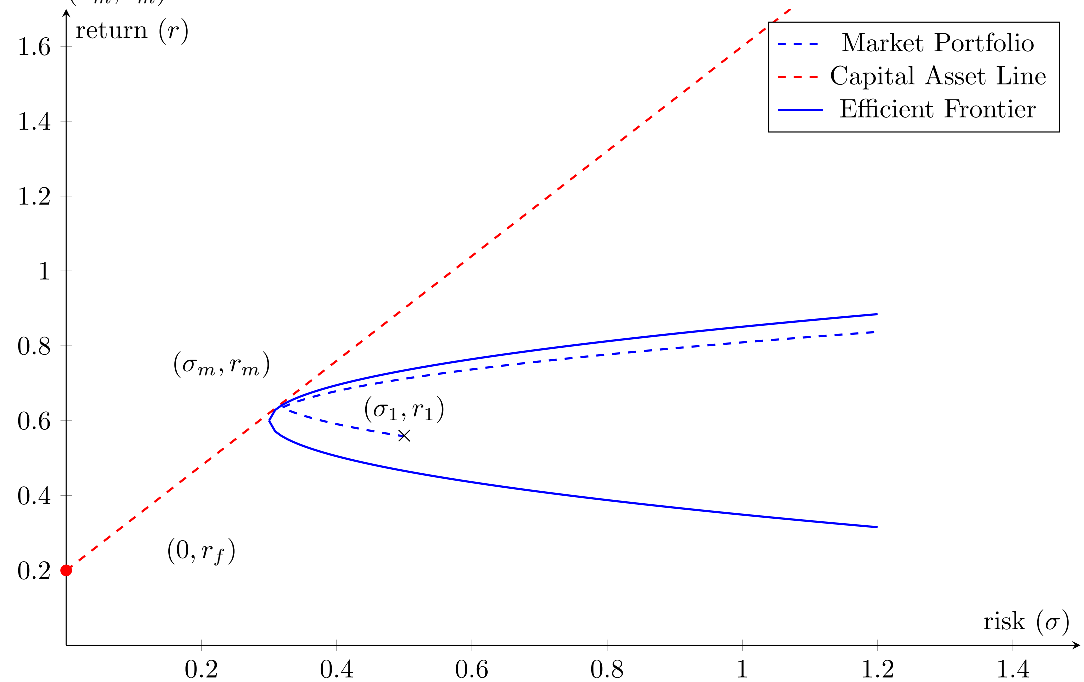
1 The Beta Factor of a Portfolio

Consider a collection of stocks on the minimal variance line. These are the stocks that have the maximum return with the smallest variance. If we plot these stocks on the risk-return axis, they will form the efficient frontier, which is a hyperbola. If, in addition to these points on the efficient frontier, we have a bond with no risk and a risk-free rate of return r_f and connect the bond to the efficient frontier, this line will intersect the efficient frontier at a point that has risk (σ_m, r_m).



The portfolio which contain the market portfolio and the s_1 stock with point (σ_1, r_1) must form a hyperbola which is also tangent to the capital asset line at the point (σ_m, r_m)
The slope of this line can be found: $\lambda S_1 + (1-\lambda)M$
Return: $\lambda r_1 + (1-\lambda) r_m$
Risk: $\sqrt{\lambda^2\sigma_1^2 + (1-\lambda)^2\sigma_m^2 + 2\lambda(1-\lambda)Cov(K_1, K_m)}$
By differentiating and setting lambda = 0 we can find the slope of risk:

$$(f(\lambda))^{\frac{1}{2}} \xrightarrow{\frac{d}{d\lambda}} \frac{1}{2}(f(\lambda))^{-\frac{1}{2}} \cdot f'(\lambda)$$

Slope of Return: $r_1 - r_m$

$$\text{Slope} = \frac{r_1 - r_m}{\frac{1}{2}\left[\sigma_m^2\right]^{-1/2} \cdot (-2\sigma_m^2 + 2C_w(k_1, k_m))} = \frac{r_1 - r_m}{\sigma_m\left[\text{Cov}(k_1, k_m) - \sigma_m^2\right]} \Rightarrow \beta = \frac{\text{Cov}(k_1, k_m)}{\text{Var}(k_m)}$$