PHYS 4900 Interim

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Introduction

In classical mechanics, it is often prudent to solve problems using the Hamiltonian formulation of classical mechanics. In the Hamiltonian formulation of classical mechanics, the most important quantity to calculate is the Hamiltonian. The Hamiltonian of a system is calculated in terms of the Lagrangian of that same system. Traditionally, the Hamiltonian is calculated under the assumption that the Lagrangian of the system is a function of only some generalized coordinates q_i, their first derivatives with respect to time $\dot{q}_i$, and time t. In 1850, Mikhail Ostrogradsky developed a method for constructing the Hamiltonian of a system whose Lagrangian depends on higher derivatives of q_i[1]. In 2015, R.P. Woodard wrote a paper, "The Theorem of Ostrogradsky", which explains the method of Ostrogradsky for constructing Hamiltonians [2].

Proposal

The goal of my research this semester was to produce Maple code which implemented the algorithm explained by Woodard in "The Theorem of Ostrogradsky" in order to calculate a Hamiltonian given a Lagrangian and the highest order time derivatives present in the Lagrangian. Another major goal was to document the code so that it can be used effectively by physicists later on. As a starting point for my research, Matthew Pontius had already produced Maple code which had the ability to construct a Hamiltonian for a system whose Lagrangian depends on 2 time derivatives of one dependent variable q. My first goal was to extend Matthew's code to compute Hamiltonians for systems whose Lagrangians depend on some arbitrary, specifiable integer N time derivatives of a single dependent variable q. Then, I aimed to make the code compute Hamiltonians for Lagrangians which contained an arbitrary, specifiable amount of dependent variables $q_1, q_2, \dots, q_k$, with the highest order time derivative of each dependent variable present in the Lagrangian being N. After that, I aimed to modify the code to compute Hamiltonians for a Lagrangian which may depend on any arbitrary, specifiable number of dependent variables, and an arbitrary, specifiable highest order time derivative for that dependent variable, i.e. $L = L(q_1, \frac{dq_1}{dt}, ..., \frac{d^{N_1}q_1}{dt^{N_1}}, ..., q_k, \frac{dq_k}{dt}, ..., \frac{d^{N_k}q_k}{dt^{N_k}})$. Finally, I aim to add error-handling to make the code more user-friendly, and write a Maple help-file which will document the code and make it more usable for physicists going forward.

Activity

Through the course of the semester, I met up with Dr. Torre weekly to gain insight on how he wanted the code to operate, help me to work out errors in my code, and figure out what needed to happen next. First, I began by familiarizing myself with the algorithms in Woodard. Once I felt comfortable with what the general algorithm should be for computing a Hamiltonian for a system whose Lagrangian depends on N time derivatives of the dependent variable, I began implementing the algorithm in Maple. Then, I generalized the algorithm in Woodard to an arbitrary, specifiable number of dependent variables, and implemented that into my algorithm, with the assumption that the maximum order of time derivative with respect to each dependent variable was the same. Then I eliminated that simplifying assumption by changing one of the procedure parameters to be a list rather than a scalar so that each dependent variable could have its own highest order derivative present in the Lagrangian. I tested my code by selecting a set of simple Lagrangians such that the method of Ostrogradsky could easily be applied by hand to find the Hamiltonian, but that also covered all the possibilities of user input. Matthew Pontius, Dr. Torre, and I worked together to debug some of the errors in the algorithm. Then, we thought it prudent to write another function which, given a Hamiltonian and the canonical pairs present in the Hamiltonian, outputs Hamilton's equations so that the motion of the system can be described. It was important that the Hamilton's equations output by this procedure were in a format compatible with Maple's `dsolve` command. After implementing this, I began on the Maple help-files for both procedures to document them for future use.

Results

The outcome of this semester's research has been

1. `Hamiltonian(L,frame,orders)`: A Maple procedure which, given a Lagrangian "L", a differential geometry jet frame "frame", and an ordered list of highest order time

derivatives present in the Lagrangian "orders", supplies the user with the corresponding Hamiltonian.

2. `HamiltonEquations(Hamiltonian, canonicalPairs, canonicalNames:=canonicalPairs)`: A Maple procedure which, given a Hamiltonian, a list of 2-tuples "canonicalPairs", and an optional list of 2-tuples "canonicalNames", outputs the Hamilton equations for the system, replacing each occurrence of an element of canonicalPairs with its corresponding element in canonicalNames.
3. Partially done Maple help-files for both `Hamiltonian` and `HamiltonEquations`

Conclusions

I exceeded the research goals this semester by not only supplying the most general `Hamiltonian` algorithm that I hoped for, but also supplying `HamiltonEquations`, a tool which will be very useful for any Hamiltonian mechanics done in Maple.

Logical next steps for this research are to firstly, finish writing robust Maple help-files for both of the Maple procedures that I coded this semester and secondly, to coauthor a research paper by Dr. Torre which will derive the method of Ostrogradsky in a more formal way.

References

[1] M. Ostrogradsky, Mem. Ac. St. Petersbourg VI 4 (1850) 385.

[2] Woodard, R. P., The Theorem of Ostrogradsky, arXiv:1506.02210, 2015