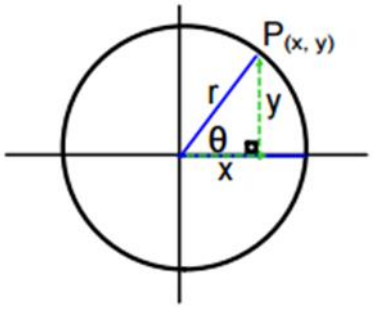
TRIGONOMETRIC FUNCTIONS

Definitions of trig ratios and functions

In Trigonometry there are six trigonometric ratios that relate the angle measures of a right triangle to the length of its sides. (Remember a right triangle contains a 90° angle)

A right triangle can be formed from an initial side *x* and a terminal side *r*, where *r* is the radius and hypotenuse of the right triangle. (see figure below) The Pythagorean Theorem tells us that $x^2 + y^2 = r^2$, therefore $r = \sqrt{x^2 + y^2}$. θ (theta) is used to label a non-right angle. The six trigonometric functions can be used to find the ratio of the side lengths. The six functions are sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot). Below you will see the ratios formed by these functions.



sin $\theta = \frac{y}{r}$, also referred to as $\frac{opposite\ side}{hypotenuse}$

cos $\theta = \frac{x}{r}$, also referred to as $\frac{adjacent\ side}{hypotenuse}$

tan $\theta = \frac{y}{x}$, also referred to as $\frac{opposite\ side}{adjacent\ side}$

These three functions have 3 reciprocal functions

csc $\theta = \frac{r}{y}$, which is the reciprocal of sin θ

sec $\theta = \frac{r}{x}$,which is the reciprocal of cos θ

cot $\theta = \frac{x}{y}$, which is the reciprocal of tan θ

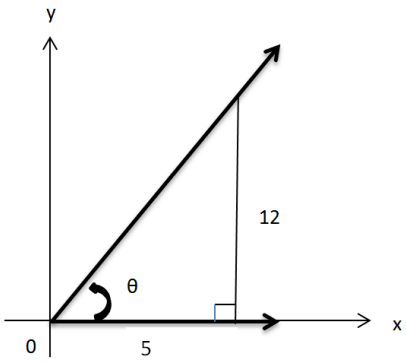
You may recall a little something called SOH-CAH-TOA to help your remember the functions!

SOH… Sine = opposite/hypotenuse

…CAH… Cosine = adjacent/hypotenuse

…TOA Tangent = opposite/adjacent

Example: Find the values of the trigonometric ratios of angle θ



Before we can find the values of the six trig ratios, we need to find the length of the missing side. Any ideas? Good call, we can use r = $\sqrt{x^2 + y^2}$ (from the Pythagorean Theorem)

r = $\sqrt{5^2 + 12^2}$ = $\sqrt{25 + 144}$ = $\sqrt{169}$ = 13

Now we can find the values of the six trig functions

sin θ = $\frac{opposite}{hyptoenuse} = \frac{12}{13}$

csc θ = $\frac{hyptoenuse}{opposite} = \frac{13}{12}$

cos θ = $\frac{adjacent}{hypotenuse} = \frac{5}{13}$

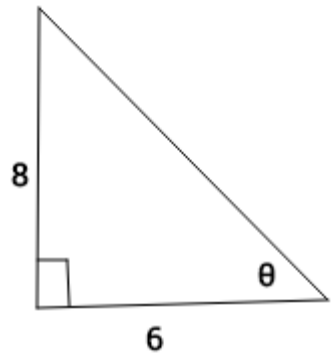
sec θ = $\frac{hypotenuse}{adjacent} = \frac{13}{5}$

tan θ = $\frac{opposite}{adjacent} = \frac{12}{5}$

cot θ = $\frac{adjacent}{opposite} = \frac{5}{12}$

Example 5

a) Use the triangle below to find the six trig ratios



First use Pythagorean Theorem to find the hypotenuse

$a^2 + b^2 = c^2$, where a and b are legs of the right triangle and c is the hypotenuse

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

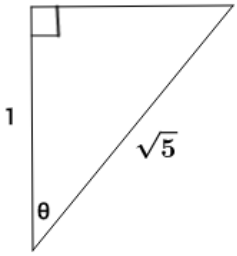
$\sin\theta = \frac{o}{h} = \frac{8}{10} = \frac{4}{5}$ $\quad \csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$

$\cos\theta = \frac{a}{h} = \frac{6}{10} = \frac{3}{5}$ $\quad \sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$

$\tan\theta = \frac{o}{a} = \frac{8}{6} = \frac{4}{3}$ $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$

Example 6

Use the triangle below to find the six trig ratios



$1^2 + b^2 = (\sqrt{5})^2$

$1 + b^2 = 5$

$b^2 = 4$

$b = 2$

$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\quad \csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\quad \sec\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\tan\theta = \frac{2}{1} = 2$ $\quad \cot\theta = \frac{1}{2}$

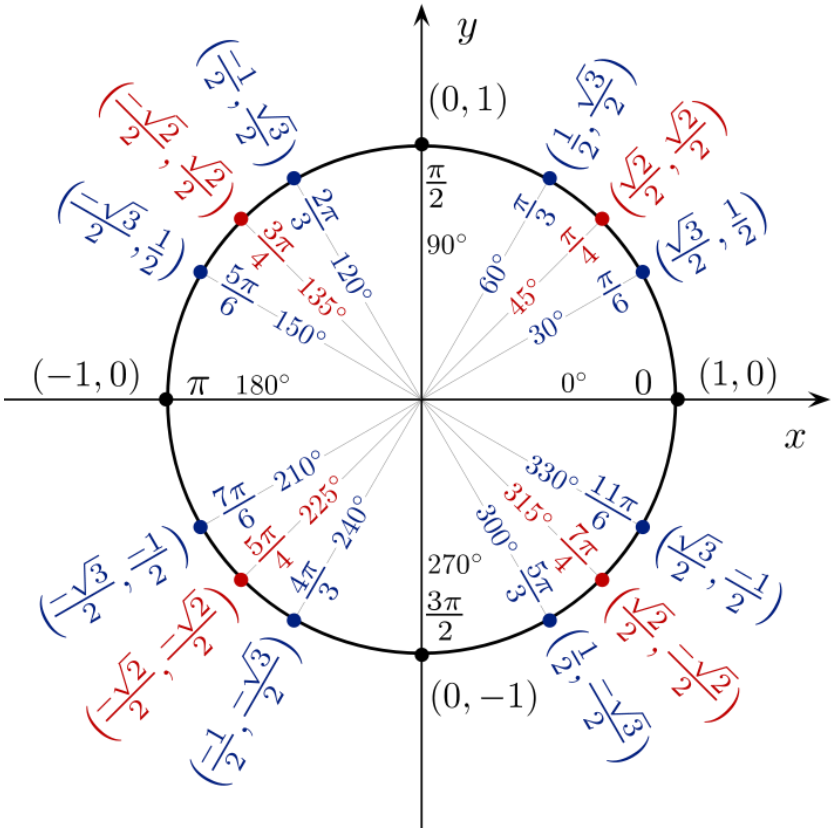
Need more help? Click below for a Khan Academy Video

Khan Academy video 2

Find the value of trig functions given an angle measure

Suppose you know the value of θ is 45°, how can this help you find the values of the six trigonometric functions?

First way: You can familiarize yourself with the unit circle we talked about.



An ordered pair along the unit circle (x, y) can also be known as (cos θ, sin θ), since the r value on the unit circle is always 1. So to find the trig function values for 45° you can look on the unit circle and easily see that sin 45° = $\frac{\sqrt{2}}{2}$, cos 45° = $\frac{\sqrt{2}}{2}$

With that information we can easily find the values of the reciprocal functions

csc 45° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$, sec 45° = $\sqrt{2}$

We can also find the tangent and cotangent function values using the quotient identities

$\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\cot 45° = 1$

Example 7

Find $\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

Example 8

Find $\tan\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

Example 9

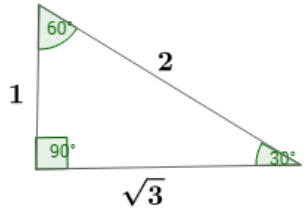
Find $\cot 240° = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

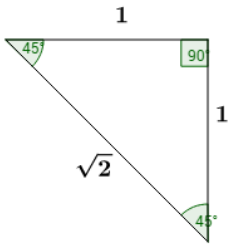
Using this method limits us to finding trig function values for angles that are accessible on the unit circle, plus who wants to memorize it!!!

Second Way: If you are given a problem that has an angle measure of 45°, 30°, or 60°, you are in luck! These angle measures belong to special triangles.

If you remember these special triangles you can easily find the ratios for all the trig functions.

Below are the two special right triangles and their side length ratios





How do we use these special right triangles to find the trig ratios?

If the θ you are given has one of these angle measures it's easy!

Example 10

Find $\sin 30°$

$\sin 30° = \frac{1}{2}$

Example 11

Find $\cos 45°$

$\cos 45° = \frac{\sqrt{2}}{2}$

Example 12

Find $\tan 60°$

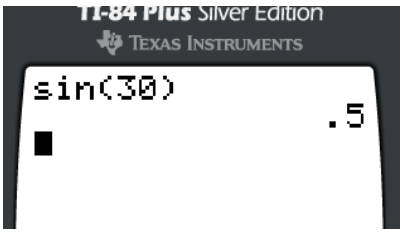
$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Third way: This is not only the easiest way, but also this way you can find trig values for angle measures that are less common. You can use your TI Graphing calculator.

First make sure your TI Graphing calculator is set to degrees by pressing mode

Next choose which trig function you need

After you choose which function you need type in your angle measure



Example 13

$\cos 55° \approx 0.5736$

Example 14

$\tan 0° = 0$

Example 15

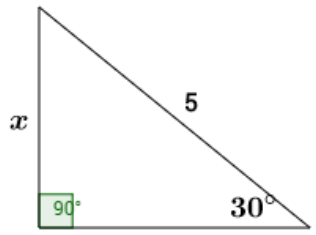
$\sin 30° = 0.5$

Find a missing side length given an angle measure

Suppose you are given an angle measure and a side length, can you find the remaining side lengths?

Yes. You can use the trig functions to formulate an equation to find missing side lengths of a right triangle.

Example 16



First we know that $\sin\theta = \frac{o}{h}$, therefore $\sin 30 = \frac{x}{5}$

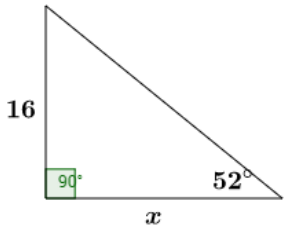
Next we solve for x, $5 \cdot \sin 30 = x$

Use your TI calculator to compute $5 \cdot \sin 30$,

And you find out $x = 2.5$

Let's see another example,

Example 17



We are given information about the opposite and adjacent sides of the triangle, so we will use tan

$$\tan 52 = \frac{16}{x}$$

$$x = \frac{16}{\tan 52}$$

$$x \approx 12.5$$

Need more help? Click below for a Khan Academy video

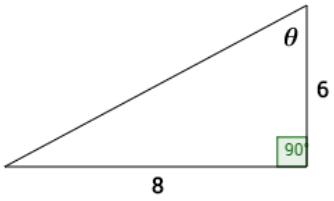
Khan Academy video 3

Find an angle measure using trig functions

Wait a minute, what happens if you have the trig ratio, but you are asked to find the angles measure? Grab your TI Graphing calculator and notice that above the sin, cos, and tan buttons, there is sin^{-1}, cos^{-1}, tan^{-1}. These are your inverse trigonometric functions, also known as arcsine, arccosine, and arctangent. If you use these buttons in conjunction with your trig ratio, you will get the angle measure for θ!

Let's see some examples of this.

Example 18



We know that $\tan\theta = \frac{8}{6}$

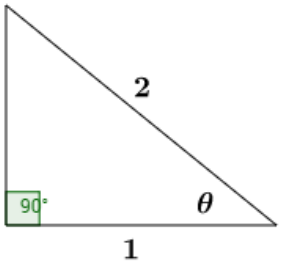
So to find the value of θ, press 2nd tan on your calculator and then type in (8/6)

$$tan^{-1}\left(\frac{8}{6}\right) \approx 53.13$$

$$\theta \approx 53.13°$$

How about another

Example 19



We are given information about the adjacent side and the hypotenuse, so we will use the cosine function

$$\cos\theta = \frac{1}{2}$$

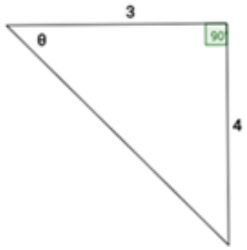
$$cos^{-1}\left(\frac{1}{2}\right) = 60$$

$$\theta = 60°$$

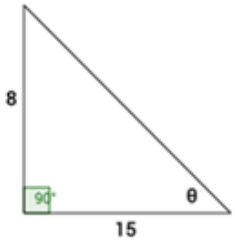
Practice Problems

Find the value of the six trigonometric functions

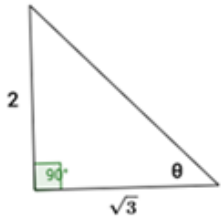
1)



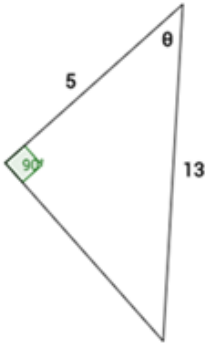
2)



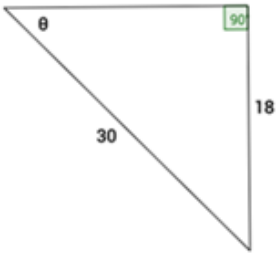
3)



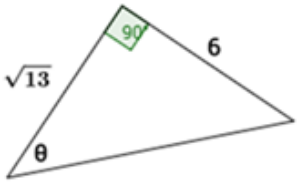
4)



5)



6)



Use the definition of the trig ratios to find the trig function indicated

7) Given: $\cos\theta = \frac{4}{5}$, find $\tan\theta$

8) Given: $\csc\theta = \frac{25}{7}$, find $\sec\theta$

9) Given: $\sin\theta = \frac{2}{3}$, $and\ \cos\theta = \frac{\sqrt{5}}{3}$, find $\cot\theta$

10) Given: $\cos\theta = \frac{\sqrt{3}}{2}$, find $\tan\theta$

Use the Unit Circle to find the values of the trig functions

11) $\cos 45°$ 12) $\sin 30°$ 13) $\sin \frac{3\pi}{4}$ 14) $\tan \frac{7\pi}{6}$

15) $\sec(-90°)$ 16) $\cot(-45°)$ 17) $\csc 150°$ 18) $\sin 270°$

19) $\cos \frac{5\pi}{4}$ 20) $\tan \frac{11\pi}{6}$

Use the special triangles (30-60-90 and 45-45-90) to find the values of the trig functions

21) $\cos 30°$ 22) $\sin 60°$ 23) $\csc 45°$ 24) $\cot 45°$

25) $\sin 30°$ 26) $\sec 45°$ 27) $\tan 60°$ 28) $\cos 45°$

Use your graphing calculator to compute the following trig values. Round to four decimal places

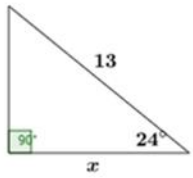
29) $\tan 43°$ 30) $\sin 13°$ 31) $\cos 120°$ 32) $\cot 79°$

33) $\sec 30°$ 34) $\cot 240°$ 35) $\sin 0$ 36) $\csc 10°$

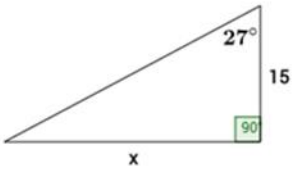
37) $\sin 30°$ 38) $\sin -330°$ 39) $\tan -36°$ 40) $\cos 135°$

Use the trig functions to find the missing side lengths. Round to the nearest hundredth.

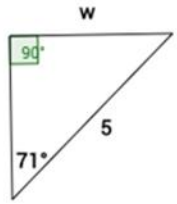
41)



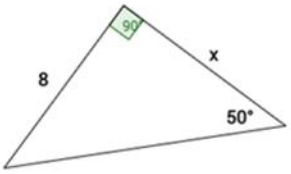
42)



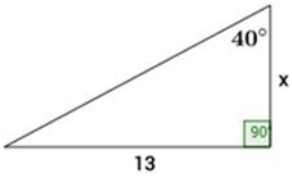
43)



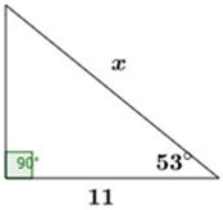
44)



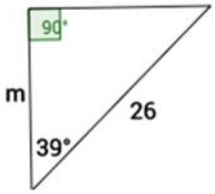
45)



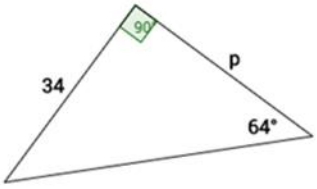
46)



47)



48)



Use the trig functions to find the value of θ. Round to the nearest degree.

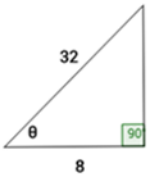
49) $\cos\theta = \frac{1}{2}$

50) $\tan\theta = \frac{30}{50}$

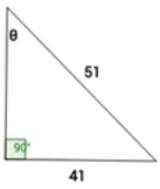
51) $\sin\theta = \frac{6}{7}$

52) $\cos\theta = \frac{42}{48}$

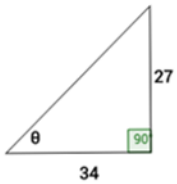
53)



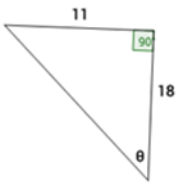
54)



55)



56)



57) $\cos\theta = \frac{48}{59}$

58) $\tan\theta = \frac{20}{17}$

59) $\sin\theta = \frac{2}{3}$

60) $\sin\theta = \frac{1}{3}$

Solutions

Find the value of the six trigonometric functions

1) $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = \frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = \frac{3}{4}$

2) $\sin\theta = \frac{8}{17}, \cos\theta = \frac{15}{17}, \tan\theta = \frac{8}{15}, \csc\theta = \frac{17}{8}, \sec\theta = \frac{17}{15}, \cot\theta = \frac{15}{8}$

3) $\sin\theta = \frac{2\sqrt{7}}{7}, \cos\theta = \frac{\sqrt{21}}{7}, \tan\theta = \frac{2\sqrt{3}}{3}, \csc\theta = \frac{\sqrt{7}}{2}, \sec\theta = \frac{\sqrt{21}}{3}, \cot\theta = \frac{\sqrt{3}}{2}$

4) $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}, \tan\theta = \frac{12}{5}, \csc\theta = \frac{13}{12}, \sec\theta = \frac{13}{5}\ \cot\theta = \frac{5}{12}$

5) $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, \tan\theta = \frac{3}{4}, \csc\theta = \frac{5}{3}, \sec\theta = \frac{5}{4}, \cot\theta = \frac{4}{3}$

6) $\sin\theta = \frac{6}{7}, \cos\theta = \frac{\sqrt{13}}{7}, \tan\theta = \frac{6\sqrt{13}}{13}, \csc\theta = \frac{7}{6}, \sec\theta = \frac{7\sqrt{13}}{13}, \cot\theta = \frac{\sqrt{13}}{6}$

Use the definition of the trig ratios to find the trig function indicated

7) $tan\ \theta = \frac{3}{4}$ 8) $sec\ \theta = \frac{25}{24}$ 9) $cot\ \theta = \frac{\sqrt{5}}{2}$ 10) $tan\ \theta = \frac{\sqrt{3}}{3}$

Use the Unit Circle to find the values of the trig functions

11) $\frac{\sqrt{2}}{2}$ 12) $\frac{1}{2}$ 13) $\frac{\sqrt{2}}{2}$ 14) $\frac{\sqrt{3}}{3}$ 15) Undefined 16) -1 17) 2 18) -1 19) $-\frac{\sqrt{2}}{2}$ 20) $-\frac{\sqrt{3}}{3}$

Use the special triangles to find the values of the trig functions

21) $\frac{\sqrt{3}}{2}$ 22) $\frac{\sqrt{3}}{2}$ 23) $\sqrt{2}$ 24) 1

25) $\frac{1}{2}$ 26) $\sqrt{2}$ 27) $\sqrt{3}$ 28) $\frac{\sqrt{2}}{2}$

** Notice by the complementary angle theorem #11 and #12 have the same value because

$\cos 30 = \sin 60$

Compute the following trig values. Round to four decimal places

29) 0.9325 30) 0.225 31) -0.5 32) 0.1944

33) 1.1547 34) 0.5774 35) 0 36) 5.7588

37) 0.5 38) 0.5 39) -0.7265 40) -0.7071

Use trig functions to find missing side lengths

41) 11.88 42) 7.64 43) 4.73 44) 6.71

45) 15.49 46) 18.28 47) 20.21 48)16.58

Use trig functions to find the value of θ

49) 60° 50) 31° 51) 59° 52) 29°

53) 76° 54) 54° 55) 38° 56) 31°

57) 36° 58) 50° 59) 42° 60) 19°

Back to Table of Contents.