Spracovanie obrazu, grafika a multimédiá

Cosine transform

DCT
Discrete Cosine Transform

DCT
Discrete Cosine Transform

Introduced by Ahmed, Nataran, Rao 1974

efficient for data compression in spectral domain

only real number (not complex)

algorithm for fast calculation

used in MP3, JPEG

DCT

DCT-I, DCT-II, DCT-III, DCT-IV

DCT II - definition

Koeficienty, ktoré vypočítame prostredníctvom DCT sú na rozdiel od DFT iba z oboru reálnych čísiel.

$$y_n = c_n \sum_{k=0}^{N-1} \cos \frac{\pi n(2k+1)}{2N} x_k, \quad \text{mit } c_0 = \frac{1}{\sqrt{N}},\ c_n = \sqrt{\frac{2}{N}}$$

Unlike DFT, the DCT coefficients are only real numbers.

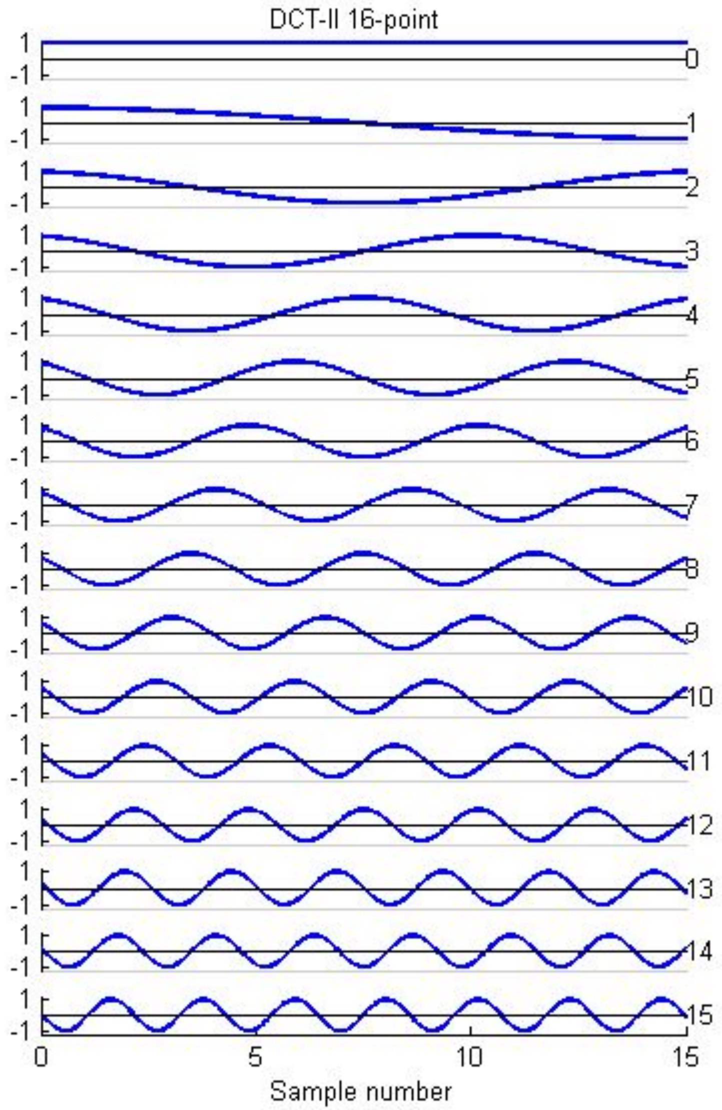
DCT Transformation matrix

$$y_n = c_n \sum_{k=0}^{N-1} \cos \frac{\pi n(2k+1)}{2N} x_k, \quad \text{mit } c_0 = \frac{1}{\sqrt{N}},\ c_n = \sqrt{\frac{2}{N}}$$

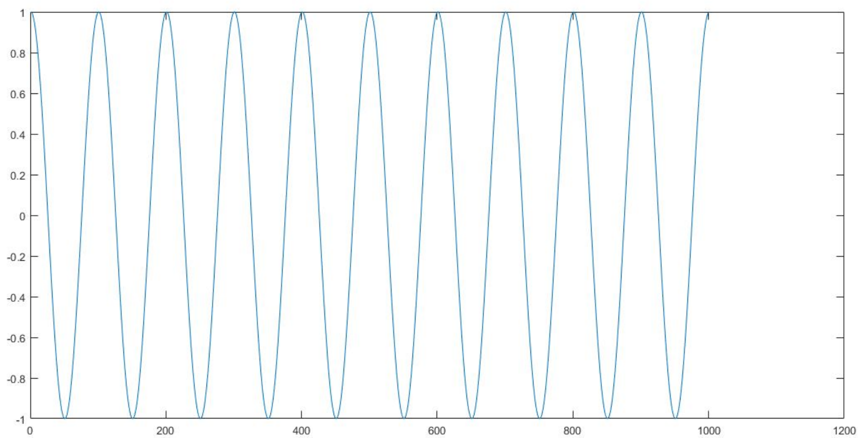
$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{pmatrix} = \underbrace{\sqrt{\frac{2}{N}} \begin{pmatrix} 1/\sqrt{2} & \dots & 1/\sqrt{2} \\ \cos\frac{\pi}{2N} & \dots & \cos\frac{\pi(2N-1)}{2N} \\ \vdots & \ddots & \vdots \\ \cos\frac{\pi(N-1)}{2N} & \dots & \cos\frac{\pi(N-1)(2N-1)}{2N} \end{pmatrix}}_{\text{1D-DCT-Matrix } C} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix}$$

DCT basis functions

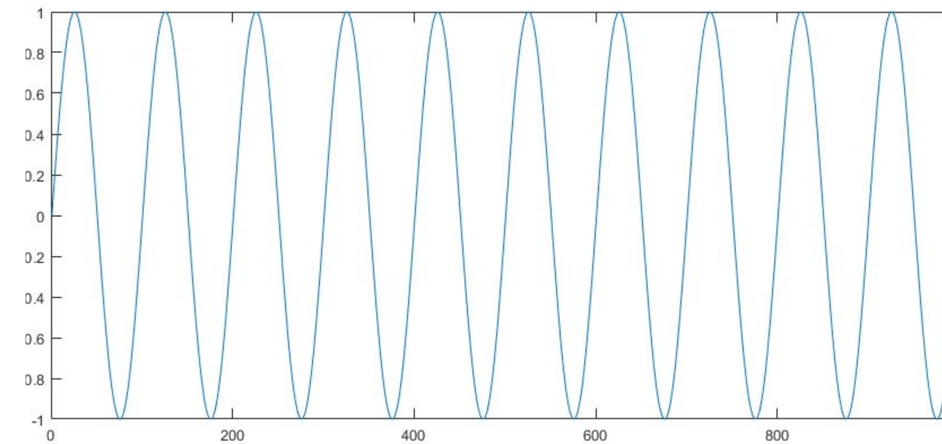
Visualisation of 16 basis functions



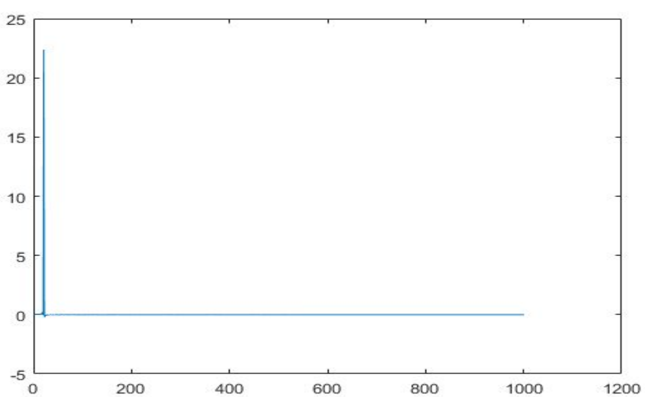
Example



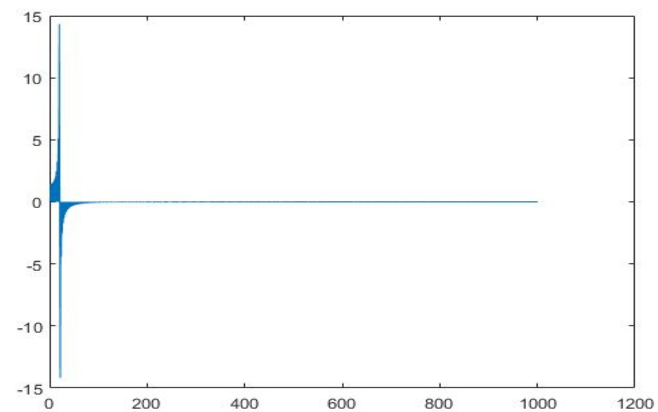
signalCos = cos(2 * pi * Frequency * SampleTime);
SamplingFrequency = 1000;   Frequency = 10



signalSin = sin(2 * pi * Frequency * SampleTime);
SamplingFrequency = 1000;   Frequency = 10



dct(signalCos)



dct(signalSin)

Data reduction in spectral domain

Data reduction in spectral domain

Important approach!

- removing of redundant and irrelevant components in signal.

Lossy data compression in spectral domain- basic steps

Basic simple algorithm:

1. calculate short time spectral coefficients (DFT or DCT)
2. Reduce less relevant spectral coefficients

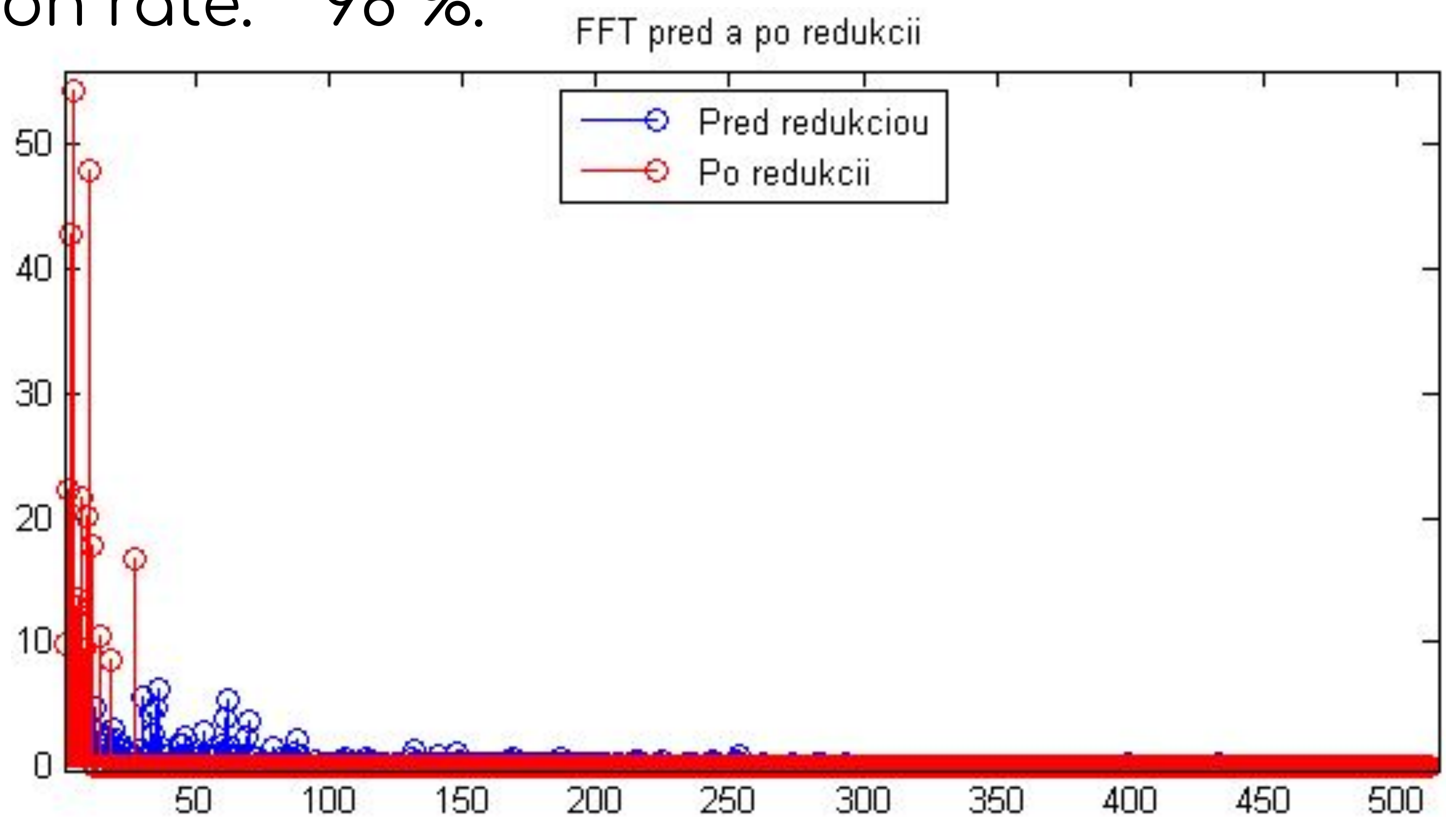
   Simple example: **coefficients < Thresh** are **set to 0**.
3. *...save or transmit the whole reduced spectrum...*
4. calculate inverse transform from reduced spectrum

!LOSS compression

Data reduction in spectral domain - example

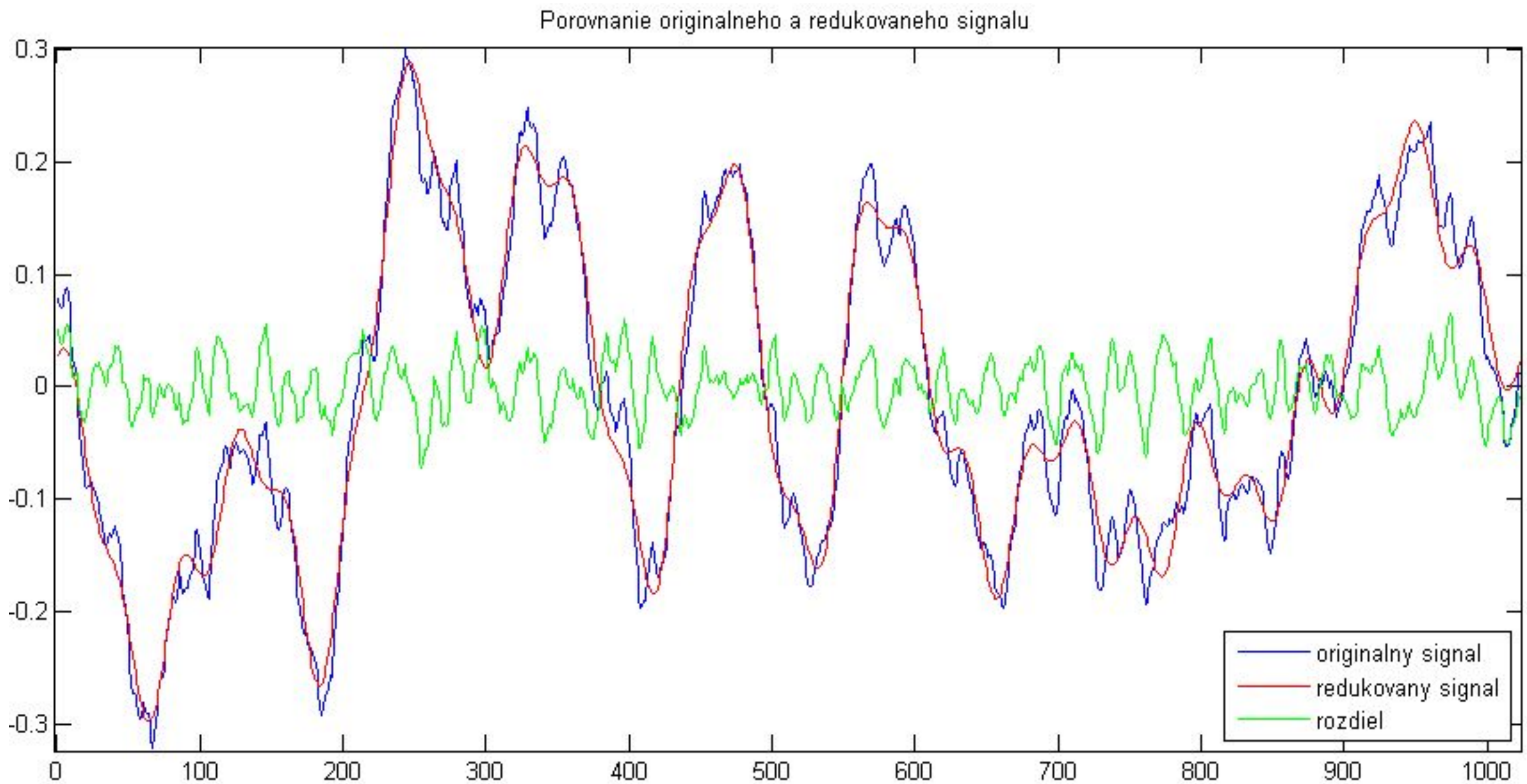
Fourier spectral coefficients

Compression rate: 98 %.

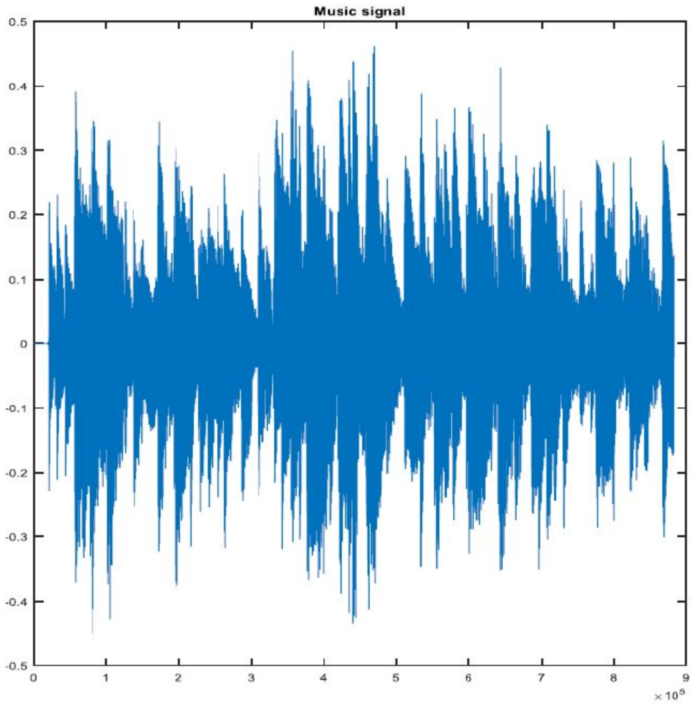


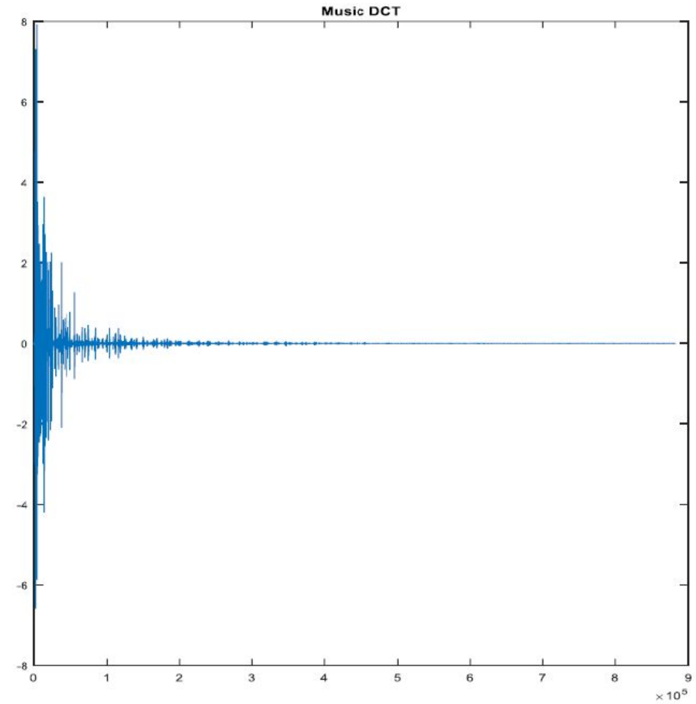
Data reduction in spectral domain - example

Audio data in the time domain

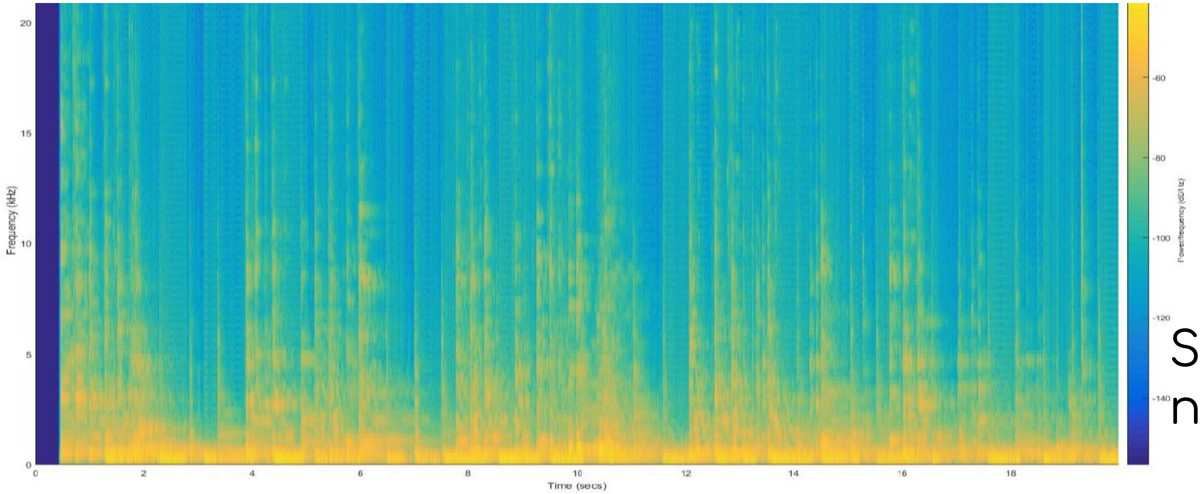


Example - music





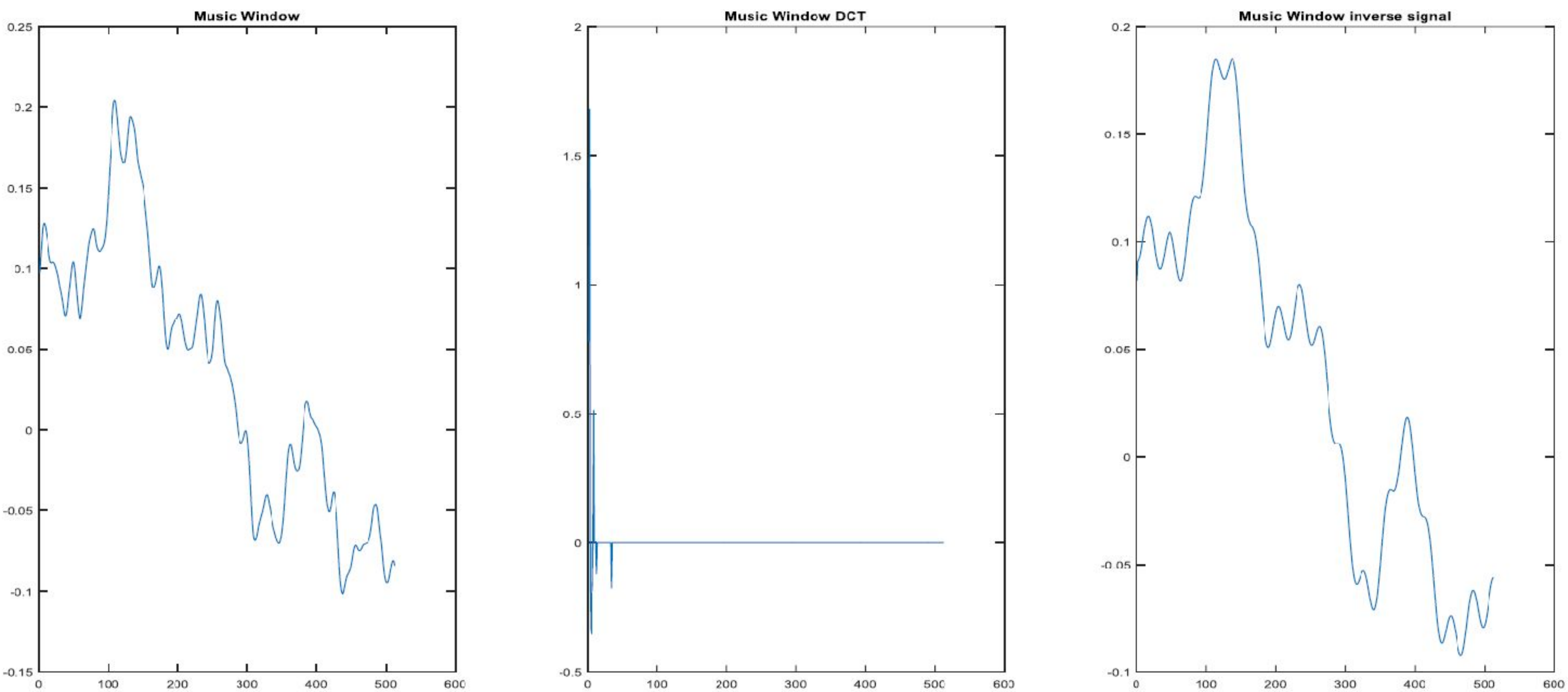
Signál nahrávky a jeho kosínové spektrum



Spektrogram nahrávky

Example – music data reduction

V získanom spektre boli vynulované koeficienty, ktoré boli v absolútnej menšie ako zvolený prah (0.1).



Signál, jeho upravené kosínové spektrum a rekonštruovaný signál

Karhunen-Loève Transform (KLT)

Karhunen-Loève Transform (KLT) Principal Components Analysis (PCA)

Data compression - remove the redundancy by decorrelation of the data

(The basis vectors of the new space define the linear transformation of the data.)

The basis vectors of the KLT are the eigenvectors of the image covariance matrix.

-> decorrelates the data

The KLT is the optimal transformation in terms of the statistical data compression

Correlation

Correlation

Intuitive approach:

Cross-correlation allows assessment of the degree of similarity between two signals.

Autocorrelation is the correlation of a signal with itself.

Discrete Cross-Correlation

In signal processing, cross-correlation is a measure of similarity of two waveforms as a function of a time-lag applied to one of them.

$$r(x) = \sum_{m=-\infty}^{\infty} f(m)h(m-x) \qquad\qquad x = 0, \pm 1, \pm 2 \ldots$$

in practice, only a finite segment of one realization of the infinite-length random process is available.

Autocorrelation

Autocorrelation is the correlation of a signal with itself at different points in time.

For a deterministic discrete-time sequence, *x(n)*, the autocorrelation is computed using the following relationship:

$$r(x) = \sum_{m=-\infty}^{\infty} f(m)f(m-x) \qquad x = 0, \pm 1, \pm 2 \dots$$

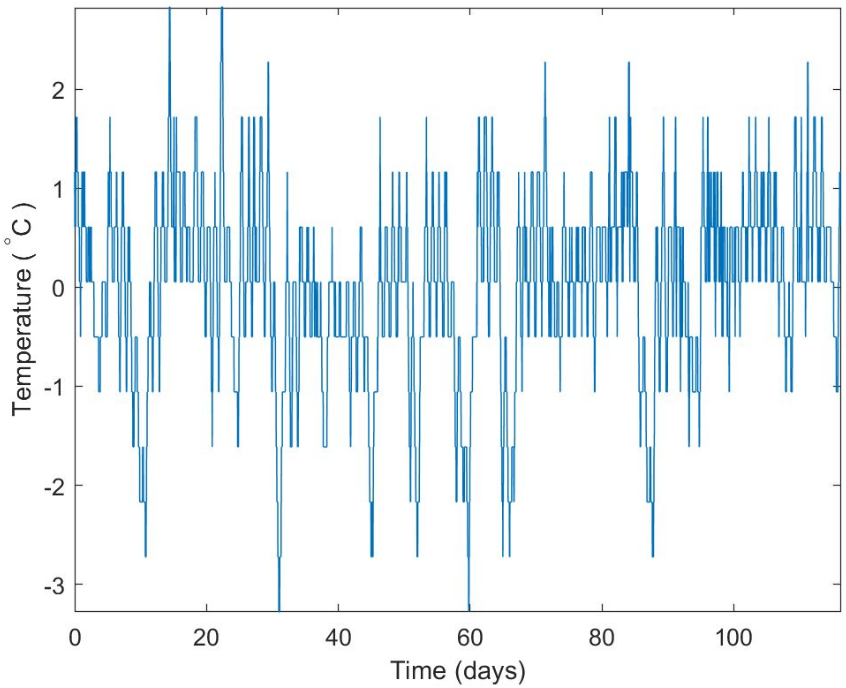
Find Periodicity Using Autocorrelation

Autocorrelation can help verify the presence of periodical cycles and determine their durations.

Example:

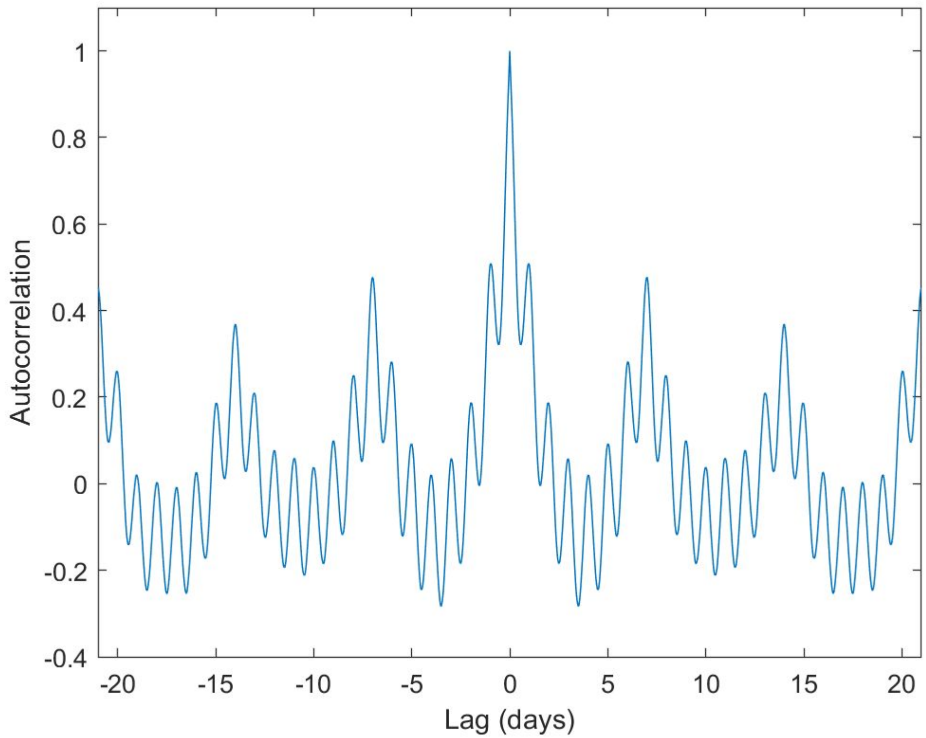
Consider a set of temperature data collected by a thermometer inside ar

The temperature does seem to oscillate, but the lengths of the cycles cannot be read out easily.



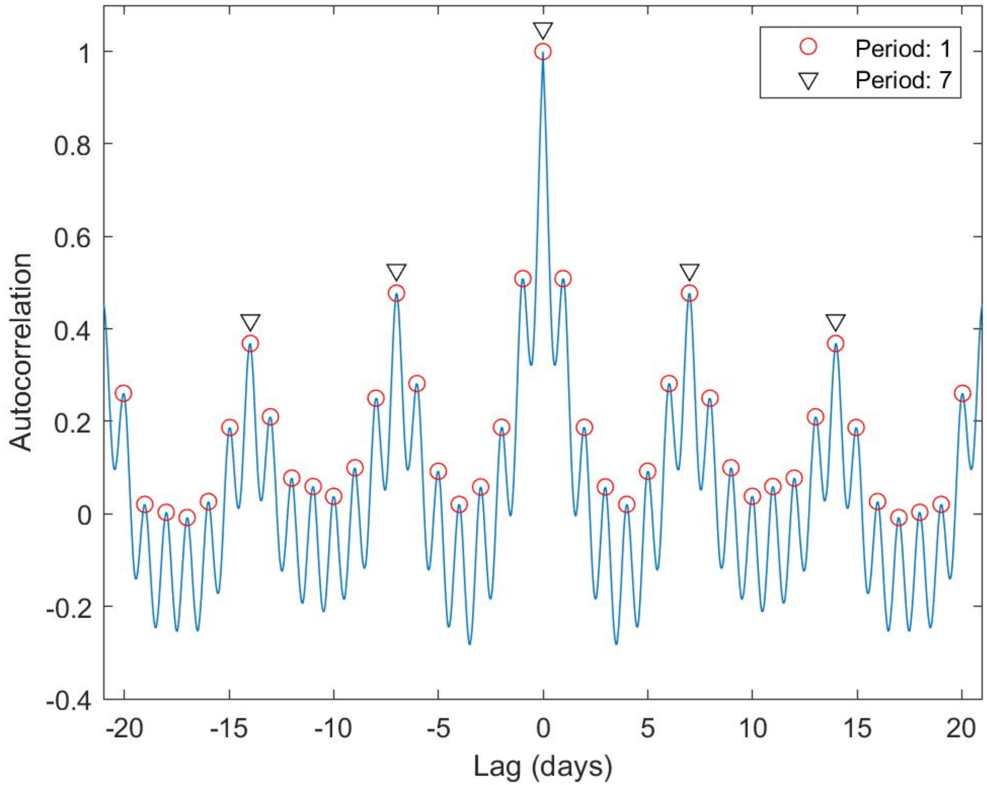
Find Periodicity Using Autocorrelation

Compute the autocorrelation of the temperature such that it is unity at zero lag. Restrict the positive and negative lags to three weeks. Note the double periodicity of the signal.

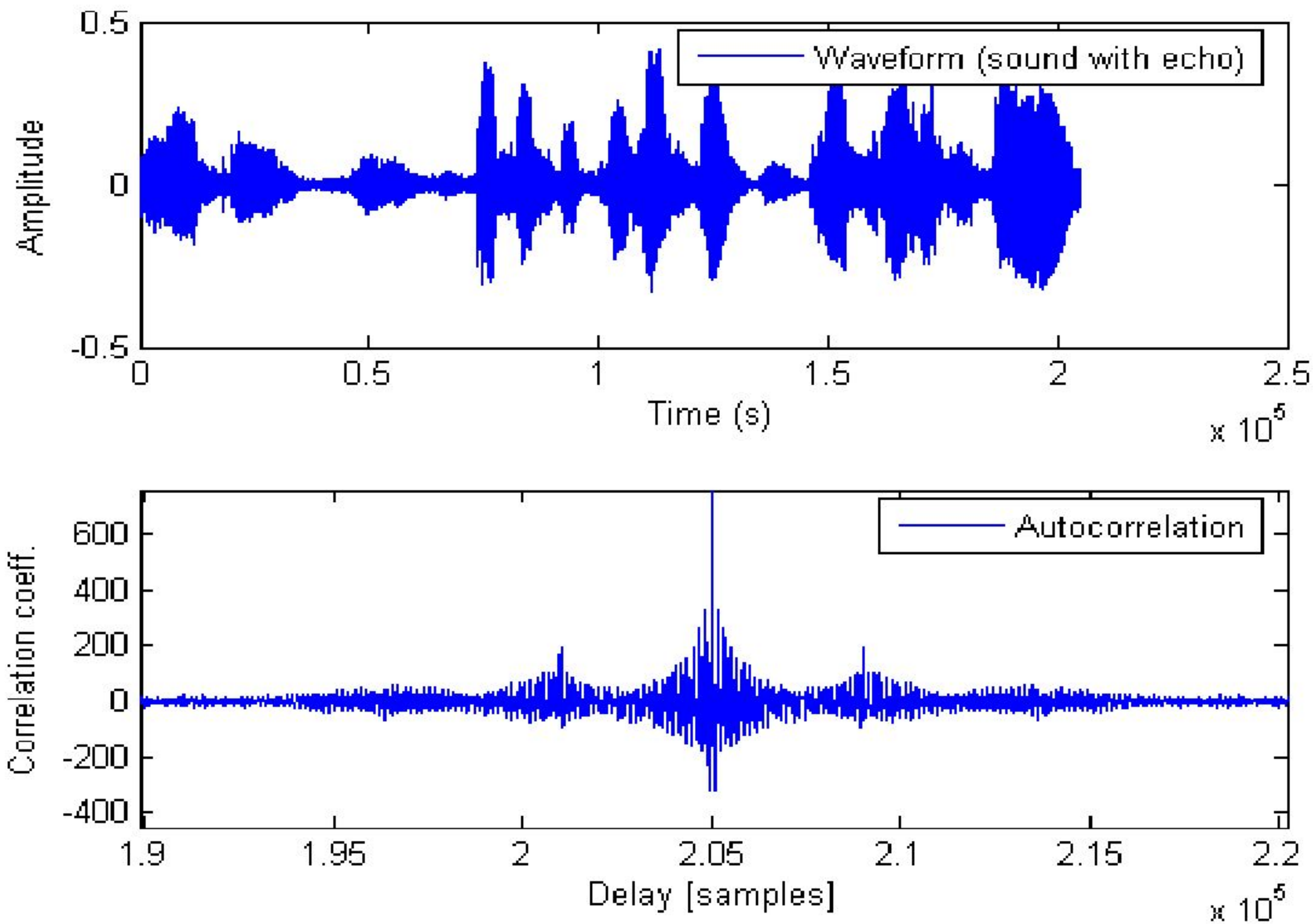


Example: Find Periodicity Using Autocorrelation

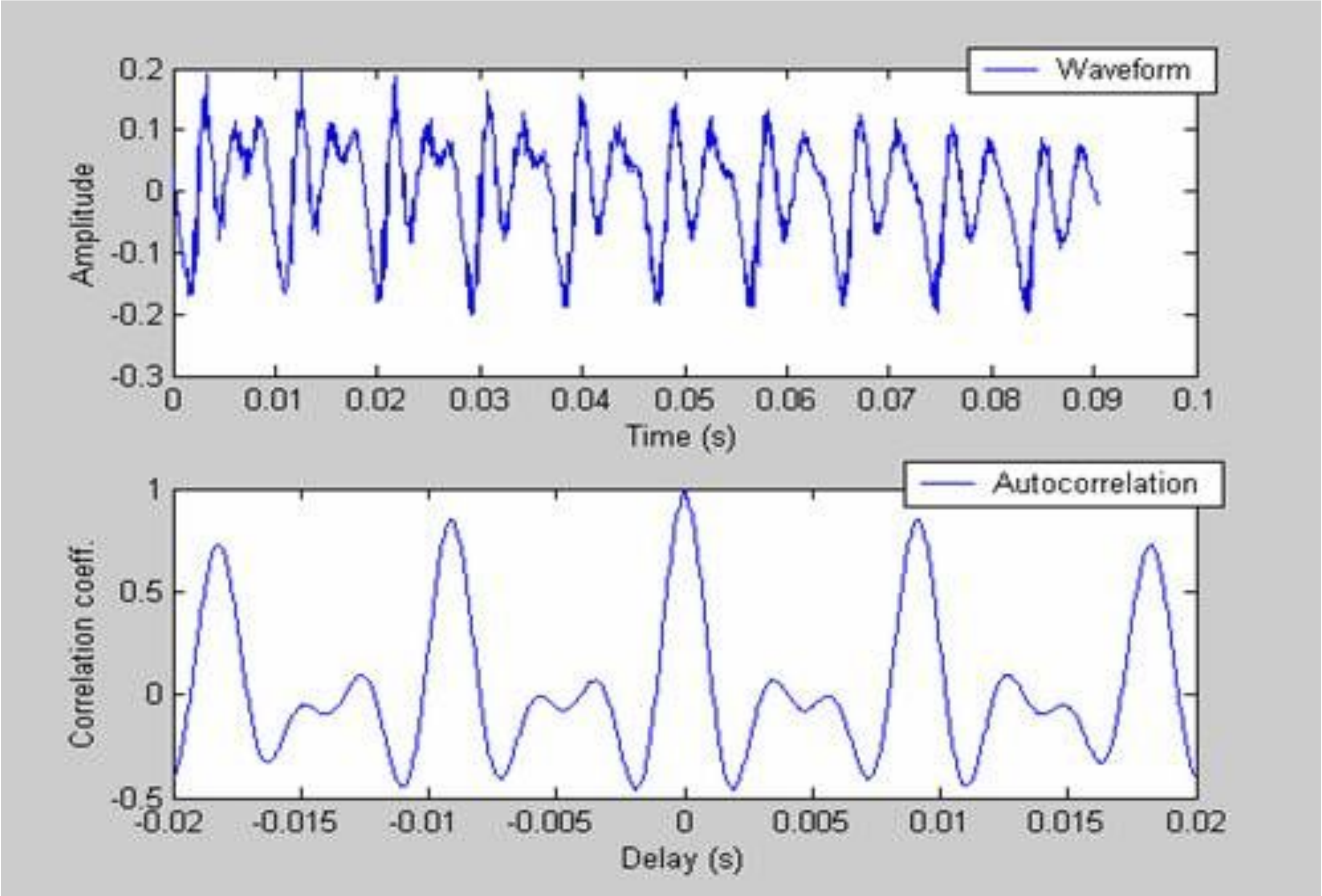
the autocorrelation oscillates both daily and weekly.



Discrete autocorrelation
Example: sound with echo

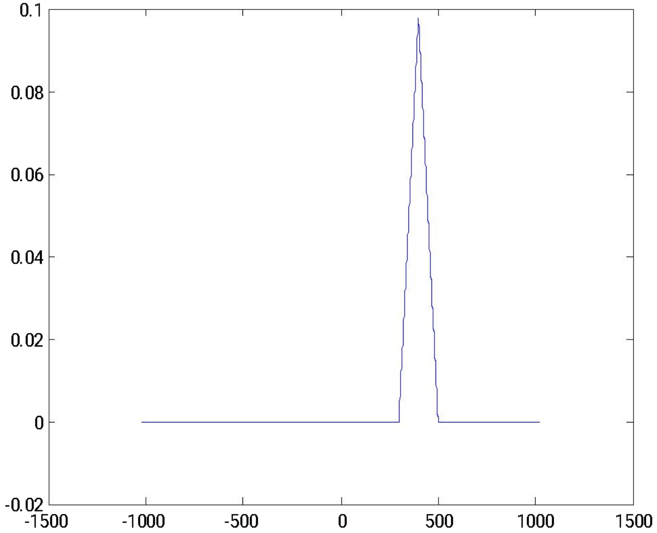


Fundamental Frequency - Pitch period or the first harmonics



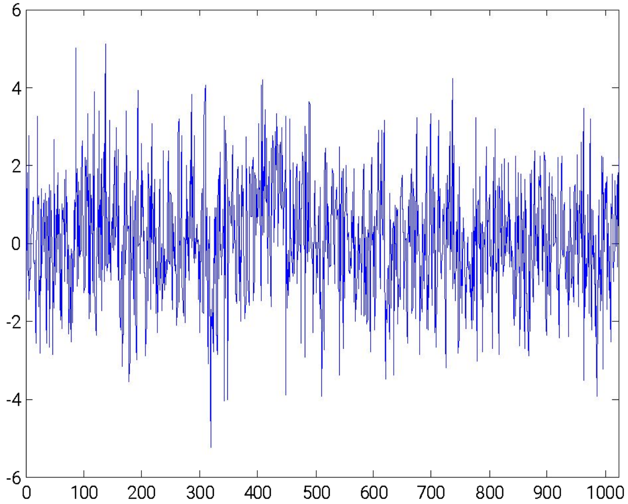
Example of a application: Sonar and Radar Ranging

The cross-correlation of the transmitted and received signals shows they are correlated with a 400 sample delay



Example of a application: Sonar and Radar Ranging

The noisy sonar signal



Cross-correlation of the transmitted signal with the noisy echo clearly shows a correlation at a delay of 400 samples