AlphaZero Versus *To Knot or Not to Knot*

Using AI to Solve Combinatorial Games

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Mathematics

April 2024

Structure

- Structure.
- What is a Knot?
- What is To Knot or Not to Knot?
- What is AlphaZero?
- AlphaKnot Zero and its performance.
- Implications.
- Future Work.

What is a Knot?

Definition: A simple closed curve in $\mathbb{R}^3$

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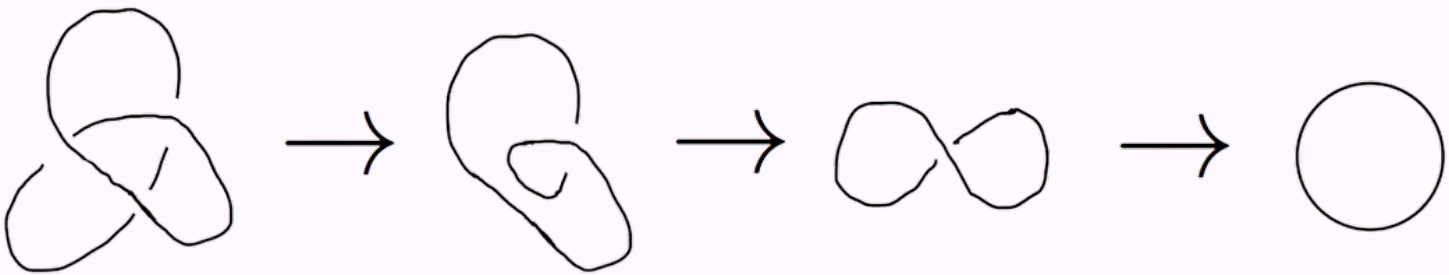
Definition: A *simple* (Doesn't Intersect Itself) *closed* (No Endpoints.) curve in $\mathbb{R}^3$



To Knot or Not to Knot

- SMALL Summer 2009 REU at Williams College
- **Combinatorial** Game
- Played on Pseudo Diagrams/Knot Shadows

To Knot or Not to Knot

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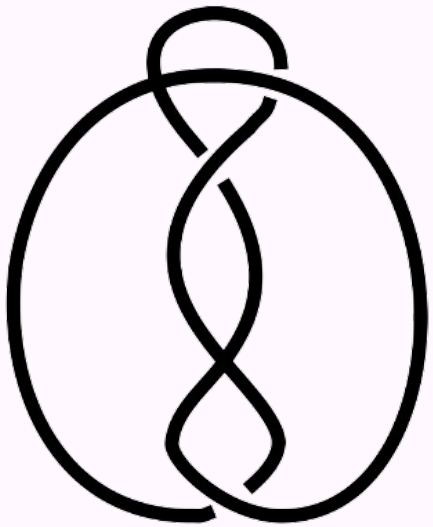
- Two Player Combinatorial Game
- Played on **Pseudo Diagrams**/Knot Shadows



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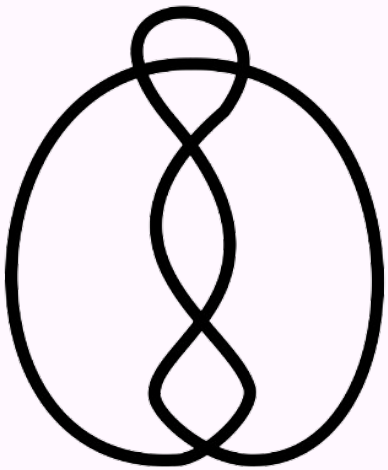
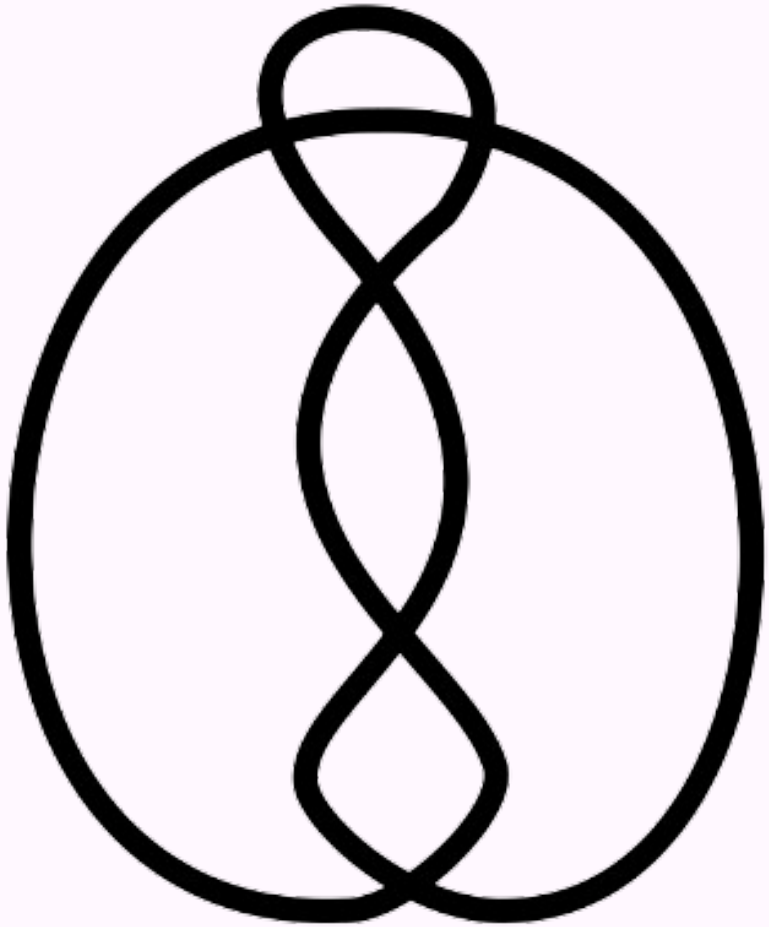
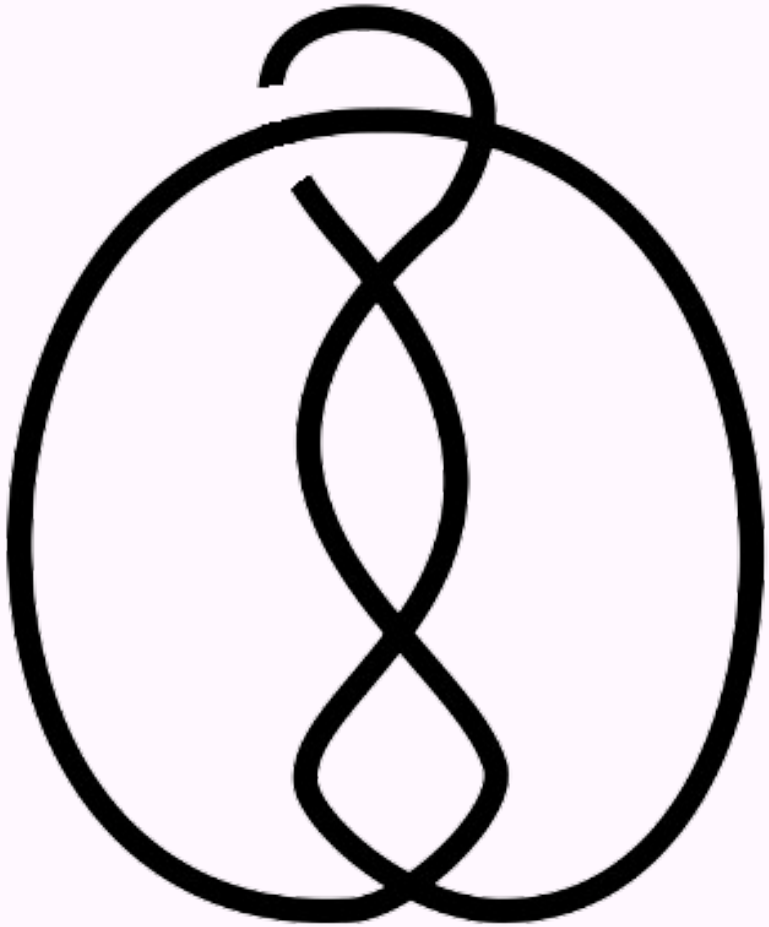


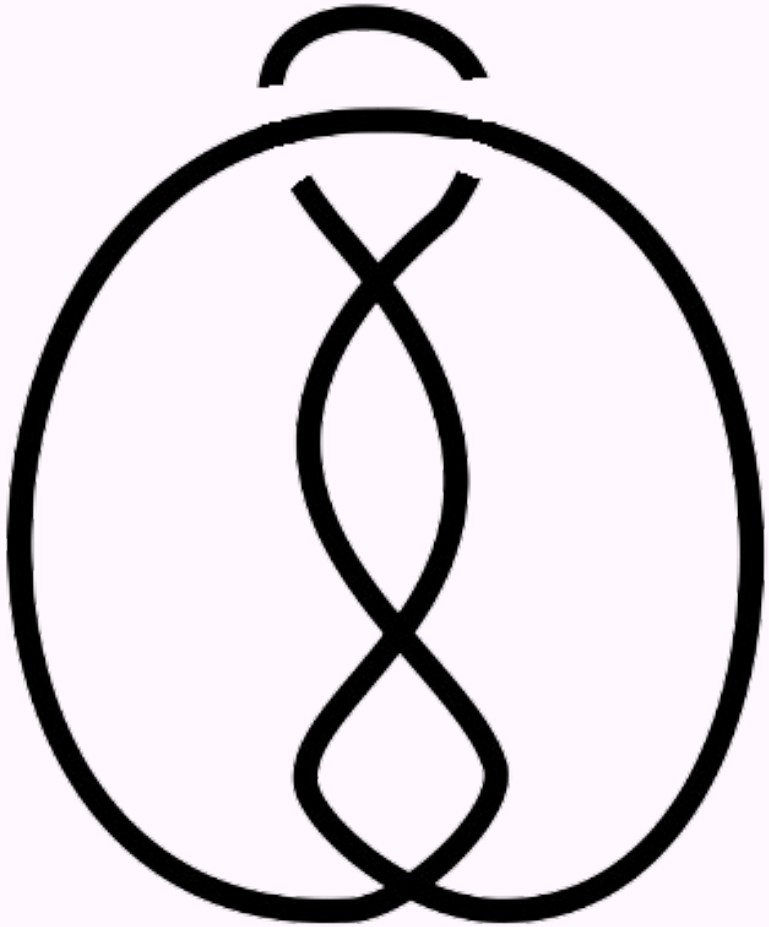
To Knot or Not to Knot - Player 1



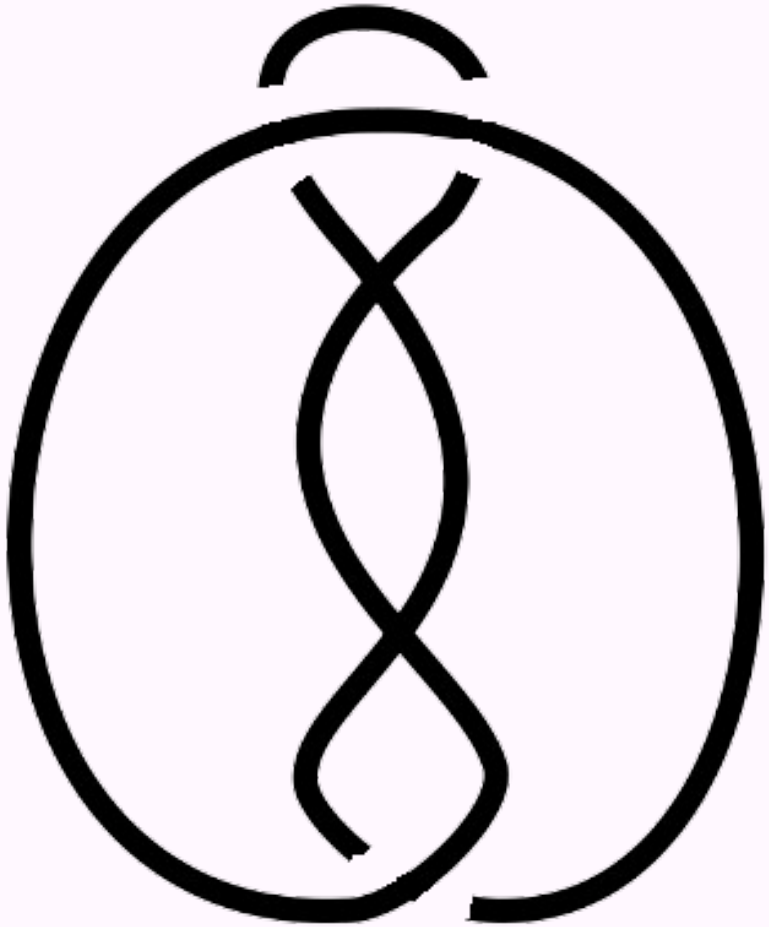
To Knot or Not to Knot - Player 1



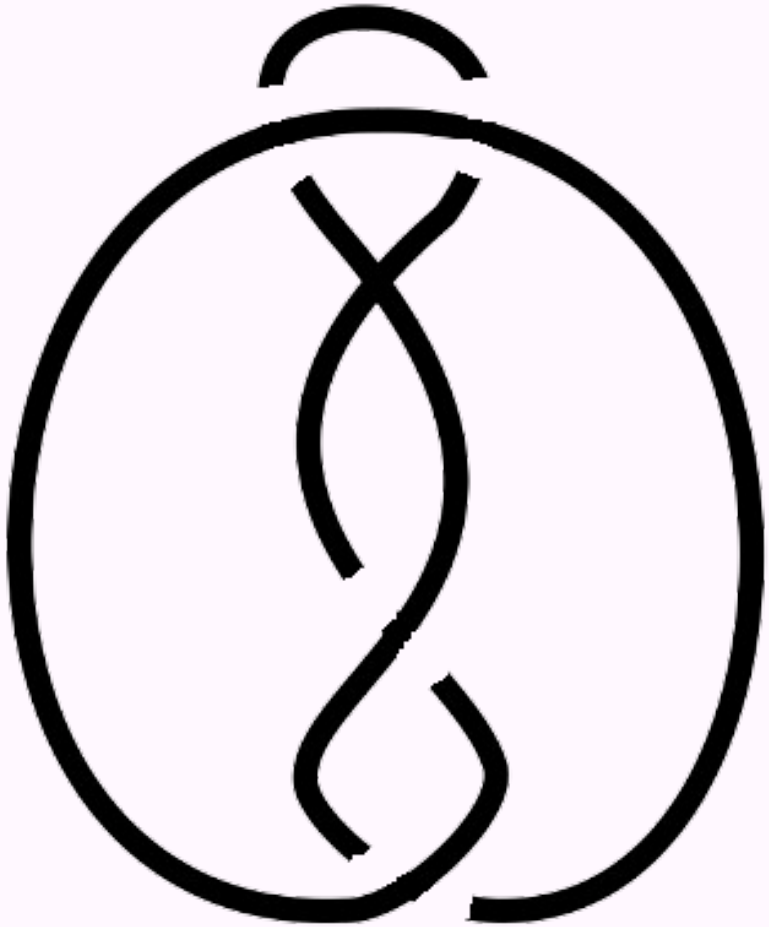
To Knot or Not to Knot - Player 2



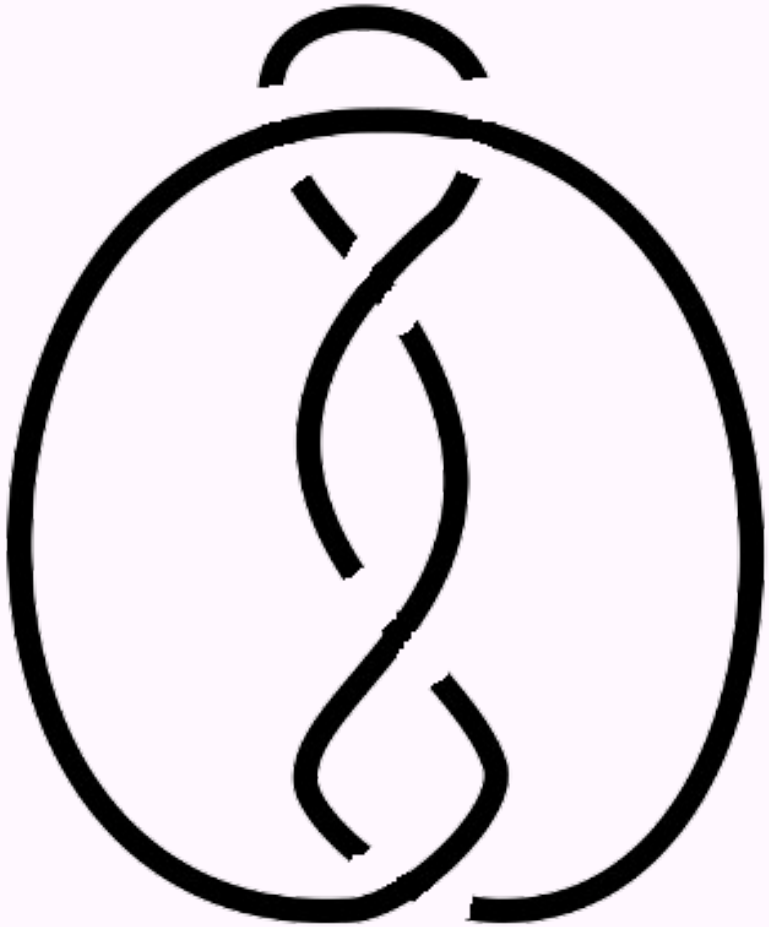
To Knot or Not to Knot - Player 1



To Knot or Not to Knot - Player 2

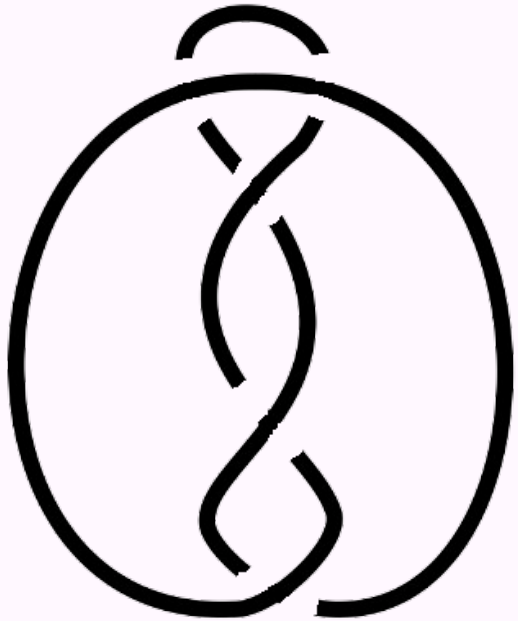


To Knot or Not to Knot - Player 1

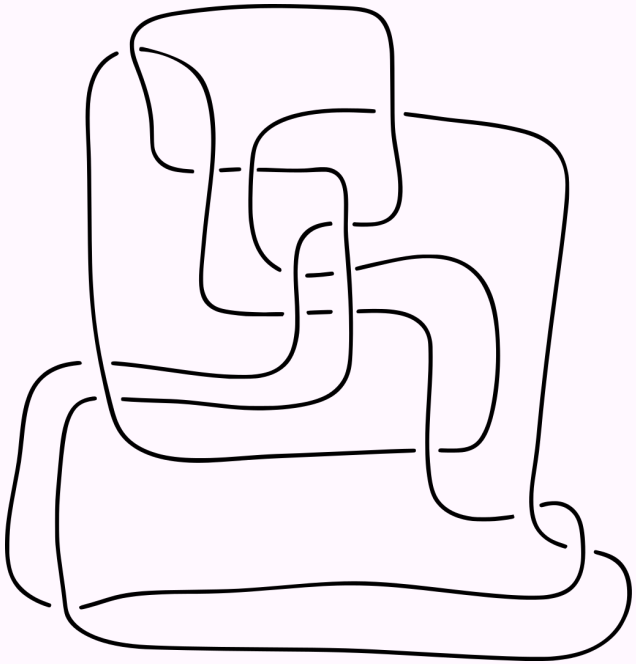


To Knot or Not to Knot

- Player 1 wins if this is the unknot.
- Player 2 wins if this is not the unknot.

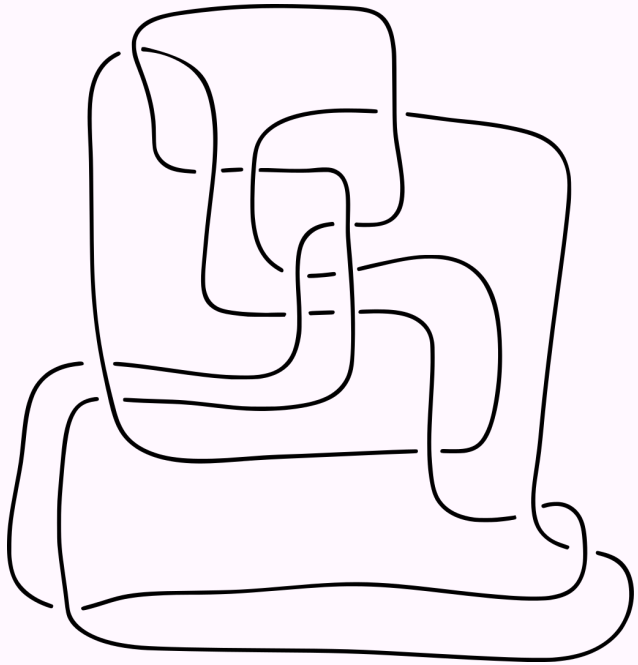


To Knot or Not to Knot - Who wins?



To Knot or Not to Knot - Who wins?

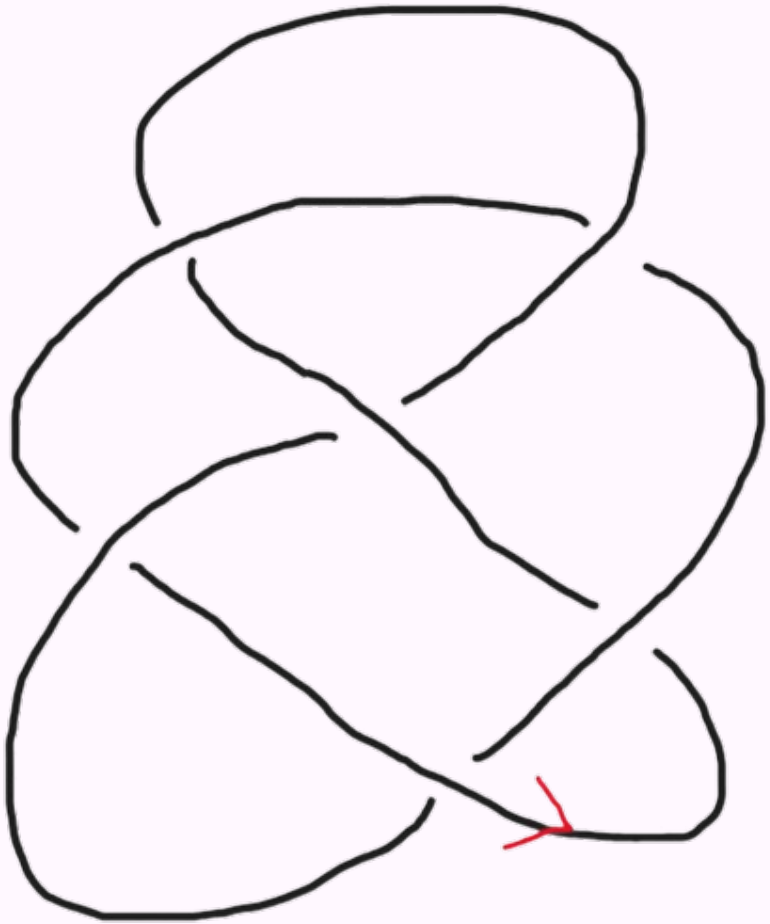
- The Jones Polynomial and the Alexander Polynomial.
- The unknot is the only knot with Alexander and Jones Polynomials equal to 1.



AlphaZero

- AlphaZero is an AI framework that learns entirely through self play, i.e. No training data
- It was first developed by DeepMind to learn how to play Go as AlphaGo Zero
- Its chess rating is around 3500 and it is the best Go player.
- It has been generalized such that it is easily integrated with any combinatorial game.

AlphaKnot Zero *Planar Diagrams*



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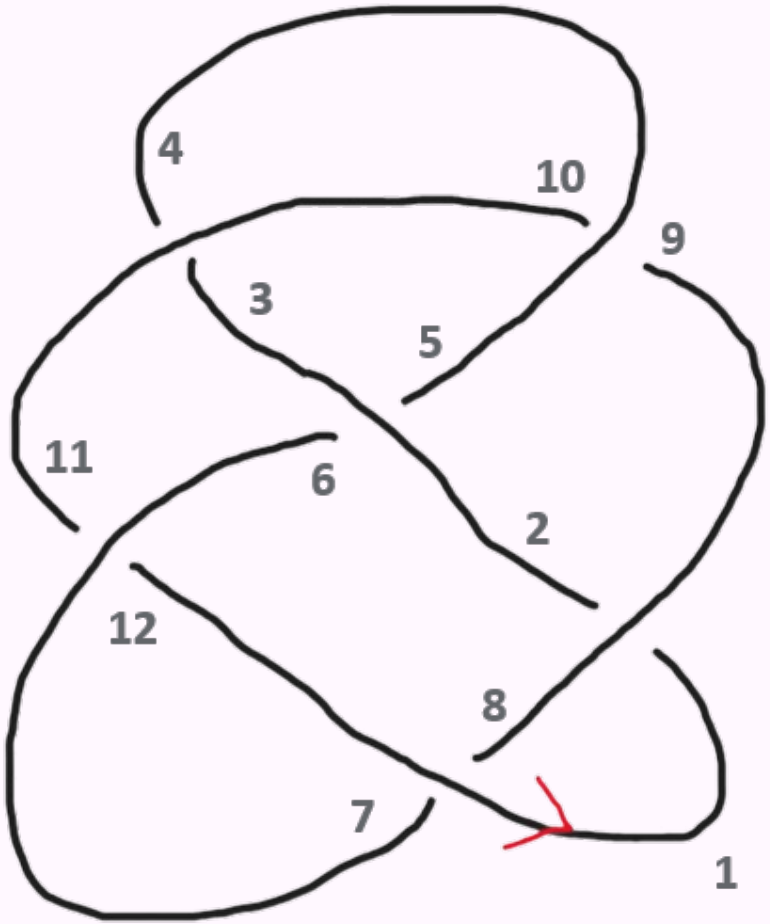
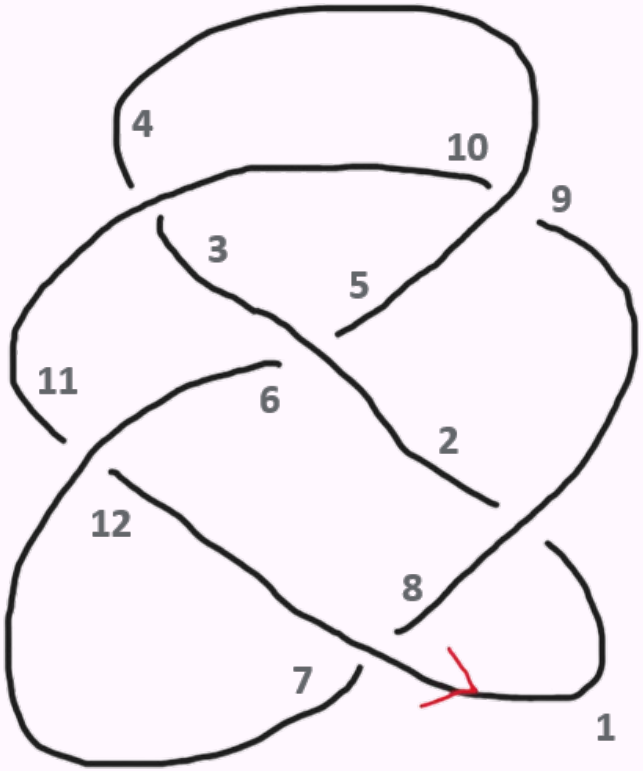


AlphaKnot Zero *Planar Diagrams*

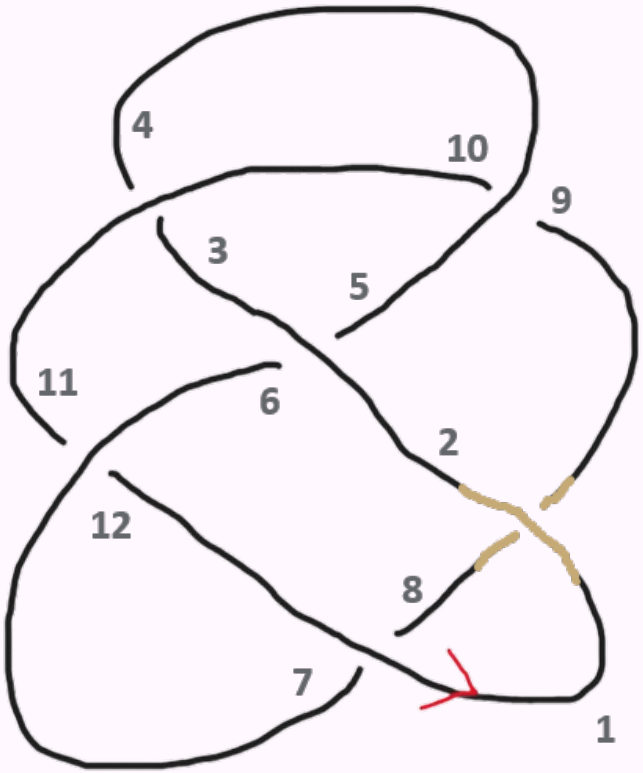
$$[[1, 9, 2, 8], [3, 10, 4, 11], [5, 3, 6, 2], [7, 1, 8, 12], [9, 4, 10, 5], [11, 7, 12, 6]]$$



AlphaKnot Zero *Planar Diagrams*

$$[A, B, C, D] \rightarrow [D, A, B, C]$$

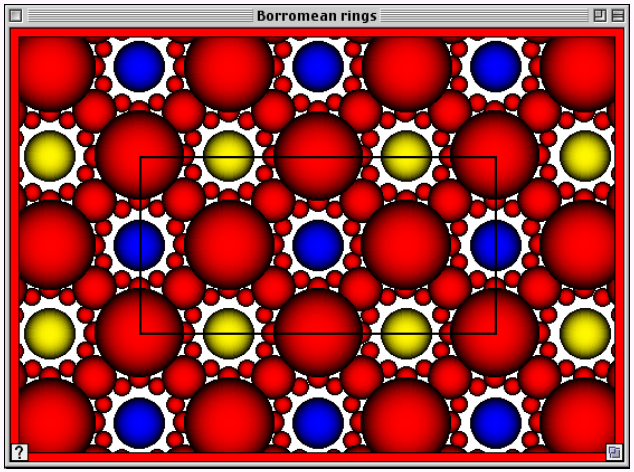
$$[1, 9, 2, 8] \rightarrow [8, 1, 9, 2]$$



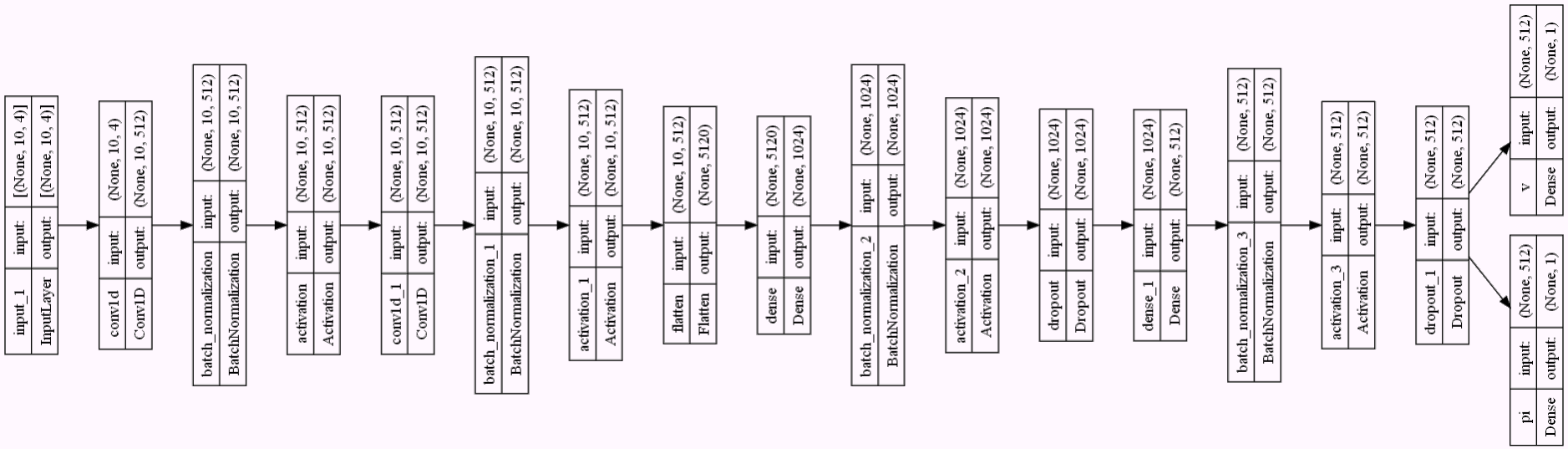
AlphaKnot Zero *Computing Unknot Detection*

- We use SnapPy/SnapPea to compute the Alexander and Jones Polynomials of the final PDcode.

  Colin Adams, Bill Arveson, Pat Callahan, Joe Christy, Dave Gabai, Charlie Gunn, Martin Hildebrand, Craig Hodgson, Diane Hoffoss, A. C. Manoharan, Al Marden, Dick McGehee, Rob Meyerhoff, Lee Mosher, Walter Neumann, Carlo Petronio, Mark Phillips, Alan Reid, Makoto Sakuma, Marc Culler, Nathan Dunfield, and Jeffrey Weeks



AlphaKnot Zero *Model Structure*



AlphaKnot Zero *Planar Diagrams*

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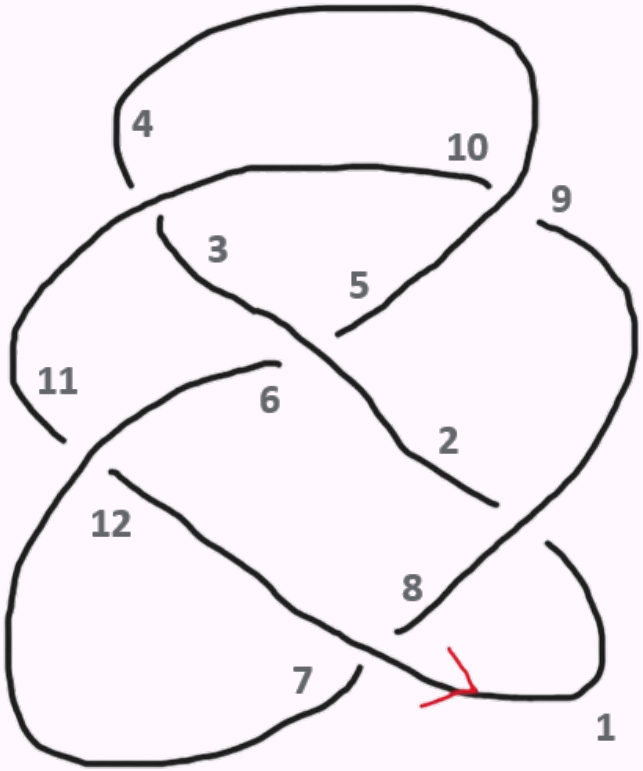
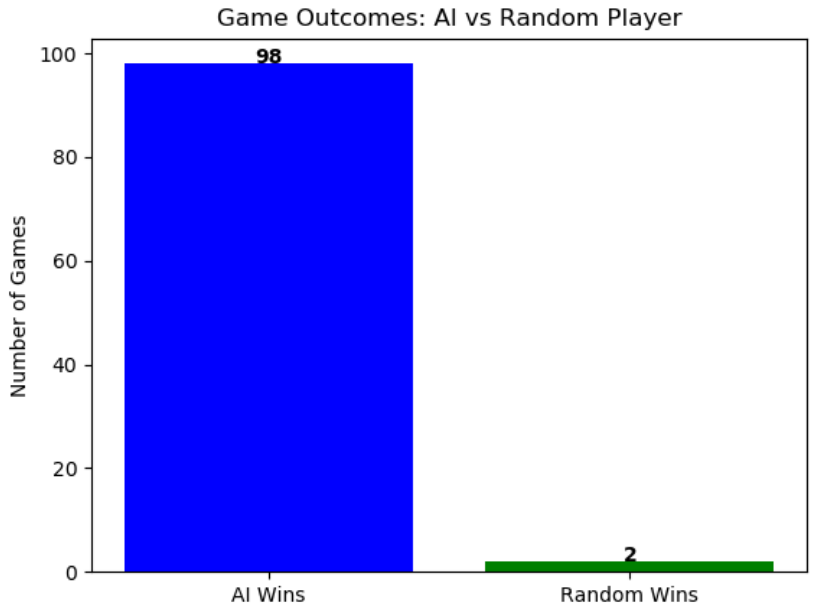
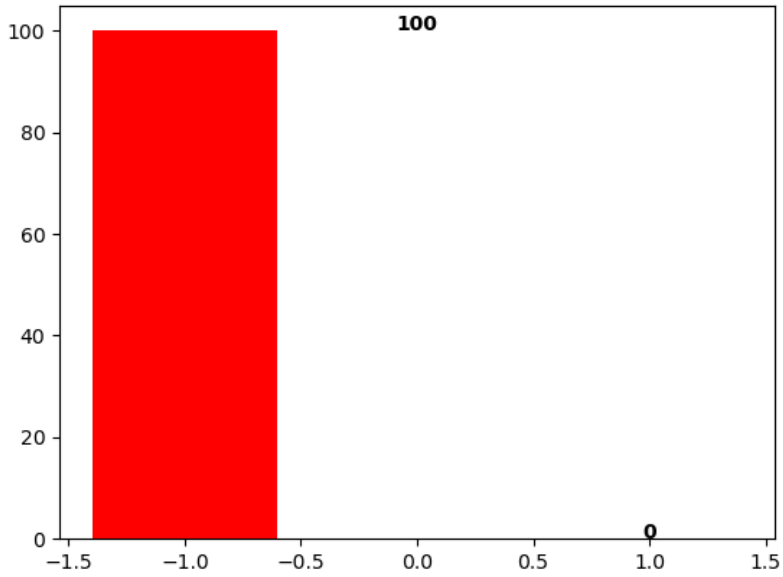


AlphaKnot Zero - Player One Unknotter Goes First

AlphaKnot Zero was trained for four hours on weak hardware.





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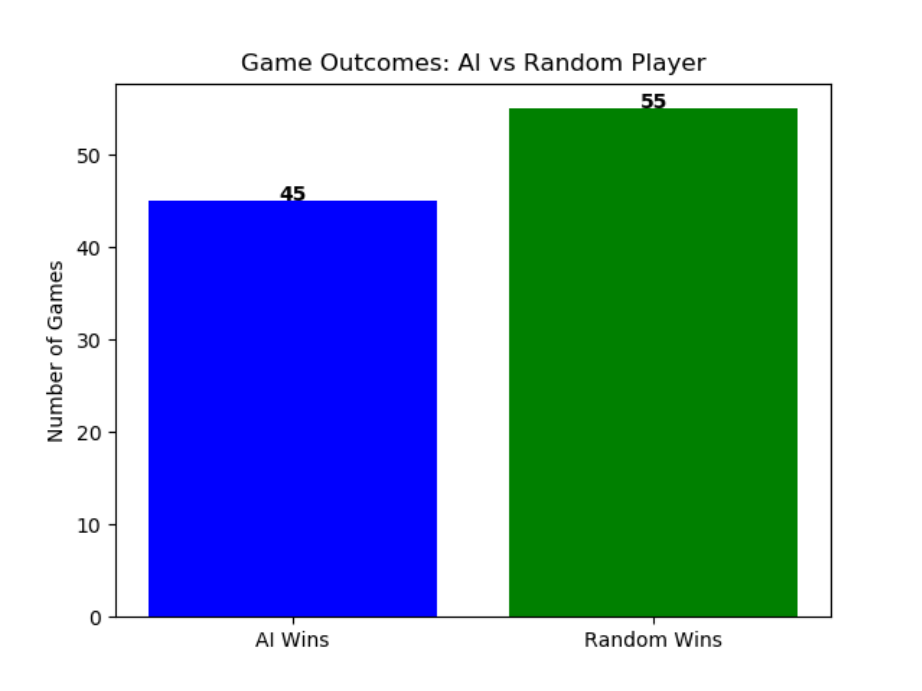

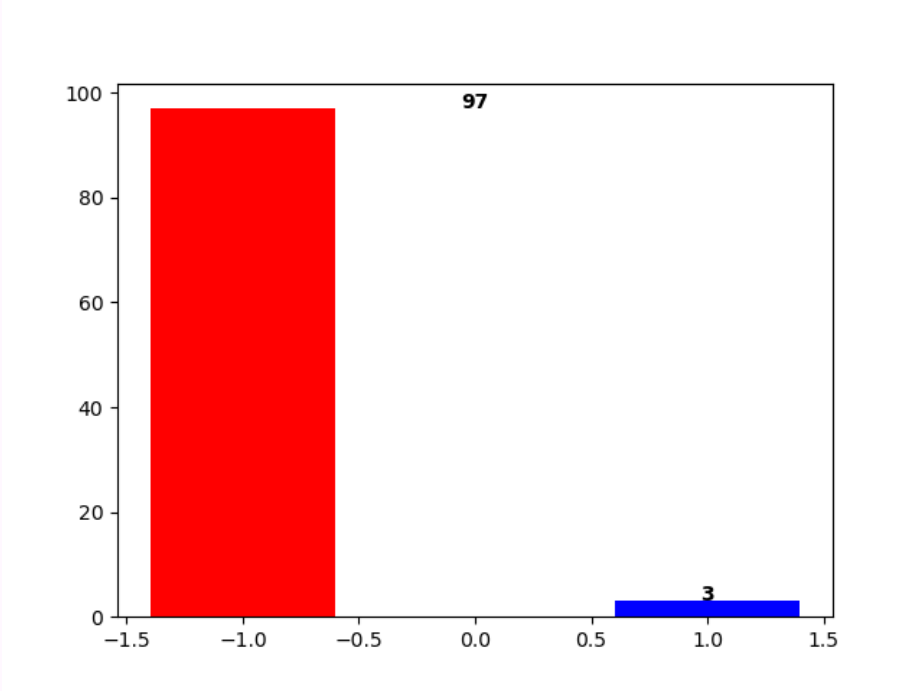

AlphaKnot Zero

- When the unknotter goes first there's probably a winning strategy for the knotter.
- When the knotter goes first ?? is more complicated and further training is needed.
- Second player probably has an advantage

More work

- Automatically classify as many boards (knots) as possible up to winning strategies using Vassar's super computer.
- Lower the training time.
- Analyze how the AI plays and extract optimal strategies.
- Prove these strategies and solve the game.