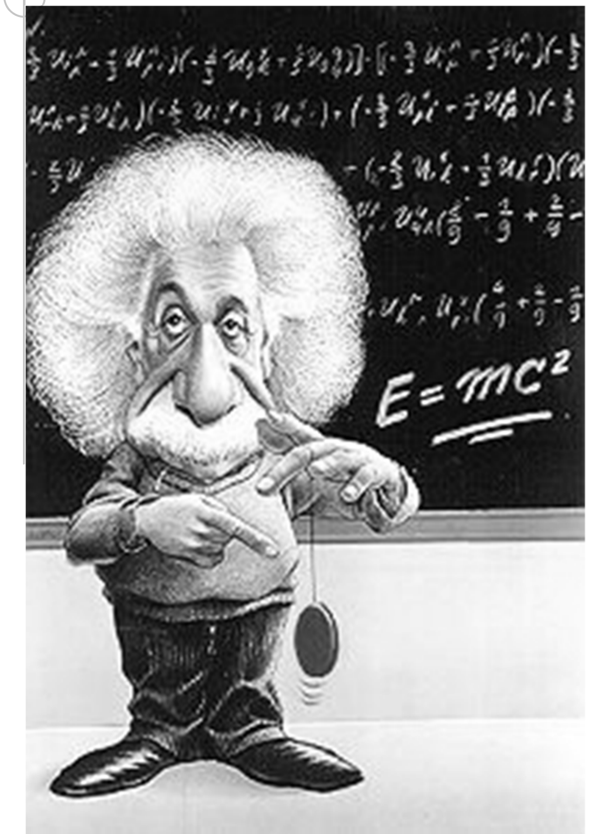
Chapter 11

Relativity

1931年，愛因斯坦與卓別林

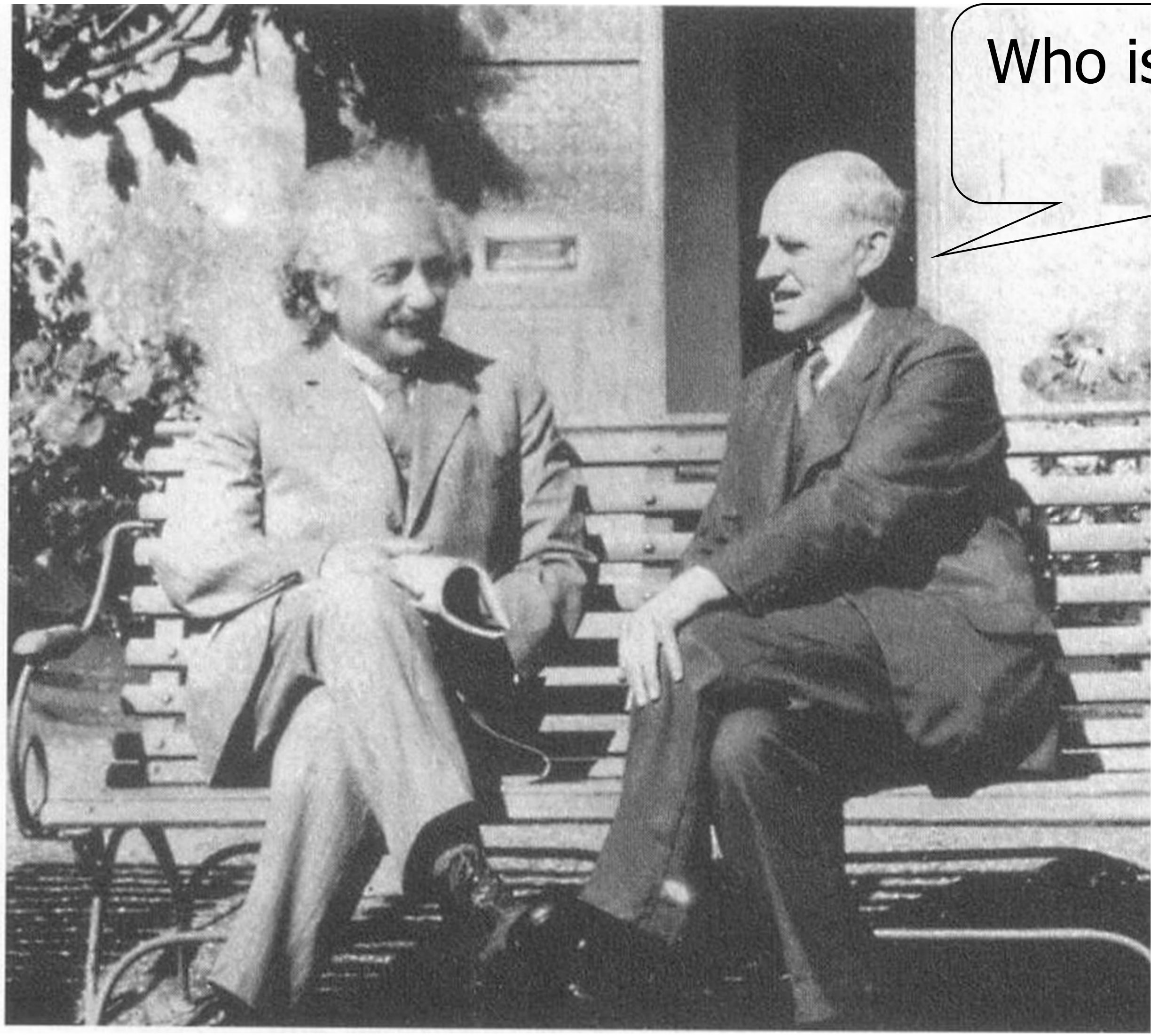


「我們兩個都是名人，但是成名的原因卻有所不同，我出名是因為任何人都看懂我在做甚麼，可是你的成名卻因為沒有人知道你在做甚麼。」

差利 卓別林 (Charlie Chaplin, 1889-1977)

《星空的思索》,p.131

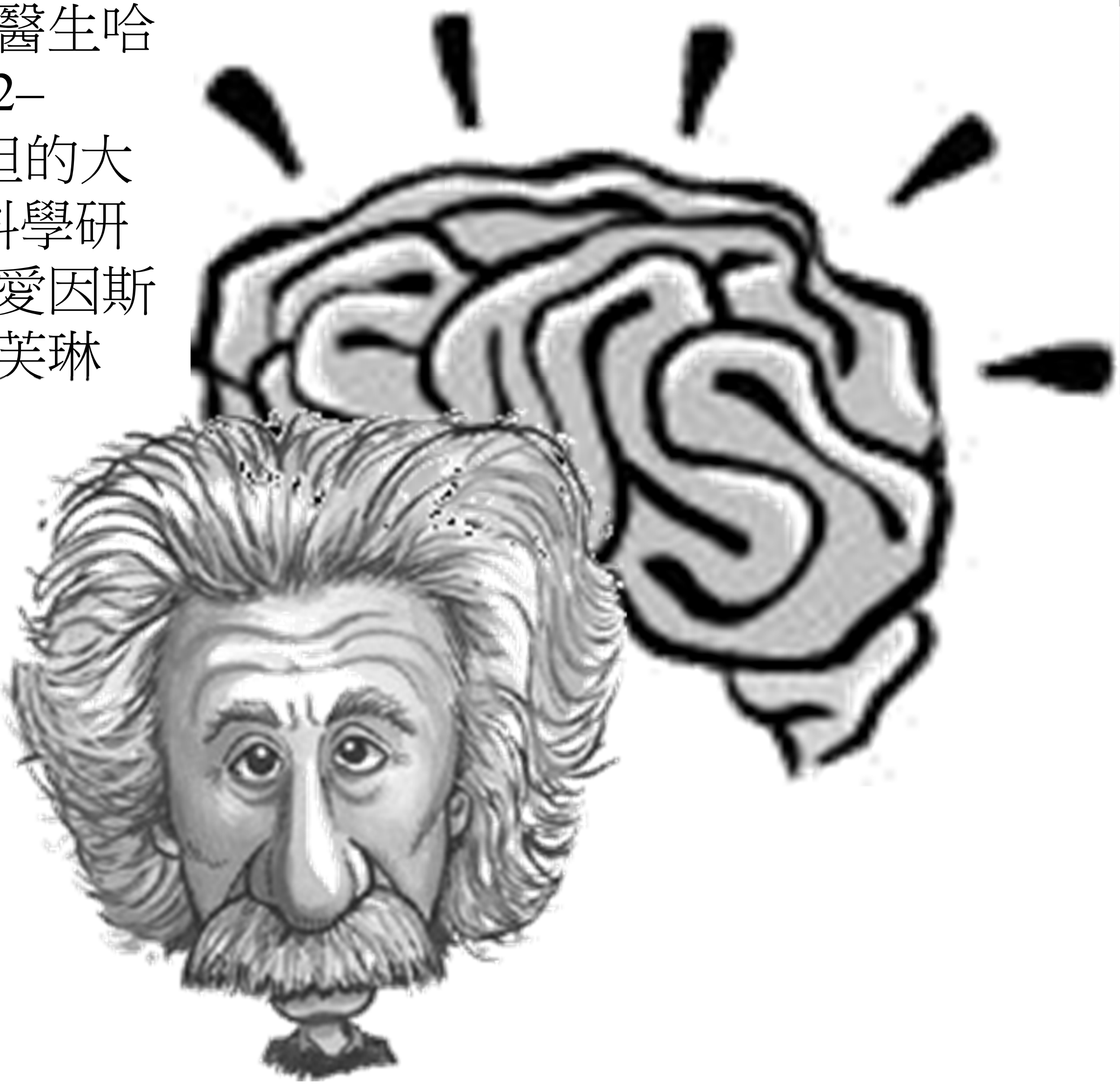
Who is the third one?



愛因斯坦和愛丁頓(Arthur Eddington, 1882-1944)
（攝於1930 年）

普林斯頓醫院的病理科醫生哈維 (Thomas Harvey, 1912–2007) ，竊取了愛因斯坦的大腦，分成了240份，作科學研究。至哈維80歲時才將愛因斯坦的大腦歸還給孫女艾芙琳 (Evelyn Einstein)。

Marian Diamond發現愛因斯坦的大腦中的神經膠質細胞 (glia) 確實比一般人的多。



Science American, April 2004, pp.54-61；《星空的思索》, p.130

Relativity

11.1 The nature of light

11.2 The rebirth of ether

11.3 Einstein's revolution

11.4 Lorentz transformation

11.5 Concepts of Space-time

11.6 Paradoxes of relativity

11.7 Velocity, Mass, Energy and Time reversal

Relativity

11.8 Difficulties of Newton's theory of gravitation

11.9 The principle of equivalence and the general relativity

11.10 Bending of light by gravity

11.11 Curved space-time

11.12 Some consequences

11.13 Experimental evidence

The nature of light

What is light?

- ✓ Def: distance traveled per unit time

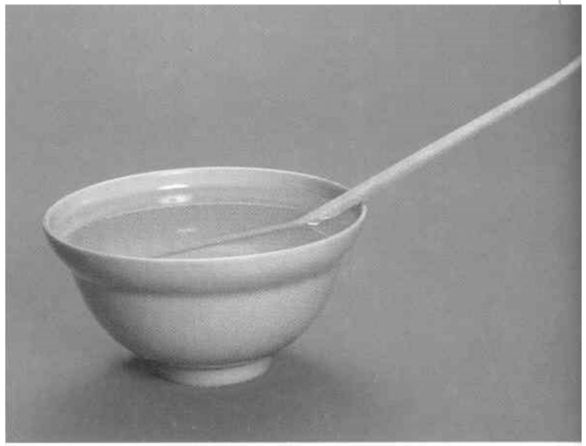
$$\text{speed} = \frac{\text{distance}}{\text{time taken}} \quad \text{unit:}[\mathrm{ms^{-1}}]$$

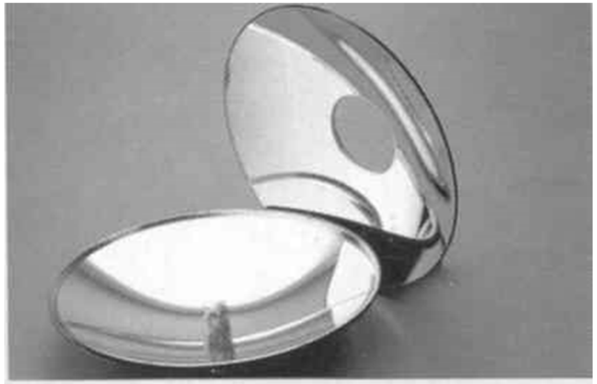
- ✓ example: running ~ 10 $\mathrm{ms^{-1}}$; car ~ 28 $\mathrm{ms^{-1}}$; aeroplane ~ 280 $\mathrm{ms^{-1}}$; sound ~ 330 $\mathrm{ms^{-1}}$; light ~ ???

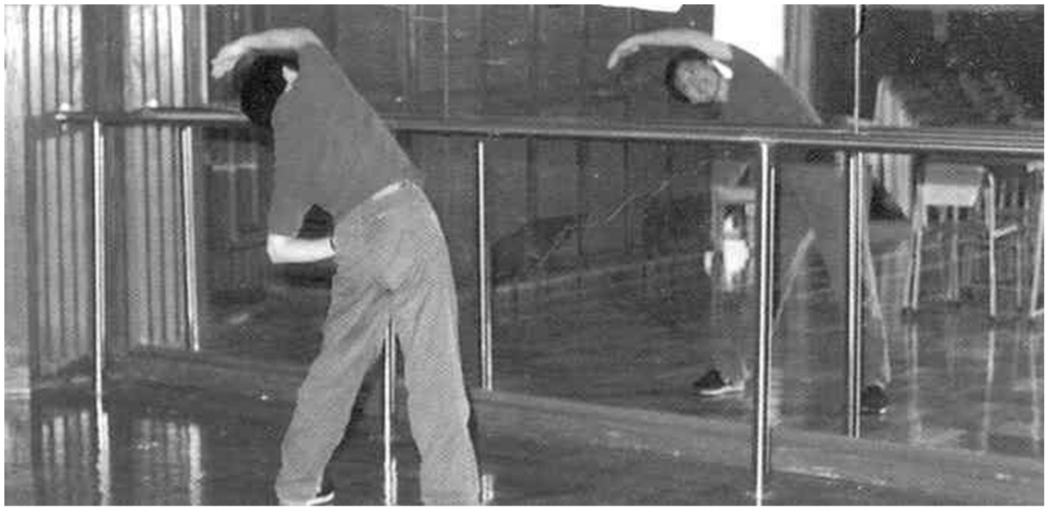
299,792,458 $\mathrm{ms^{-1}}$

So what is light ??

- ✓ Newton: Light is particle because some properties of light are explained by particle picture, e.g., reflection, refraction.
- ✓ But has not constructed a theory.

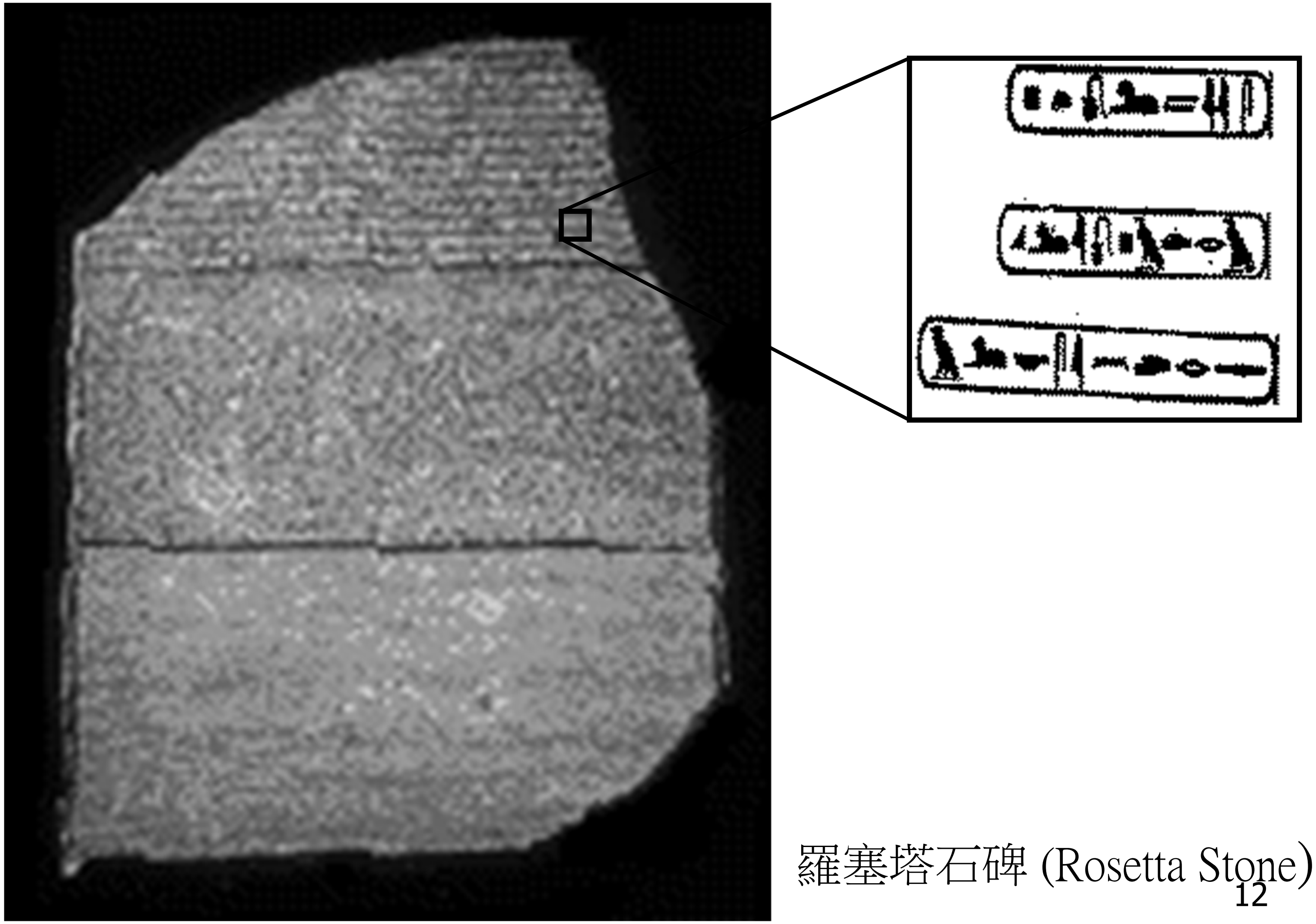






Young, Thomas (1773 - 1829)

- ✓ British physicist, physician and Egyptologist (埃及古物學家)
- ✓ assisted in deciphering the Egyptian hieroglyphics (象形文字系統) inscribed on the Rosetta Stone (羅塞塔石碑).



羅塞塔石碑 (Rosetta Stone)

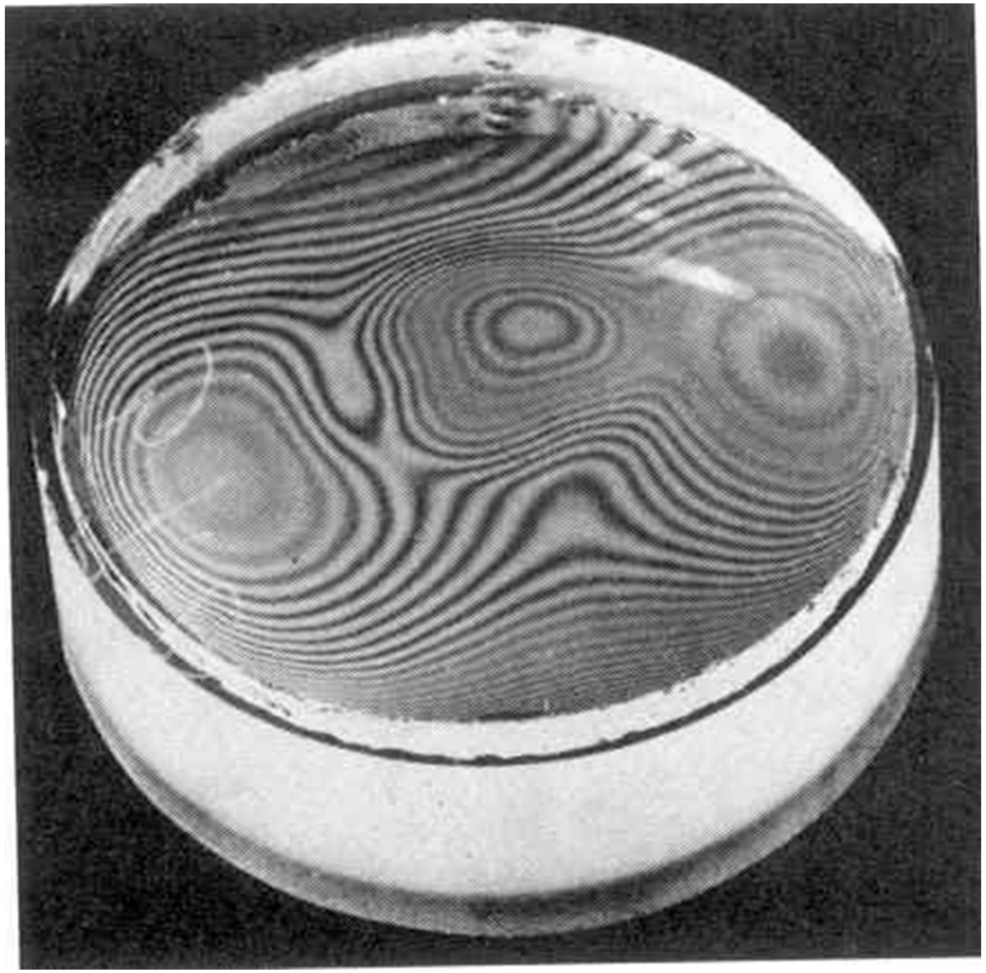
Young, Thomas (1773 - 1829)

- ✓ Best known for contributions in the optics.
- ✓ In 1801, Thomas Young: light is wave. Light have all properties of wave (reflection, refraction, diffraction and interference).

water ripples show all properties of wave

Interference of water waves

- ✓ In 1801, famous young's double slit experiment.
- ✓ Thomas Young: Light has all properties of waves.
- ✓ *Light is wave*

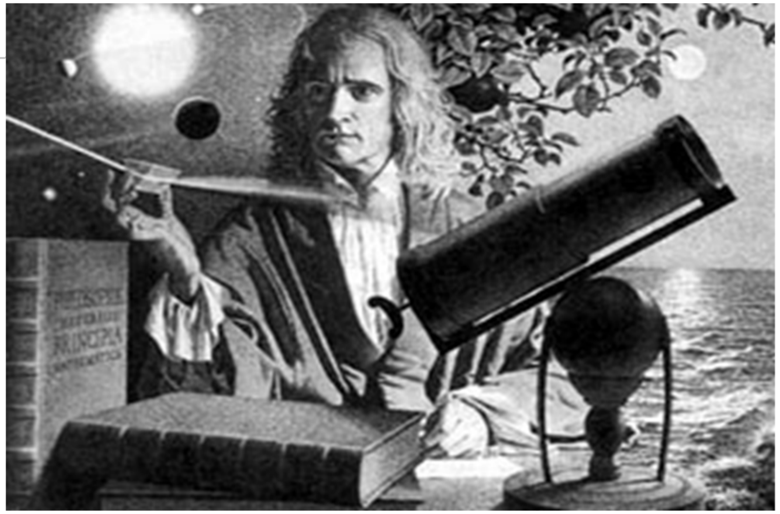


- ✓ Maxwell was born in Scotland 蘇格蘭, and was educated at the universities of Edinburgh and Cambridge. He became the first professor of experimental physics at Cambridge
- ✓ In 1873 he constructed four equations that describe the nature of electromagnetic waves (e.g., light, radio waves, x-rays).

James Clerk Maxwell (1831-1879)

- ✓ also developed the kinetic theory of gases that explains the physical properties and nature of a gas.
- ✓ investigation of colour vision and the principle of thermodynamics.

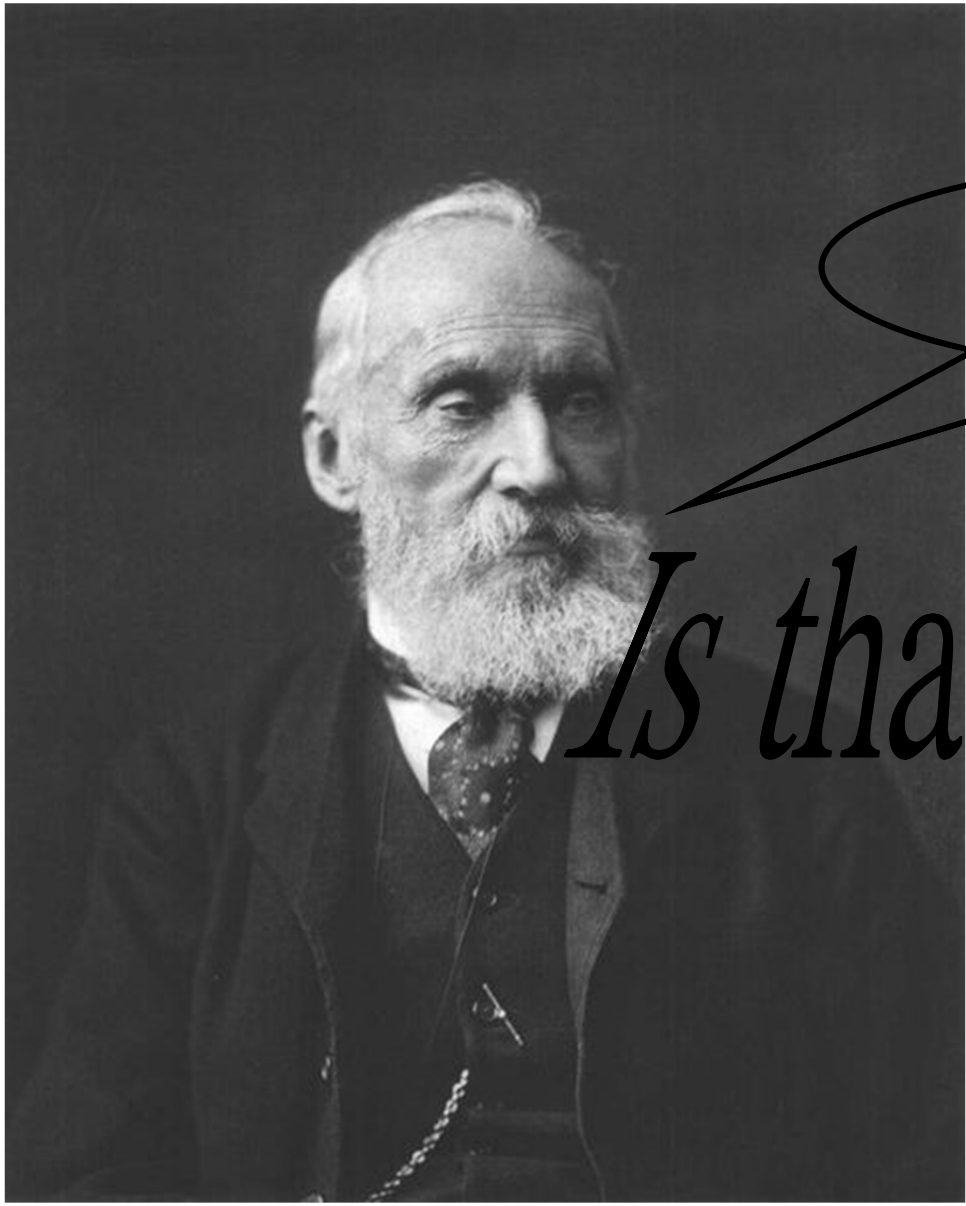
James Clerk Maxwell (1831-1879)



Newton, Isaac (1642--1727)

By the end of the 19th century, scientists regarded physical phenomena can be explained by Newton's law of gravitation and Maxwell's EM (light) theory.

James Clerk Maxwell (1831-1879)



Lord Kelvin (1824-1907)

Most of the works about physics had been done

Is that true!?

In 1900, Kelvin gave a lecture titled *Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light* and said…

The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science, Series 6, volume 2, page 1 (1901)

At the end of his lecture, he said that there were still two “dark clouds ”.

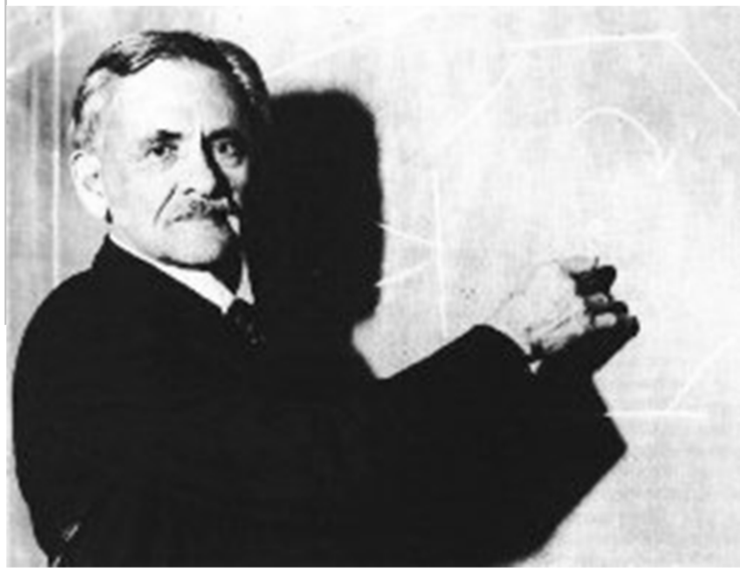
- One of that cloud is the Michelson-Morley experiment. The result cannot be explained using classical physic. The postulation of special relativity was based on that experiment

- Another cloud is the experiment of blackbody (黑體輻射). That experiment cannot be explained by the classical physic too. By solving the result of that experiment and the works of many scientist, quantum mechanics was finally built up.

The rebirth of ether

- ✓ Maxwell predicted the light speed = 299,792,458 m/s.
- ✓ What is this speed measured against? Stationary ether?!
- ✓ Physicists expected: as the speed of the Earth is changing (relative to the Earth, speed of ether is changing), so light speed should vary from time to time.

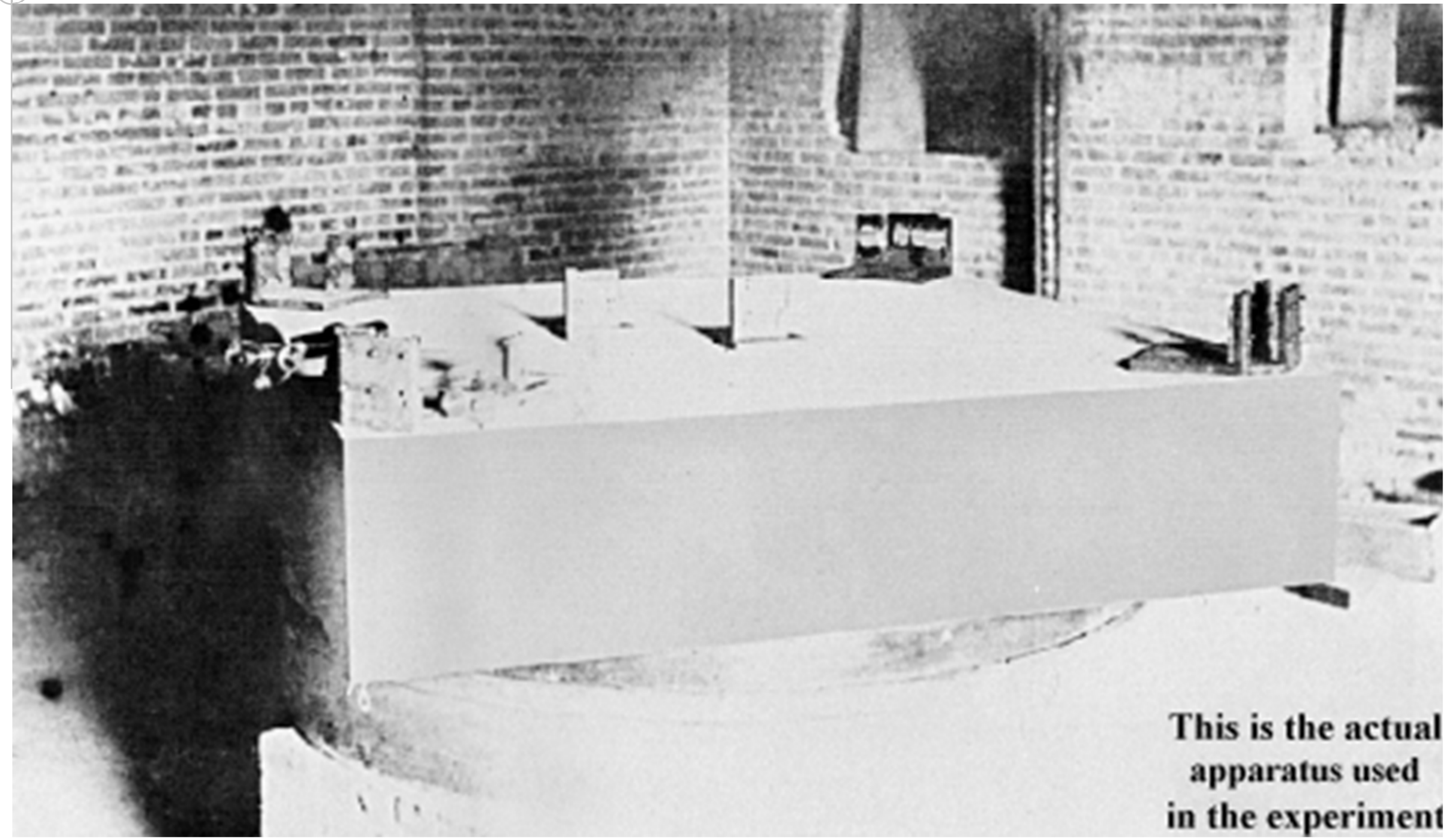
But how to test it?



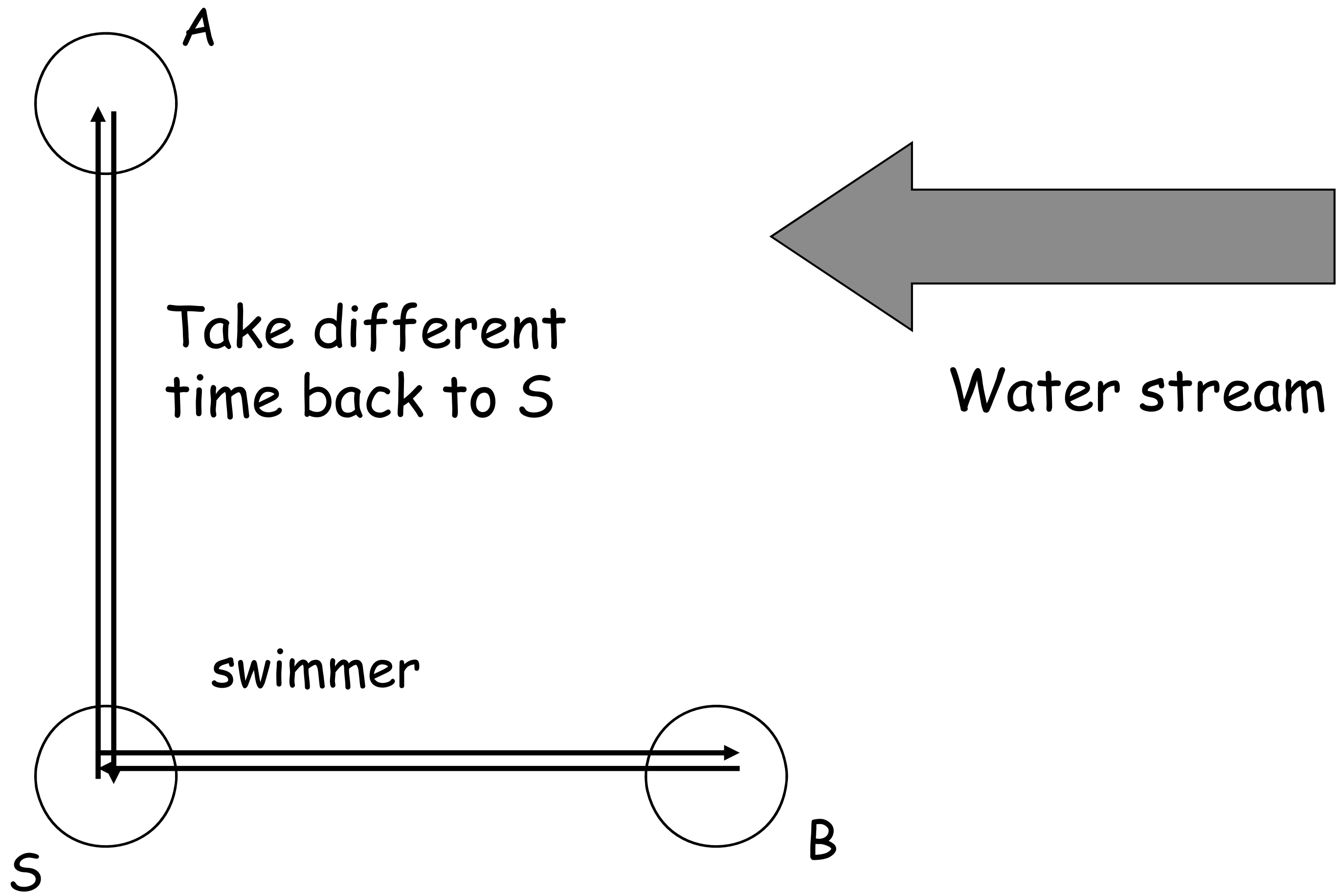
Michelson, Albert Abraham
(1852 - 1931)

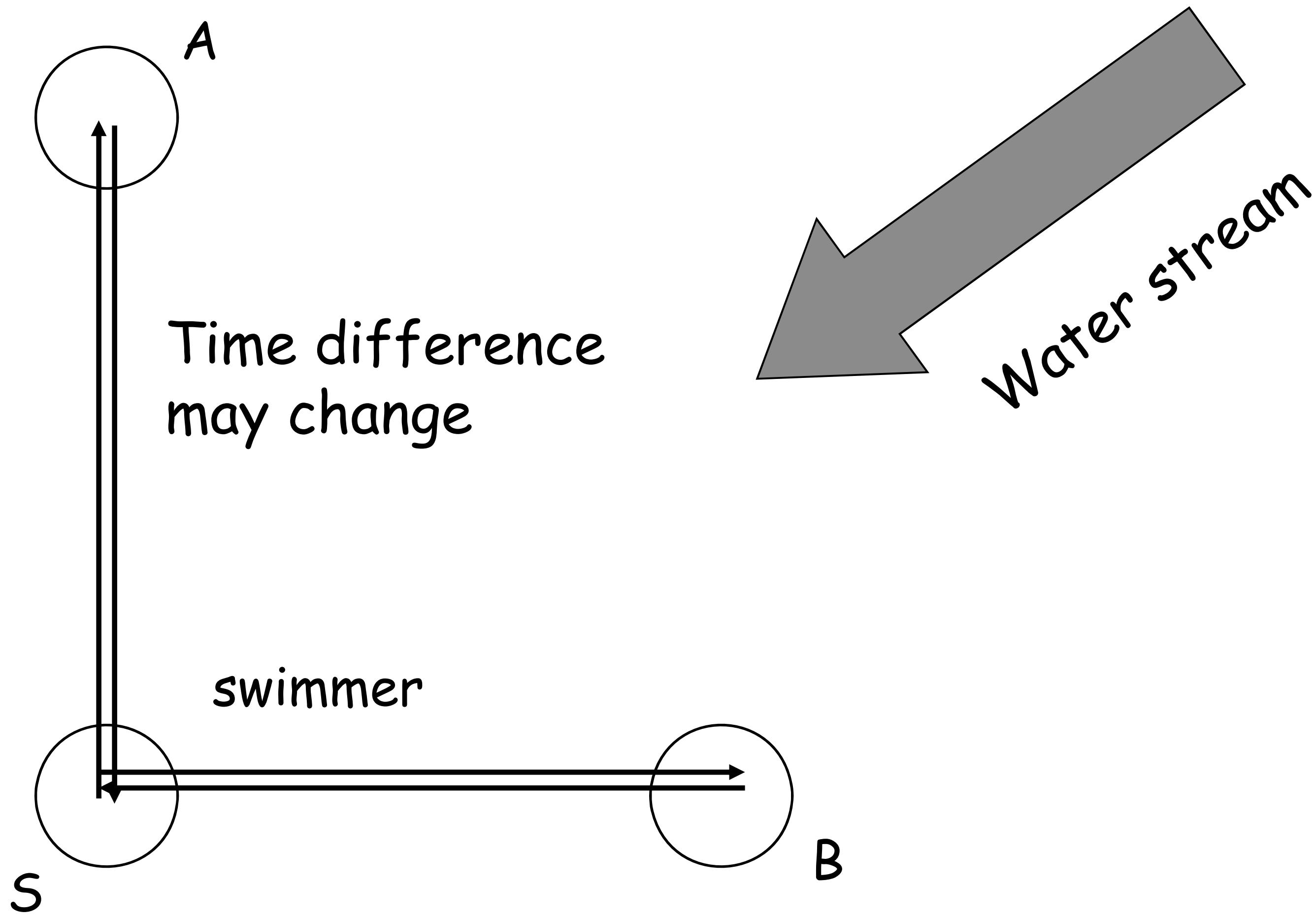
- ✓ German-born American physicist
- ✓ Best known for his famous experiment (with American chemist Edward Williams Morley): Michelson-Morley experiment
- ✓ accurate measurement of speed of light on the Earth.
- ✓ In 1907 he was awarded the Nobel Prize in physics for developing extremely precise instruments and conducting important investigations with them

What was the experiment about?

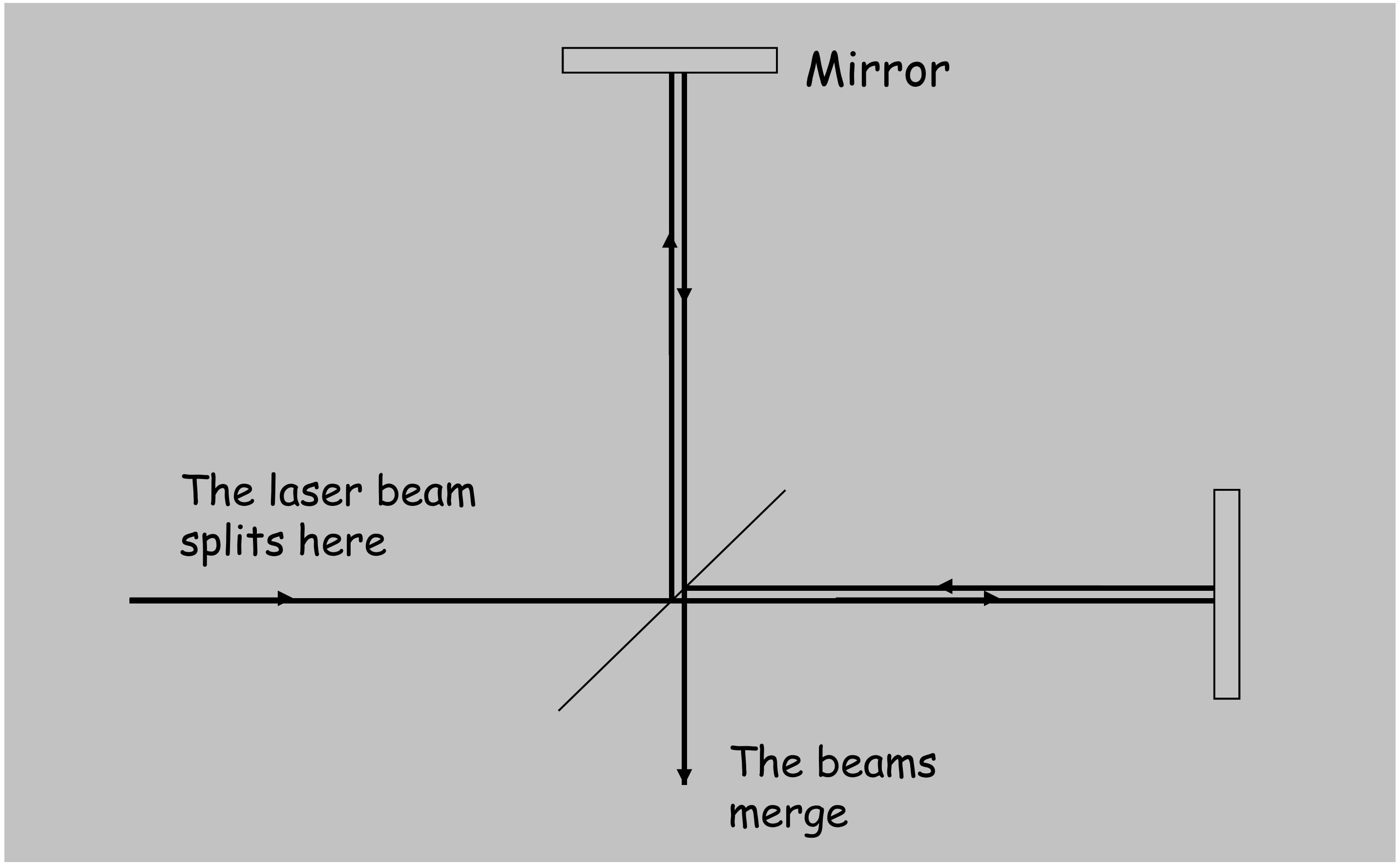


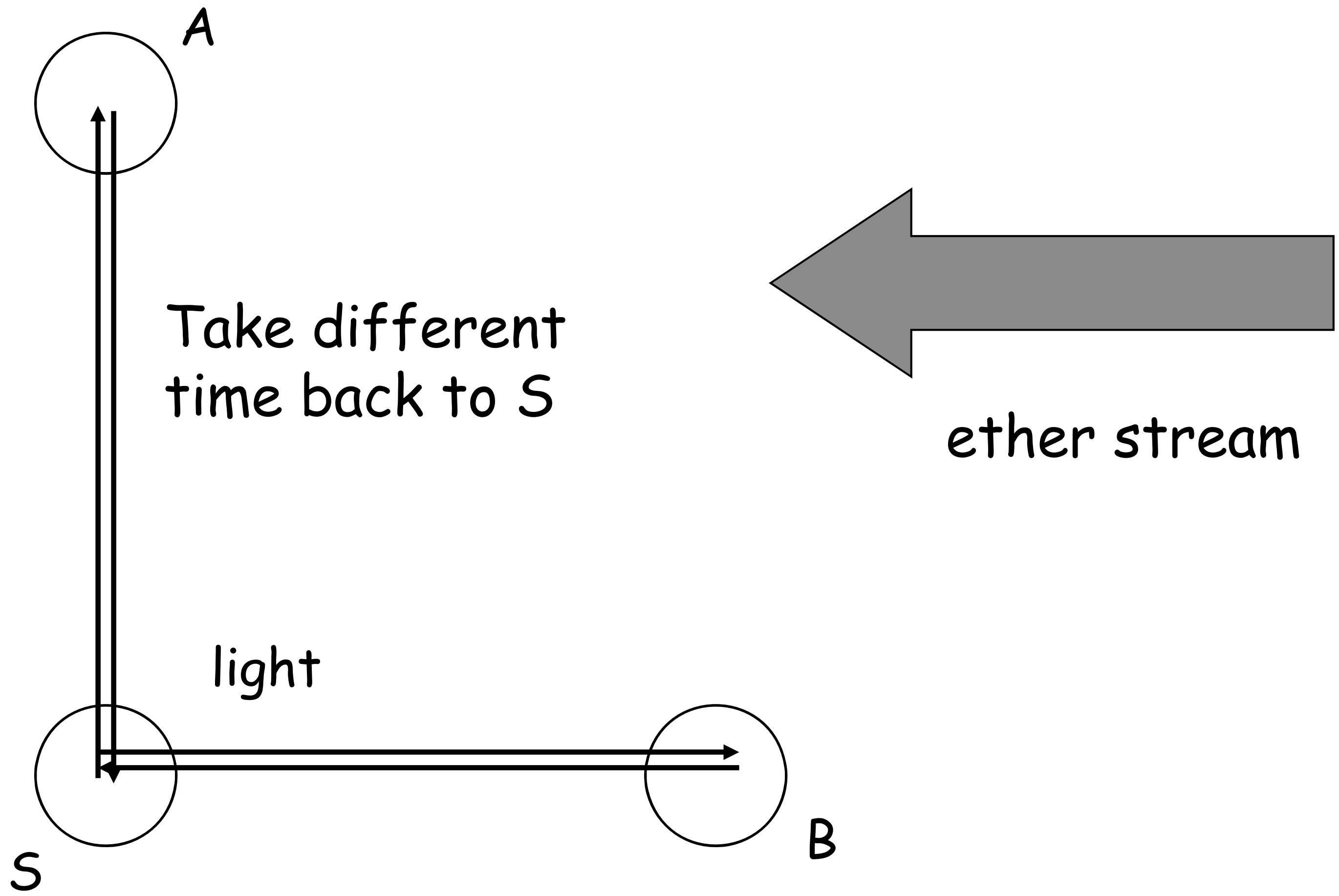
http://www.upei.ca/~phys221/nem/Apparatus/apparatus.html

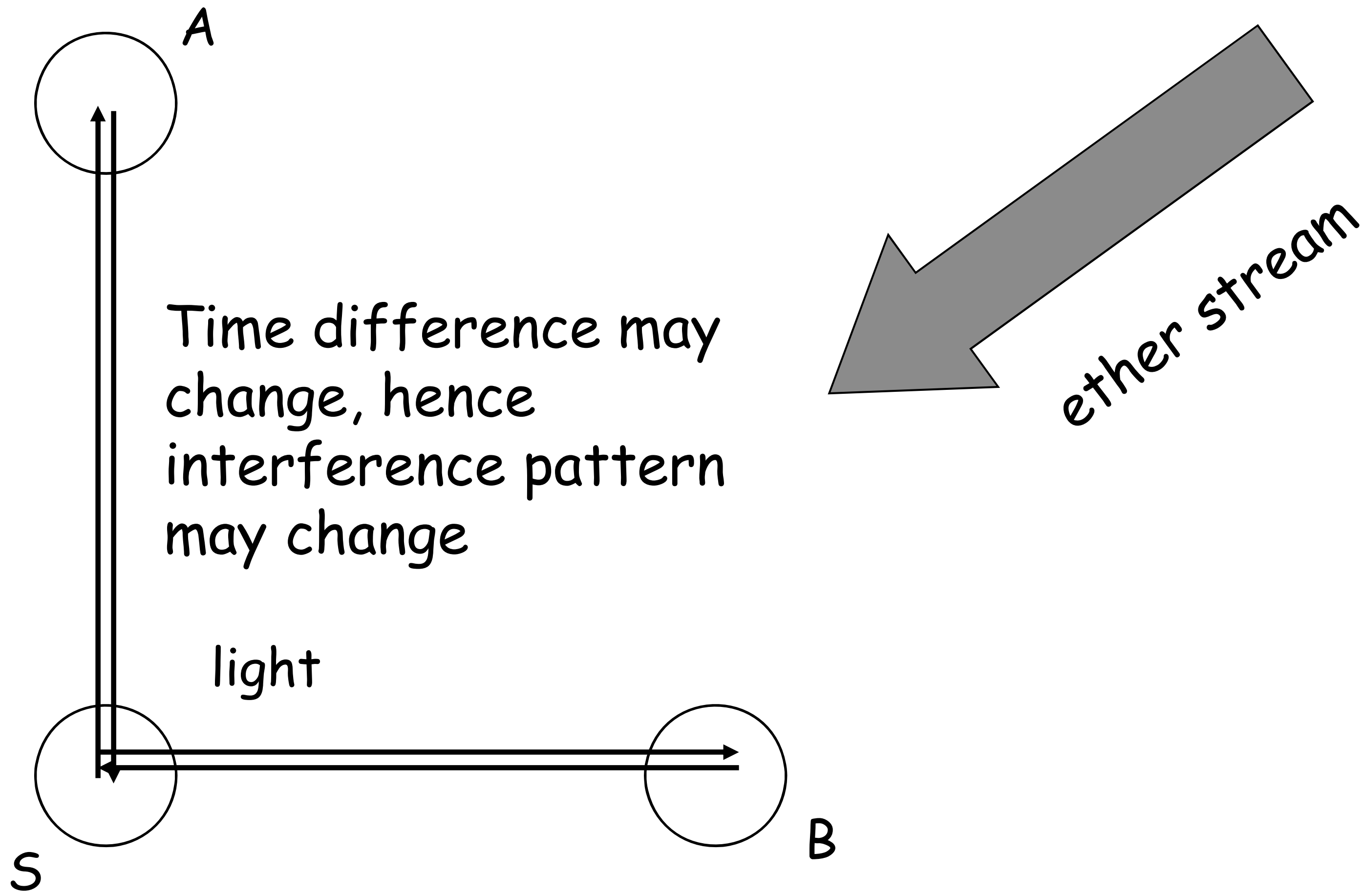

A
Take different
time back to S
Water stream
swimmer
S
B

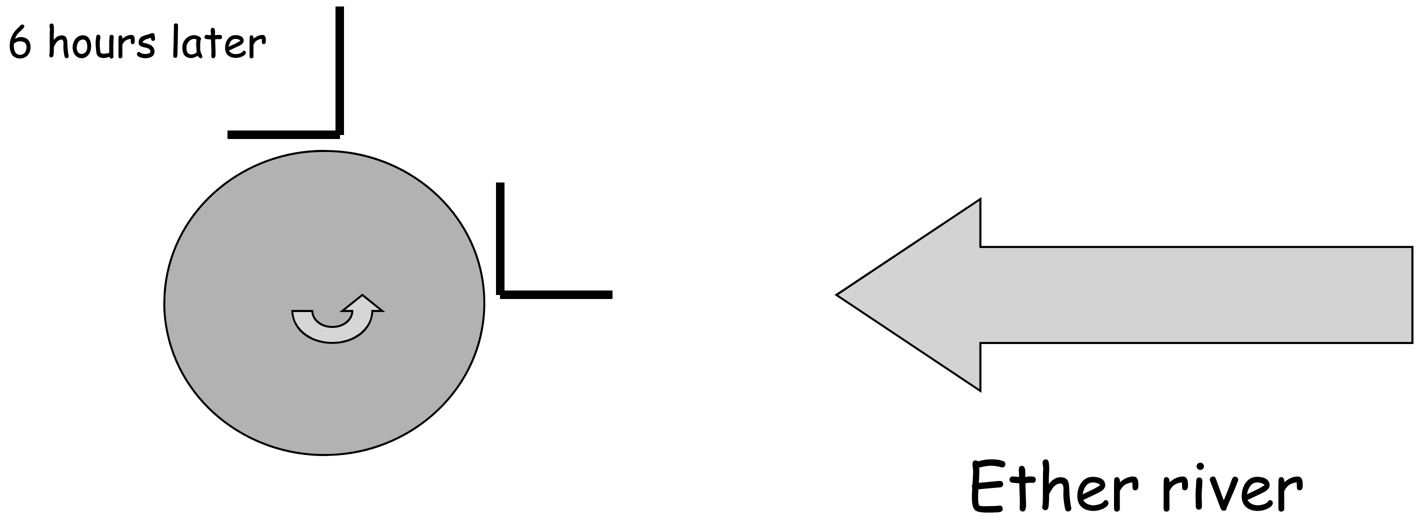

A
Time difference
may change
Water stream
swimmer
S
B

Michelson-Morley experiment



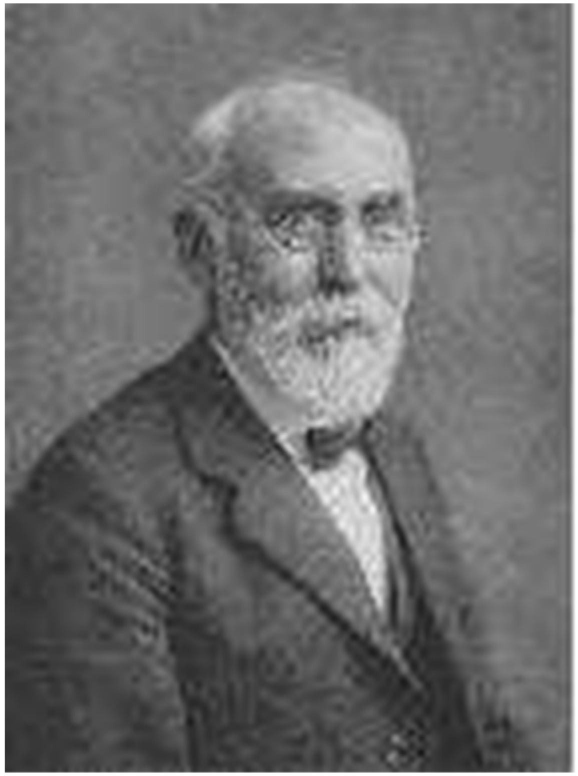

A
Take different
time back to S
ether stream
light
S
B


A
Time difference may change, hence interference pattern may change
light
ether stream
S
B



expected: interference pattern changed.
But the experiments show the pattern is
always the same !! Why??

- ✓ The FitzGerald-Lorentz contraction hypothesis: Due to some *unknown reasons*, an object contracts along its direction of motion.
- ✓ Try to retain the concept of ether and explain the M-M experiment
- ✓ But experiments did not support.

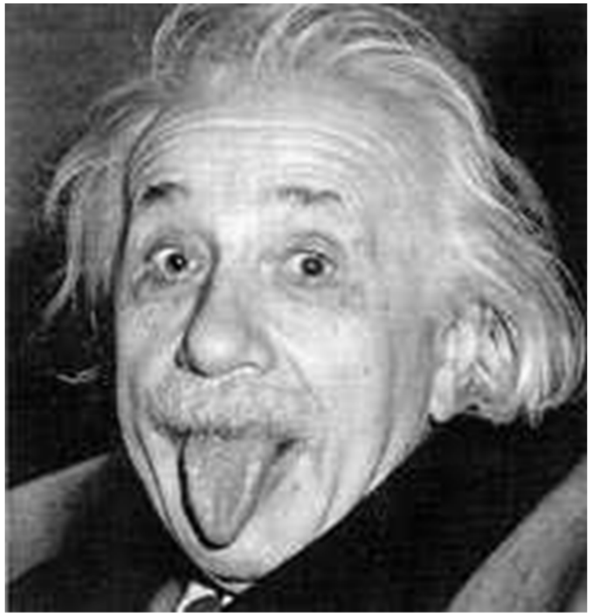


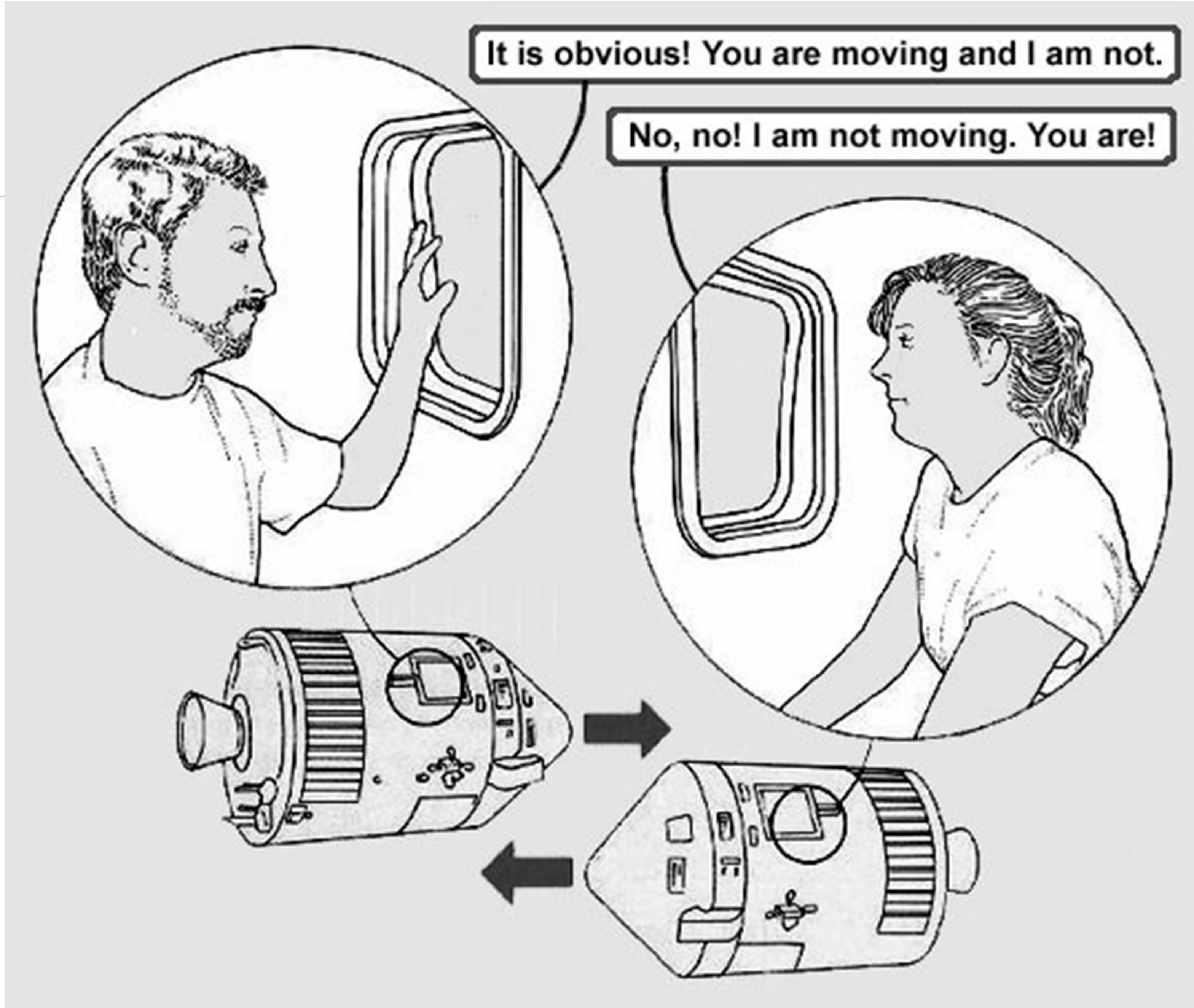
Einstein's revolution

- ✓ In 1903, Einstein announced his proposal.
  - ❖ Ether is a hopeless concept.
  - ❖ Electromagnetic waves propagate without medium.
  - ❖ Light speed is a constant no matter the measurer is moving or not.
- ✓ Published a paper "On the Electrodynamics of Moving Bodies" — later called Special relativity.

Postulates of special relativity

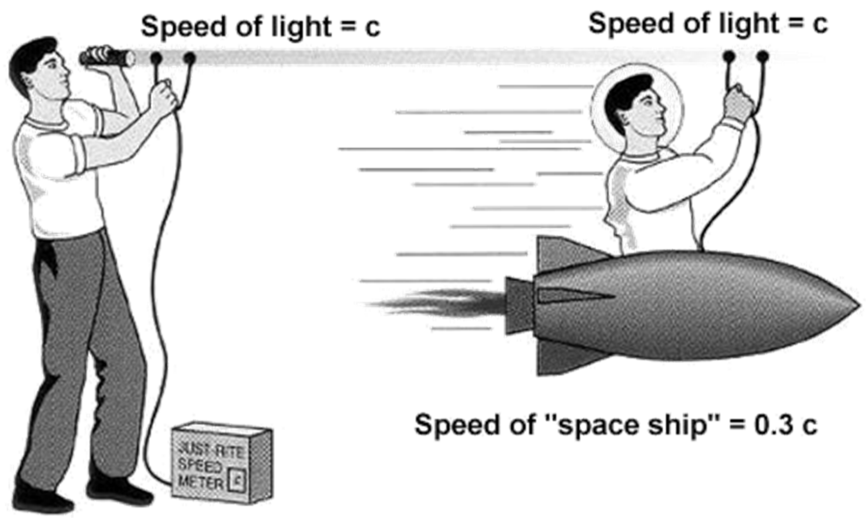
- ✓ **The Principle of relativity:** The laws of physics are the same in all inertial systems (that is at rest, or constant velocity). *No preferred* inertial system exists.

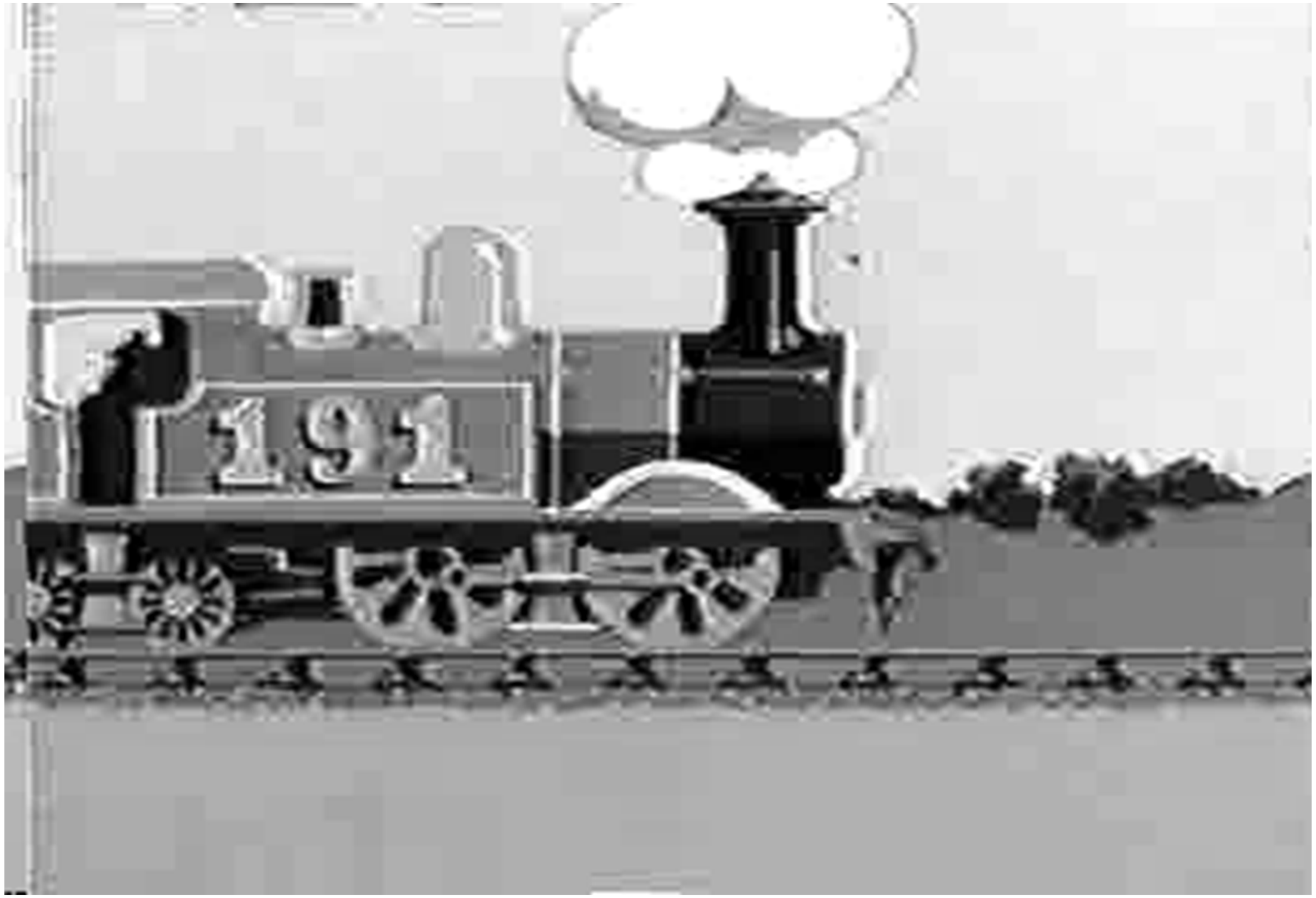



It is obvious! You are moving and I am not.
No, no! I am not moving. You are!

Postulates of special relativity

✓ **The constancy of the speed of light:**
The speed of light in free space has the same value *c* in all inertial systems.

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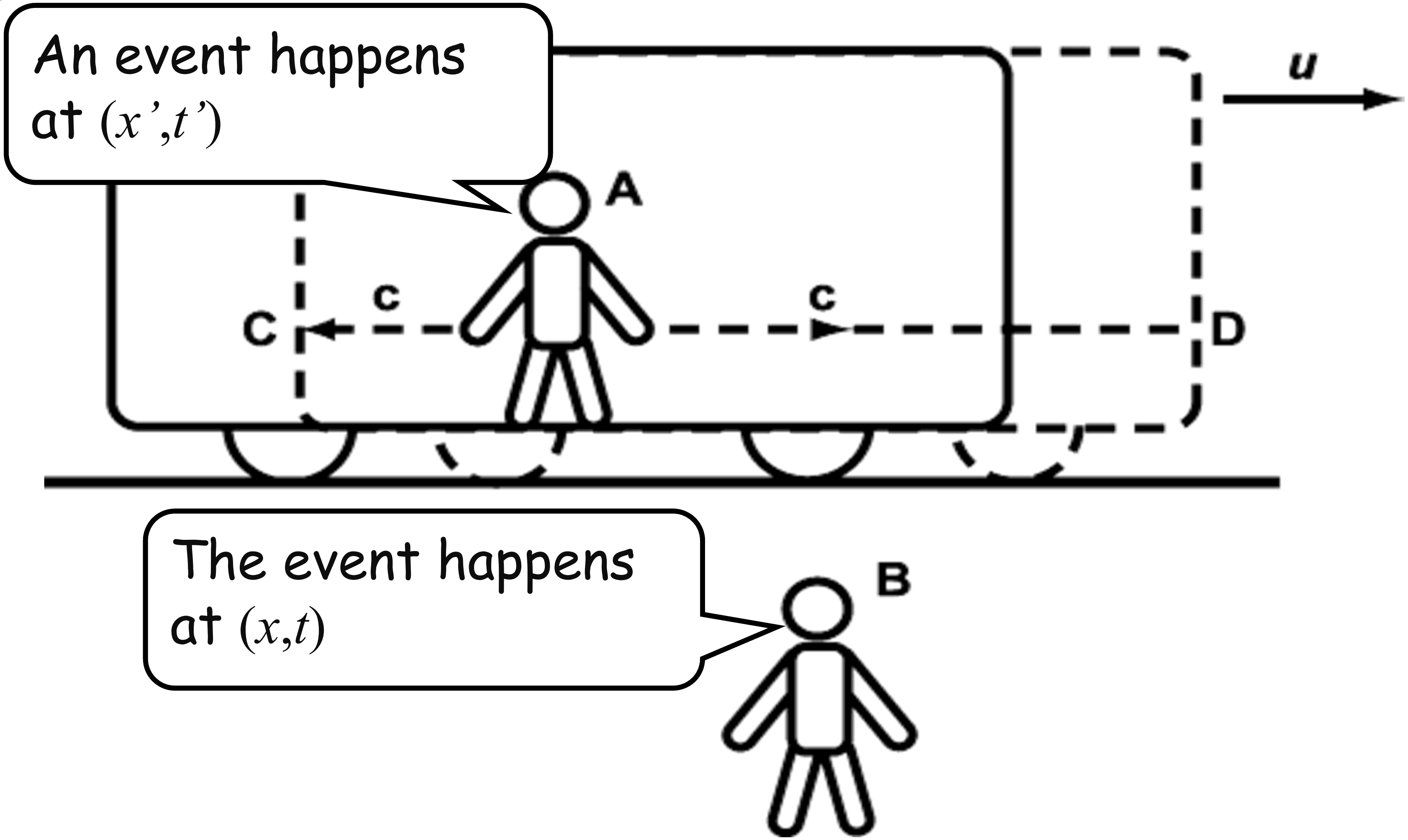
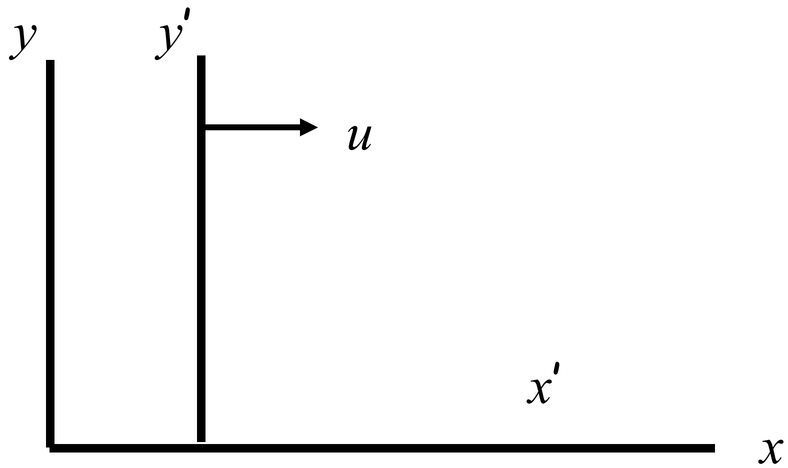
Lorentz transformation


An event happens at (x',t')
u
A
c
c
C
D
The event happens at (x,t)
B

Galilean transformation

S' moving along x with speed u

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$



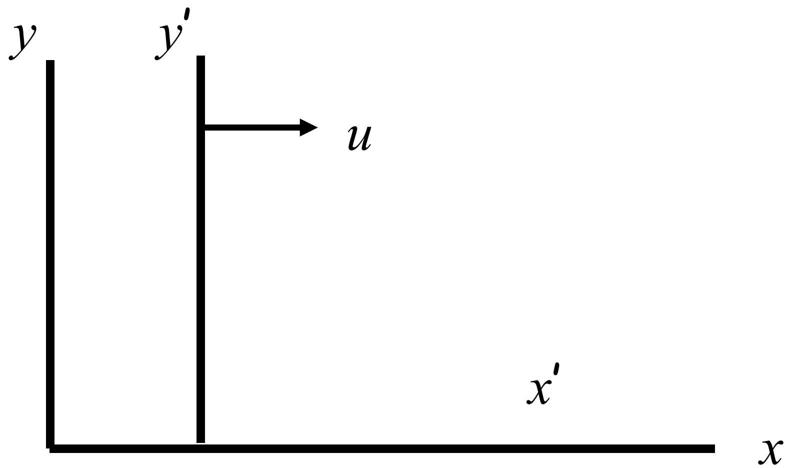
Is that still correct in relativity?

In general, a transformation between the two systems:

$$x' = a_{11}x + a_{12}t$$
$$t' = a_{21}x + a_{22}t$$

Consider an event at $x' = 0$ then $x = ut$. Hence,

$$x' = a_{11}\left(x - ut\right)$$
$$t' = a_{21}x + a_{22}t$$



Suppose light leaves the origin of S', it must travels at speed of light, i. e., $x'^2 - c^2 t'^2 = 0$

It becomes

$$\left(a_{11}^2 - a_{21}^2 c^2\right) x^2 - 2\left(a_{21} a_{22} c^2 + a_{11}^2 u\right) tx - \left(a_{22}^2 c^2 - a_{11}^2 u^2\right) t^2 = 0$$

As observed in S, it must also travels at speed of light, i. e., $x^2 - c^2 t^2 = 0$

$$a_{11}^2 - a_{21}^2 c^2 = 1$$

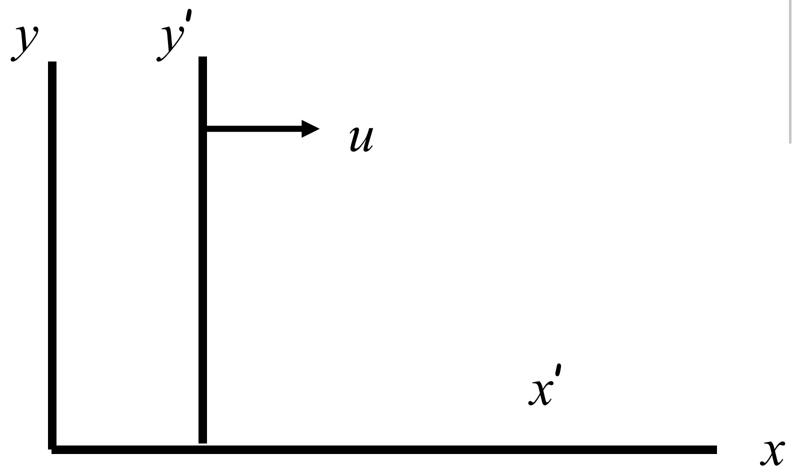
$$a_{21} a_{22} c^2 + a_{11}^2 u = 0$$

$$a_{22}^2 c^2 - a_{11}^2 u^2 = c^2$$

Then solve for a_{11}, a_{21} and a_{22}.

$$a_{11} = \gamma, a_{21} = -\frac{\beta\gamma}{c}, a_{22} = \gamma$$

where, $\gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = u/c$



Therefore, the Lorentz transformation reads

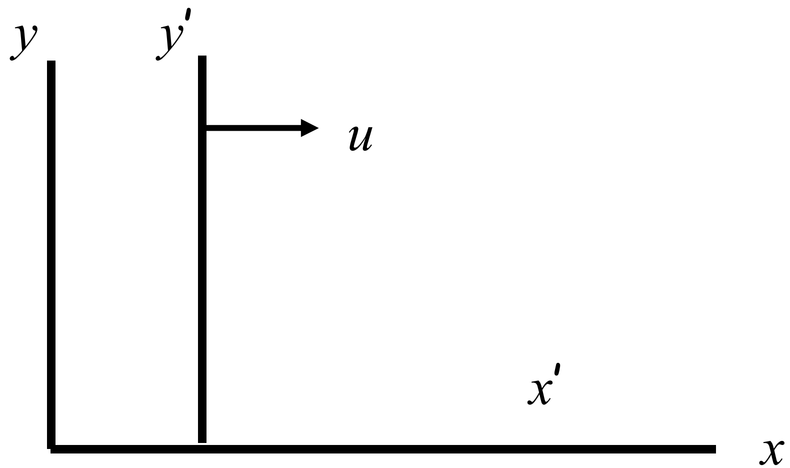
$$\begin{aligned} x' &= \gamma(x - ut) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \beta x/c) \end{aligned} \qquad \begin{aligned} x &= \gamma(x' + ut') \\ y &= y' \\ z &= z' \\ t &= \gamma(t' + \beta x'/c) \end{aligned}$$

the Lorentz transformation:

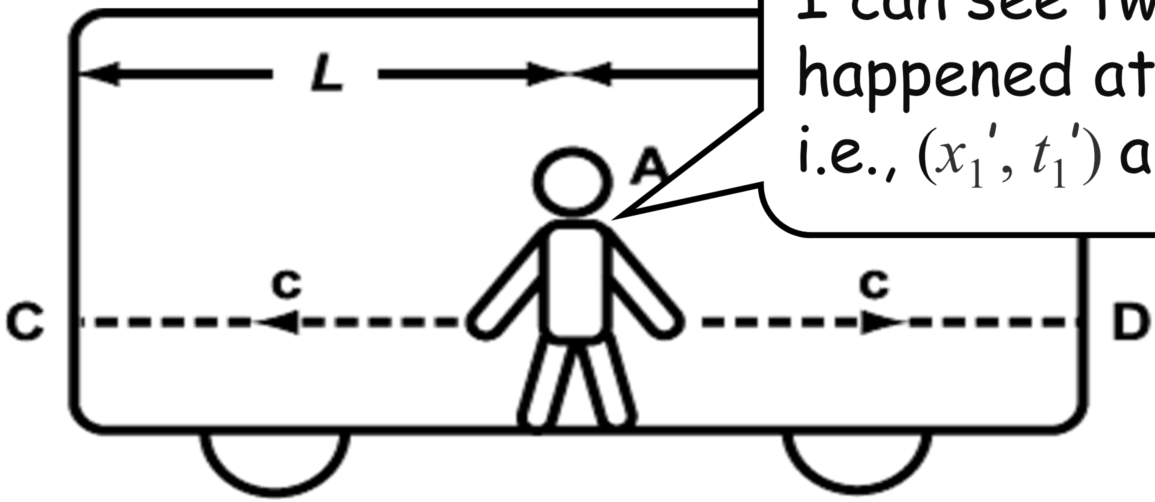
$$
\begin{aligned}
x' &= \gamma(x - ut) & \qquad x &= \gamma(x' + ut') \\
y' &= y & y &= y' \\
z' &= z & z &= z' \\
t' &= \gamma(t - \beta x/c) & t &= \gamma(t' + \beta x'/c)
\end{aligned}
$$

Galilean transformation:

$$
\begin{aligned}
x' &= x - ut \\
y' &= y \\
z' &= z \\
t' &= t
\end{aligned}
$$



Simultaneity



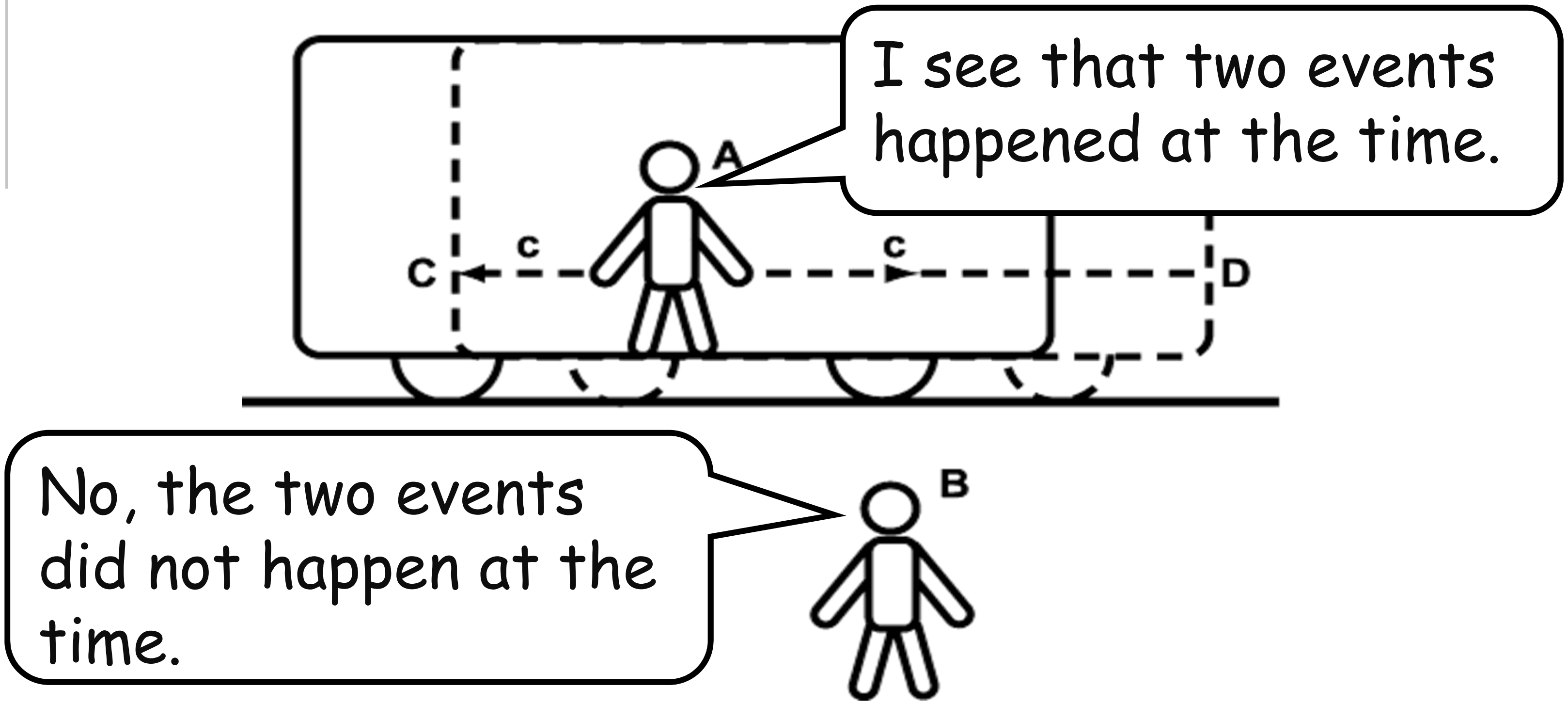
I can see two events happened at the time, i.e., (x_1', t_1') and (x_2', t_2') .

Will they happen at the same time as observed in S ?

Simultaneity

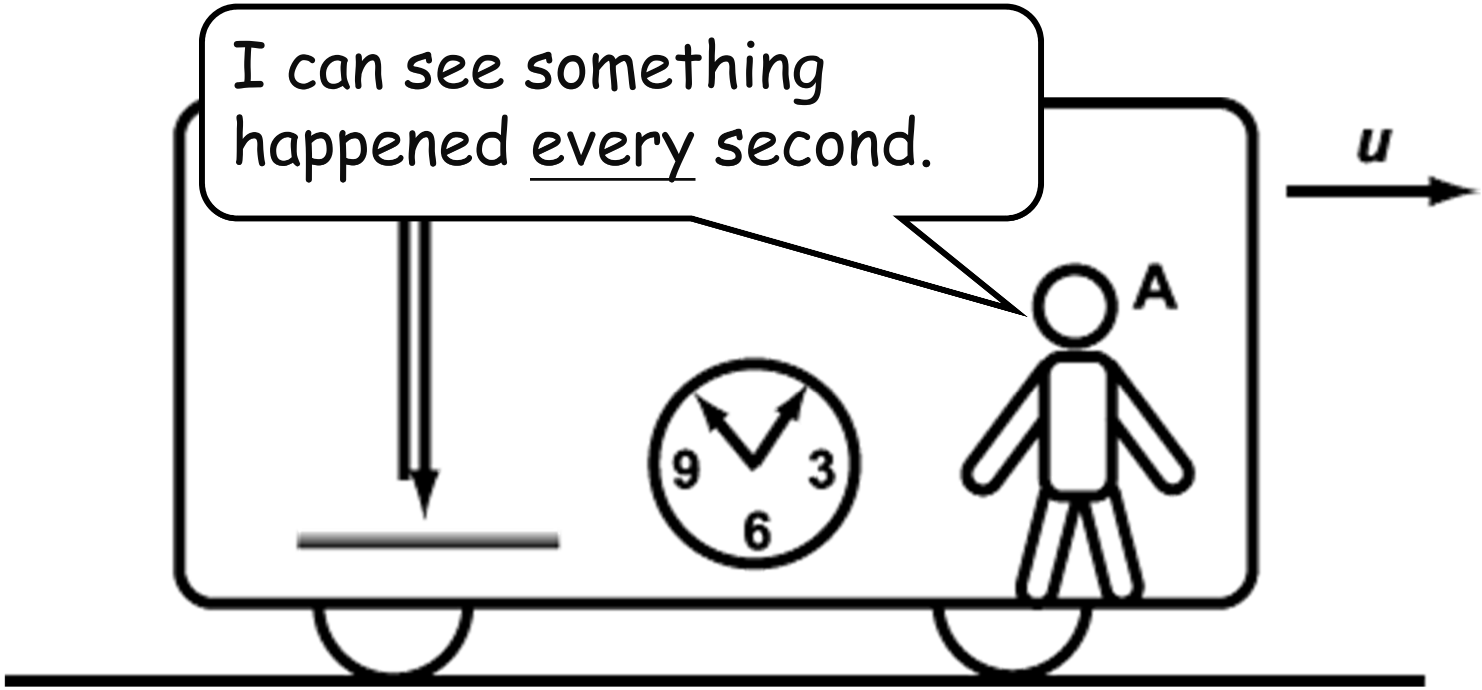
From the Lorentz transformation: $t = \gamma\left(t' + \beta x'/c\right)$

In S frame: $t_2 - t_1 = \gamma\left(t_2' - t_1'\right) + \gamma\beta\left(x_2' - x_1'\right)/c = \gamma\beta\left(x_2' - x_1'\right)/c$



Simultaneity depends on frames of reference.

Time dilation

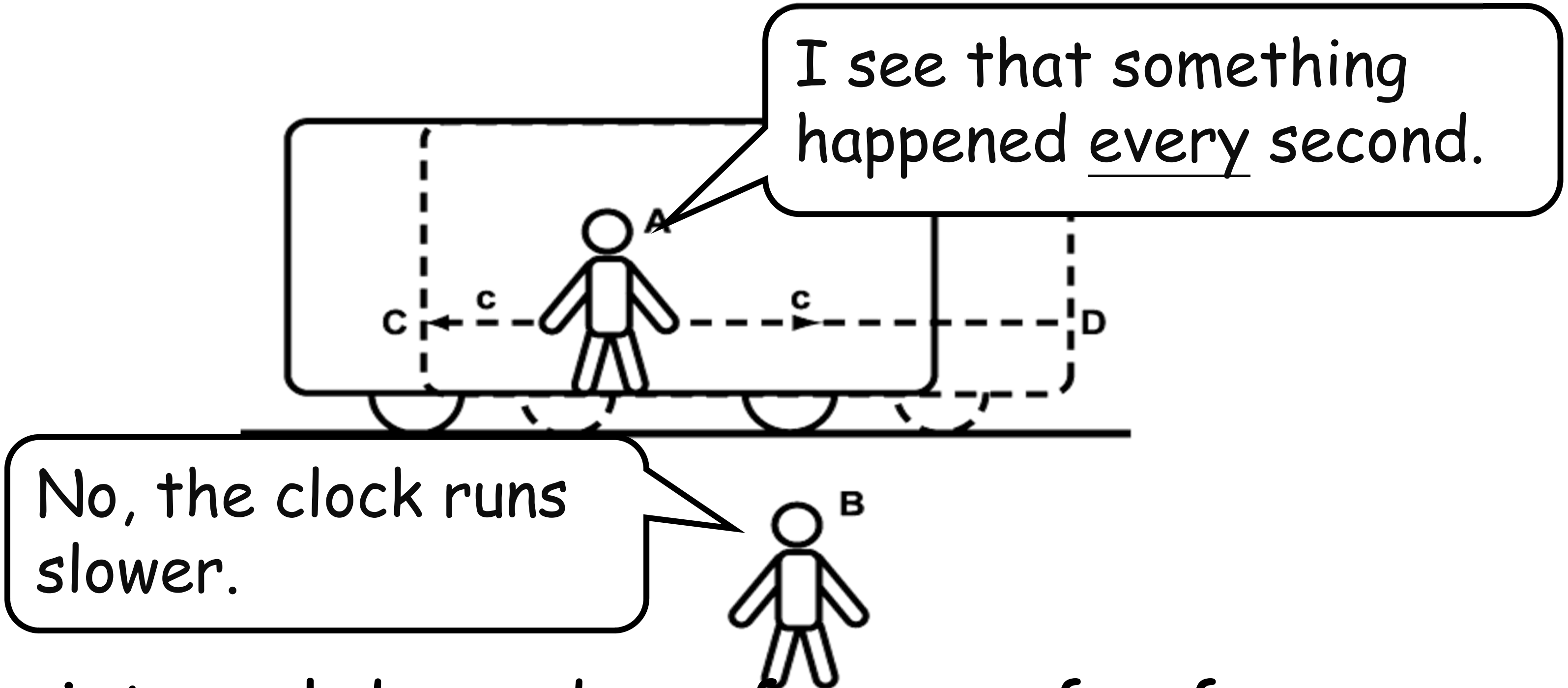


Will it be the same as observed in S?

Time dilation

Suppose a clock is at $x' = 0$, two events are defined as $(0, t_1')$ and $(0, t_2')$ as observed by A:

In S frame: $t_2 - t_1 = \gamma\left(t_2' - t_1'\right) + \gamma\beta\left(x_2' - x_1'\right)/c \Rightarrow t_2 - t_1 = \gamma\left(t_2' - t_1'\right)$



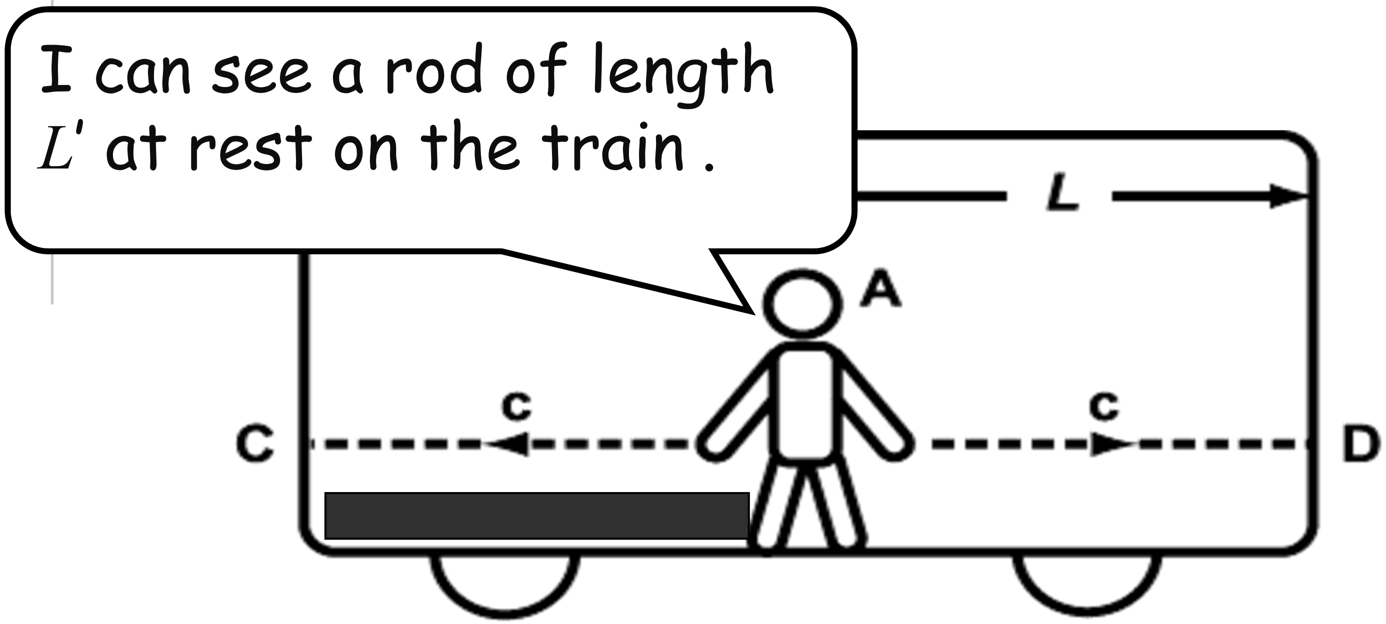
Time interval depends on frames of reference.

Time dilation

1) On the other hand, relative to A (on the train), he saw B's clock running slower. So which clock runs slower? A: on train or B: platform.
2) when speed of the train $u \rightarrow c$, $\gamma \rightarrow$ ◎, hence $\Delta t \rightarrow$ ◎, i.e., motions on train seem frozen.
3) In particular, relative to A (on train) his clock at rest (two events happened at same place), A measured *Proper Time Interval*

- *For proper time interval Δt another inertial frame observes measure $\gamma\Delta t$*

Lorentz contraction



Is it the same as observed by B?

Lorentz contraction

As observed by B, he has to measure the length as some particle time, says at time t. Therefore, two events are defined as (x_1, t) and (x_2, t).

From the Lorentz transformation:

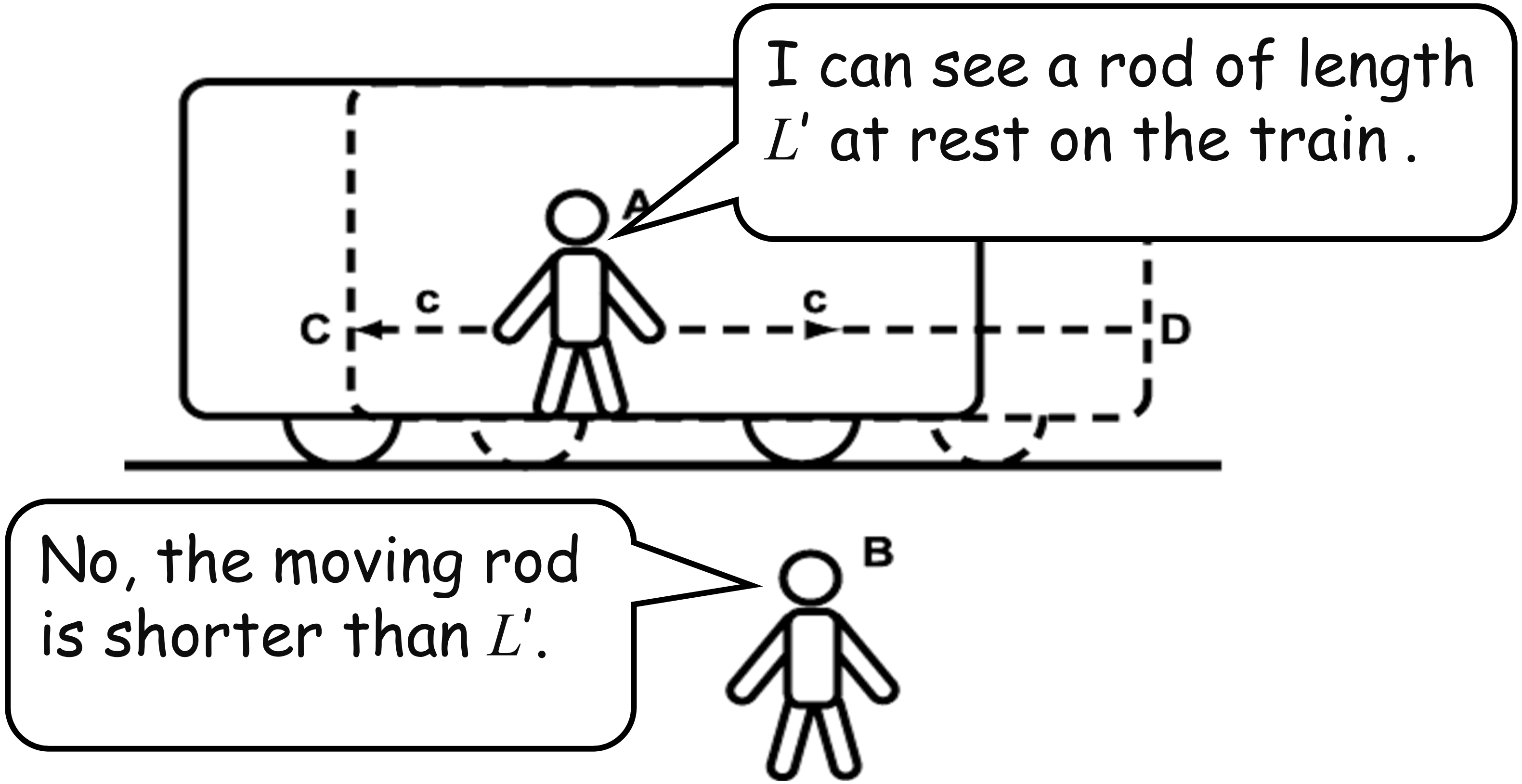
$$x_2' - x_1' = \gamma(x_2 - x_1) - \gamma u(t_2 - t_1) = \gamma(x_2 - x_1)$$

$$L = L'/\gamma$$

B finds the rod shorter than L'.

Question: the two events happened at the same time in S, it is not necessarily at the same time in S′. Why $x_2' - x_1'$ still equal to L'?

Lorentz contraction



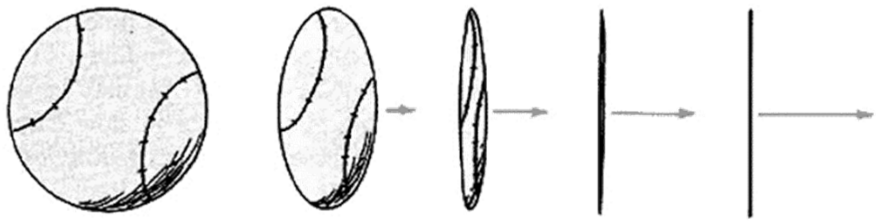
Length depends on frames of reference.

Lorentz contraction

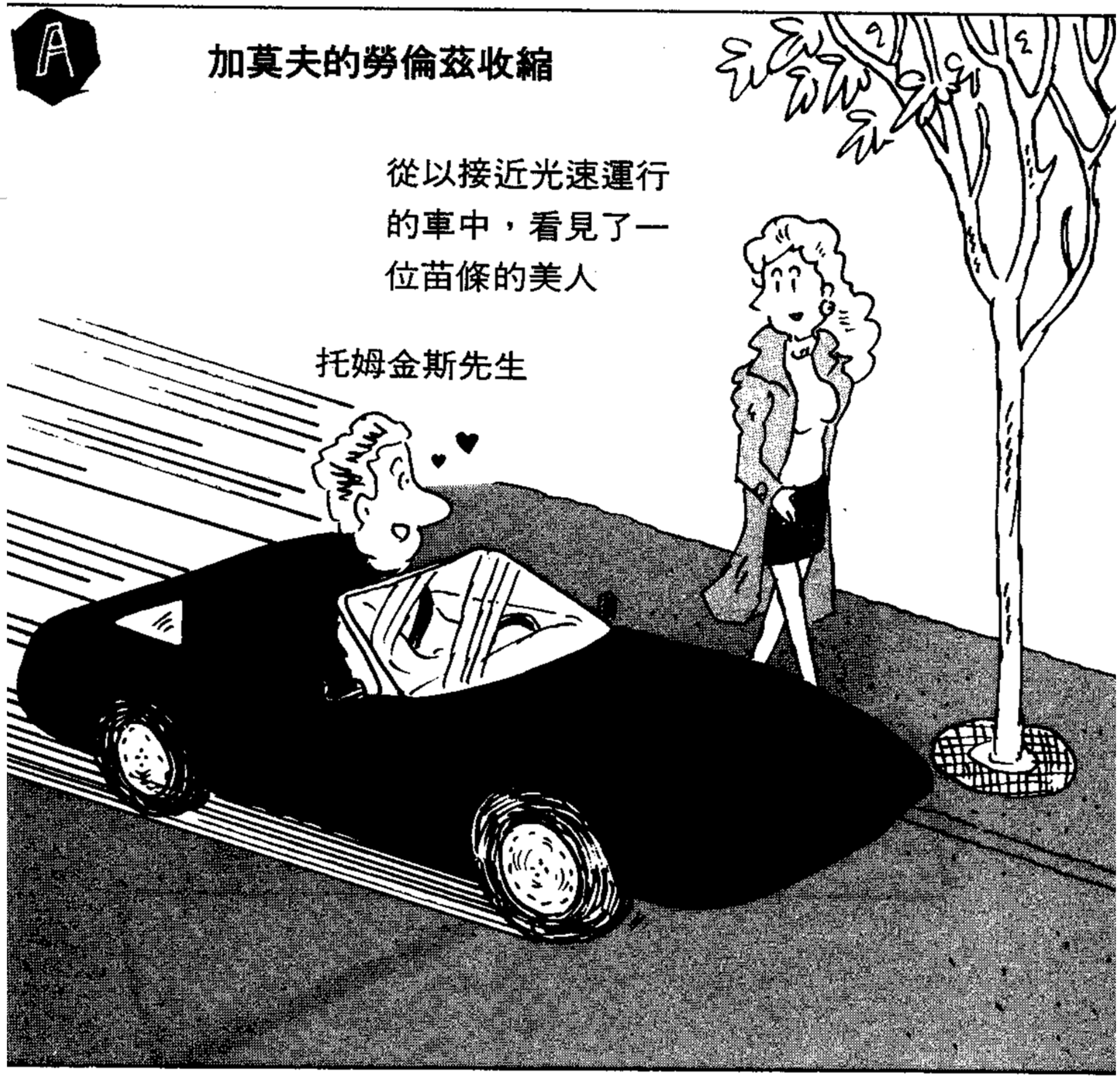
- ✓ In particular, relative to A the rod is at rest, the length of the rod as measured by A is called *proper length*.
- *For a proper length L_0, other inertial system observers measure the length as L_0/γ.*

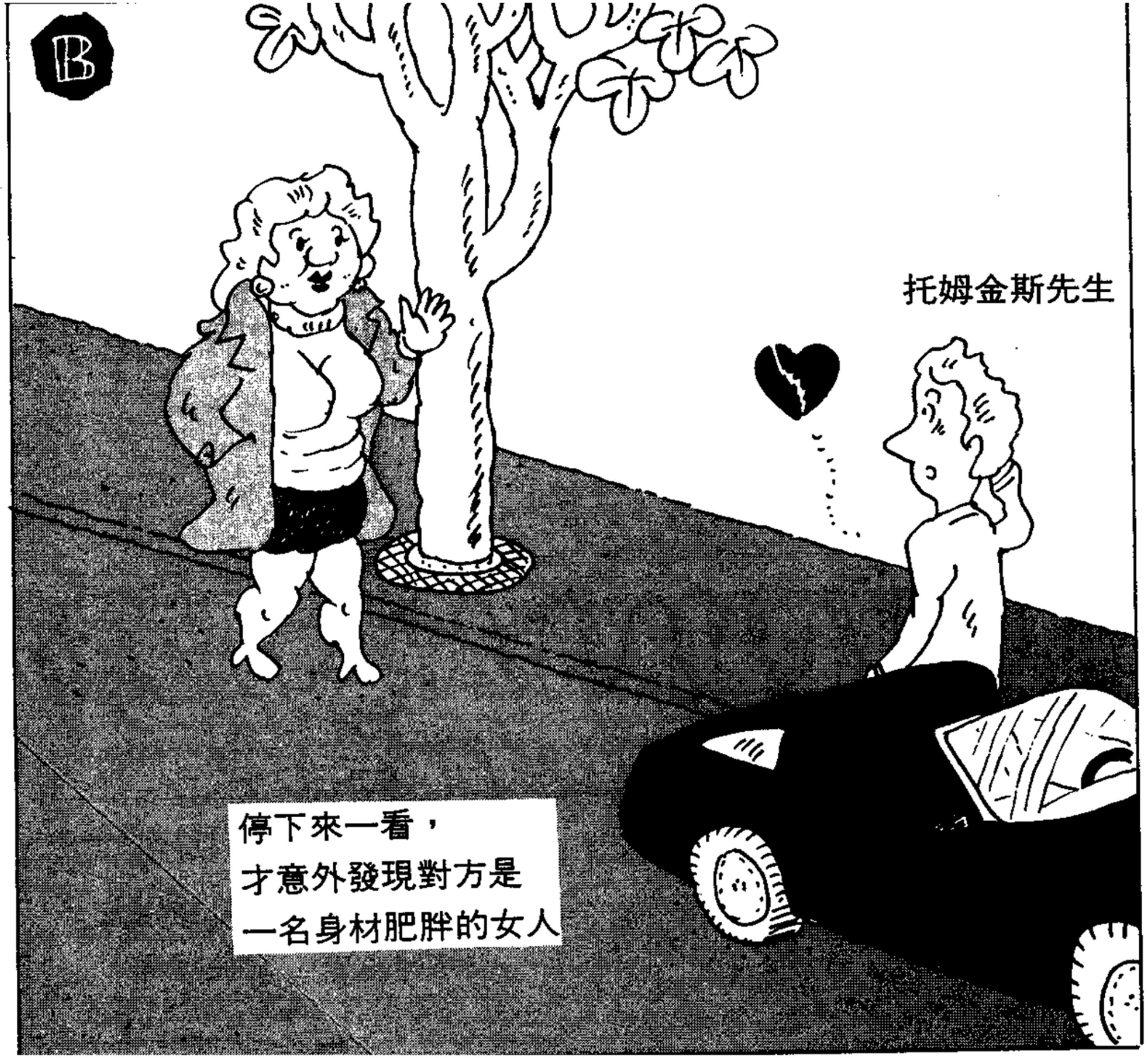
Lorentz contraction

✓ The length of a moving object becomes shorter along the direction of its motion when measured by a stationary observer.



✓ Contraction occurs for objects made up of any material


A
加莫夫的勞倫茲收縮
從以接近光速運行的車中，看見了一位苗條的美人
托姆金斯先生


B
托姆金斯先生
停下來一看，
才意外發現對方是
一名身材肥胖的女人

- ✓ In daily experiences, speeds much smaller than c.
- ✓ Example: train moves at $u = 100\ \mathrm{km/h} = 28\ \mathrm{m/s}$; hence $\gamma \sim 1$. Effects of time dilation and Lorentz contraction too small to be observed!
- ✓ However, the effects become noticeable for particles at speeds close to c.

Question ?

life-time of pions ~ 1.77×10^{-8} s at rest frame. When produced, its speed $0.99c$ and travels 37 m before decayed.

- ✓ Consistent ?
- ✓ How explain ?

Answer

- ✓ Consistent ? NO.
  Expected distance $0.99c \times 1.77 \times 10^{-8} = 5.26$ m.
- ✓ How explain ?

Proper length $= 37$ m as measured in lab. frame

But relative to the pion, the length shortened by $\gamma = 7.089$, i.e., length ~ 5.22 m only

Concepts in relativity

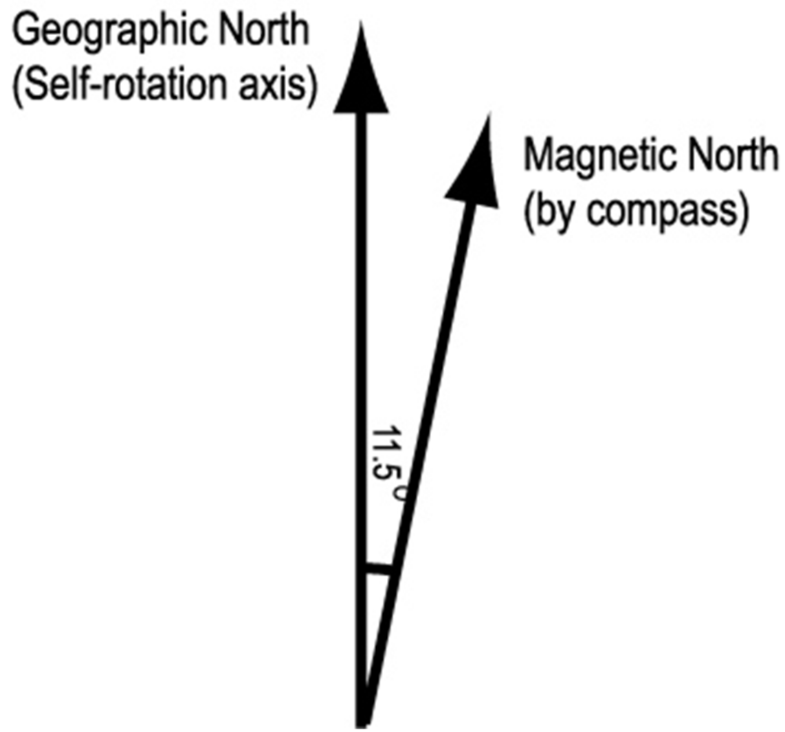
Lorentz contraction
Time dilation

occur TOGETHER

Space-time

- ✓ Length contraction and time dilation happen together.
- ✓ Instead of space and time, we say space-time.
- ✓ Space and time are mixed up, Space-time is really one thing!

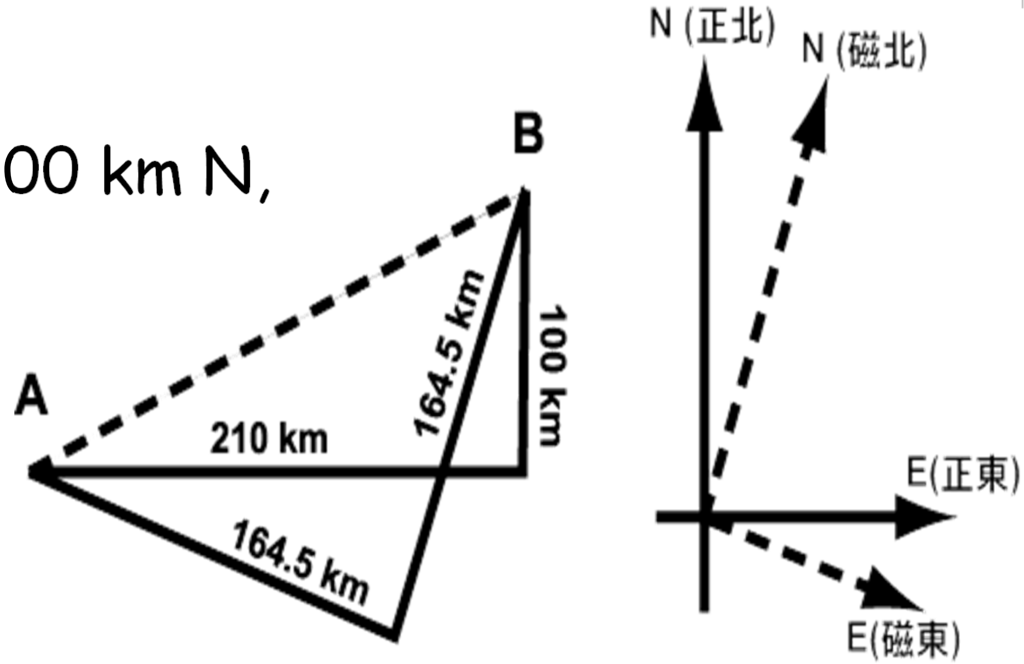
Analogy: magnetic North versus true North



use either system to specify a direction

Analogy: magnetic North versus true North

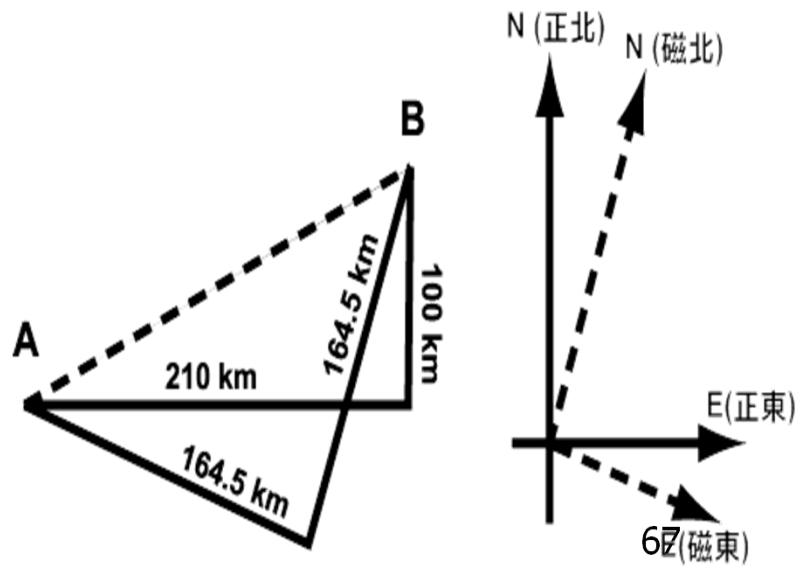
✓ Polaris N : B is at (100 km N, 210 km E) of A



✓Compass N : B is at (164.5 km N, 164.5 km E) of Point A.

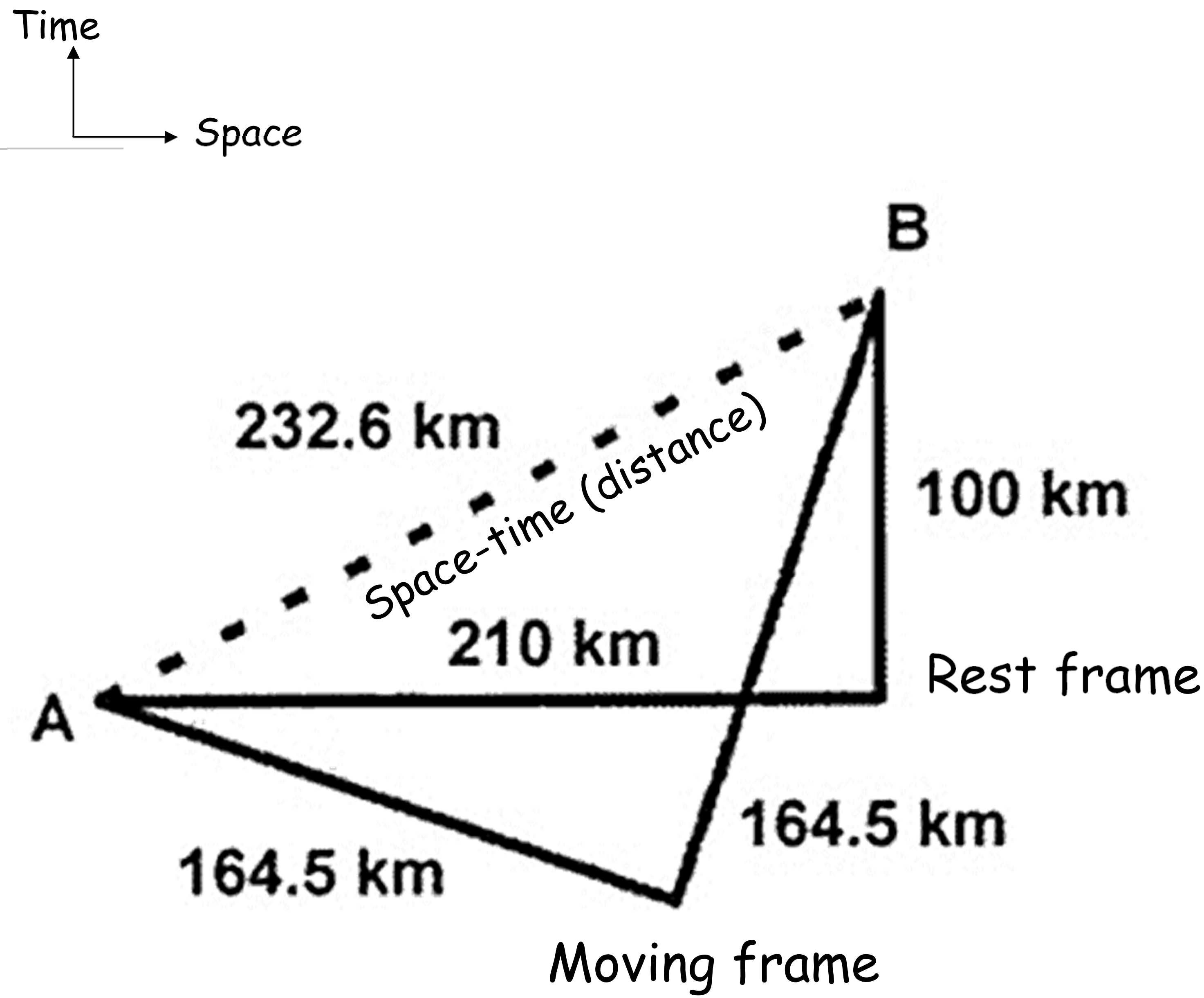
Analogy: magnetic North versus true North

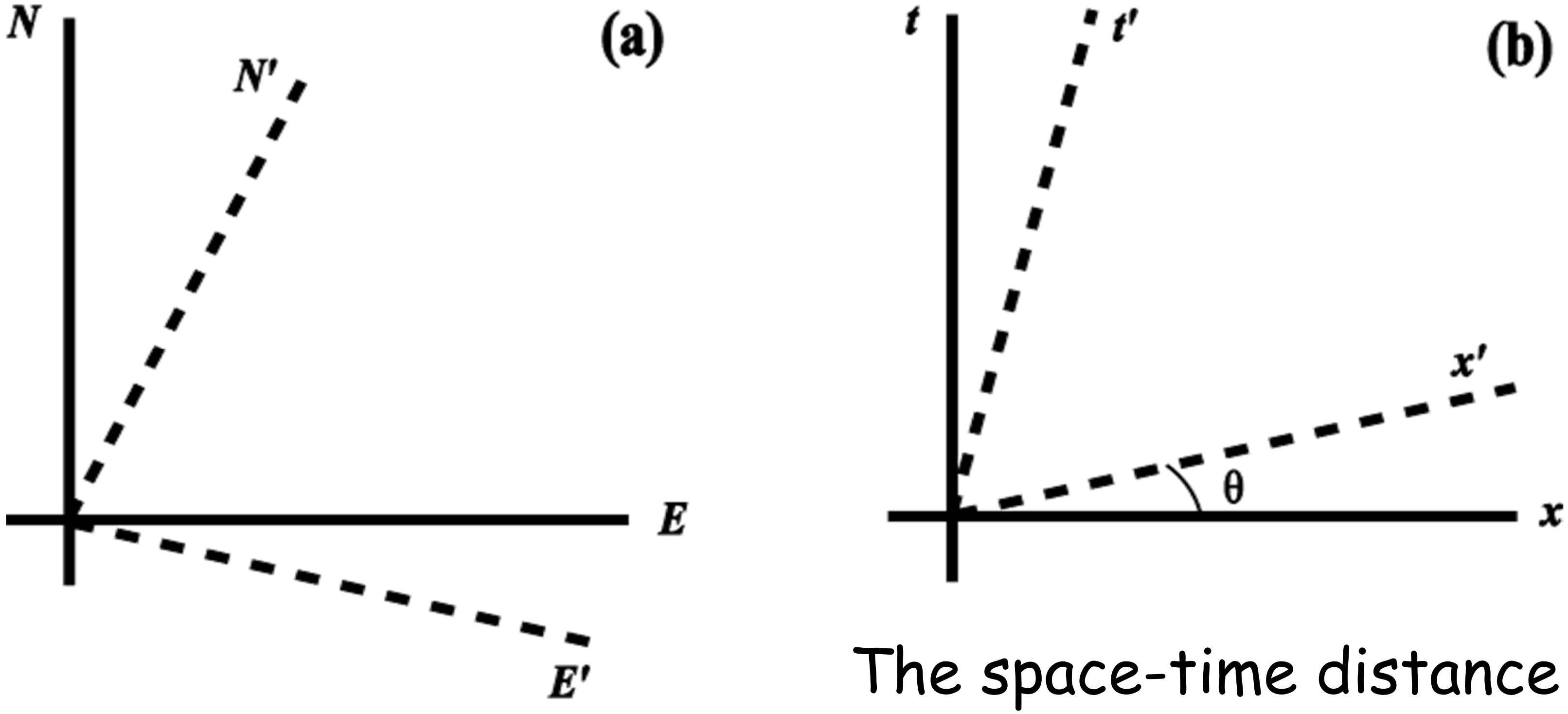
- ✓Which is correct?! Both are correct
- ✓distance between A and B is the same for both cases !



Comparison between NE and space-time

- ✓ Any point: a combination of N and E $\Leftrightarrow$ Any event: a combination of space and time.
- ✓ Shift from another coordinate: different E and N $\Leftrightarrow$ view from another inertial frame: observe contraction and time dilation.
- ✓ Distance AB remains unchanged $\Leftrightarrow$ Distance in space-time remains unchanged (not space or time alone as in Newtonian).


Time
Space
B
232.6 km
Space-time (distance)
100 km
210 km
Rest frame
A
164.5 km
164.5 km
Moving frame



The distance between two points is invariant (unchanged) for any coordinate system.

The space-time distance between two events is invariant for inertial frame observers.

Mathematically, one may define a space-time distance between two events, namely (x_1, t_1) and (x_2, t_2) as $\Delta S^2 = \Delta x^2 - c^2 \Delta t^2$.

Another inertial system observer measures the coordinates of the same events as (x_1', t_1') and (x_2', t_2') respectively and the space-time distance $\Delta S'^2 = \Delta x'^2 - c^2 \Delta t'^2$

Can you prove space-time distance is invariant (constant) under the Lorentz transformation? That is $\Delta S^2 = \Delta S'^2$

The universe is 4-dimensional
(3 space and 1 time)

What is UNIVERSE (宇宙)？

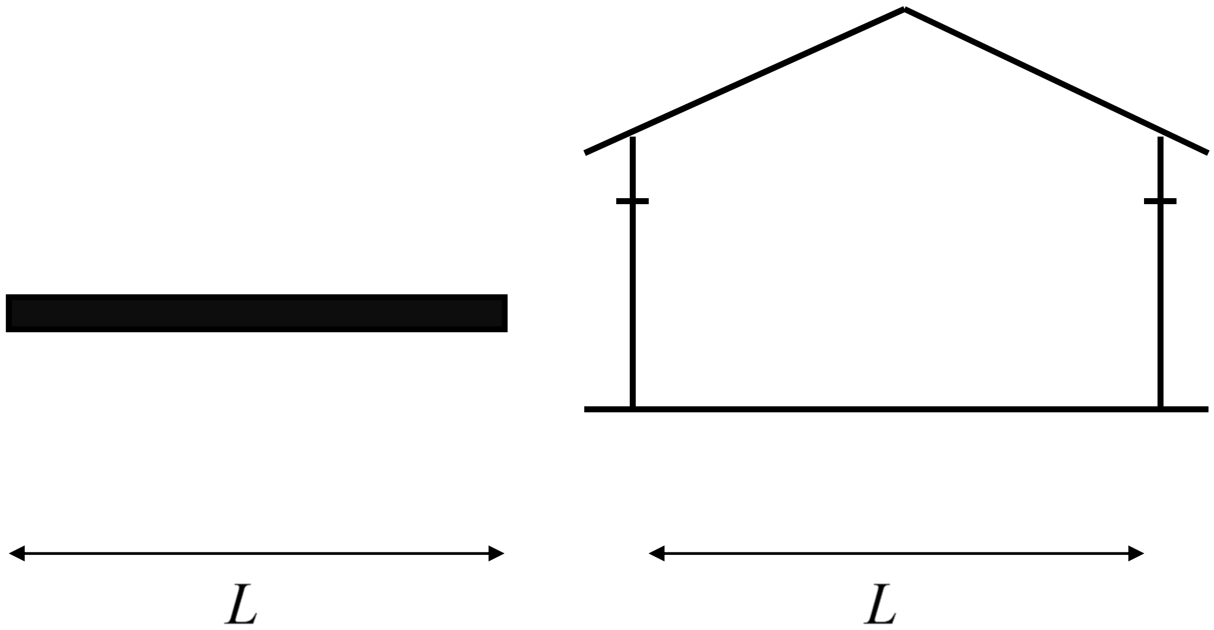
宙：古往今來 – time

宇：上下四方 - space

Paradox in Relativity

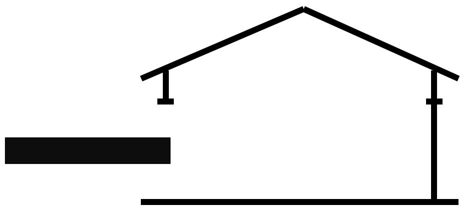
The Pole and Barn

- ✓ Suppose both proper length of pole and width of the barn are L

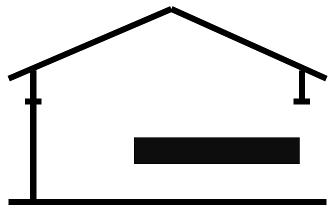


The Pole and Barn

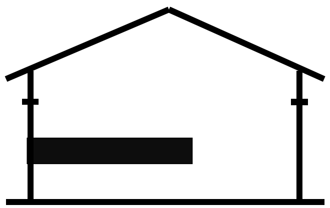
✓Relative to Barn,



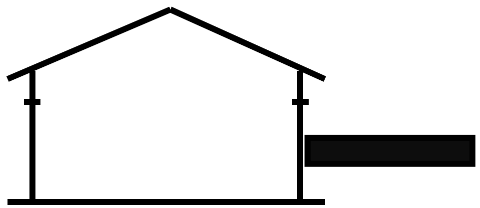
$t = t_1$



$t = t_3$



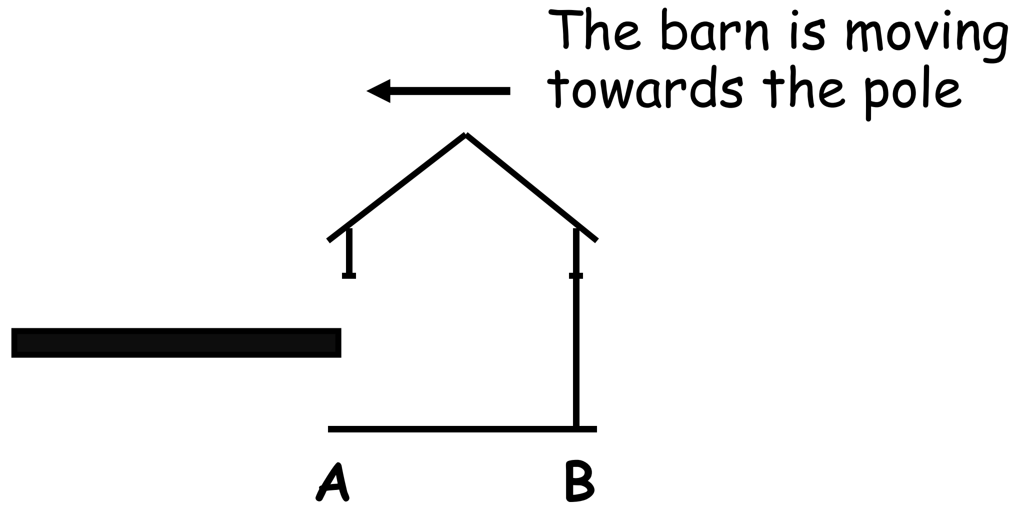
$t = t_2$



$t = t_4$

The Pole and Barn

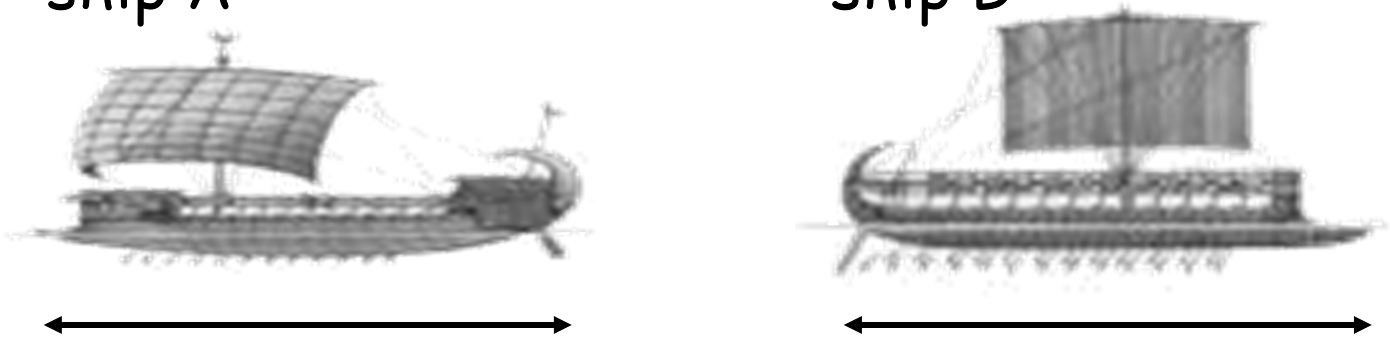
- ✓ Relative to Pole, The barn is shorter than the pole, and is moving towards.



What's happen?

Warship problem

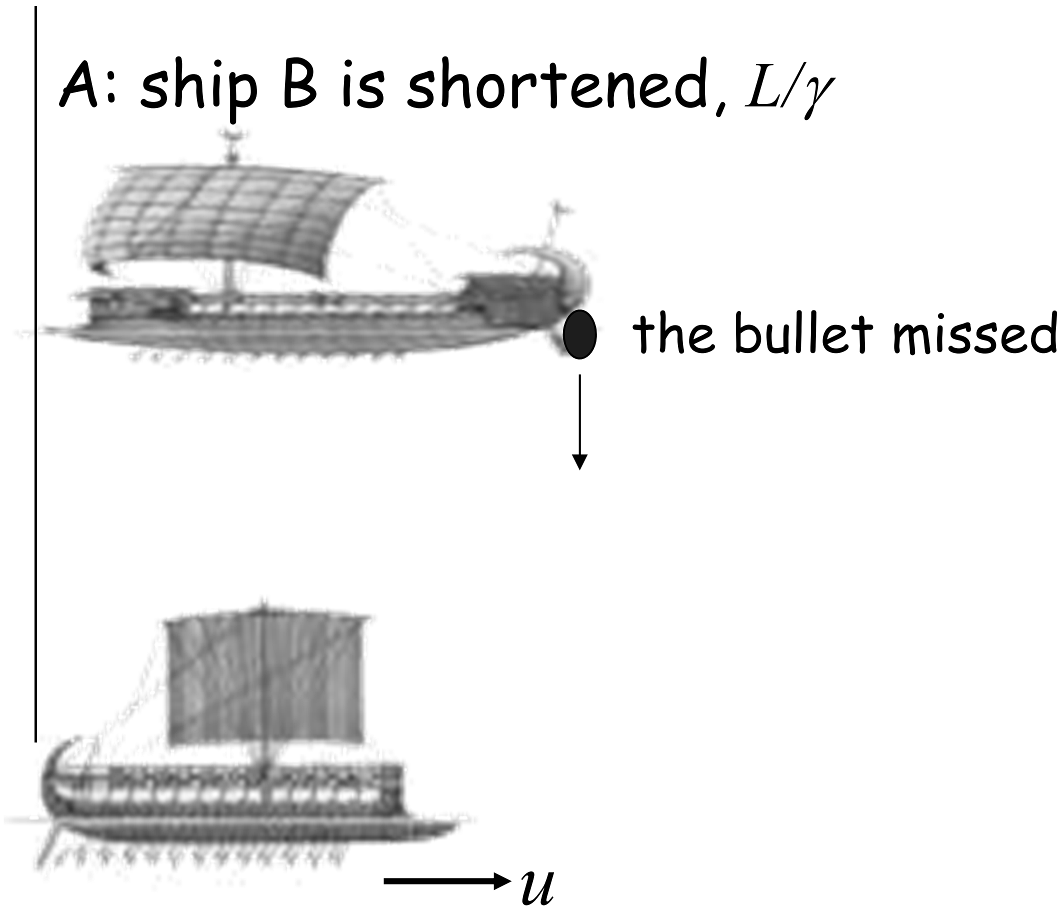
ship A ship B



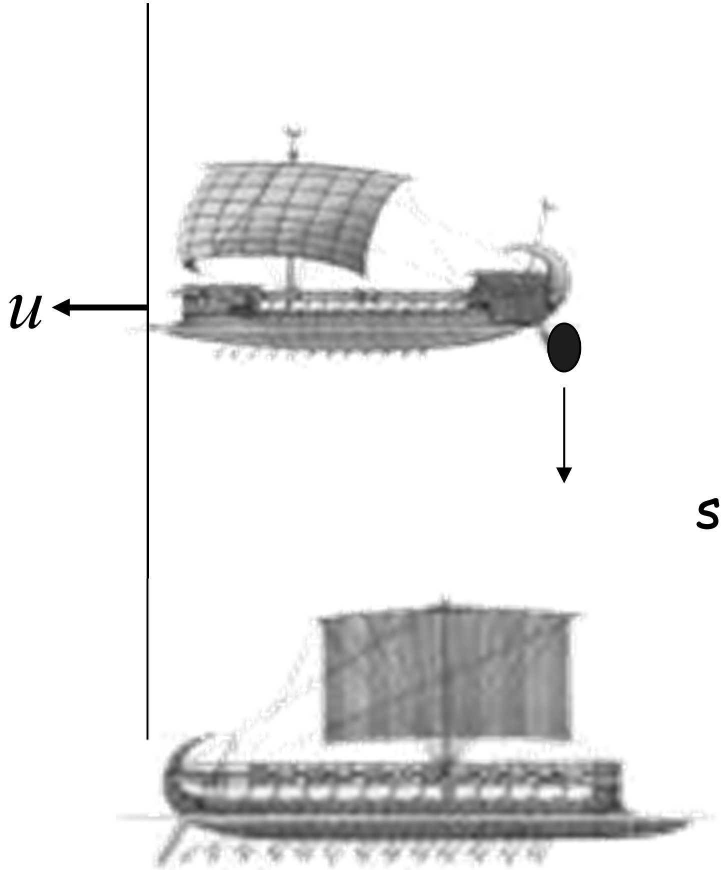
Two ships are of proper length L

Warship problem

A: ship B is shortened, L/γ



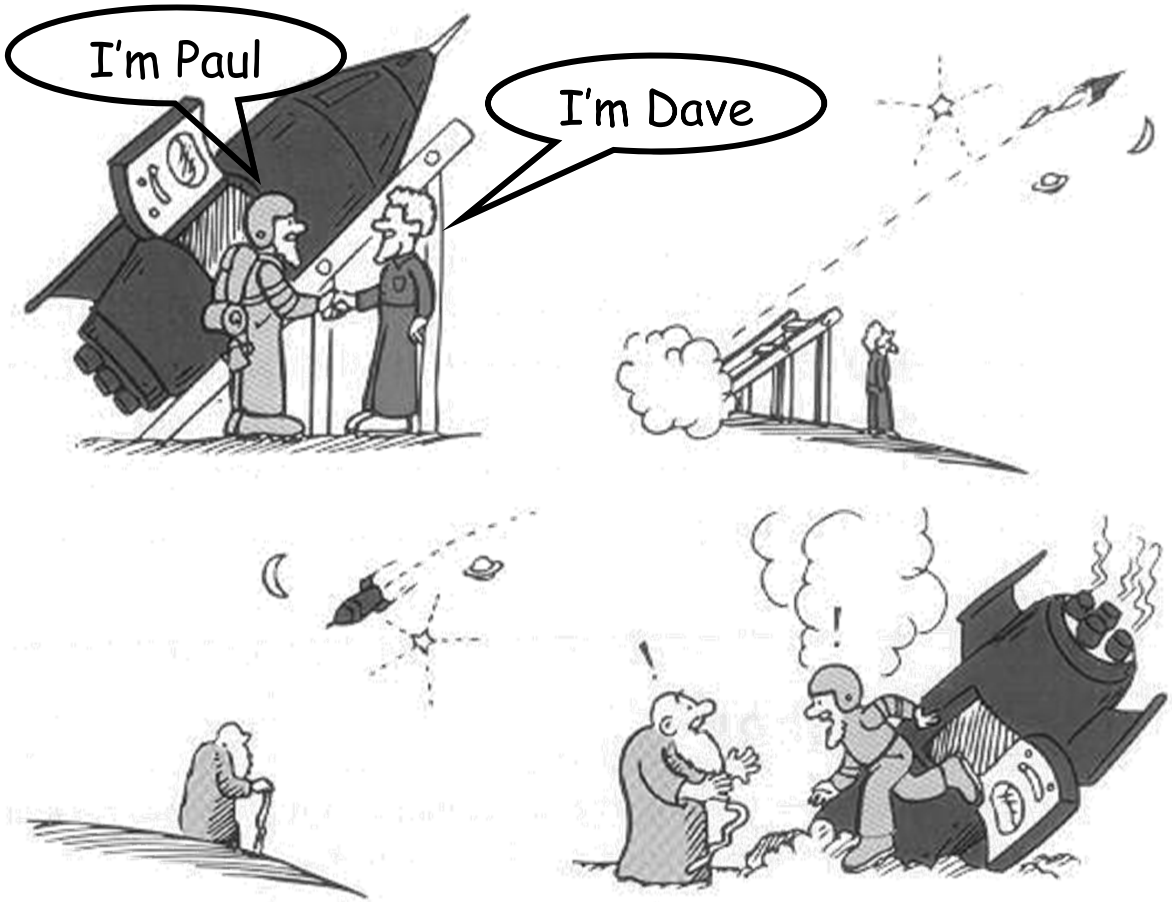
Warship problem



ship B is hit by the bullet.

B: ship A is shortened, L/γ

The twin paradox

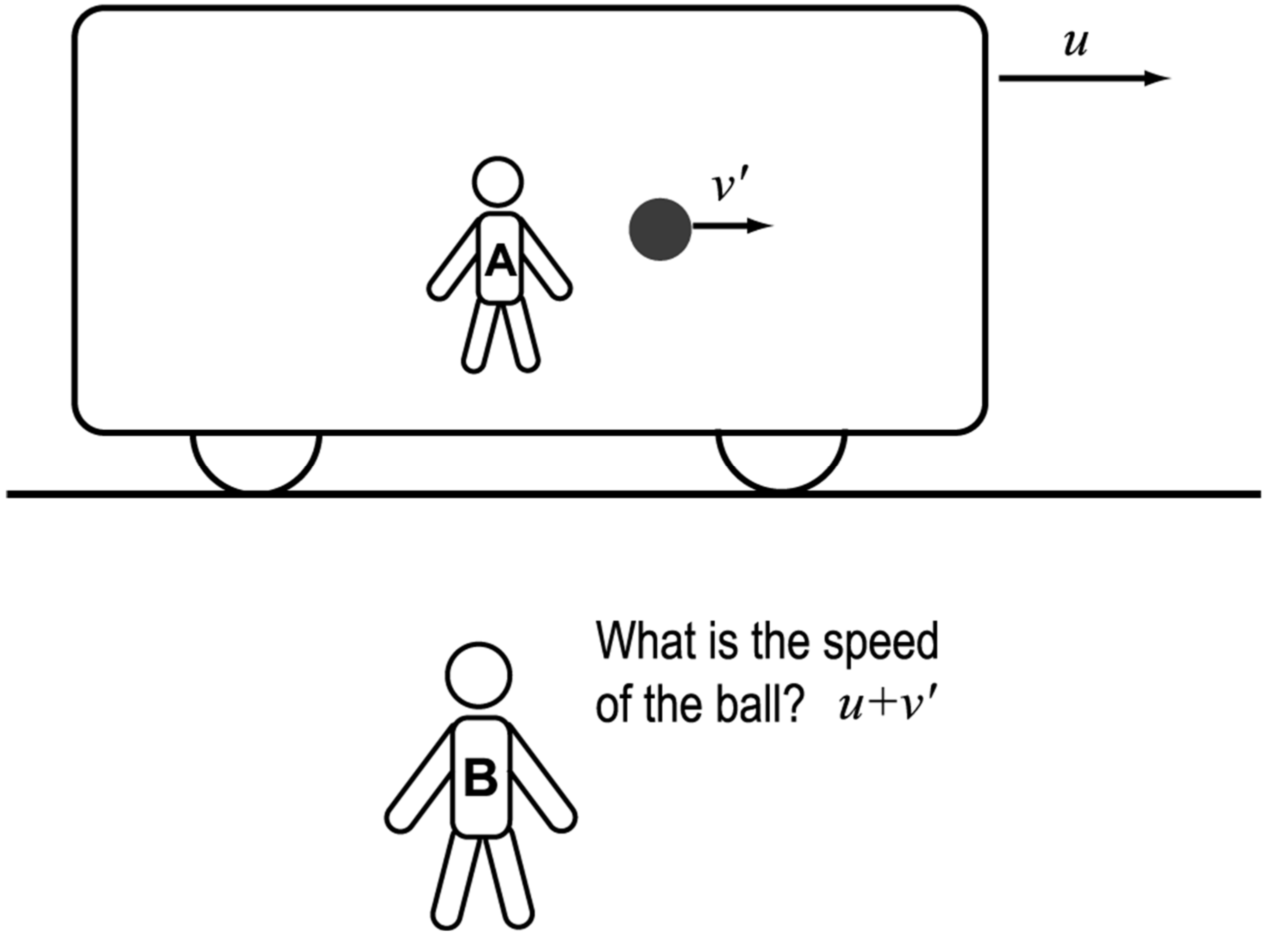


- ✓Easy! Dave on the earth observes clocks on the fast-moving shuttle run slower; hence, Paul is younger when he returns
- ✓But the motion is relative. Paul on the shuttle observes clocks on the "fast-moving" earth run slower. Dave should be younger! What's wrong?

- ✓Note that Dave remains the same frame, while Paul has experienced two different frames, separated by a turning point.
- ✓Hence, NO symmetry between Paul and Dave.
- ✓As a result, Dave is younger when Paul returns back to the earth.

Velocity, Mass, energy and Time reversal

Velocity addition



Velocity addition

A sees the ball are at E_1: $(0,0)$ and E_2: $(v't', t')$

By Lorentz transformation: relative to B,

$E_1 : (0,0)$ are $E_2 : (x = \gamma(v't' + ut'), t = \gamma(t' + \beta v't'/c))$

Therefore, the speed is

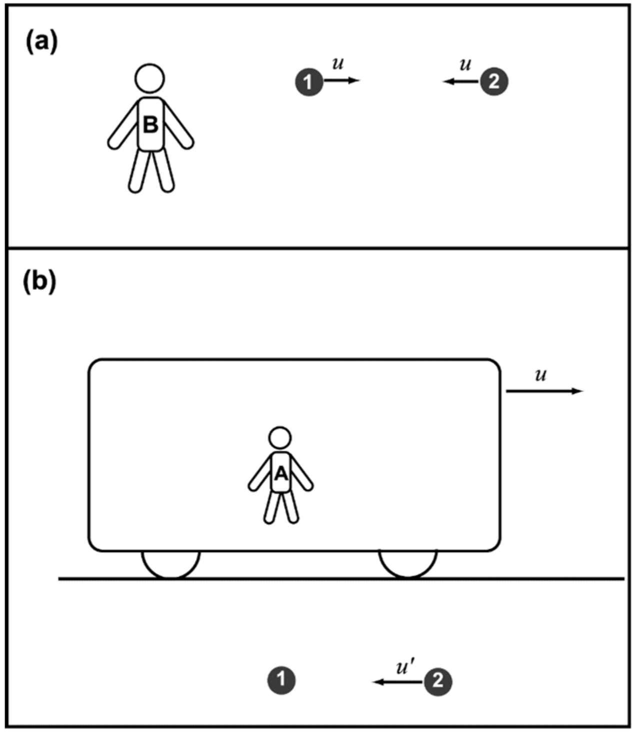
$$v = x/t = \frac{v' + u}{1 + uv'/c^2}$$

Momentum and Mass

Def. $momentum = mass \times velocity$

Example: Totally inelastic collision

$$p_i = mu - mu = 0$$



$$p_f = 0$$

$$p_i' = 0 - m\frac{2u}{1+u^2/c^2} = \frac{-2mu}{1+u^2/c^2}$$

$$p_f' = -2mu$$

So momentum is not conserved?!

Momentum and Mass

- ✓ Define *relativistic momentum*: $p = \gamma m_0 u$ in order to preserve momentum conservation, where m_o is rest mass (zero speed)
- ✓ Hence, mass increases with speed, $m = \gamma m_0$ mass depends on frames of reference.

Speed of light as a limit

✓ As speed of an object increasing and approaching c, Lorentz factor γ increasing and approaching infinite

❖ Infinite temporal dilation and indefinite spatial contraction

❖ Mass of the object tends to infinite, need infinite to accelerate the object to c !

✓ Hence c is the limit of speed

Equivalence of Mass and Energy

✓Kinetic energy of a particle:

$$K.E. = \int F ds = \int \frac{dp}{dt} ds$$, where $F = \frac{dp}{dt} = \frac{d}{dt}(\gamma m_0 u)$

$$= \int u dp$$

$$= up - \int p du$$

$$= \gamma m_0 u^2 + \frac{1}{\gamma} m_0 c^2 - m_0 c^2$$

$$= \gamma m_0 c^2 - m_0 c^2$$

So total energy $E = \gamma m_0 c^2$ can be written as

$E = m_0 c^2 + K.E.$, where rest mass energy is $m_0 c^2$.

Equivalence of Mass and Energy

✓Total energy E of a particle: $E = \gamma m_0 c^2$

$$E = m_0 c^2 \left(1 - \frac{u^2}{c^2}\right)^{-1/2}$$

$$\approx m_0 c^2 \left(1 + \frac{u^2}{2c^2}\right) = m_0 c^2 + \frac{1}{2} m_0 u^2,$$

Rest mass energy

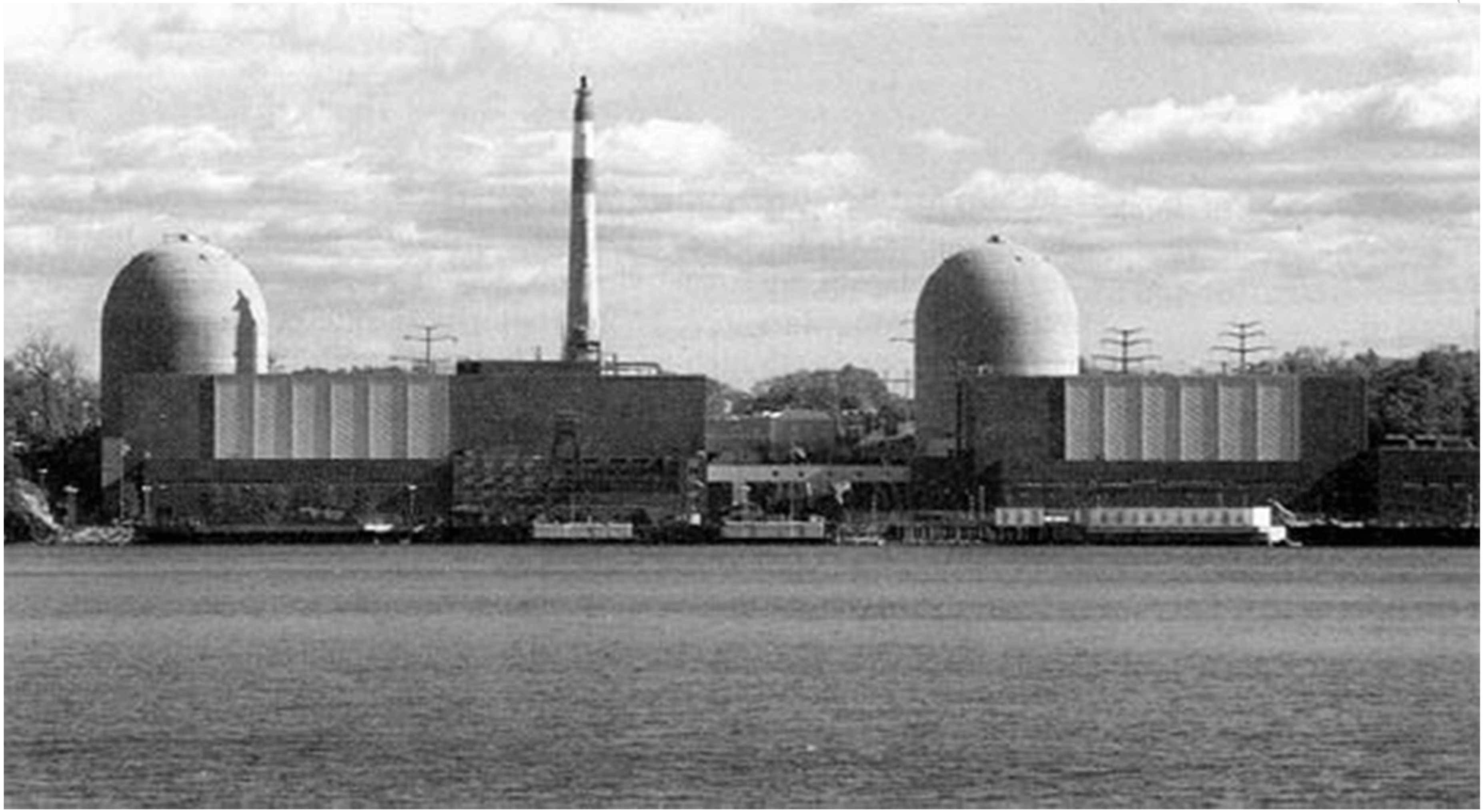
Newtonian
Kinetic energy

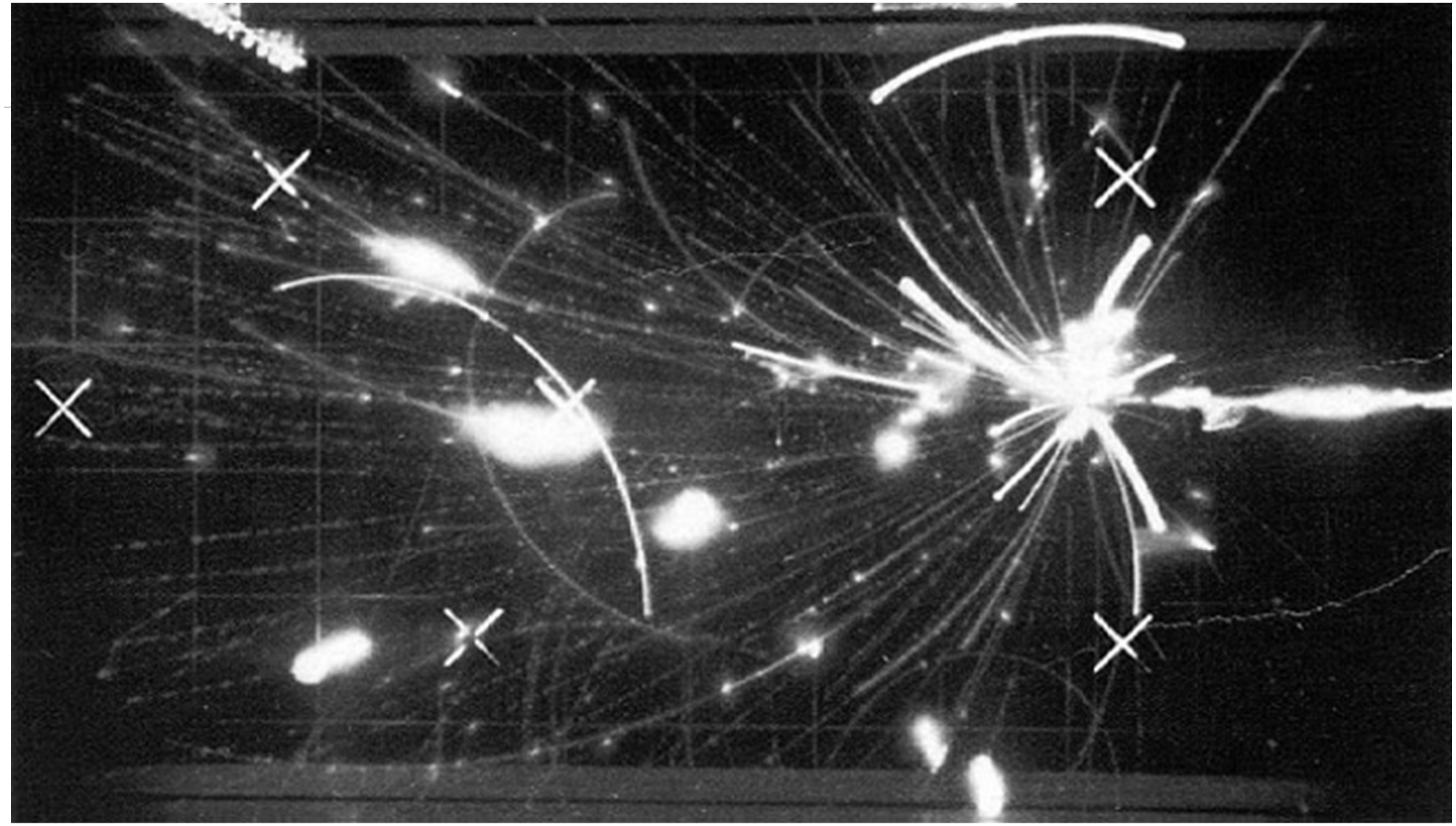
Equivalence of Mass and Energy

- ✓ proof: $E^2 = p^2c^2 + m_0^2c^4$

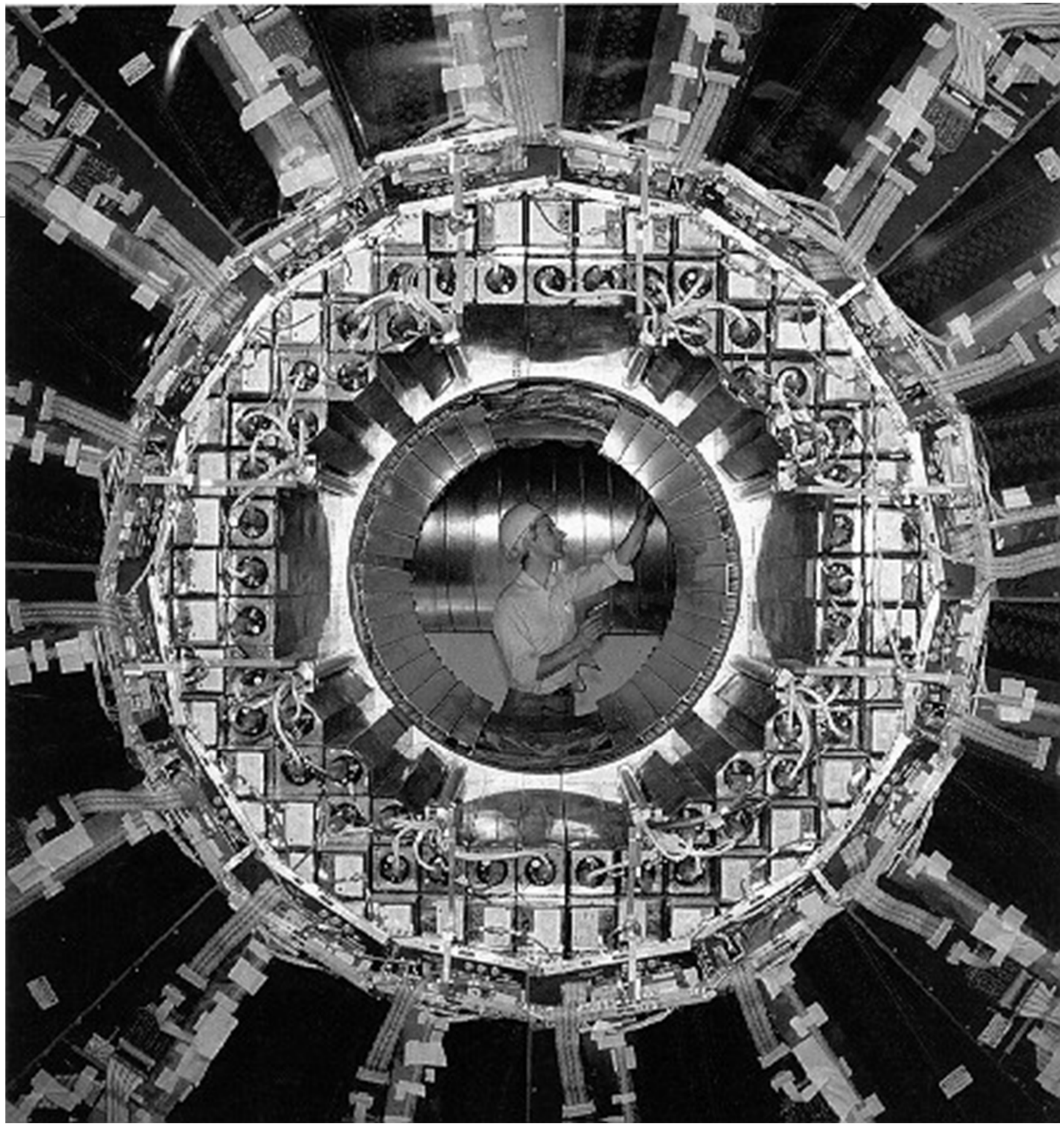
  You are now able to show that the momentum is conserved in the above example.

  In particular, light has no rest mass, hence $E = pc$. Thus light has momentum
- ✓ Mass and energy are equivalent! Application in nuclear plants; nuclear weapons





Collisions of high-energy particles



Particle accelerator (粒子加速器)

Rigid body

- ✓ Rigid body is an incompressible solid; its shape does not change
- ✓ Diamond are hard, but still not rigid.

Is the concept of "rigid body" consistent with special relativity?

Rigid body

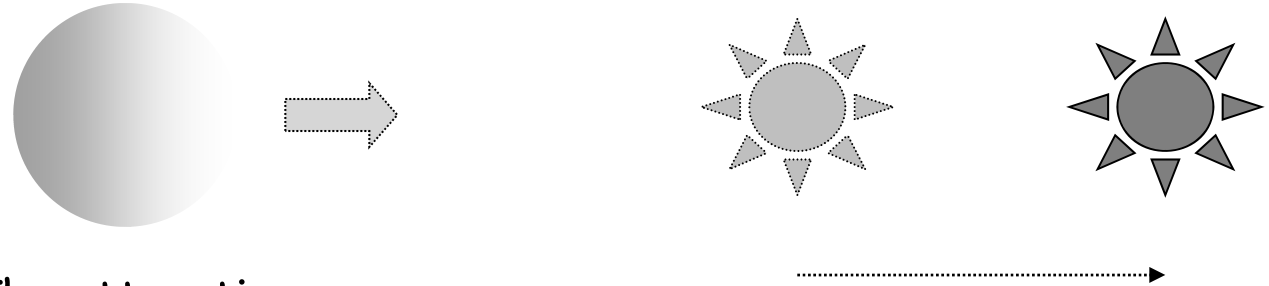
- ✓ If the pen is *rigid*, pushing one end will make the other end move immediately
- ✓ Speed of the signal is infinite; however, it is not allowed by special relativity.
- ✓ Hence, NO rigid body in SR.

Time reversal

- ✓ if we can travel faster than light, *Time reversal* , e.g., Tachyons: particles faster than light.
- ✓ May exist but have not been detected.
- ✓ Still believe that time has only one direction — forward.

Difficulties of Newton’s theory of gravitation

✓Newton's theory of gravitation: gravitational force is assumed to be transmitted instantaneously.



The attraction
decreases immediately

but it violates relativity: c is speed limit

- ✓ Also, special theory is a theory of inertial frames. A theory of *non-inertial frames* is needed.
- ✓ New theory of gravitation is on call.

General theory of relativity

The principle of equivalence and the general relativity

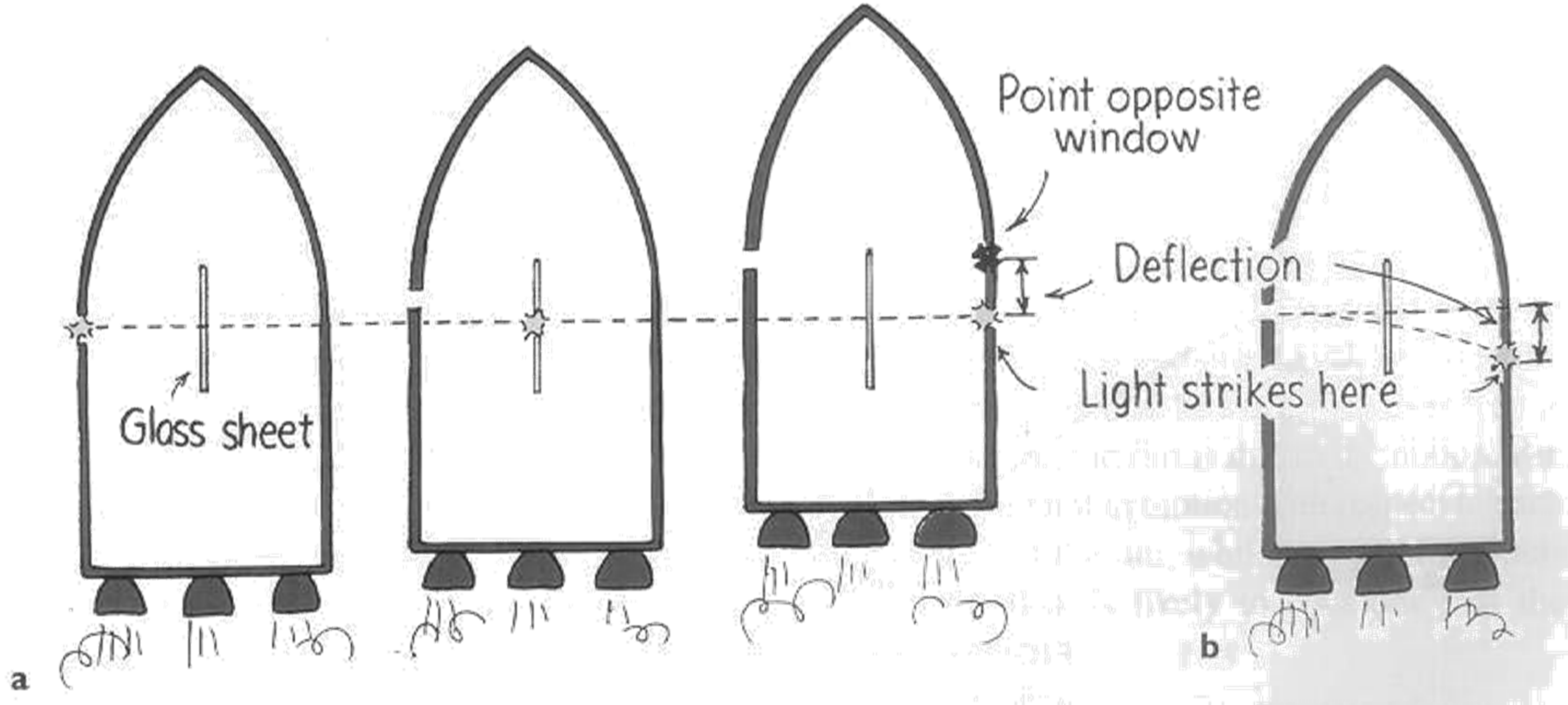
The principle of equivalence states

Observers cannot distinguish locally between forces due to acceleration and uniform gravitational forces due to the presence of a massive body.

“There is gravity!”

“There is no gravity!”

Bending of light by gravity

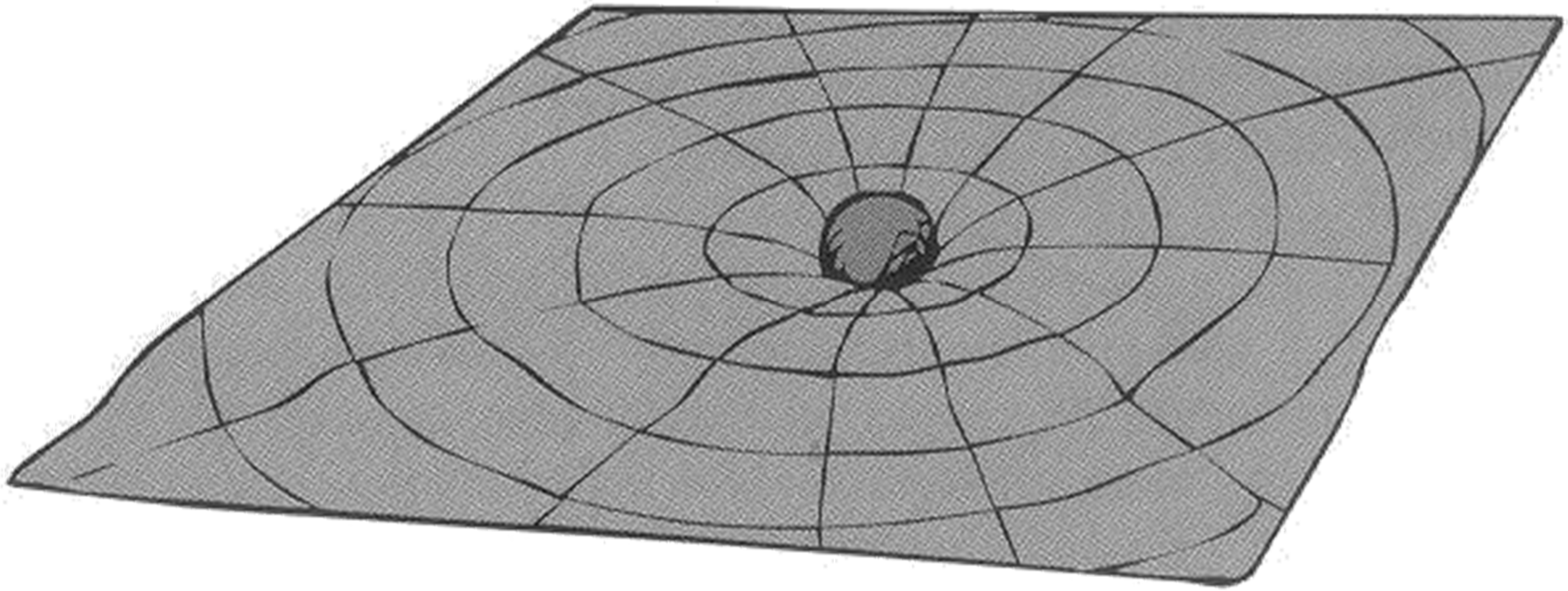


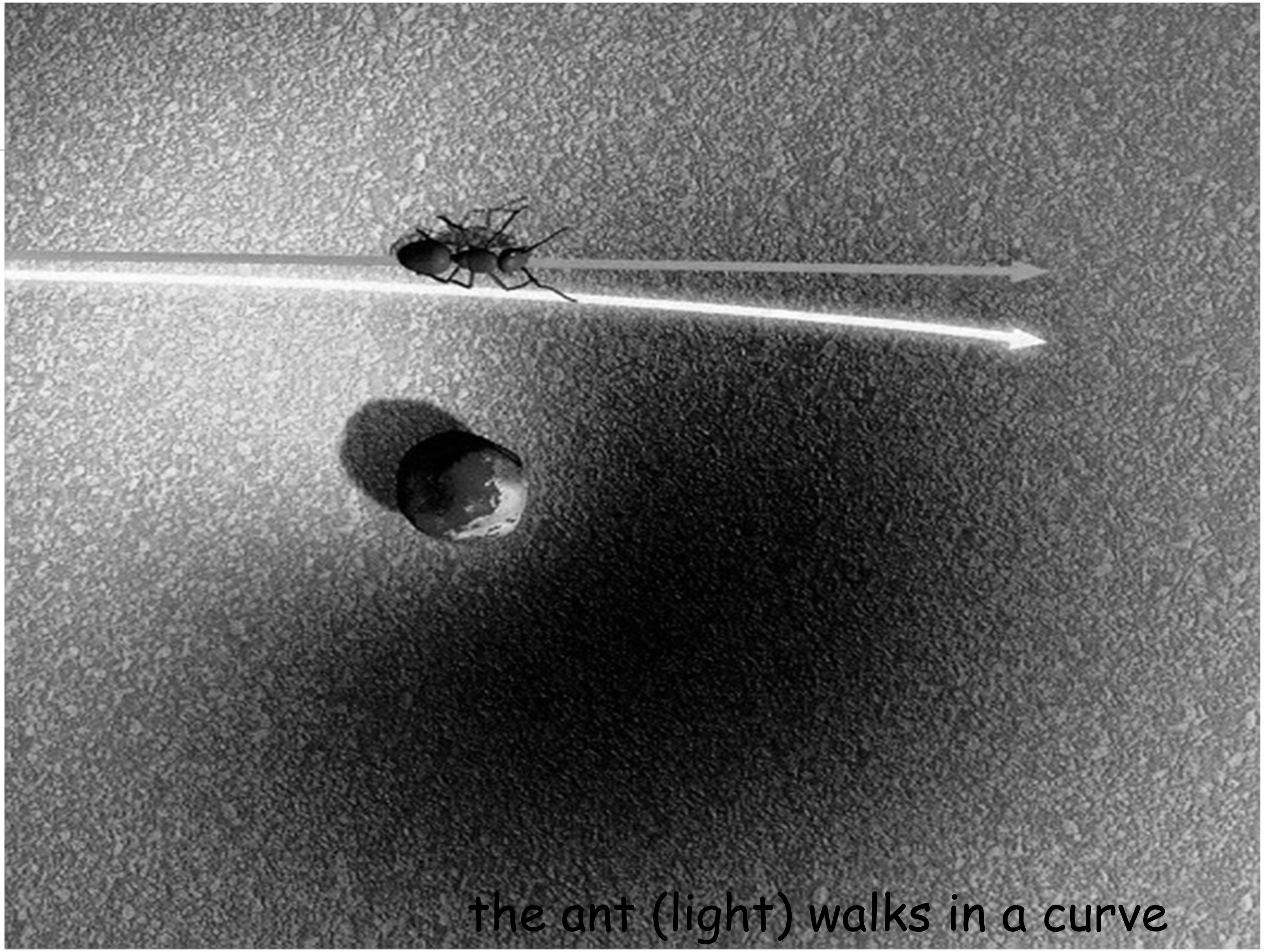
✓Light has no mass, Newton's theory could not explain the bending of light by gravity

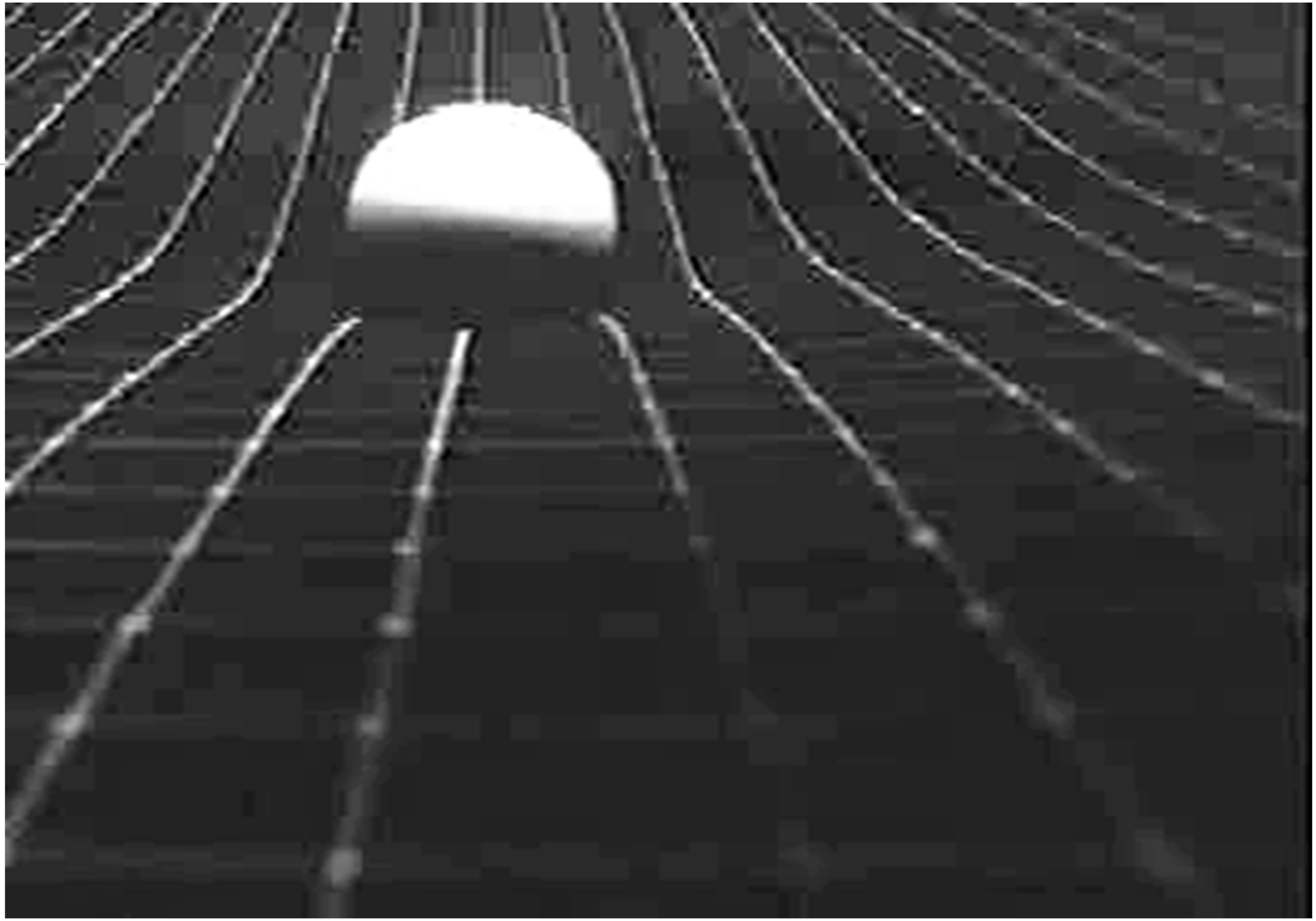
✓In Einstein's language, the light bends because it travels in a curved space-time.

Curved space-time

- ✓ According to GR, mass causes space-time to curve.
- ✓ Object in a curved space-time moves in a curve.
- ✓ As if they are pulled by some "forces" — gravity.
- ✓ Greater mass ↓ greater curvature.

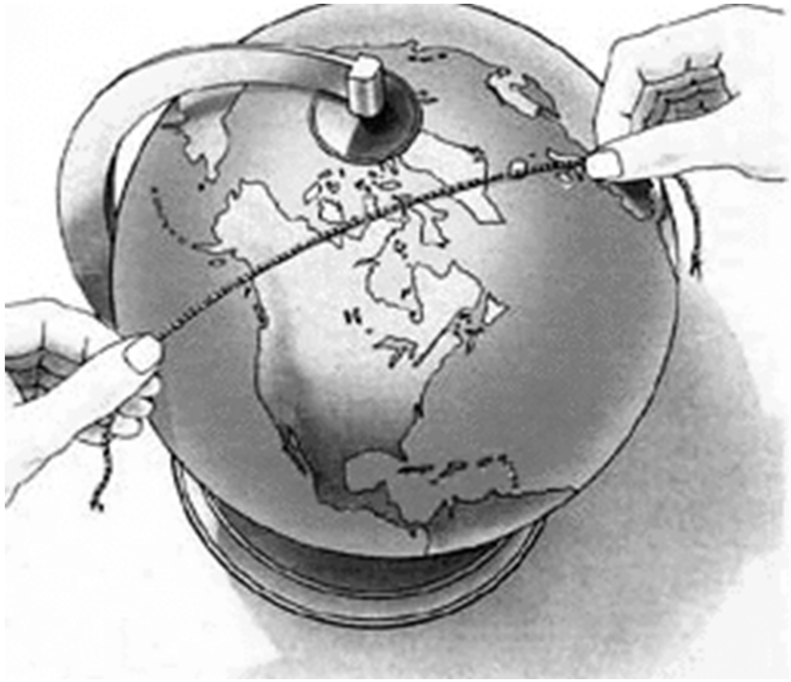



the ant (light) walks in a curve



How does gravity influence space and time?

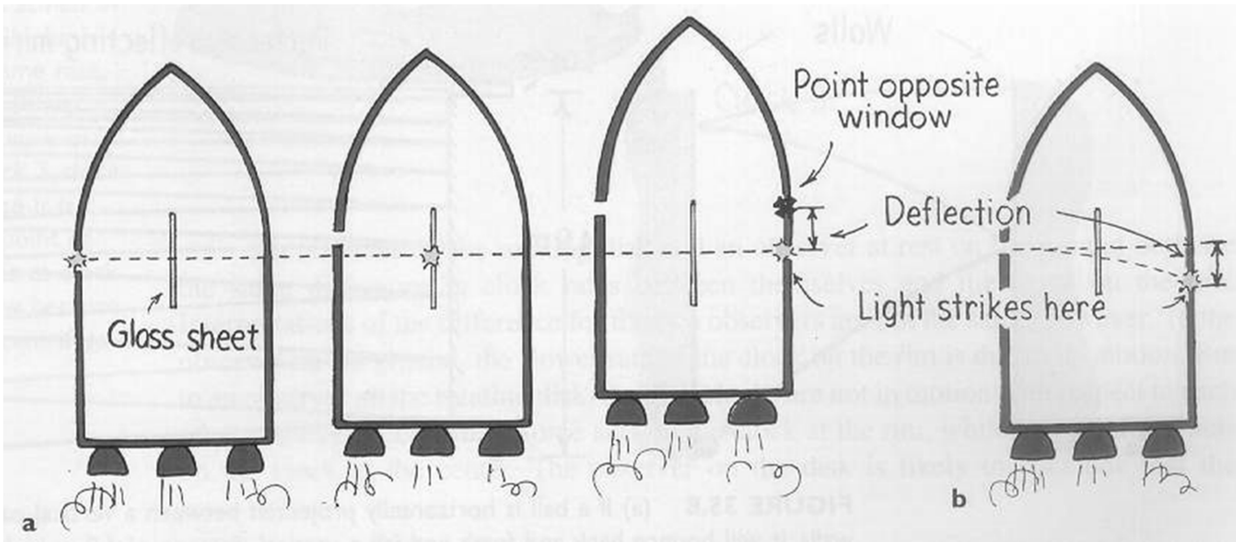
- ✓An object always takes the shortest paths (*geodesic*) in space-time
- ✓Analogy: geodesic on a sphere



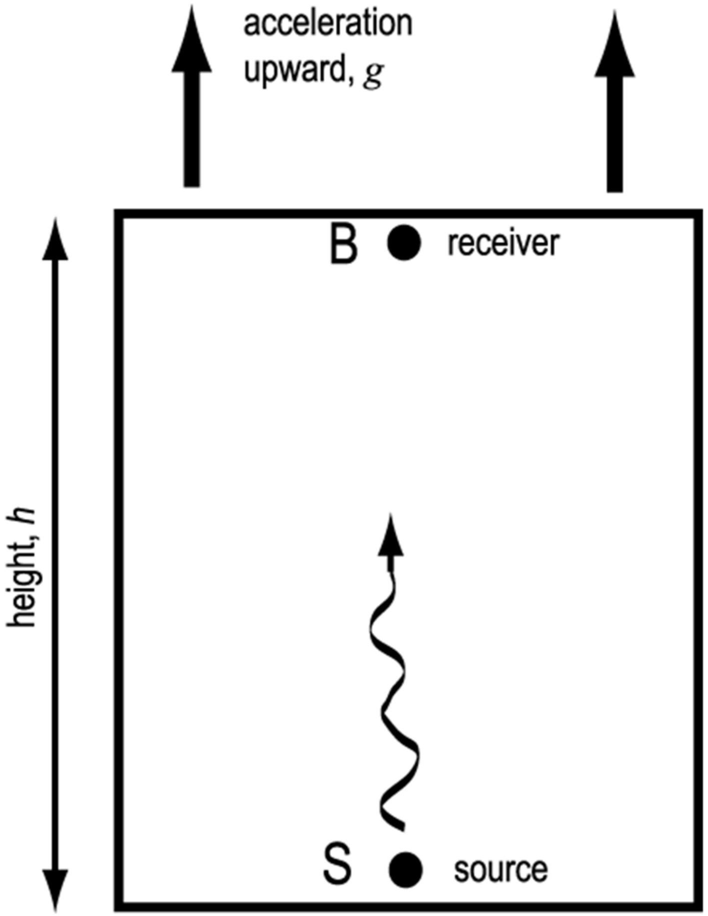
Some consequences

Light is deflected by masses

✓According to GR, *any objects*, including light, will deflect in a curved space-time due to the presence of massive body



Gravitational red shift



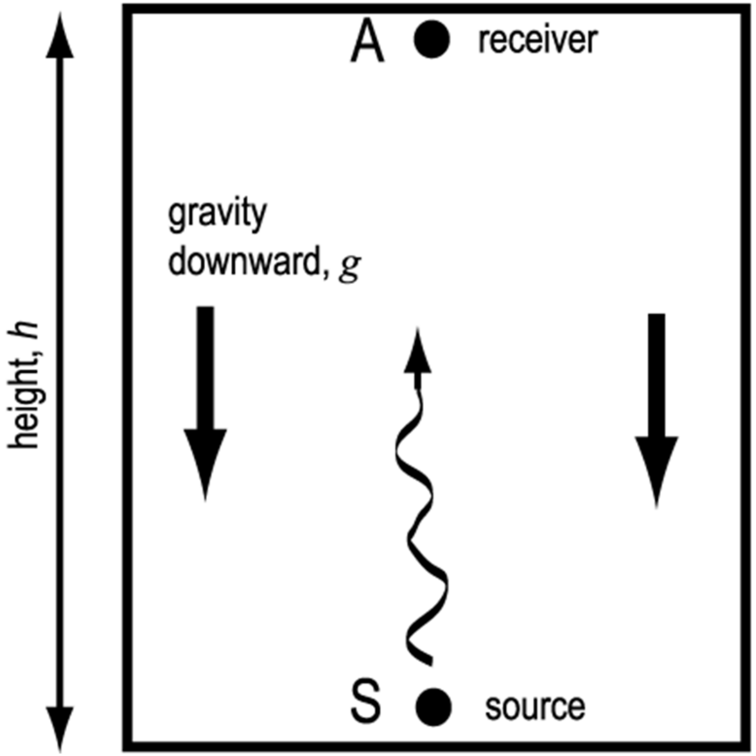
Light takes $\Delta t = h/c$ for the pulse reaching the ceiling

During that time, the lift is at a speed of $v = g\Delta t = gh/c$

Classical Doppler shift: B receives a frequency-shifted light

$$\frac{\Delta f}{f_0} = \frac{-v}{c} = \frac{-gh}{c^2}$$

Gravitational red shift



By equivalent principle, A receives red-shifted light:

$$\Delta f / f_0 = -gh / c^2$$

$$f = (1 - gh / c^2) f_0$$

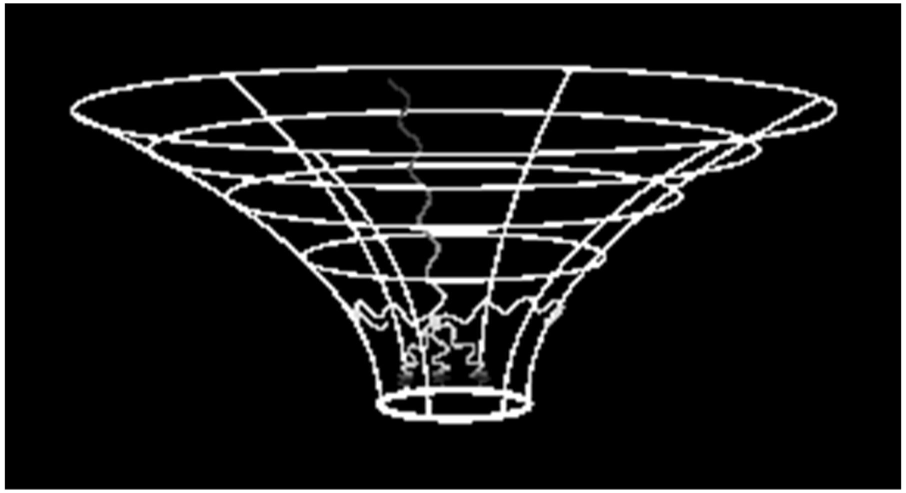
Note: only true for uniform field g

Gravitational red shift

In terms of *change* in gravitational potential $\Delta\phi$,

✓Light is red-shifted when it moves opposite to the direction of a gravitation field.

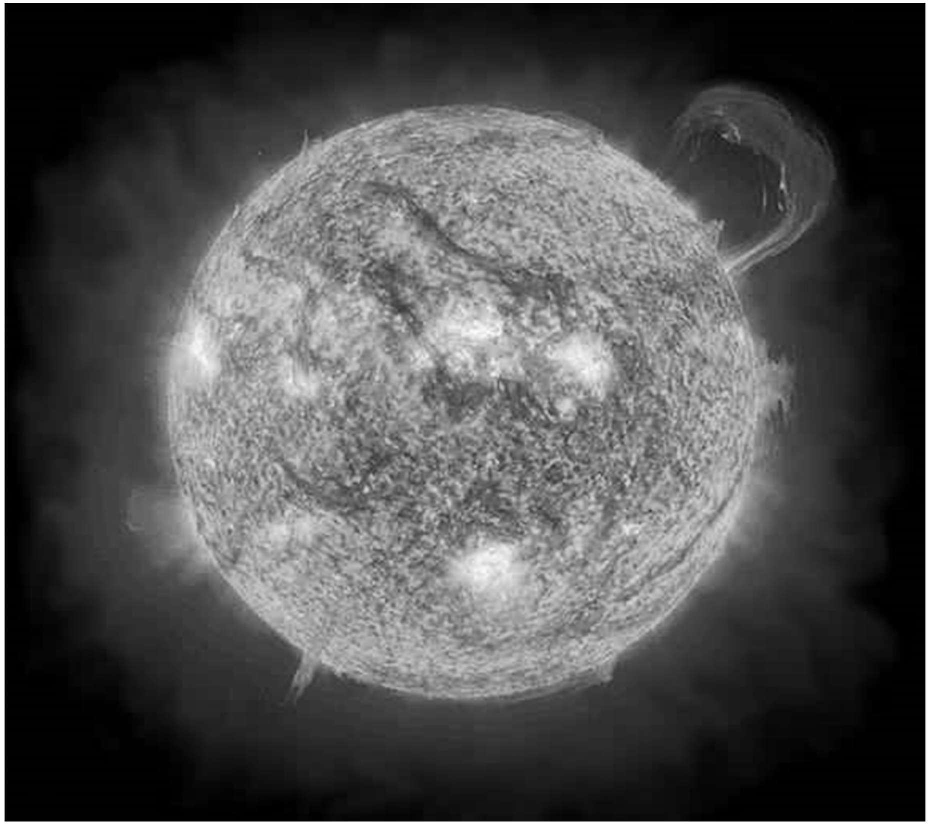
✓Similarly, light that moves in the direction of a gravitation field is blue-shifted.



Gravitational red shift

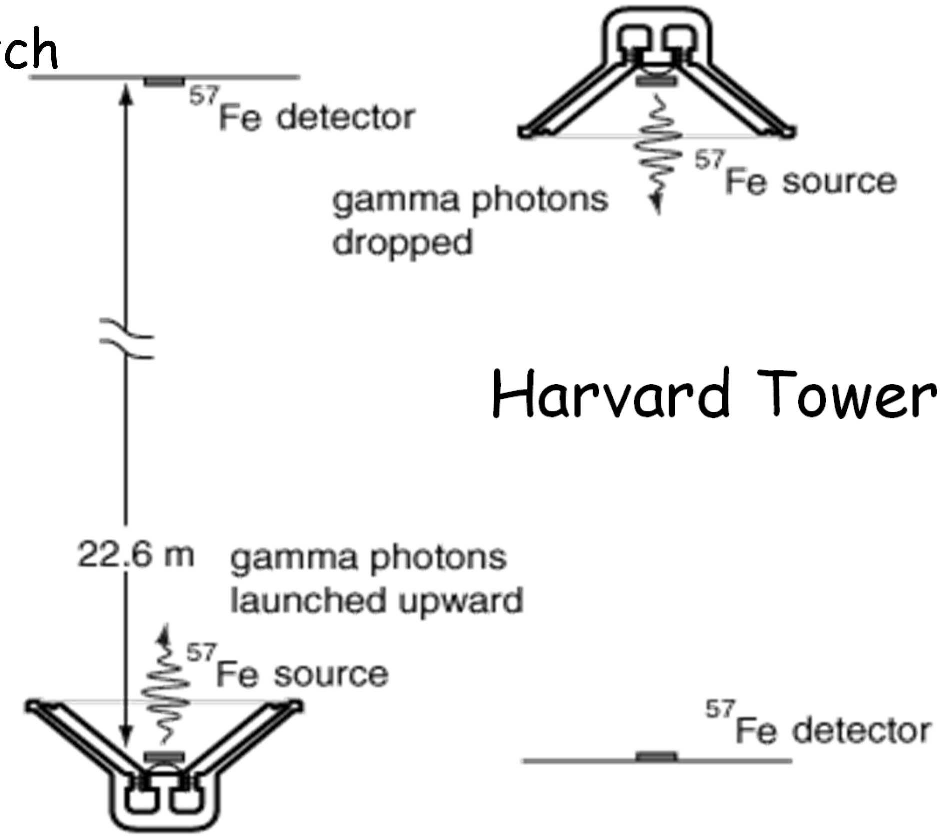
Example, for a light from the surface of the Sun (to the Earth), the change in gravitational potential is roughly equal to

$$\Delta\phi / c^2 = GM / rc^2 = 2.1\times10^{-6}$$



Gravitational red shift

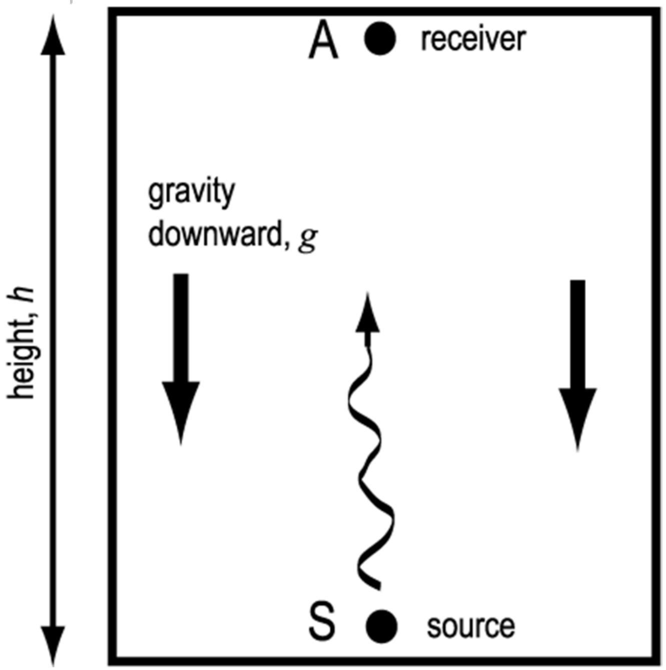
Lower pitch
received



Harvard Tower Experiment

higher pitch
received

Gravitational time dilation



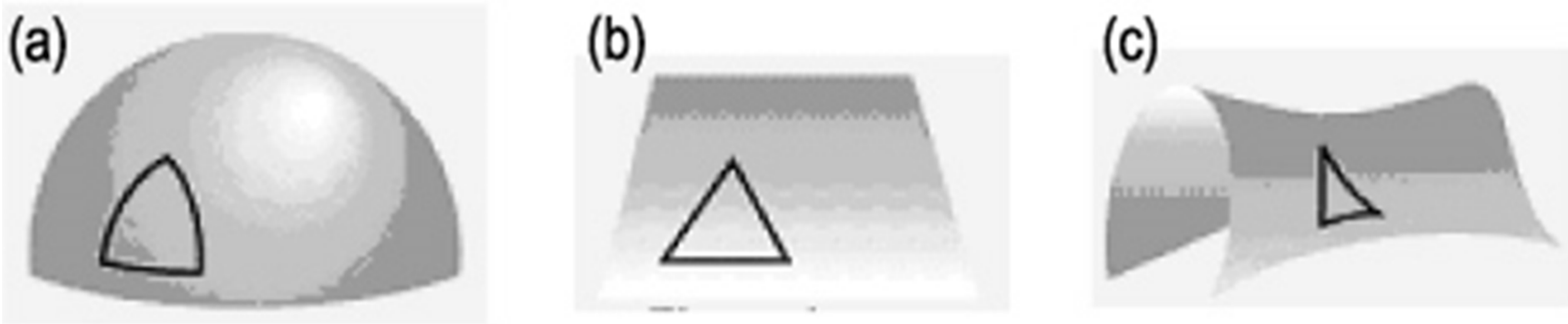
One clock at S, another at A; A receives a red-shifted light. That means S emits N pulses and A receives all, but at different rate. How come?

One would conclude that the two clocks keep time at different rates.

In other words, the clock at source runs slower compared with the clock at A.

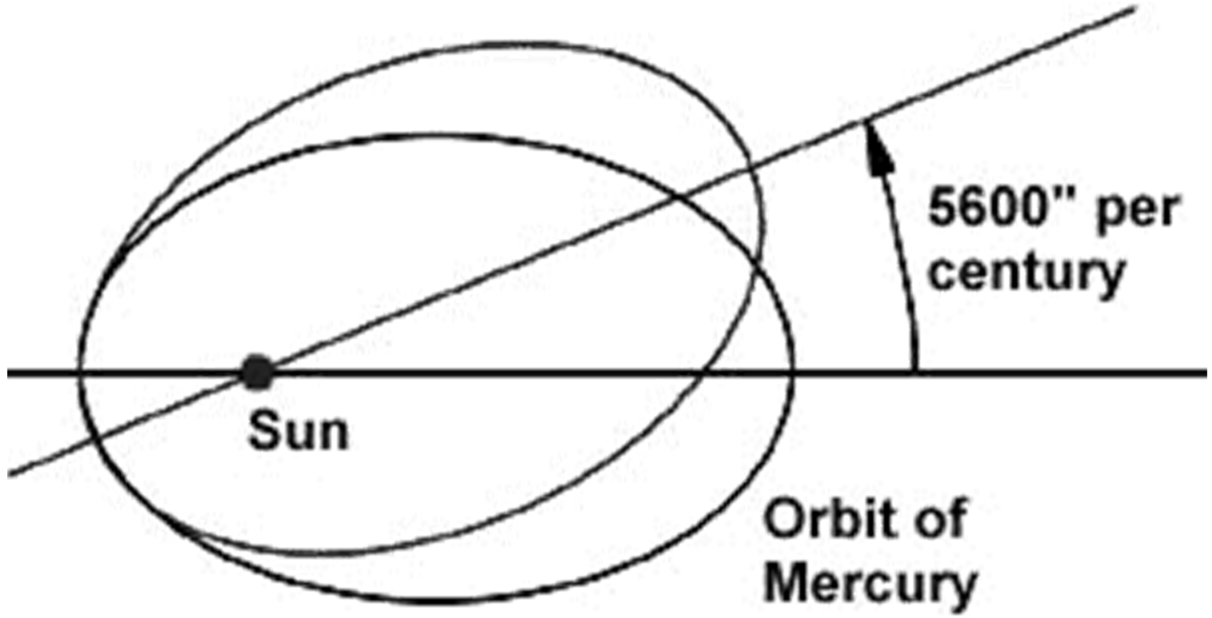
Clocks close to a massive body run slow compared to ones that are farther away. The effect is known as gravitational time dilation.

Gravity and space

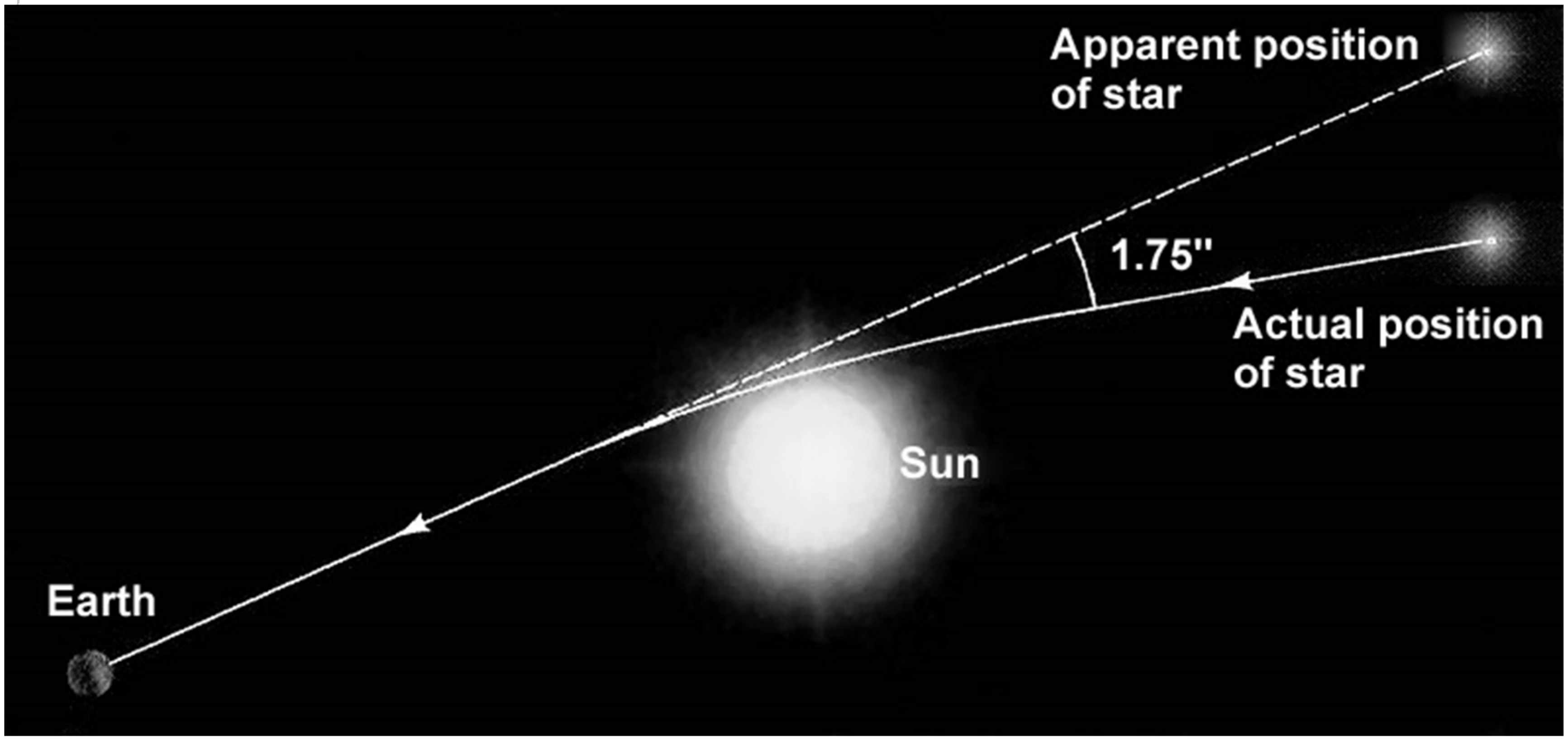


Two-dimensional analogy. Under the influence of gravity, the sum of interior angles may be (a) greater, (b) equal to, or (c) less than 180 degrees.

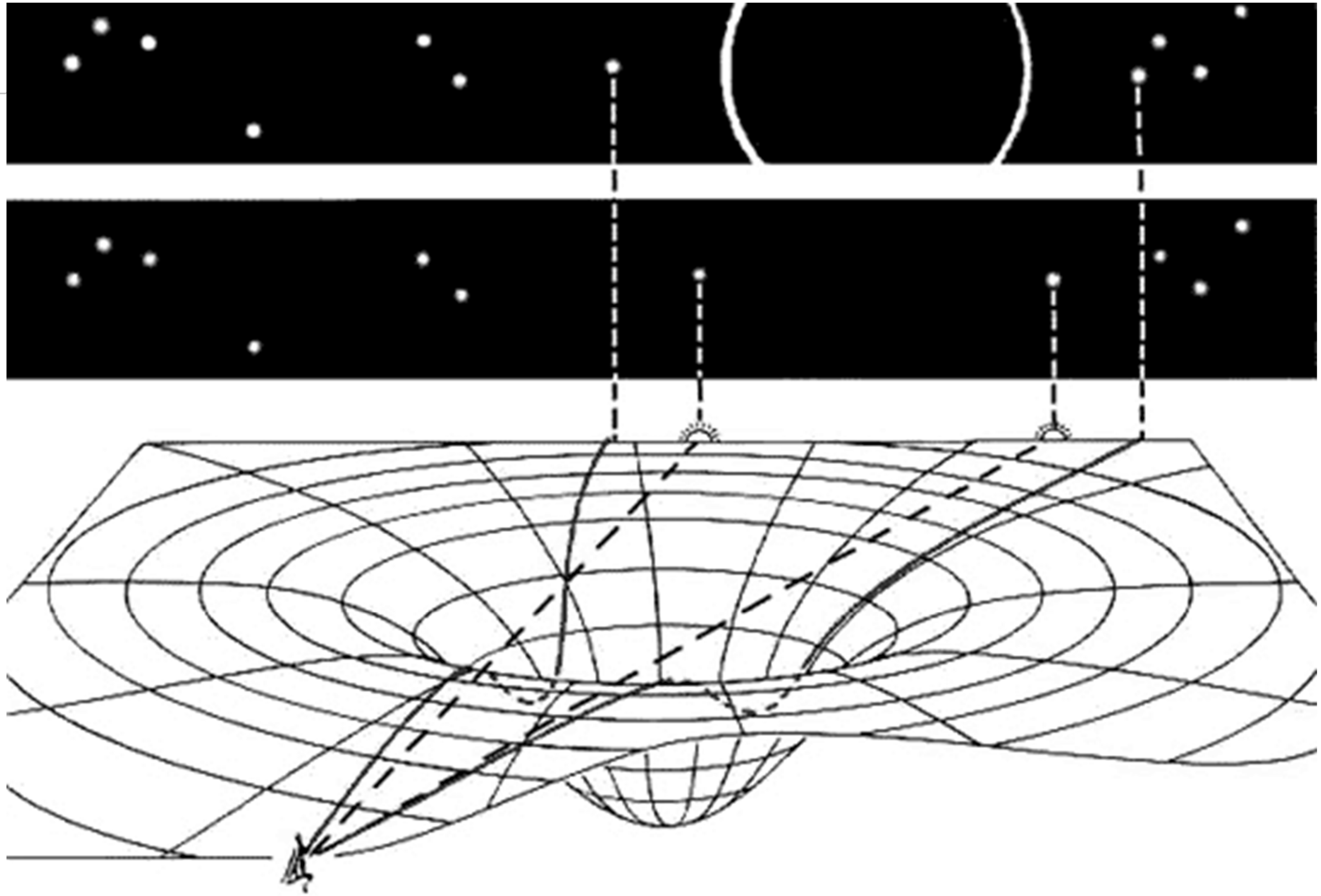
Experimental evidence



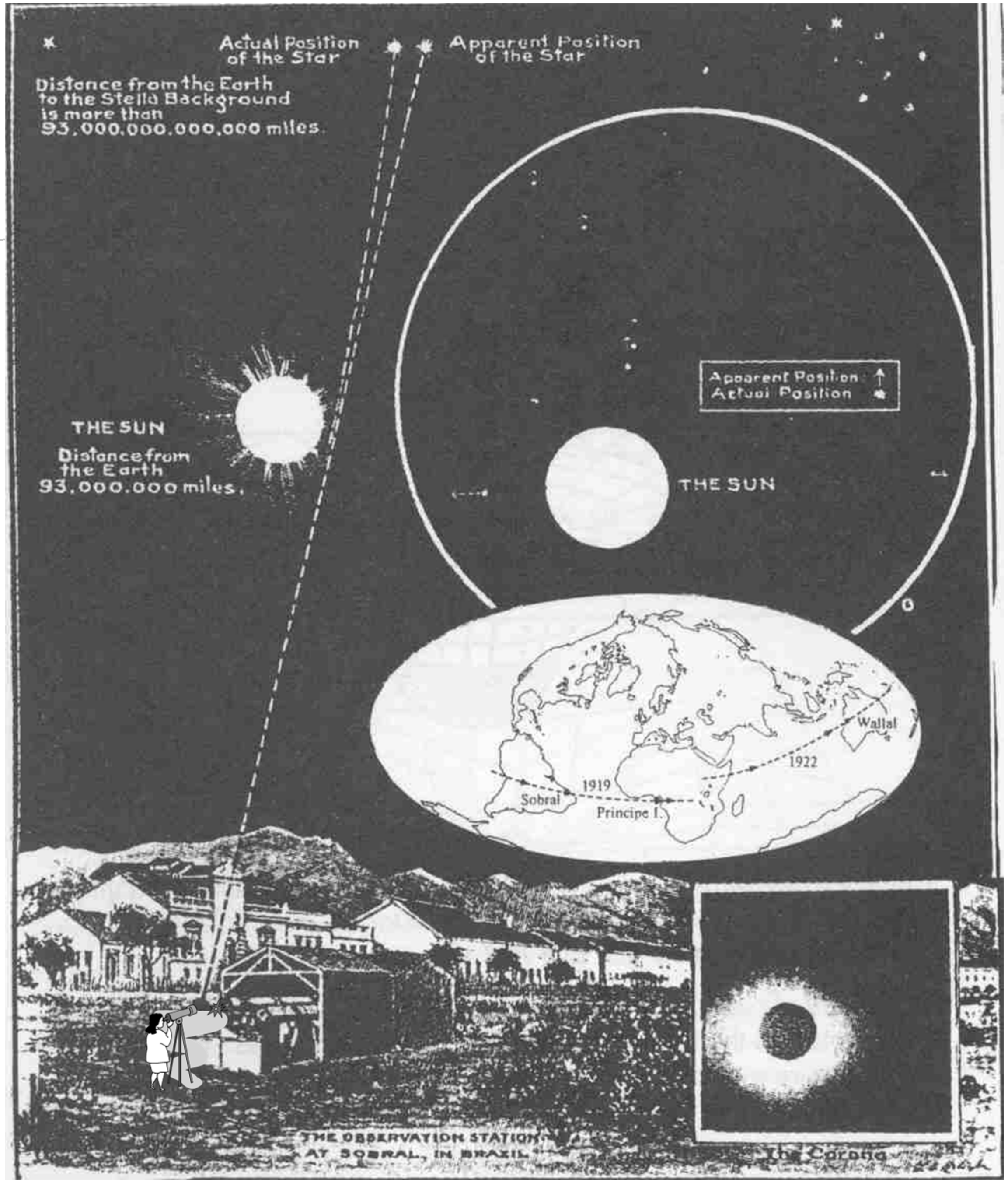
✓Newton can explain the *precession of Mercury orbit* is about 5600″ per century
✓But discrepancy of 43″ per century only explained by GR


Apparent position
of star
1.75''
Actual position
of star
Sun
Earth

Eclipse of 29 May 1919



Einstein predicted shifted by ~1.75″



Eclipse:
29 May 1919