

Compressed Coding in a Shallow Neural Network with Random Weights

1 Summary

The mean response of neurons to parameters of sensory stimuli is characterized by tuning curves. Classically, these response profiles are described by simple, monomodal or monotonic, smooth functions. Interesting coding properties emerge with more complex tuning curves: as an example, grid cells, with their spatially periodic responses, generate a precise code [1]. These multi-peaked tuning curves allow for combinatorial patterns of activity to represent spatial locations with fewer neurons than in a population with monomodal tuning curves.

Is periodicity necessary for enhanced coding, or do similar properties emerge more generally in populations of neurons with complex tuning curves? To address this question, we consider a simple circuit that produces complex but unstructured tuning curves, namely, a feedforward neural network with random connectivity, in which information is compressed from a first layer to a second one of smaller size. The random connectivity generates irregular tuning curves; these represent richer ‘sensors’ of the stimulus (as compared to monomodal tuning curves), but result in ambiguous coding which can yield catastrophic errors. Combining analytical and numerical methods, we show that, as a function of its parameters, this simple network interpolates between a classical code robust to noise and a combinatorial code, highly accurate but prone to catastrophic errors. Efficient coding implies an optimal point that specifies the spatial scale of tuning curve irregularities, as a function of the compression of the information between network layers.

Spatial coding in the monkey motor cortex can be viewed as an instantiation of such a ‘compressed coding’. We reanalyze ~~earlier~~ data [2] by extending our model to higher-dimensional stimuli and fitting it to neural recordings. The approach enables us to compare the performance of a compressed coding model of spatial representation in motor cortex with the classical model that assumes smooth, linear tuning curves. This example illustrates the possible use of complex tuning curves in population coding. More generally, our work proposes a new angle on efficient population coding beyond the classical model with simple tuning curves.

2 Additional details

Tuning curves are used broadly in neuroscience to describe the response of neurons to parameters of sensory stimuli. Their shape is often described by simple, smooth functions, like Gaussians or sigmoids, and a large body of literature addresses the problem of optimizing the parameters of these functions with respect to an objective or loss function such as the estimation error [3]. In classical models, this error is small in large populations of neurons; typically, it decreases as an inverse power of the population size.

However, more efficient coding schemes exist in terms of error suppression with the size of the population. A paradigmatic example is given by grid cells in the entorhinal cortex, which collectively encode the position of an animal moving in an environment by means of spatially periodic tuning curves with different periods. Sreenivasan and Fiete [1] showed how in such populations the error in the position estimate decreases exponentially with the number of neurons, thus outperforming classical codes. This phenomenon can be intuited from a geometrical representation introduced by Shannon [4], on the basis of which a coding scheme ~~can be regarded~~ as a map that associates each realization of the stimulus with a point in the space of the joint neural activities of the population. For a population of grid cells, due to the periodicity of their responses, fully combinatorial patterns of activity actually correspond to encoded stimuli. This results in a denser coverage of the activity space and, consequently, in a more accurate local representation of the encoded sensory **variable** as compared to the case of an equisized population of classically tuned neurons. However, this coding scheme comes with the drawback that two distant stimuli can be mapped to nearby activity patterns and thereby become **indistinguishable** due to the presence of noise that corrupts neural responses. As a result, in such a code, large or ‘global’ errors in stimulus estimation can occur.

While grid cells are widely studied in the literature, there may be other ~~coding schemes~~ that share similar coding properties. In order to see if a highly designed structure as periodicity is necessary for enhanced coding, we consider a two-layer feedforward neural network with unstructured, i.e., random, connectivity. In the first layer, a large population of L sensory neurons encodes a one-dimensional stimulus into a high-dimensional representation by means of Gaussian tuning curves of width σ , whose centers are arranged uniformly over the stimulus space. This population projects onto a smaller layer of N neurons by means of random i.i.d. synaptic weights, which effectively generate a set of irregular tuning curves in the second layer. The average joint activity of neurons in this layer, as parametrized by the stimulus, can be represented as a one-dimensional manifold embedded in the N -dimensional space of neural responses (Fig.1). We model the **trial to trial** variability as i.i.d Gaussian noise; this noise will perturb the responses away from the manifold. The tuning width, σ , of neurons in the first layer governs the scale of the irregularities in the manifold, that is, the extent to which responses preserve the distance between two nearby stimuli. If σ is small, neurons in the second layer generate uncorrelated random responses regardless of the distance between stimuli, and the manifold reduces to an

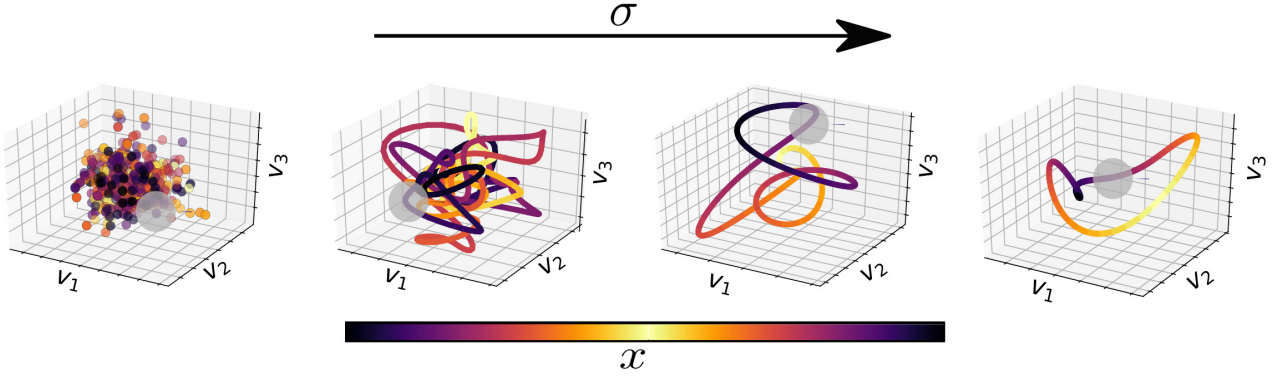


Figure 1: **Geometrical view of a random coding scheme.** Example of manifold described by the mean joint activity of three neurons, for increasing value of σ . The three axes represent the firing rate of three sample neurons and the manifold is color coded according to the corresponding stimulus value as shown in the legend. Trial to trial variability in the population response is represented as a grey cloud of noisy responses around the mean value. As σ is lowered, the manifold increases in length, resulting in a higher discrimination of nearby stimuli inside the same ball of noise. On the other hand, due to the complex winding, the number of segments coding for distant stimuli, represented by different colors, is higher, which leads to a larger probability of ‘global’ errors.

ensemble of scattered points in the activity space. As σ grows, irregularities are smoothed out, and nearby stimuli evoke increasingly correlated responses.

In order to study the coding properties of the network, we quantified the coding accuracy through the Mean Square Error (MSE) in the stimulus estimate as obtained from an ideal decoder. ~~This decoder will associate to each noisy response, the stimulus corresponding to the closest point on the manifold.~~ We showed that the error can be viewed as the sum of two qualitatively distinct contributions:

$$\varepsilon^2 = \varepsilon_l^2 + \varepsilon_g^2. \quad (1)$$

‘Local errors’, quantified by ε_l^2 , occur when estimates are close to the true stimulus value; this kind of error scales with the tuning width, $\varepsilon_l \sim \frac{\sigma}{N}$, and the tuning width in turn rules the local curvature of the manifold. ‘Global errors’ are controlled by the winding of the response manifold, and scale as $\varepsilon_g \sim \frac{1}{\sigma} \exp(-N)$. In a regime of compression, i.e., when $N \ll L$, there is a trade-off between ε_l and ε_g that yields optimal tuning parameters, for which the total error us suppressed exponentially with ~~population size~~. We corroborated the analytical results with numerical simulations; .

Spatial coding in monkey motor cortex can be viewed as an instantiation of the coding scheme just outlined. Neurons in this area encode the spatial position of the monkey’s hand; classically, their responses have been fitted to a function of the cosine of the angle between the target position and a ‘preferred direction’, that varies from neuron to neuron. In a recent study, Lalazar et al. [2] proposed a new fit of tuning curves that took into account the heterogeneity among neurons on a finer scale. Motivated by the similarity of their model with a generalization of our shallow random network that encodes a three-dimensional stimulus, we used the latter to capture the monkey cortex data. Despite being simpler, our model yielded a faithful fit of the neural recording; it also allowed us to quantify spatial coding in cortical populations of neurons, and, in particular, to compare coding when tuning curves take ~~their~~ classical, simpler form and the more complex form generated in our network. By varying the population size and the magnitude of the single-neuron noise, we identified the parameter regions in which tuning curve irregularities are beneficial to spatial coding, in spite of the ambiguity that they may generate. Our work illustrates the power of complex tuning curves and compressed coding in the absence of a highly structured design of a network, and suggests, correspondingly, that a brain area achieve a balance between high encoding accuracy and robustness to noise.

References

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