

# 2014 Rocky Mountain Regional Programming Contest

## Solution Sketches

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- xxx submissions, xxx correct, first correct: xxx minutes
- Straightforward
- Process each task in order, keeping track of the total number of minutes so far.

- xxx submissions, xxx correct, first correct: xxx minutes
- Straightforward
- Just apply the rules one at a time, stopping as soon as a decision is known.
- No need to look at month/day of a date, just the year.

# Plane Ticket Pricing

- xxx submissions, xxx correct, first correct: xxx minutes
- Dynamic Programming
- Let  $f(n, w)$  = the maximum revenue that can be obtained when there are  $n$  seats left and  $w$  weeks before the flight.
- Base case:  $f(n, w) = 0$  when  $n \leq 0$  or  $w < 0$ .
- Recursion:  
$$f(n, w) = \max_{1 \leq i \leq K_w} \{f(n - s_{i,w}, w - 1) + p_{i,w} \cdot s_{i,w}\}$$
- Minor adjustment above if  $n < s_{i,w}$ .
- Complexity:  $O(NWK)$

# Facility Locations

- xxx submissions, xxx correct, first correct: xxx minutes
- We can model this as a bipartite graph: one set of nodes are the clients, and the other set of nodes are the potential locations.
- We connect a client to a location if the cost is 0.
- The locality property implies that each connected component is a complete bipartite subgraph—so each connected component can be served by just one facility.
- i.e. If the number of connected components is at most  $k$ , then it is possible.

# Repeated Substrings

- xxx submissions, xxx correct, first correct: xxx minutes
- Use suffix arrays and the Longest Common Prefix (LCP) array.
- Whenever  $LCP[i] > LCP[i - 1]$ , the difference is the number of unique substrings repeated.
- Sum up all such differences.
- For the first sample input “aabaab”

Prefix	LCP
aab	0
aabaab	3
ab	1
abaab	2
b	0
baab	1

# Landline Telephone Network

- xxx submissions, xxx correct, first correct: xxx minutes
- Form the weighted undirected graph as given.
- Without any insecure buildings, this is just the standard Minimum Spanning Tree problem.
- The insecure buildings must be leaves in the spanning tree, the other ones can be internal nodes or leaves.
- Compute the MST without the insecure buildings. For each insecure building, connect it to the MST using the cheapest edge.



# Aquarium Tank

- xxx submissions, xxx correct, first correct: xxx minutes
- Two possible approaches (among others):
- First approach: “walk up” the polygon and figure out how high the water goes.
- Second approach: guess the height and compute the resulting volume. Use binary search to refine the height.
- Either way: need to intersect polygon with horizontal lines, and compute the area of a polygon or trapezoid.

# Restaurant Ratings

- xxx submissions, xxx correct, first correct: xxx minutes
- Approach 1: make use of the fact that the number of integer solutions  $x_1, \dots, x_n \geq 0$  such that  $x_1 + \dots + x_n = r$  is  $\binom{r+n-1}{r}$ .
  - We can find the number of ratings less than the given total.
  - Use the above to find the number of worse ratings with the same total, but with the same first  $k$  ratings
- Approach 2: Dynamic Programming
  - State is  $(a, k, s)$  :  $a$  is rating already worse?,  $k$  rating index,  $s$  remaining rating sum.
  - $$f(1, k, s) = \sum_{i=0}^s f(1, k+1, s-i)$$
  - $$f(0, k, s) = f(0, k+1, s-r_k) + \sum_{i=0}^{r_k-1} f(1, k+1, s-i) + \sum_{i=r_k+1}^s f(1, k+1, s-1-i)$$
  - base case  $f(x, n, y) = f(x, y, 0) = 1$ , answer is  $f(0, 0, S)$  where  $S$  is the total rating

# Locked Treasure

- xxx submissions, xxx correct, first correct: xxx minutes
- The answer is  $\binom{n}{m-1}$ .
- For each subset of  $m - 1$  bandits, there must be at least one lock that they cannot open (lower bound).
- For each subset of  $m - 1$  bandits, have one lock such that the keys are distributed to all others who are not in the subset. Any group of  $m$  bandits must have a key to every lock (upper bound).

# Yet Satisfiability Again!

- xxx submissions, xxx correct, first correct: xxx minutes
- Exhaustive search.
- Just try all possible  $2^n$  truth-value assignment to the variables and test if the clauses are all satisfied.