

# Normal Conversion of Lagrangian to Hamiltonian

$$L = L(q, \dot{q}, t)$$

let the conjugate momentum  $p$  be given by

$$p = \frac{dL}{d\dot{q}}$$

The Hamiltonian can be computed via

$$H = p\dot{q} - L$$

## Generalization

Let the Lagrangian be given by

$$L = L(q, \dot{q}, \ddot{q}, \dots, q^{(n)}, t)$$

Let  $q_1 = q, q_2 = \dot{q}, q_3 = \ddot{q}$ , up to  $q_n$ . The conjugate momenta are given by

$$p_i = \sum_{j=i}^n \left(-\frac{d}{dt}\right)^{j-i} \frac{\partial L}{\partial q^{(j)}}$$

The Hamiltonian is then given by

$$H = \sum_{i=1}^n p_i \dot{q}_i - L$$

and the Hamilton Equations are given by

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

and

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

## Multiple Dependent Variables

If your Lagrangian has multiple dependent variables (like  $x$  and  $y$ ), simply add a summation term for each dependent variable

Label dependent variables  $q^1, q^2, q^3, \dots, q^m$

Hamiltonian is given by

$$H=\sum_{j=1}^m\sum_{i=1}^np_i^jq_i^j-L$$