

(I)

1. The notation $p_e(x/x_j)$ is not good.

2. A simpler reasoning:

(a) For any realization W there are equiprobable realizations that correspond to permuting stimuli \Rightarrow

averaged over W \rightarrow $MSE = P_{\text{error}} \times \frac{1}{L} \sum_{i=1}^L \langle (x_i - \hat{x}_i)^2 \mid \hat{x}_i \neq x_i \rangle_W$.

and $P(\hat{x}_i = x_j \mid \hat{x}_i \neq x_i) = \frac{1}{L-1} \quad \forall j \neq i.$

(b) $P_{\text{error}} = 1 - P_{\text{no error}}$
 $= 1 - (\dots)^{L-1}$ etc.

(c) For $\overline{e^2}$:

$$* \langle (x_i - \hat{x}_i)^2 \rangle = \frac{1}{L-1} \sum_{\substack{j=1 \\ j \neq i}}^L \left(\frac{i-j}{L} \right)^2$$

$$* \overline{e^2} = \frac{1}{L \cdot (L-1)} \sum_{i \neq j} \left(\frac{i-j}{L} \right)^2$$

3. Don't use "in the first step...". Either "the equality is obtained by..." or something like this; or reformulate differently.

(II)

1. The sections "Local error" and "global error" should be sub-sections of "Broad tuning curves" (not sections on the same level).
2. Instead of defining local & global errors, say that here we calculate the MSE in the case in which the response curve is smooth on the scale of the noise (large enough σ , write the inequality). Then errors can be with nearby stimulus (local) or with distant stimulus because resp. curve loops back (global).

$$MSE = MSE_{\text{local}} + MSE_{\text{global}}$$

$$3. \quad MSE_{\text{local}} = \int_0^1 dx \frac{\eta^2}{[v'(x)]^2} \quad \text{and}$$

$$[v'(x)]^2 = \sum_{i,j,i'} W_{ij} W_{ij'} u'(x-c_j) u'(x-c_{j'})$$

$$\approx \sum_{ij} (W_{ij})^2 [u'(x-c_j)]^2 \quad (\text{for large } L)$$

Now, by central-limit theorem: $\sum_j (W_{ij})^2 [u'(x-c_j)]^2$ is a Gaussian rand. var. with mean \approx

$$\sum_i L \cdot \frac{1}{L} \cdot \int_0^1 dy \frac{(x-y)^2}{\sigma^4} A^2 \exp\left(-\frac{(x-y)^2}{\sigma^2}\right) = \frac{\sqrt{\pi}}{2} \frac{NA^2}{\sigma}$$

& calculate variance; but check & mention the Lindeberg condition.

4. For MSE_{global} , I would simply say that we approximate the response curve by a set of $\frac{1}{6}$ uncorrelated segments. Then the only thing that changes in the previous derivation is the prefactor: $L \rightarrow \frac{1}{6}$.

III

1. Call this section "Influence of input noise on population coding" and place it after the section on multi-dimensional stimuli.
2. This identity is always true. L large implies that the second term is small; reformulate.
3. It is not a development in $\frac{N}{L}$; furthermore, it is not clear whether you do a development in $\frac{1}{L}$ or in $\frac{\xi}{\tilde{\eta}}$. The text suggests that it is the former (and indeed there is no reason that it be the latter since ξ & η can be of the same order). But this seems contradicted by your results, since both 1st & 2nd order terms are of order $\frac{N^2}{L^2}$. So is it really a correct expansion? Are all order terms scaling like $\frac{N^2}{L^2}$?
4. Problem related to point 3 above: in a correct expansion, with $T \approx 1 + \epsilon + \epsilon^2$, for $\frac{1}{T}$ you would need to include both 1st-order correction in $\epsilon + \epsilon^2$ and second-order correction in ϵ .

IV

1. All the definitions / model explanations should come in the earlier section.
2. Define clearly the MSE. For a multi-dimensional stimulus, there can be various definitions.