Chapter 9 Post-main-sequence evolution

Main-sequence and post-main-sequence evolutionary tracks of stars of various masses were first computed and published in the mid-1960s. Modern calculations involves more sophisticated models. The location of the present-day Sun is depicted by a solar symbol ($\odot$) between points 1 and 2 on the 1 $M_\odot$ track. A schematic diagram of the evolution of 1-solar-mass star from the zero-age main sequence to the formation of a white dwarf star is shown in Fig. 9-1. An evolutionary track includes various phases of an evolution: Zero-Age-Main-Sequence (ZAMS), Sub-Giant Branch (SGB), Red Giant Branch (RGB), Early Asymptotic Giant Branch (E-AGB), Thermal Pulse Asymptotic Giant Branch (TP-AGB), Post-Asymptotic Giant Branch (Post-AGB), Planetary Nebula formation (PN formation), and P-white dwarf phase leading to white dwarf phase. In the following, some significant phases will be discussed here and the next chapter. Bradley has a more detailed discussion.[1]

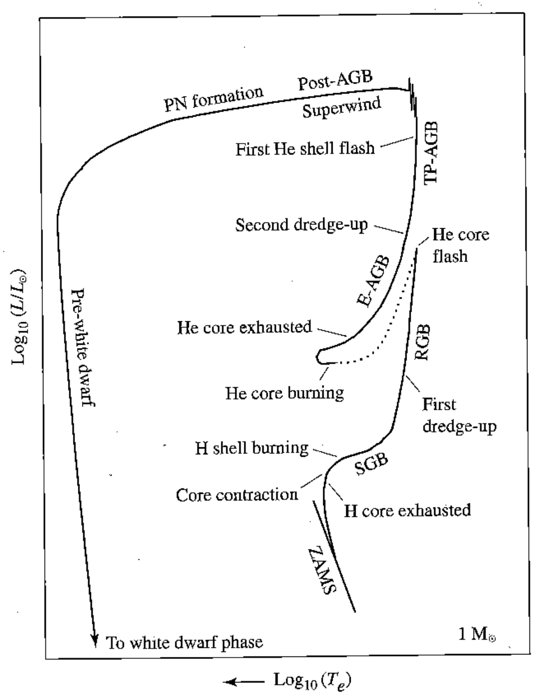

Fig. 9-1: A schematic diagram of the evolution of 1-solar-mass star from the zero-age main sequence to the formation of a white dwarf star. (Adapted from FIGURE 13.4 in Bradley, p.458)

[1] Bradley W. Carroll, Dale A. Ostlie, *An Introduction to Modern Astrophysics* (San Francisco: Pearson, 2007), Chapter 13 "Main Sequence and Post-Main-Sequence Stellar Evolution".

9.1 Red giants

After fusing hydrogen gas in the core for a long time, the core becomes mostly helium (ashes), surrounded by hydrogen layers. As the energy production rate by the hydrogen fusion at the core decreases, (Fig. 9-2) the outward energy flow from the core decreases. Consequently, the temperature gradient becomes smaller and smaller, and an **isothermal** (constant temperature) core is formed. The drop in fusion rate also leads to a decrease in thermal pressure, and the core contracts due to gravity. ("Core contraction" in Fig. 9-1)

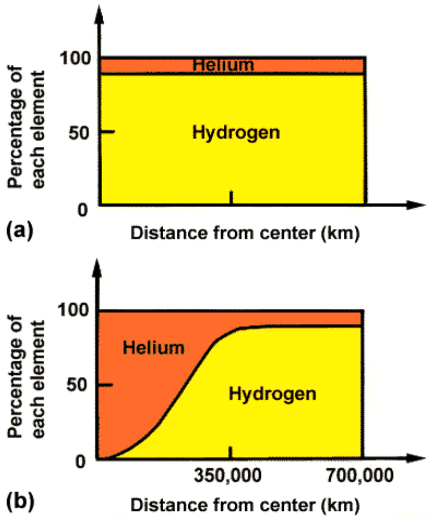

Fig. 9-2: Theoretical estimates of the changes in the composition of a Sun-like star. Hydrogen and helium abundances are shown at birth (top), and after about 10 billion years (bottom), when helium ashes fill the star's core.

While the gas in the core is being compressed, the temperature of the core increases. The central temperature of the star becomes so hot that nuclear fusion of hydrogen gas in a region just outside the core is ignited. It is called **shell hydrogen burning**. ("H shell burning" in Fig. 9-1 and Fig. 9-3)

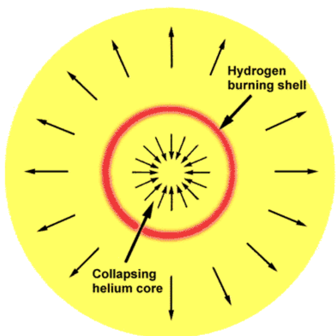

Fig. 9-3: When a star runs out of hydrogen at its centre, it ignites a hydrogen-burning shell.

Rapid contraction of the core (hence denser and hotter) releases gravitational energy which causes the outer layers to expand. At the same time, the temperature and density of the hydrogen-burning shell also increase, hence the shell hydrogen burning is more rapidly and again causes the outer layers to expand. As a result, the outer layer becomes less dense and cooler, resulting in redward evolution on the H-R diagram. This phase is known as the subgiant branch, ("SGB" in Fig. 9-1) together with Red Giant Branch, ("RGB" in Fig. 9-1) the star becomes a **red giant**.[2] Both α Tauri (Aldebaran) and α Orion (Betelgeuse) are examples of red giants. In addition, by Stefan-Boltzmann law, the luminosity of the star is given by $L = 4\pi R^2 \sigma T^4$. (***Box 5.1***) Takes the Sun as an example,[3] the surface temperature drops by about 50% as it becomes a red giant; whereas the radius of the red giant is about 250 times greater than the current solar radius. (Fig. 9-4) The much larger total surface area increases the luminosity.

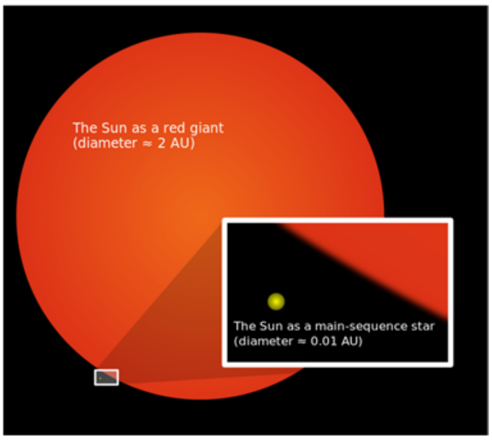

Fig. 9-4: Relative size of the Sun today (yellow dot) and as a red giant (big red circle). (Credit: Wikimedia Commons)

Mass loss in red giant

Red giants undergo substantial *mass loss* because of their large radii and thus weak surface gravity, which makes it quite easy for gases to escape into space. The mass loss could be detected by observing narrow absorption lines which are slightly blue-shifted by the Doppler effect. (***Box 6.1***) A typical red giant loses roughly 10^{-7} solar mass of matter (with a

[2] Bradley W. Carroll, Dale A. Ostlie, *An Introduction to Modern Astrophysics* (San Francisco: Pearson, 2007), pp.457 – 461.

[3] The numbers that we quote here are approximate, although we are pretty sure that the red-giant Sun will engulf Mercury, Venus and the Earth. For a more updated calculation of the post-main-sequence evolution of the Sun, see, e.g., Schröder & Connon Smith, 2008, MNRAS, 386, 155 (preprint is here: http://arxiv.org/abs/0801.4031)

speed of about 10 km s^{-1}) per year. For comparison, the Sun's present-day mass loss rate is only about 10^{-14} solar mass per year. Because of the partial mixture of materials between regions near the core and the outer layers, heavier elements are found in stellar atmosphere (from the observation of spectral lines). It is an evidence of the stellar evolution theory.

Helium fusion at the core

As the temperature of the core reaches about 10^8 K, the **triple-α process** (3 ^{4}He nuclei fuse into 1 ^{12}C nucleus) begins in the core:[4]

$$^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be} + \gamma$$

$$^8\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C} + \gamma$$

Large amount of energy is produced from the helium fusion process in the core, leading to the expansion of the core. Both the core's and surface's temperature rise rapidly. The star now moves towards the left (hotter) across the H-R diagram along the horizontal branch. ("He core burning" in Fig. 9-1) In addition, the rate of the triple-α process is very dramatic temperature dependence and proportional to T^{41} at temperature near 10^8 K. For instance, an increase of only 10% in temperature raises the energy output rate by more than 50 times. Energy released by triple-α process is not as much as hydrogen fusion, and the star will not be stable for a long time.

Nucleosynthesis

If a star is sufficiently massive, still higher central temperature can be obtained and many other nuclear products become possible. Examples of available reaction include carbon burning reactions near 6×10^8 K, and oxygen burning reaction near 10^9 K. For stars of mass heavier than 4 solar masses, the cycles of burning heavier and heavier atoms repeat until an *iron core* is developed. The star produces various elements during this process, called **nucleosynthesis**. It is the origin of most elements. The repeated cycles mean that the star moves left and right across the H-R diagram for many times. A schematic diagram of the evolution of 5-solar-mass star from the zero-age main sequence to the formation of a white dwarf star is shown in Fig. 9-5.

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[4] Bradley W. Carroll, Dale A. Ostlie, *An Introduction to Modern Astrophysics* (San Francisco: Pearson, 2007), pp.312 – 313.

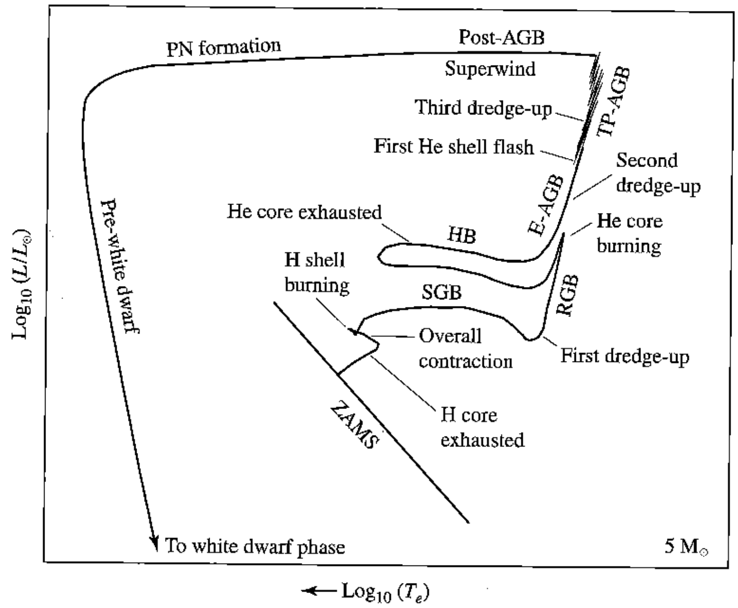


Fig. 9-5: A schematic diagram of the evolution of 5-solar-mass star from the zero-age main sequence to the formation of a white dwarf star. The diagram labelled according to Fig. 9-1. (Adapted from FIGURE 13.5 in Bradley, p.459)

9.2 Variable stars

Stars with changing brightness are called variable stars. One example is eclipsing variable stars, which refers to binaries that periodically block each other. Here we will focus on another type of variables that is related to post-main-sequence evolution of stars.

Giants moving left and right in the H-R diagram may enter a nearly vertical region called the **instability strip**, (Fig. 9-6) the stars will then pulsate, and the outer layers of the stars will oscillate. They are called **variables**, which have periodic variations in brightness, size, etc.

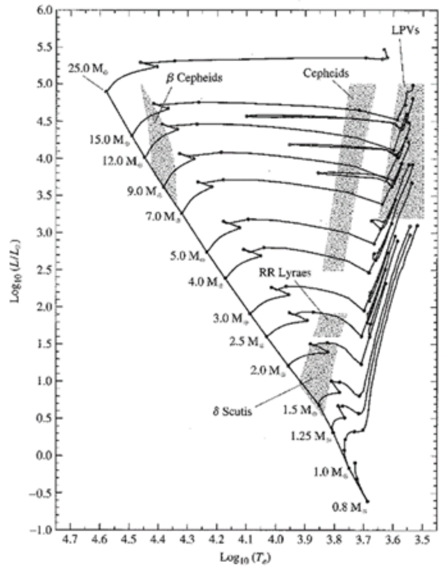


Fig. 9-6: The instability strip (shaped) is a region of the H-R diagram in which stars are unstable and pulsate as variable stars. (Adapted from Bradley, p.490)

A Cepheid variables is a type of star that pulsates radially, varying in both diameter and temperature and producing changes in brightness. Periodic variables with radii changes by 5 - 10%, brightness changes by 0.1 - 2 magnitudes in days. (Fig. 9-7) For example, The pulsation period of Polaris is about 4 days, its brightness change by 0.1 magnitude. Why do Cepheid variable pulsate? See ***Box 9.2*** for the details. Yet, Cepheid variables are important because the period can be measured even at distances of millions of parsecs. **Period-luminosity correlation:** A massive Cepheid has a large inertia, so it can oscillate slowly. Thus, the period of a Cepheid provides us information about its mass, and hence its luminosity (*the mass-luminosity relation*). A massive Cepheid crosses the strip at higher luminosity than lower-mass stars, and because of their higher mass and larger radius, they pulsate with longer periods.

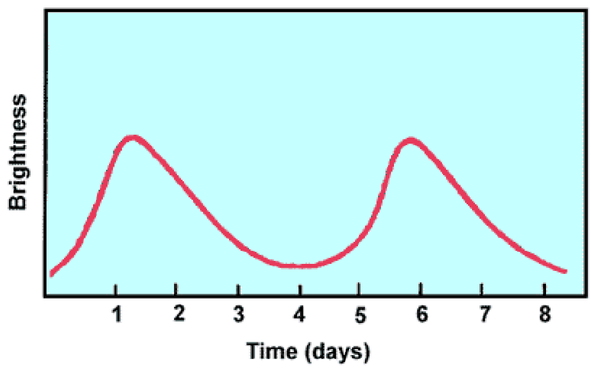


Fig. 9-7: The brightness of a Cepheid varies periodically with time.

Determination of the distance to Cepheids: It is an important method to measure the distances of galaxies [5]. Since the mean absolute magnitude may be determined from a *calibrated period-luminosity diagram* (Fig. 9-8). The period-luminosity relation is obtained directly from observation. The distance of the Cepheid is then found by comparing the mean absolute magnitude with apparent magnitude.[6]

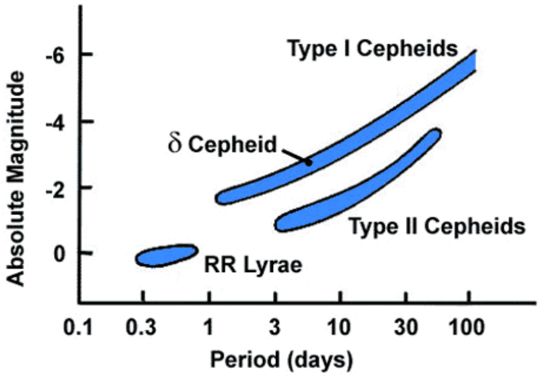
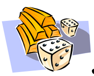


Fig. 9-8: Mean luminosity of Cepheids is related to their periods. The longer the period, the more luminous is the Cepheid. The two kinds of Cepheids, namely, type I and II, have different period-luminosity correlation.



Box 9.2 Why does a Cepheid pulsate periodically? (*for reference only*)

Between the outer layer (not ionized) and the fusion region (completely ionized), a zone of partially ionized helium exists. If a star contracts, the compression will ionize the helium in the partially ionized helium zone. Ionized helium is opaque to radiation, so these layers trap heat very effectively. The star would eventually expand again due to the heat. On the other hand, the expansion of the star causes the helium ions to recombine with electrons. The gas becomes more transparent and releases the trapped energy. The star then falls inward, recompressing the helium, and the whole cycle of contraction and expansion begins all over again.

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[5] In 1912, Henrieta Swan Leavitt studied Cepheid variables in the Small Magellanic Cloud and found the relation between the luminosity and period. In 1923, Edwin Hubble found the Cepheid variable star V1 in M31 (the Andromeda Galaxy). That and the subsequent observations allowed astronomers to measure large cosmic distances, and led to the discovery that distant celestial objects are moving away from us.

[6] See Chapter 6: Measuring stars

The details of Cepheids pulsations depend on the amount of *metal* (heavy elements), because even trace amounts of these elements can have a large effect on the opacity of the stellar gases. Hence, Cepheids are classified according to their metal content. If the star is metal-rich, it is called a **Type I Cepheid**; if it is a metal-poor, it is called **Type II Cepheid**. (Fig. 9-8)

9.3 Star clusters

Stars in the same star cluster are formed from the same cloud, so they should have similar ages and compositions. All stars in a cluster are approximately the same distance from the Earth. Hence, the *relative* apparent luminosity represents the *relative* absolute luminosity. Each star in the same cluster may differ in luminosity and surface temperatures; therefore, they locate at difference positions on the H-R diagram. More massive members (more luminous) in the cluster evolve faster, leave the main sequence and become giants. It happens while lower mass members still lie on the main sequence. Observations of star clusters do confirm this evolution picture that more massive stars have shorter life spans on the main sequence (Figs. 9-9, 10). It is possible to estimate the cluster′s age by locating the **turn-off point** of the cluster′s H-R diagram.

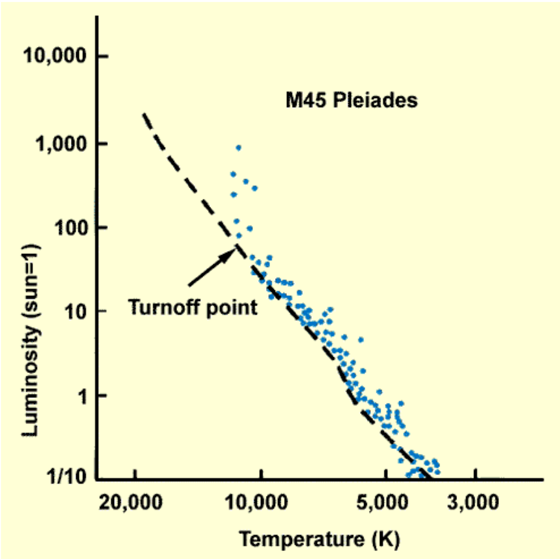


Fig. 9-9: The H-R diagram of the open cluster Pleiades. It is a young cluster, so most stars are still on the main sequence.

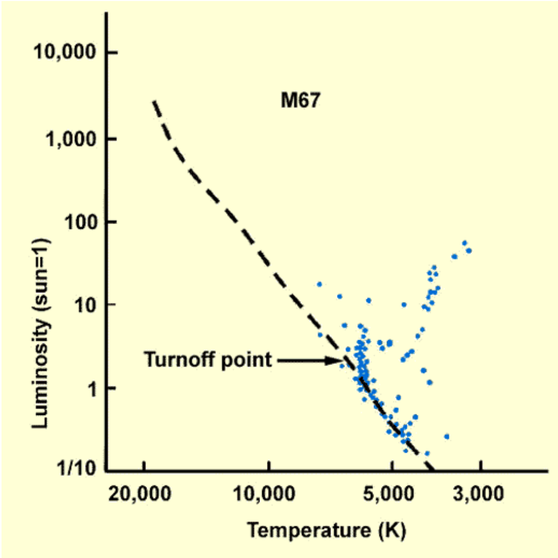


Fig. 9-10: The H-R diagram of the open cluster M67, which is about 5 billion years old. All of its massive stars have "died" and left the main sequence.

There are two kinds of clusters (Figs. 9-11, 12). An **open cluster** contains 10-1000 stars. It is loosely bound, less densely populated, younger, and distributed close to the plane of our Galaxy. A **globular cluster** contains 10^5-10^6 stars. It is densely packed, spherical shape, older (~10-15 billion years), and distributed in the halo of our Galaxy.

Fig. 9-11: The open cluster Pleiades (M45) in Taurus is visible to naked eye. The stars are surrounded by a faint nebula caused by reflection of starlight from dust in the nebula.

Fig. 9-12: The globular cluster M13 in Hercules contains about hundreds of thousands of stars in a region of only about 45 pc in diameter. This cluster is at least 10 billion years old. (Credit: M. Burali, T. Capecchi, M. Mancini (Osservatorio MTM))

9.4 Evolution of binary stars

More than half of all stars on the sky are members of *binaries***.** In some binary systems, the less massive star is a giant, while the more massive one is still on the main sequence. The situation, known as the **Algol paradox**, seems to violate the stellar evolution theories. Why does it happen?

As expected, the more massive star in a binary leaves the main sequence first and swells to a red giant. In cases where the two stars in the binary are not far apart, some of the gas in the red giant's outer layer would transfer to its companion (***Box 9.3***). Hence, the companion becomes more massive while still on the main sequence. (Fig. 9-13) On the other hand, if both stars in a binary are close together and become red giants, they may merge to form a spinning **supergiant**.

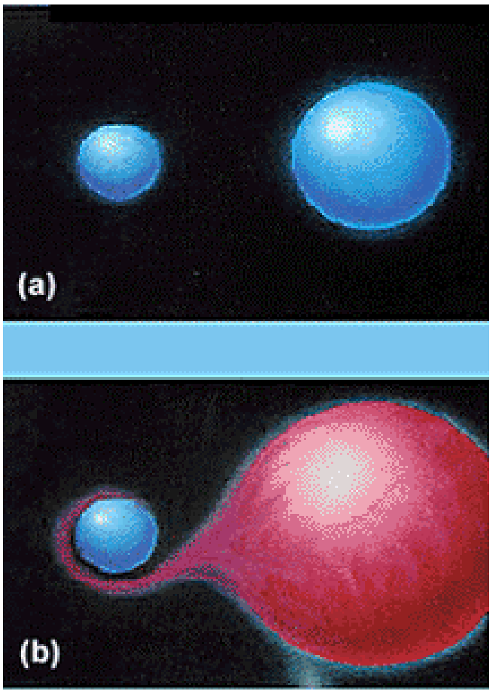
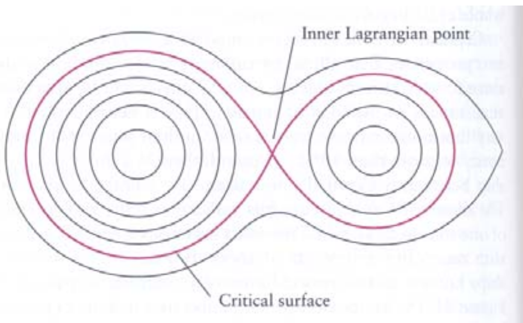


Fig. 9-13: (a) The star on the right is more massive. (b) The more massive star evolves into a giant first, transferring mass to its companion.

Box 9.3 **Mass transfer occur in binaries** *(for reference)*

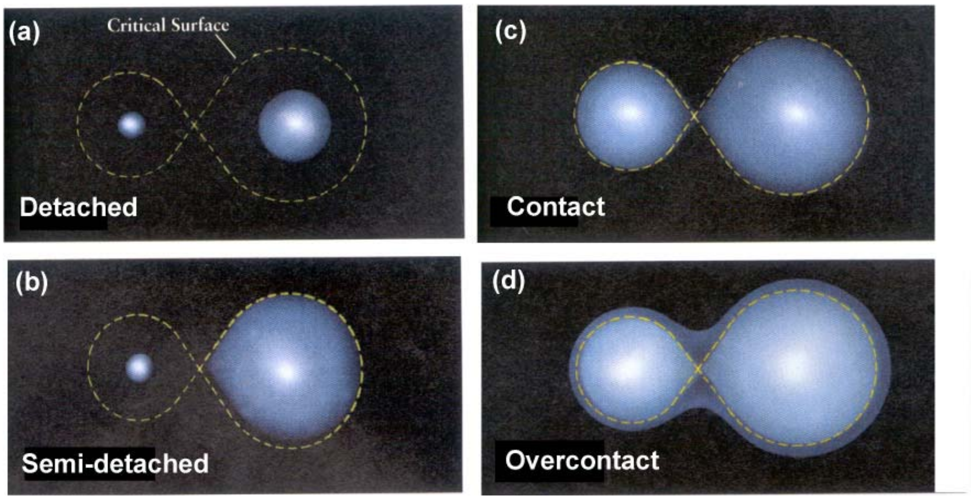
If you consider the gravitational field of a two-body system, the **equipotential surfaces** will be series of curves as shown in the figure. The coloured red curve is called the **critical surface**. Each half of the curve is known as the **Roche lobe**. In a binary, if a star expands to fill the Roche lobe, its mass will transfer to its companion easily.



Equipotential contours.

In many binaries, the stars are so far apart that even during their red-giant stages the surfaces of the stars remain well inside their Roche lobes. Each star thus lives out its life as if it were single and isolated. Such a binary is referred to a **detached binary** as shown in the following figure.

When one star expands and becomes a red giant, and fills its Roche lobe, the binary is a **semi-detached binary**. Mass transfer is often observed in such binaries. In the unlikely situation in which both stars exactly fill their Roche lobes, the system is known as a **contact binary**. A more possible scenario is that they overflow their Roche lobes, giving rise to a common envelope of gas. Such a system is called an **overcontact binary**.



(a) Detached, (b) semi-detached, (c) contact, and (d) overcontact binaries.