

Chapter 6

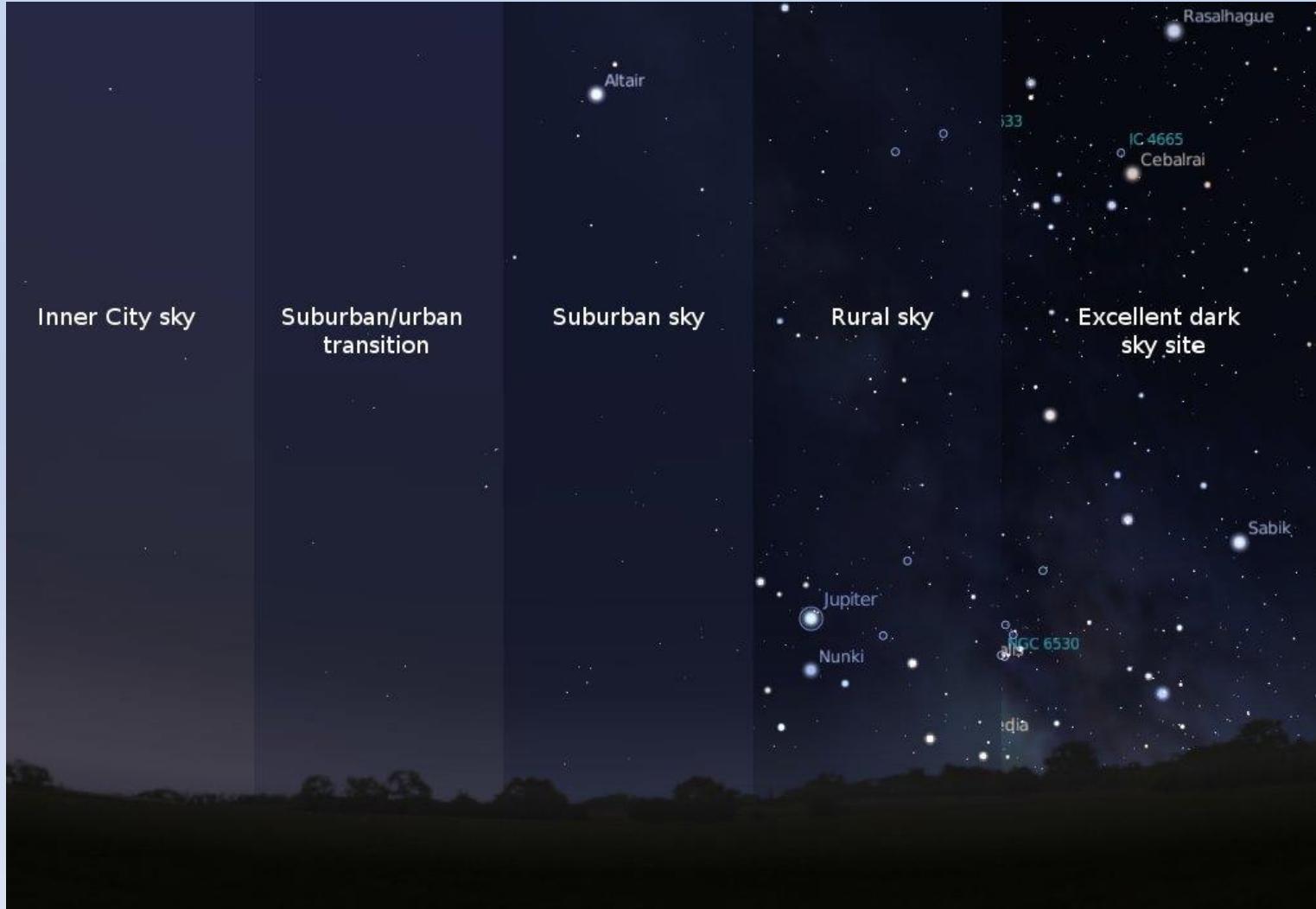
Measuring Stars

Measuring Stars

- 6.1 Telescopes
- 6.2 Distances and motions of stars
- 6.3 Magnitude scales
- 6.4 Surface temperature of stars
- 6.5 Physics of stellar spectra
- 6.5 Hertzsprung-Russell diagram
- 6.6 Binary stars

6.1 Telescopes

Image: Stellarium



Even without light pollution, some stars are too dim for us to see.

6.1 Telescopes

- ✓ Main purpose is for collecting light
 - basically a funnel of light (generally, EM radiation; or more generally, radiation or particle)



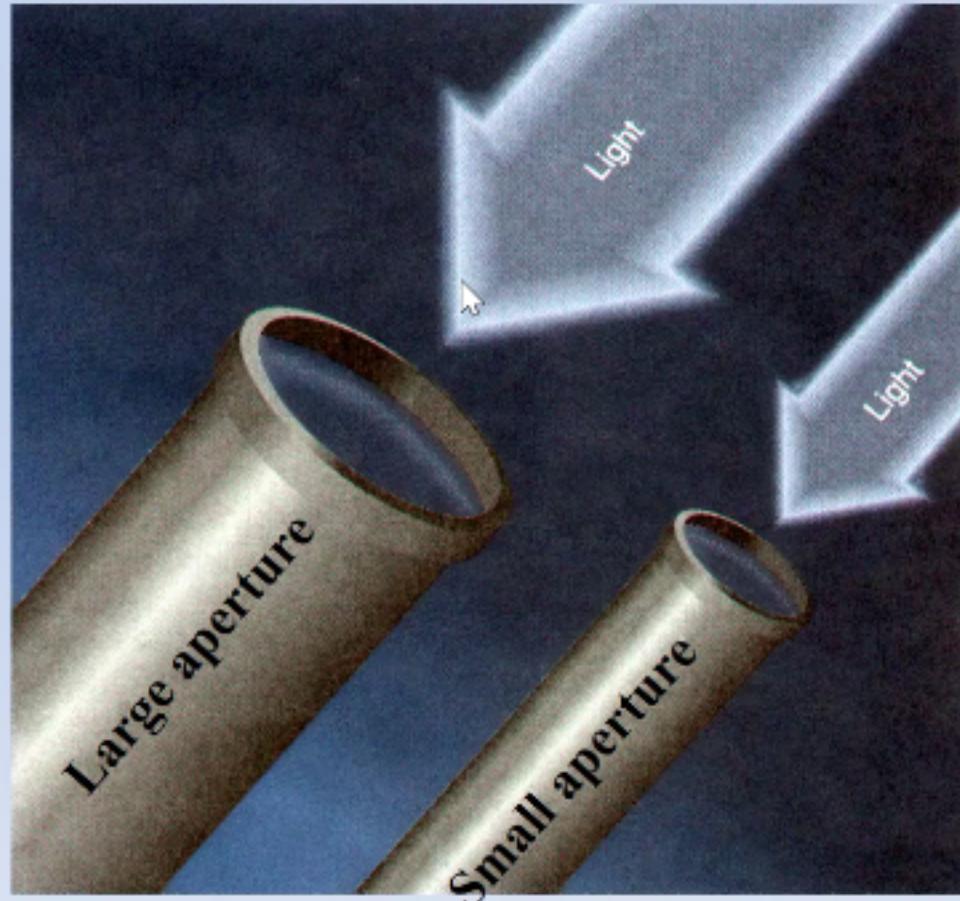
- ✓ The light-gathering power is directly proportional to the area of objective lens/mirror (thus fixed)



diameter of human
pupil \sim 7 mm

Light collection power =
$$\left(\frac{\text{lens diameter}}{\text{pupil diameter}} \right)^2$$

For example, for a 100-mm telescope, the images are \sim 204x brighter than naked eyes



A large aperture telescope can collect more light.

Types of optical tubes

1. Refractors (折射式望遠鏡)
2. Reflectors (反射式望遠鏡)
3. Catadioptrics (折反式望遠鏡)





The biggest refractor in the world in Yerkes Observatory (USA), a telescope with an aperture of 102 cm.





... | The largest reflector in the world, Gran Telescopio Canarias (Spain), aperture 10.4 m.





Reflectors: The Hubble Space Telescope (HST), the first space telescope that was launched into low Earth orbit in 1990, aperture 2.4 m.



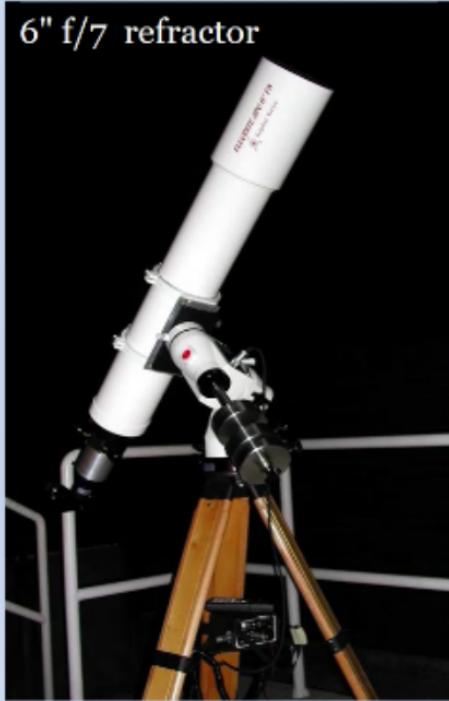


Catadioptric, aperture 8"





6" f/7 refractor



5" f/6 Newtonian reflector



f-number: $\frac{\text{focal length, } f}{\text{diameter, } d}$

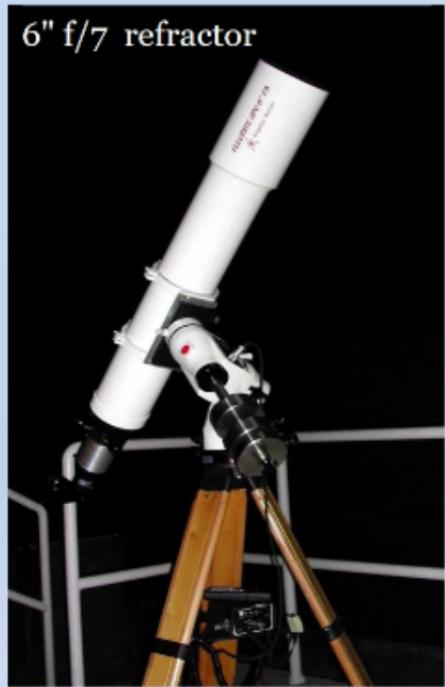
6" f/7 APO refractor

diameter of the
aperture = 6"





6" f/7 refractor



5" f/6 Newtonian reflector



The focal length of the objective lens:
750 mm (=5x25.4x6)

The focal length of the objective
lens: 1000 mm (=6x25.4x7)

8" f/10 Schmidt-Cassegrain

The focal length of the objective
lens: 2000 mm (=8x25.4x10)



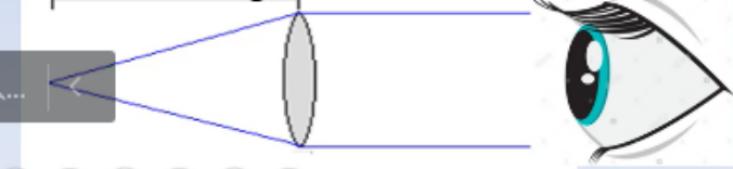


Accessories

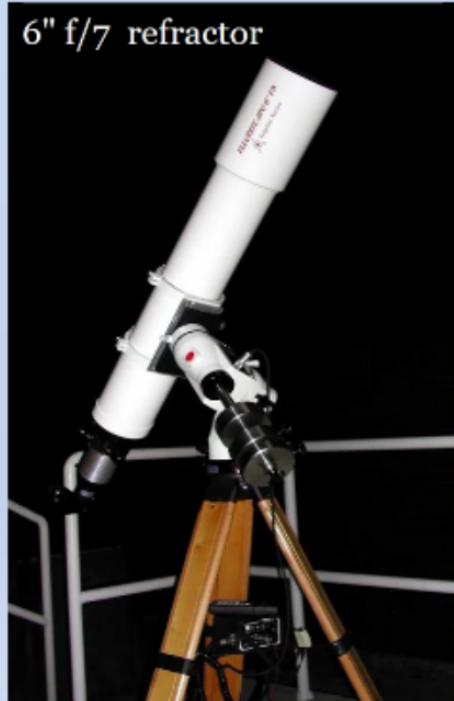
Eyepieces



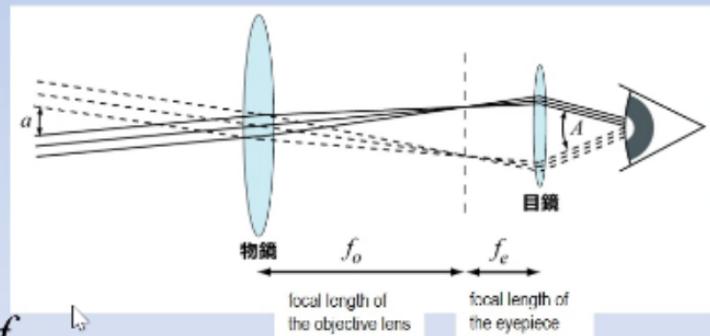
focal length



6" f/7 refractor



5" f/6 Newtonian reflector



Magnification: $m = \frac{A}{a} = \frac{f_o}{f_e}$



$$\text{Magnification: } m = \frac{A}{a} = \frac{f_o}{f_e}$$

For a 6" f/7 telescope, the focal length of the objective lens is 1000 mm, then

$m \sim 50$ for a 20 mm eyepiece,

$m \sim 100$ for a 10 mm eyepiece, and

$m \sim 200$ for a 5 mm eyepiece.

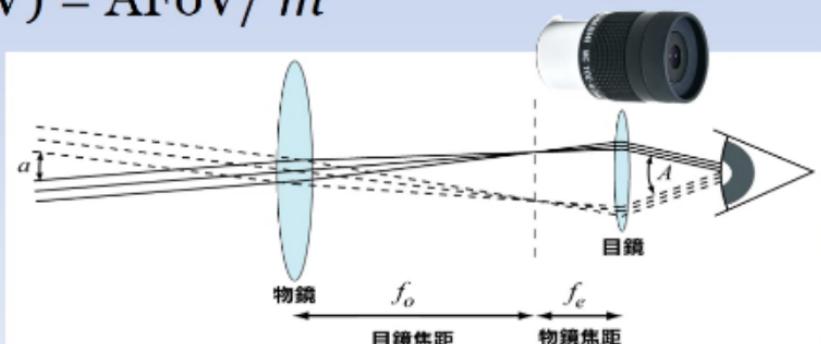
The shorter the focal length of a eyepiece, the larger the magnification is.



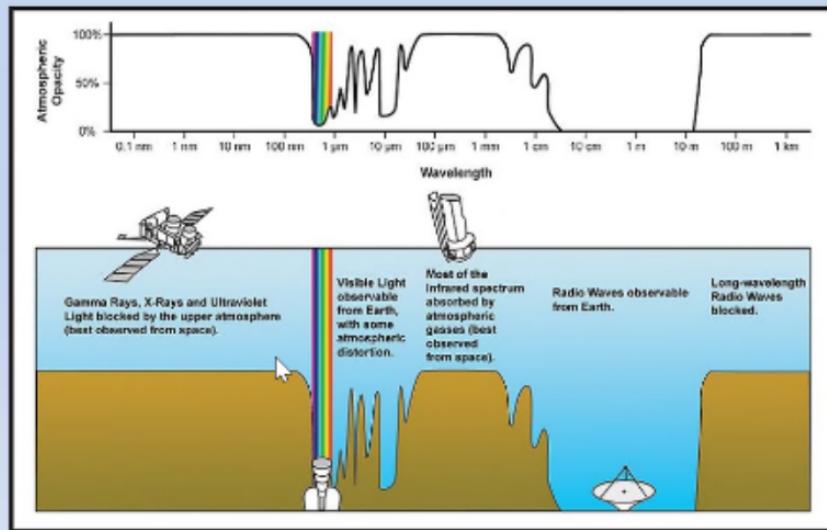
Accessories

Eyepieces

- commonly used: 5 mm, 10 mm, 15 mm
- Magnification: $m = f_o/f_e$, shorter focal length, larger magnification
- Apparent Field of View (AFoV) $\sim 50^\circ$ (typical); True FoV (TFoV) = AFoV/ m



Receiving invisible light



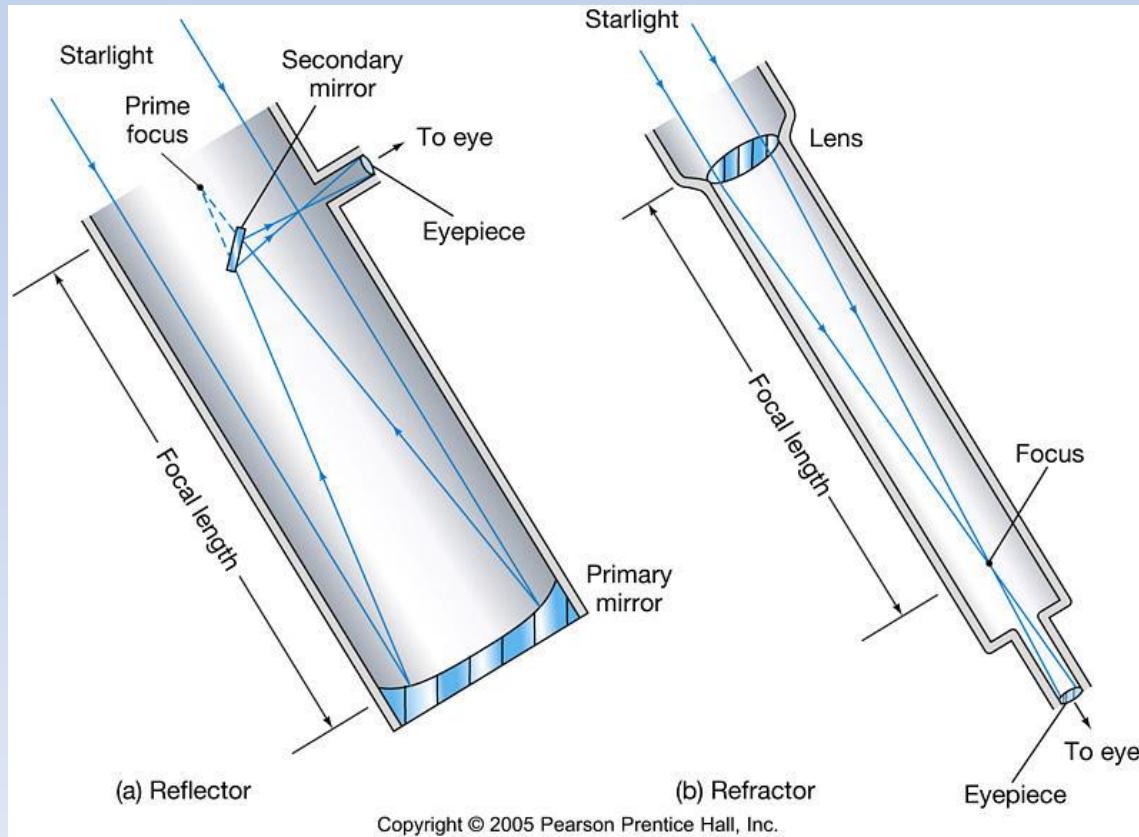
Infra-red, ultra-violet, and x-ray are easily absorbed by the atmosphere; infra-red is also absorbed by water.

To receive those signals, telescopes are built on dry regions, high mountains, or in the space.

6.1 Telescopes

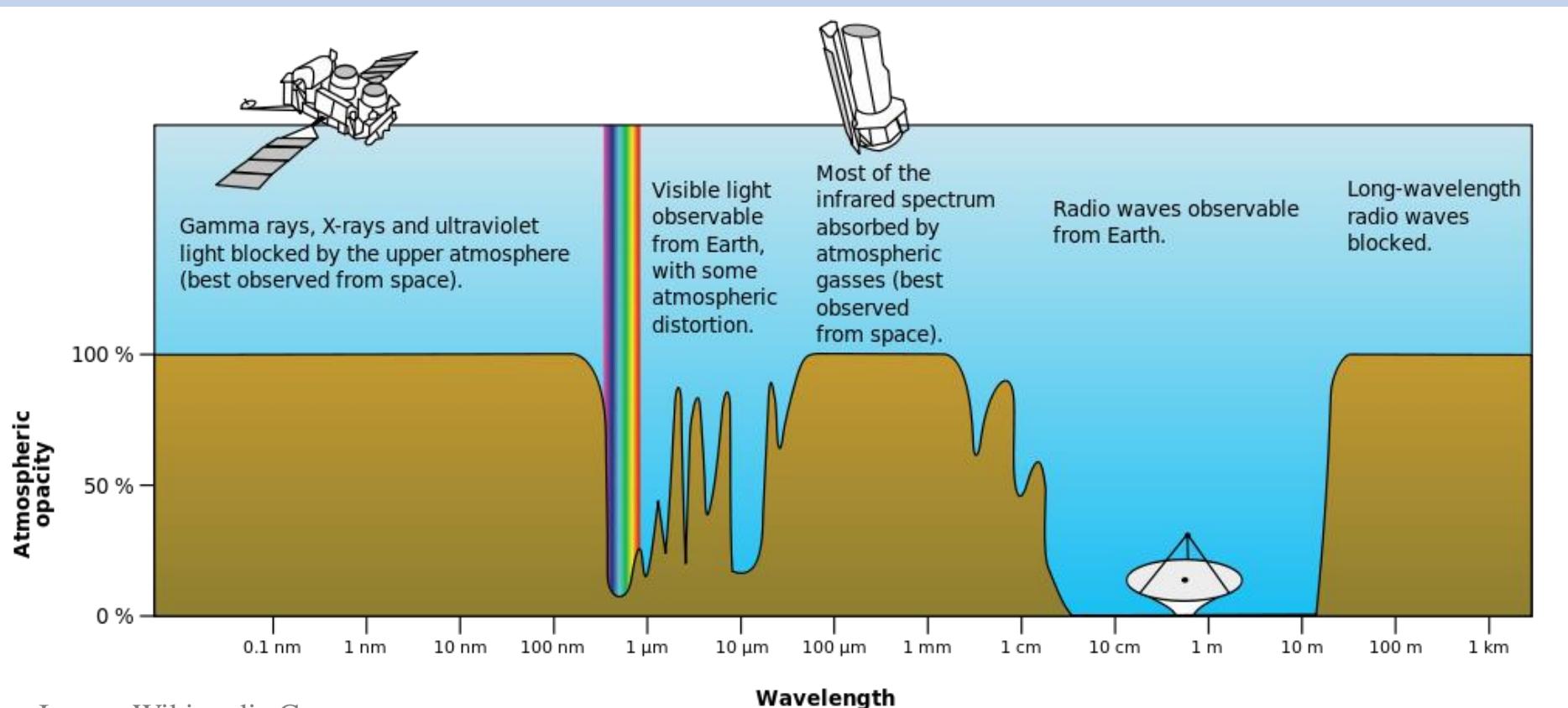
- ✓ Magnification
= (apparent angular size)/(real angular size)
- ✓ can be changed by replacing the eyepiece

Here are two of the different designs to focus light for optical telescope.



6.1 Telescopes

- ✓ Angular resolution: ability to resolve fine details
- ✓ In a lot of situations, the resolving power depends on the atmospheric condition (the "seeing")



6.1 Telescopes

- ✓ Water vapor is the main culprit at a lot of wavelength
- ✓ Need to place them at dry location, ...

ALMA, 5000m above sea level, in Chile



Image: ESO/NAOJ/NRAO

South Pole Telescope



Image: U of Chicago

6.1 Telescopes

- ✓ or up in the sky or even space (necessary for UV, X-ray, gamma ray, ...)

SOFIA in flight



Image: NASA

Hubble Space Telescope



Image: NASA

6.1 Telescopes

✓ or by doing some tricks.

Using adaptive optics with VLT

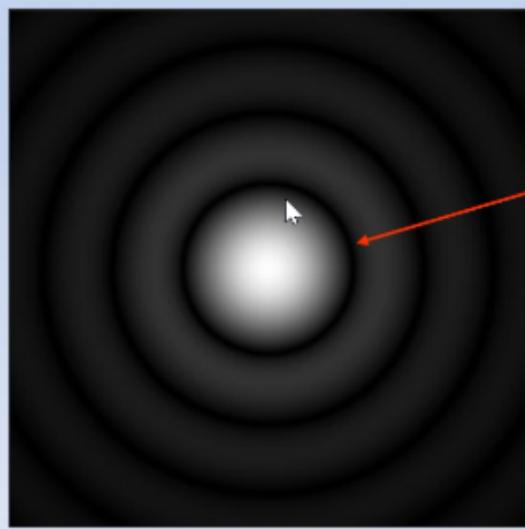


Image: ESO

Limitations: Resolving power of a telescope

- Finite size of a telescope's aperture leads to diffraction of light.

The image of a point source.



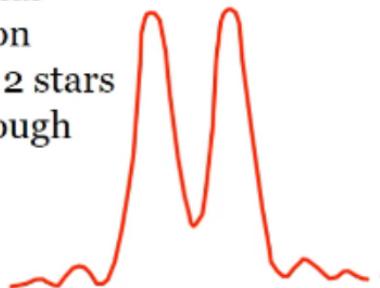
Airy disk (艾里斑)

Image: Wikimedia Commons

Limitations: Resolving power of a telescope

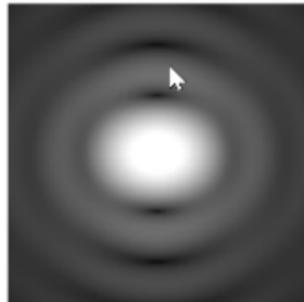
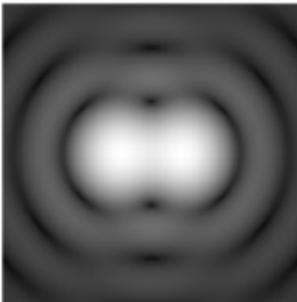
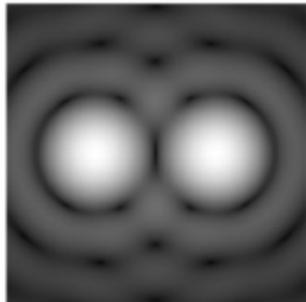
Resolved

the angular separation between 2 stars is far enough

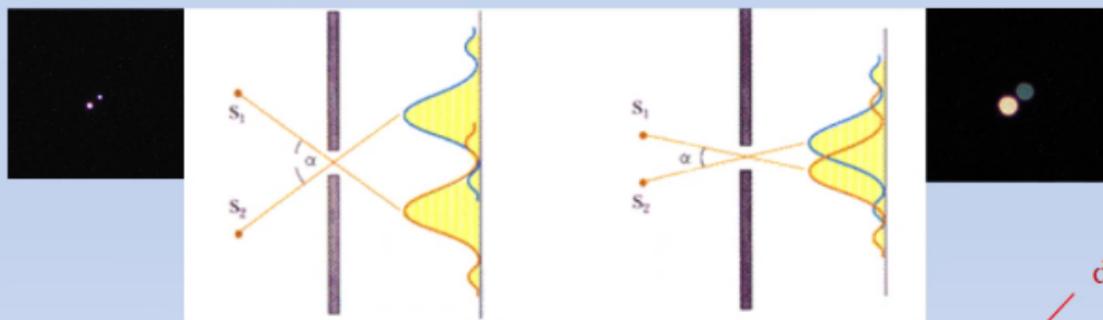


Unresolved

the separation is close, a telescope cannot resolve them.



✓ Limitations: Resolving power of a telescope



If the angular separation between 2 stars is far enough, a telescope can resolve them

However, if the separation is close, a telescope might not resolve them. Theoretically, the resolving power of a telescope: $\alpha = 1.22\lambda/d$ (in radian)
For $\lambda = 550 \text{ nm}$, the resolution of a 0.3m aperture telescope ~ 0.47 arcseconds.

arc second: 弧秒; 角秒 (等于second, arc-second)

6.1 Telescopes

- ✓ Yet, there is a limit of the angular resolution.
- ✓ Finite size of a telescope's aperture leads to diffraction of light.
- ✓ Can't resolve two sources if their Airy disks overlap
- ✓ Rayleigh criterion:

$$\theta = 1.22 \frac{\lambda}{D}$$

where D is diameter of the telescope's aperture

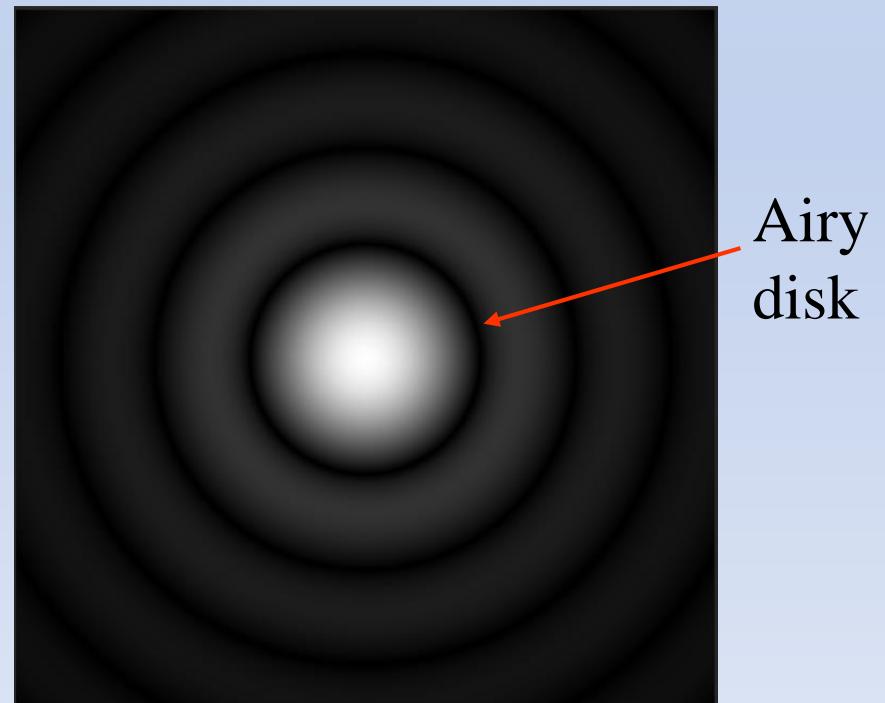


Image: Wikimedia Commons

6.1 Telescopes

✓ There's a trick to increase effective aperture.

Interferometry

VLA



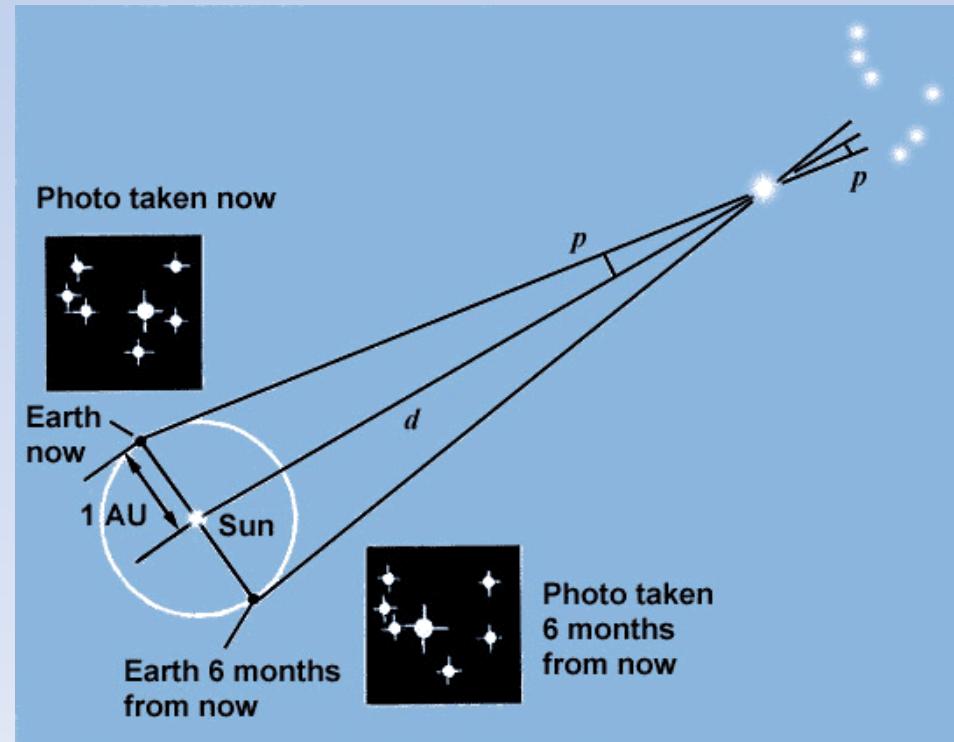
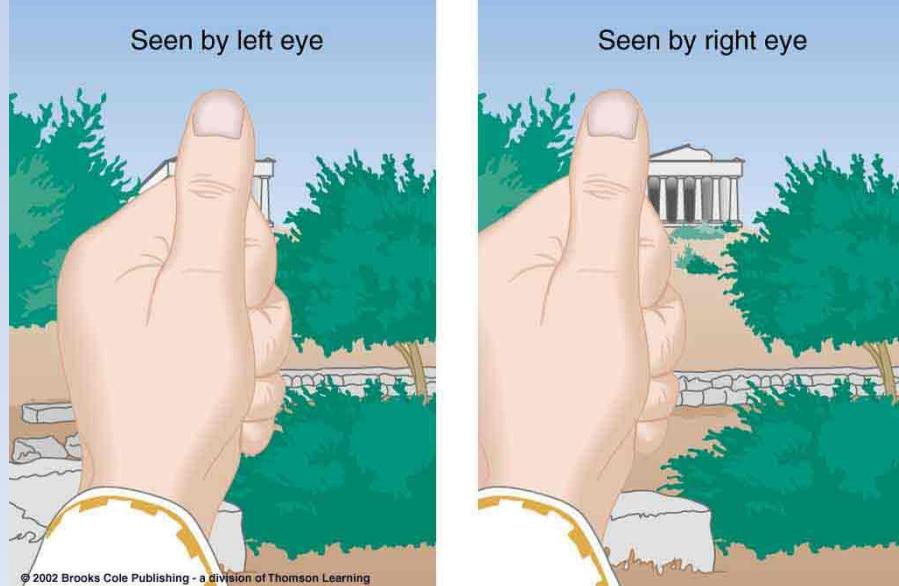
VLBI



6.2 Distances and motions of stars

Method of parallax (aka Stellar parallax)

- ✓ Earth moves around the Sun → observed from Earth, a nearby star appears to move in an ellipse in a year
- ✓ Apparent position change depends on its distance from Earth



6.2 Distances and motions of stars

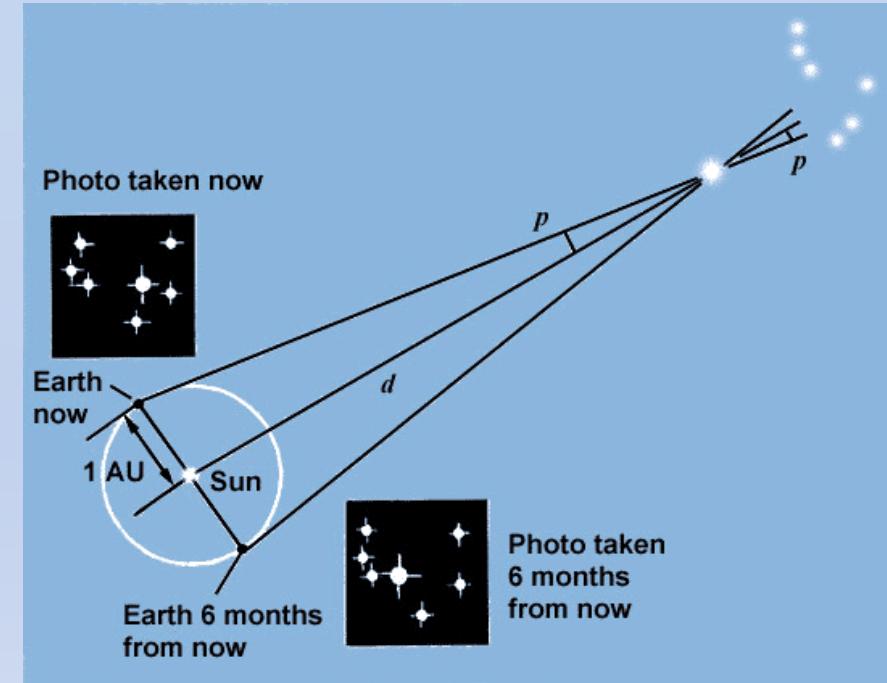
Method of parallax

- ✓ Parallax p : half of the total shift in the angular position of a star.

$$d = 1/p$$

- ✓ Angle p is in arc-second; distance d is in parsec

$$1\text{ pc} = 206265 \text{ AU} \approx 3.26 \text{ ly}$$

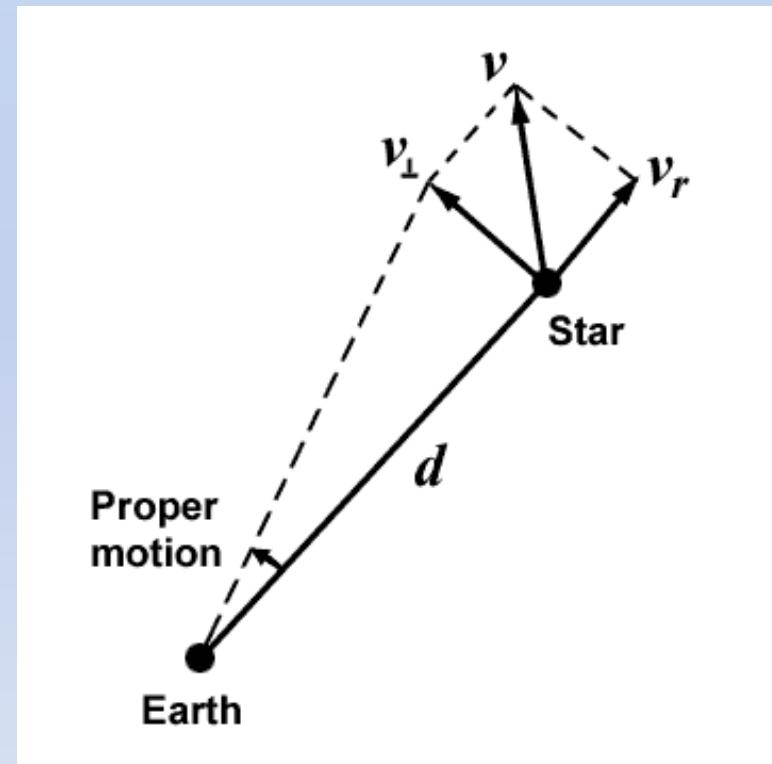


- ✓ Earth's atmosphere limits the angular resolution
- ✓ Farthest star measured by this method ~ 100 pc
(space-based Hipparcos (1989) measured distance up to ~ 1600 pc, Gaia (2013) up to $\sim 10^7$ pc)

6.2 Distances and motions of stars

Stellar Motions

- ✓ Star moves very slowly in the sky.
- ✓ Velocity can be broken into components **parallel** and **perpendicular** to our line of sight.
- ✓ The component perpendicular to our line of sight, also known as the tangential velocity v_{\perp} , is related to the **proper motion**.



6.2 Distances and motions of stars

Stellar Motions

- ✓ Proper motion (μ): the change in the angular position in one year
- ✓ The tangential velocity v_{\perp} , in km/s is given by

$$v_{\perp} = 4.74 \mu \times d,$$

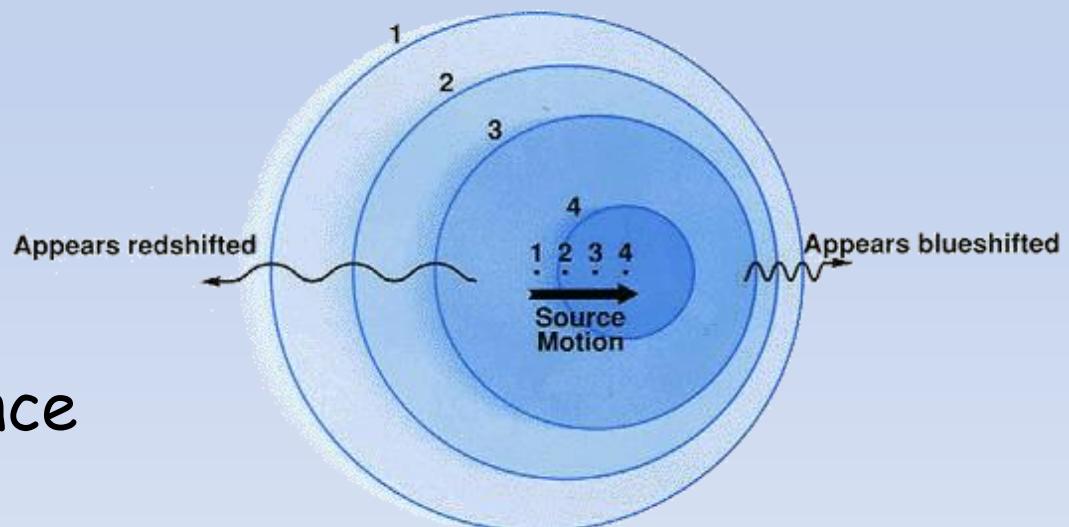
where μ measured in arcseconds; in d parsecs

- ✓ The radial component of a star's motion parallel to our line of sight is its radial velocity v_r
- ✓ Determined by the Doppler shifts of the star's spectral lines.



Box 6.1 Doppler effect

- ✓ Waves get compressed in the direction of motion of the source, wavelength smaller, frequency higher; hence called **blue shifted**
- ✓ Opposite effect observed behind the source. That is, wavelength longer, frequency lower; hence called **red shifted**





Box 6.1 Doppler effect

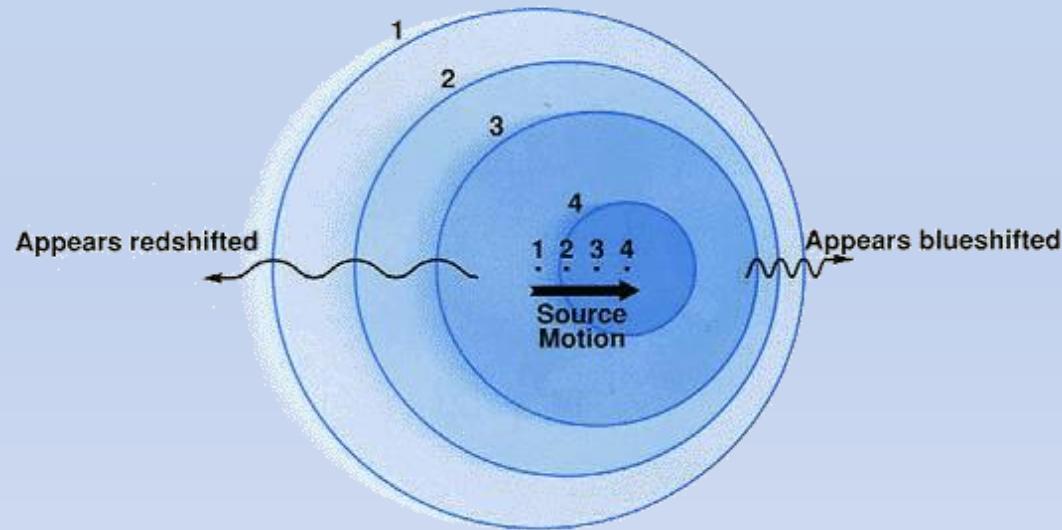
✓ For a wave propagating in a medium

$$\lambda = \left(\frac{c + v_s}{c - v_r} \right) \lambda_0$$

✓ For light, if $v \sim c$,

$$\lambda = \lambda_0 \sqrt{\frac{1+v/c}{1-v/c}}$$

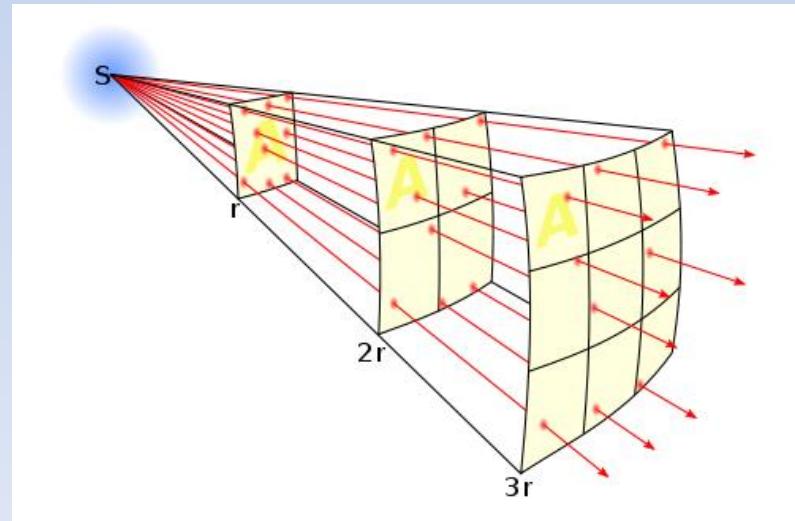
which reduces to the classical equation
for smaller velocity.



6.3 Magnitude scales

- ✓ **Luminosity L :** defined total radiation energy (all wavelengths) emitted by a star per unit time (unit W). For the Sun, $L_{\odot} = 3.90 \times 10^{26}$ W
- ✓ **Apparent Brightness $B(r)$:** amount of light energy received per unit time per unit area at a distance r from the star, (unit W m^{-2}). Also known as flux.
- ✓ **Inverse-Square Law:**

$$B(r) = \frac{L}{4\pi r^2}$$



6.3 Magnitude scales

- ✓ Apparent magnitude m : measure a star's brightness compared with a reference star, Vega

$$m = -2.5 \log(B/B_{\text{Vega}})$$

apparent magnitude of the reference star, $m_{\text{Vega}} = 0$

- ✓ *does not* measure the star's intrinsic luminosity
- ✓ The magnitude difference: $m_1 - m_2 = -2.5 \log(B_1/B_2)$

For example, 5 magnitudes ~ 100 times in brightness.

6.3 Magnitude scales

- ✓ **Absolute Magnitude M** : Magnitude measured *as if* the star was at a distance of 10 pc from the Earth.
- ✓ By Inverse-Square Law:

$$B(r)/B(10) = 10^2 / r^2$$

$$\log [B(r)/B(10)] = 2 - 2 \log r,$$

where the distance r is measured in pc. Hence, the **distance modulus** is $m - M = -5 + 5 \log r$

- ✓ *can* measure the star's intrinsic luminosity
- ✓ Bolometric magnitude: consider the whole spectrum

6.4 Surface temperature of stars

Why do stars have different colours?

e.g., reddish Betelgeuse (獵戶 α), and bluish Bellatrix (獵戶 γ)

- ✓ The colour reveals a star's surface temperature. But how are they related?

6.4 Surface temperature of stars

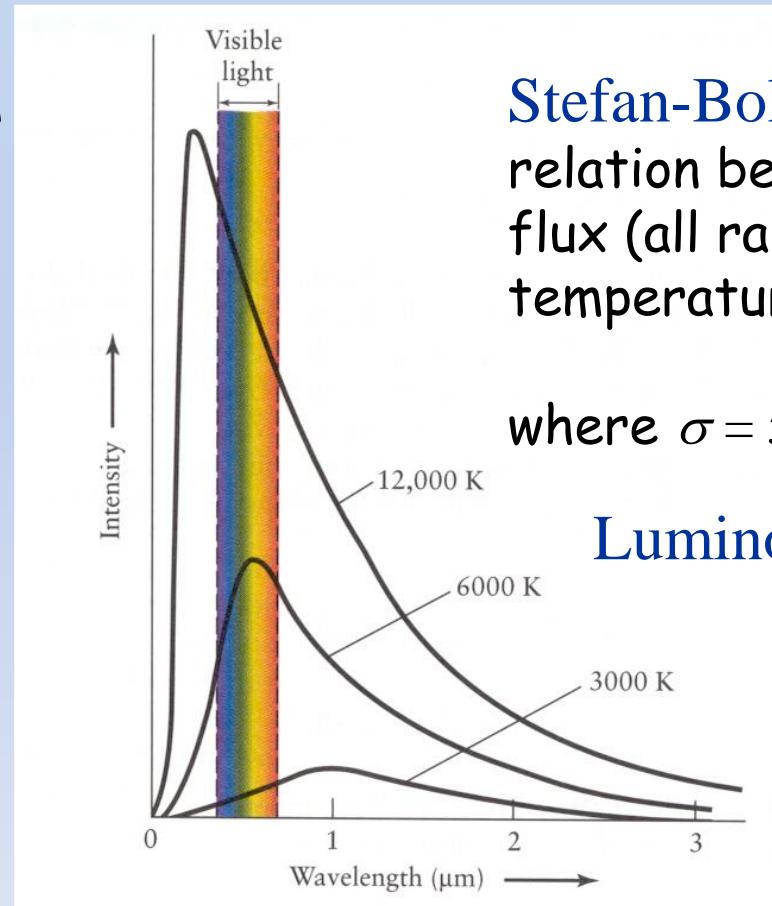
- ✓ In the star's core, nuclear reactions takes place and emits high-energy radiations, e.g., γ -rays
- ✓ The radiation absorbed and re-emitted inside the Sun. It takes about 10^5 years for the radiation to reach the surface!!
- ✓ Deep into a star, atoms fully ionized in a state of *plasma*
- ✓ Because of collisions of charged particles and photons, a continuous spectrum (EM waves of continuous wavelengths) observed, like blackbody radiation

Revision: blackbody radiation

Wien's law: relation between the wavelength at **max** emission and the temperature

$$\lambda_{\text{max}} = \frac{0.0029}{T}$$

where λ_{max} in meter;
 T in K



Stefan-Boltzmann law:
relation between the energy flux (all ranges of λ) and its temperature

$$F = \sigma T^4$$

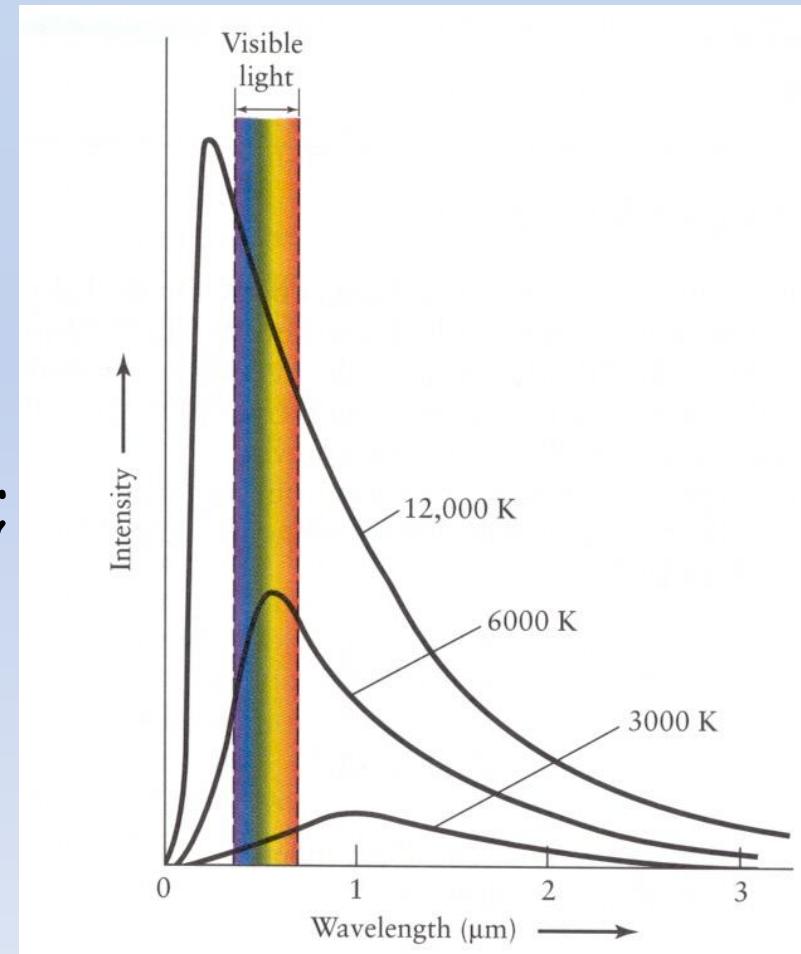
where $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

Luminosity: $L = 4\pi R^2 \sigma T^4$

- ✓ Shape of spectrum depends on its temperature

6.4 Surface temperature of stars

- ✓ According to Wien's law, blue stars are of higher surface temperatures than red stars
- ✓ the shape of spectrum depends on its temperature; hence, it is revealed by comparing the relative intensity in neighboring wavelength bands



6.4 Surface temperature of stars

UVB photometric system :

- ✓ there are lots of info in different wavelengths of the EM signal
- ✓ brightness is measured in passbands - frequency ranges allowed selectively by certain filters

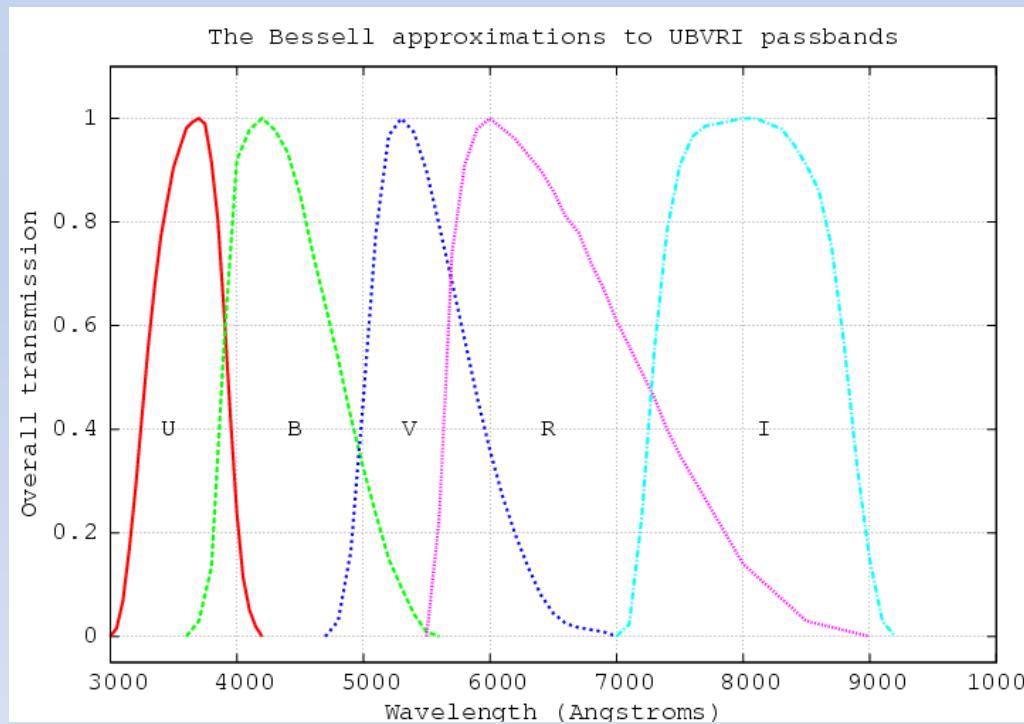
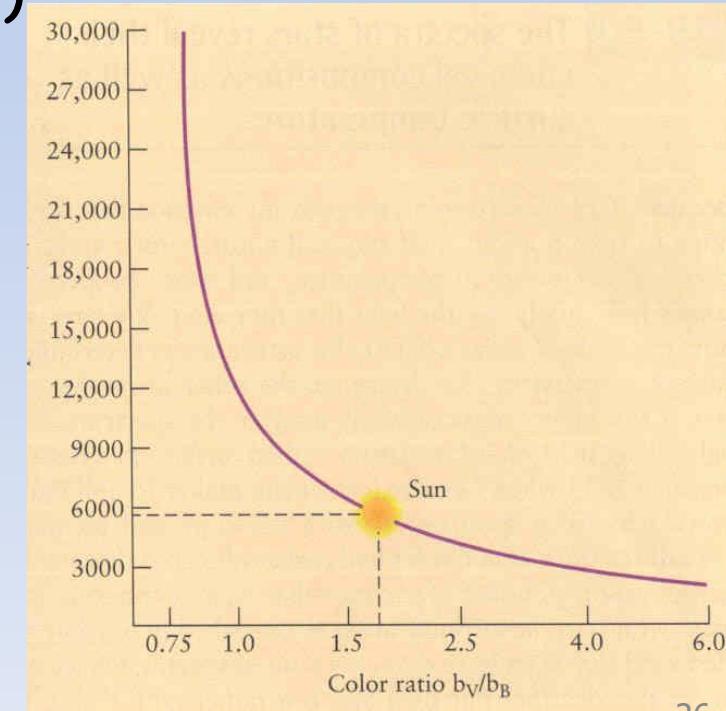


Image: RIT

6.4 Surface temperature of stars

UBV photometric system :

- ✓ A common technique to measure surface temperature
- ✓ measure apparent brightness in 3 passbands:
U-band (ultraviolet, b_U), **B**-band (blue, b_B) and
V-band (visual, peak at yellow, b_V)
- ✓ Colour ratios b_V/b_B and b_B/b_U indicate surface temperature.
For the Sun, $b_V/b_B = 1.77$ and
 $b_B/b_U = 1.10$, corresponds to
 $T = 5800\text{K}$
- ✓ Similarly, we can use the magnitudes to define the colour difference B-V and U-B



6.5 Physics of stellar spectra

How could we know the physical nature of a star,
e.g., temperature, structure, chemical
composition, etc.?

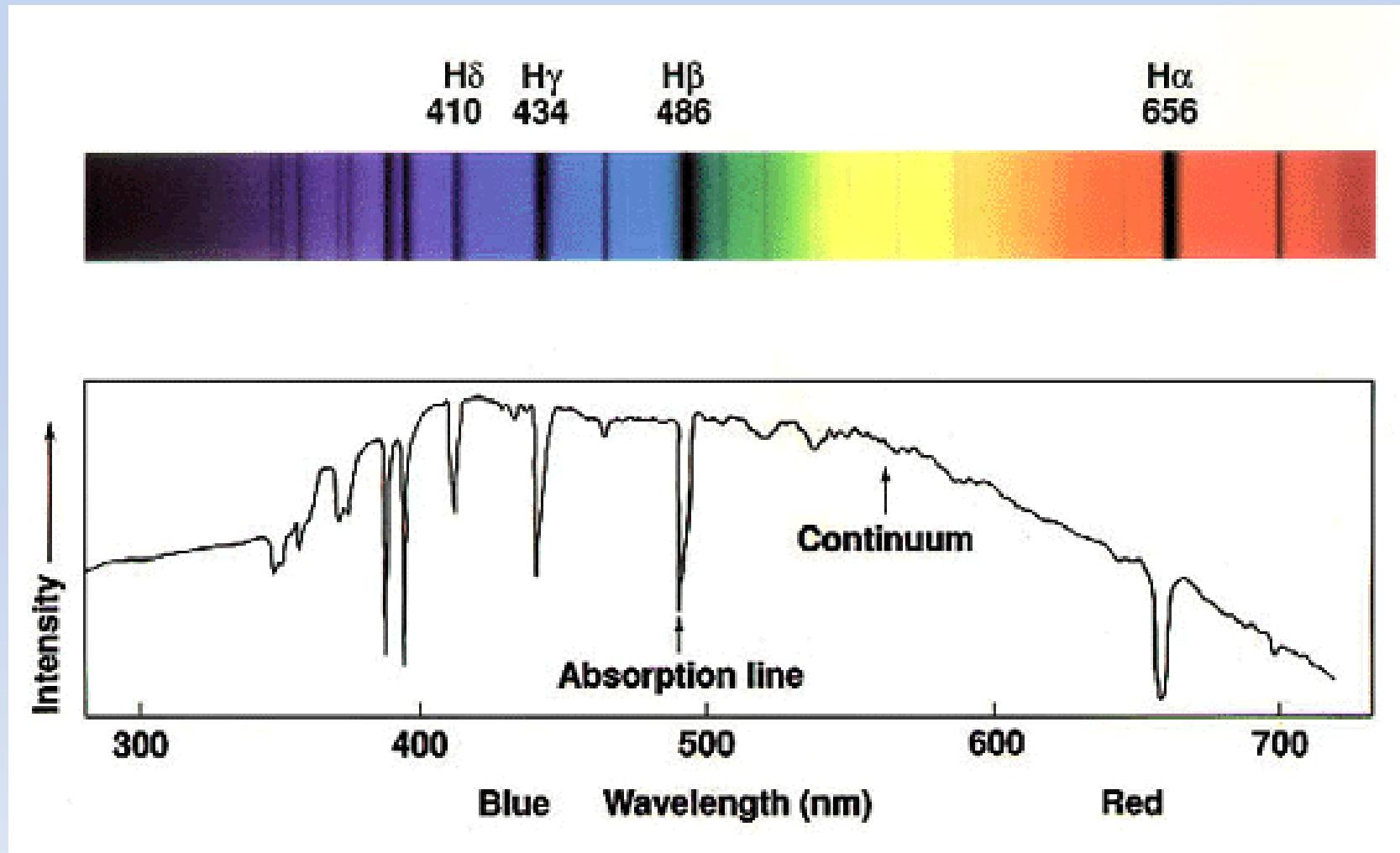
- ✓ Stars are so distant, only information we can measure is the *spectra of stars*

6.5 Physics of stellar spectra

- ✓ Continuous blackbody spectrum
 - produced at low-lying levels of the stars where the gases are relatively hot and dense
- ✓ Absorption lines
 - produced when continuum radiation flows outward through the cooler, less dense, upper layers of star's atmosphere. Atoms in the upper layers absorbs radiation at specific wavelengths;
 - hence, the spectra can tell what the elements are and their temperature

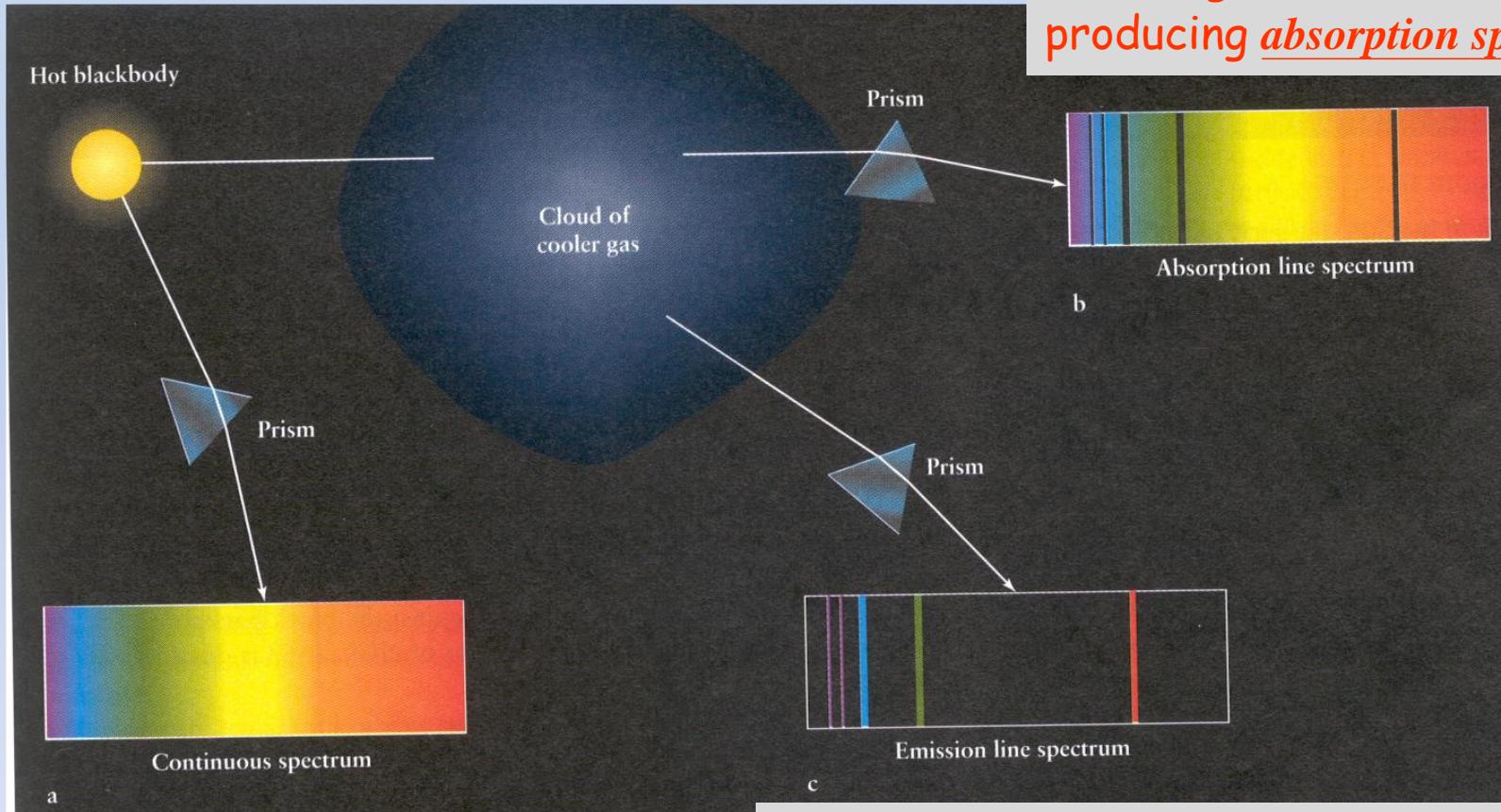
6.5 Physics of stellar spectra

吸收光谱





Box 6.2 Types of Spectra



hot, glowing object produces a continuous spectrum of light

the cloud absorbs and re-emits radiation of specific wavelengths. Observe emission spectrum against dark background

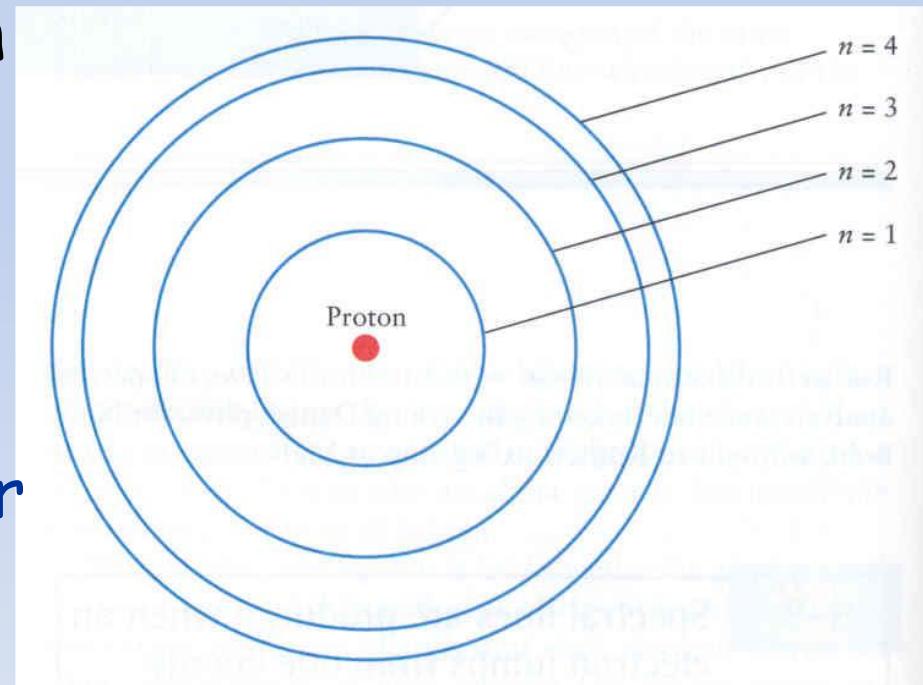
passing through cloud of cool gas, certain wavelengths are absorbed, producing absorption spectrum



Box 6.2 Types of Spectra

Bohr Model of atom

- ✓ Photon **absorbed** when an electron jumps from **lower energy orbit** to another **higher energy orbit**
- ✓ Photon **emitted** when an electron jumps from **higher energy orbit** to another **lower energy orbit**





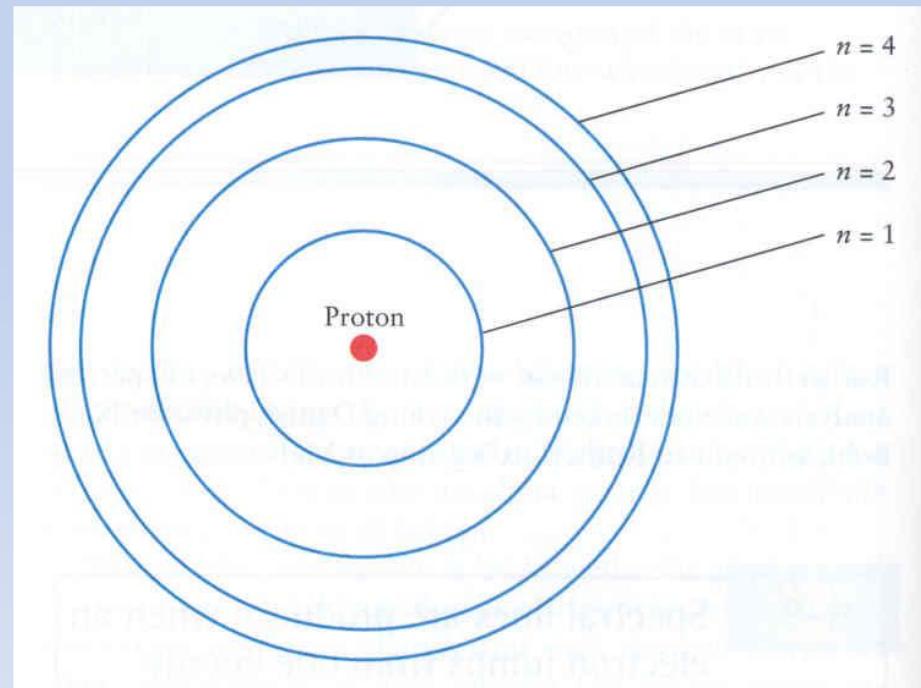
Box 6.2 Types of Spectra

Bohr Model of atom

✓ For hydrogen atom, wavelength of absorbed or emitted radiation is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{N^2} - \frac{1}{n^2} \right)$$

$R = 1.097 \times 10^7 \text{ m}^{-1}$, N is the number of inner orbit, and n is the number of outer orbit.



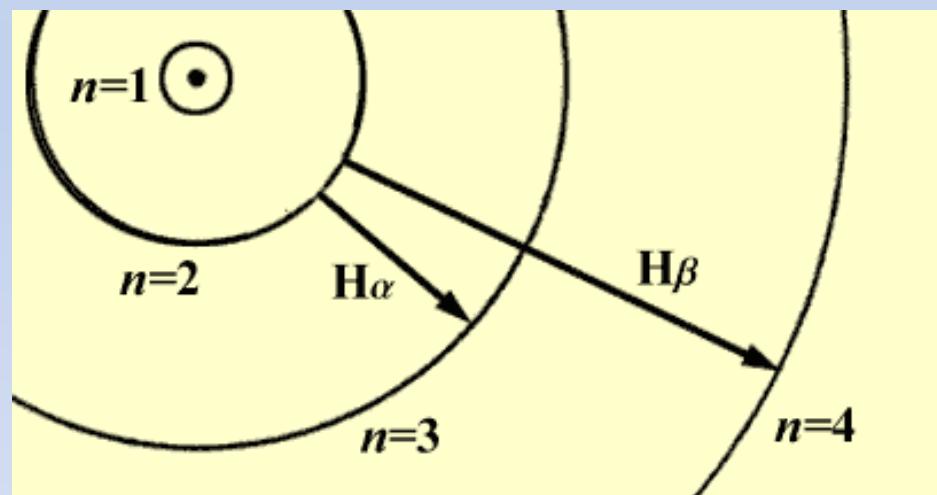


Box 6.2 Types of Spectra

✓ If electron transition between the 2nd orbit ($N = 2$) and a higher orbit, it is *Balmer series*, e.g., $n = 3$ (H_{α} , $\lambda \sim 656.3$ nm); $n = 4$ (H_{β} , $\lambda \sim 486.2$ nm).

✓ If electron transition between the 1st orbit ($N = 1$) and a higher orbit, it is *Lyman series (ultraviolet)*

✓ If electron transition between the 3rd orbit ($N = 3$) and a higher orbit, it is *Paschen series (infrared)*



6.5 Physics of stellar spectra

Harvard spectral classification

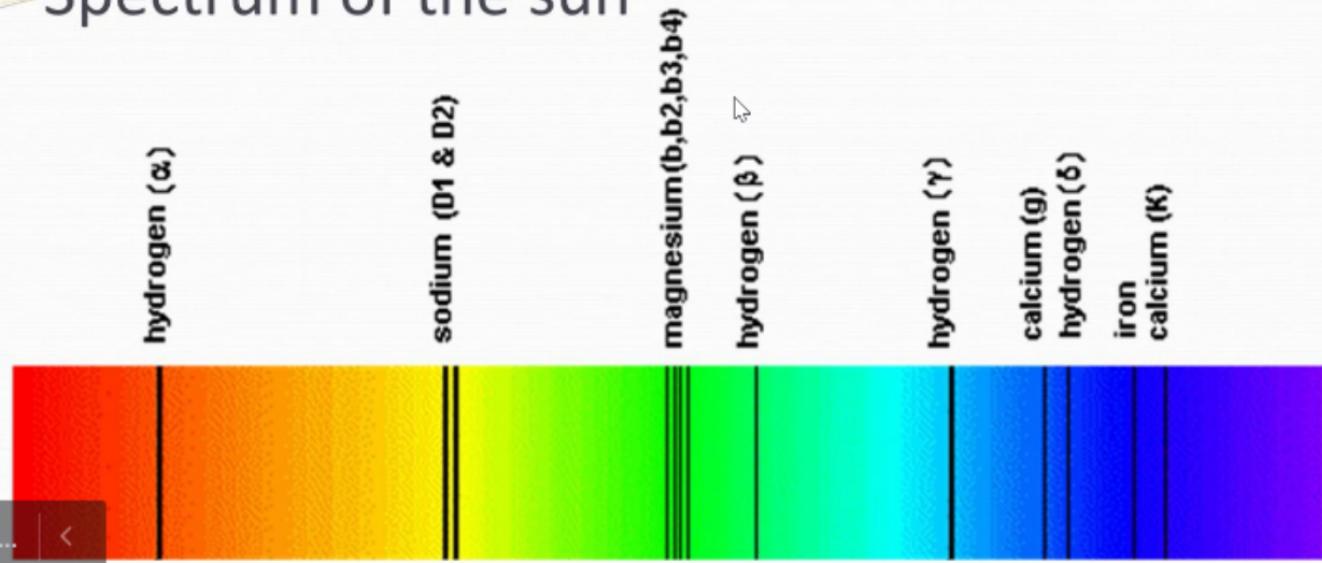
According to star's surface temperature, spectra classified into 7 *spectral classes*

Oh! Be A Fine Girl (Guy) Kiss Me

- ✓ Hottest: O stars; Coldest: M stars
- ✓ Each class divided into 10 sub-classes. For example, spectral class F includes F0, F1, .. F9.9; and then G0, ...
- ✓ Our Sun is a G2 star.

Hence, the absorption lines tell the chemical composition of the star concerned.

Spectrum of the sun



Williamina
Fleming



Catalogued roughly
5000 stars per
month between
1911-1915

Made a major
contribution to the
system of stellar
classification (O, B,
A, F, G, K, M) in
1911-1915



Annie Jump Cannon (1863–1941)

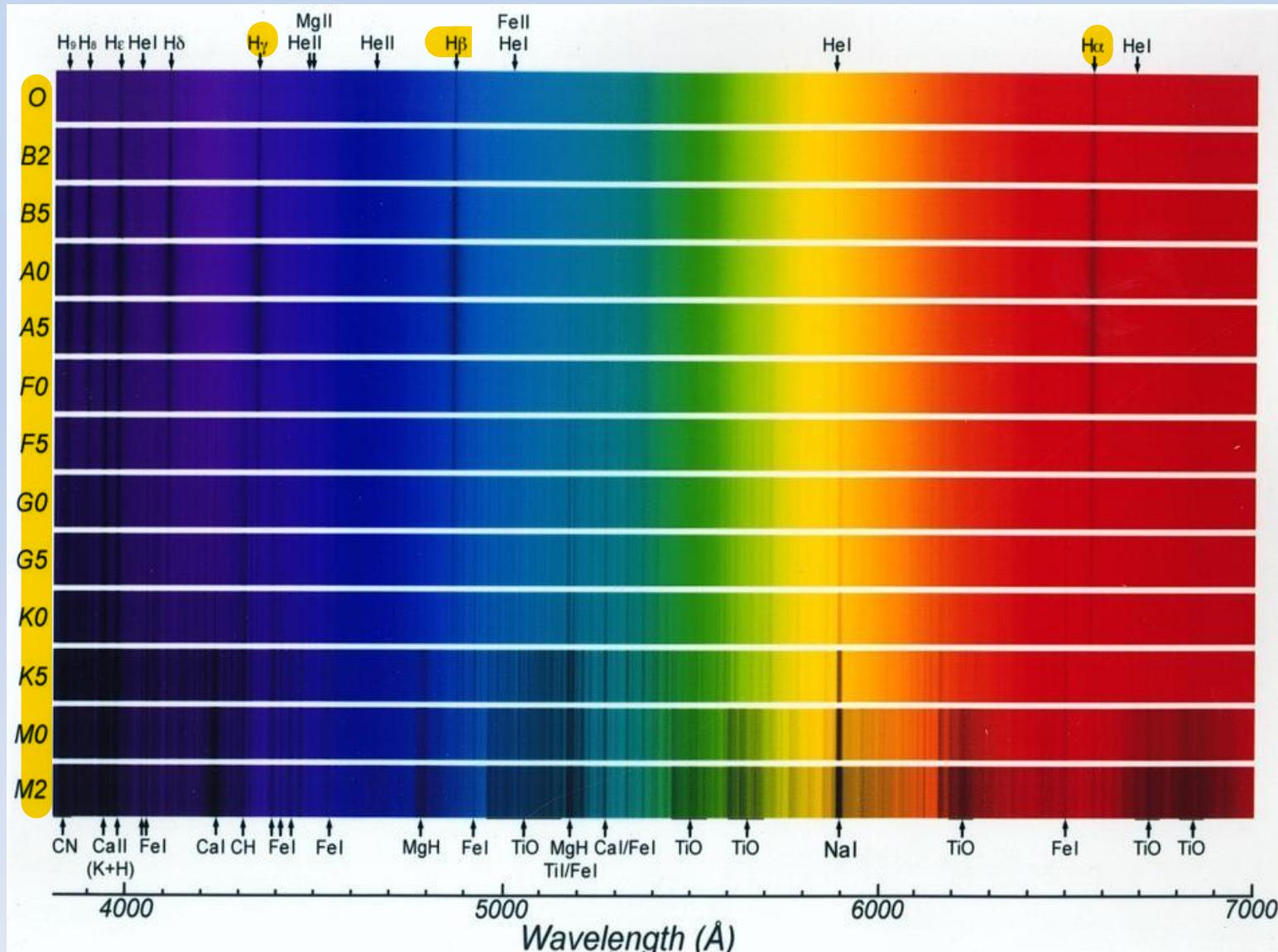
6.5 Physics of stellar spectra

Spectral Class	Spectral Characteristics	Surface Temperature (K)
O	Hottest blue star with few lines. Strong He II absorption lines.	> 30,000
B	Hot blue-white stars. He I absorption strongest at B2. HI (Balmer) absorption lines becoming stronger.	20,000
A	White stars. Balmer absorption lines strongest at A0. Ca II absorption lines becoming stronger.	8,500
F	Yellow-white stars. Ca II lines continue to strengthen as Balmer lines continue to weaken.	6,500
G	Yellow stars. Ca II lines continue becoming stronger. Fe I, other neutral lines becoming stronger.	5,500
K	Cool orange star. Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.	4,500
M	Coolest red stars. Spectra dominated by molecular absorption bands, especially titanium oxide. Neutral metal absorption lines remain strong.	3,000

6.5 Physics of stellar spectra

Question: Hydrogen is the most abundant elements in the universe. Why don't Balmer lines show up in all stars' spectra?

6.5 Physics of stellar spectra



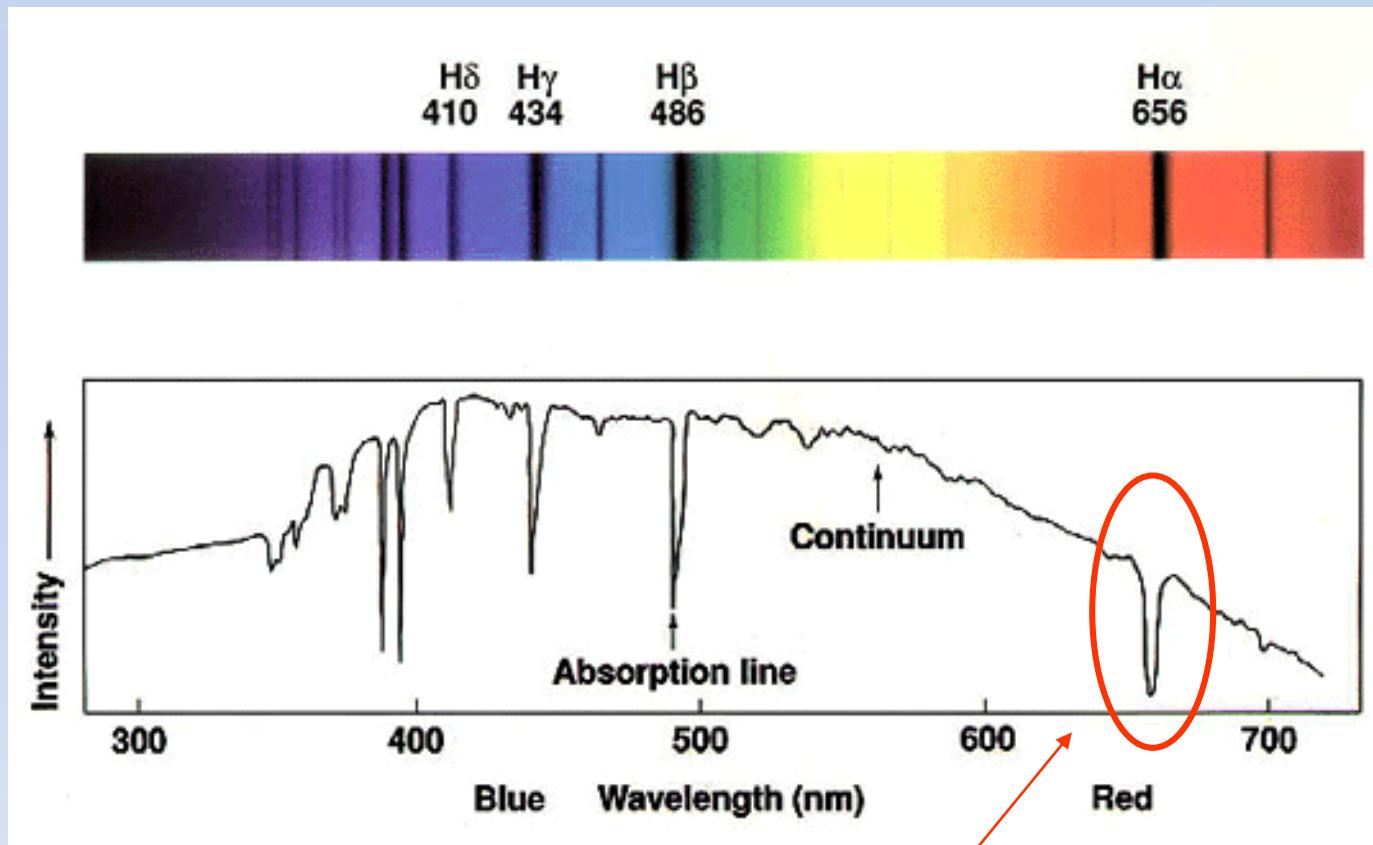
6.5 Physics of stellar spectra

- ✓ Balmer lines come from electron transition between $N = 2$ and a higher energy shell
- ✓ For a hot star, hydrogen atoms are ionized easily by high-energy radiation. When hydrogen's electron is torn away, no hydrogen spectral lines produced. Therefore, Balmer lines are relatively weak in a hot star
- ✓ For a cold star, electron in the lowest orbit ($N = 1$), the corresponding transition cannot produce the Balmer series

6.5 Physics of stellar spectra

Structure spectral lines

吸收的不是一条线，而是一个宽度

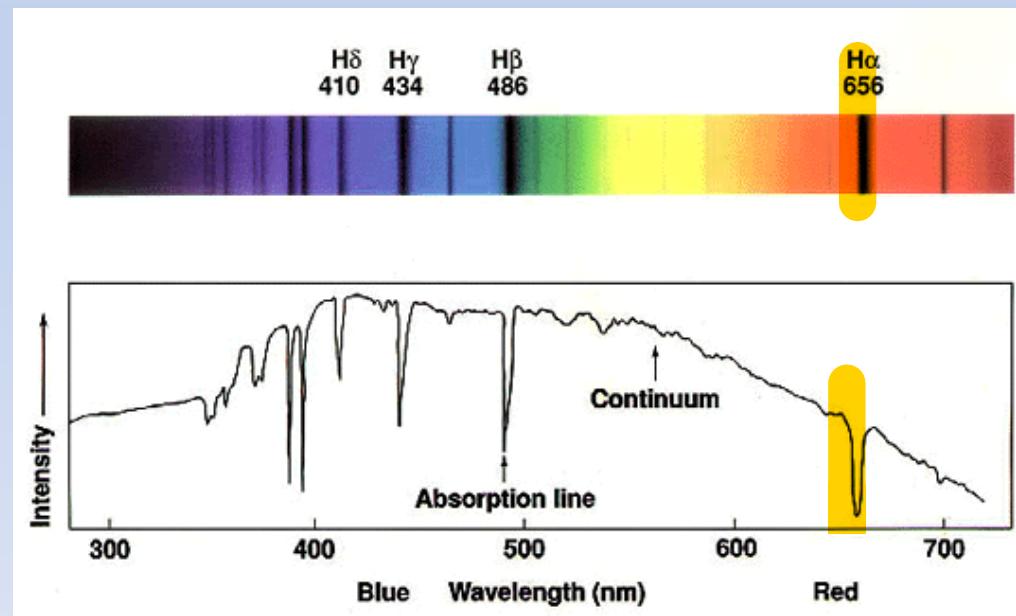


Detailed structure of a
line called *Line profile*

6.5 Physics of stellar spectra

Structure spectral lines

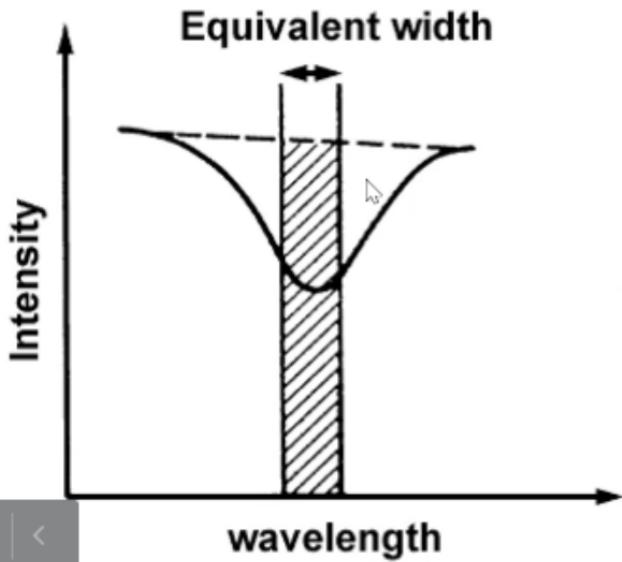
- ✓ Area of the spectral line indicates how much energy is absorbed from continuum
- ✓ proportional to amount of atoms in the star's atmosphere to absorb the photons
- ✓ In a broad and deep spectral line more energy is being absorbed. There are hence more atoms.



6.5 Physics of stellar spectra

Structure spectral lines

温度影响宽度，宽度用来估计星体H的数量



In practice, one calculate the equivalent width which is defined in a way to preserve the same area as the observed line:

- ✓ Indicate the strength of an absorption line;
- ✓ for easy comparison.

Other factors determine the structure of spectral lines: Doppler broadening

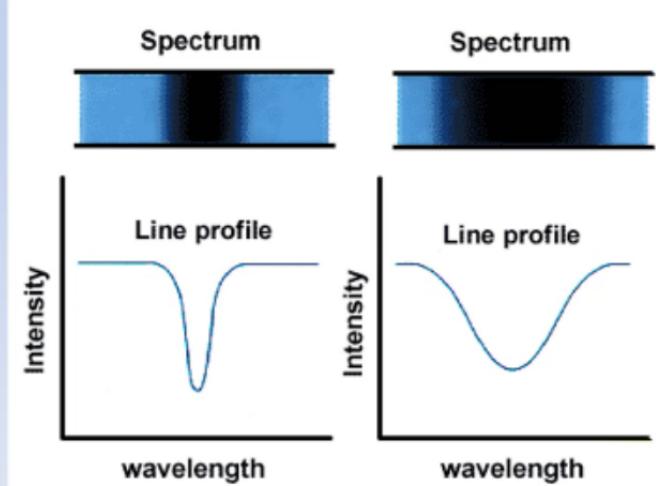
- ✓ Also known as **thermal broadening** – broaden the spectral line due to random thermal motions of particles.

- ✓ Assume $v \ll c$, $\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$

$$v_{\text{rms}} = \sqrt{3kT/m}$$

- ✓ width of spectral line:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{2}{c} \sqrt{\frac{3kT}{m}}$$



Other factors determine the structure of spectral lines: **Doppler broadening**

Exercise: Calculate the Doppler broadening of the H_α line of the Sun. Given that the surface temperature of the Sun is 5770 K, the frequency of H_α is $\lambda = 6563 \text{ Å}$, the mass of hydrogen atom is $m = 1.67 \times 10^{-27} \text{ kg}$, the Boltzmann constant is

$$k = 1.380 \times 10^{-23} \text{ J K}^{-1}$$

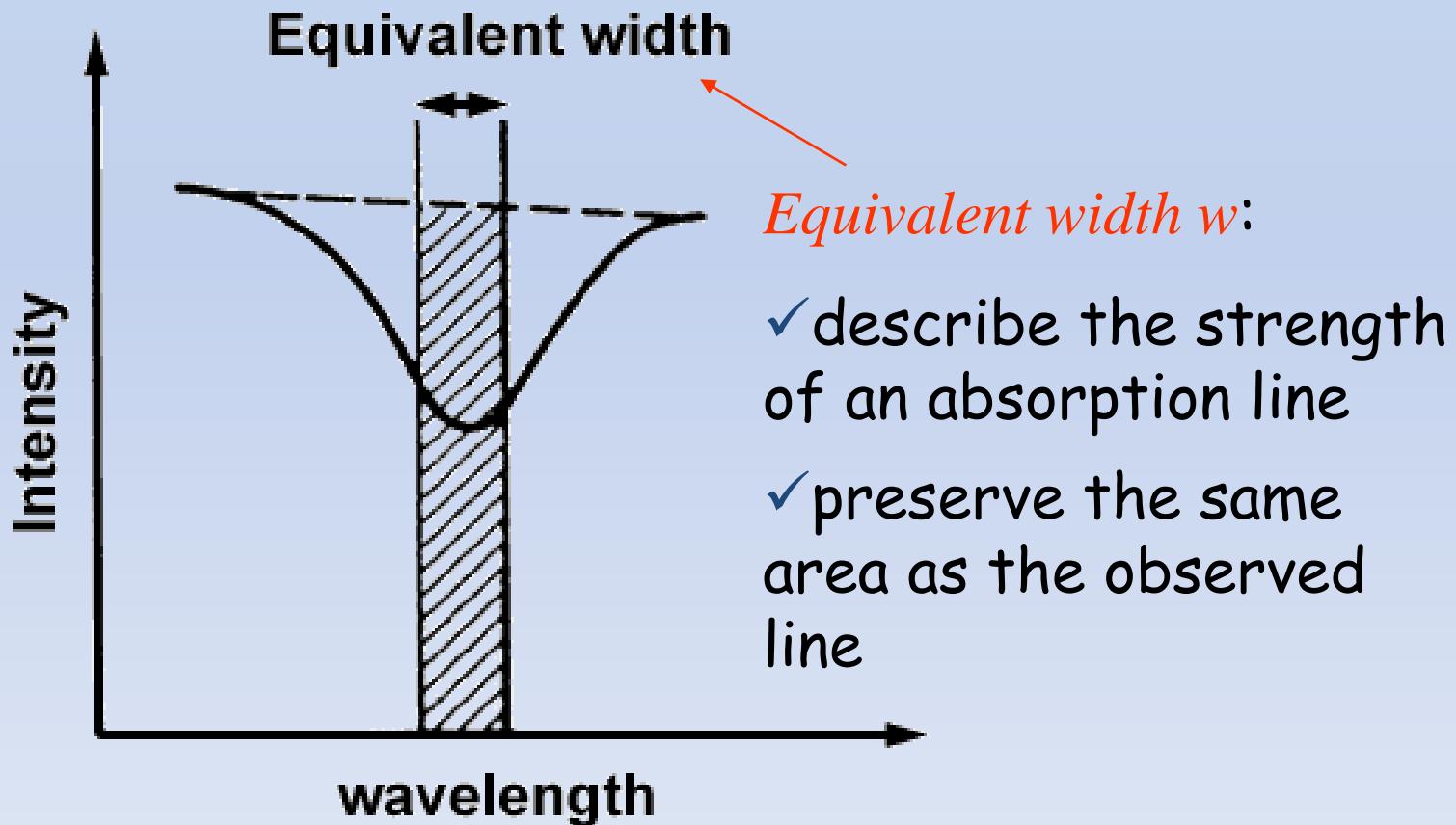
$$\Delta\lambda = \lambda \cdot 10^{-10}$$

$$\Delta\lambda \approx 0.52 \text{ Å}$$



6.5 Physics of stellar spectra

Structure spectral lines



6.5 Physics of stellar spectra

Structure spectral lines

- ✓ **Doppler broadening** – broaden the spectral line due to random thermal motions of particles.

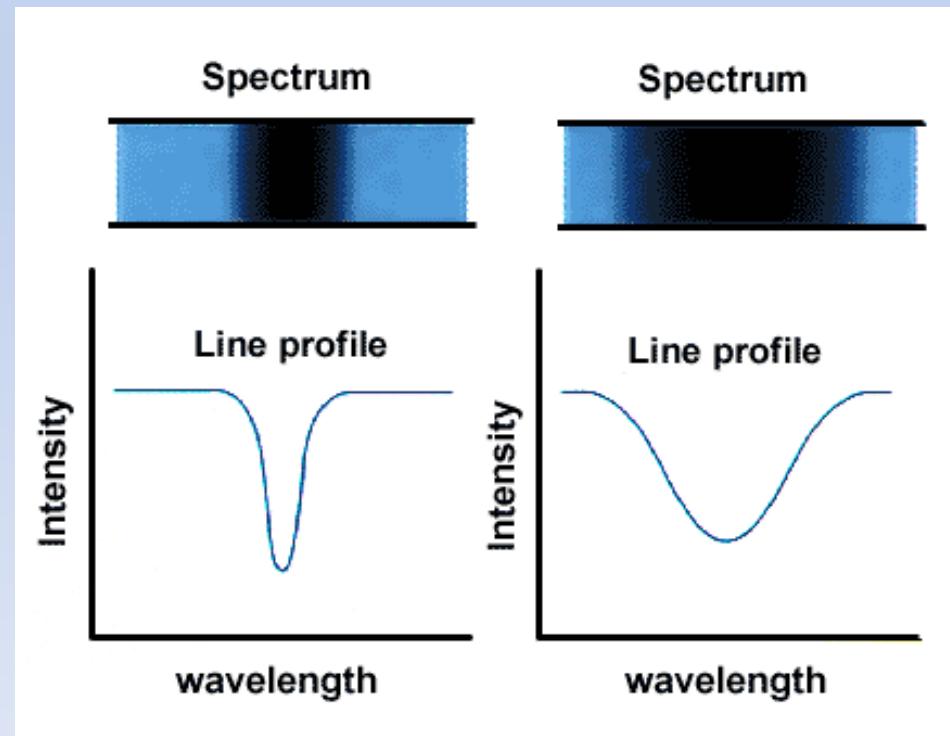
- ✓ Assume $v \ll c$, $\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$

- ✓ By Maxwell distribution,

$$v_{\text{rms}} = \sqrt{3kT / m}$$

- ✓ width of spectral line:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{2}{c} \sqrt{\frac{3kT}{m}}$$



6.5 Physics of stellar spectra

Structure spectral lines

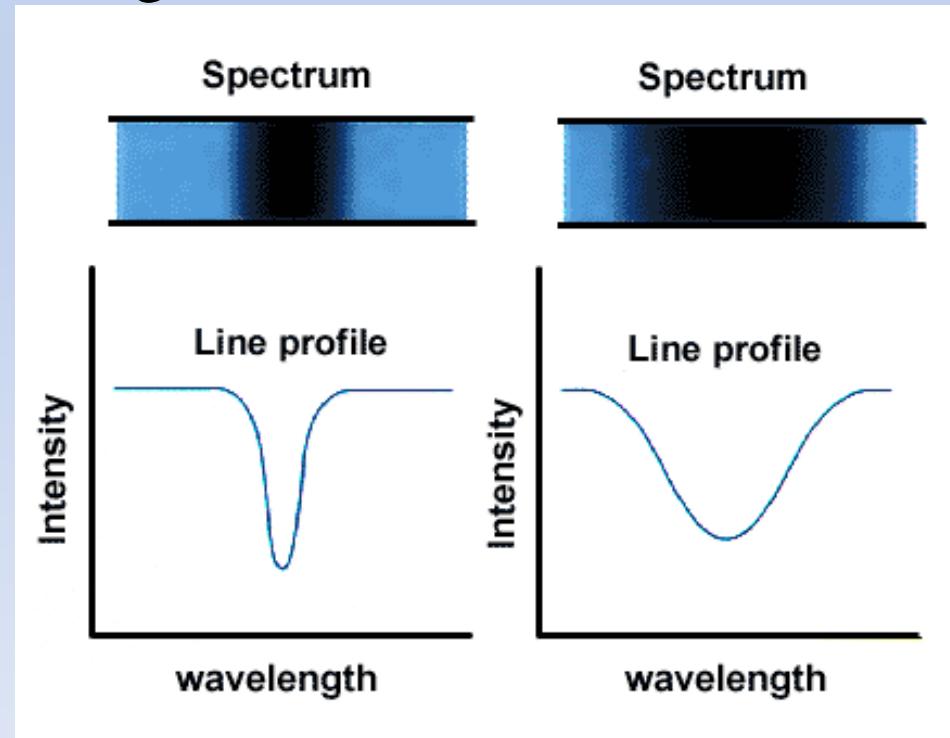
- ✓ For example, H_α line of the Sun

$T = 5770\text{ K}$, $m = 1.67 \times 10^{-27} \text{ kg}$, $\lambda = 6563\text{ Å}$,

$k = 1.380 \times 10^{-23} \text{ J K}^{-1}$

$\Delta\lambda \approx 0.52\text{ Å}$

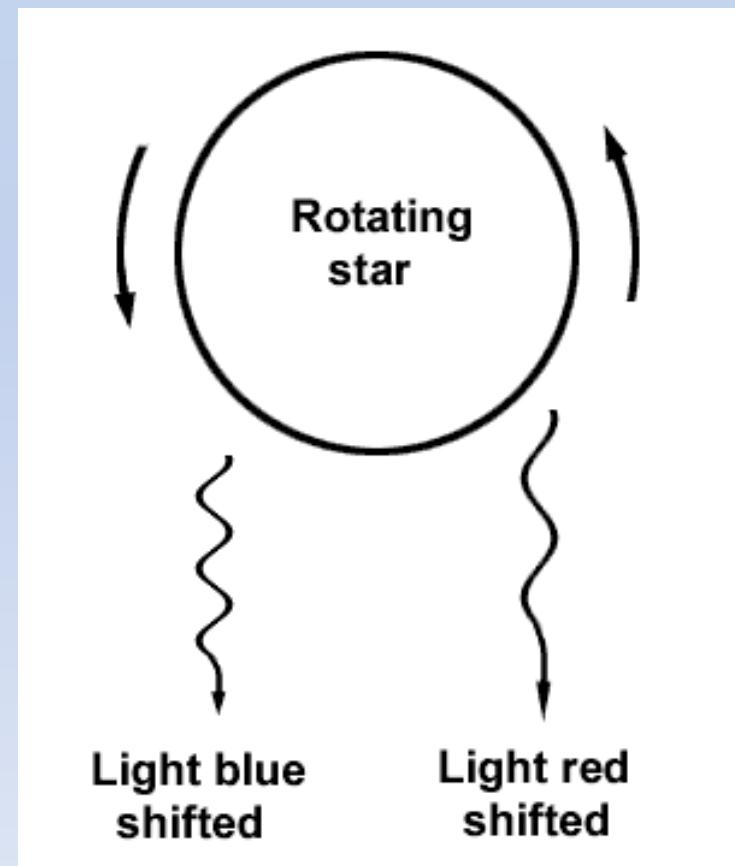
- ✓ 1000 times $>$ *natural broadening* (from the uncertainty principle)



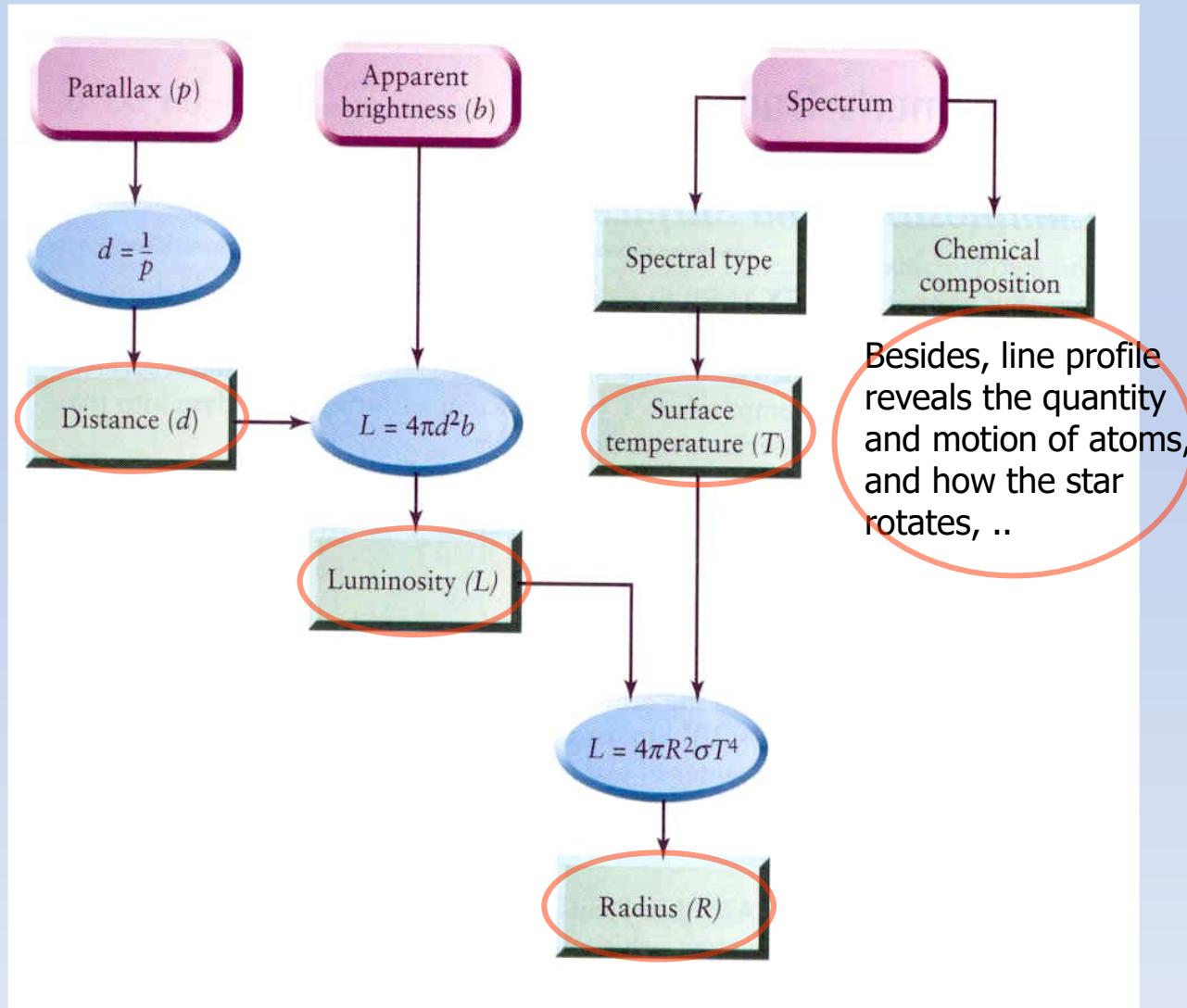
6.5 Physics of stellar spectra

Structure spectral lines

- ✓ Rotational broadening - light coming from a rotating star is Doppler shifted
- ✓ Astronomers deduce how fast the star is spinning

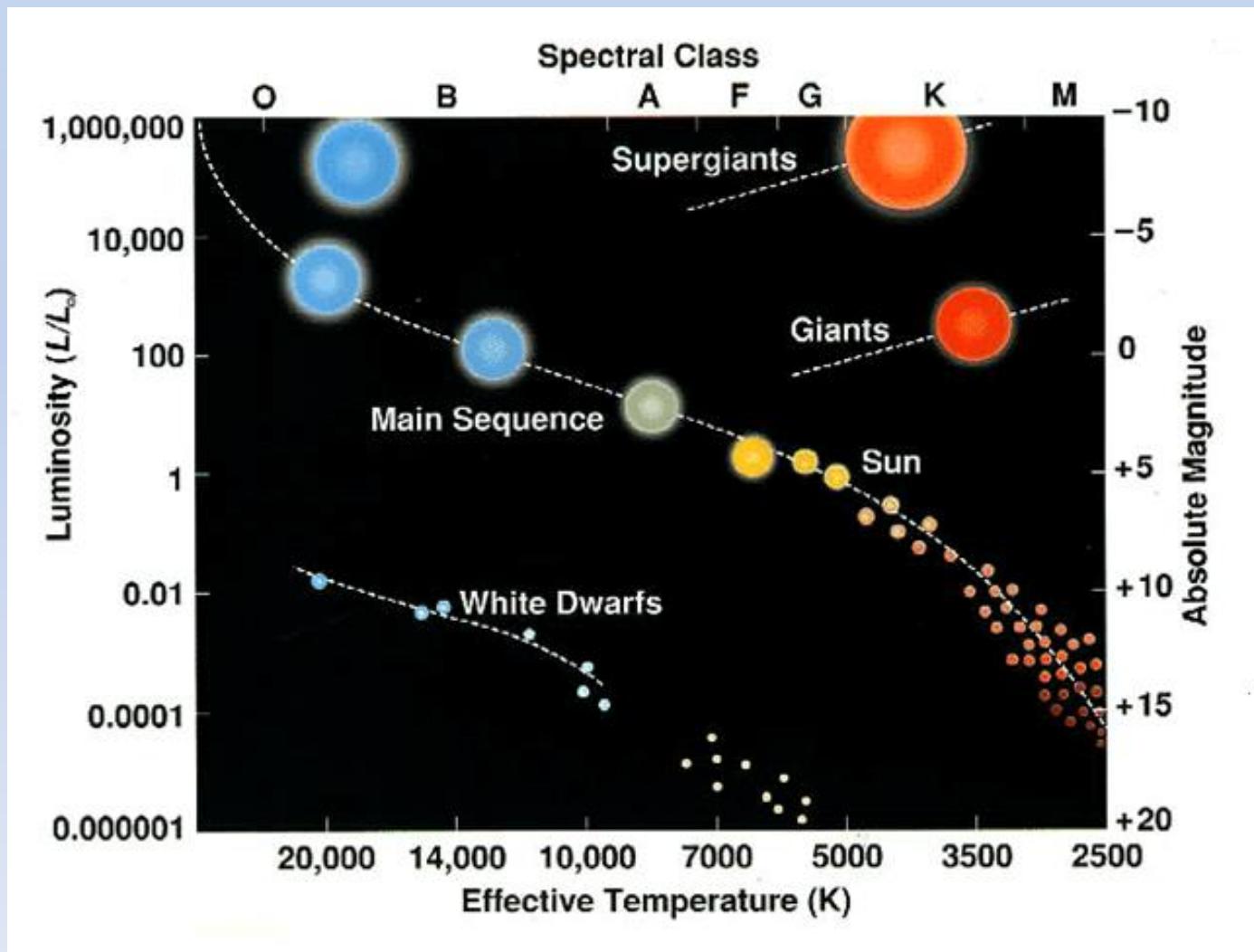


6.5 Physics of stellar spectra

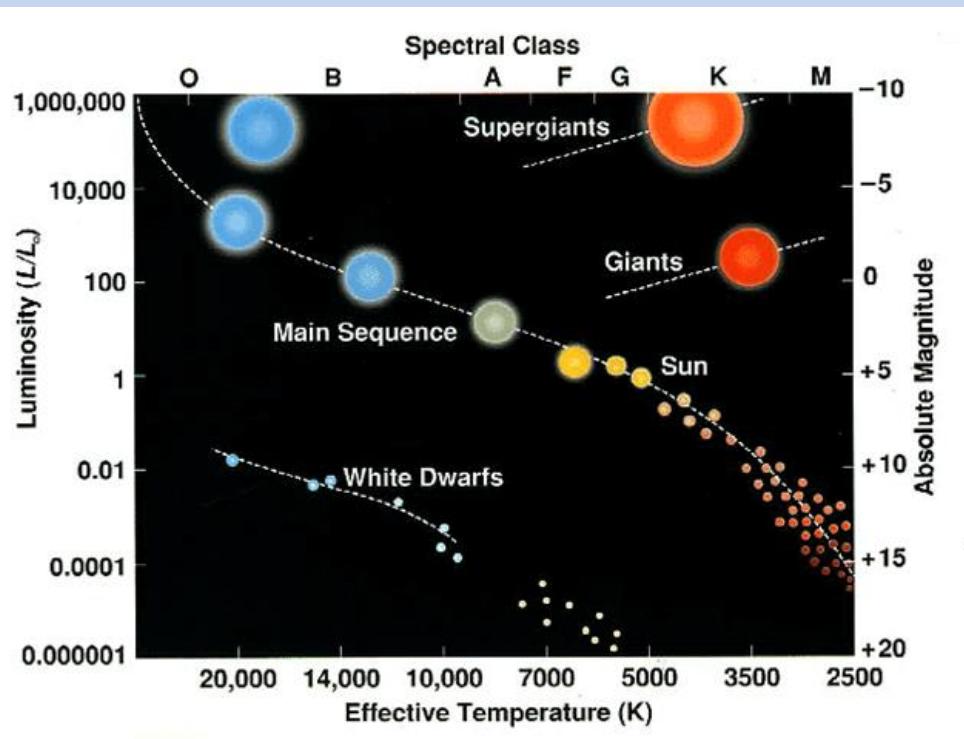


6.6 Hertzsprung-Russell (H-R) diagram

Luminosity vs spectral classes (surface temperature)



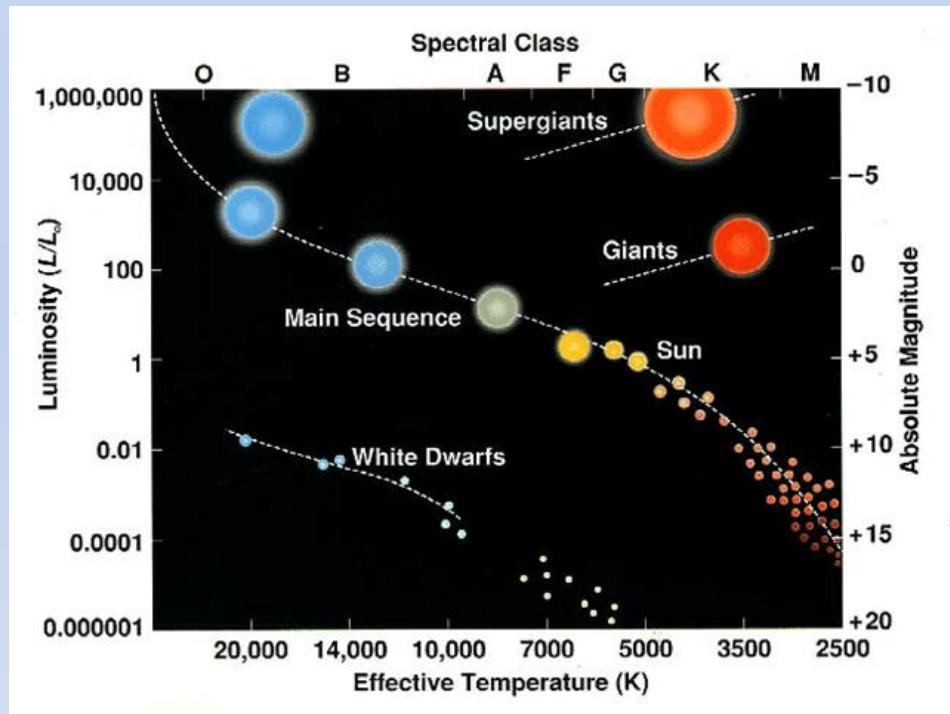
6.6 Hertzsprung-Russell (H-R) diagram



- ✓ Top: luminous stars; bottom: faint stars
- ✓ left: hot stars; right: cold stars
- ✓ As star evolves, temperature and luminosity changes, so its position on the H-R diagram also changes.

6.6 Hertzsprung-Russell (H-R) diagram

- ✓ *Main sequence*: A belt from upper left to lower right, 90% of all stars.
- ✓ cool stars are faint and small; hot stars are bright and large.



How large?

A comparison of star sizes

Red Dwarf
Lower limit:
0.08 solar
masses



Our Sun
1 solar mass



Blue-white
Supergiant
150 solar masses



Red Giant
Very old stars that
evolve from stars of
<5 solar masses



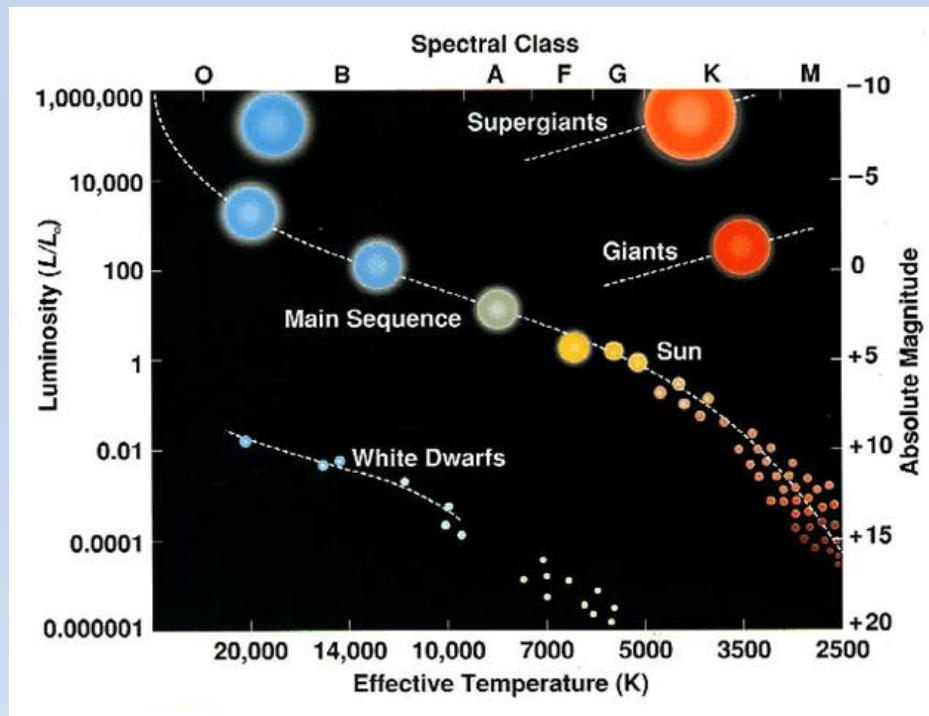
The stars on the H-R diagram have vastly different masses and sizes.

6.6 Hertzsprung-Russell (H-R) diagram

- ✓ *Giants* at the upper right corner, they are cool but luminous
- ✓ must have large surface area $R/R_{\odot} \sim 10-100$
- ✓ *Supergiants* are even larger: $R/R_{\odot} \sim 100-1000$



和太阳的比例



6.6 Hertzsprung-Russell (H-R) diagram

✓ Stefan-Boltzmann's law

$$L = 4\pi R^2 \sigma T^4$$

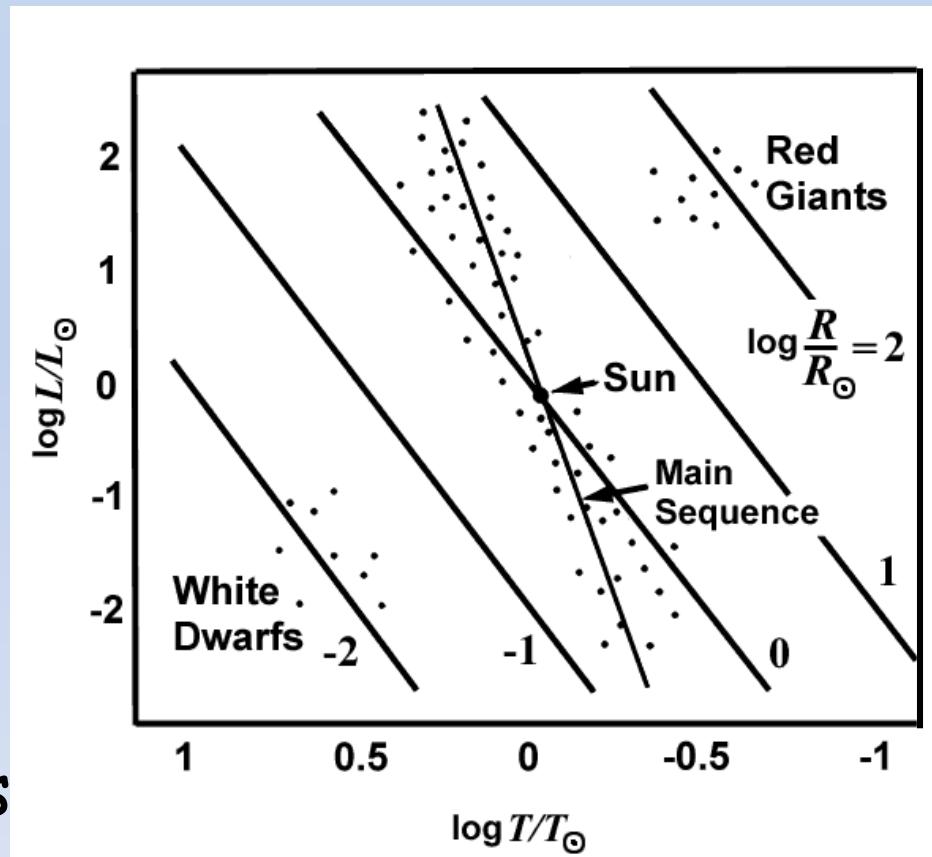
$$\frac{L}{L_\odot} = \frac{R^2 T^4}{R_\odot^2 T_\odot^4}$$

$$\log\left(\frac{L}{L_\odot}\right) = 4 \log\left(\frac{T}{T_\odot}\right)$$

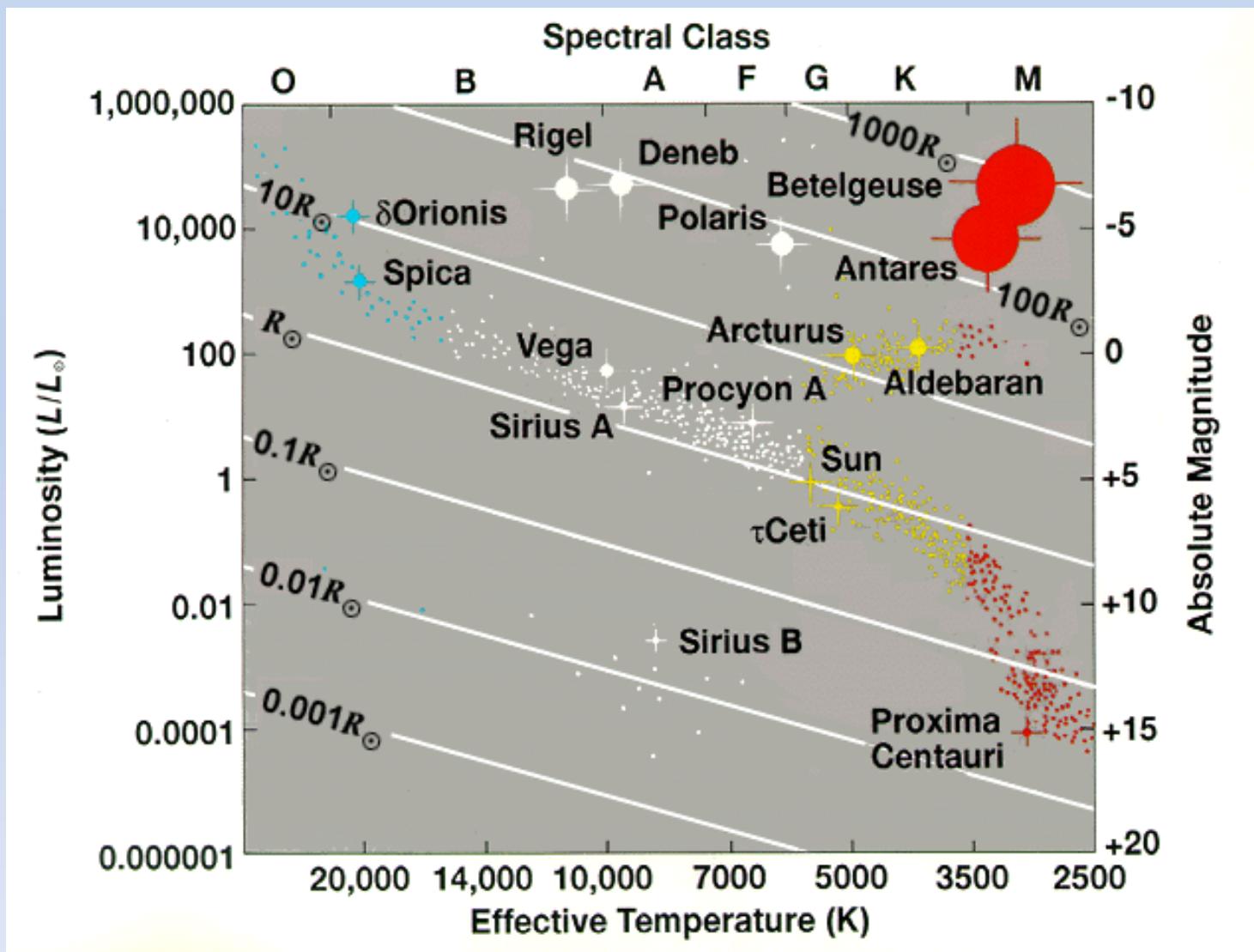
$$+ 2 \log\left(\frac{R}{R_\odot}\right)$$

✓ Lines of constant radius

$$c = 2 \log\left(\frac{R}{R_\odot}\right)$$

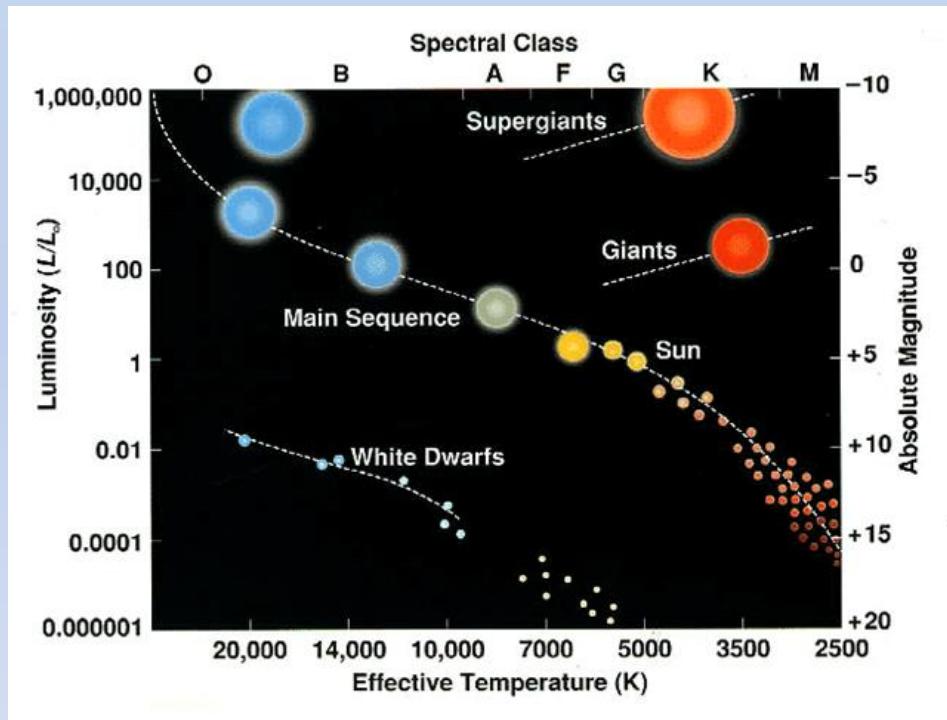


6.6 Hertzsprung-Russell (H-R) diagram

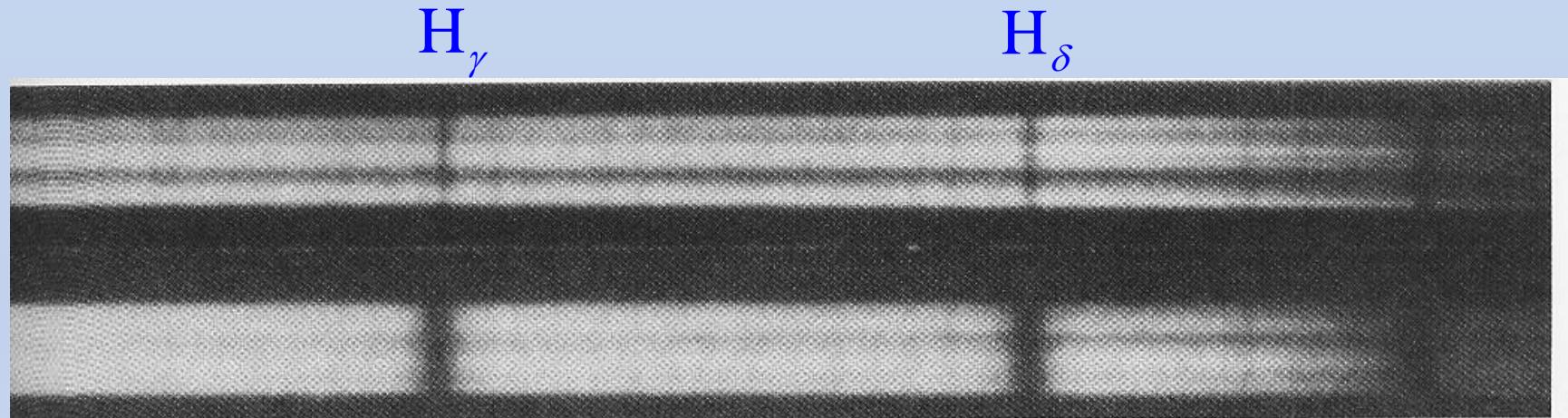


6.6 Hertzsprung-Russell (H-R) diagram

- ✓ *White dwarfs* lie in the lower left, they are hot but faint
- ✓ must be very small (~ size of Earth)



Turns out, there is degeneracy in our definition...



(a)

(b)

- (a) B8 supergiant Rigel with $L = 100,000L_\odot$, one of the brightest stars in Orion
- (b) B8 main-sequence star Algol with $L = 100L_\odot$, the second brightest star in Perseus

6.6 Hertzsprung-Russell (H-R) diagram

✓ *Luminosity classes*

classify the stars according to their luminosity (practically the difference in spectral lines)

Ia *Bright supergiant*

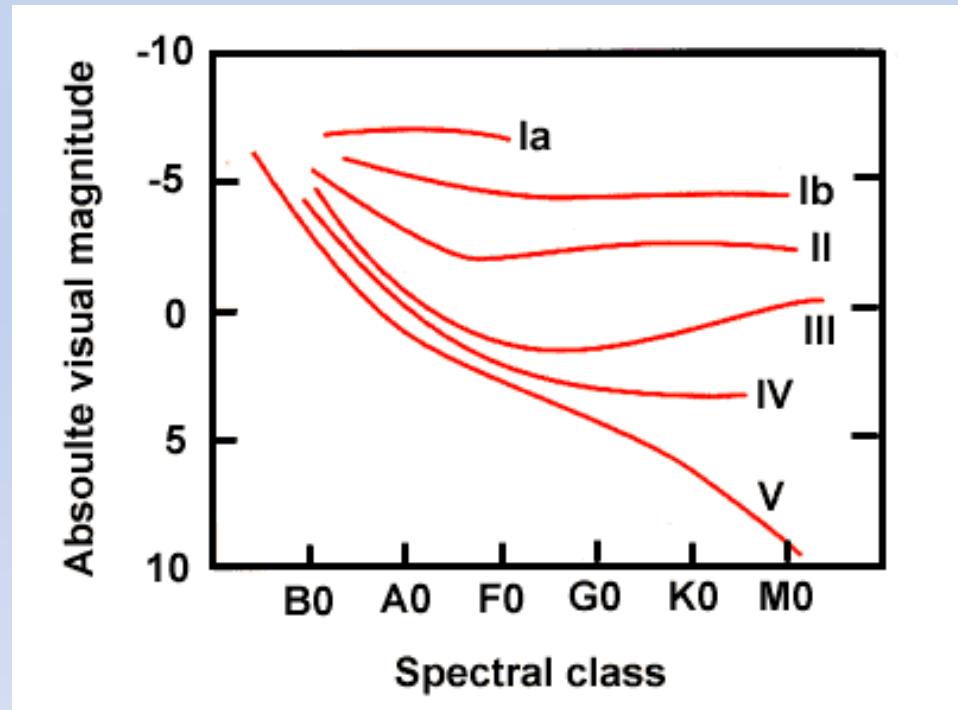
Ib *Supergiant*

II *Bright giant*

III *Giant*

IV *Subgiant*

V *Main-sequence star*

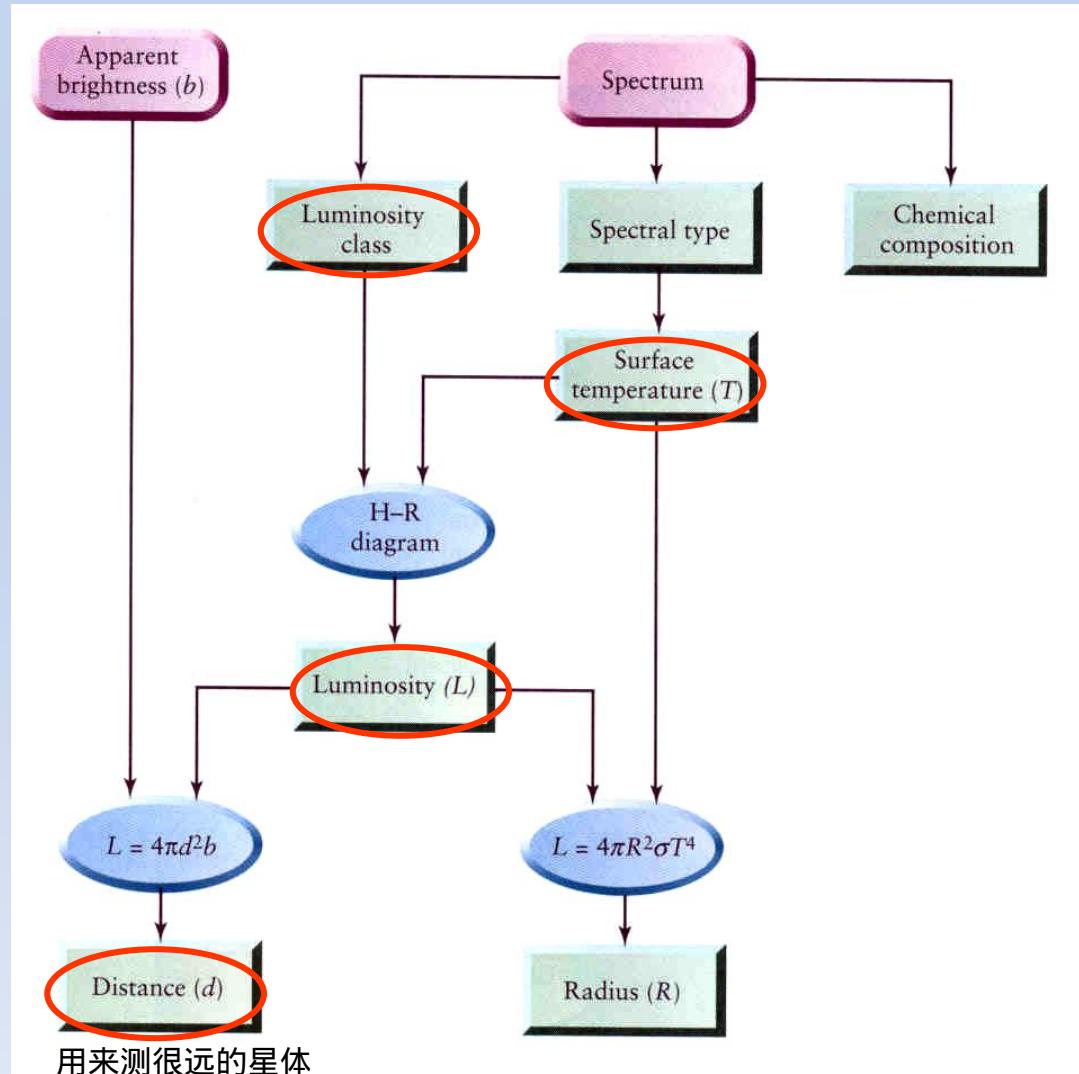
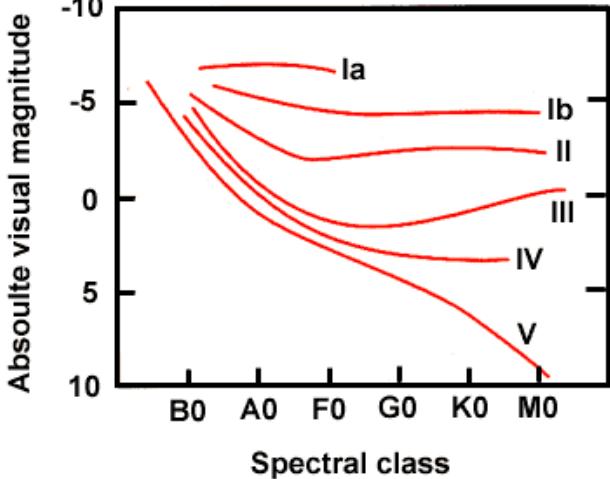


6.6 Hertzsprung-Russell (H-R) diagram

Question: How to determine the distance of a distant star from us?

6.6 Hertzsprung-Russell (H-R) diagram

Method of spectroscopic parallax



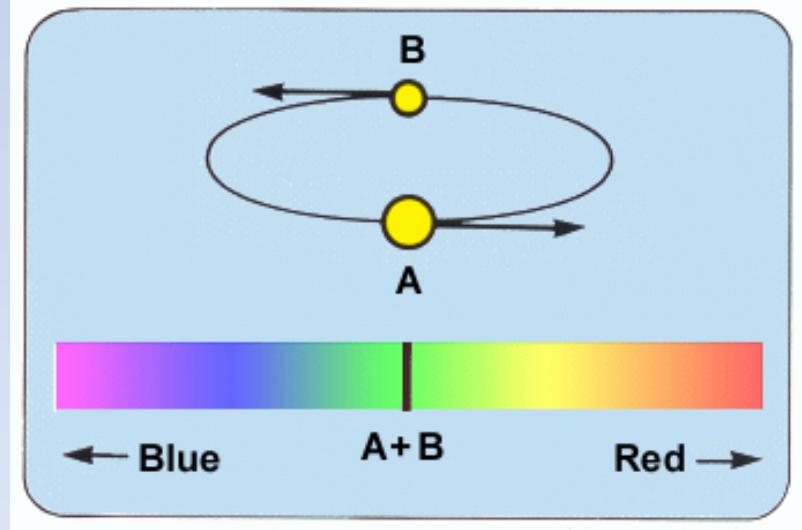
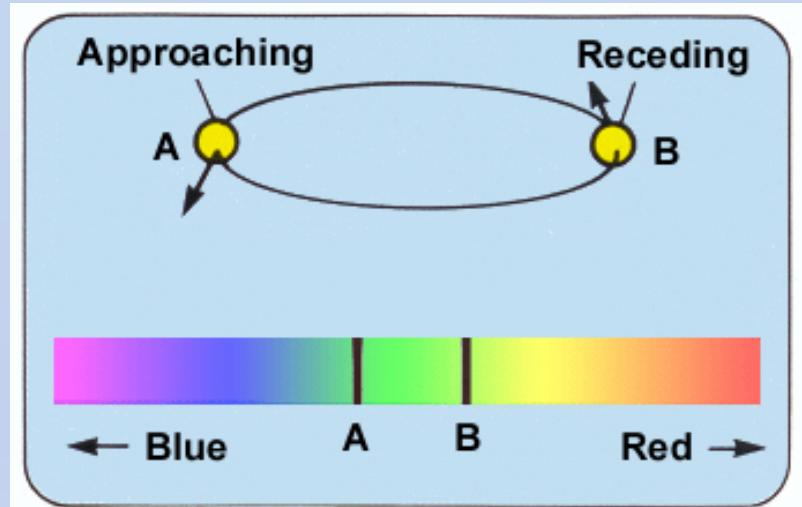
6.7 Binary systems

- ✓ A pair of stars appears at nearby the same position is called *double stars*
- ✓ *Optical double stars*: lie along nearly the same line of sight but are actually at very different distances from us
- ✓ Pairs of stars actually orbiting each other - *binary stars*, or *binaries*

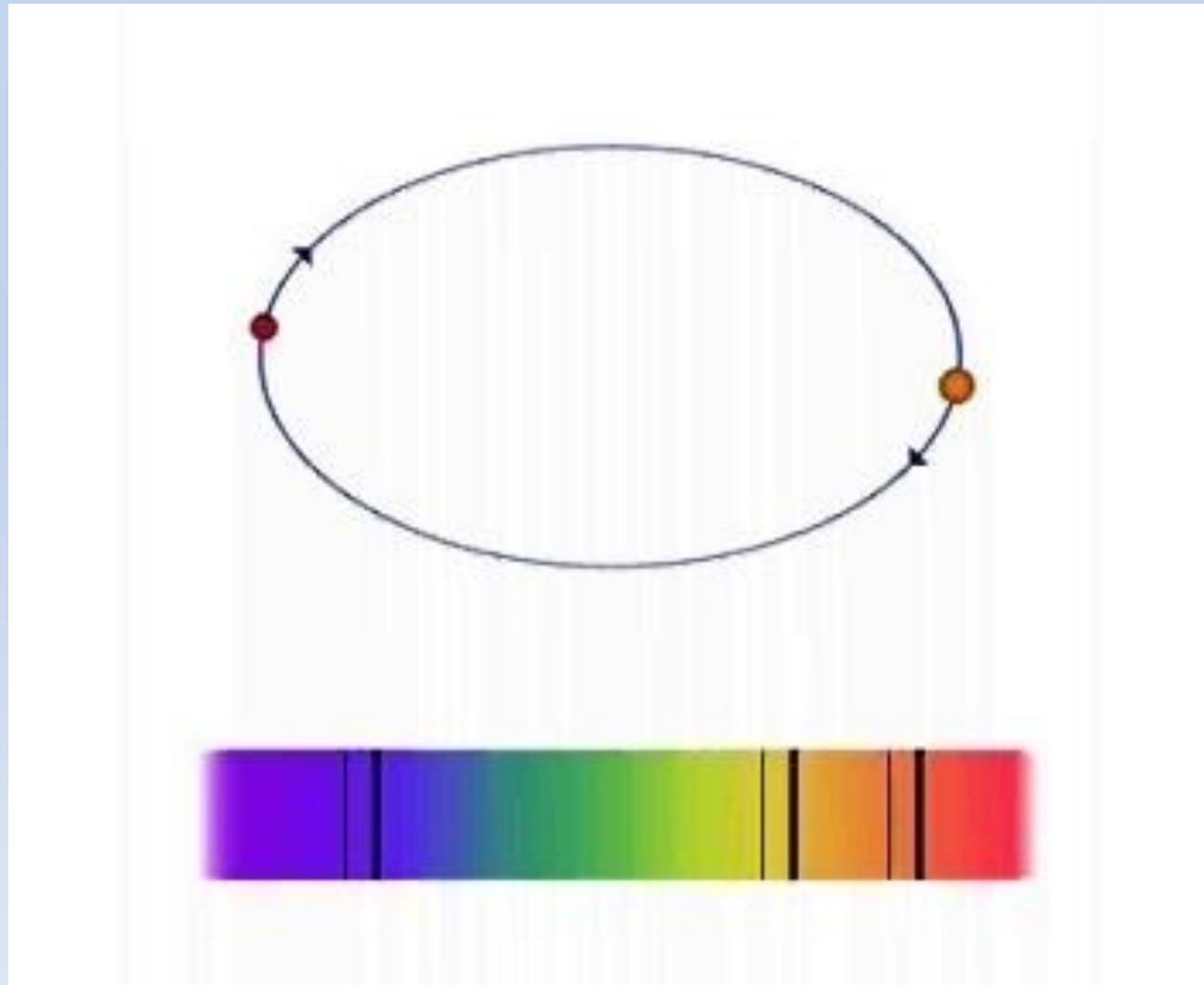
6.7 Binary systems

✓ *Visual binaries*: Double stars gravitationally connected, can be resolved by optical telescopes

✓ *Spectroscopic binaries*: Gravitationally connected binaries too close to be resolved by optical telescopes, but spectra show periodic Doppler shifts



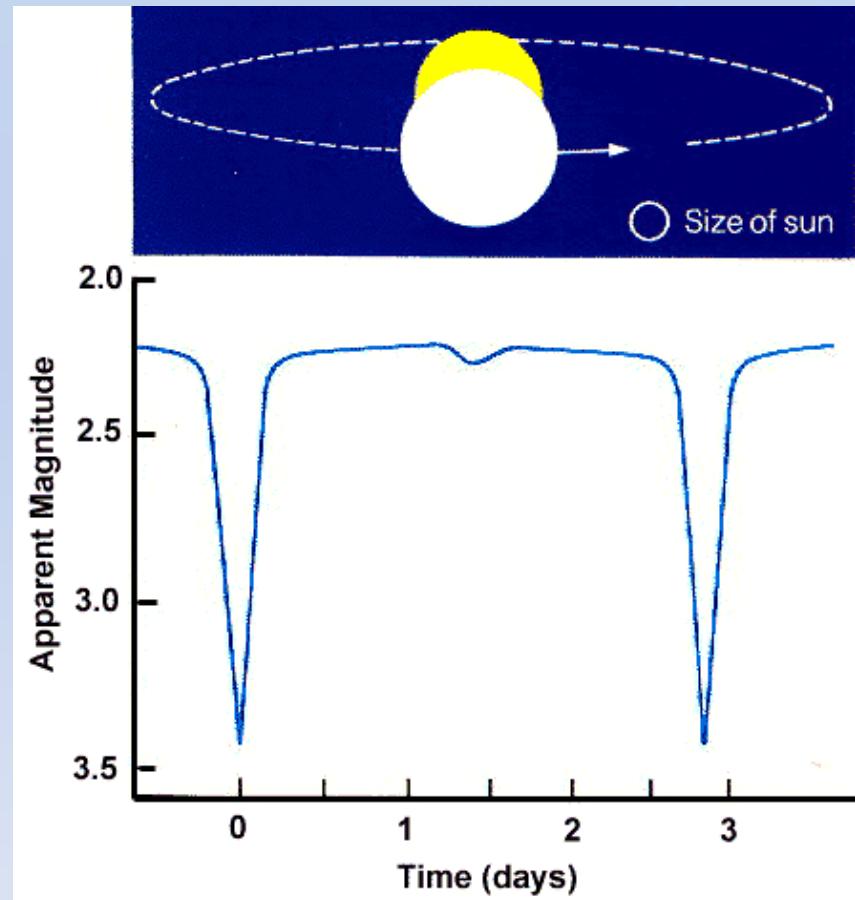
6.7 Binary systems



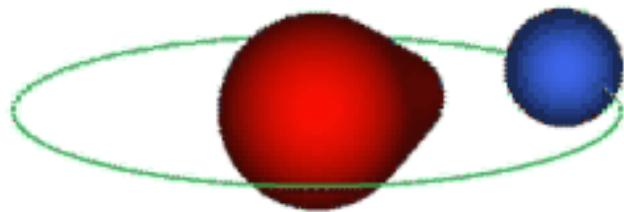
6.7 Binary systems

Eclipsing Binaries

- ✓ Most binary stars are too close to be resolved by optical telescopes,
- ✓ but may show periodic variation in brightness by eclipses



6.7 Binary systems



EclBinary2.mpg

6.7 Binary systems

Eclipsing Binaries

- ✓ Analysis of light curves allows one to determine the **ratio** of luminosity and radii of the component stars

