

Dynamic Programming: When your big problem is just too hard to solve

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<https://github.com/depstein/programming-competitions>

In [mathematics](#), [computer science](#), [economics](#), and [bioinformatics](#), **dynamic programming** is a method for solving complex problems by breaking them down into simpler subproblems. It is applicable to problems exhibiting the properties of [overlapping subproblems](#)^[1] and [optimal substructure](#) (described below). When applicable, the method takes far less time than naive methods that don't take advantage of the subproblem overlap (like [depth-first search](#)).

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Don't repeat work

Optimal Substructure

- By combining the optimal solutions to subproblems, you can solve the overall problem

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Overlapping Subproblems

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Overlapping Subproblems

- To find the optimal solution, the subproblem must be referred to multiple times



1, 1, 2, 3, 5, 8, 13, 21, 34, ...

```
import java.io.*;
import java.util.*;

public class fibonacci {
    public static void main(String[] args) {
        for(int i=0;i<50;i++) {
            System.out.printf("The %dth Fibonacci number is %d.\n", i, fib(i));
        }
    }

    public static long fib(int n) {
        if(n==0 || n==1) {
            return 1;
        }
        return fib(n-1) + fib(n-2);
    }
}
```

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The 30th Fibonacci number is 1346269.
The 31th Fibonacci number is 2178309.
The 32th Fibonacci number is 3524578.
The 33th Fibonacci number is 5702887.
The 34th Fibonacci number is 9227465.
The 35th Fibonacci number is 14930352.
The 36th Fibonacci number is 24157817.
The 37th Fibonacci number is 39088169.
The 38th Fibonacci number is 63245986.
The 39th Fibonacci number is 102334155.
The 40th Fibonacci number is 165580141.
The 41th Fibonacci number is 267914296.
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Slows... waaaaay... doooooooooooooooooooooown

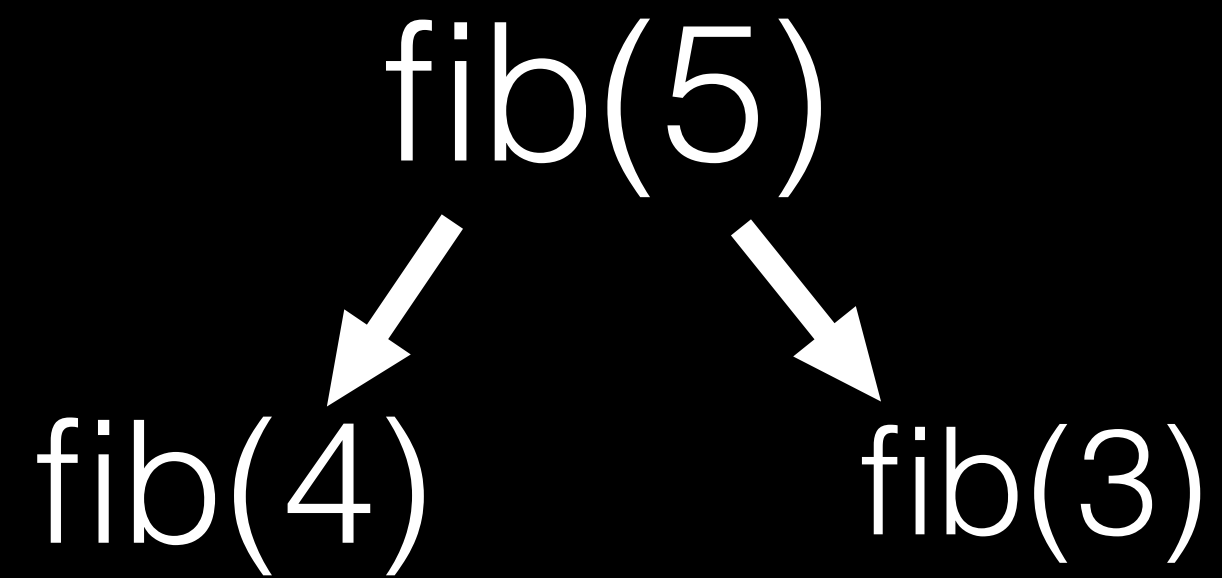
```
import java.io.*;
import java.util.*;

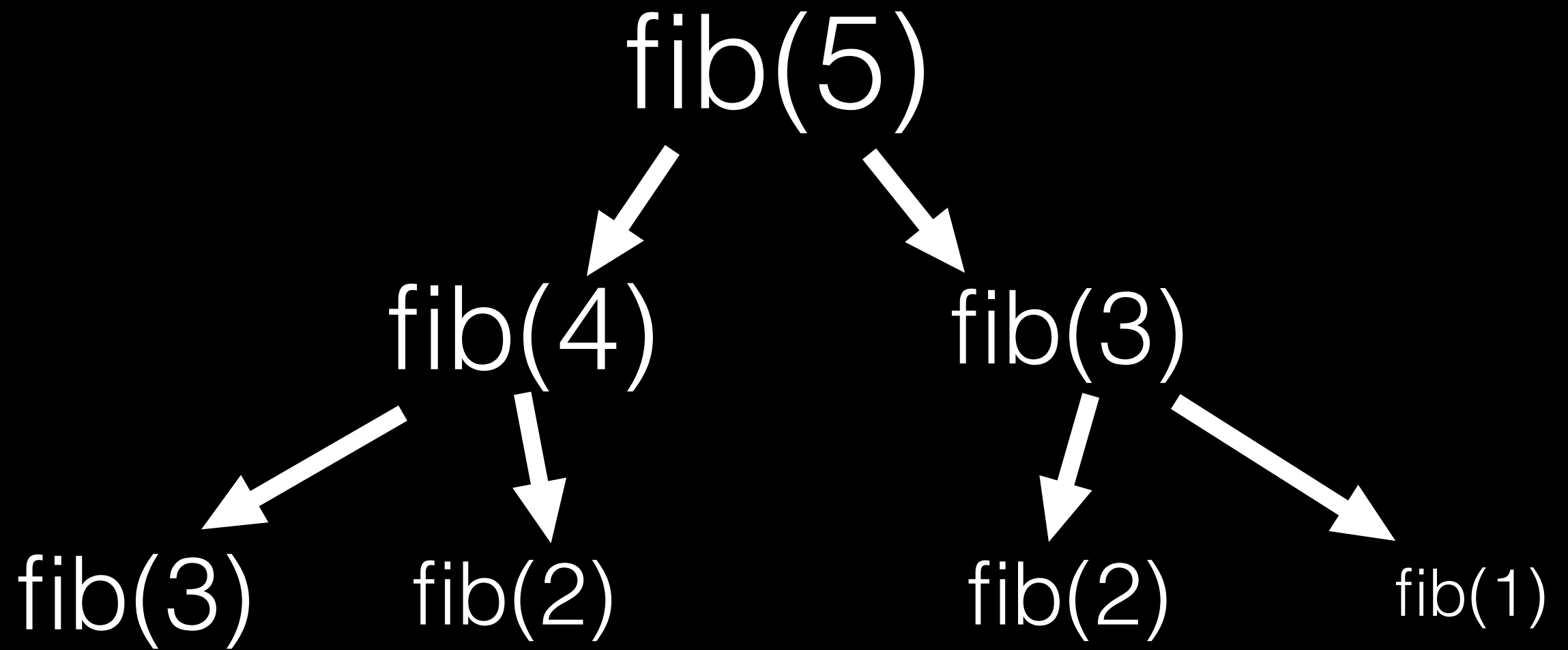
public class fibonacci {
    public static void main(String[] args) {
        for(int i=0;i<50;i++) {
            System.out.printf("The %dth Fibonacci number is %d.\n", i, fib(i));
        }
    }

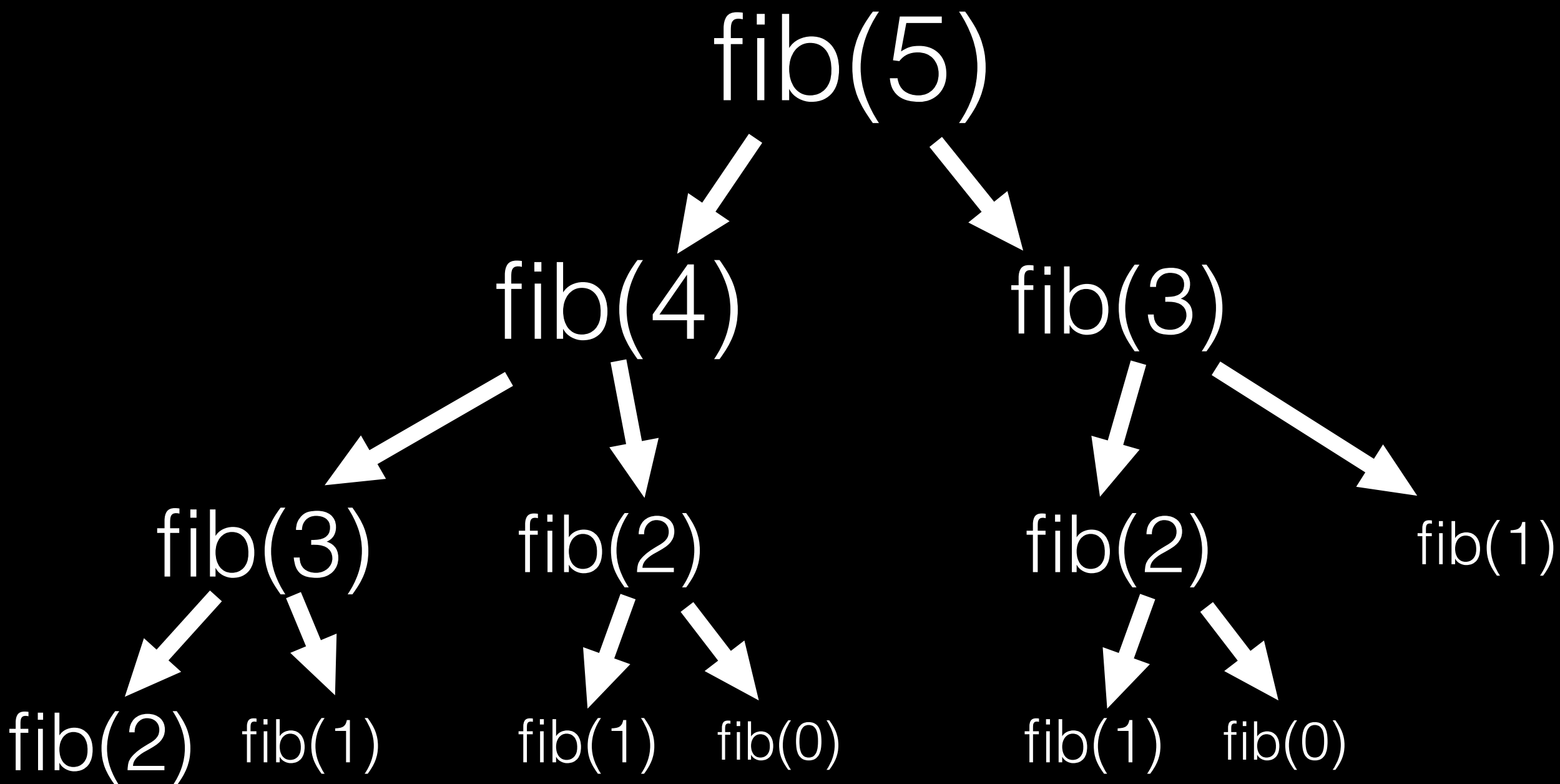
    public static long fib(int n) {
        if(n==0 || n==1) {
            return 1;
        }
        return fib(n-1) + fib(n-2);
    }
}
```

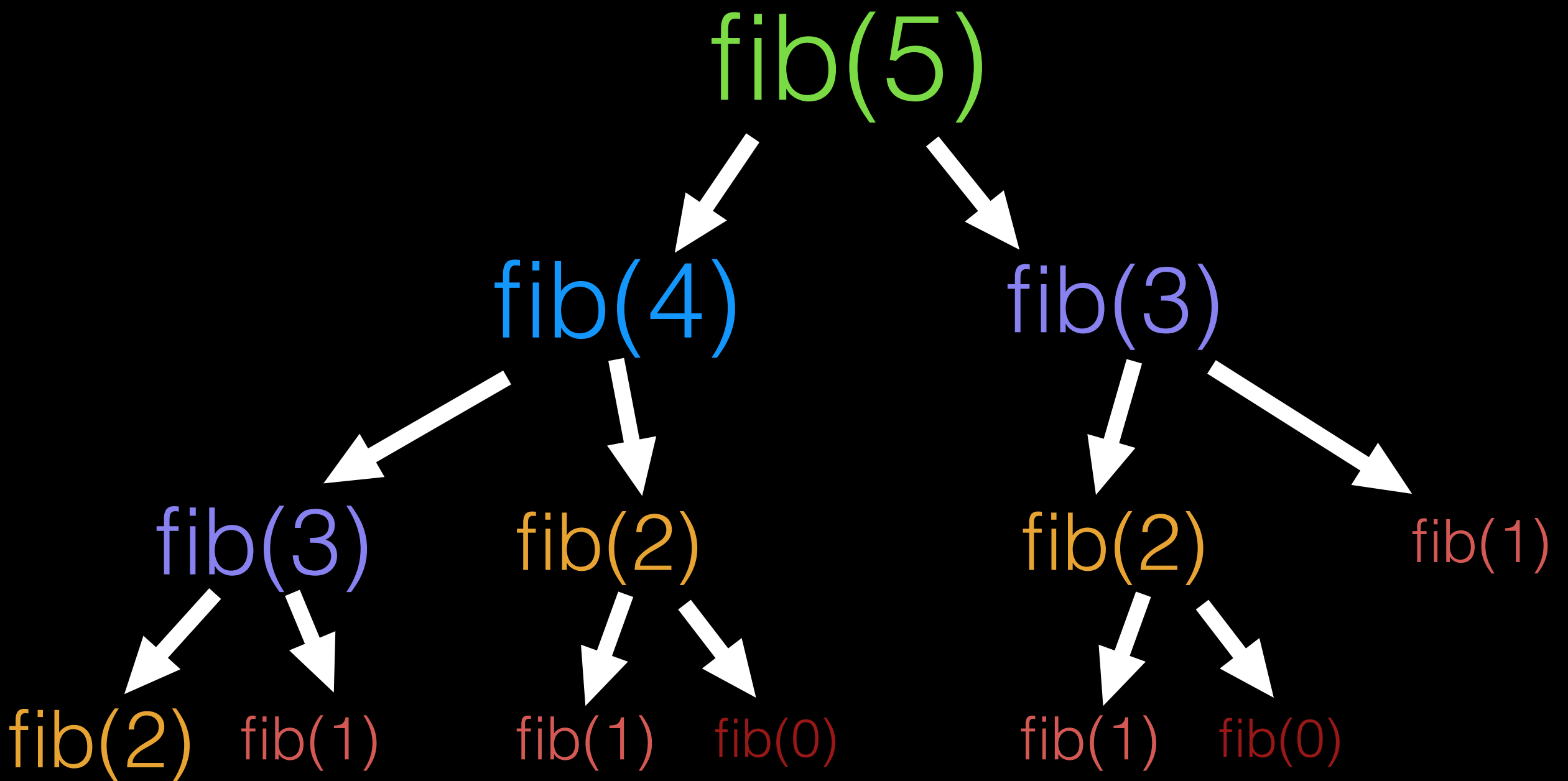
Optimal Substructure!

fib(5)









Overlapping subproblems!

Memoization

In **computing**, **memoization** is an **optimization** technique used primarily to speed up **computer programs** by keeping the results of expensive **function calls** and returning the cached result when the same inputs occur again.

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```
import java.io.*;
import java.util.*;

public class fibonacci {
    public static long[] memoize = new long[50];

    public static void main(String[] args) {
        memoize[0] = 1;
        memoize[1] = 1;

        for(int i=0;i<50;i++) {
            System.out.printf("The %dth Fibonacci number is %d.\n", i, fib_dp(i));
        }
    }

    public static long fib_dp(int n) {
        if(memoize[n] != 0) { //0 is the default value for the array elements
            return memoize[n];
        }

        memoize[n] = fib_dp(n-1) + fib_dp(n-2);
        return memoize[n];
    }
}
```

How about a harder
problem?

Longest Common Subsequence

TATGT

CATAG

Longest Common Subsequence

TATGT

CATAG

```
import java.io.*;
import java.util.*;

public class lcs {
    public static void main(String[] args) {
        String s1 = "TATGT";
        String s2 = "CATAG";
        System.out.printf("The longest common subsequence is of length %d.\n", lcs(s1, s2, 0,
0));
    }

    public static int lcs(String s1, String s2, int s1index, int s2index) {
        if(s1index == s1.length() || s2index == s2.length()) {
            return 0;
        }
        if(s1.charAt(s1index) == s2.charAt(s2index)) {
            return 1 + lcs(s1, s2, s1index+1, s2index+1);
        }
        return Math.max(lcs(s1, s2, s1index+1, s2index), lcs(s1, s2, s1index, s2index+1));
    }
}
```

```
import java.io.*;
import java.util.*;

public class lcs {
    public static void main(String[] args) {
        String s1 = "TATGT";
        String s2 = "CATAG";
        System.out.printf("The longest common subsequence is of length %d.\n", lcs(s1, s2, 0, 0));
    }

    public static int lcs(String s1, String s2, int s1index, int s2index) {
        if(s1index == s1.length() || s2index == s2.length()) {
            return 0;
        }
        if(s1.charAt(s1index) == s2.charAt(s2index)) {
            return 1 + lcs(s1, s2, s1index+1, s2index+1);
        }
        return Math.max(lcs(s1, s2, s1index+1, s2index), lcs(s1, s2, s1index, s2index+1));
    }
}
```



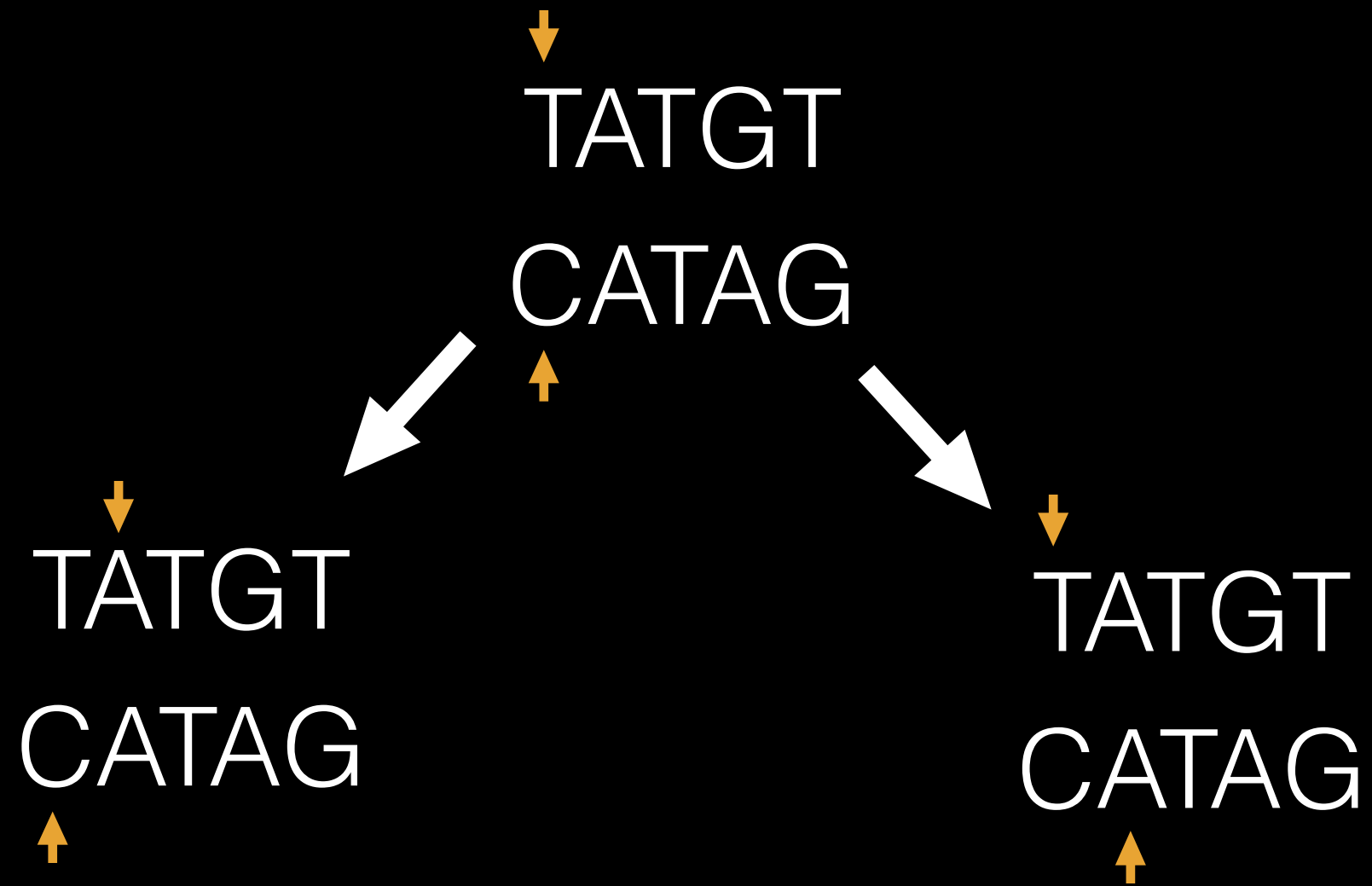
Optimal Substructure!

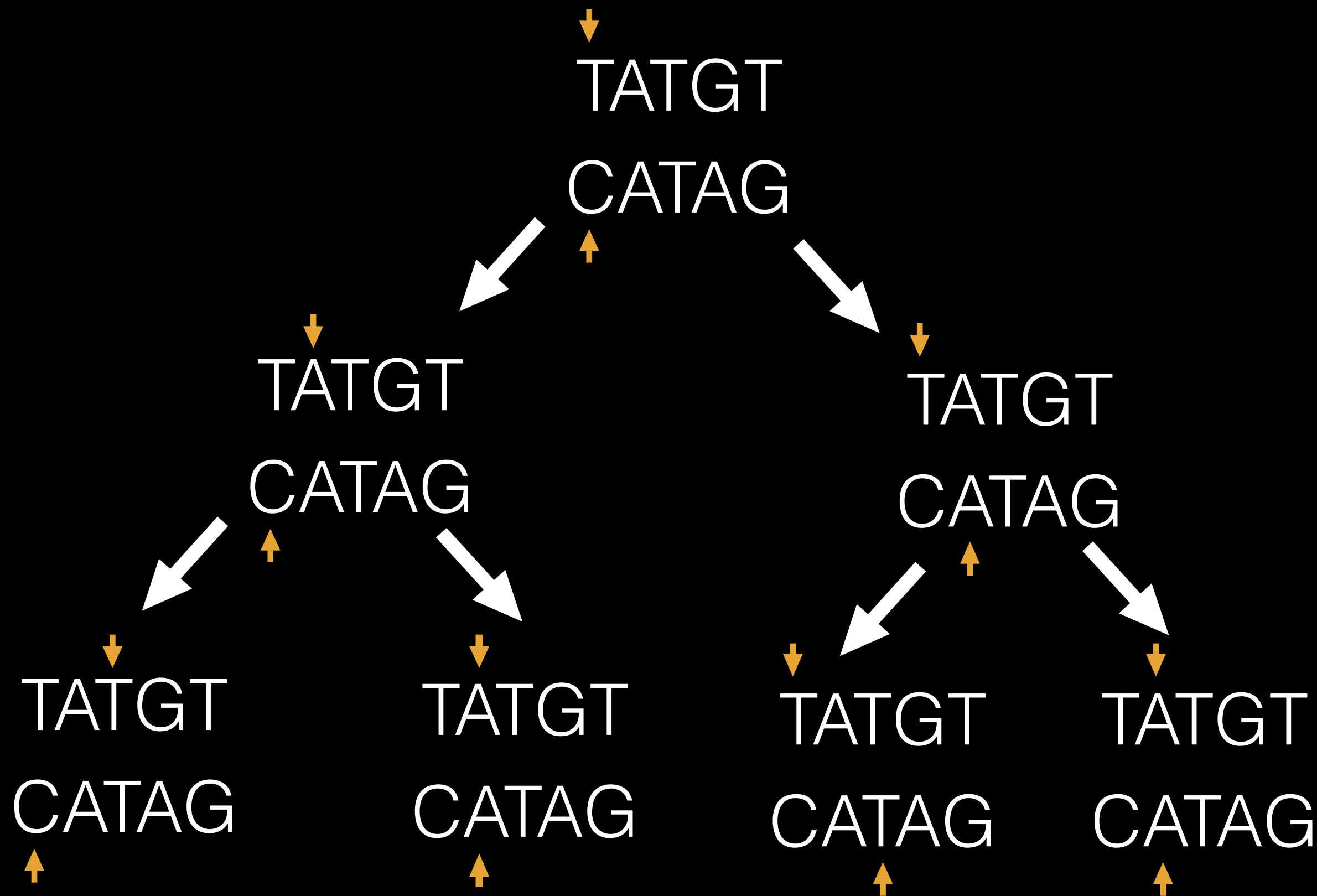


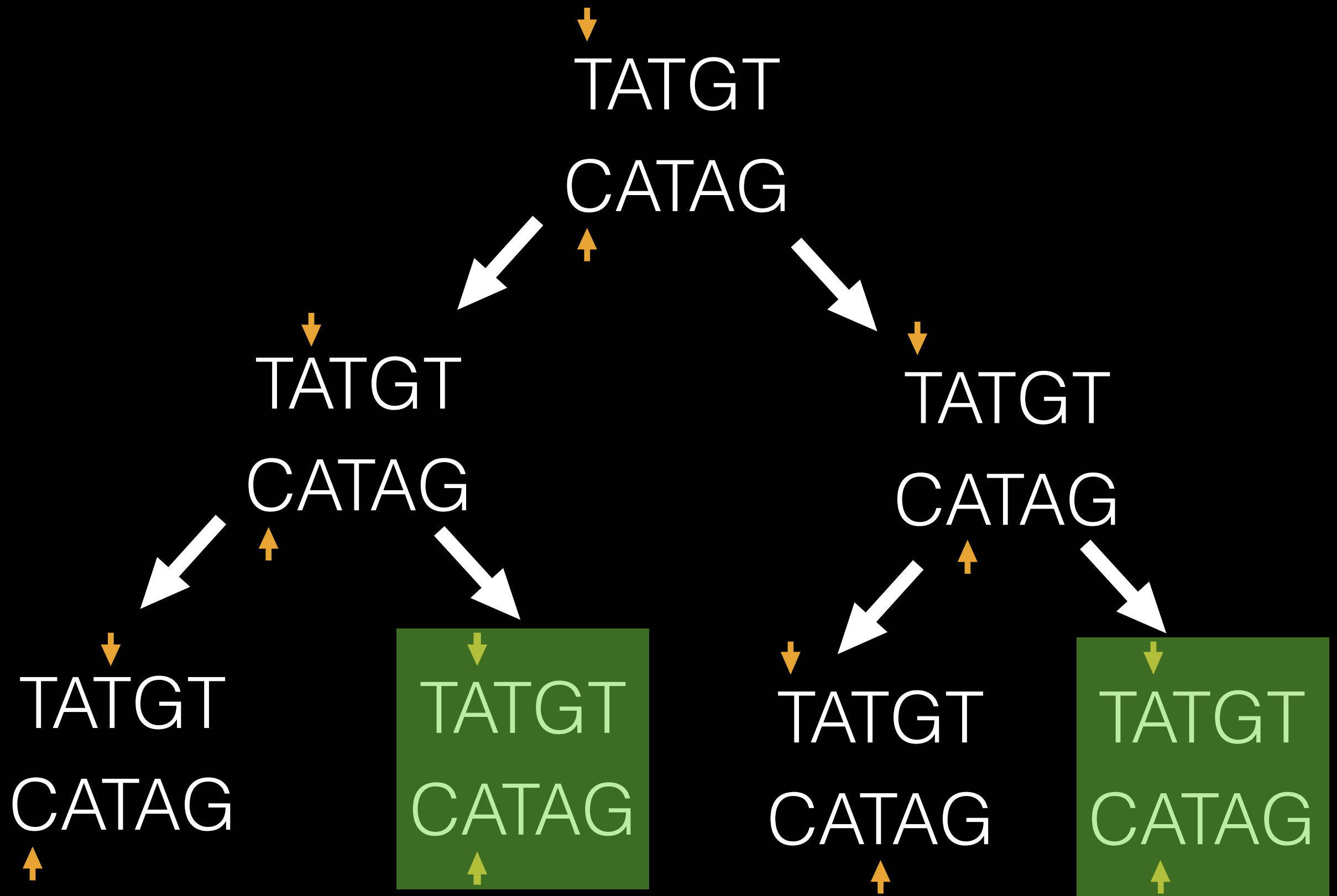
TATGT

CATAG









Overlapping subproblems!

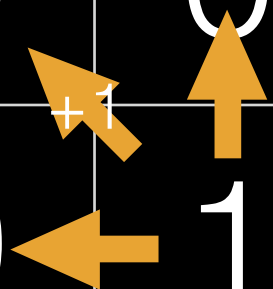
	/	C	A	T	A	G
/						
T						
A						
T						
G						
T						

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0					
A	0					
T	0					
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0				
A	0					
T	0					
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
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A	0					
T	0					
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1		
A	0					
T	0					
G	0					
T	0					



	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1	1	1
A	0					
T	0					
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1	1	1
A	0	0	1	1	2	2
T	0					
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1	1	1
A	0	0	1	1	2	2
T	0	0	1	2	2	2
G	0					
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1	1	1
A	0	0	1	1	2	2
T	0	0	1	2	2	2
G	0	0	1	2	2	3
T	0					

	/	C	A	T	A	G
/	0	0	0	0	0	0
T	0	0	0	1	1	1
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T	0	0	1	2	2	2
G	0	0	1	2	2	3
T	0	0	1	2	2	3

```

import java.io.*;
import java.util.*;

public class lcs {
    public static int[][] memoize = new int[6][6];

    public static void main(String[] args) {
        for(int i=1;i<6;i++) {
            for(int j=1;j<6;j++) {
                memoize[i][j] = -1;
            }
        }

        String s1 = "TATGT";
        String s2 = "CATAG";
        System.out.printf("The longest common subsequence is of length %d.\n", lcs_dp(s1, s2, 5, 5));
    }

    public static int lcs_dp(String s1, String s2, int slindex, int s2index) {
        if(memoize[slindex][s2index] == -1) {
            if(s1.charAt(slindex-1) == s2.charAt(s2index-1)) { // the first row & column are all 0's, so charAt
is off-by-1
                memoize[slindex][s2index] = 1 + lcs_dp(s1, s2, slindex-1, s2index-1);
            } else {
                memoize[slindex][s2index] = Math.max(lcs_dp(s1, s2, slindex-1, s2index), lcs_dp(s1, s2, slindex,
s2index-1));
            }
        }
        return memoize[slindex][s2index];
    }
}

```