2.6 a b A^+A

a) $A = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$

b) $A = \begin{pmatrix}1-i\\ 2\\ \sqrt{2}i\end{pmatrix}$

a) $\left(\left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}\right)^*\right)^T = \left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}\right)^T = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-i}{\sqrt{2}}\end{pmatrix}^T = \left(\frac{1}{\sqrt{2}} \quad \frac{-i}{\sqrt{2}}\right)$

b) $\begin{pmatrix}1+i\\ 2\\ \sqrt{2}i\end{pmatrix}^T = (1+i \quad 2 \quad \sqrt{2}i)$

2.6 a) $A^+A = \left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}\right)^+ \left(\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}\right) = \left(\frac{1}{\sqrt{2}} \quad \frac{-i}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} - \frac{i^2}{2} = 1$

b) $\begin{pmatrix}1-i\\ 2\\ \sqrt{2}i\end{pmatrix}^+ = (1+i \quad 2 \quad \sqrt{2}(-i))$

$\cdot \begin{pmatrix}1-i\\ 2\\ \sqrt{2}i\end{pmatrix} = (1+i)(1-i) + 4 + \sqrt{2}(-i)\cdot\sqrt{2}i =$

$1 - i + i - i^2 + 4 + \sqrt{2}(-i)\cdot 2i =$

$6 + \sqrt{4\cdot 1\cdot i^2} = = 6 + \sqrt{4+1} = 8$

2.8 a) $A^+A = (1 \quad 0)\begin{pmatrix}1\\ 0\end{pmatrix} = 1$

$B^+B = (0 \quad 1)\begin{pmatrix}0\\ 1\end{pmatrix} = 1$

$A^+A - B^+B = 0$

b) $A^+A = \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$

$B^+B = \left(\frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$

$A^+A - B^+B = 1$

a) $A^+A = (1 \quad 0)\begin{pmatrix}1\\ 0\end{pmatrix} = 1$

$B^+B = (0 \quad 1)\begin{pmatrix}0\\ 1\end{pmatrix} = 1$

$AA^+ - BB^+ = \begin{pmatrix}1\\ 0\end{pmatrix}(1 \quad 0) - \begin{pmatrix}0\\ 1\end{pmatrix}(0 \quad 1) = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} - \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

b) $A^+A = \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$

$B^+B = \left(\frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$

$AA^+ - BB^+ = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}\left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right) -$

$\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}\left(\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}\right) = \begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix} -$

$\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

2.9 c) $A^+A = \begin{pmatrix} 1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} = \frac{1}{2} - \frac{i^2}{2} = 0$

$B^+B = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix} = \frac{1}{2} - \frac{i^2}{2} = 0$

$AA^+ - BB^+ = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix} - \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \end{pmatrix} =$

$= \begin{pmatrix} \frac{1}{2} & -\frac{i}{2} \\ \frac{i}{2} & \frac{1}{2} \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

3.1 b) $\begin{pmatrix} (1+i)-(1+i) \\ (1-i)-(1+i) \end{pmatrix} = \begin{pmatrix} 0 \\ -2i \end{pmatrix}$ ... $\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \rightarrow$ ... $0 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ldots$

3.4 a) ~~$\begin{pmatrix} 1+2i \\ i \\ 0 \end{pmatrix}$ ... $=$~~

$|w\rangle^+|v\rangle = \begin{pmatrix} 3-2i & -i & 0 \end{pmatrix}\begin{pmatrix} 1-i \\ 2+3i \\ 5 \end{pmatrix}$ $(3-2i)(1-i) - i(2+3i) =$
$3 - 3i - 2i - 2 - 2i + 3 =$
$4 - 7i$

3.5 a) $(10)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$ ortho

b) $(1/\sqrt{2} \quad 1/\sqrt{2})\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$ ortho

c) $\frac{1}{\sqrt{3}}(1 \; 1 \; 1)\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{3}} \cdot 1 = \frac{1}{\sqrt{3}}$ nicht ortho

d) $\frac{1}{\sqrt{2}}(1 \; -i)\begin{pmatrix} 1 \\ -i \end{pmatrix} = 1 + i^2 = 1 - 1 = 0$ ortho

3.7 a d)

a) $\begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = (1/\sqrt{2}\,|0\rangle - 1/\sqrt{2}\,|1\rangle)$

d) $\begin{pmatrix} -1 \\ 0 \\ 2+i \\ -i \end{pmatrix} = (-1\,|0\rangle + 0.|1\rangle + (2+i)\,|2\rangle + -i\,|3\rangle)$