Homework 1: Linear Programming

Due: 11:59pm on Wednesday 2/8/23

A few reminders:

- You are encouraged to discuss homework problems with classmates and even work in groups.
- However, the work you turn in must be your own. You must not communicate files containing code or answers to homework questions to each other.
- Please submit the answers in the same order as in the assignment.
- Please denote the start of each question in your Julia workbook using a LARGE FONT.
- Submit a pdf file with the contents of your notebook, showing the output obtained by running your code cells. To obtain a version suitable for submission, choose `File->Print Preview` to obtain a nice version in a new tab, then print the contents of this tab to a pdf file.

1. [2 points] Warm-up. Model the following problem in JuMP.

$$\begin{array}{ll} \underset{x_1,x_2,x_3}{\text{maximize}} & 5x_1 - x_2 + 15x_3 \\ \text{subject to:} & 2x_1 \geq x_2 + \frac{1}{2}x_3 \\ & 0 \leq x_j \leq 4, \; j \in \{1,2,3\} \end{array}$$

(a) Write Julia code to solve this problem using `HiGHS`, `ECOS`, and `SCS` solvers. Do all give the same result?

(b) Which solver is fastest (use the `@time` macro in front of the `optimize!()` statement to check)? Can you say why?

(c) Consider changing the first constraint to $2x_1 \geq x_2 + \frac{1}{2}x_3 + \alpha$, where α is some real number. For all values of α larger than a certain threshold value, the problem with become infeasible. (In Julia, the solver will terminate with status `INFEASIBLE`.) What is this threshold value of α? Can you explain by examining the constraints why this value makes the problem infeasible?

2. [2 points] Standard Form Recall that in class we defined the "standard form" of a linear program to be

$$\max \; c^T x \quad \text{subject to } Ax \leq b, \;\; x \geq 0.$$

Consider the following linear program which is NOT in standard form:

$$\begin{aligned} & \min \; 3z_1 + 2z_2 - z_3 \\ \text{subject to} \;\; & 2z_1 - z_2 + 3z_3 = 10, \\ & 0 \leq z_2 \leq 5, \quad -5 \leq z_3 \leq 20. \end{aligned}$$

a) Transform the given LP into our standard form in terms of some vector variable x. How are the components of x related to the components of $z = (z_1, z_2, z_3)$? Write out explicity the quantities A, b, c for the standard form.

b) Solve both versions of the LP using JuMP and show that you can recover the optimal z and objective value by solving your transformed version of the LP.

3. [2 points] Polyhedron modeling. We saw that the set of x such that $Ax \leq b$ where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ describes a polyhedron. Find A and b such that $Ax \leq b$ describes the pyramid-shaped (that is, $x \in \mathbb{R}^3$) that is defined by the following five vertices:

$$(1,1,0),\ (1,-1,0),\ (-1,-1,0),\ (-1,1,0),\ (0,0,2).$$

(It might help to sketch this figure!)

4. [3 points] Alloy blending. The company Steelco has received an order for 500 tons of steel to be used in shipbuilding. The steel must have the following characteristics:

Chemical Element	Minimum Grade (%)	Maximum Grade (%)
Carbon (C)	2	3
Copper (Cu)	0.4	0.6
Manganese (Mn)	1.2	1.65

The company has seven different raw materials in stock that may be used for the production of this steel. The following table lists the grades, available amounts and prices for all materials:

Raw Material	C%	Cu%	Mn%	Availability in tons	Cost in $/ton
Iron alloy 1	2.5		1.3	400	200
Iron alloy 2	3		0.8	300	250
Iron alloy 3		0.3		600	150
Copper 1		90		500	220
Copper 2		96	4	200	240
Aluminum 1		0.4	1.2	300	200
Aluminum 2		0.6		250	165

Determine the composition of the steel that minimizes the production cost.

(See the starter code for this problem posted on Canvas, which illustrates the use of dictionaries. You can paste this code into your solution and add another cell with model and solution.)