Tim Cheeseman
CS 260-003
Summer 2014
Assignment 1 Sample Solution

1.10)

f_1 and f_2

$f_1(n) \in O(f_2(n))$
$n^2 \leq c(n^2 + 1000 \cdot n)$ with $n_0 = 0$, $c = 1$

$f_1(n) \in \Omega(f_2(n))$
$n^2 \geq c(n^2 + 1000 \cdot n)$ with $n_0 = 1$, $c = \frac{1}{1001}$

$f_2(n) \in O(f_1(n))$
$n^2 + 1000 \cdot n \leq c \cdot n^2$ with $n_0 = 1$, $c = 1001$

$f_2(n) \in \Omega(f_1(n))$
$n^2 + 1000 \cdot n \geq c \cdot n^2$ with $n_0 = 0$, $c = 1$

---

f_1 and f_3

$f_1(n) \notin O(f_3(n))$
$f_1(n) \notin \Omega(f_3(n))$
$f_3(n) \notin O(f_1(n))$
$f_3(n) \notin \Omega(f_1(n))$

$f_3(n)$ oscillates between being a lower bound and upper bound for $f_1(n)$ and cannot be exclusively either.

---

f_1 and f_4

$f_1(n) \in O(f_4(n))$
$f_1(n) \leq c \cdot f_4(n)$ with $n_0 = 101$, $c = 1$

$f_1(n) \notin \Omega(f_4(n))$
There is no constant c that can make $n^2 \geq c \cdot n^3$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$, it will always dominate f_1.

$f_4(n) \notin O(f_1(n))$
There is no constant c that can make $n^3 \leq c \cdot n^2$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$, it will always dominate f_1.

$f_4(n) \in \Omega(f_1(n))$
$f_4(n) \geq c \cdot f_1(n)$ with $n_0 > 101$, $c = 1$

f_2 and f_3

$f_2(n) \notin O(f_3(n))$
$f_2(n) \notin \Omega(f_3(n))$
$f_3(n) \notin O(f_2(n))$
$f_3(n) \notin \Omega(f_2(n))$

$f_3(n)$ oscillates between being a lower bound and upper bound for $f_2(n)$ and cannot be exclusively either.

---

f_2 and f_4

$f_2(n) \in O(f_4(n))$
$n^2 + 1000 \cdot n \le f_4(n)$ with $n_0 = 101$, $c = 1$

$f_2(n) \notin \Omega(f_4(n))$
There is no constant c that can make $n^2 + 1000 \cdot n \ge c \cdot n^3$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$, it will always dominate f_2.

$f_4(n) \notin O(f_2(n))$
There is no constant c that can make $n^3 \le c \cdot (n^2 + 1000 \cdot n)$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$, it will always dominate f_2.

$f_4(n) \in \Omega(f_2(n))$
$f_4(n) \ge n^2 + 1000 \cdot n$ with $n_0 = 101$, $c = 1$

---

f_3 and f_4

$f_3(n) \in O(f_4(n))$
$f_3(n) \le c \cdot f_4(n)$ with $n_0 > 101$, $c = 1$

$f_3(n) \notin \Omega(f_4(n))$
There is no constant c that can make $n \ge c \cdot n^3$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$ and $f_3(n) = n$ for infinitely many odd n, f_4 will dominate f_3 for infinitely many n.

$f_4(n) \notin O(f_3(n))$
There is no constant c that can make $n^3 \le c \cdot n$ true for infinitely many n. As $f_4(n) = n^3$ for all $n > 100$ and $f_3(n) = n$ for infinitely many odd n, f_4 will dominate f_3 for infinitely many n.

$f_4(n) \in \Omega(f_3(n))$
$f_4(n) \ge c \cdot f_3(n)$ with $n_0 > 101$, $c = 1$

1.12)

a) **O(n^3)**

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} 1 = \sum_{i=1}^{n}\sum_{j=1}^{n} n = \sum_{i=1}^{n} n^2 = n^3$$

b) **O(n^3)**

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} j \le \sum_{i=1}^{n-1} n^2 \le n^3$$

c) **O(n^2)**

Let's define a function to be the running time of the conditional: $f(n,i) = 1$ when i is even and $f(n,i) = n$ when i is odd (the combination of the inner two for loops is O(n-i+i) = O(n))

The running time is $\sum_{i=1}^{n} f(n,i) = \frac{n}{2}1 + \frac{n}{2}n = \frac{n^2+n}{2}$ which is O(n^2)

d) **O(2^n)**

$T(n) = 1$ when n = 1 and $T(n) = 2 \cdot T(n-1)$ when n > 1.

$T(n-1) \; = 2 \cdot 2 \cdot T(n-2)$

$T(n-2) \; = 2 \cdot 2 \cdot 2 \cdot T(n-3)$

…

$T(n-(n-1)) \; = 2 \cdot 2^{n-1} = 2^n$