径***a*** $\rightarrow$ ***b***）

半径为 ***r*** 处，取宽 **d*r*** 的线元， 旋转时等效圆电流为 **d*I***

$$\mathrm{d}I=\frac{\lambda\mathrm{d}r}{2\pi/\omega}\qquad \mathrm{d}B_3=\frac{\mu_0\mathrm{d}I}{2r}=\frac{\mu_0}{2r}\frac{\lambda\omega\mathrm{d}r}{2\pi}\qquad\odot$$

$$B_3=\int_a^b\frac{\mu_0}{2r}\frac{\lambda\omega\mathrm{d}r}{2\pi}=\frac{\mu_0\lambda\omega}{4\pi}\ln\frac{b}{a}\qquad\odot$$

$$B=B_1+B_2+B_3+B_4$$

线段4：同理 $B_4=\frac{\mu_0\lambda\omega}{4\pi}\ln\frac{b}{a}$

$$=\frac{1}{2}(1+\frac{1}{\pi}\mathrm{In}\frac{b}{a})\mu_0\lambda\omega$$

总结

1. 毕—萨定律应用

- 载流圆线圈轴线上的磁场 $B=\dfrac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$
- 载流圆线圈的圆心处 $B=\dfrac{\mu_0 I}{2R}$
- 载流直螺线管轴线上的磁场 $B=\dfrac{\mu_0 nI}{2}\left(\cos\beta_2-\cos\beta_1\right)$
- 无限长载流螺线管 $B=\mu_0 nI$

2. 运动电荷的磁场

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}_0}{r^2}$$