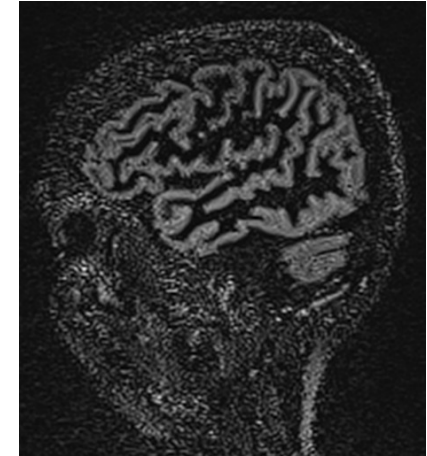
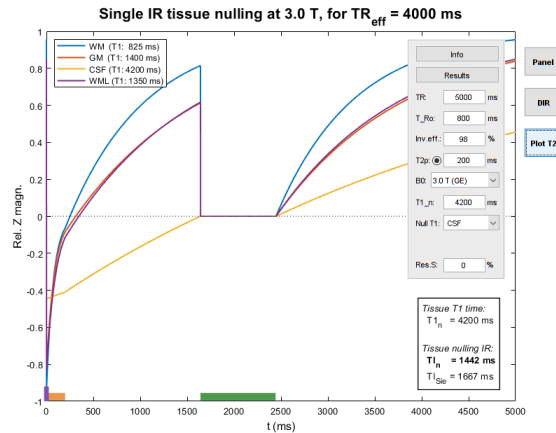


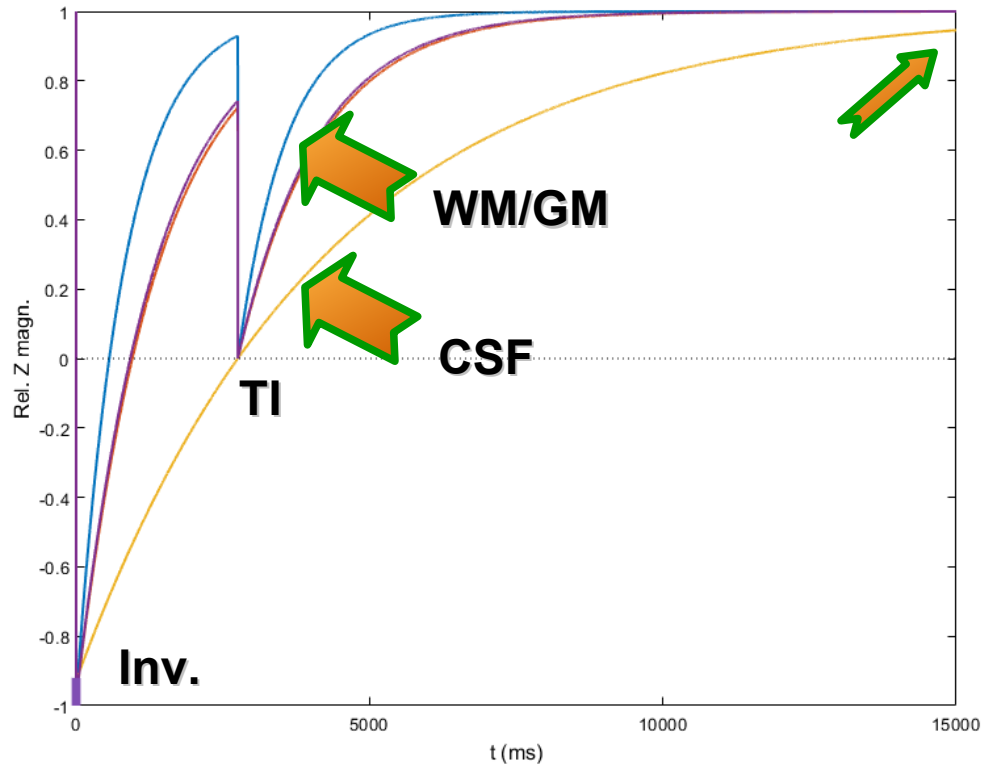
MR Inversion Recovery Calculation

Simulering og optimalisering
av inversjonssekvenser
i MatLab



Øystein B Gadmar & Wibeke Nordhøy

FLAIR – Fluid Attenuated Inv. Recovery



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} ; M_t = M_0 - (M_0 - M_1)E_t$$

$$S \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - (1 - -1)E_{TI} = 1 - 2E_{TI}$$

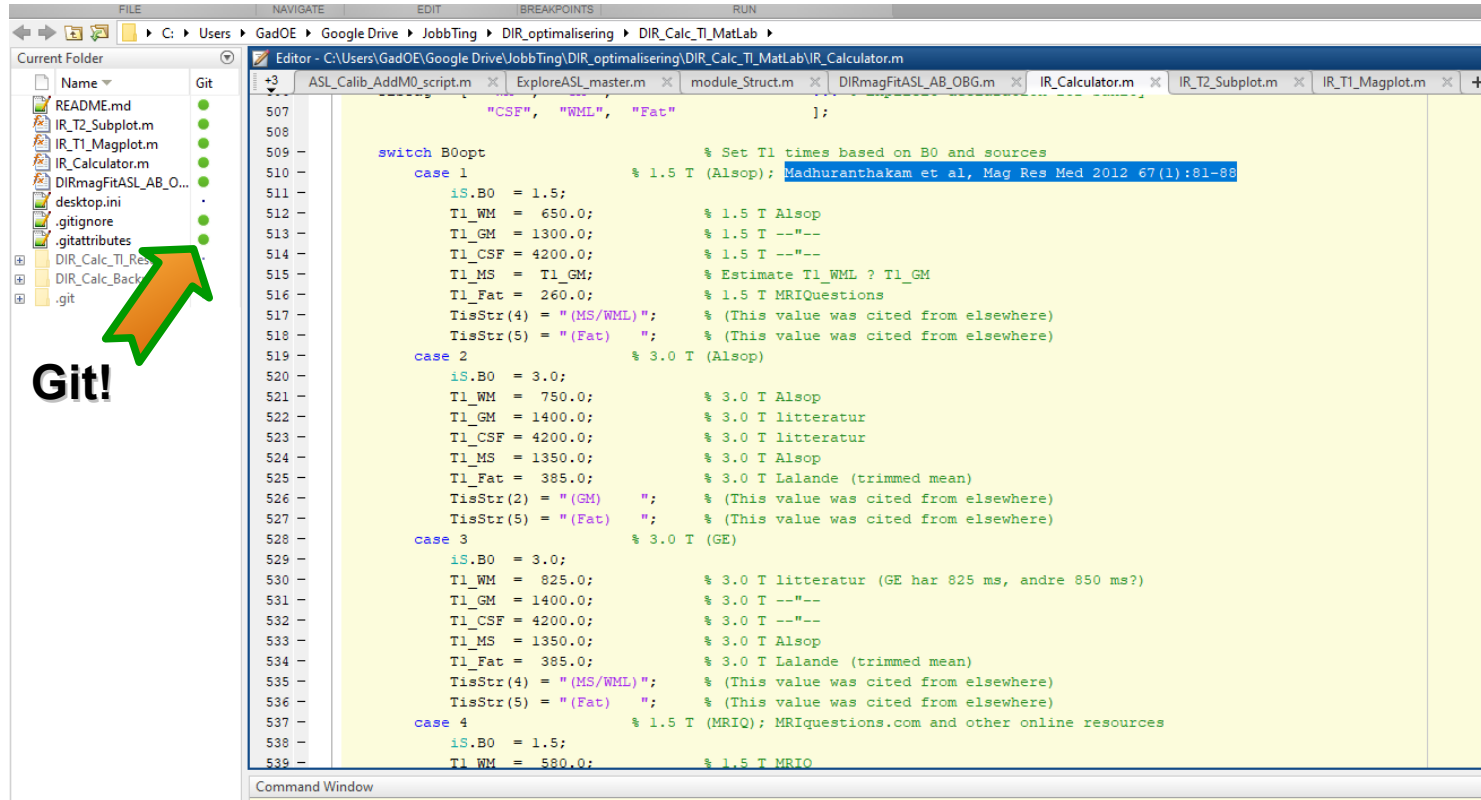
↓

$$S_0 = 0 \Rightarrow E_{TI} = e^{-\frac{TI}{T1}} = \frac{1}{2}$$

$$\Rightarrow TI = \underline{T1 \cdot \ln(2)}$$

Q.E.D.

Our IR Calculator for MatLab

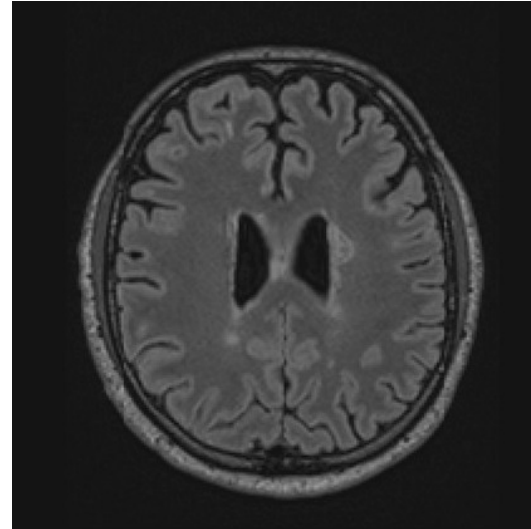
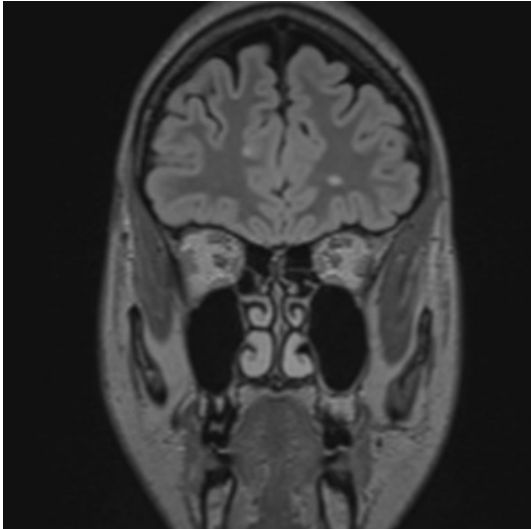


The screenshot displays the MATLAB IDE interface. The left sidebar shows the 'Current Folder' and 'Git' columns. A green arrow points to the 'Git' column, with the text 'Git!' next to it. The main editor window shows the 'IR_Calculator.m' script, which is a MATLAB script for calculating T1 times based on B0 and sources. The script includes comments and code for various T1 values and sources.

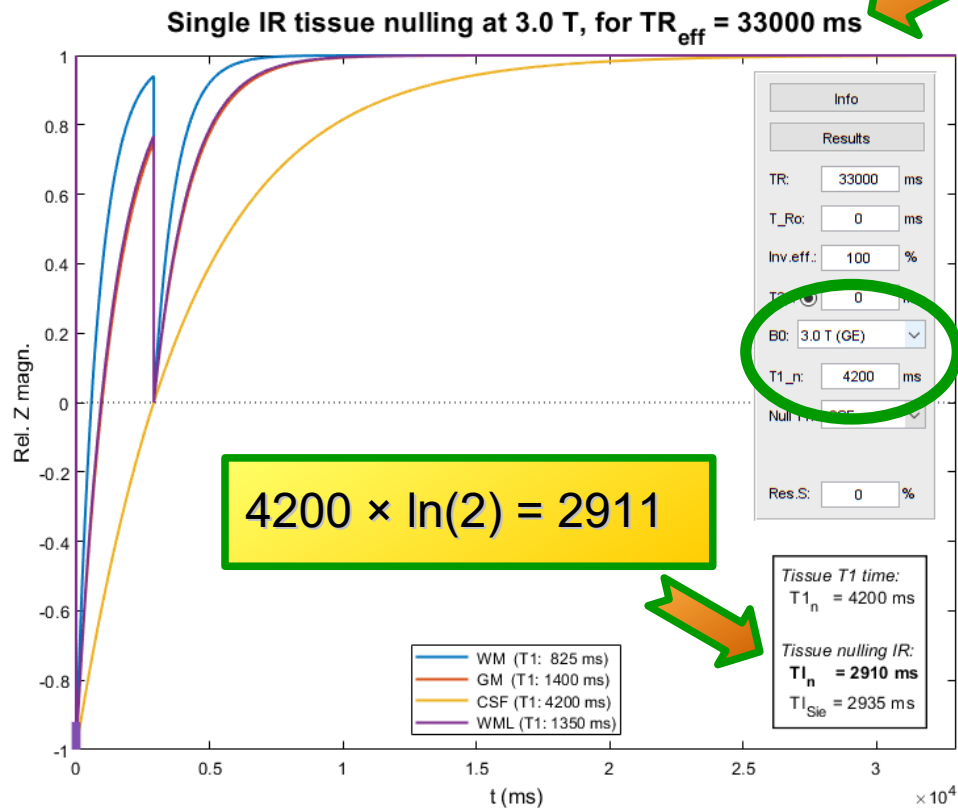
```
507         "CSF", "WML", "Fat"    ];
508
509     switch B0opt                % Set T1 times based on B0 and sources
510     case 1                      % 1.5 T (Alsop); Madhuranthakam et al, Mag Res Med 2012 67(1):81-88
511         iS.B0 = 1.5;
512         T1_WM = 650.0;          % 1.5 T Alsop
513         T1_GM = 1300.0;         % 1.5 T ---
514         T1_CSF = 4200.0;        % 1.5 T ---
515         T1_MS = T1_GM;          % Estimate T1_WML ? T1_GM
516         T1_Fat = 260.0;         % 1.5 T MRIQuestions
517         TisStr(4) = "(MS/WML) "; % (This value was cited from elsewhere)
518         TisStr(5) = "(Fat) ";   % (This value was cited from elsewhere)
519     case 2                      % 3.0 T (Alsop)
520         iS.B0 = 3.0;
521         T1_WM = 750.0;          % 3.0 T Alsop
522         T1_GM = 1400.0;         % 3.0 T litteratur
523         T1_CSF = 4200.0;        % 3.0 T litteratur
524         T1_MS = 1350.0;        % 3.0 T Alsop
525         T1_Fat = 385.0;         % 3.0 T Lalande (trimmed mean)
526         TisStr(2) = "(GM) ";   % (This value was cited from elsewhere)
527         TisStr(5) = "(Fat) ";   % (This value was cited from elsewhere)
528     case 3                      % 3.0 T (GE)
529         iS.B0 = 3.0;
530         T1_WM = 825.0;          % 3.0 T litteratur (GE har 825 ms, andre 850 ms?)
531         T1_GM = 1400.0;         % 3.0 T ---
532         T1_CSF = 4200.0;        % 3.0 T ---
533         T1_MS = 1350.0;        % 3.0 T Alsop
534         T1_Fat = 385.0;         % 3.0 T Lalande (trimmed mean)
535         TisStr(4) = "(MS/WML) "; % (This value was cited from elsewhere)
536         TisStr(5) = "(Fat) ";   % (This value was cited from elsewhere)
537     case 4                      % 1.5 T (MRIQ); MRIQuestions.com and other online resources
538         iS.B0 = 1.5;
539         T1_WM = 580.0;          % 1.5 T MRIO
```

Fluid Attenuated Inversion Recovery

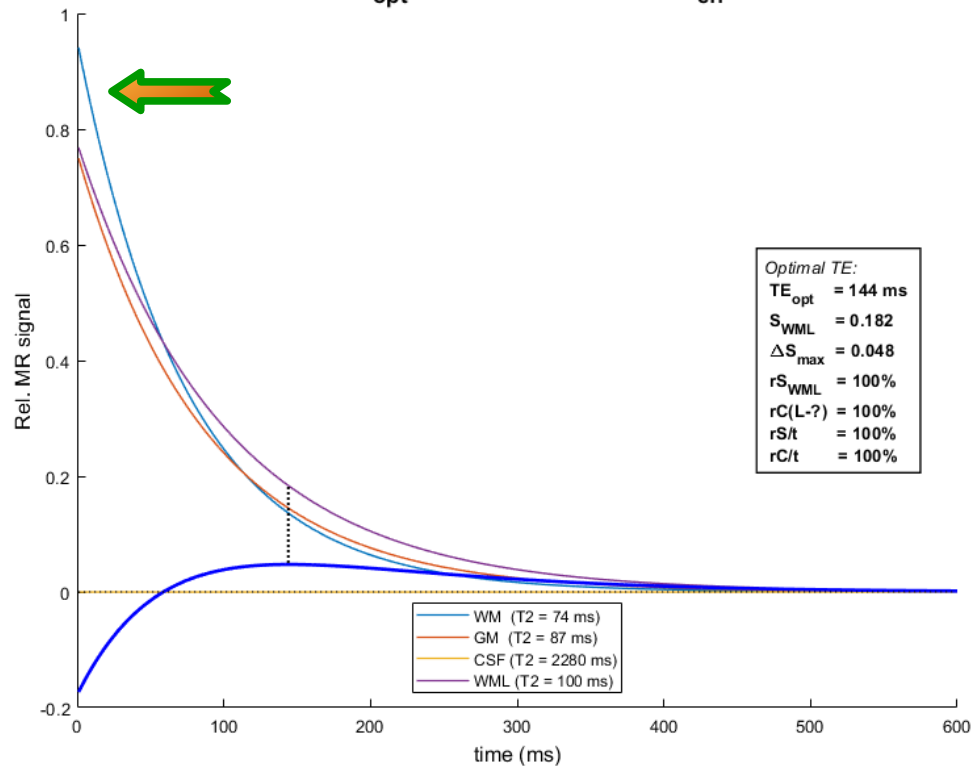
- One IR pulse, then a TI delay before readout
- Ours: 3D T2-weighted readout (SPACE/CUBE/VIEW)
- T1 weighting from IR counteracts the desired T2 contrast...



“Simple” FLAIR



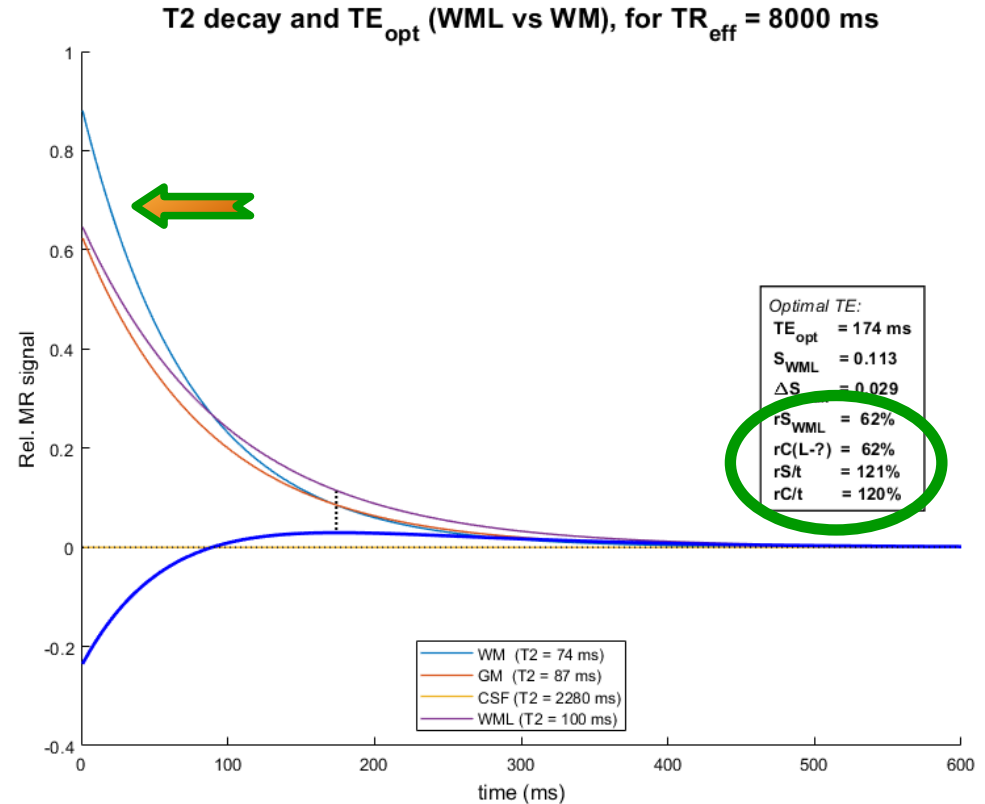
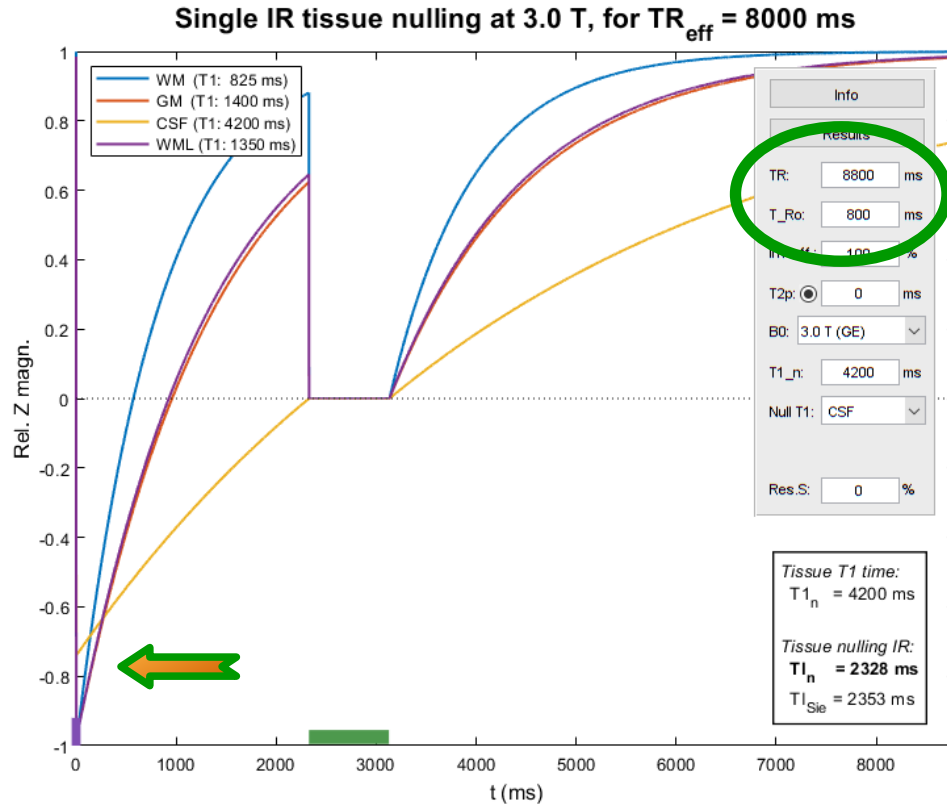
T2 decay and TE_{opt} (WML vs WM), for $TR_{eff} = 33000$ ms



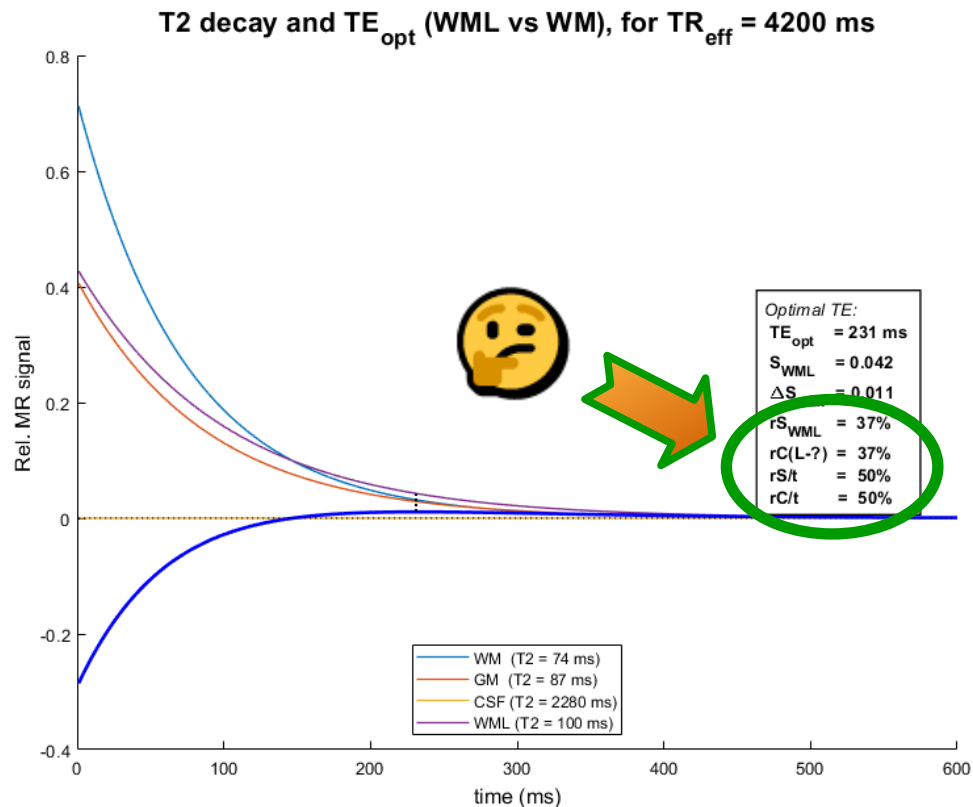
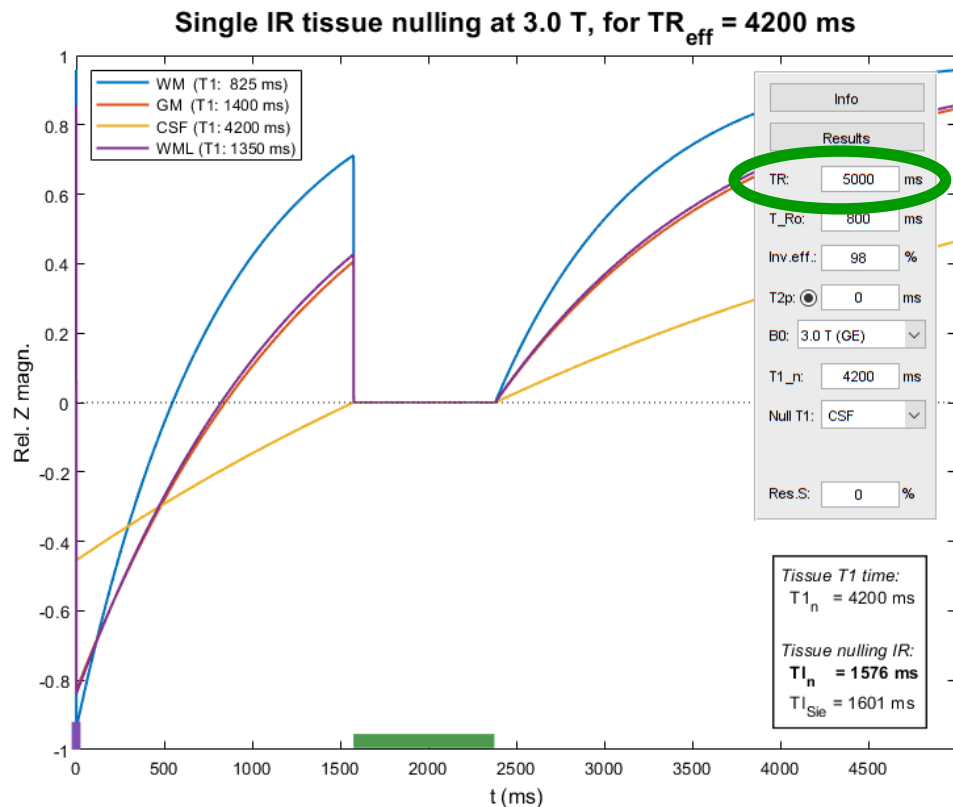
Notes On Noise

- S = Signal strength at TE
- C = Difference between S_{WML} and S_{WM} (usually)
- N = Noise $\propto \sqrt{t}$
- $\text{CNR} = C/N \propto t/\sqrt{t} = \sqrt{t}$
- Contrast efficiency: $\text{CNR}/t \propto C/\sqrt{\text{TR}}$
- CNR/t – a good end point (but also look to SNR/t)

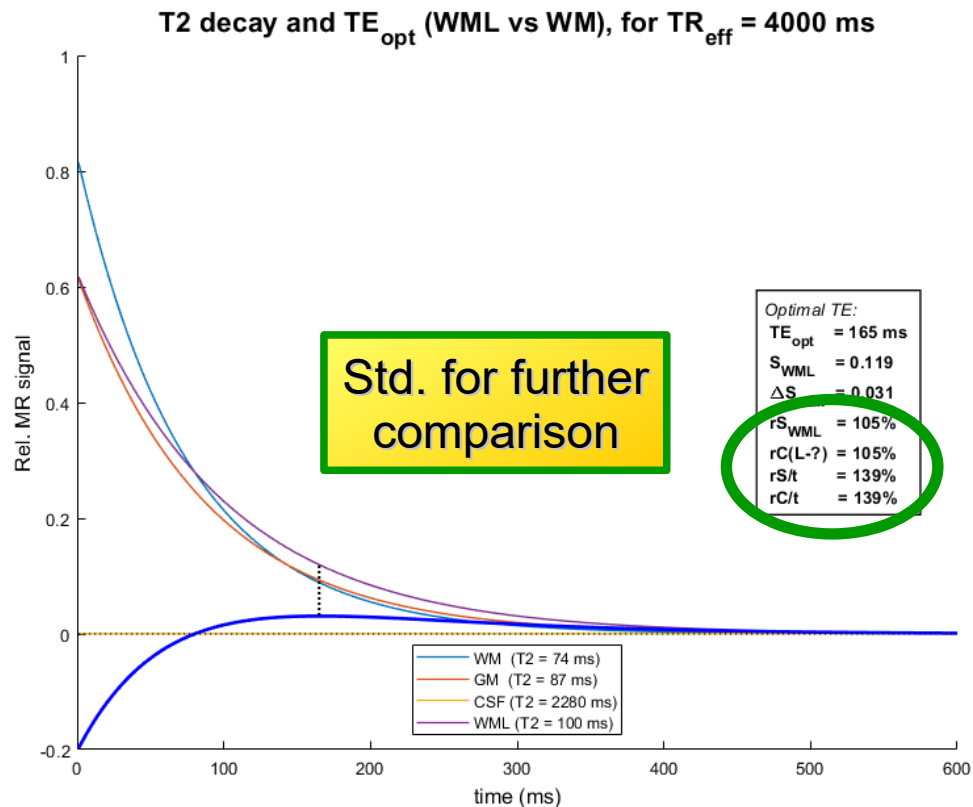
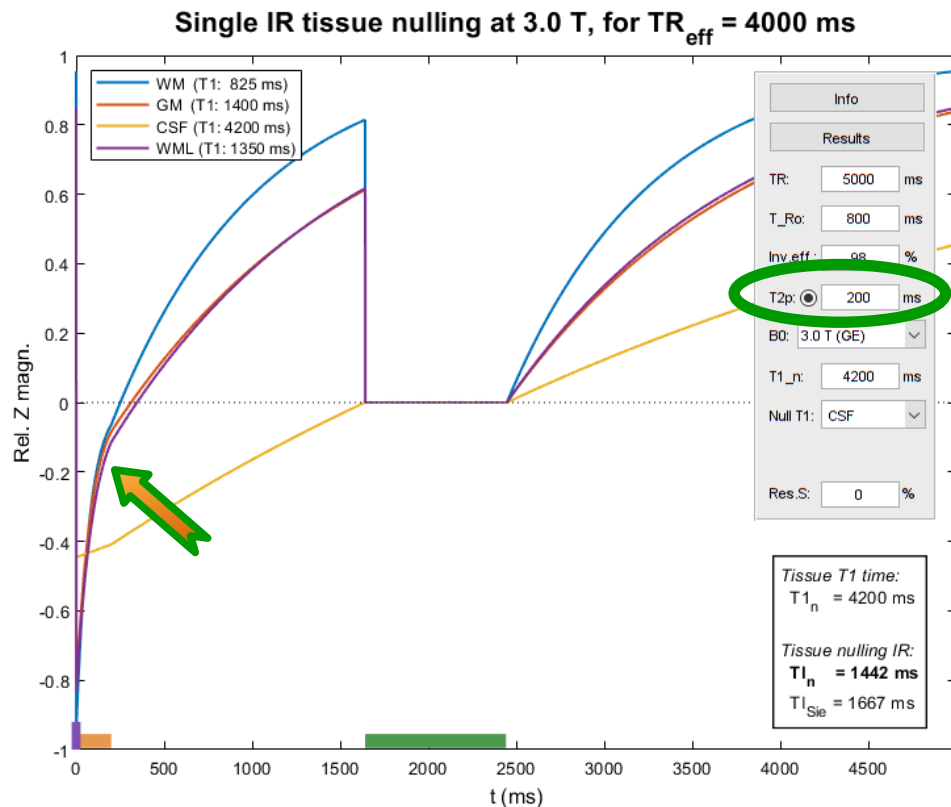
Realistic 3D FLAIR



“Vendor” 3D FLAIR w/ short TR???

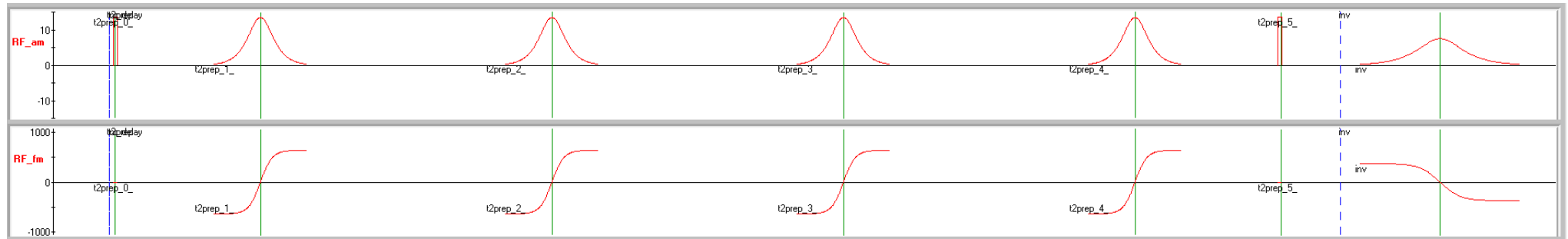


“Vendor” 3D FLAIR w/ T2 Prep.!

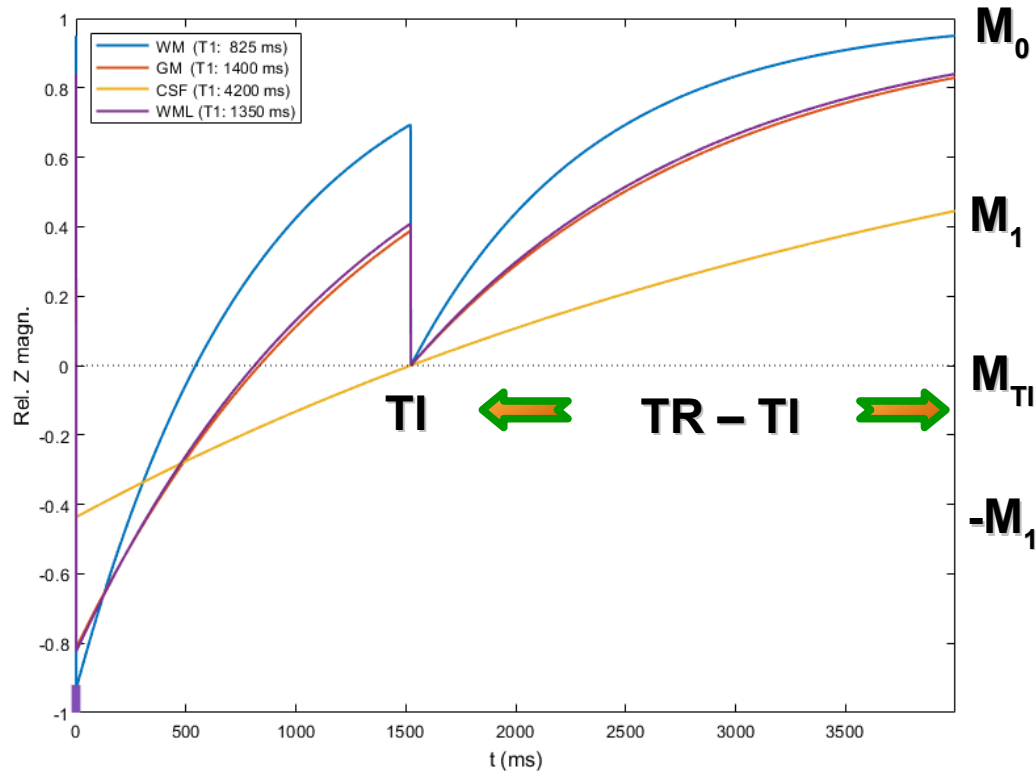


T2 prep. – How?

- $90^\circ_x - [180^\circ_y]_n - -90^\circ_x$ to achieve spin lock
- T2P and inversion are interchangeable
- Pure T2 decay used for Mz preparation
- CSF hardly affected (long T2), but tissues relax much



IR-Calc (0th: In Real Time)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} ; M_t = M_0 - (M_0 - M_1) E_t$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}}$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \frac{M_1}{M_0}\right) E_{TI} = (1 + E_{TR}) - (1 + 1) E_{TI}$$

↓

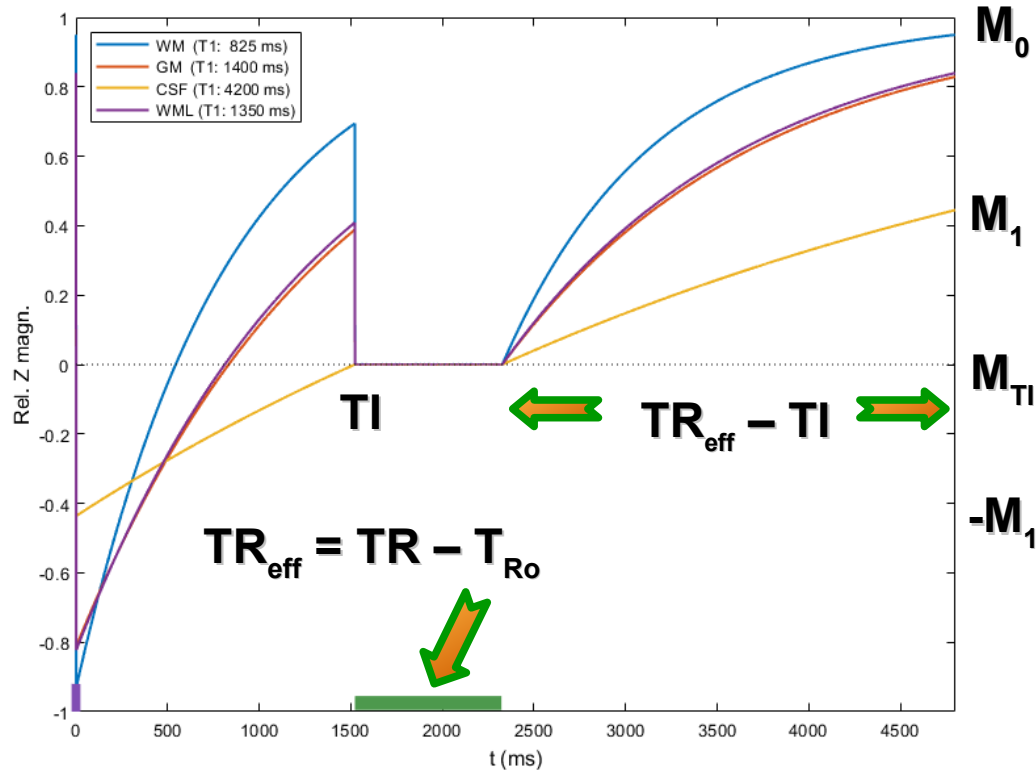
$$S_0 = 0 \Rightarrow E_{TI_n} = e^{-\frac{TI_n}{T1}} = \frac{1 + E_{TR}}{2}$$

$$\Rightarrow TI_n = T1 \left[\ln(2) - \ln(1 + E_{TR}) \right] ; E_{TR} = e^{-\frac{TR}{T1}}$$



Q.E.D.

IR-Calc (1st: Effective TR)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T_1}}$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} ; (TR \rightarrow TR_{eff})$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \frac{M_1}{M_0}\right) E_{TI} = \frac{(1 + E_{TR}) - (1 + 1) E_{TI}}{1}$$

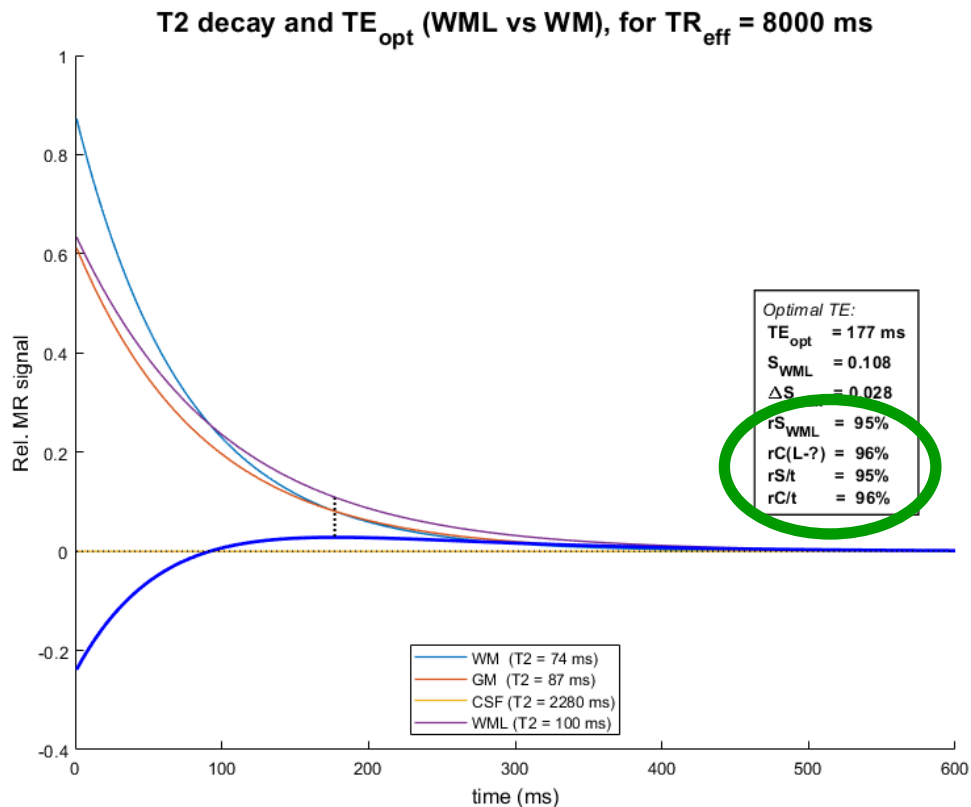
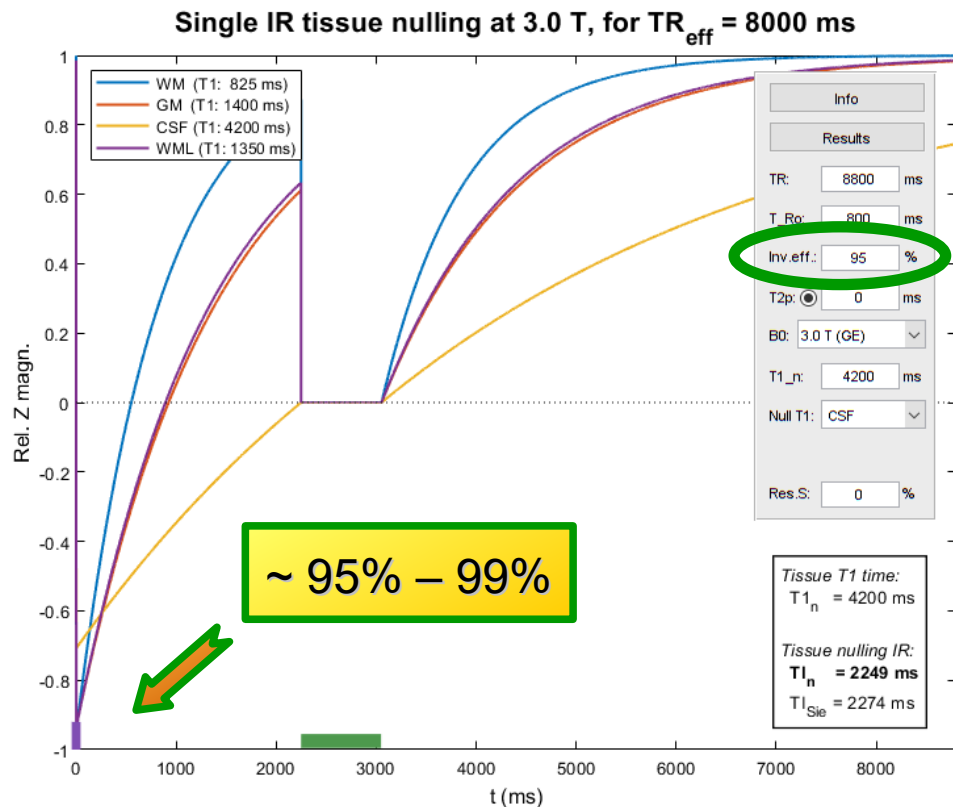
↓

$$S_0 = 0 \Rightarrow E_{TI_n} = e^{-\frac{TIn}{T_1}} = \frac{1 + E_{TR}}{2}$$

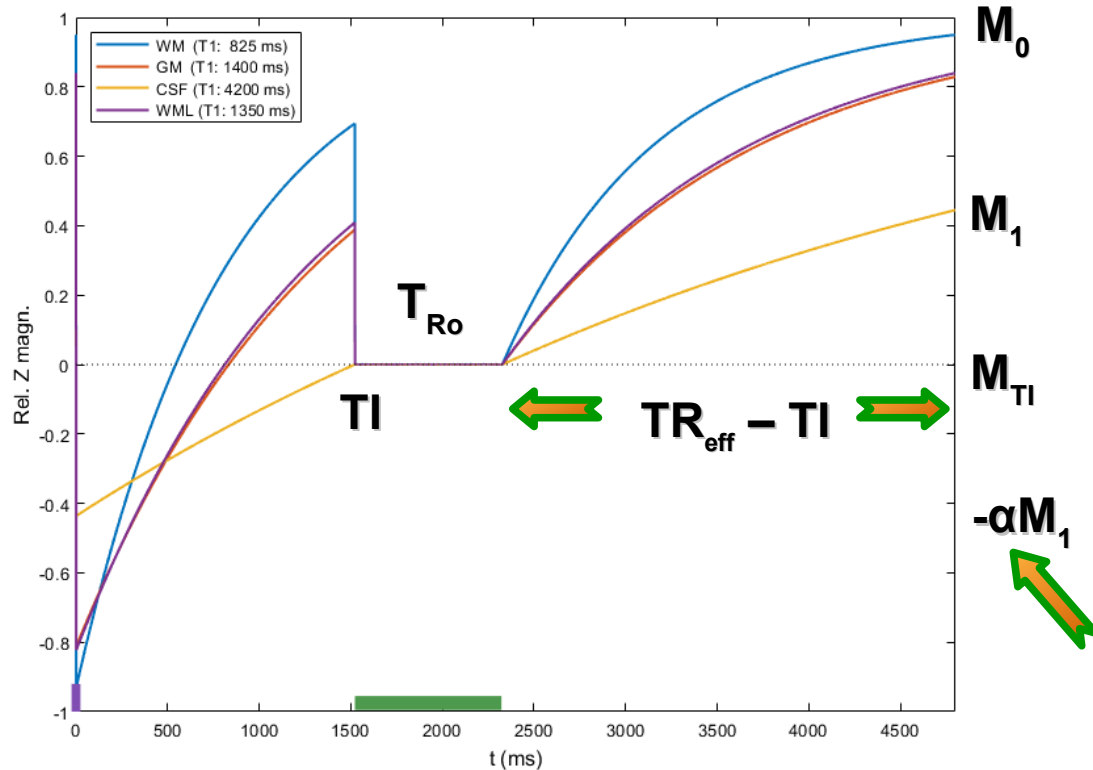
$$\Rightarrow TIn = T1 \left[\ln(2) - \ln(1 + E_{TR}) \right] ; E_{TR} = e^{-\frac{TR_{eff}}{T_1}}$$

Q.E.D.

The efficiency of inversion efficiency?



IR-Calc (2nd: Inefficient Inversion)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} ; \alpha \stackrel{\text{def}}{=} -\cos(FA_{inv})$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} ; (TR \rightarrow TR_{eff})$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_1}{M_0}\right) E_{TI} = (1 + \alpha E_{TR}) - (1 + \alpha) E_{TI}$$

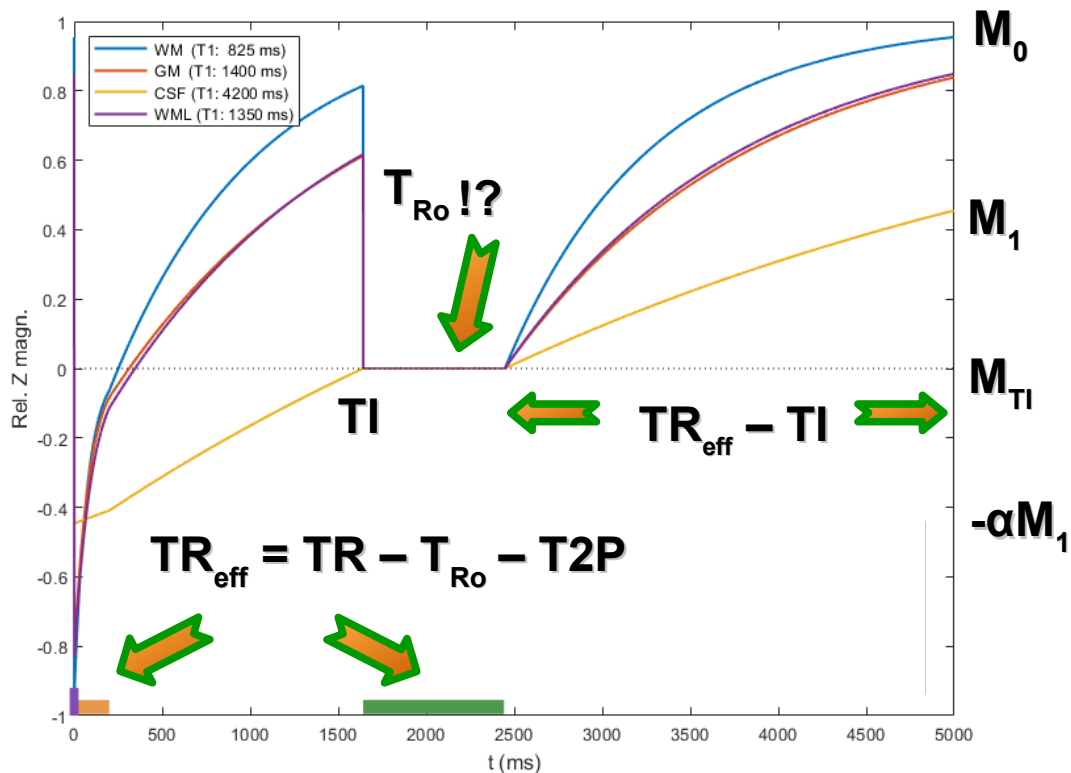
↓

$$S_0 = 0 \Rightarrow E_{TI_n} = e^{-\frac{TI_n}{T1}} = \frac{1 + \alpha E_{TR}}{1 + \alpha}$$

$$\Rightarrow TI_n = T1 \left[\ln(1 + \alpha) - \ln(1 + \alpha E_{TR}) \right]$$

Q.E.D.

IR-Calc (3rd: Be T2 Prepared!)



$$E_t \stackrel{\text{def}}{=} e^{-\frac{t}{T1}} ; \alpha \stackrel{\text{def}}{=} -\cos(FA_{inv}) ; E_{T2P} \stackrel{\text{def}}{=} e^{-\frac{T2P}{T2}}$$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI)} = 1 - \frac{E_{TR}}{E_{TI}} ; (TR \rightarrow TR_{eff})$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_1}{M_0} \right) E_{TI} = \frac{(1 + \alpha E_{TR}) - (1 + \alpha) E_{TI}}{1}$$

↓

$$S_0 = 0 \Rightarrow E_{Tin} = e^{-\frac{TI n}{T1}} = \frac{1 + \alpha E_{TR}}{1 + \alpha}$$

$$\Rightarrow TI_n = T1 \left[\ln(1 + \alpha) - \ln(1 + \alpha E_{TR}) \right]$$

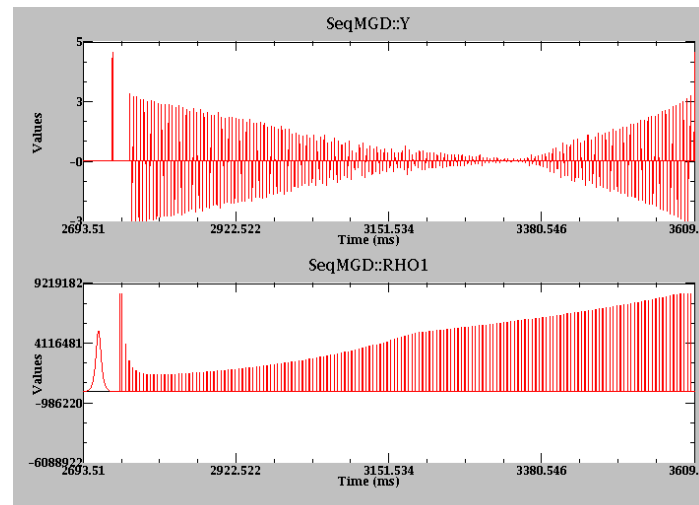
↓

$$\alpha \rightarrow \alpha E_{T2P} \Rightarrow TI_n = T1 \left[\ln(1 + \alpha E_{T2P}) - \ln(1 + \alpha E_{T2P} E_{TR}) \right]$$

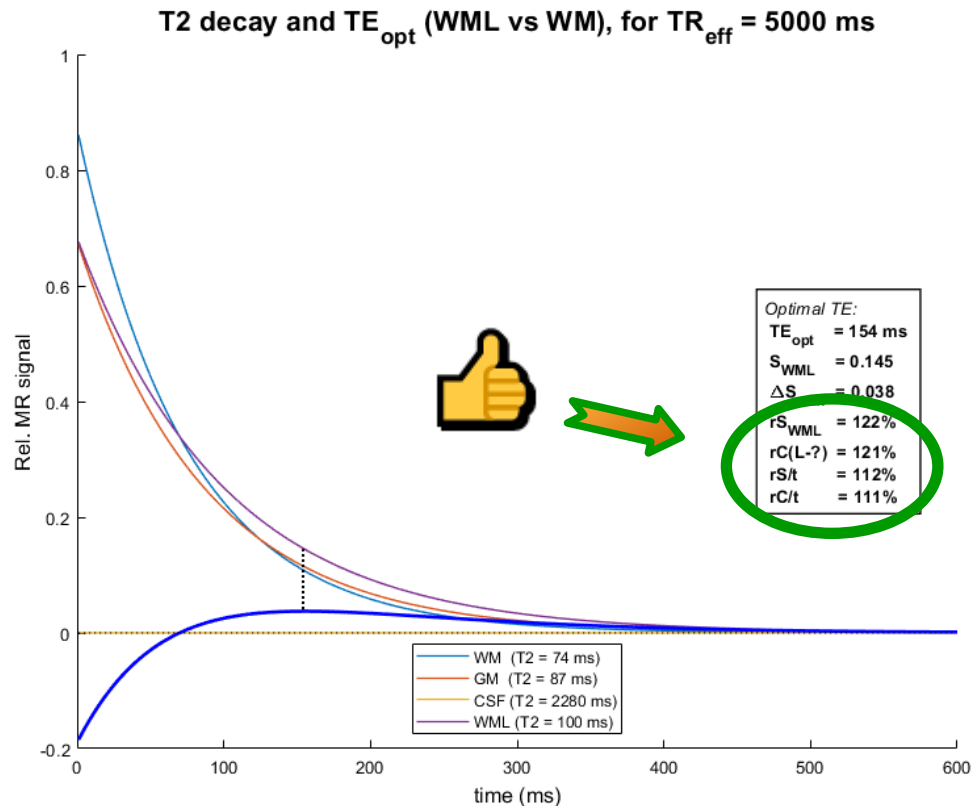
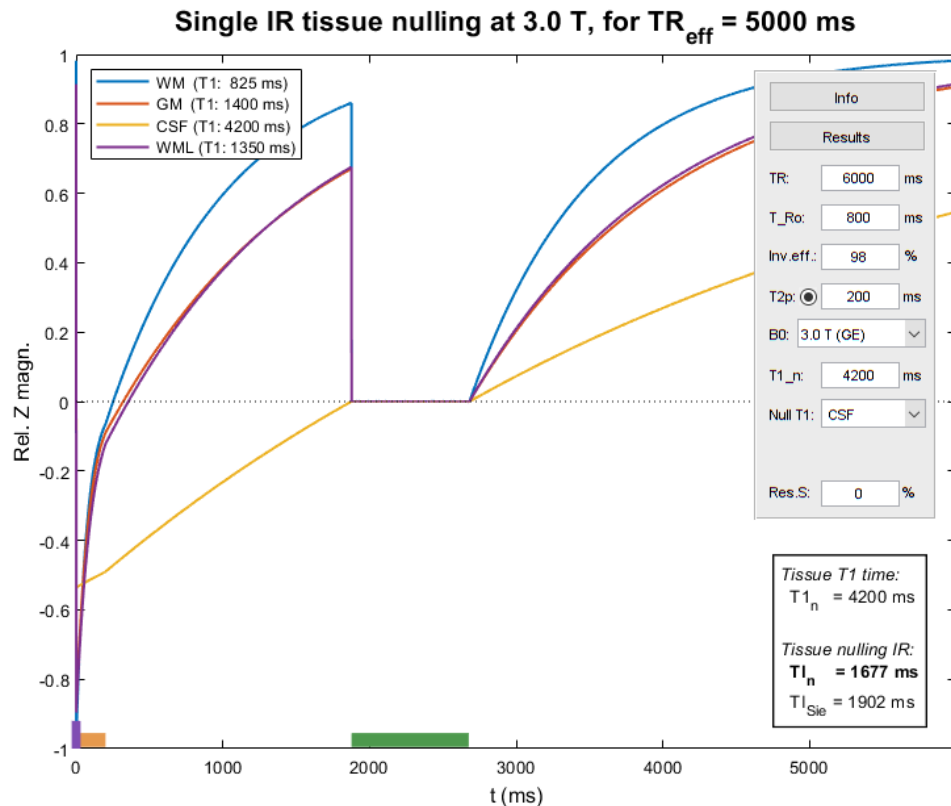
Q.E.D.

3D TSE readout – How?

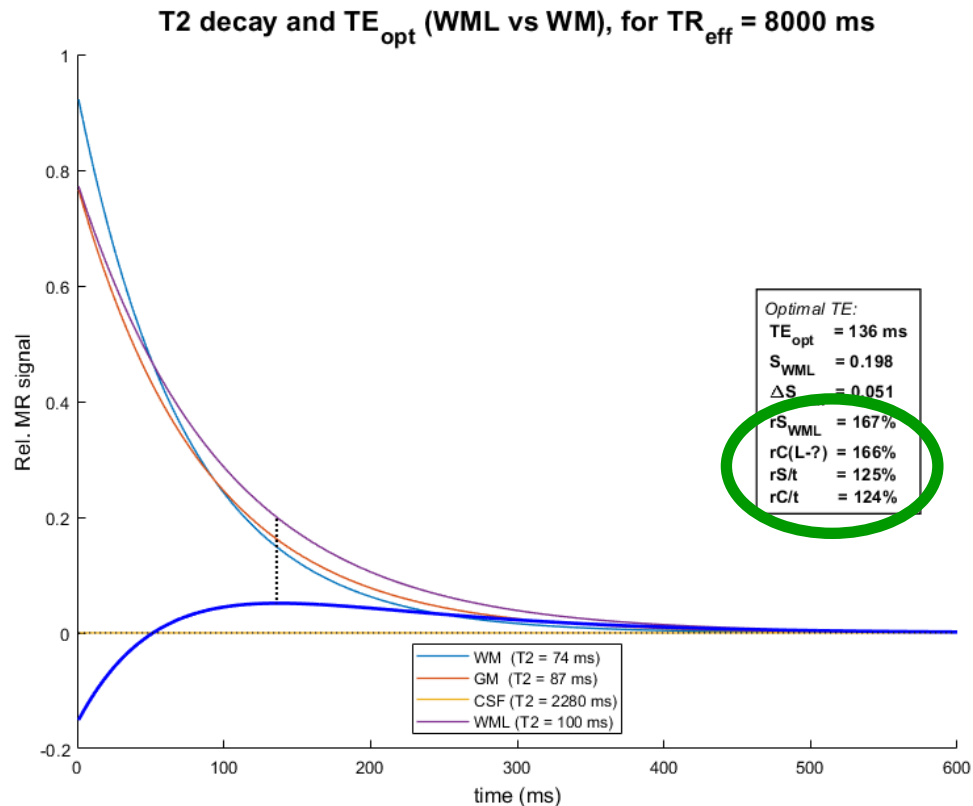
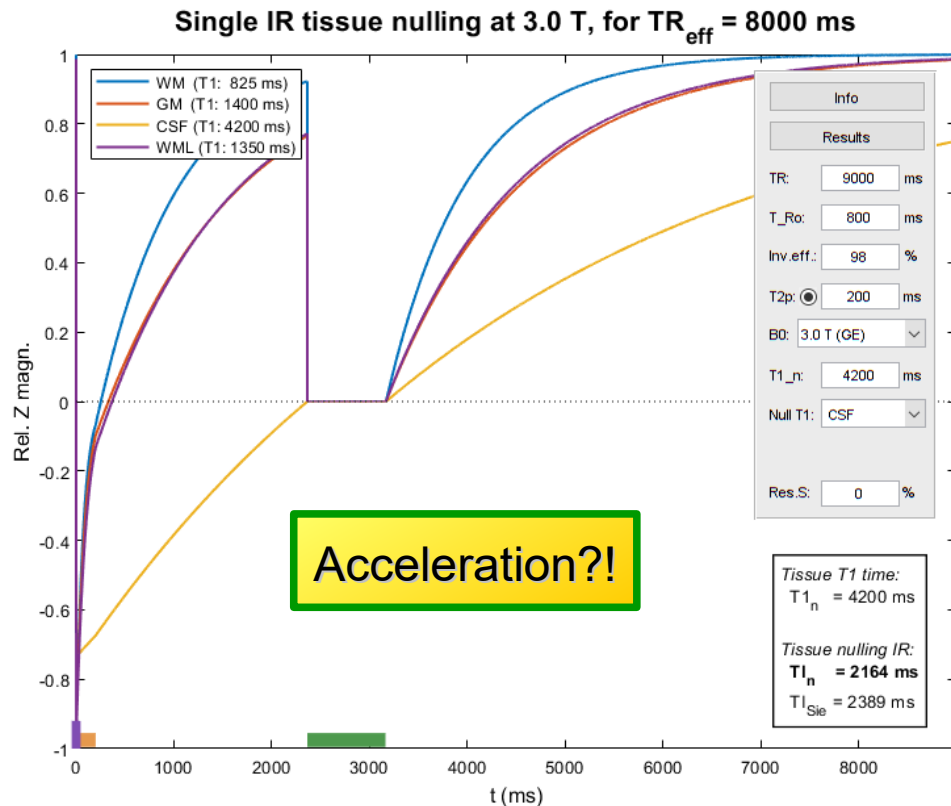
- Complex flip angle sweep to preserve magnetization
- Complex profile ordering to avoid artefacts/blurring
- TE_{eff} estimated; much shorter than zero crossing time!
- Contrast mainly T2 – some T1?!
- M_z *probably* used up over T_{R0} ?
- We're working to understand it...!



TR 6 s – “One up” on the vendor

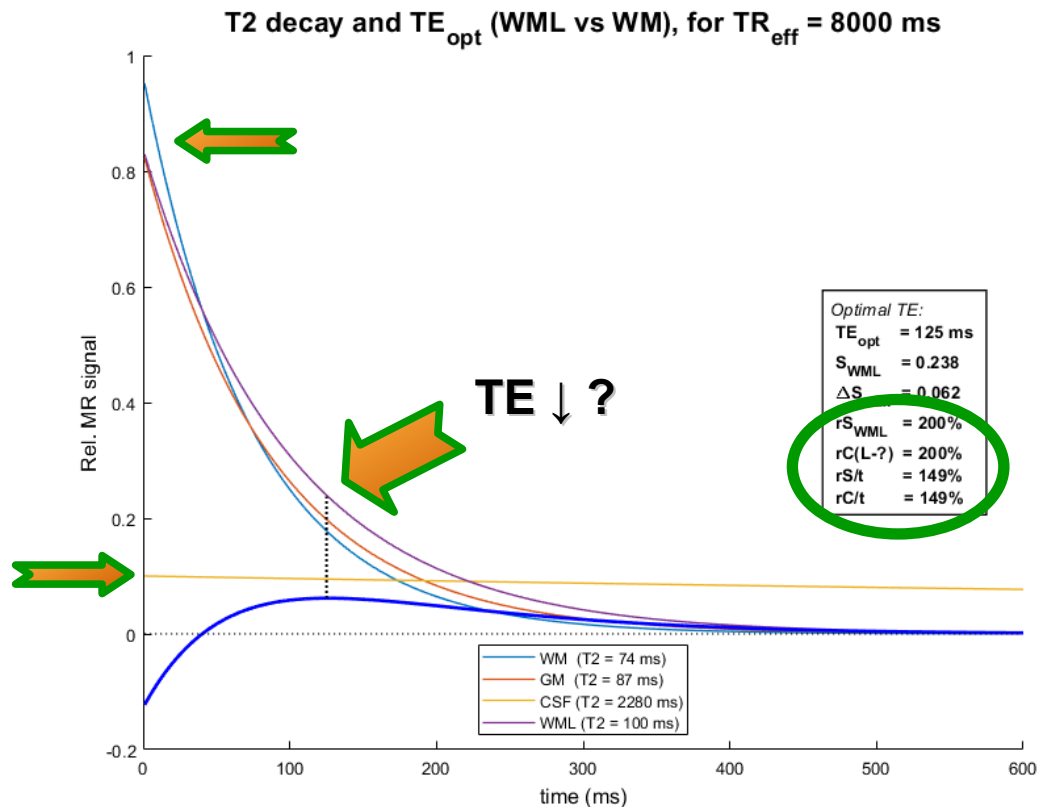
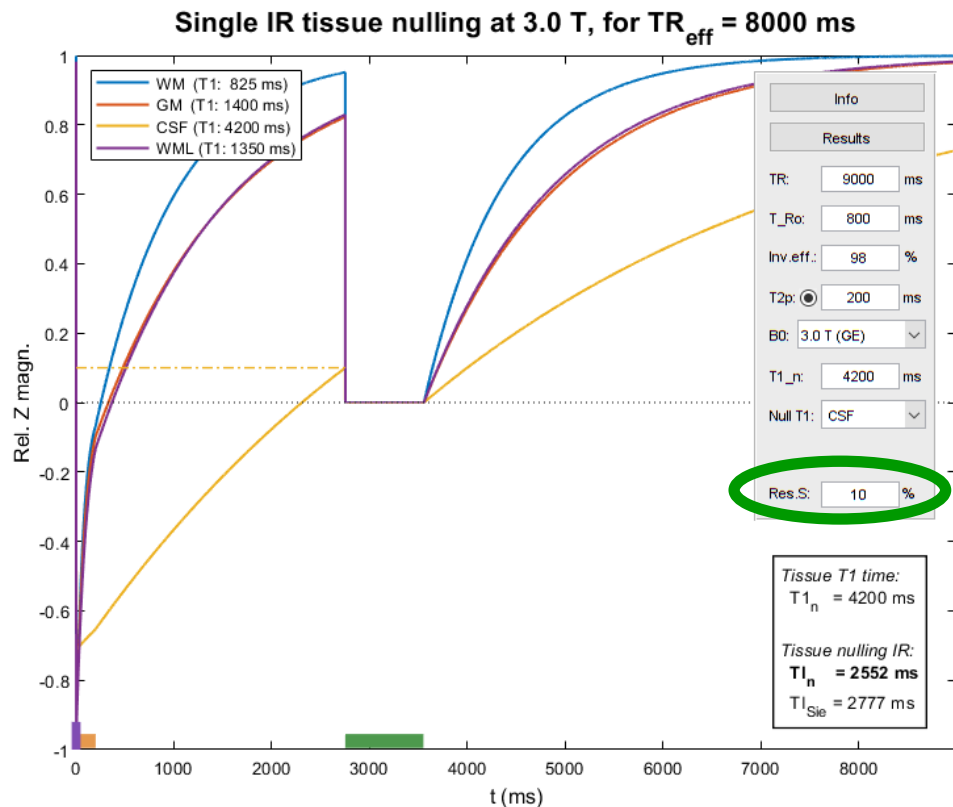


TR 9 s – FLAIR And Back Again?!

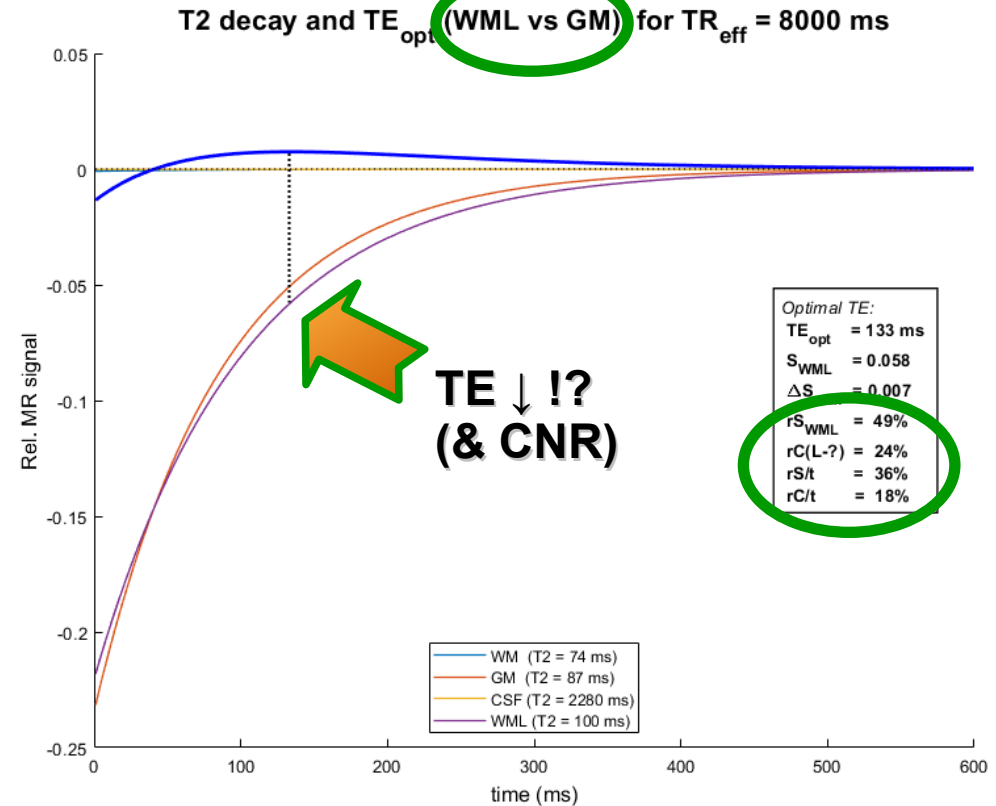
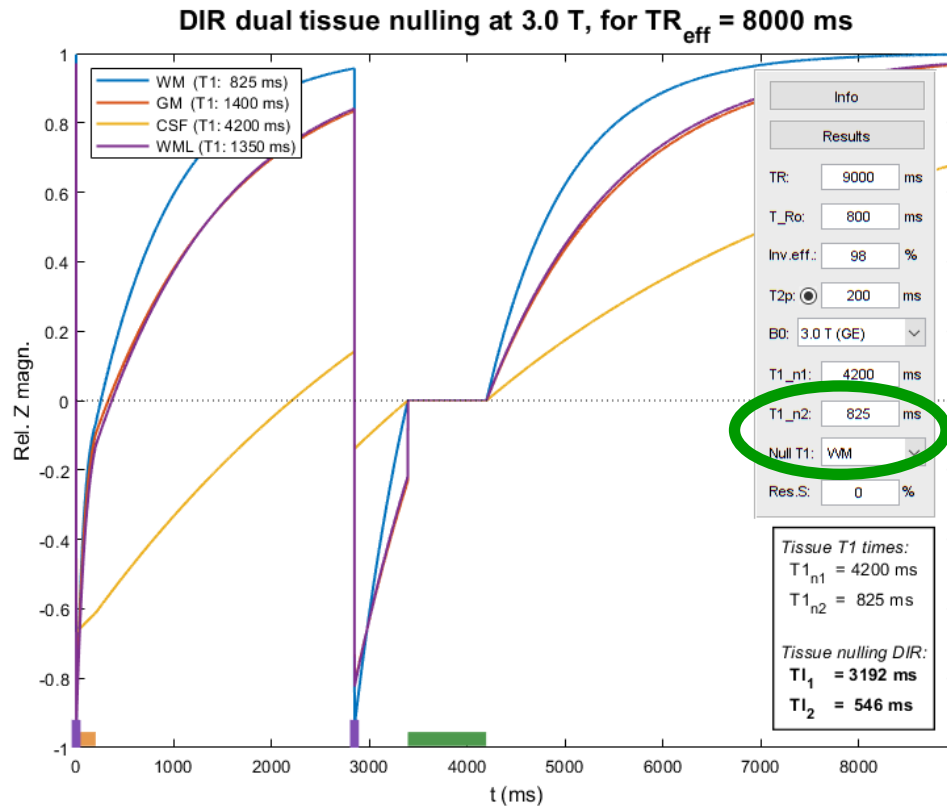


Fluid Attenuated IR ... by how much?

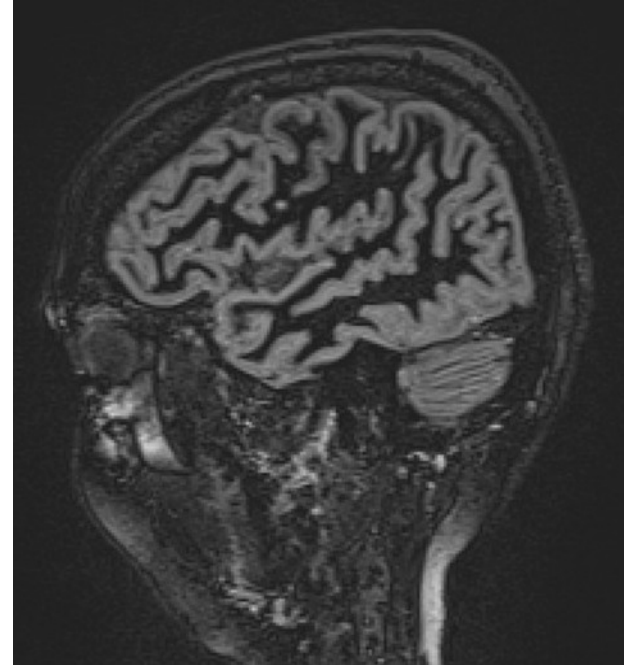
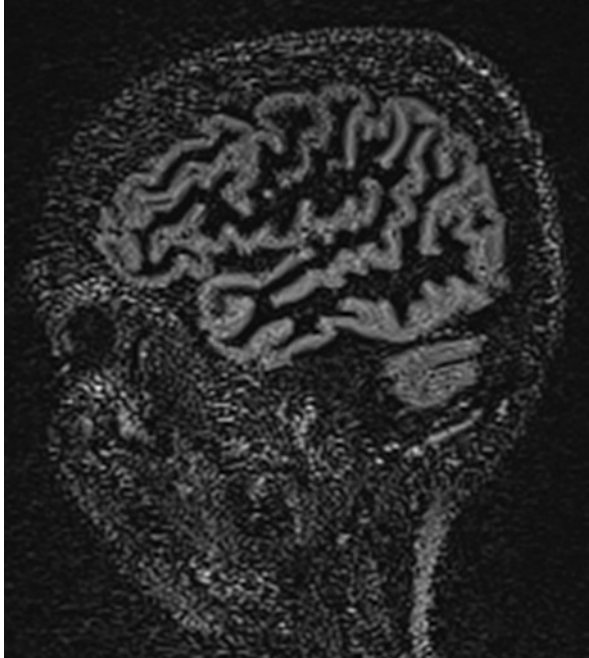
"Images that computers like"



Dual IR: Two for the price of WM (and SNR)

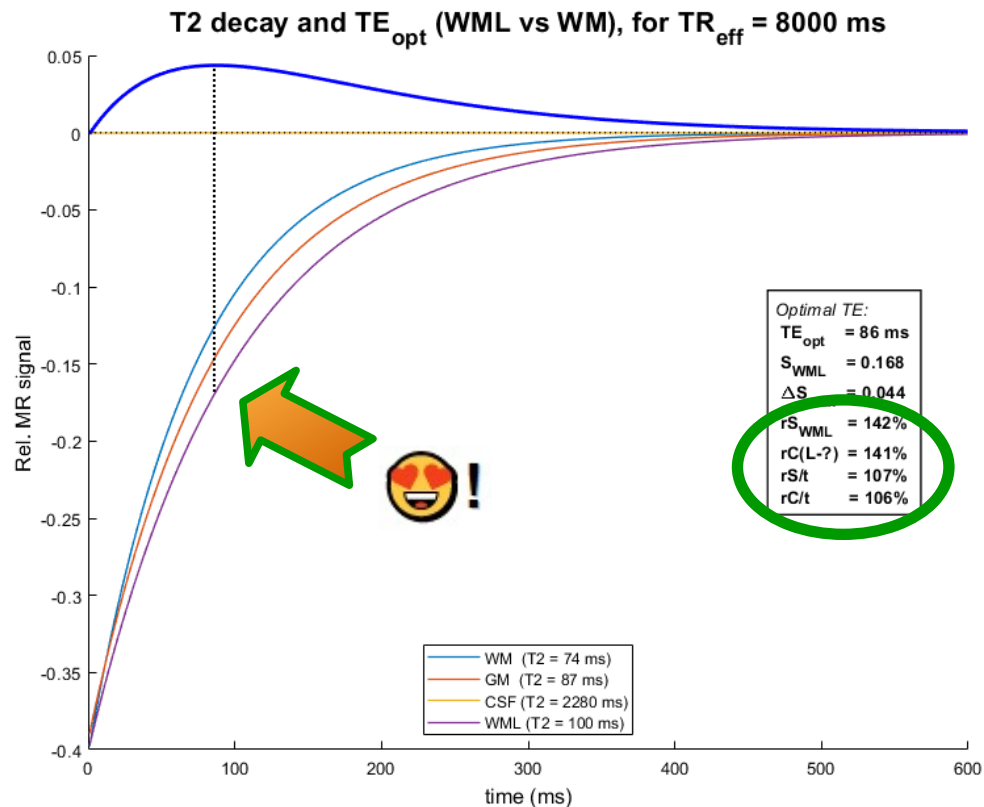
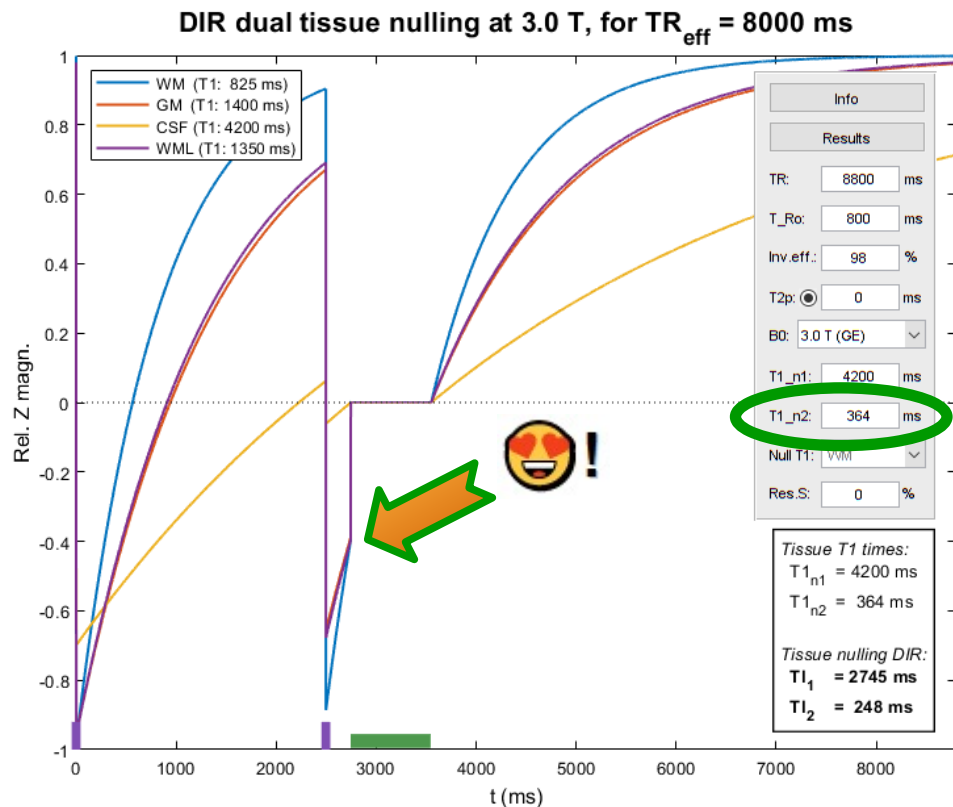


DIR-WM – A Pulse Too Far?

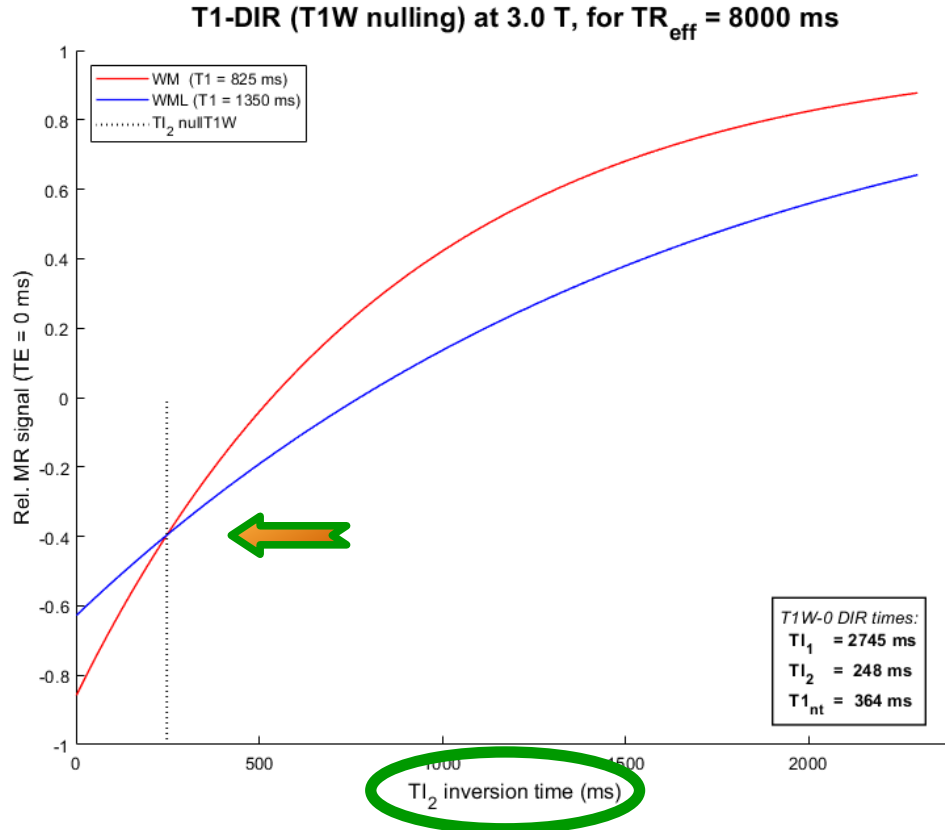


CNR/t bad. Better at low TE – at the cost of WML–GM CNR

T1-nulled DIR – A New Hope



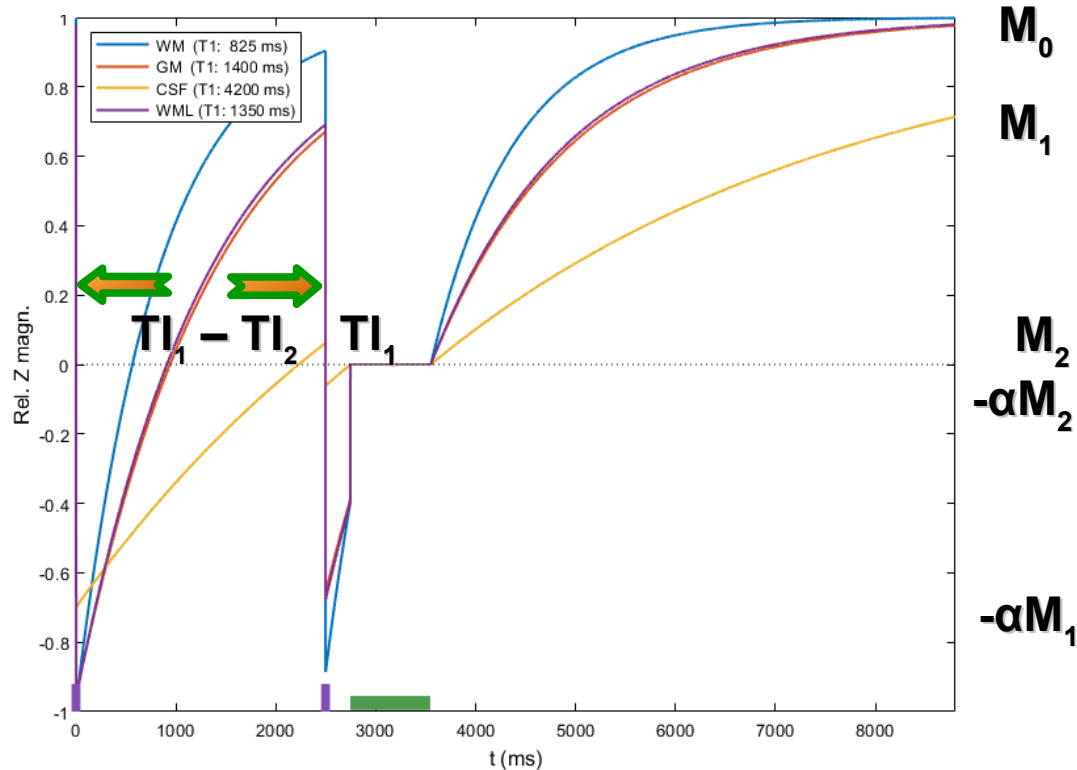
T1-nulled DIR – How?



- Find the TI_1/TI_2 pair that null the difference between the DIR signal for T_1 times of WM and WML
- Find the T_1 that is nulled at these TI_1/TI_2
- WM will be brighter, but SNR/CNR better
- Optimal TE will be free of T_1 effects

[Madhuranthakam et al, Mag Res Med 2012 67(1):81-88]

DIR-Calc (Inversion juggling)



$$\frac{M_1}{M_0} = 1 - E_{(TR-TI_1)} = 1 - \frac{E_{TR}}{E_{TI_1}} ; (TR \rightarrow TR_{eff})$$

$$\frac{M_2}{M_0} = 1 - \left(1 + \alpha \frac{M_1}{M_0} \right) E_{(TI_1-TI_2)} = \frac{1}{E_{TI_2}} \left[E_{TI_2} + \alpha E_{TR} - (1 + \alpha) E_{TI_1} \right]$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_2}{M_0} \right) E_{TI_2}$$

$$S_0 = \frac{1 - (1 + \alpha) E_{TI_2} + \alpha (1 + \alpha) E_{TI_1} - \alpha^2 E_{TR}}{1 - (1 + \alpha) E_{TI_2} + \alpha (1 + \alpha) E_{TI_1} - \alpha^2 E_{TR}}$$

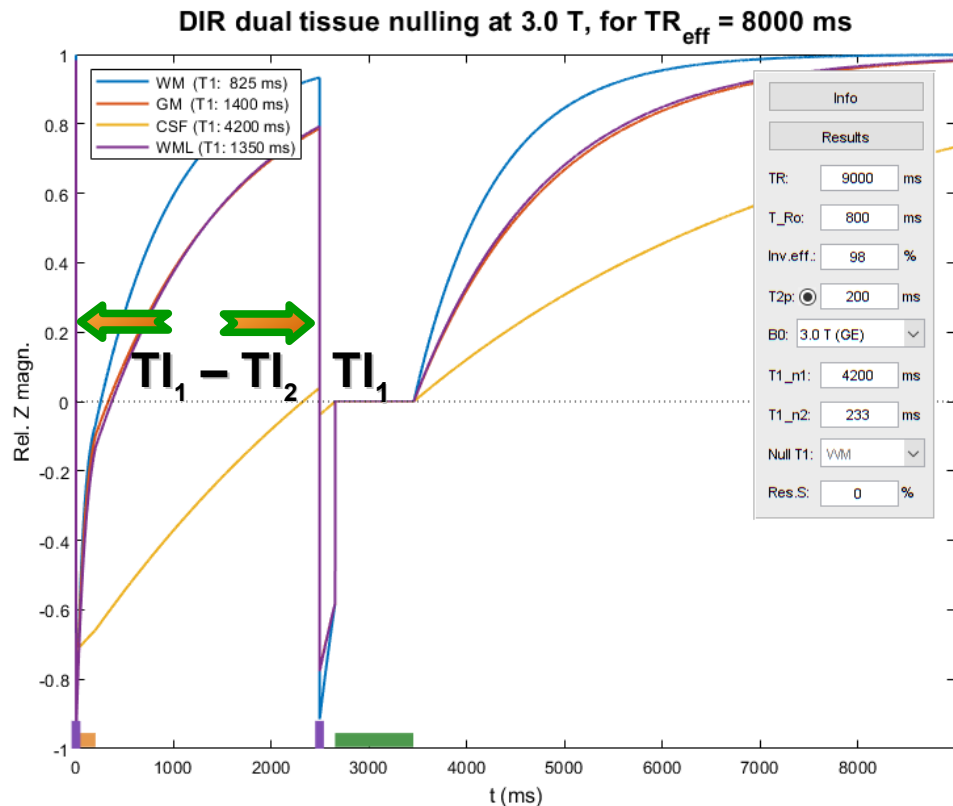
↓

$$S_0 = 0 \Rightarrow$$

$$TI_{in} = \frac{TI \ln \left(\left[\alpha (1 + \alpha) \right] / \left[-1 + (1 + \alpha) E_{TI_2} + \alpha^2 E_{TR} \right] \right)}{1}$$

Q.E.D.

DIR-Calc (Oh My T2P... version)



M_0

M_1

M_2

$-\alpha M_2$

$-\beta M_1$

$$\frac{M_1}{M_0} = 1 - E_{(TR-TI_1)} = 1 - \frac{E_{TR}}{E_{TI_1}} ; (TR \rightarrow TR_{eff}) ; \beta \stackrel{\text{def}}{=} \alpha E_{T2P}$$

$$\frac{M_2}{M_0} = 1 - \left(1 + \beta \frac{M_1}{M_0} \right) E_{(TI_1-TI_2)} = \frac{1}{E_{TI_2}} \left[E_{TI_2} + \beta E_{TR} - (1 + \beta) E_{TI_1} \right]$$

$$S_0 \stackrel{\text{def}}{=} \frac{M_{TI}}{M_0} = 1 - \left(1 + \alpha \frac{M_2}{M_0} \right) E_{TI_2}$$

$$S_0 = 1 - (1 + \alpha) E_{TI_2} + \alpha (1 + \beta) E_{TI_1} - \alpha \beta E_{TR}$$

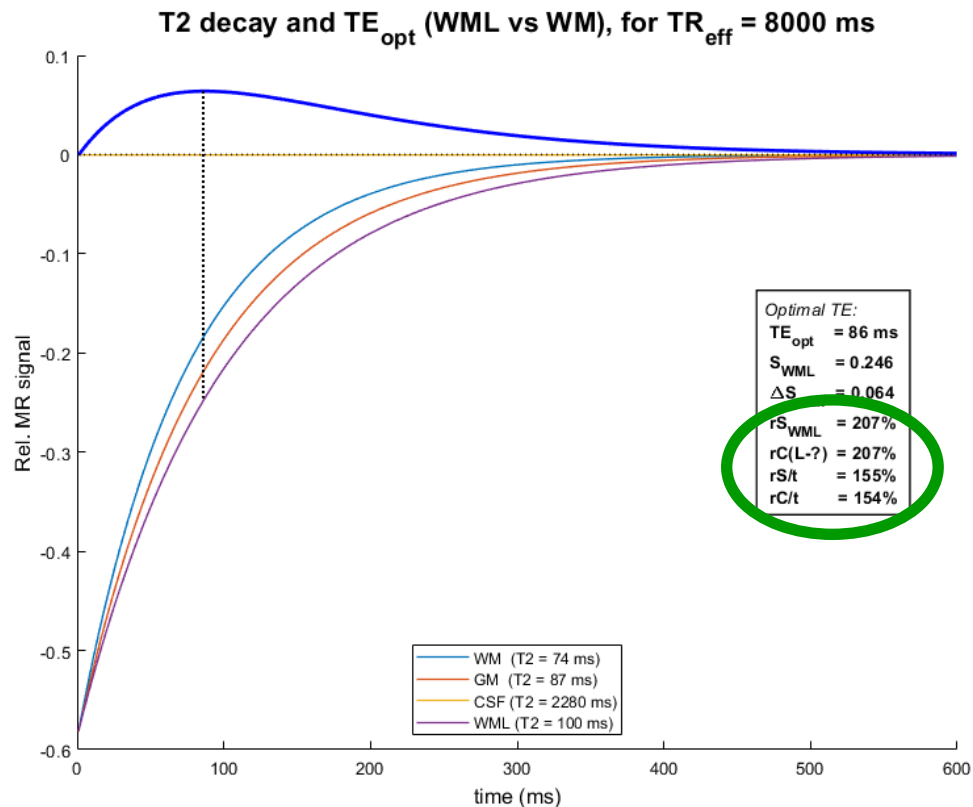
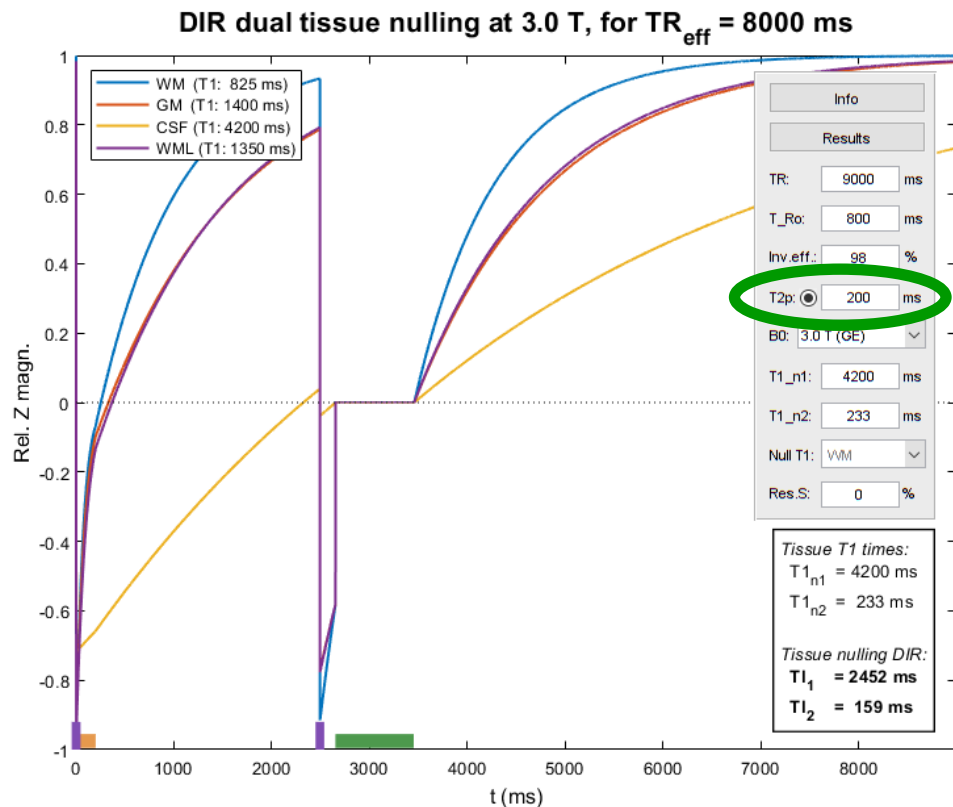
↓

$$S_0 = 0 \Rightarrow$$

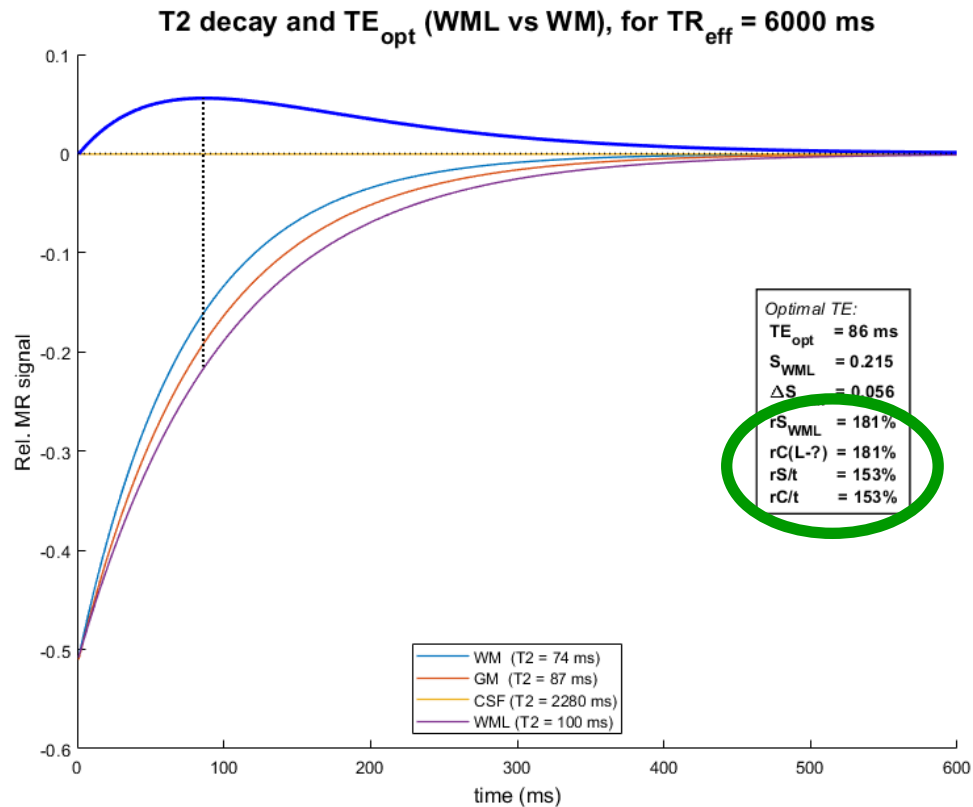
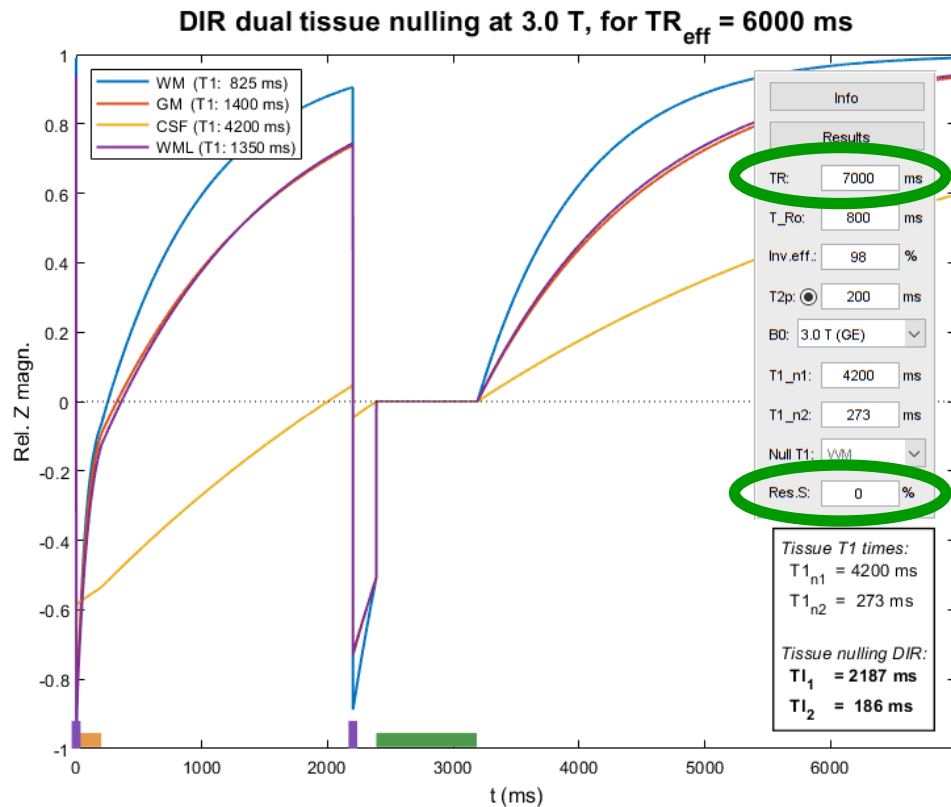
$$TI_{1n} = \frac{TI_1 \ln \left(\left[\alpha (1 + \beta) \right] / \left[-1 + (1 + \alpha) E_{TI_2} + \alpha \beta E_{TR} \right] \right)}{Q.E.D.}$$

Q.E.D.

TrueT2-DIR – From Zero To Hero!

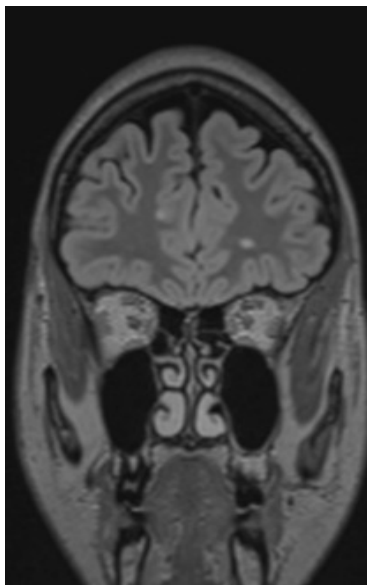
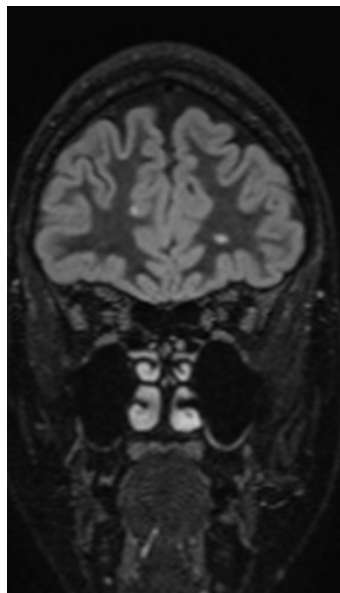


TrueT2-DIR – From Zero To Hero!

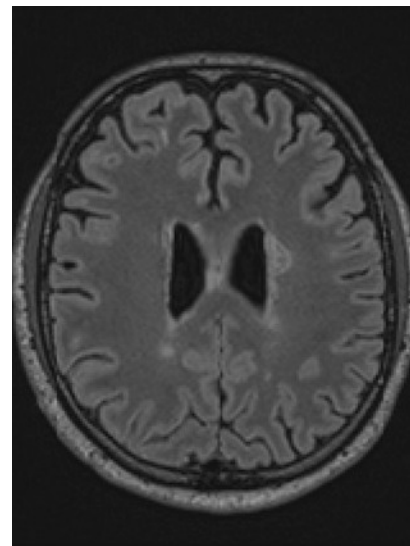
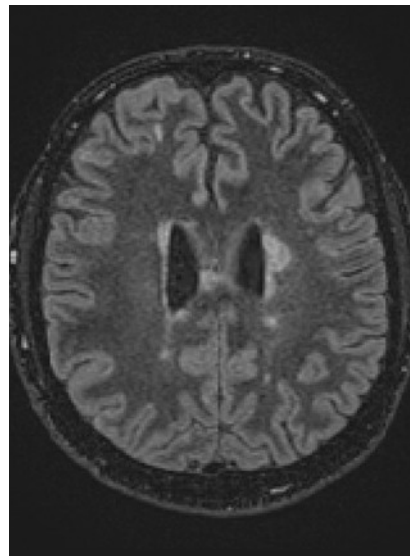


TrueT2-DIR vs FLAIR images

TrueT2



FLAIR



CNR/t at least as good as FLAIR; better WML–GM CNR?!

DIR 4 Dummies!



	A	B	C	D	E	F	G	H	I	J	K	L
1												
2				TrueT2-DIR T1 calculator for 3T GE using the IR-Calc simulator								
3				~ Simulations and regressions by Øystein Bech and Wibeke Nordhøy ~								
4												
5												
6												
7				T11 [ms] = $\frac{1377 \times \ln(\text{TR}) + 226 \times \ln(\text{T1_L}) + 372 \times \ln(\text{T1_W}) + -13990}{\dots}$								
8												
9				T12 [ms] = $\frac{-150 \times \ln(\text{TR}) + 142 \times \ln(\text{T1_L}) + 236 \times \ln(\text{T1_W}) + -1084}{\dots}$								
10												
11				T1n [ms] = $\frac{-221 \times \ln(\text{TR}) + 207 \times \ln(\text{T1_L}) + 346 \times \ln(\text{T1_W}) + -1596}{\dots}$								
12												
13												
14												
15												
16												
17												
18												
19												
20												
21												
22												
23												
24												
25												
26												
27												
28												

Editable parameters:

TR	8000 ms
T1_WML	1400 ms
T1_WM	850 ms

Calculated parameters:

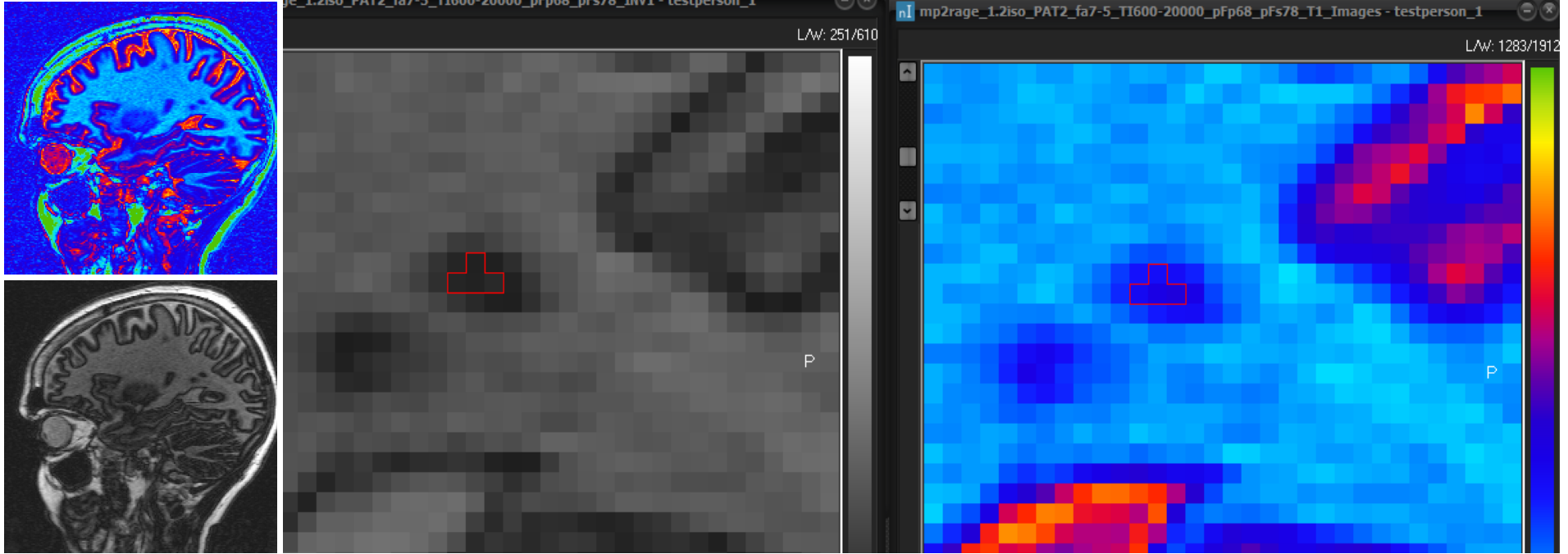
T11	2536 ms
T12	189 ms
T1_null	249 ms

Non-editable parameters:

B0/System	3 T GE
T1_CSF	5000 ms
T1_GM	1400 ms
T1_WM	800–900 ms
T1_WML	1200–1600 ms
TR	5000–9000 ms
T_Readout	850 ms
T_T2prep	200 ms
T_Inv	16 ms
Inv. Eff.	100 %

Accessible spreadsheet tools, based on simulation regressions

DIR 4 Dummies!?

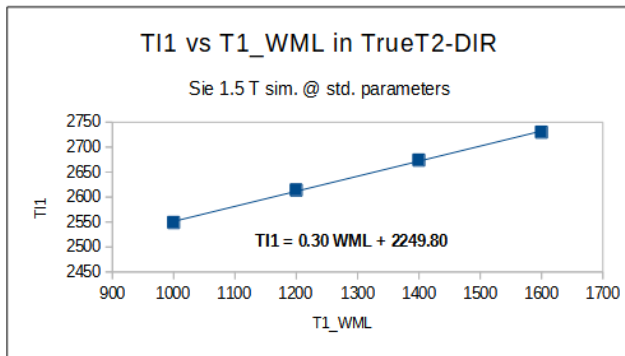


Measuring WM and WML T1 values w/ MP2RAGE MRI

DIR 4 Dummies???

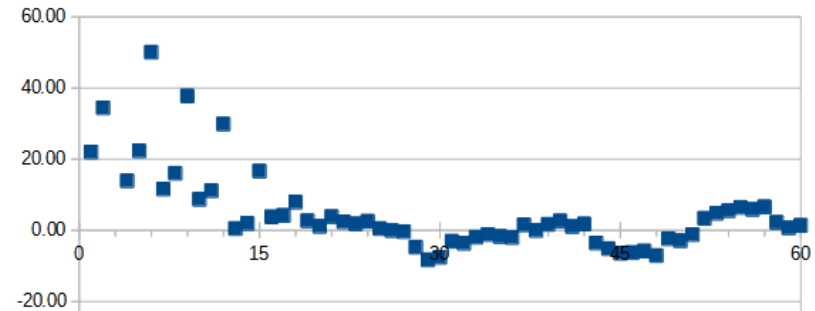


T1_WM	T1_WML	TR	T1n	T11	T12	rS=rC	r[S]C/NR/t
600	1000	8000	169	2520	133	122	122
660	1000	8000	193	2549	150	117	117
750	1000	8000	229	2593	175	110	110
850	1000	8000	270	2642	203	102	102
600	1200	8000	218	2579	167	104	104
660	1200	8000	247	2614	187	100	100 (standard)
750	1200	8000	289	2665	216	94	94
850							
600							
660							
750							
850							
600							
660							
750							
850							



Regression		WM&WML&TR vs. T11		@ T1_GM 1400 ms; T1_CSF 5000 ms			
Regression Statistics				LINEST raw output			
R ²	1.000	Regression	Logarithmic	1377.191	226.070	372.295	-13990.063
Standard Error	4.26	Confidence	0.950	3.059	5.405	13.211	100.987
Count of X vari	3	TINV ± factor	2.02	1.000	4.26	#N/A	#N/A
Observations	45			68425.1	41	#N/A	#N/A
Adjusted R ²	1.000			3728656.1	744.7	#N/A	#N/A
Analysis of Variance (ANOVA)							
	df	SS	MS	F	Signific. F	Sim.	Est.
Regression	3	3728656.1	1242885.4	68425.1	7.47E-76	2375	2373.3
Residual	41	744.7	18.2			2516	2513.4
Total	44	3729400.8				2537	2536.0
						2559	2557.2
	Coefficients	Standard Error	t-Statistic	P-value	Lower 95%	Upper 95%	±
Intercept	-13990	101	-138.5	2.2E-56	-14194	-13786	204
LN(T1_WM)	372.3	13.2	28.2	1.9E-28	345.6	399.0	26.7
LN(T1_WML)	226.1	5.4	41.8	3.0E-35	215.2	237.0	10.9
LN(TR)	1377.2	3.1	450.3	2.3E-77	1371.0	1383.4	6.2

TI1 residuals from logarithmic regression,
sorted by T1_WML - TR - T1_WM



- Good regressions may be hard to get right, but...
- Their results are easy to use!