CPSC 354 Report

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Contents

1 The MU-Puzzle

MI $\rightarrow$ MU

Rule 1: If you possess a string whose last letter is `I`, add `U`.

Rule 2: Suppose you have `Mx`, you may add `Mxx`.

Rule 3: If `III` occurs in one of the strings, you may make a new string with `U` in place of `III`.

Rule 4: If `UU`, you can drop it.

MI
MII Mxx
MIIII Mxx
MIIIIIIII Mxx
MUIIU MIU
$\varnothing$

MI $\rightarrow$ use Mxx rule ∞ times
MIIII...

No matter what Rule you use you will never be able to get 0 Mod3, because I will always be 1 mod 3 or 2 mod 3

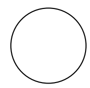
`MUUU`
`MIII`

Rule 1 does not affect # of I's.
Rule 2 does not give 0 mod 3.
Rule 3 does not solve the problem as removing 3 I's does not change the output of mod3.
Rule 4 does not change the # of I's.

We can never get rid of all of the I's, 0 mod 3 is not possible. Thus you cannot get MU from MI.

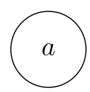
2 Rewriting Assignment

1. $A = \{\}$
   $R = \{\}$

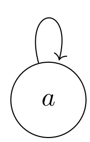
   This diagram is terminating because there are no infinite loops, confluent because all paths lead to the same result, and has a unique normal form as there is only one final state.

2. $A = \{a\}$
   $R = \{\}$

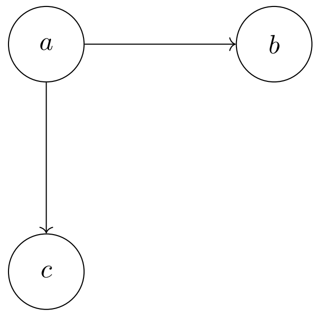
   This diagram is terminating because there are no infinite loops, confluent because all paths lead to the same result, and has a unique normal form as there is only one final state.

3. $A = \{a\}$
   $R = \{(a, a)\}$

   This diagram is not terminating due to the presence of infinite loops, confluent because all paths merge, but does not have a unique normal form as multiple results are possible.

4. $A = \{a, b, c\}$
   $R = \{(a, b), (a, b)\}$

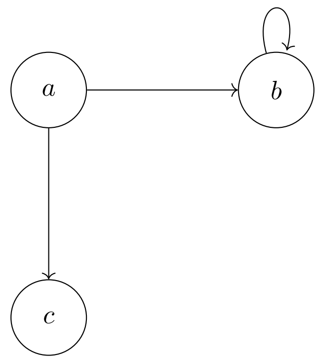


   This diagram is terminating as there are no infinite loops, not confluent because paths diverge, and does not have a unique normal form due to multiple end states.

5. $A = \{a, b\}$
   $R = \{(a, a), (a, b)\}$

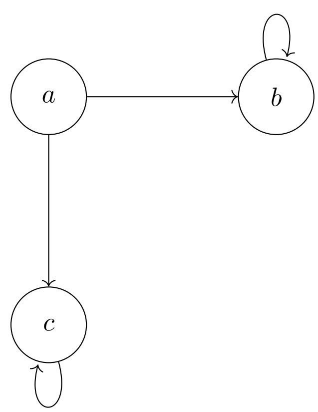
   This diagram is not terminating due to the presence of infinite loops, confluent because all paths lead to b, it has a unique normal form as the single end state is b.

6. $A = \{a, b, c\}$
   $R = \{(a, b), (b, b), (a, c)\}$



This diagram is not terminating due to the presence of infinite loops, not confluent because paths diverge, and has a unique normal form due to having a single end state on c.

7. $A = \{a, b, c\}$
   $R = \{(a, b), (b, b), (a, c), (c, c)\}$



This diagram is not terminating due to the presence of infinite loops, not confluent because paths diverge, and does not have a unique normal form due to no end states.

Properties

T	C	U	Example(s)	Explanation
True	True	True	1, 2	These examples terminate, are confluent, and have a unique normal form because all paths lead to a single final state without divergence or loops.
True	True	False	None	This state is impossible because confluence implies a unique normal form when termination is true.
True	False	True	None	This state is impossible because non-confluence means paths diverge, which contradicts having a unique normal form.
True	False	False	4	This example terminates but is not confluent due to diverging paths and does not have a unique normal form as multiple end states exist.
False	True	True	5	This example does not terminate because a is a loop, but since it loops on itself it can still be confluent and point to b to result in a unique normal form.
False	True	False	3	This example does not terminate but is confluent because all paths merge, though it does not have a unique normal form due to multiple results.
False	False	True	None	This state is impossible because non-termination and non-confluence contradict having a unique normal form.
False	False	False	6, 7	These examples do not terminate, are not confluent due to diverging paths, and do not have a unique normal form as no single end state exists.

T: Terminating, C: Confluent, U: Unique Normal Form

3 String Rewriting Excercise 5 and 5b

$$\begin{aligned} ab &\to ba \\ ba &\to ab \\ aa &\to \varepsilon \\ b &\to \varepsilon \end{aligned}$$

rewrite steps for abba and bababa:

$$\begin{aligned} &abba \to abab \to \infty \\ &abba \to aaba \to aa \to \varepsilon \\ &baba \to \infty \end{aligned}$$

$$\begin{aligned} ababa &\to \infty \\ bababa &\to aaba \to aa \to a \\ bababa &\to \infty \end{aligned}$$

1. The ARS is not terminating because there is an infinite loop in the first two rules, as they are inverses of each other.

2. Two equivalence classes: $b \to \varepsilon$ means b's don't matter, and $aa \to \varepsilon$ means a's cancel in pairs. Normal forms: Even # of a's $\Rightarrow$ ε Odd # of a's $\Rightarrow$ a

3. You can make the ARS terminating without changing the equivalence classes by removing one of the rules involved in the loop (either $ab \to ba$ or $ba \to ab$).

4. Semantic question the ARS can answer: "Is the number of a's even or odd?" This works because the complete invariant is the parity of # of a's.

5. If instead $aa \to a$, this changes the question from even-or-odd to: No a's $\Rightarrow \varepsilon$ At least one $a \Rightarrow a$