(4.2) $\gamma \cdot \gamma^* = e^{\frac{2\pi i}{3}} \cdot e^{-\frac{2\pi i}{3}} = e^0 = \underline{1}$

d) $\gamma + \gamma^* = e^{\frac{2\pi i}{3}} + e^{-\frac{2\pi i}{3}}$

$= \cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi + \cos -\frac{2}{3}\pi + i \sin -\frac{2}{3}\pi$

$= -1/2 + i\sqrt{3}/2 - 1/2 - i\sqrt{3}/2$

$= \underline{-1}$

$$\frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \gamma & \gamma^* \\ 1 & \gamma^* & \gamma \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \gamma^* & \gamma \\ 1 & \gamma & \gamma^* \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 & 1+\gamma^*+\gamma & 1+\gamma+\gamma^* \\ 1+\gamma+\gamma^* & 3 & 1+\gamma\gamma+\gamma^*\gamma^* \\ 1+\gamma+\gamma^* & 1+\gamma\gamma+\gamma^*\gamma^* & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\gamma \cdot \gamma = \cos \frac{4}{3}\pi + i \sin 4/3\pi = -1/2 + i\sqrt{3}/2$

$\gamma^* \cdot \gamma^* = \cos -4/3\pi + i \sin -4/3\pi = -1/2 - i\sqrt{3}/2$

$\gamma \cdot \gamma + \gamma^* \gamma^* = \underline{-1}$

g) $\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&0&1 \\ 0&0&1&0 \end{pmatrix} = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \Rightarrow$ je unitárna

(4.3) c) $1/2 \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} = \{\frac{1}{2}(|0\rangle + \sqrt{3}|1\rangle) ; \frac{1}{2}(-\sqrt{3}|0\rangle + |1\rangle)\}$

$\frac{1}{4}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+3 & 0 \\ 0 & 3+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(4.7) e) $\left(\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}\frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{3} \end{pmatrix}\right)^+ = \frac{1}{2}\begin{pmatrix} a + i\sqrt{3}b \\ -b^* + i\sqrt{3}a^* \end{pmatrix}^+ = \underline{\underline{\frac{1}{2}(a^* - i\sqrt{3}b^* ; -b - i\sqrt{3}a)}}$

$\langle\psi|U^+ = \frac{1}{2}(1 ; -i\sqrt{3})\begin{pmatrix} a^* & -b \\ b^* & a \end{pmatrix} = \underline{\underline{\frac{1}{2}(a^* - b^* i\sqrt{3} ; -b - i\sqrt{3}a)}}$

$\overline{(U|\psi\rangle)^+ = \langle\psi|U^+}$

$(|\psi\rangle\langle+|)^+ = \left(\frac{1}{2}\begin{pmatrix} 1 \\ i\sqrt{3} \end{pmatrix}\frac{1}{\sqrt{2}}(1 \;\; 1)\right)^+ = \left(\frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i\sqrt{3} & i\sqrt{3} \end{pmatrix}\right)^+ = \underline{\underline{\frac{1}{2\sqrt{2}}\begin{pmatrix} 1 ; -i\sqrt{3} \\ 1 ; -i\sqrt{3} \end{pmatrix}}}$

$|+\rangle\langle\psi| = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{2}(1 \;\; -i\sqrt{3}) = \underline{\underline{\frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & -i\sqrt{3} \\ 1 & -i\sqrt{3} \end{pmatrix}}}$

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c)

$(U|\psi\rangle)^+ = \left(\begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}\begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}\right)^+ = \begin{pmatrix} a\psi_0 + b\psi_1 \\ -b^*\psi_0 + a^*\psi_1 \end{pmatrix}^+ = \underline{(a^*\psi_0^* + b^*\psi_1^* ; -b\psi_0^* + a\psi_1^*)}$

$\langle\psi|U^+ = (\psi_0^* \;\; \psi_1^*)\begin{pmatrix} a^* & -b \\ +b^* & a \end{pmatrix} = \underline{\underline{(a^*\psi_0^* + b^*\psi_1^* ; -b\psi_0^* + a\psi_1^*)}}$

$(U|\psi\rangle)^+ = \langle\psi|U^+$

4.7

c) $(|\psi\rangle\langle +|)^{\dagger} = \left(\begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\right)^{\dagger} = \left(\frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0 & \psi_0\\ \psi_1 & \psi_1\end{pmatrix}\right)^{\dagger} = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0^* & \psi_1^*\\ \psi_0^* & \psi_1^*\end{pmatrix}$

$|+\rangle\langle\psi| = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}\begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0^* & \psi_1^*\\ \psi_0^* & \psi_1^*\end{pmatrix}$

$(|\psi\rangle\langle +|)^{\dagger} = |+\rangle\langle\psi|$