CS/ECE/ISyE 524 - Fall 2023 - HW 10 - Starter Code

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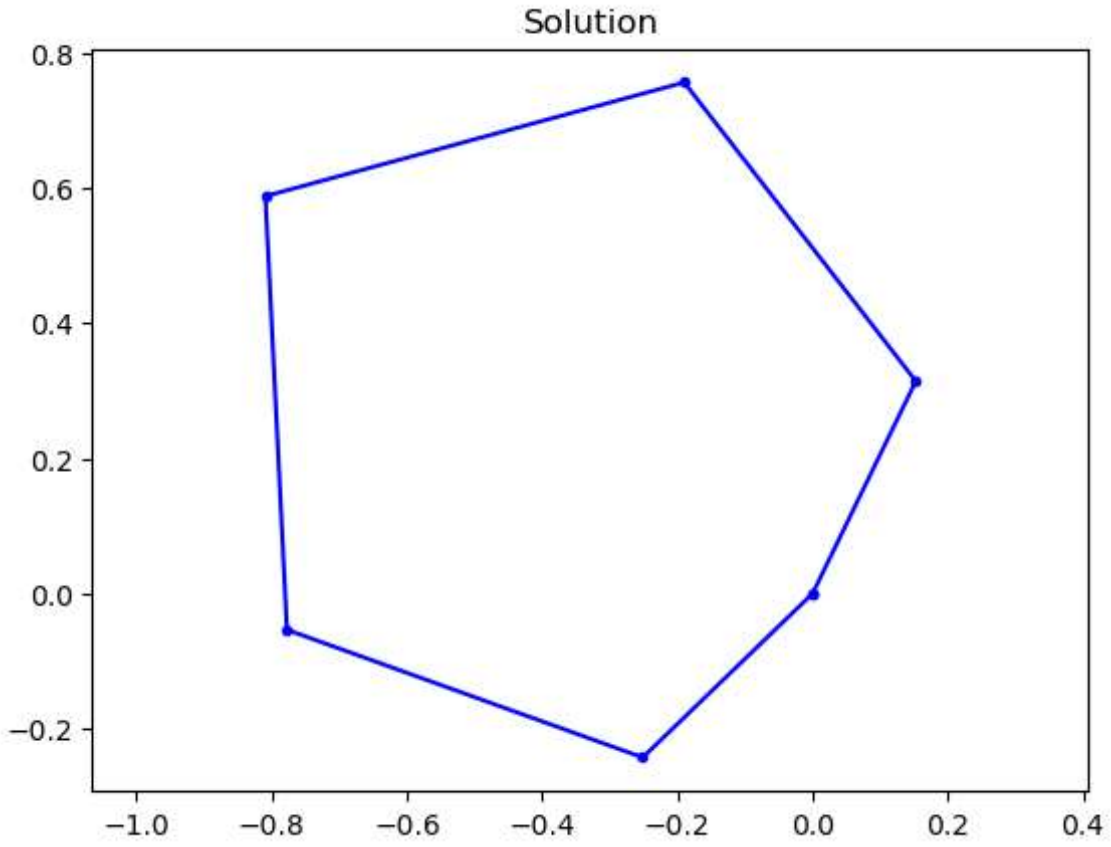
1. Hexagon construction

In [5]:
```
# Question 1 starter

using Random, JuMP, Ipopt

m = Model(Ipopt.Optimizer)

# locations of the six vertices.
@variable(m, x[1:6])
@variable(m, y[1:6])

# your code here
n = 6

@constraint(m, x[1] == 0)
@constraint(m, y[1] == 0)

@NLobjective(m, Max, 0.5*sum( x[i]*y[i+1]-y[i]*x[i+1] for i=2:n-1))

# add ordering constraint to the vertices
for i = 2:n-1
    @NLconstraint(m, x[i]*y[i+1]-y[i]*x[i+1] >= 0 )
end

for i = 1:n
    for j = i:n
        @NLconstraint(m, (x[i]-x[j])^2 + (y[i]-y[j])^2 <= 1)
    end
end

Random.seed!(2367877)

set_start_value.(x,rand(n))
set_start_value.(y,rand(n))

# setvalue(x,rand(n))
# setvalue(y,rand(n))

optimize!(m)

# print optimal objective
println("area=",objective_value(m))

# plot solution from optimal values of (x,y) coordinates.
using PyPlot
X = [value.(x); 0]
Y = [value.(y); 0]
axis("equal")
```

```
plot(X,Y,"b.-")
title("Solution");
```

```
This is Ipopt version 3.14.4, running with linear solver MUMPS 5.5.1.

Number of nonzeros in equality constraint Jacobian...:        2
Number of nonzeros in inequality constraint Jacobian.:       88
Number of nonzeros in Lagrangian Hessian.............:      144

Total number of variables............................:       12
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        2
Total number of inequality constraints...............:       25
        inequality constraints with only lower bounds:        4
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:       21

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
   0  2.5267500e-02 9.56e-01 1.11e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00
0
   1  7.2142785e-02 1.86e-01 7.08e-01  -1.0 9.56e-01    -  9.79e-01 8.06e-01h
1
   2  1.3685851e-01 0.00e+00 1.07e+00  -1.7 1.86e-01   0.0 5.92e-01 1.00e+00h
1
   3  1.4563864e-01 0.00e+00 1.85e-01  -1.7 6.95e-02   0.4 1.00e+00 1.00e+00h
1
   4  1.9215749e-01 1.82e-33 1.45e-01  -2.5 8.88e-02  -0.1 8.87e-01 1.00e+00h
1
   5  3.8952427e-01 9.70e-33 1.09e-01  -2.5 3.67e-01  -0.5 1.00e+00 1.00e+00h
1
   6  4.1162541e-01 1.38e-33 1.04e-01  -2.5 1.32e-01  -0.1 1.00e+00 1.00e+00h
1
   7  5.0121782e-01 8.67e-03 7.81e-02  -2.5 2.98e-01  -0.6 1.00e+00 5.73e-01f
1
   8  6.2013002e-01 4.94e-02 5.92e-02  -2.5 1.54e+00  -1.1 6.05e-01 2.02e-01f
1
   9  6.3333894e-01 2.49e-31 5.58e-02  -2.5 2.38e-01  -0.6 1.00e+00 1.00e+00h
1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
  10  6.5763856e-01 1.37e-02 3.83e-02  -2.5 3.82e-01  -1.1 1.00e+00 3.09e-01h
1
  11  6.5771439e-01 3.27e-27 7.05e-03  -2.5 6.48e-02  -1.6 1.00e+00 1.00e+00f
1
  12  6.7377186e-01 6.21e-04 7.27e-04  -3.8 7.43e-02  -2.1 9.48e-01 9.54e-01h
1
  13  6.7499784e-01 1.75e-04 8.43e-05  -5.7 1.26e-02    -  1.00e+00 9.68e-01h
1
  14  6.7497054e-01 3.61e-28 2.58e-06  -5.7 8.91e-04  -2.5 1.00e+00 1.00e+00h
1
  15  6.7498144e-01 5.08e-26 5.65e-10  -8.6 4.42e-05    -  1.00e+00 1.00e+00h
1

Number of Iterations....: 15

                                   (scaled)                 (unscaled)
Objective...............:  -6.7498143500990837e-01    6.7498143500990837e-01
Dual infeasibility......:   5.6541463178128026e-10    5.6541463178128026e-10
Constraint violation....:   5.0830639782791767e-26    5.0830639782791767e-26
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
```

```
Complementarity.........:   2.8313618569865823e-09    2.8313618569865823e-09
Overall NLP error.......:   2.8313618569865823e-09    2.8313618569865823e-09


Number of objective function evaluations             = 16
Number of objective gradient evaluations             = 16
Number of equality constraint evaluations            = 16
Number of inequality constraint evaluations          = 16
Number of equality constraint Jacobian evaluations   = 16
Number of inequality constraint Jacobian evaluations = 16
Number of Lagrangian Hessian evaluations             = 15
Total seconds in IPOPT                               = 0.013

EXIT: Optimal Solution Found.
area=0.6749814350099084
```



2. Fertilizer influence model

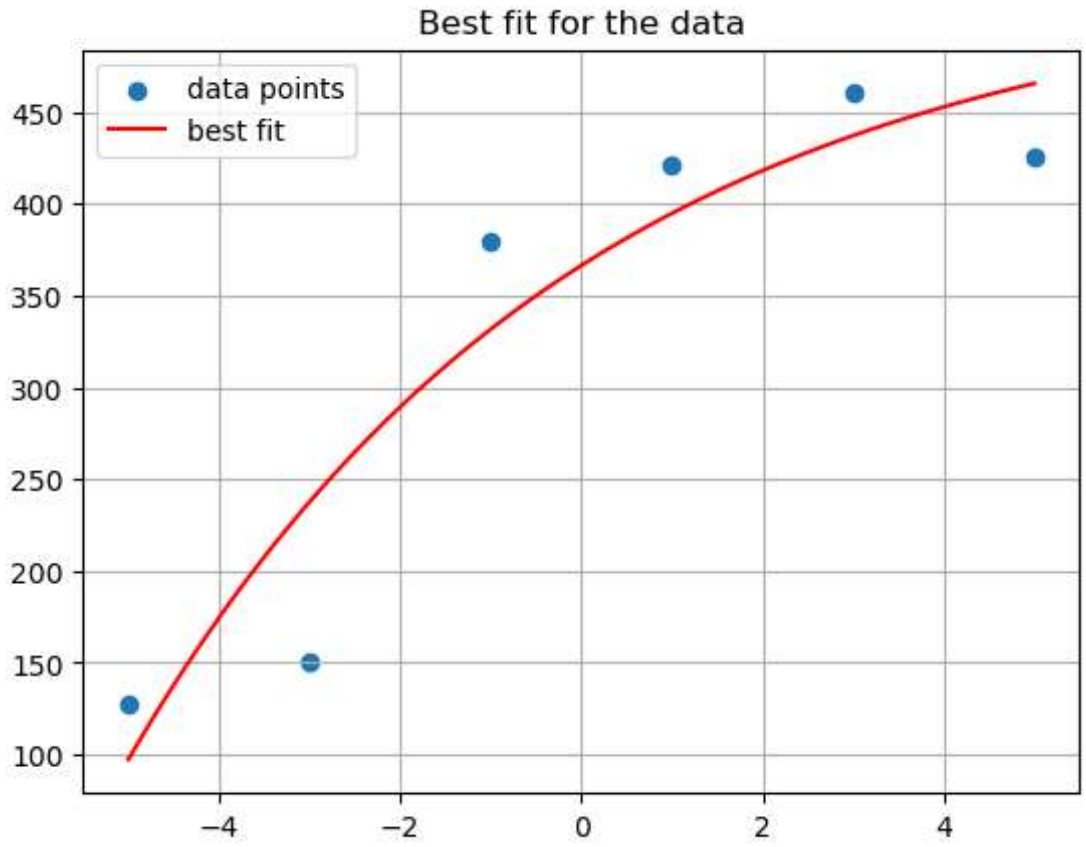
In [12]:

```julia
# Question 2 starter


using JuMP, Ipopt

x =     [ -5  -3  -1   1   3   5]
ymeas = [127 151 379 421 460 426]
N = length(x)


# your code here
m = Model(Ipopt.Optimizer)

@variable(m, k[1:3])
expected_k = [500 -200 -1]
```

```
set_start_value.(k, expected_k)

@NLexpression(m, y[i in 1:N], k[1] + k[2] * exp(k[3] * x[i]))

@NLobjective(m, Min, sum((y[i] - ymeas[i])^2 for i in 1:N))

optimize!(m)
println(value.(k))

# Plot the data and the best-fit curve
using PyPlot

xv = range(-5, stop=5, length=100)
# plot some silly curve (replace this with the real thing)
yv = value.(k)[1] .+ value.(k)[2] .* exp.(value.(k)[3] .* xv)

scatter(x, ymeas, label="data points")
plot(xv, yv, "r-", label="best fit")
grid()
legend(loc="best"); title("Best fit for the data")

;
```

```
This is Ipopt version 3.14.4, running with linear solver MUMPS 5.5.1.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian.............:        6

Total number of variables............................:        3
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        0
                     variables with only upper bounds:        0
Total number of equality constraints.................:        0
Total number of inequality constraints...............:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
   0  8.2578724e+05 0.00e+00 1.00e+02  -1.0 0.00e+00    -  0.00e+00 0.00e+00
0
   1  7.9098305e+05 0.00e+00 4.31e+01  -1.0 1.11e-01   2.0 1.00e+00 1.00e+00f
1
   2  7.7368507e+05 0.00e+00 1.84e+01  -1.0 1.22e-01   1.5 1.00e+00 1.00e+00f
1
   3  7.6462135e+05 0.00e+00 7.52e+00  -1.0 1.43e-01   1.0 1.00e+00 1.00e+00f
1
   4  7.5669591e+05 0.00e+00 2.34e+00  -1.0 6.22e-01   0.6 1.00e+00 1.00e+00f
1
   5  7.5214915e+05 0.00e+00 3.36e+00  -1.0 3.25e-01   1.0 1.00e+00 1.00e+00f
1
   6  7.3630462e+05 0.00e+00 2.55e+01  -1.0 4.44e-01   1.4 1.00e+00 1.00e+00f
1
   7  7.3348859e+05 0.00e+00 7.86e+00  -1.0 4.13e-02   1.8 1.00e+00 1.00e+00f
1
   8  7.3268256e+05 0.00e+00 1.60e+00  -1.0 4.79e-02   1.4 1.00e+00 1.00e+00f
1
   9  7.3160820e+05 0.00e+00 1.27e+00  -1.0 1.36e-01   0.9 1.00e+00 1.00e+00f
1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
  10  1.0987204e+05 0.00e+00 2.99e+02  -1.0 3.81e+02    -  1.00e+00 1.00e+00f
1
  11  5.6246423e+04 0.00e+00 8.34e+01  -1.0 3.78e-01   1.3 1.00e+00 1.00e+00f
1
  12  4.6991507e+04 0.00e+00 1.86e+01  -1.0 1.78e-01   0.8 1.00e+00 1.00e+00f
1
  13  4.5480730e+04 0.00e+00 5.13e-01  -1.0 3.26e-01   0.4 1.00e+00 1.00e+00f
1
  14  4.4525878e+04 0.00e+00 3.00e+01  -1.7 1.08e+01    -  1.00e+00 5.00e-01f
2
  15  4.0430426e+04 0.00e+00 2.12e+01  -1.7 6.55e+00    -  1.00e+00 1.00e+00f
1
  16  3.9007772e+04 0.00e+00 1.63e+00  -1.7 4.62e-01  -0.1 1.00e+00 1.00e+00f
1
  17  3.7663598e+04 0.00e+00 3.73e+01  -1.7 1.36e+01    -  1.00e+00 5.00e-01f
2
  18  3.2613903e+04 0.00e+00 3.23e+01  -1.7 7.37e+00    -  1.00e+00 1.00e+00f
1
  19  3.0652798e+04 0.00e+00 3.77e+00  -1.7 6.75e-01  -0.6 1.00e+00 1.00e+00f
1
```

```
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
  20  2.9040108e+04 0.00e+00 4.44e+01  -1.7 2.83e+01    -  1.00e+00 5.00e-01f
2
  21  2.3552089e+04 0.00e+00 3.64e+01  -1.7 9.88e+00    -  1.00e+00 1.00e+00f
1
  22  2.1706354e+04 0.00e+00 4.18e+00  -1.7 8.62e-01  -1.1 1.00e+00 1.00e+00f
1
  23  1.8903288e+04 0.00e+00 2.83e+01  -1.7 4.26e+01    -  1.00e+00 5.00e-01f
2
  24  1.6006615e+04 0.00e+00 4.59e-01  -1.7 1.42e+01    -  1.00e+00 1.00e+00f
1
  25  1.5213482e+04 0.00e+00 3.23e+01  -1.7 3.69e+01    -  1.00e+00 1.00e+00f
1
  26  1.3697207e+04 0.00e+00 5.82e+00  -1.7 1.22e+01    -  1.00e+00 1.00e+00f
1
  27  1.3434765e+04 0.00e+00 2.30e+00  -1.7 3.88e+01    -  1.00e+00 5.00e-01f
2
  28  1.3393324e+04 0.00e+00 9.80e-01  -1.7 1.01e+01    -  1.00e+00 1.00e+00f
1
  29  1.3390124e+04 0.00e+00 7.86e-02  -1.7 3.13e+00    -  1.00e+00 1.00e+00f
1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  l
s
  30  1.3390093e+04 0.00e+00 1.12e-03  -2.5 3.65e-01    -  1.00e+00 1.00e+00f
1
  31  1.3390093e+04 0.00e+00 1.14e-07  -5.7 3.80e-03    -  1.00e+00 1.00e+00f
1
  32  1.3390093e+04 0.00e+00 4.37e-14  -8.6 5.00e-07    -  1.00e+00 1.00e+00f
1

Number of Iterations....: 32

                                   (scaled)                 (unscaled)
Objective...............:   3.0954349427064827e+00    1.3390093119479570e+04
Dual infeasibility......:   4.3732226821005179e-14    1.8917489796876907e-10
Constraint violation....:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity.........:   0.0000000000000000e+00    0.0000000000000000e+00
Overall NLP error.......:   4.3732226821005179e-14    1.8917489796876907e-10


Number of objective function evaluations             = 58
Number of objective gradient evaluations             = 33
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 32
Total seconds in IPOPT                               = 0.019

EXIT: Optimal Solution Found.
[523.3055386212443, -156.94784350151693, -0.19966456906074545]
```


Best fit for the data
data points
best fit
450
400
350
300
250
200
150
100
−4
−2
0
2
4

In [ ]: