BoogiepopT Core

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1 Introduction

This report is a mathematical reference and documentation of details of the functionality from BoogiepopT Core. BoogiepopT Core is a C# library targeting .NET Standard 2.0 containing structure and algorithms for media recommendation. This report assumes all necessary mathematical knowledge as it serves as a reference. A report explaining the functionality of BoogiepopT Core may be available in the future.

Part I

Topological Methods

2 Metric Space on Collections

First we recall the definition of a metric on a set.

Definition 2.0.1 (Metric)**.** A *metric* on a set X is a map

$$d\colon X \times X \to \mathbb{R}_{\geqslant 0},$$

satisfying the following three properties:

M1. $d(x, x) = 0$ for all $x \in X$ and $d(x, y) > 0$ for distinct elements $x, y \in X$.

M2. (Symmetry) $d(x, y) = d(y, x)$, for all $x, y \in X$.

M3. (Triangle inequality) $d(x, z) \leqslant d(x, y) + d(y, z)$, for all $x, y, z \in X$.

A set X together with a metric d is called a *metric space* and denoted (X, d). Often however when a set X is understood to have a metric, then the set X itself will be called a metric space. Furthermore the number $d(x, y)$ is called the *distance* between x and y.

The concept of a metric on a set is that of a well behaved distance function on that set. However what this distance means and what it may be useful for depends on the specific definition of the metric, this includes the set on which the metric is defined.

In BoogiepopT Core we've been able to define a metric that turns any collection of objects into a metric space, given that it is possible to calculate or extract some extra information from these objects.

2.1 Definition of the Metric

First we will provide a formal definition of the metric and prove that it is indeed a metric. Afterwards a concrete example will be provided in which this metric is made specific and used in a specific context. In here some of the main ideas of the metric are also mentioned and discussed. In short, as a preparation to the reader, the beginning will be very abstract, whereas the latter half will be more concrete and less formal.

As mentioned we start with the formal definition.

Definition 2.1.1. Let $\mathcal{B}$ be a set, i.e. a collection of objects. Let

$$s\colon \mathcal{B} \to V$$

be a map, where $(V, |\cdot|)$ is a normed vector space. Moreover let

$$c\colon \mathcal{B} \times \mathcal{B} \to \mathbb{R}_{\geqslant 0}$$

be a metric. Finally we define

$$d_{s,c}\colon \mathcal{B} \times \mathcal{B} \to \mathbb{R}_{\geqslant 0}$$

by

$$d_{s,c}(A, B) \coloneqq |s(A) - s(B)| + c(A, B).$$

The map $d_{s,c}$ is the metric that we wish to obtain.

Theorem 2.1.2. *Let $(V, |\cdot|)$ be a normed vector space, $s\colon \mathcal{B} \to V$ a map and $c\colon \mathcal{B} \times \mathcal{B} \to \mathbb{R}_{\geqslant 0}$ a metric. The map $d_{s,c}$ is a metric.*

Proof. Follows directly from the properties of the norm $|\cdot|$ and the metric c. □

The significance of $d_{s,c}$ lies in the specification of s and c. An example is due to see the strength of the metric and to get an idea of how to apply it.

Example 2.1.3 (Metric on Tags)**.** Suppose that we have a collection $\mathcal{F}$ of our favourite media. Let $\mathcal{B} = \text{Tags}(\mathcal{F})$ be the set of all tags that a media in $\mathcal{F}$ could have. In order to use the metric defined in definition 2.1.1 we need to concretely specify s and c. One way of thinking of s is by seeing it as a function that computes an associated quantity or extracts a statistic from an object in $\mathcal{B}$, in this case a tag. And one way of interpreting c is that it defines an inherent distance between the objects one wishes to compare.

We will define $s(t)$ as the amount of media in $\mathcal{F}$ that have the tag t. Note that $s\colon \text{Tags}(\mathcal{F}) \to \mathbb{R}$, where $\mathbb{R}$ is indeed a normed vector space with the absolute value function as norm. We define $c\colon \text{Tags}(\mathcal{F}) \times \text{Tags}(\mathcal{F}) \to \mathbb{R}$ as follows:

$$c(t, g) = \begin{cases} 0 & , t = g \\ 2 & , \text{otherwise} \end{cases}.$$

To show that c is a metric is a minor exercise.

With s and c defined, the behaviour of our metric $d_{s,c}$ is determined. The metric behaves in a way such that the difference is small when the tags have a similar amount of media occurrences in $\mathcal{F}$, whereas the difference is big when the tags have a disparate amount of media occurrences.

Note that because of c the smallest distance two distinct tags can have is two. This value has been a rather arbitrary choice, it could be any constant greater than 0.

As illustrated by example 2.1.3, the meaning of $d_{s,c}$ is determined by the set (or collection) of objects, the map s and the metric c.

3 Recommendation Method: Metric Lift

Metric Lift is a flexible recommendation method with various nuances and room for variations or generalisations. This makes it a very exciting recommendation method and provides a great starting point in developing more recommendation methods. We will however focus mainly on the definition of Metric Lift in BoogiepopT Core.

3.1 Definition of Metric Lift

Let $\mathcal{G}$ be a set of objects that satisfies the following property:

O1. For all $X \in \mathcal{G}$ there exists a set $\mathcal{B}(X) \subset \mathcal{B}$, where $\mathcal{B}$ is a metric space.

Now let $(A, B) \in \mathcal{G} \times \mathcal{G}$ be a selected element from the cartesian product of $\mathcal{G}$ with itself.

Remark. The set $\mathcal{G}$ is the set from which we'd want a recommendation from. The idea behind achieving this recommendation is to mimic a recommendation pattern. This recommendation pattern is defined by $(A, B) \in \mathcal{G} \times \mathcal{G}$, which one can think of as a recommendation from A to B. Mimicking the recommendation pattern is done by considering a set of associated objects to A and a set of associated objects to B, where the elements in these sets are from the same metric space. These are respectively the sets $\mathcal{B}(A)$, and $\mathcal{B}(B)$. The recommendation pattern then gets encoded as a vector of distances, the distances being calculated for all elements in the set $\mathcal{B}(A) \times \mathcal{B}(B)$. We will now work out these latter details.

Consider the vector space $\mathbb{R}^k$, with $k = |\mathcal{B}(A) \times \mathcal{B}(B)|$, where each dimension of $\mathbb{R}^k$ is indexed by an element of $\mathcal{B}(A) \times \mathcal{B}(B)$. Write $\pi_{(a,b)} \colon \mathbb{R}^k \to \mathbb{R}$ for the projection function that projects onto the component indexed by $(a, b) \in \mathcal{B}(A) \times \mathcal{B}(B)$.

Now define the preference vector $\mathrm{prf} \in \mathbb{R}^k$ by

$$\pi_{(a,b)}(\mathrm{prf}) = d(a, b),$$

where d is the metric on $\mathcal{B}$ and $(a, b) \in \mathcal{B}(A) \times \mathcal{B}(B)$.

In general for $X, Y \in \mathcal{G}$ define $v(X, Y) \in \mathbb{R}^k$ as

$$\pi_{x,y} \circ v(X, Y) = \begin{cases} d(x, y) & \text{, if } (x, y) \in \mathcal{B}(A) \times \mathcal{B}(B) \cap \mathcal{B}(X) \times \mathcal{B}(Y) \\ 0 & \text{, otherwise} \end{cases}.$$

Remark. The function v is what encodes any arbitrary pair of elements from $\mathcal{G}$, one could say any arbitrary recommendation pattern from $\mathcal{G}$, to a vector of distances in which only the associated objects pairing appearing in $\mathcal{B}(A) \times \mathcal{B}(B)$ are taken into account. Since the encoding is done by vectors in $\mathbb{R}^k$ the most obvious way to compare them is by using the inner product.

For any $C, D \in \mathcal{G}$ one then compares (C, D) to (A, B) by

$$\Delta = \|\mathrm{prf}\|^2 - \langle \mathrm{prf}, v(C, D) \rangle, \tag{3.1}$$

where $\|\cdot\|$ is the euclidean norm and $\langle \cdot, \cdot \rangle$ is the standard inner product on $\mathbb{R}^k$. There is no need for an absolute value in 3.1, since for any $X, Y \in \mathcal{G}$ it holds that $\langle \mathrm{prf}, v(X, Y) \rangle \leqslant \|\mathrm{prf}\|^2$. Concisely this follows from the definition of $v(X, Y)$ for $X, Y \in \mathcal{G}$, as $\pi_{x,y} \circ v(X, Y)$ is either something that appears in prf or it's 0.

The behaviour of Metric Lift is largely determined by the choice of associated objects to the objects in $\mathcal{G}$, and the metric that one has on the set of all such associated objects. What follows is a quick example of an application of Metric Lift.

Example 3.1.1 (Metric Lift From Metric on Tags)**.** Let $\mathcal{G}$ be $\mathcal{F}$ from in example 2.1.3. Then for $X \in \mathcal{F}$ let $\mathcal{B}(X)$ be the set of all the tags of X. Indeed we have for any $X \in \mathcal{F}$ that $\mathcal{B}(X) \subset \mathcal{B} = \mathrm{Tags}(\mathcal{F})$. Moreover by example 2.1.3 we have the metric $d_{s,c}$ on $\mathrm{Tags}(\mathcal{F})$ which makes it into a metric space. Here we will apply Metric Lift by selecting two media A, B from $\mathcal{F}$ and defining prf using (A, B). Now for every $X \in \mathcal{F} - \{A, B\}$ we calculate Δ using (B, X). Then pick the media D such that (B, D) has the lowest Δ.

Bibliography