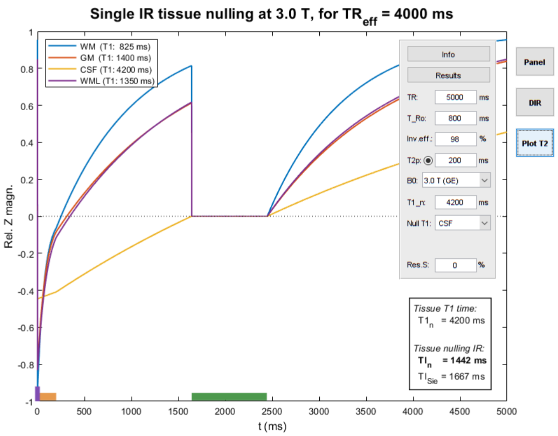
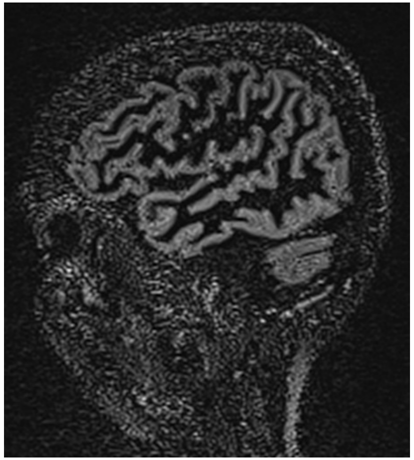
MR Inversion Recovery Calculation

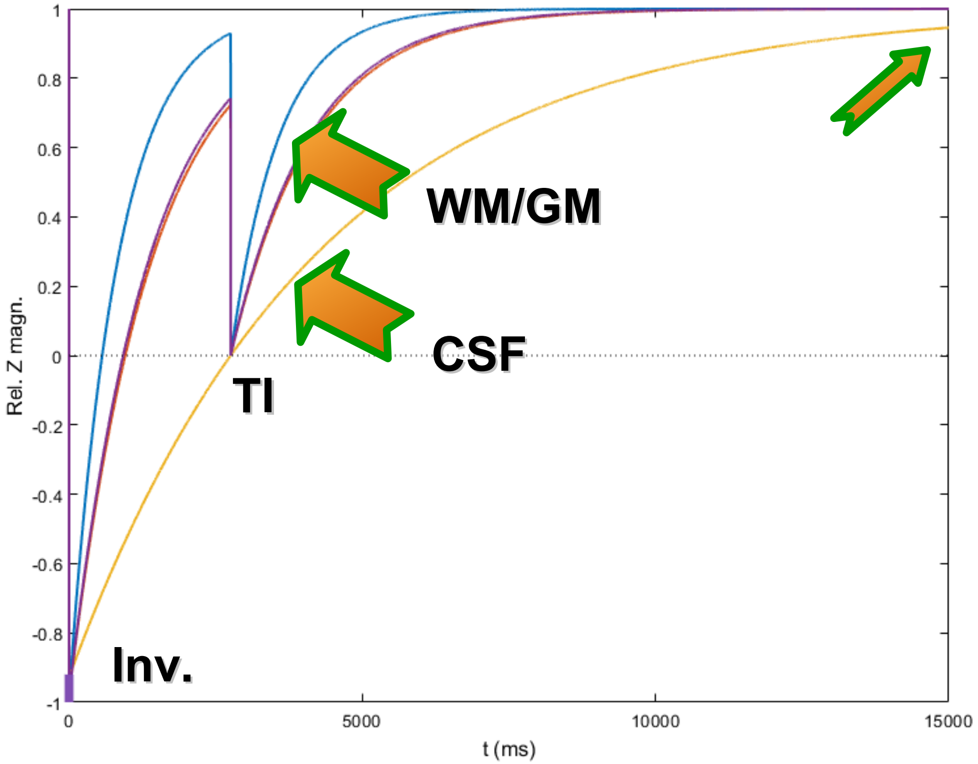
Simulering og optimalisering av inversjonssekvenser i MatLab





Øystein B Gadmar & Wibeke Nordhøy

FLAIR – Fluid Attenuated Inv. Recovery



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} \quad ; \quad M_t = M_0 - \left(M_0 - M_1\right) E_t$$

$$S \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 - -1\right) E_{TI} = 1 - 2\,E_{TI}$$

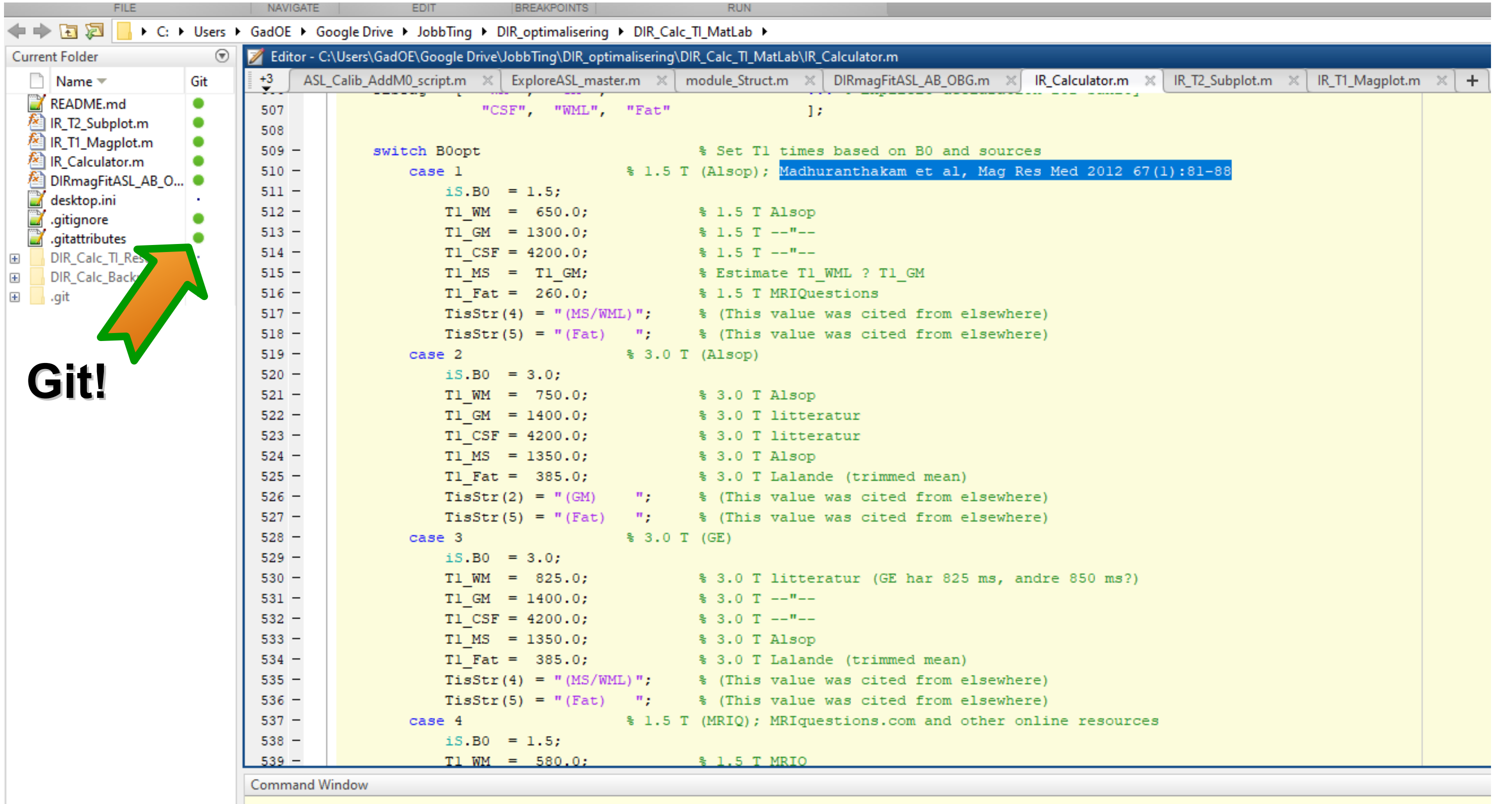
$$\downarrow$$

$$S_0 = 0 \quad \Rightarrow \quad E_{TI} = e^{-\frac{TI}{T1}} = \frac{1}{2}$$

$$\Rightarrow \quad TI = \underline{T1 \cdot \ln\left(2\right)}$$

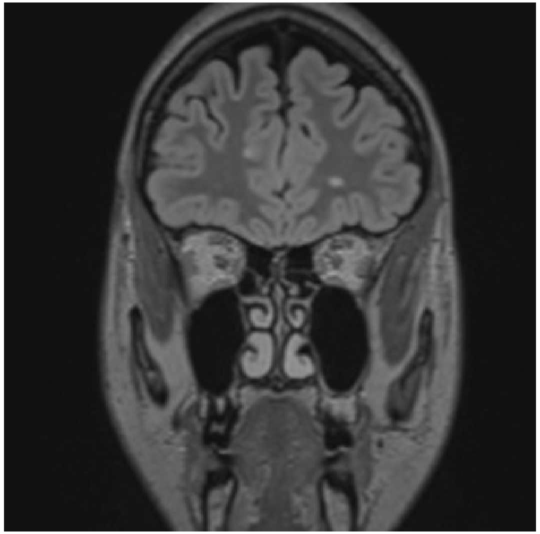
$$Q.E.D.$$

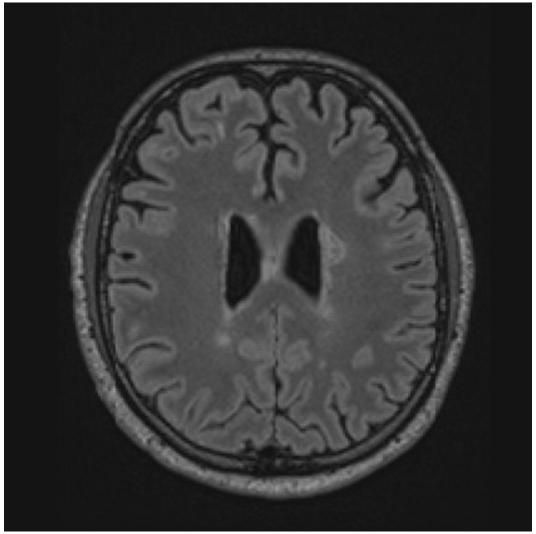
Our IR Calculator for MatLab



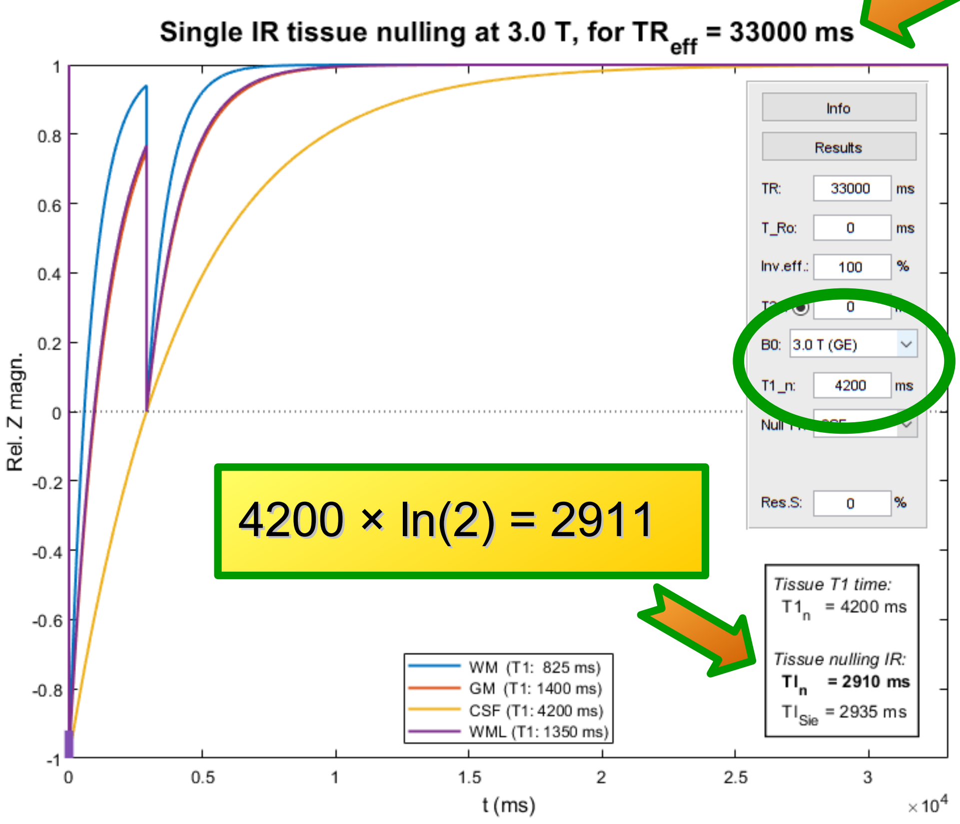
Fluid Attenuated Inversion Recovery

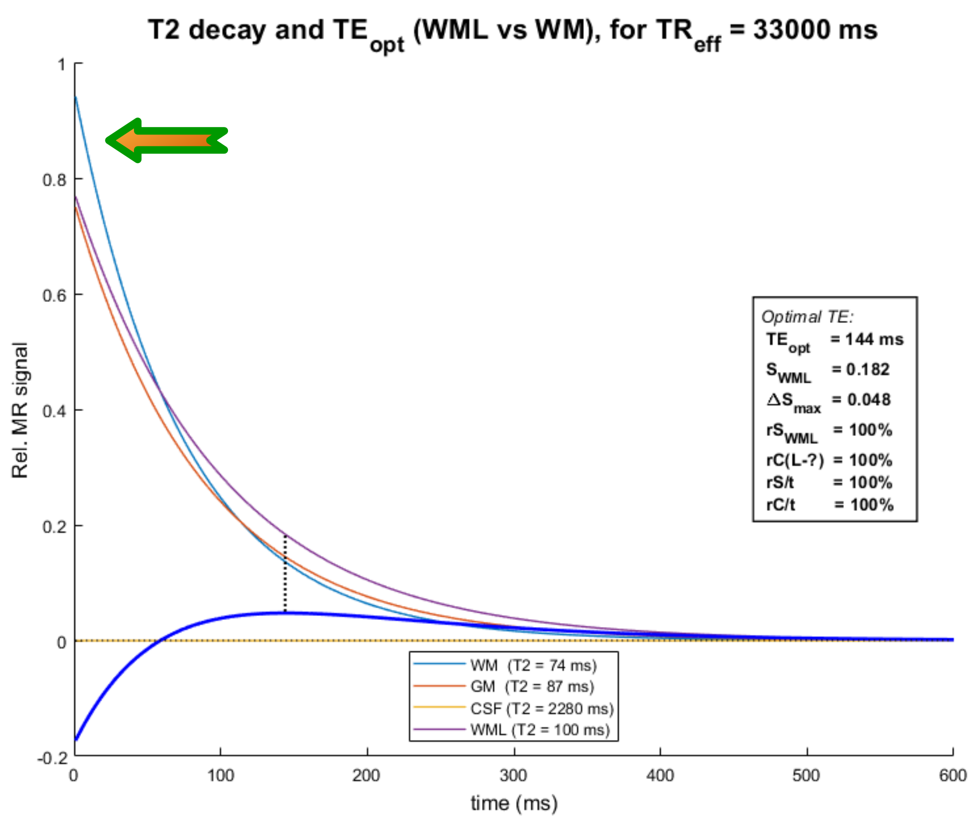
- One IR pulse, then a TI delay before readout
- Ours: 3D T2-weighted readout (SPACE/CUBE/VIEW)
- T1 weighting from IR counteracts the desired T2 contrast...





"Simple" FLAIR

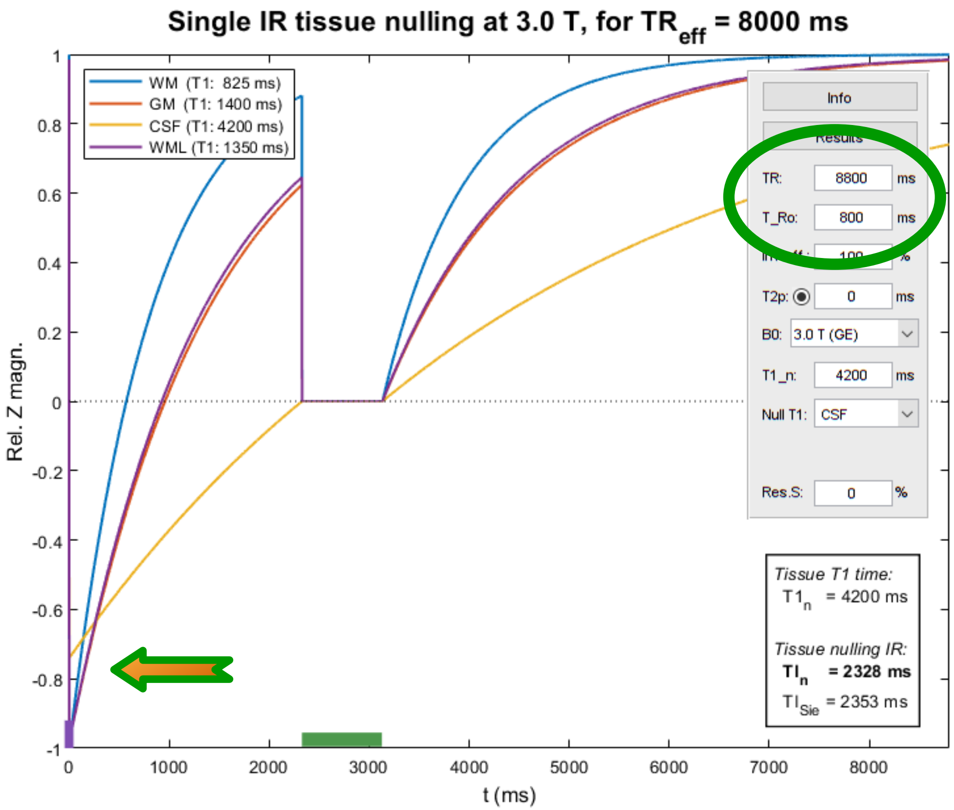


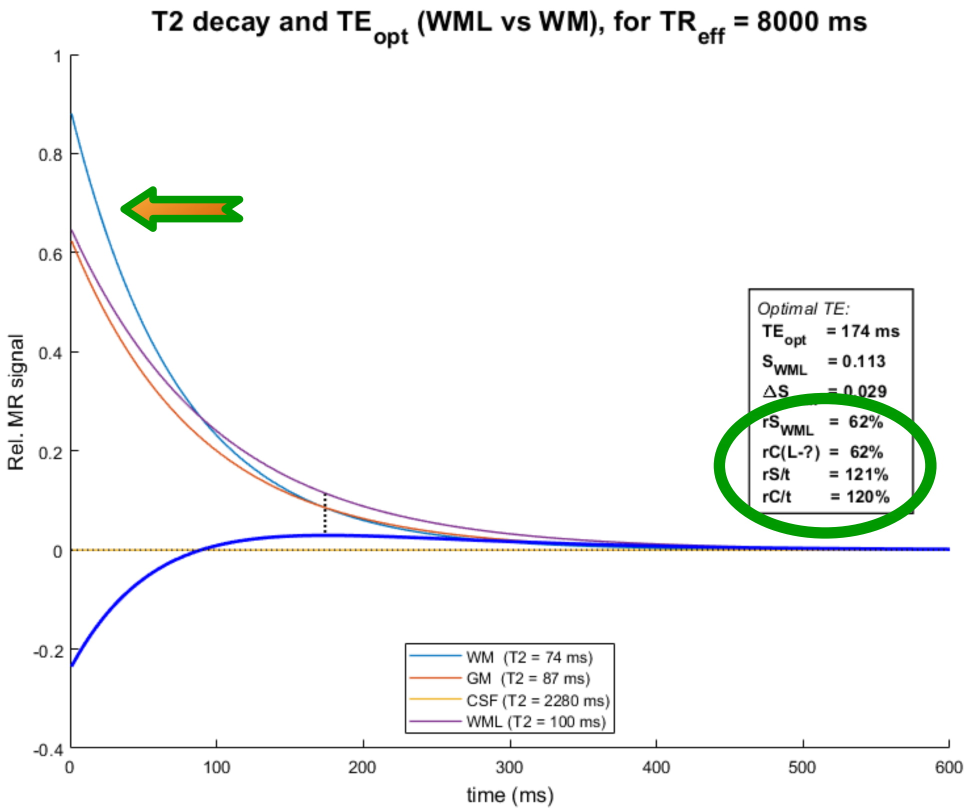


Notes On Noise

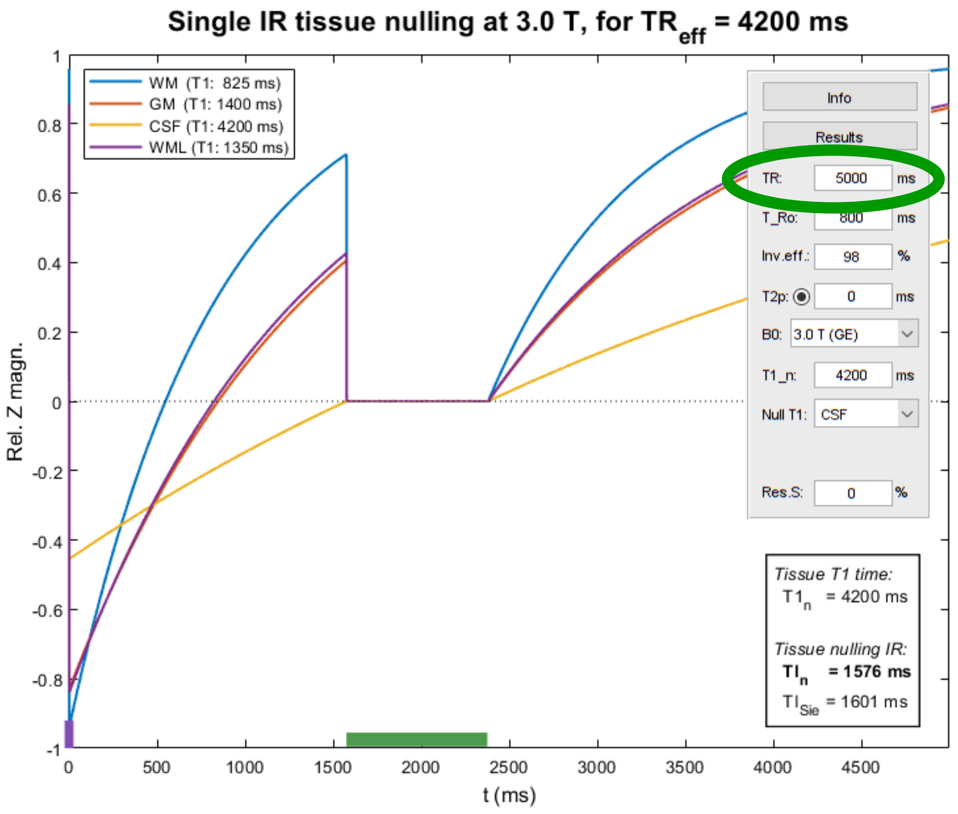
- S = Signal strength at TE
- C = Difference between S_{WML} and S_{WM} (usually)
- N = Noise $\propto$ sqrt(t)
- CNR = C/N $\propto$ t/sqrt(t) = sqrt(t)
- Contrast efficiency: CNR/t $\propto$ C/sqrt(TR)
- CNR/t – a good end point (but also look to SNR/t)

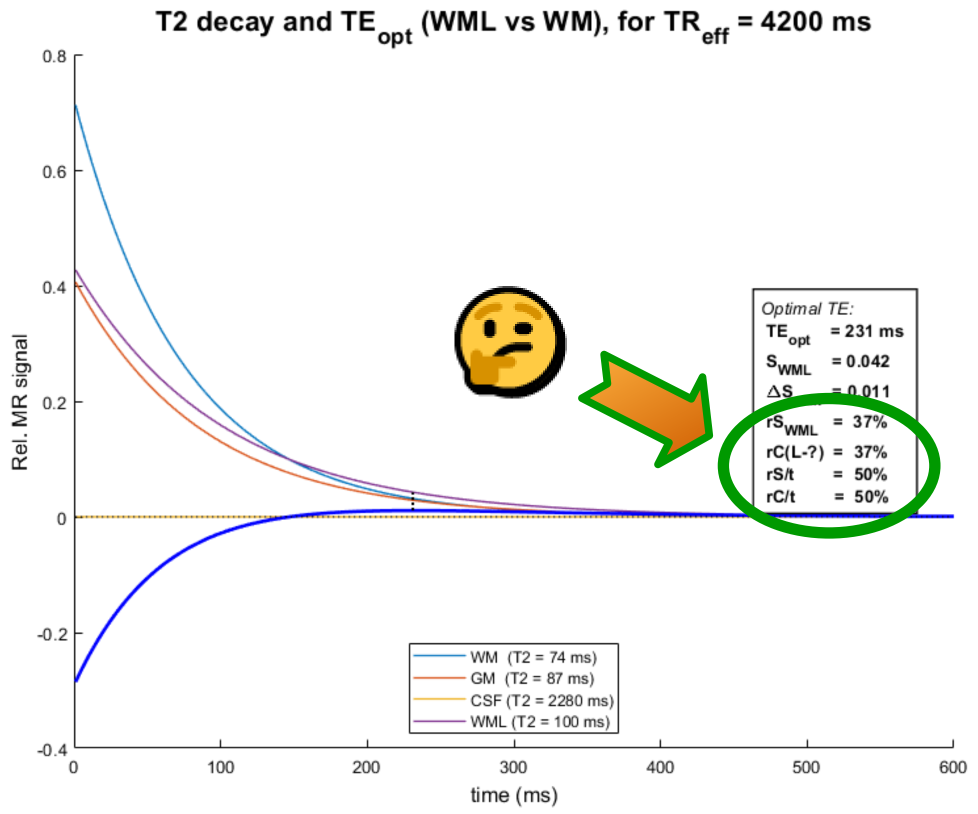
Realistic 3D FLAIR



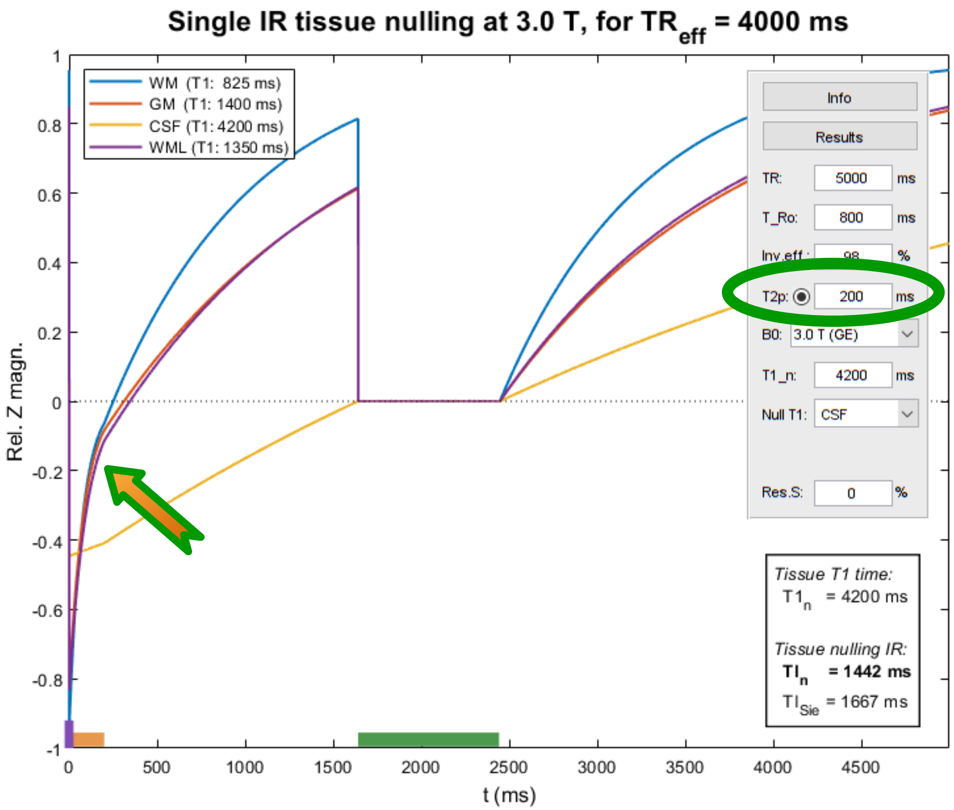


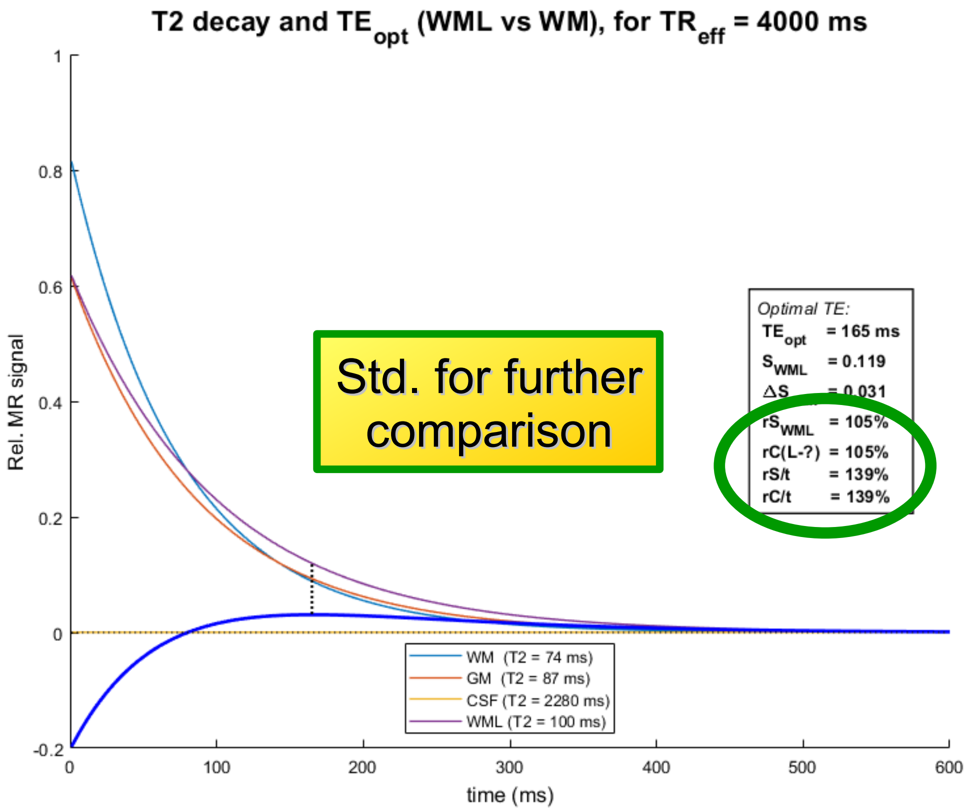
“Vendor” 3D FLAIR w/ short TR???





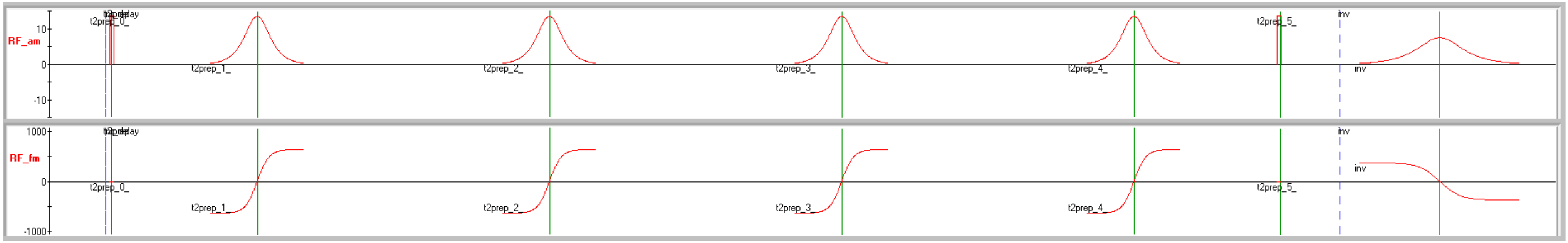
“Vendor” 3D FLAIR w/ T2 Prep.!



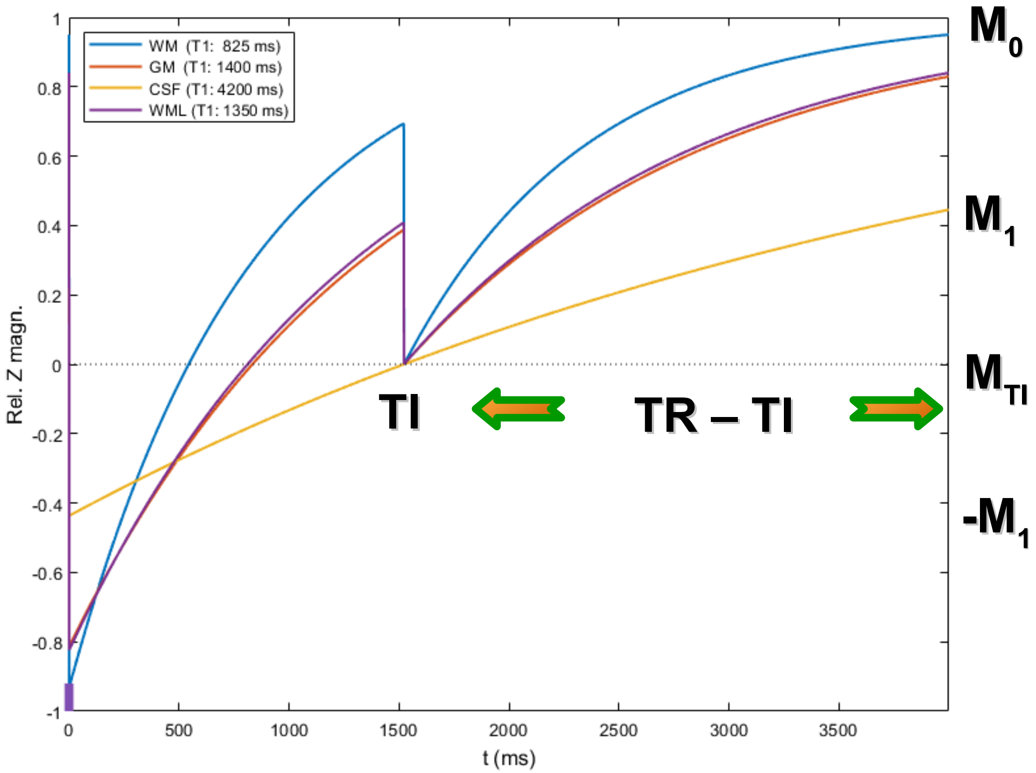


T2 prep. – How?

- $90°x$ – [$180°y$]$_n$ – $-90°x$ to achieve spin lock
- T2P and inversion are interchangeable
- Pure T2 decay used for Mz preparation
- CSF hardly affected (long T2), but tissues relax much



IR-Calc (0^{th}: In Real Time)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} \;\; ; \;\; M_t = M_0 - \left(M_0 - M_1\right) E_t$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}}$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \frac{M_1}{M_0}\right) E_{TI} = \underline{\left(1 + E_{TR}\right) - \left(1 + 1\right) E_{TI}}$$

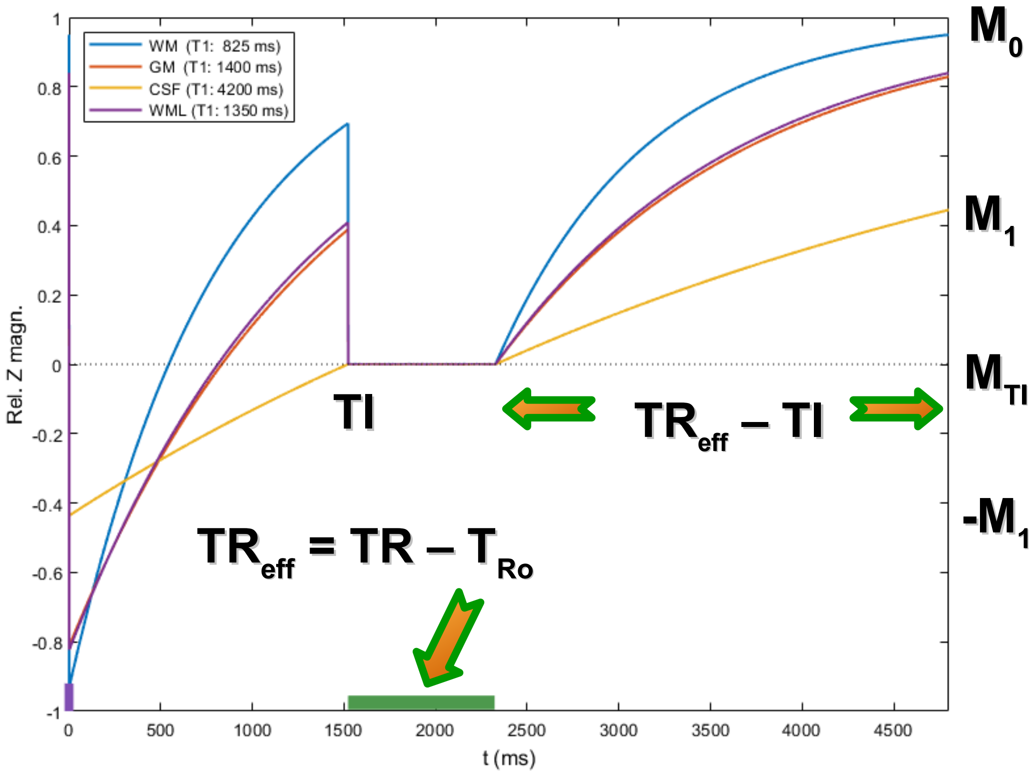
$$\downarrow$$

$$S_0 = 0 \;\Rightarrow\; E_{TIn} = e^{-\frac{TIn}{T1}} = \frac{1 + E_{TR}}{2}$$

$$\Rightarrow\; TI_n = \underline{T1\left[\, \ln\left(2\right) - \ln\left(1 + E_{TR}\right) \right] \; ; \;\; E_{TR} = e^{-\frac{TR}{T1}}}$$

$Q.E.D.$

IR-Calc (1st: Effective TR)



$$E_t \overset{\text{def}}{=} e^{-\frac{t}{T1}}$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} \quad ; \quad (TR \rightarrow TR_{eff})$$

$$S_0 \overset{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \frac{M_1}{M_0}\right) E_{TI} = \underline{(1+E_{TR}) - (1+1)\, E_{TI}}$$

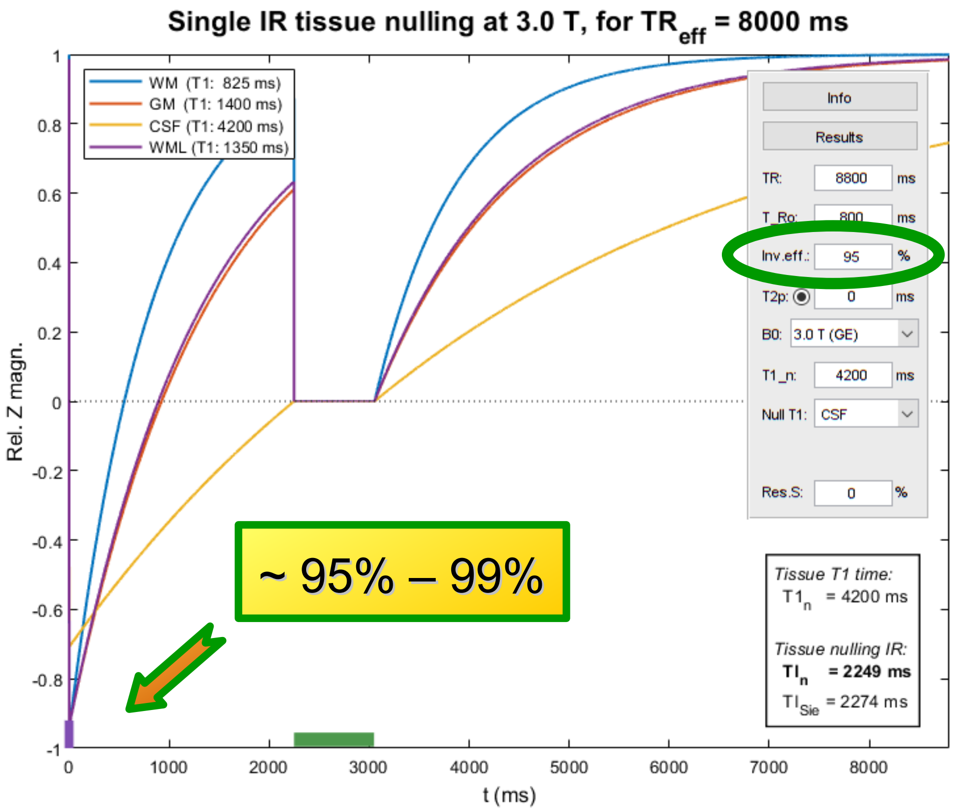
$$\downarrow$$

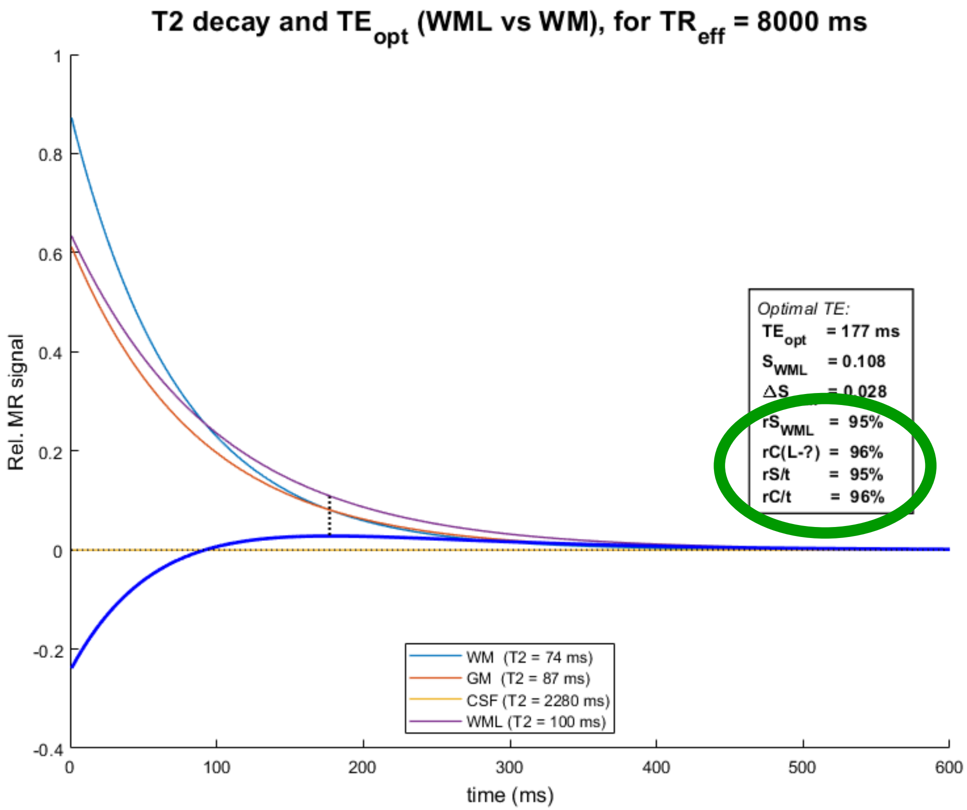
$$S_0 = 0 \;\Rightarrow\; E_{TIn} = e^{-\frac{TIn}{T1}} = \frac{1+E_{TR}}{2}$$

$$\Rightarrow\; TI_n = \underline{T1\left[\ln(2) - \ln(1+E_{TR})\right] \; ; \; E_{TR} = e^{-\frac{TReff}{T1}}}$$

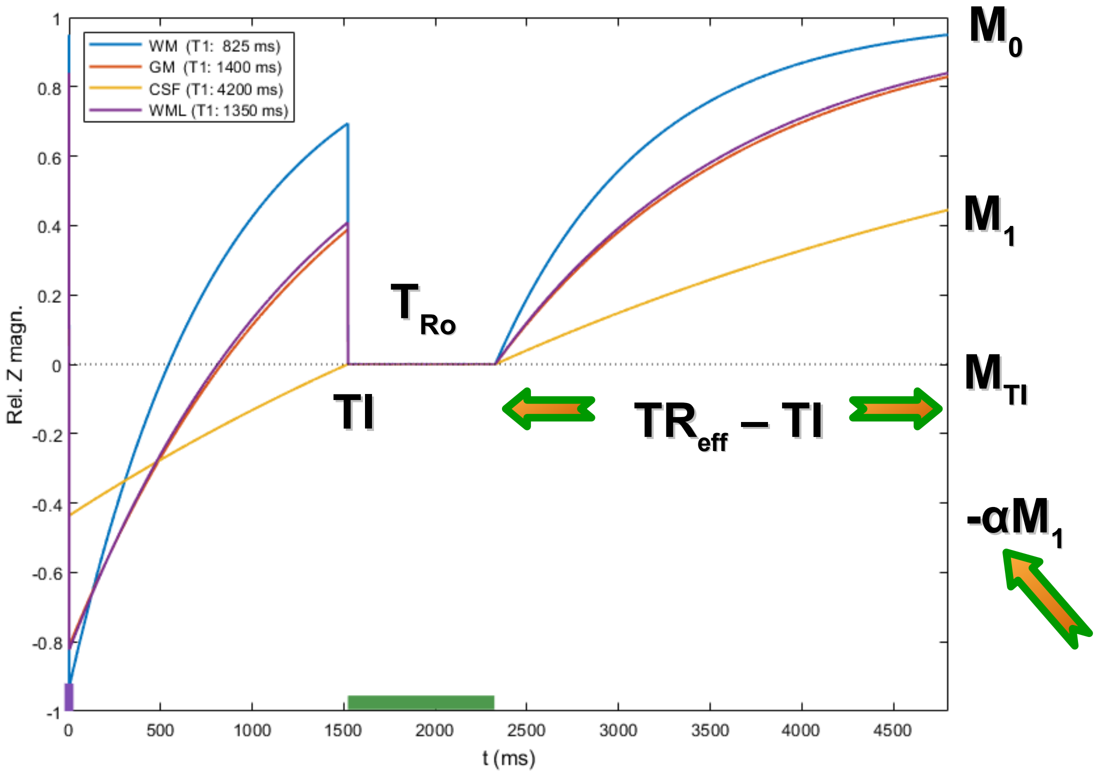
$$Q.E.D.$$

The efficiency of inversion efficiency?





IR-Calc (2nd: Inefficient Inversion)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} \;\; ; \;\; \alpha \stackrel{\text{def}}{=} -\cos\left(FA_{inv}\right)$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} \;\; ; \;\; \left(TR \rightarrow TR_{eff}\right)$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha\frac{M_1}{M_0}\right)E_{TI} = \underline{\left(1+\alpha E_{TR}\right) - \left(1+\alpha\right)E_{TI}}$$

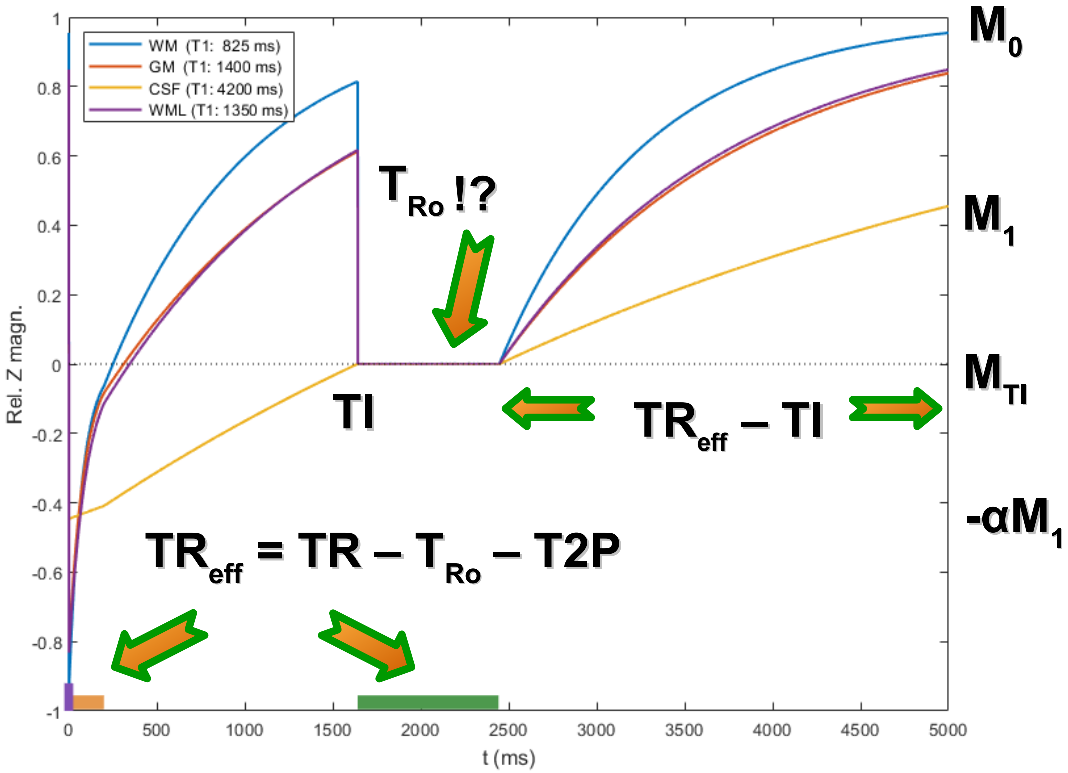
$$\downarrow$$

$$S_0 = 0 \;\Rightarrow\; E_{TIn} = e^{-\frac{TIn}{T1}} = \frac{1+\alpha E_{TR}}{1+\alpha}$$

$$\Rightarrow\; TI_n = \underline{T1\left[\,\ln\left(1+\alpha\right) - \ln\left(1+\alpha E_{TR}\right)\right]}$$

$$Q.E.D.$$

IR-Calc (3rd: Be T2 Prepared!)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} \;;\; \alpha \stackrel{\text{def}}{=} -\cos\left(FA_{inv}\right) \;;\; E_{T2P} \stackrel{\text{def}}{=} e^{-\frac{T2P}{T2}}$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} \;;\; \left(TR \rightarrow TR_{eff}\right)$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1+\alpha\frac{M_1}{M_0}\right)E_{TI} = \underline{\left(1+\alpha E_{TR}\right) - \left(1+\alpha\right)E_{TI}}$$

$$\downarrow$$

$$S_0 = 0 \;\Rightarrow\; E_{TIn} = e^{-\frac{TIn}{T1}} = \frac{1+\alpha E_{TR}}{1+\alpha}$$

$$\Rightarrow\; TI_n = \underline{T1\left[\ln\left(1+\alpha\right) - \ln\left(1+\alpha E_{TR}\right)\right]}$$

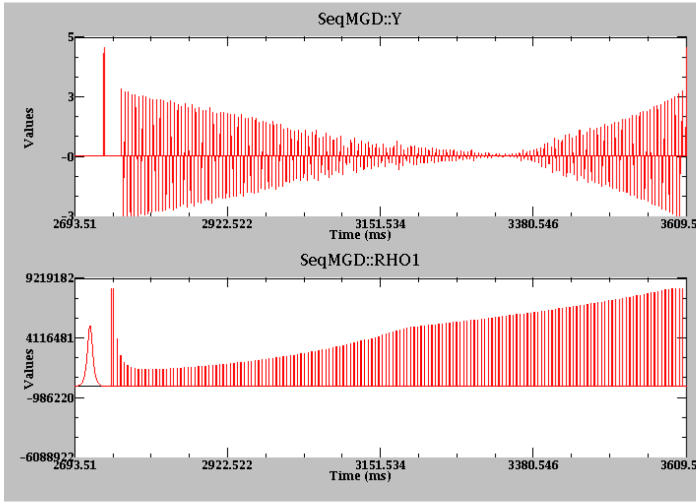
$$\downarrow$$

$$\alpha \rightarrow \alpha E_{T2P} \;\Rightarrow\; TI_n = \underline{T1\left[\ln\left(1+\alpha E_{T2P}\right) - \ln\left(1+\alpha E_{T2P}E_{TR}\right)\right]}$$

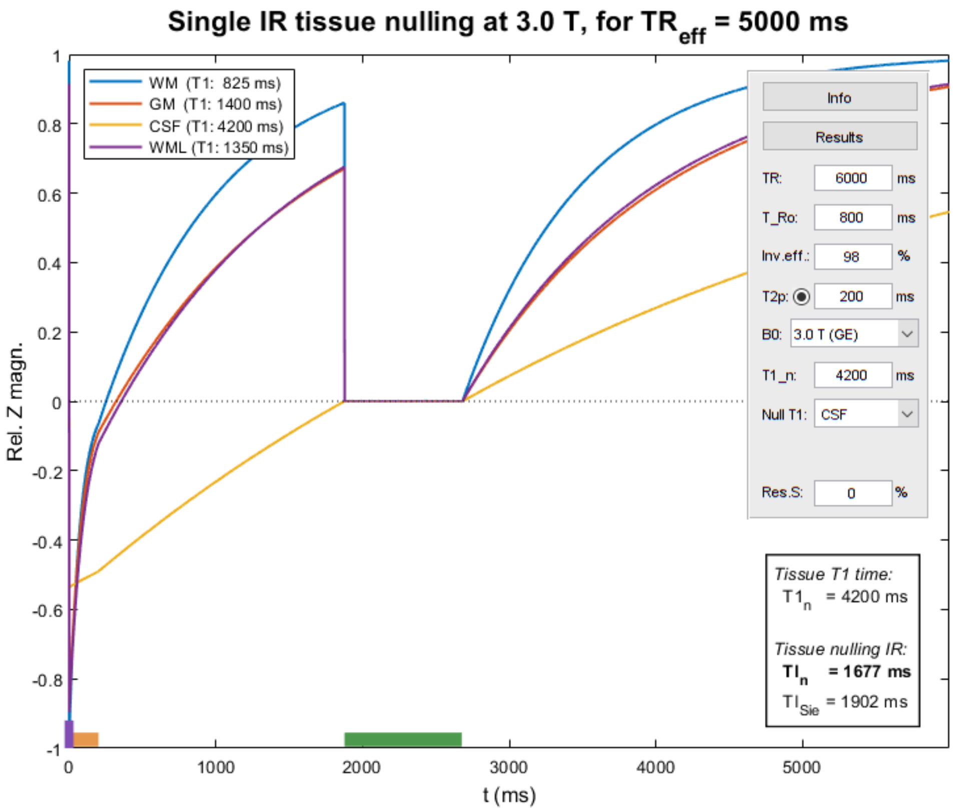
$$Q.E.D.$$

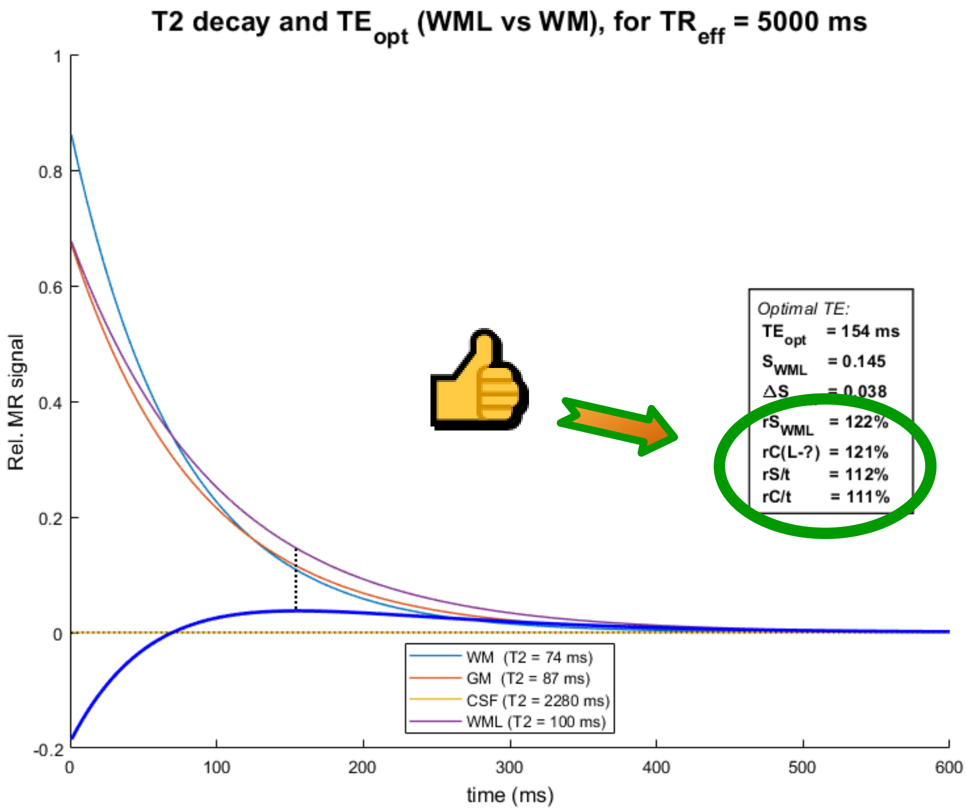
3D TSE readout – How?

- Complex flip angle sweep to preserve magnetization
- Complex profile ordering to avoid artefacts/blurring
- TE_{eff} estimated; much shorter than zero crossing time!
- Contrast mainly T2 – some T1?!
- Mz *probably* used up over T_{Ro}?
- We're working to understand it…!

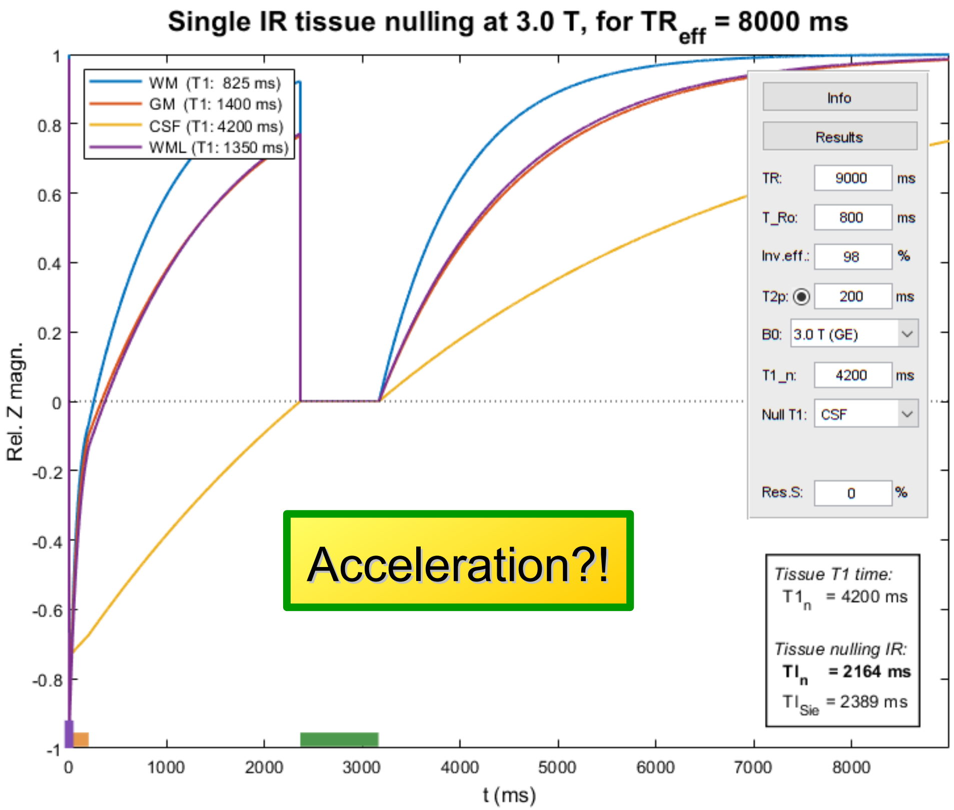


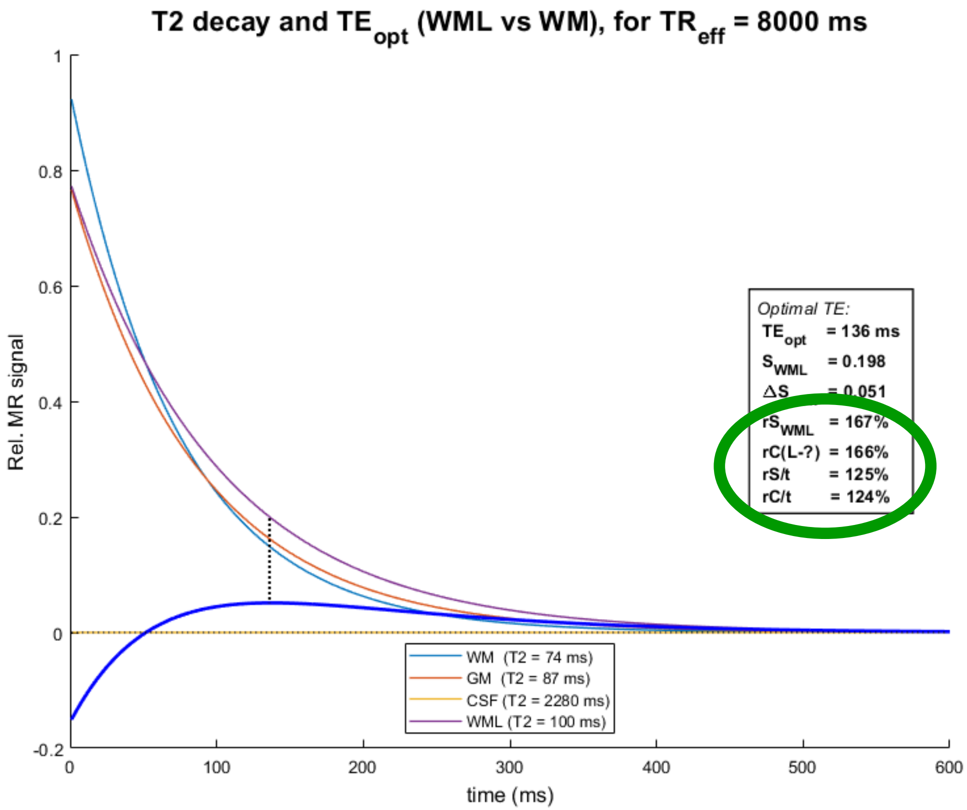
TR 6 s – "One up" on the vendor





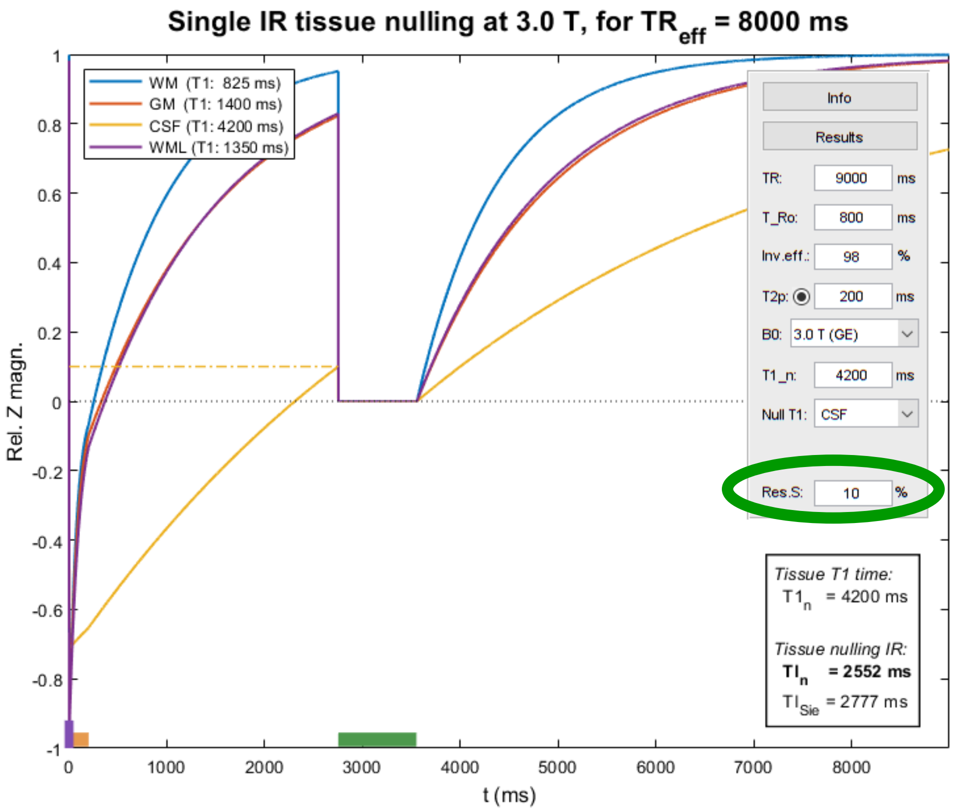
TR 9 s – FLAIR And Back Again?!

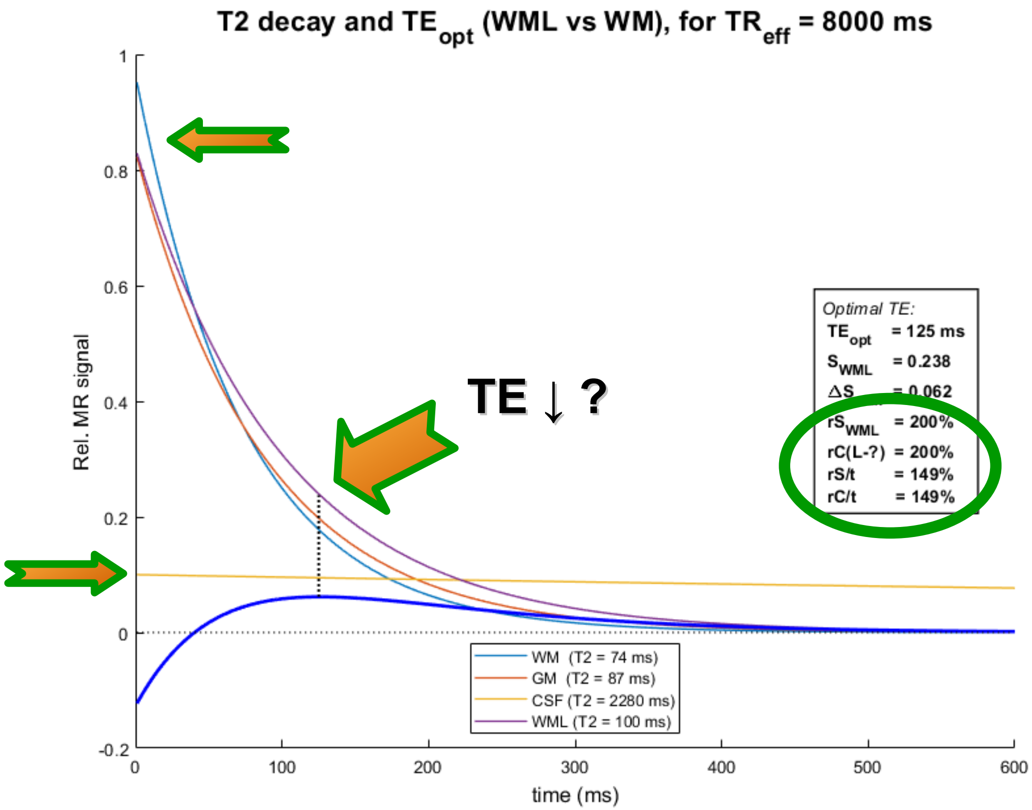




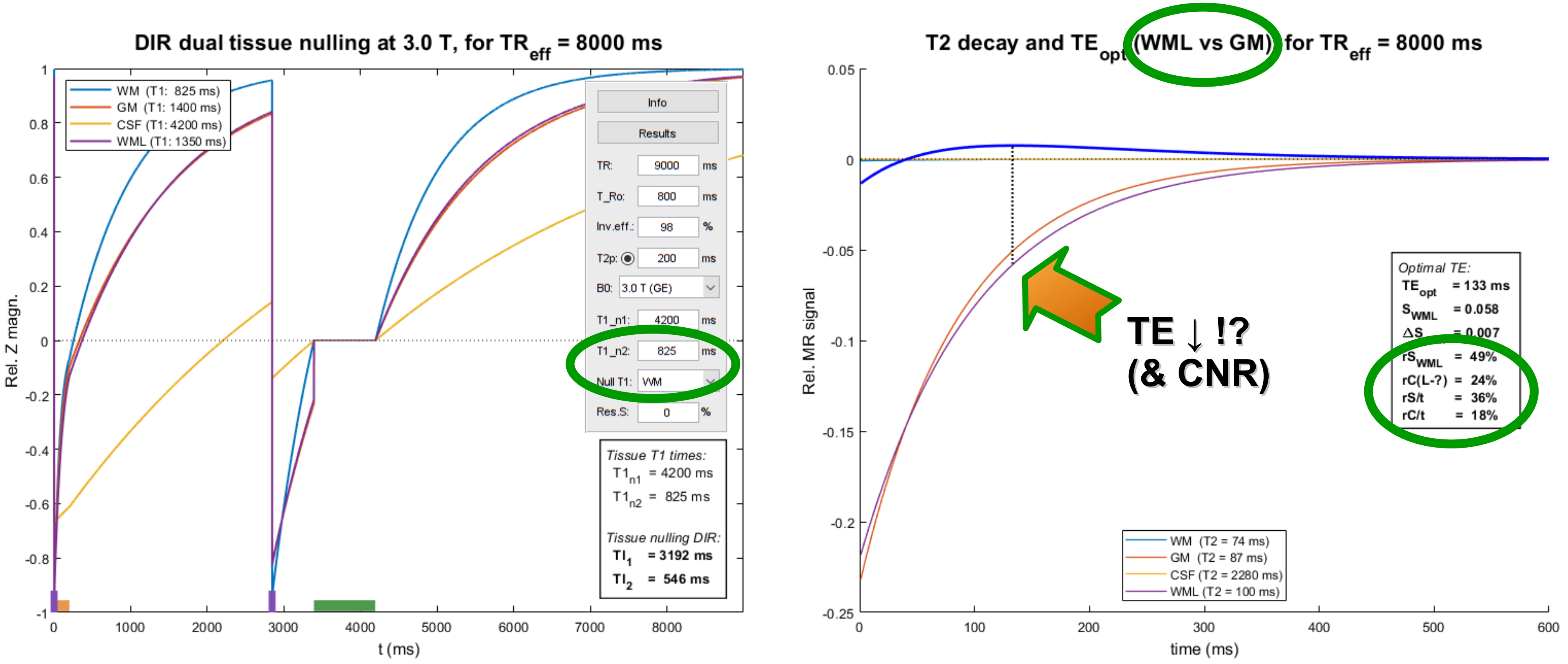
Fluid *Attenuated* IR … by how much?

“Images that computers like”

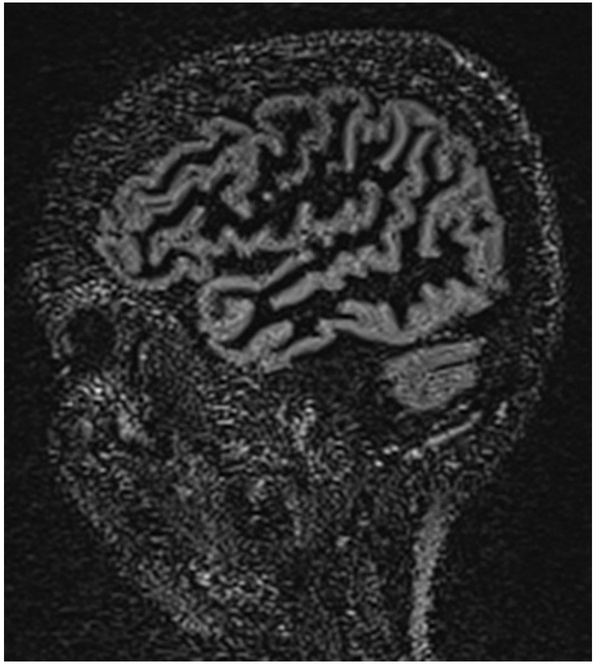


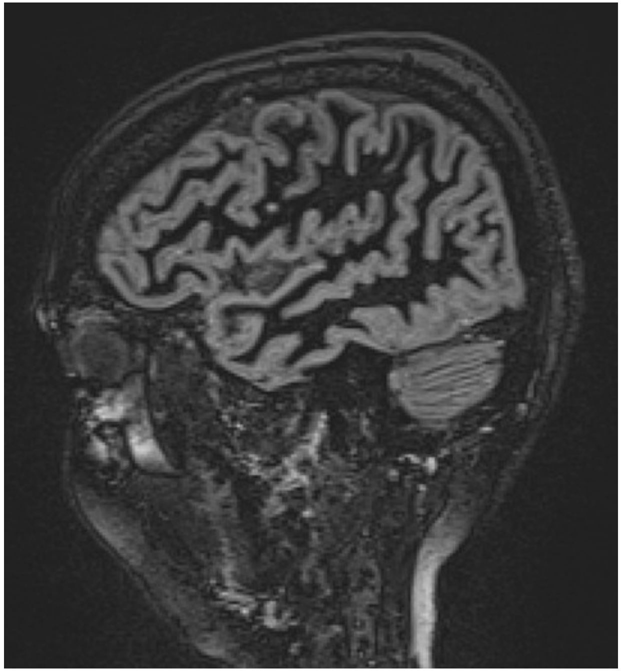


Dual IR: Two for the price of WM (and SNR)



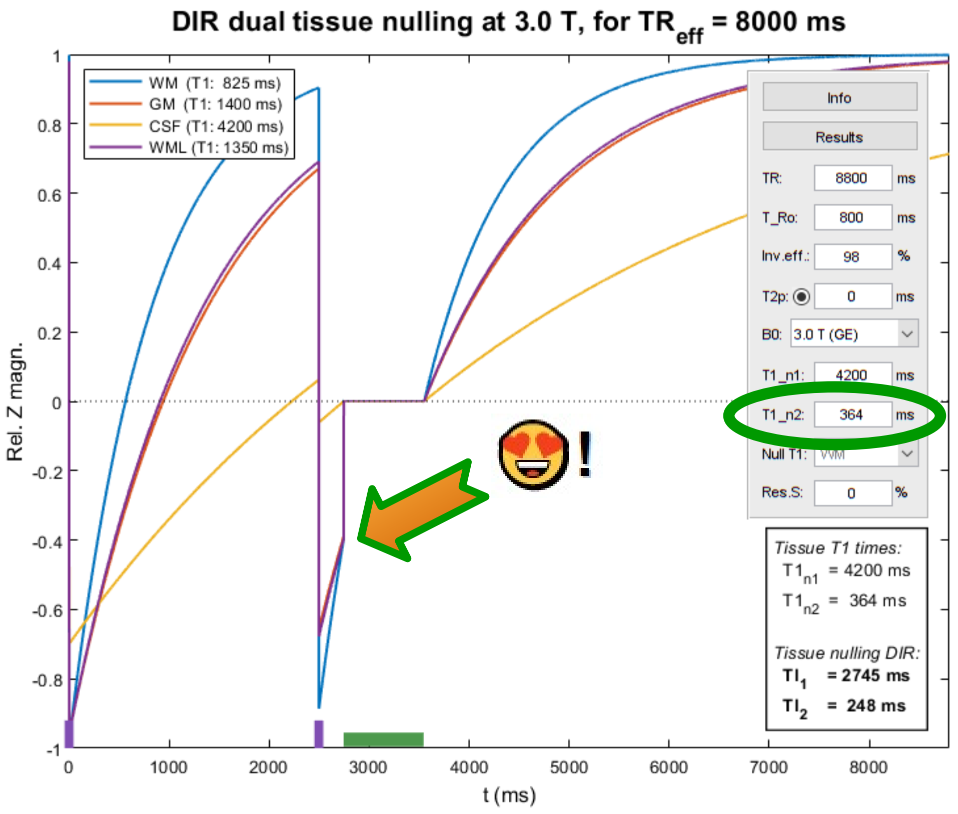
DIR-WM – A Pulse Too Far?

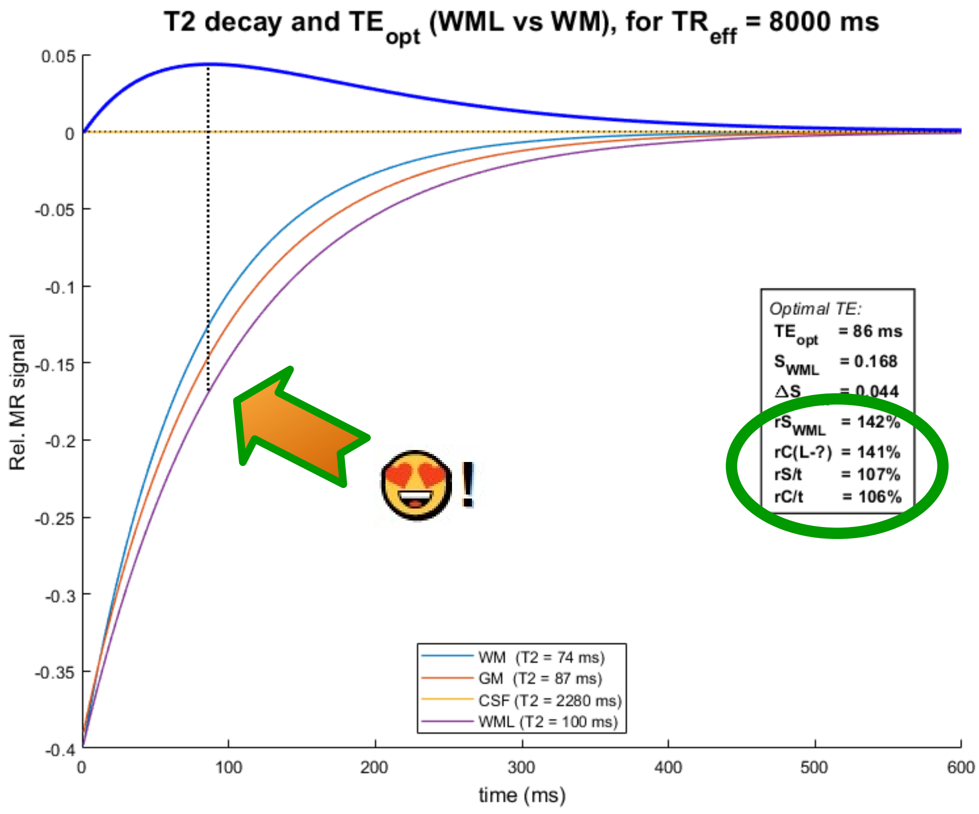




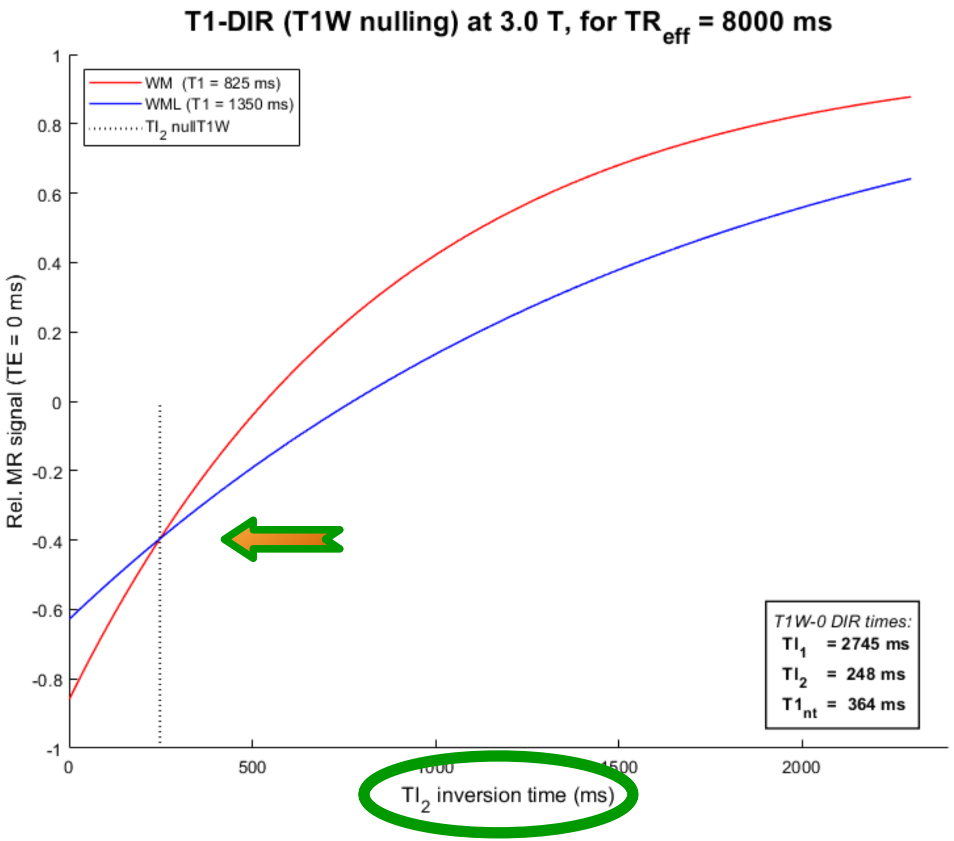
CNR/t bad. Better at low TE – at the cost of WML–GM CNR

T1-nulled DIR – A New Hope





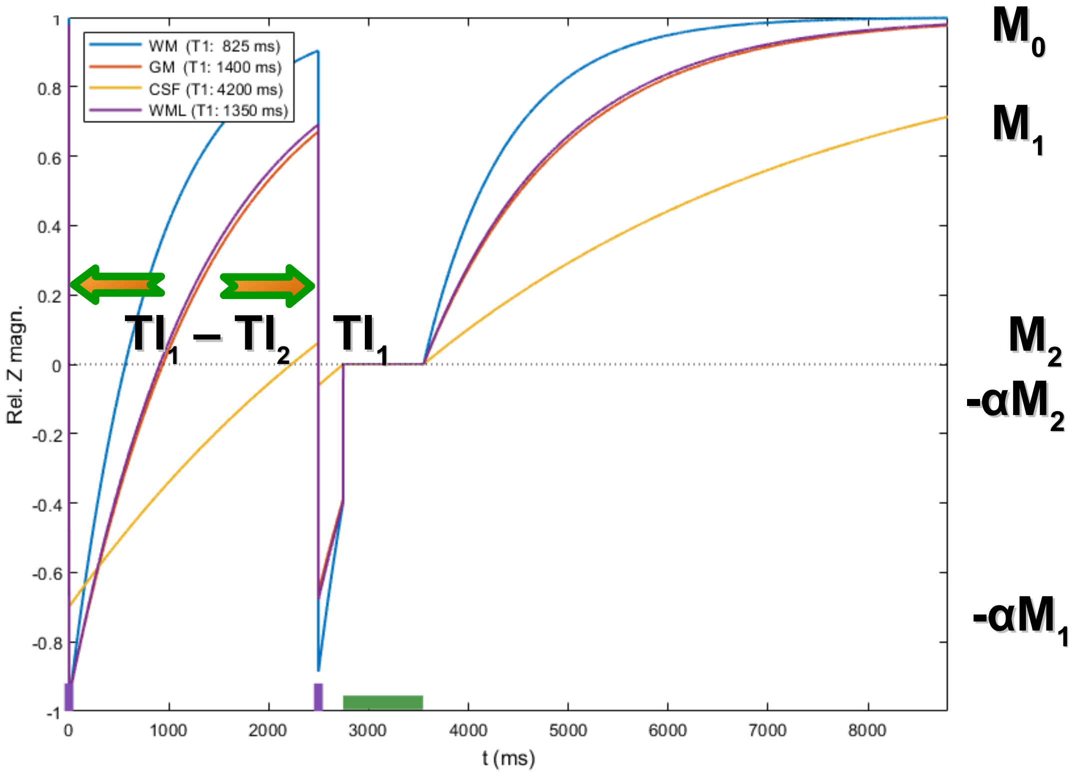
T1-nulled DIR – How?



- Find the TI_1/TI_2 pair that null the difference between the DIR signal for T1 times of WM and WML
- Find the T1 that is nulled at these TI_1/TI_2
- WM will be brighter, but SNR/CNR better
- Optimal TE will be free of T1 effects

[Madhuranthakam et al, Mag Res Med 2012 67(1):81-88]

DIR-Calc (Inversion juggling)



$$\frac{M_1}{M_0} = 1 - E_{(TR-TI_1)} = 1 - \frac{E_{TR}}{E_{TI_1}} \;\; ; \;\; (TR \rightarrow TR_{eff})$$

$$\frac{M_2}{M_0} = 1 - \left(1 + \alpha \frac{M_1}{M_0}\right) E_{(TI_1 - TI_2)} = \frac{1}{E_{TI_2}} \left[E_{TI_2} + \alpha E_{TR} - (1+\alpha) E_{TI_1} \right]$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_2}{M_0}\right) E_{TI_2}$$

$$S_0 = \underline{1 - (1+\alpha) E_{TI_2} + \alpha (1+\alpha) E_{TI_1} - \alpha^2 E_{TR}}$$

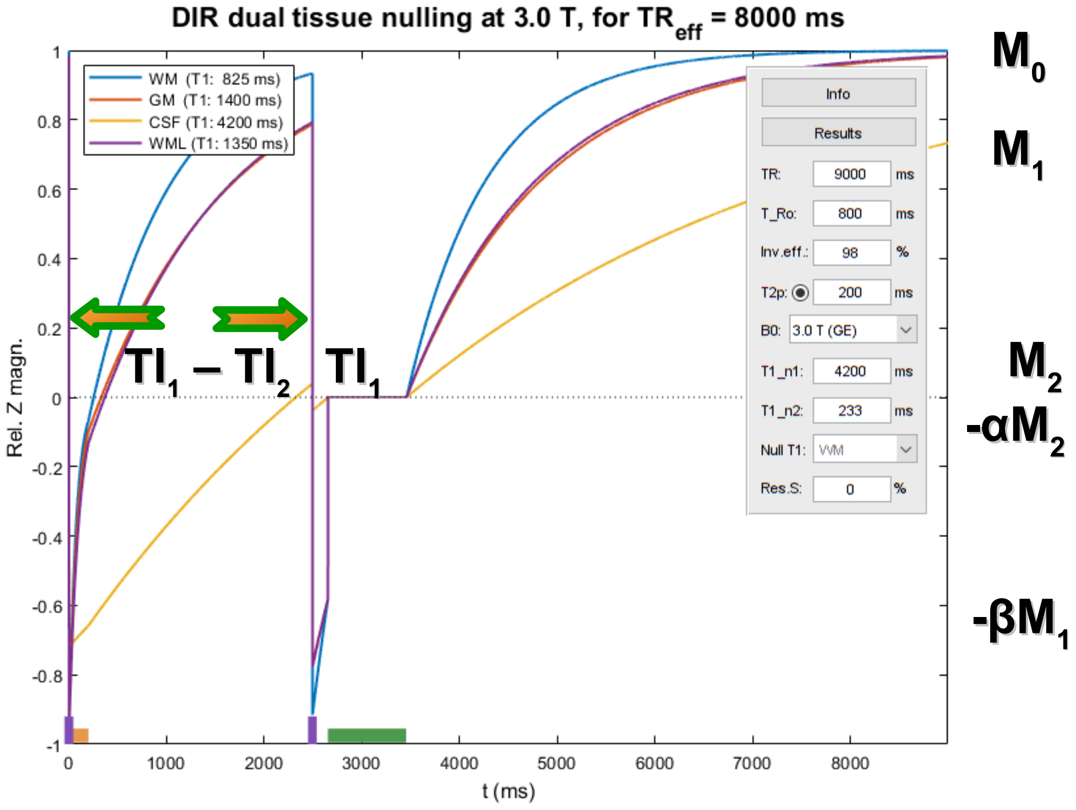
$$\downarrow$$

$$S_0 = 0 \;\; \Rightarrow$$

$$TI_{1n} = \underline{T1 \ln\left([\alpha(1+\alpha)] / \left[-1 + (1+\alpha) E_{TI_2} + \alpha^2 E_{TR} \right] \right)}$$

$Q.E.D.$

DIR-Calc (Oh My T2P... version)



$$\frac{M_1}{M_0} = 1 - E_{(TR-TI_1)} = 1 - \frac{E_{TR}}{E_{TI_1}} \;\; ; \;\; \left(TR \rightarrow TR_{eff}\right) \;\; ; \;\; \beta \overset{\text{def}}{=} \alpha E_{T2P}$$

$$\frac{M_2}{M_0} = 1 - \left(1 + \beta \frac{M_1}{M_0}\right) E_{(TI_1 - TI_2)} = \frac{1}{E_{TI_2}} \left[E_{TI_2} + \beta E_{TR} - (1+\beta) E_{TI_1} \right]$$

$$S_0 \overset{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_2}{M_0}\right) E_{TI_2}$$

$$S_0 = \underline{1 - (1+\alpha) E_{TI_2} + \alpha (1+\beta) E_{TI_1} - \alpha\beta E_{TR}}$$

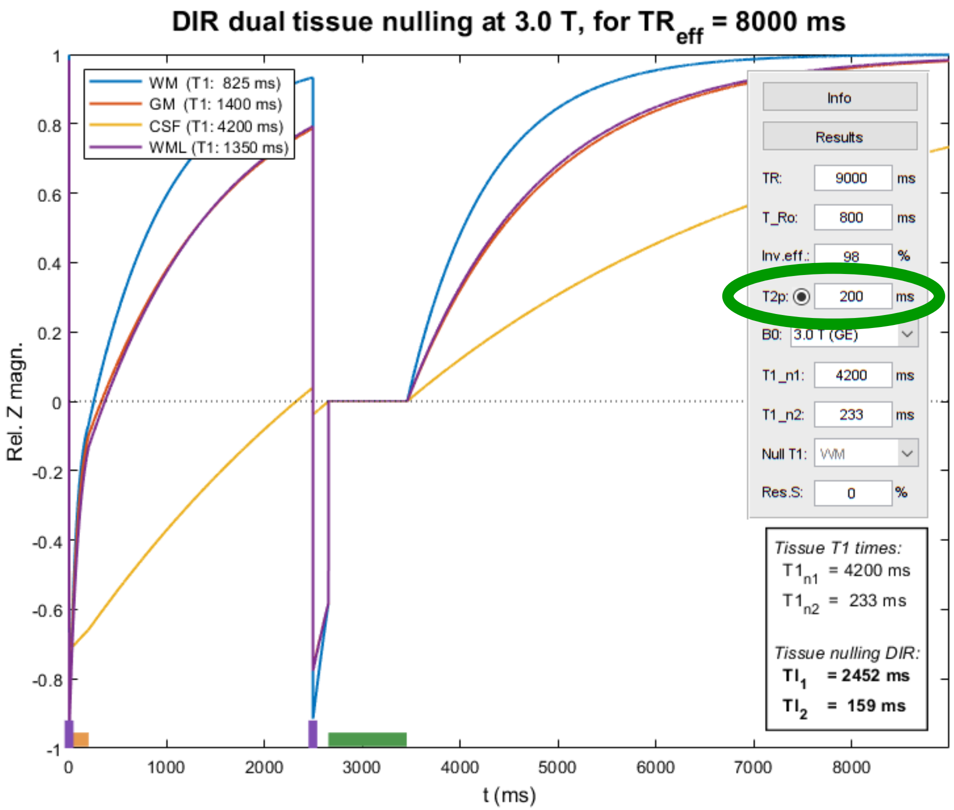
$$\downarrow$$

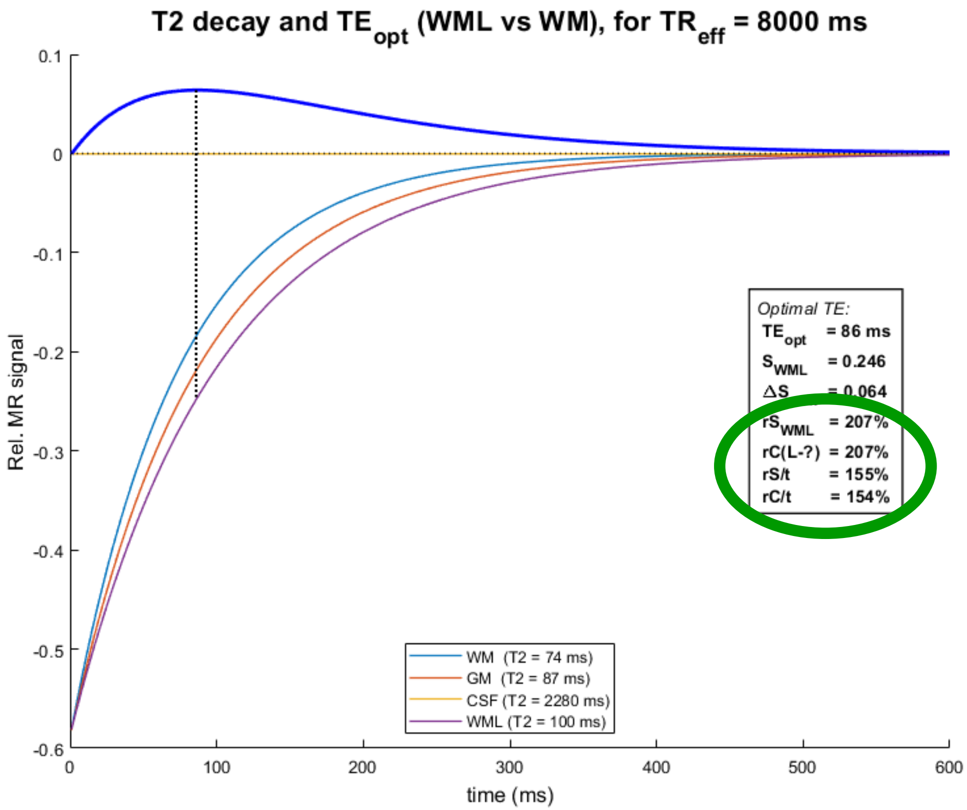
$$S_0 = 0 \;\; \Rightarrow$$

$$TI_{1n} = \underline{T1 \ln\left(\left[\alpha (1+\beta) \right] / \left[-1 + (1+\alpha) E_{TI_2} + \alpha\beta E_{TR} \right] \right)}$$

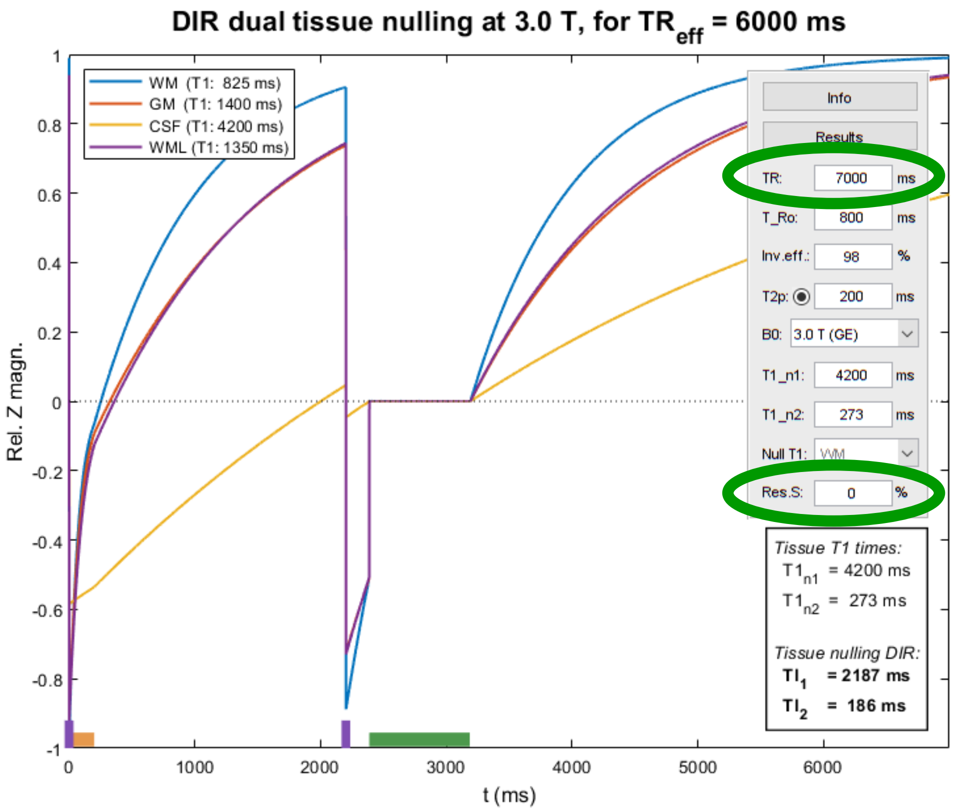
$$Q.E.D.$$

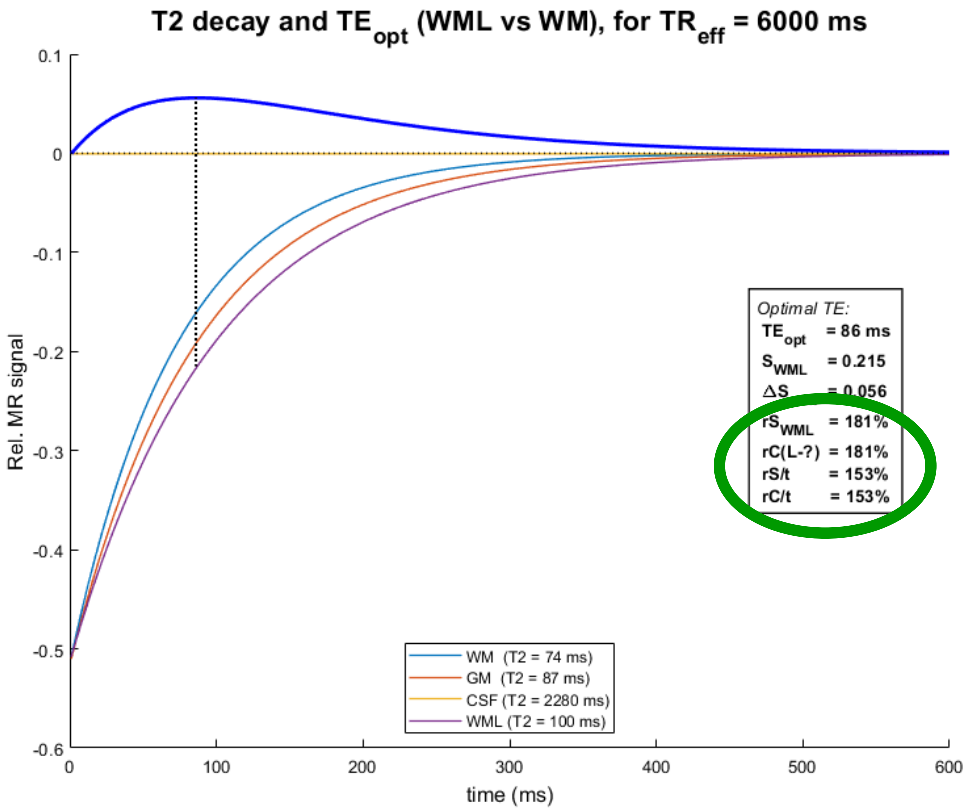
TrueT2-DIR – From Zero To Hero!





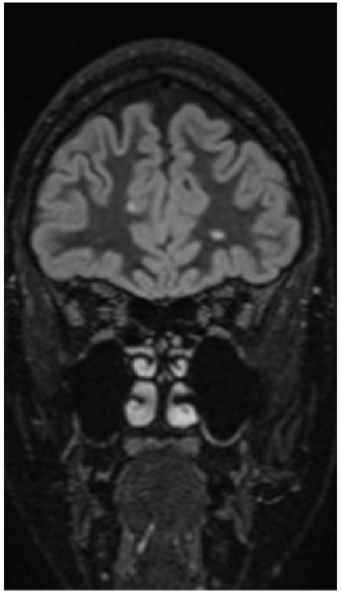
TrueT2-DIR – From Zero To Hero!

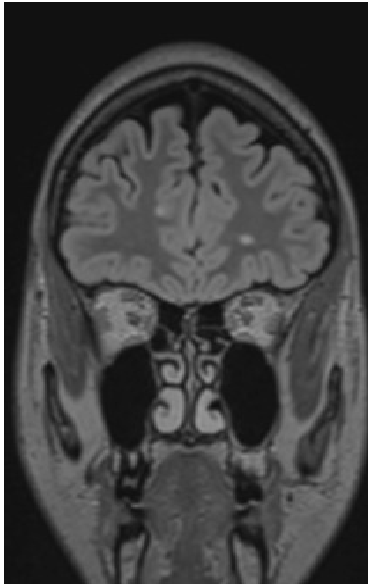




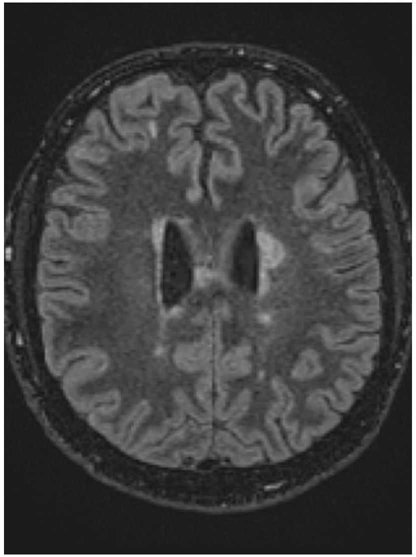
TrueT2-DIR vs FLAIR images

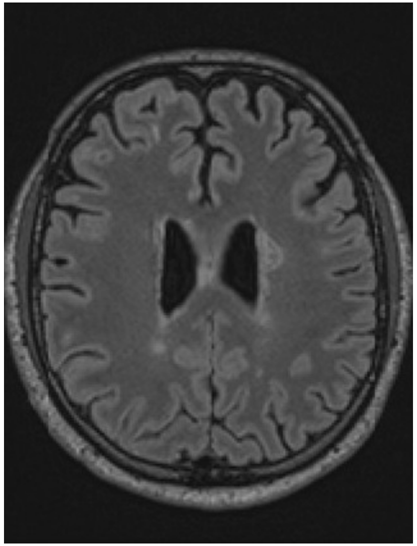
TrueT2





FLAIR





CNR/t at least as good as FLAIR; better WML–GM CNR?!

DIR 4 Dummies! 😁🤞

TrueT2-DIR TI calculator for **3T GE** using the IR-Calc simulator

~ Simulations and regressions by Øystein Bech and Wibeke Nordhøy ~

TI1 [ms] =	1377	× ln(TR) +	226	× ln(T1_L) +	372	× ln(T1_W) +	-13990
TI2 [ms] =	-150	× ln(TR) +	142	× ln(T1_L) +	236	× ln(T1_W) +	-1084
T1n [ms] =	-221	× ln(TR) +	207	× ln(T1_L) +	346	× ln(T1_W) +	-1596

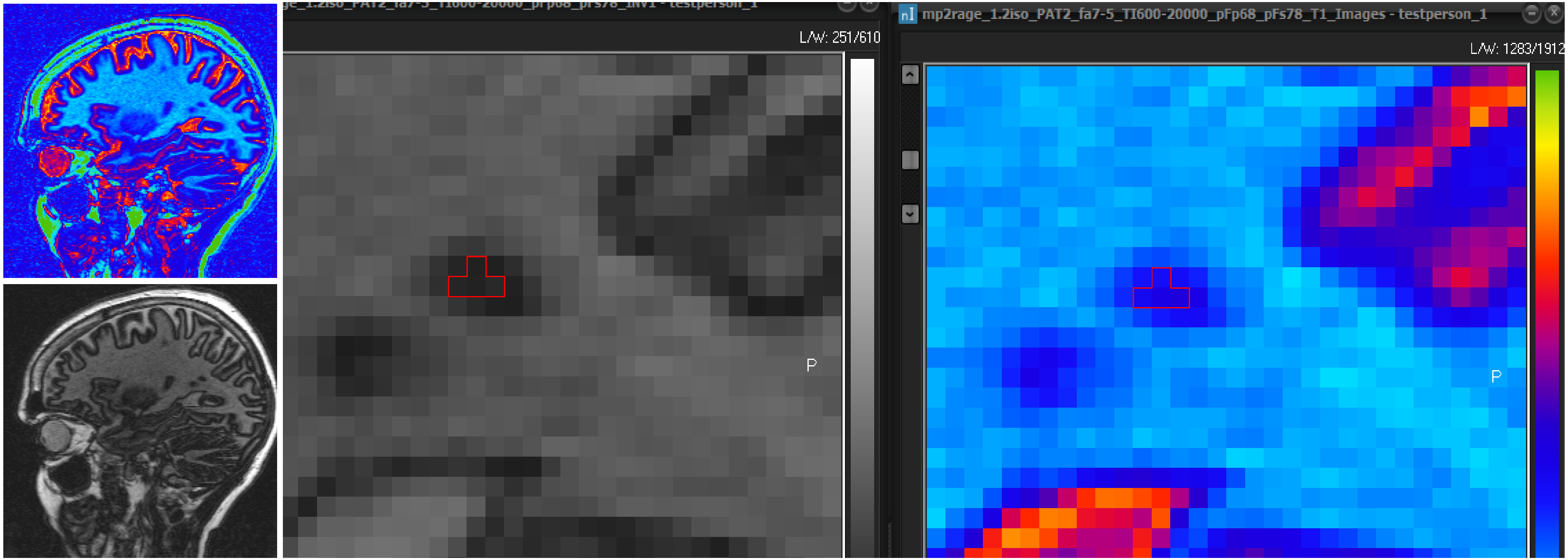
Editable parameters:		
TR	8000	ms
T1_WML	1400	ms
T1_WM	850	ms

Calculated parameters:		
TI1	2536	ms
TI2	189	ms
T1_null	249	ms

Non-editable parameters:		
B0/System	3	T GE
T1_CSF	5000	ms
T1_GM	1400	ms
T1_WM	800–900	ms
T1_WML	1200–1600	ms
TR	5000–9000	ms
T_Readout	850	ms
T_T2prep	200	ms
T_Inv	16	ms
Inv. Eff.	100	%

Accessible spreadsheet tools, based on simulation regressions

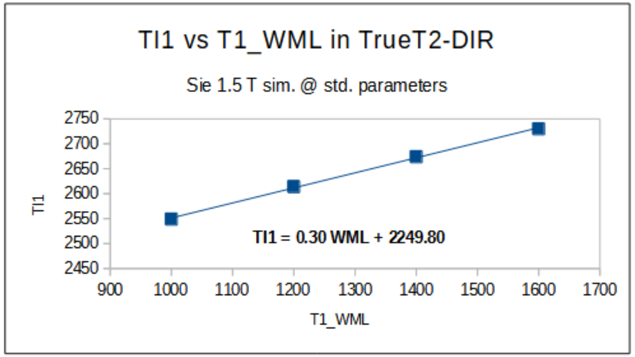
DIR 4 Dummies!? 😎



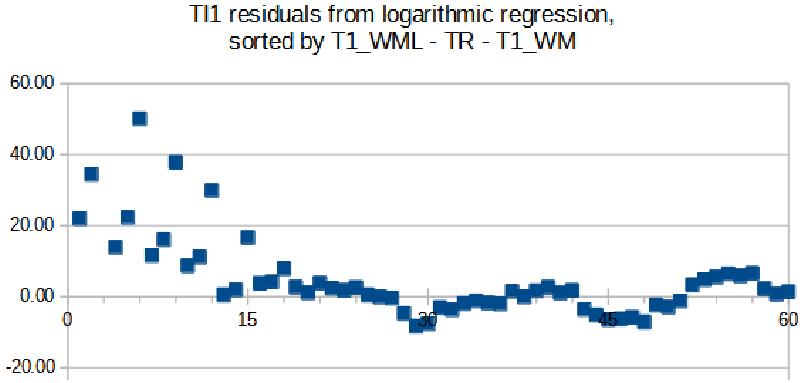
Measuring WM and WML T1 values w/ MP2RAGE MRI

DIR 4 Dummies???

T1_WM	T1_WML	TR	T1n	TI1	TI2	rS≈rC	r[S|C]NR/t	
600	1000	8000	169	2520	133	122	122	
660	1000	8000	193	2549	150	117	117	
750	1000	8000	229	2593	175	110	110	
850	1000	8000	270	2642	203	102	102	
600	1200	8000	218	2579	167	104	104	
660	1200	8000	247	2614	187	100	100	(standard)
750	1200	8000	289	2665	216	94	94	
850								
600								
660								
750								
850								
600								
660								
750								
850								



Regression	WM&WML&TR vs.		TI1	@ T1_GM 1400 ms; T1_CSF 5000 ms			
Regression Statistics				LINEST raw output			
R^2	1.000	Regression M▸	Logarithmic	1377.191	226.070	372.295	-13990.063
Standard Error	4.26	Confidence le▸	0.950	3.059	5.405	13.211	100.987
Count of X vari▸	3	TINV ± factor	2.02	1.000	4.26	#N/A	#N/A
Observations	45			68425.1	41	#N/A	#N/A
Adjusted R^2	1.000			3728656.1	744.7	#N/A	#N/A
Analysis of Variance (ANOVA)						Sim.	Est.
	df	SS	MS	F	Signific. F	2375	2373.3
Regression	3	3728656.1	1242885.4	68425.1	7.47E-76	2516	2513.4
Residual	41	744.7	18.2			2537	2536.0
Total	44	3729400.8				2559	2557.2
	Coefficients	Standard Er▸	t-Statistic	P-value	Lower 95%	Upper 95%	±
Intercept	-13990	101	-138.5	2.2E-56	-14194	-13786	204
LN(T1_WM)	372.3	13.2	28.2	1.9E-28	345.6	399.0	26.7
LN(T1_WML)	226.1	5.4	41.8	3.0E-35	215.2	237.0	10.9
LN(TR)	1377.2	3.1	450.3	2.3E-77	1371.0	1383.4	6.2



- Good regressions may be hard to get right, but…
- Their results are easy to use!