The Ostrogradsky Construction in Maple

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Introduction

In Hamiltonian mechanics, the Hamiltonian of a system is computed in terms of the Lagrangian of that same system. The traditional calculation of the Hamiltonian operates under the assumption that the Lagrangian of the system depends only on first order time derivatives of the generalized coordinate q. In 1850, Mikhail Ostrogradsky developed a method for constructing the Hamiltonian of a system whose Lagrangian depends on higher order time derivatives of q [1]. Later, R. P. Woodard would write a paper titled "The Theorem of Ostrogradsky" [2], which outlines the method Ostrogradsky proposed for constructing such Hamiltonians.

Method

Prior to this semester, Matthew Pontius produced Maple code which can construct a Hamiltonian for a system whose Lagrangian depends on 2 time derivatives of q. The objective of this semester's research is to convert the algorithm outlined by Woodard in "The Theorem of Ostrogradsky" into Maple code to be able to compute Hamiltonians for systems whose Lagrangian depends on some arbitrary specifiable integer N time derivatives of q. If this is accomplished, we will move on to producing code which can construct Hamiltonians whose Lagrangian depends on N time derivatives of multiple dependent variables $q_1, q_2, \ldots, q_M$. Then, the code will have to be adjusted so that for each dependent variable q_i the code computes the Hamiltonian based on N_i time derivatives of q_i. Following this adjustment, the code will be made bulletproof by adding error handling.

In order to accomplish these goals, I must first gain familiarity with the programming features of Maple, such as the syntax of instantiating data structures, procedures, and loops. Then, I will need to gain familiarity with the Differential Geometry package and the features it offers. Finally, I will need to fully

understand the algorithm that Woodard outlines in “The Theorem of Ostrogradsky” so that I can effectively convert it to Maple code.

Timeline

September 4 – Familiarize myself with Woodard’s “The Theorem of Ostrogradsky”

September 11 – Examine Matt’s existing code and figure out what it is doing

September 18 – Begin work on code for N derivatives and M dependent variables

September 25 – Continue work on code

October 2 – Finish code for N derivatives and independent variables

October 9 – Adjust the code to be compatible with specification of order for each dependent variable.

October 16 – Debugging, error handling

October 23 – More error handling

References

[1] M. Ostrogradsky, Mem. Ac. St. Petersbourg VI 4 (1850) 385.

[2] Woodard, R. P., The Theorem of Ostrogradsky, arXiv:1506.02210, 2015