# CS-412 Research Proposal

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## I. Context

All integer programming problems can be written in the following form.

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \quad \leq \quad \mathbf{b} \end{aligned}$$

where $A \in M_{m\times n}(\mathbb{R})$, $\mathbf{b} \in \mathbb{R}^m$, $\mathbf{c} \in \mathbb{R}^n$, and $\mathbf{x} \in \mathbb{N}_0^n$ where $m, n \in \mathbb{N}_0$. Notice by only modifying the domain of the decision variable to $\mathbf{x} \in \mathbb{F}_{\geq 0}^n$, we now have the general form for every linear programming problem. Furthermore, modifying the domain of the decision variable to $\mathbf{x} \in \mathbb{B}_2^n$ yields the general form for every binary knapsack problem. Given this, I have sufficient reason to believe that every integer programming problem can be reduced to a system of a linear programming problem and a binary knapsack problem.

## II. Linear-Binary Reduction

Let $\mathbf{x}^* \in \mathbb{R}^n$ be an optimal solution to the following linear programming problem.

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \quad \leq \quad \mathbf{b} \end{aligned}$$

Notice $\mathbf{x}^* = \mathbf{x}_q + \mathbf{x}_r$ where $\mathbf{x}_q \in \mathbb{N}_0^n$ and $\mathbf{x}_r \in [0, 1)^n$. Consequently, $\mathbf{x}_q$ is a feasible solution to the corresponding integer programming problem. We now construct the following binary knapsack problem.

$$\begin{aligned} \text{maximize} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \quad \leq \quad \mathbf{b} - A\mathbf{x}_q \end{aligned}$$

Notice $\mathbf{x}_r$ is an optimal solution to the above problem's linear counterpart. Let $\mathbf{x}_b$ be an optimal solution to this binary knapsack problem. Equivalently, $\mathbf{x}_b$ is an optimal solution the above's integer programming counterpart. Consequently, $\mathbf{x}_q + \mathbf{x}_b$ is a feasible solution the integer programming counterpart to our original linear programming problem. We will call this process the linear-binary reduction.

## III. Goals

It is yet to be proven that the solution, $\mathbf{x}_q+\mathbf{x}_b$ acquired from linear-binary reduction is an optimal one. As such, to do so is a goal of this research project. Furthermore, it could stand to reason that $\mathbf{x}_r$ could be used to more efficiently solve for $\mathbf{x}_b$ in many, if not all, cases. To find such a method is also a goal of this research project. Finally, given the second goal can be met, we wish to find a non-trivial set of integer programming problems such that $\mathbf{x}_r$ can be used to solve $\mathbf{x}_b$ in polynomial time.

## IV. Motivation

If the above goals are met, then the result will be a process that can be used to efficiently solve integer programming problems. Furthermore, it will show that a non-trivial subset of integer programming problems (which are NP-hard) are in P. It is important to note this does not show P = NP as it does not reduce all integer programming problems to P nor could it because this process is dependent on binary knapsacks (which are also NP-hard). It would; however, also show that a non-trivial subset of binary knapsack problems are reducible to P. I have sufficient motivation to believe that there also exists a non-trivial subset of integer programming problems for which this process does not reduce to $P$. By the same reasoning, this does not show P $\neq$ NP. The end goal efficiently solves integer programming problems, and in some non-trivial cases, solves them in polynomial time, which I hope will grant further insight into the P-NP problem and motivate further research into its solution or lack thereof.

## V. IDEAS

What does the Fourier Transform of such problems looks like? What does the row-wise convolution of the primal, $A\mathbf{x} = \mathbf{b}$? What about the column-wise convolution? What about the same operations on the dual, $A^T\mathbf{y} = \mathbf{c}$.

## VI. NOTATION

The following are clarifications regarding notation.

$$
\begin{aligned}
\mathbb{N}_0 &:= \text{the natural numbers including } 0 \\
\mathbb{B}_n &:= \mathbb{N}_0 \cap [0, n) \text{ where } n \in \mathbb{N} \\
\mathbb{F} &:= \text{a field} \\
M_{m\times n}(\mathbb{F}) &:= \text{the space of } m \text{ by } n \text{ matrices over } \mathbb{F} \\
S[x] &:= \text{the set of polynomials with coefficients in } S \\
S[x; T] &:= S[x] \text{ with exponents in } T \\
S[x; n] &:= S[x; \mathbb{B}_{n+1}] \text{ where } n \in \mathbb{N}_0
\end{aligned}
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