Institute of Molecular and Clinical Ophthalmology Basel

www.iob.ch

Random Compressed Coding with Neurons

We study the coding properties of a population of neurons with complex, but unstructured `tuning curves'. Such a scheme allows to encode stimuli with a precision which grows exponentially with the population size, outperforming classical codes. Re-analyzing previous data, we show the benefits of such `compressed' schemes in the spatial tuning of monkey motor neurons.

Simone Blanco Malerba[1,2], Mirko Pieropan[1], Yoram Burak[3], Rava Azeredo da Silveira[1,2]

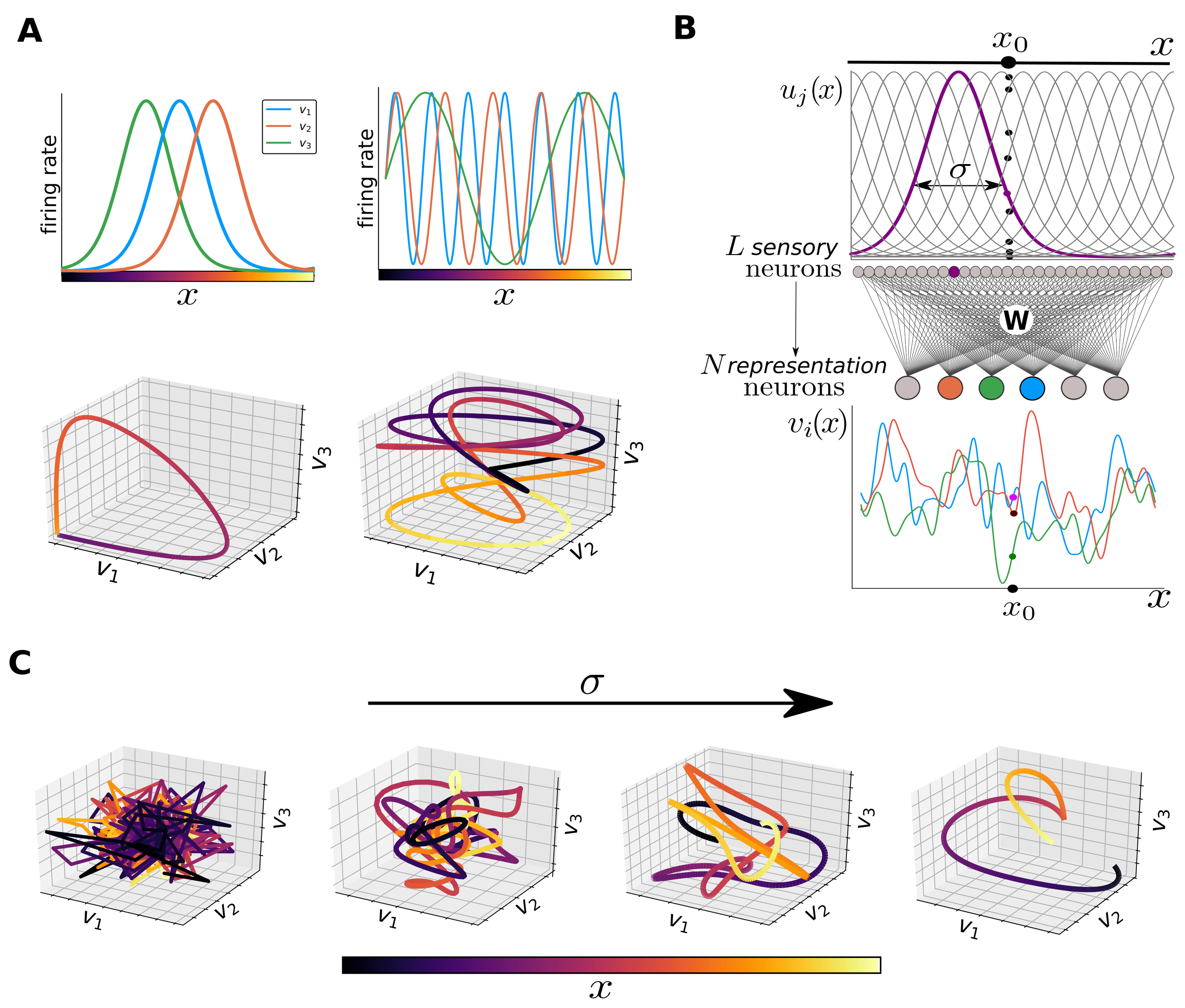
[1]Ecole Normale Supérieure de Paris, Paris [2] Institute of Molecular and Clinical Ophthalmology Basel (IOB), Basel [3] The Hebrew University of Jerusalem, Jerusalem

Research question

The mean response of neurons as a function of sensory stimuli is often described through simple, unimodal or monotonic `tuning curves'. Nevertheless, interesting coding properties emerge when considering complex response profiles. As an example, grid cells, with spatially periodic tuning curves, generate a precise combinatorial code, representing a large range of locations with high accuracy. Here we ask if periodicity is necessary for enhanced coding, by analyzing a network which produces complex, irregular tuning curves. As these are the result of merely random connectivity, the model can be thought of as a benchmark for the analysis of population coding with complex tuning curves.

The geometry of neural codes

- **Simple vs complex tuning curves.** Tuning curves map the 1-*D* stimulus space into a curve in the *N-D* space of neural population activity (A). Noise (η^2) creates a region of uncertainty about the true stimulus identity; the local accuracy is proportional to the ratio between the size of this region and the total length of the population activity curve. Unimodal tuning curves produce a simple smooth curve. Complex (periodic) tuning curves obtain a serpentine shape, achieving a higher local accuracy by `stretching' the response curve. On the other hand, the curve is very tangled and it happens that nearby responses are evoked by distant stimuli; this may lead to large, catastrophic (global) errors.
- **Random feedforward network.** We consider a two-layer neural network (B). A layer of sensory neurons respond to stimuli with classical, bell-shaped tuning curves. These neurons are connected with random weights to a smaller population of *N* representation neurons, which exhibit irregular response profiles. The tuning width of first layer neurons, σ, controls the smoothness of these irregularities, and, consequently, the geometry of the curve in the activity space (C). By varying σ, the model interpolates between a very long curve, but also very tangled and therefore sensible to catastrophic errors, and a smooth one, with lower local accuracy but more robust to large errors.

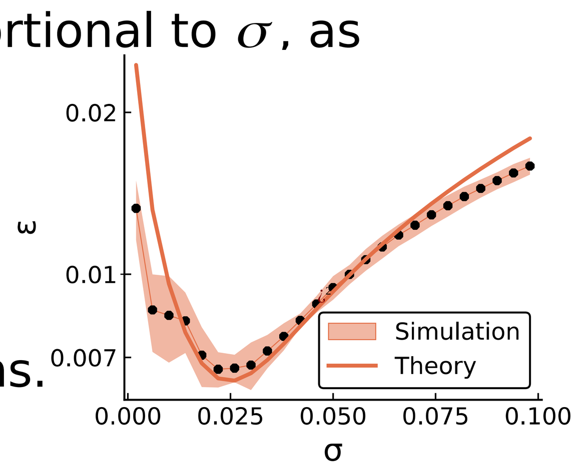


Theoretical results

The error can be described as the sum of two contributions: local, proportional to σ, and global, inversely proportional to σ, as

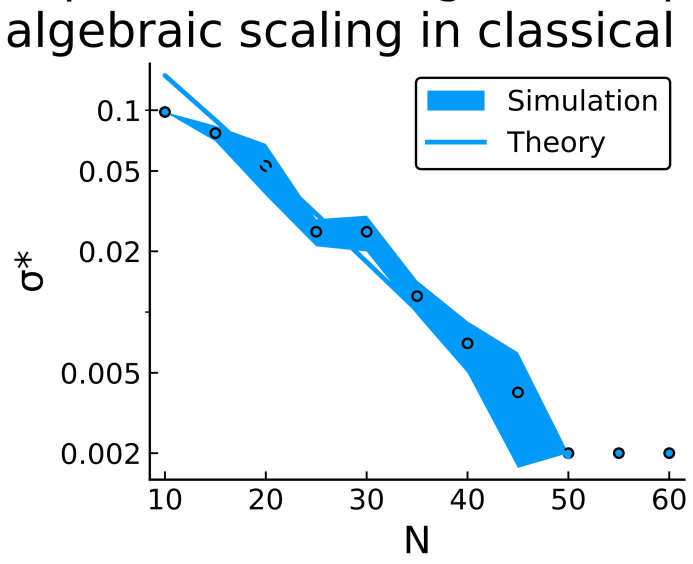
$$\varepsilon^2=\varepsilon_l^2+\varepsilon_g^2$$

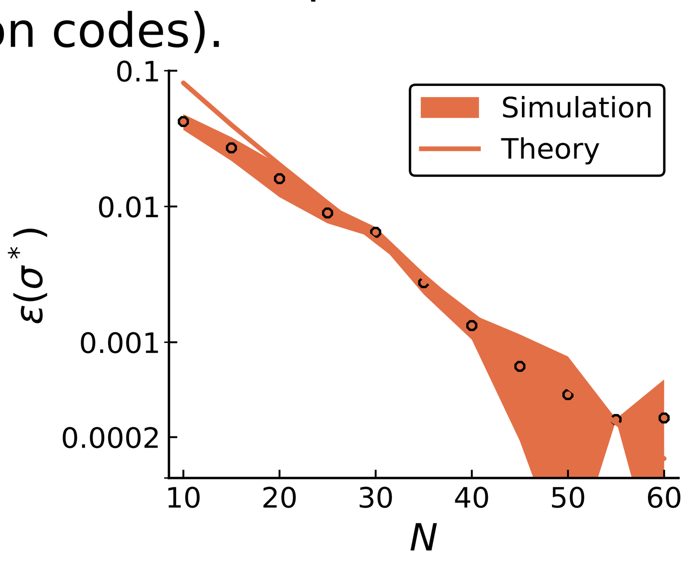
$$\approx \frac{2\sigma^2\eta^2}{RN}+\frac{1}{\sigma\sqrt{2\pi N}}\bar{\varepsilon}_g \exp\left(-\log\left(1+\frac{R}{2\eta^2}\right)\frac{N}{2}\right).$$



Optimal $\sigma^*(N)$ balances the two contributions.

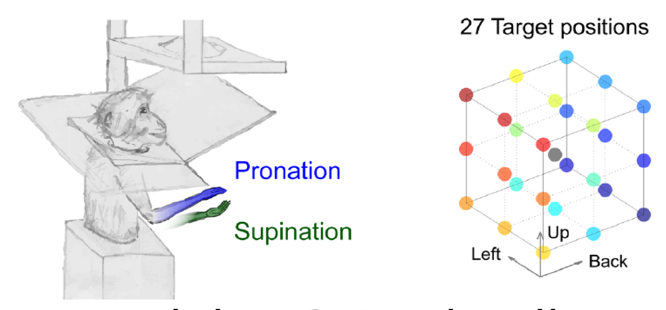
Exponential scaling of the optimal width and the optimal error (vs algebraic scaling in classical population codes).

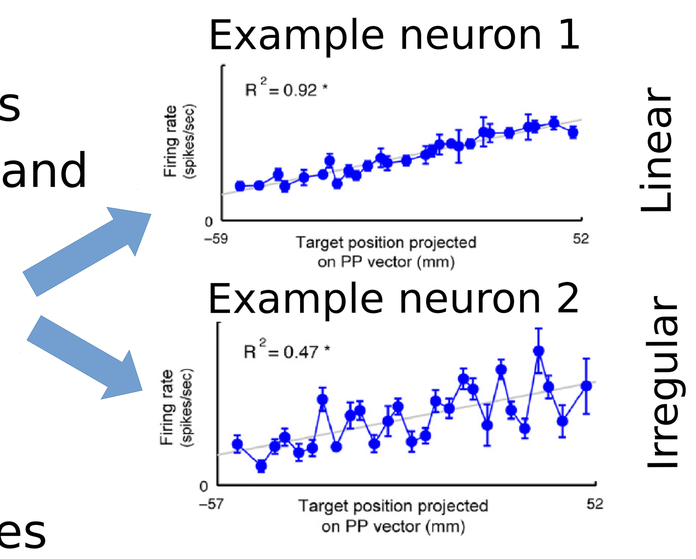




Compressed coding in monkey motor cortex

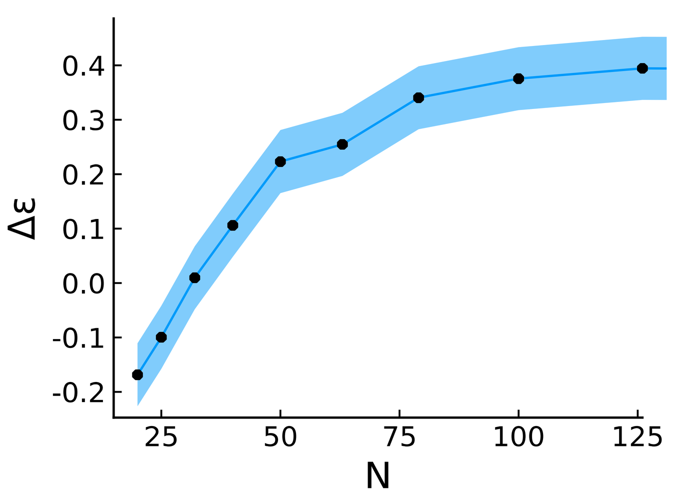
Experiment: monkey performs a static 'hold' task[1] → Motor neurons exhibit heterogeneous and irregular tuning curves as a function of hand position





- Extension of our model to 3-*D* stimuli captures this heterogeneity → Fit σ_f to reproduce summary statistics of heterogeneity of experimental tuning curves.
- What is the role of irregularities? Comparison between a population with irregular tuning curves with one with `classical' linear tuning curves: $\Delta\varepsilon=\frac{\varepsilon_{lin}-\varepsilon_{irr}}{\varepsilon_{lin}}$

→ Advantage of irregular coding scheme increases with population size.



[1] *Data from Lalazar et al., 2016*

Conclusions

- Coding with irregular tuning curves requires a trade-off between local accuracy and smoothness, which allows robustness to catastrophic errors.
- Optimality is achieved at an intermediate level of irregularity, and the error decreases exponentially with the population size.
- Tuning curves found in monkey motor cortex can be viewed as an instantiation of these principles; at sufficiently large population size, such an irregular coding scheme outperforms a classical one with smooth linear tuning curves .