A Test-suite of Non-Convex Constrained Optimization Problems from the Real-World and Some Baseline Results

Abhishek Kumar[a], Guohua Wu[b], Mostafa Z. Ali[c], Rammohan Mallipeddi[d], Ponnuthurai Nagaratnam Suganthan[e,*], Swagatam Das[f]

[a] *Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi, Varanasi, 221005, India.*
[b] *School of Traffic and Transportation Engineering, Central South University, Changsha 410075, China.*
[c] *School of Computer Information Systems, Jordan University of Science & Technology, Jordan 22110.*
[d] *School of Electronics Engineering, Kyungpook National University, Daegu 41566, Republic of Korea.*
[e] *School of Electrical Electronic Engineering, Nanyang Technological University, Singapore 639798.*
[f] *Electronics and Communication Sciences Unit, Indian Statistical Institute, Kolkata, India.*

Abstract

Real-world optimization problems have been comparatively difficult for solving because of their complex objective function with a substantial number of constraints. To deal with these problems, several metaheuristics and/or several constraint handling approaches have been suggested. To validate effectiveness and strength, a newly designed approach must be benchmarked on some complex real-world problems. Many real-world test problems have been suggested in the literature. While a list of standard problems is needed for benchmarking new algorithms in an efficient and unbiased manner. In this study, a set of 57 real-world constrained optimization problems is discussed and a benchmark suite is proposed that can be utilized to validate the constrained optimization algorithms. Reported problems have been singled out from a wide range of problems possible in various research fields. Three state-of-the-art constrained optimization methods are applied to analyze hardness of these problems. The experimental outcomes reveal that the selected problems are challenging to these algorithms.

Keywords: Real-world optimization problem; Metaheuristics; Constraint handling technique; Benchmark suite.

1. Introduction

Optimization is a numerical process used to determine the decision variables for minimizing or maximizing the objective function value while satisfying the constraints of decision-space. In most of the real-world applications, problems contain non-linear objective function and constraints with multiple local optimum, and low feasible region [1]. Solving these problems using classical algorithms is hard and tedious due to many local optimum and low feasible region.

Since the last three decades, Swarm-based Algorithms (SAs) and Evolutionary Algorithms (EAs) have drawn attention and have become a common choice as an optimization algorithm in real-world applications. One of the main advantages of these algorithms over classical numerical algorithms is that they only require evaluation of the objective function and constraints (if available) to get a piece of information about the problem. Therefore, these algorithms can deal with non-linear problems with discrete decision-space. In addition, the stochastic behaviors of search agents provide global search capability in them.

To validate the effectiveness of new algorithms, it is essential that the performance is evaluated on a good set of benchmark problems and is also compared with popular existing algorithms. Benchmark problems can be categorized into two groups: test problems or functions and real-world problems. Artificial problems are called test functions and behavior of optimization algorithms are generally evaluated on these test problems. Several benchmark suites of

*Corresponding author
Email address: `epnsugan@ntu.edu.sg` (Ponnuthurai Nagaratnam Suganthan)

test problems have been proposed for the validation of unconstrained and constrained algorithms. For unconstrained algorithms, a diverse set of test problems with different levels of difficulty are proposed in [2, 3, 4, 5]. Similarly, several collections of test problems are proposed for constrained algorithms in [6, 7, 8].

On the other hand, optimization problems originate from real-world applications are called real-world problems. For unconstrained algorithm, a set of real-world problems is reviewed and compiled in [9]. However, majority of real-world optimization problems are constrained type where presence of constraints and low feasible region may degrade the robustness and effectiveness of any optimization algorithm. Moreover, most of algorithms are normally designed for unconstrained optimization problems. Accordingly, an added mechanism called constraint handling technique (CHT) is required to handle the constraints of the real-worldproblems. Several CHTs have been suggested in the literature. Historically, the most common practice to handleconstraints has been by penalizing the fitness value of infeasible solutions. But, penalty functions used to penalize have numerous limitations. Due to limitations of the penalty factor, several CHTs have been proposed in the last two decades. Some popular CHTs among them are superiority of feasible [10], self-adaptive penalty function [11], epsilon constraint handling [12], stochastic ranking [13] and ensemble of CHTs [14].

In addition, the role of search methods in constrained algorithms needs to be analyzed on constrained optimization problems. In recent years, this research topic has become popular among researchers. Newly developed algorithms have been benchmarked on several engineering optimization problems (for example, see [15], [16], and [17]). However, in most of these studies, the chosen problems and algorithms have been complementary to each other i.e. algorithms perform well on selected problems, while may not perform well on other sets of problems. Therefore, the evaluation of algorithms needs to be done on a variety of real-world problems in a systematic manner.

The above reasons motivate us to construct a benchmark suite containing real-world optimization problems for constrained optimization algorithms. In this paper, 57 real-world constrained problems selected from different real-world applications are reviewed to create a benchmark suite. Three recently proposed state-of-the-art constrained algorithms viz. IUDE [18], ϵMAgES [19], and iLSHADE$_\epsilon$ [20], are chosen to demonstrate the difficulty level of these real-world optimization problems.

2. Real-world Constrained Optimization Problems

The real-world constrained optimization problems can be represented as follows.

$$\text{Minimize, } f(\bar{x}),\ \ \bar{x} = (x_1, x_2, .., x_n) \tag{1}$$

$$\text{Subject to: } g_i(\bar{x}) \leq 0,\ \ i = 1, .., n$$
$$h_j(\bar{x}) = 0,\ \ j = n+1, .., m$$

Generally, an equality constraint can be transformed into two inequality constraints using following equation.

$$|h_j(\bar{x})| - \epsilon \leq 0,\ \ j = n+1, .., m \tag{2}$$

where ϵ is set to a small value (near to zero).

2.1. Industrial Chemical Process Problems

Chemical engineering practice involves several non-linear constrained optimization problems [21]. Design relations of process equipment and equations of mass and heat balance introduce non-linearities in the problems. Many chemical process problems have been proposed which are highly complex and non-linear due to many non-linear inequality and equality constraints. The following problems are considered in this work.

2.1.1. Heat Exchanger Network Design (case 1) [22]

The optimal shape of the heat exchanger structure is considered in this problem. In three hot currents, one cold current is heated to reduce thecomprehensive area of heat exchange structure. The mathematical model can be described as following way.

Minimize :

$$f(\bar{x}) = 35x_1^{0.6} + 35x_2^{0.6} \tag{3}$$

subject to :

$$h_1(\bar{x}) = 200x_1x_4 - x_3 = 0,$$

$$h_2(\bar{x}) = 200x_2x_6 - x_5 = 0,$$

$$h_3(\bar{x}) = x_3 - 10000(x_7 - 100) = 0,$$

$$h_4(\bar{x}) = x_5 - 10000(300 - x_7) = 0,$$

$$h_5(\bar{x}) = x_3 - 10000(600 - x_8) = 0,$$

$$h_6(\bar{x}) = x_5 - 10000(900 - x_9) = 0,$$

$$h_7(\bar{x}) = x_4\ln(x_8 - 100) - x_4\ln(600 - x_7) - x_8 + x_7 + 500 = 0,$$

$$h_8(\bar{x}) = x_6\ln(x_9 - x_7) - x_6\ln(600) - x_9 + x_7 + 600 = 0$$

with bounds :

$$0 \le x_1 \le 10, 0 \le x_2 \le 200, 0 \le x_3 \le 100, 0 \le x_4 \le 200,$$

$$1000 \le x_5 \le 2000000, 0 \le x_6 \le 600, 100 \le x_7 \le 600, 100 \le x_8 \le 600,$$

$$100 \le x_9 \le 900.$$

2.1.2. Heat Exchanger Network Design (case 2) [23]

This is the second case of heat exchange network design problem. In this case, three nonlinear equality constraints and six linear equality constraints with a nonlinear objective function are involved in the problem. Moreover, seven additional linear inequality constraints are included due to bounds on the temperatures.

Minimize :

$$f(\bar{x}) = \left(\frac{x_1}{120x_4}\right)^{0.6} + \left(\frac{x_2}{80x_5}\right)^{0.6} + \left(\frac{x_3}{40x_6}\right)^{0.6} \tag{4}$$

subject to :

$$h_1(\bar{x}) = x_1 - 10^4(x_7 - 100) = 0,$$

$$h_2(\bar{x}) = x_2 - 10^4(x_8 - x_7) = 0,$$

$$h_3(\bar{x}) = x_3 - 10^4(500 - x_8) = 0,$$

$$h_4(\bar{x}) = x_1 - 10^4(300 - x_9) = 0,$$

$$h_5(\bar{x}) = x_2 - 10^4(400 - x_{10}) = 0,$$

$$h_6(\bar{x}) = x_3 - 10^4(600 - x_{11}) = 0,$$

$$h_7(\bar{x}) = x_4\ln(x_9 - 100) - x_4\ln(300 - x_7) - x_9 - x_7 + 400 = 0,$$

$$h_8(\bar{x}) = x_5\ln(x_{10} - x_7) - x_5\ln(400 - x_8) - x_{10} + x_7 - x_8 + 400 = 0,$$

$$h_9(\bar{x}) = x_6\ln(x_{11} - x_8) - x_6\ln(100) - x_{11} + x_8 + 100 = 0,$$

with bounds :

$$10^4 \le x_1 \le 81.9 \times 10^4, 10^4 \le x_2 \le 113.1 \times 10^4, 10^4 \le x_3 \le 205 \times 10^4,$$

$$0 \le x_4, x_5, x_6 \le 5.074 \times 10^{-2}, 100 \le x_7 \le 200, 100 \le x_8, x_9, x_{10} \le 300$$

$$100 \le x_{11} \le 400.$$

2.1.3. Haverly's Pooling Problem [23]

Haverly's Pooling problem is linear objective non-linear constrained optimization problem and has the following form.

Maximize :

$$f(\bar{x}) = 9x_1 + 15x_2 - 6x_3 - 16x_4 - 10(x_5 + x_6) \tag{5}$$

subject to :

$$h_1(\bar{x}) = x_7 + x_8 - x_4 - x_3 = 0,$$
$$h_2(\bar{x}) = x_1 - x_5 - x_7 = 0,$$
$$h_3(\bar{x}) = x_2 - x_6 - x_8 = 0,$$
$$h_4(\bar{x}) = x_9x_7 + x_9x_8 - 3x_3 - x_4 = 0,$$
$$g_1(\bar{x}) = x_9x_7 + 2x_5 - 2.5x_1 \leq 0,$$
$$g_2(\bar{x}) = x_9x_8 + 2x_6 - 1.5x_2 \leq 0,$$

with bounds :

$$0 \leq x_1, x_3, x_4, x_5, x_6, x_8 \leq 100, 0 \leq x_2, x_7, x_9 \leq 200.$$

2.1.4. Blending-Pooling-Separation problem [24]

This problem contains a feed mixture having three-component that are utilized to separate out into two multi-component outputs by employing separators and splitting /blending /pooling. The operating cost of each separator depends linearly on the flow-rate of the separator and the constraints based on mass balances relation around the individual separators, splitters, and mixers.

Minimize :

$$f(\bar{x}) = 0.9979 + 0.00432x_5 + 0.01517x_{13} \tag{6}$$

subject to :

$$h_1(\bar{x}) = x_4 + x_3 + x_2 + x_1 = 300,$$
$$h_2(\bar{x}) = x_6 - x_8 - x_7 = 0,$$
$$h_3(\bar{x}) = x_9 - x_{11} - x_{10} - x_{12} = 0,$$
$$h_4(\bar{x}) = x_{14} - x_{16} - x_{17} - x_{15} = 0,$$
$$h_5(\bar{x}) = x_{18} - x_{20} - x_{19} = 0,$$
$$h_6(\bar{x}) = x_5x_{21} - x_6x_{22} - x_9x_{23} = 0,$$
$$h_7(\bar{x}) = x_5x_{24} - x_6x_{25} - x_9x_{26} = 0,$$
$$h_8(\bar{x}) = x_5x_{27} - x_6x_{28} - x_9x_{29} = 0,$$
$$h_9(\bar{x}) = x_{13}x_{30} - x_{14}x_{31} - x_{18}x_{32} = 0,$$
$$h_{10}(\bar{x}) = x_{13}x_{33} - x_{14}x_{34} - x_{18}x_{35} = 0,$$
$$h_{11}(\bar{x}) = x_{13}x_{36} - x_{14}x_{37} - x_{18}x_{35} = 0,$$
$$h_{12}(\bar{x}) = 0.333x_1 + x_{15}x_{31} - x_5x_{21} = 0,$$
$$h_{13}(\bar{x}) = 0.333x_1 + x_{15}x_{34} - x_5x_{24} = 0,$$
$$h_{14}(\bar{x}) = 0.333x_1 + x_{15}x_{37} - x_5x_{27} = 0,$$

$$h_{15}(\bar{x}) = 0.333x_2 + x_{10}x_{23} - x_{13}x_{30} = 0,$$
$$h_{16}(\bar{x}) = 0.333x_2 + x_{10}x_{26} - x_{13}x_{33} = 0,$$
$$h_{17}(\bar{x}) = 0.333x_2 + x_{10}x_{29} - x_{13}x_{36} = 0,$$
$$h_{18}(\bar{x}) = 0.333x_3 + x_7x_{22} + x_{11}x_{23} + x_{16}x_{31} + x_{19}x_{32} = 30,$$
$$h_{19}(\bar{x}) = 0.333x_3 + x_7x_{25} + x_{11}x_{26} + x_{16}x_{34} + x_{19}x_{35} = 50,$$
$$h_{20}(\bar{x}) = 0.333x_3 + x_7x_{28} + x_{11}x_{29} + x_{16}x_{37} + x_{19}x_{38} = 30,$$
$$h_{21}(\bar{x}) = x_{21} + x_{24} + x_{27} = 1,$$
$$h_{22}(\bar{x}) = x_{22} + x_{25} + x_{28} = 1,$$
$$h_{23}(\bar{x}) = x_{23} + x_{26} + x_{29} = 1,$$
$$h_{24}(\bar{x}) = x_{30} + x_{33} + x_{36} = 1,$$
$$h_{25}(\bar{x}) = x_{31} + x_{34} + x_{37} = 1,$$
$$h_{26}(\bar{x}) = x_{32} + x_{35} + x_{38} = 1,$$
$$h_{27}(\bar{x}) = x_{25} = 0,$$
$$h_{28}(\bar{x}) = x_{28} = 0,$$
$$h_{29}(\bar{x}) = x_{23} = 0,$$
$$h_{30}(\bar{x}) = x_{37} = 0,$$
$$h_{31}(\bar{x}) = x_{32} = 0,$$
$$h_{32}(\bar{x}) = x_{35} = 0,$$

with bounds :

$$0 \le x_1, x_3, x_8, x_9, x_5, x_6, x_{14}, x_{18}, x_{10}, x_{16}, x_{13}, x_{20} \le 90,$$
$$0 \le x_2, x_4, x_7, x_{11}, x_{12}, x_{15}, x_{17}, x_{19} \le 150,$$
$$0 \le x_{21}, x_{23}, x_{24}, x_{25}, x_{27}, x_{28} \le 1,$$
$$0 \le x_{22}, x_{32}, x_{34}, x_{35}, x_{37}, x_{38} \le 1.2,$$
$$0 \le x_{26}, x_{29}, x_{30}, x_{31}, x_{33}, x_{36} \le 0.5.$$

2.1.5. Propane, Isobutane, n-Butane Nonsharp Separation [25]

This test problem contains a three-componentfeed mixture that is required toseparate products into two three-componentproducts. The problemis defined as a non-linear constrained optimization problem and has a following form.

Minimize :

$$f(\bar{x}) = c_{11} + (c_{21} + c_{31}x_{24} + c_{41}x_{28} + c_{51}x_{33} + c_{61}x_{34})x_5 + c_{12} \\ +(c_{22} + c_{32}x_{26} + c_{42}x_{31} + c_{52}x_{38} + c_{62}x_{39})x_{13}, \tag{7}$$

where,

c	$i=1$	$i=2$
c_{1i}	0.23947	0.75835
c_{2i}	−0.0139904	−0.0661588
c_{3i}	0.0093514	0.0338147
c_{4i}	0.0077308	0.0373349
c_{5i}	−0.0005719	0.0016371
c_{6i}	0.0042656	0.0288996

subject to :

$$h_1(\bar{x}) = x_4 + x_3 + x_2 + x_1 = 300,$$

$$h_2(\bar{x}) = x_6 - x_8 - x_7 = 0,$$

$$h_3(\bar{x}) = x_9 - x_{12} - x_{10} - x_{11} = 0,$$

$$h_4(\bar{x}) = x_{14} - x_{17} - x_{15} - x_{16} = 0,$$

$$h_5(\bar{x}) = x_{18} - x_{20} - x_{19} = 0,$$

$$h_6(\bar{x}) = x_6x_{21} - x_{24}x_{25} = 0,$$

$$h_7(\bar{x}) = x_{14}x_{22} - x_{26}x_{27} = 0,$$

$$h_8(\bar{x}) = x_9x_{23} - x_{28}x_{29} = 0,$$

$$h_9(\bar{x}) = x_{18}x_{30} - x_{31}x_{32} = 0,$$

$$h_{10}(\bar{x}) = x_{25} - x_5x_{33} = 0,$$

$$h_{11}(\bar{x}) = x_{29} - x_5x_{34} = 0,$$

$$h_{12}(\bar{x}) = x_{35} - x_5x_{36} = 0,$$

$$h_{13}(\bar{x}) = x_{37} - x_{13}x_{38} = 0,$$

$$h_{14}(\bar{x}) = x_{27} - x_{13}x_{39} = 0,$$

$$h_{15}(\bar{x}) = x_{32} - x_{13}x_{40} = 0,$$

$$h_{16}(\bar{x}) = x_{25} - x_6x_{21} - x_9x_{41} = 0,$$

$$h_{17}(\bar{x}) = x_{29} - x_6x_{42} - x_9x_{23} = 0,$$

$$h_{18}(\bar{x}) = x_{35} - x_6x_{43} - x_9x_{44} = 0,$$

$$h_{19}(\bar{x}) = x_{37} - x_{14}x_{45} - x_{18}x_{46} = 0,$$

$$h_{20}(\bar{x}) = x_{27} - x_{14}x_{22} - x_{18}x_{47} = 0,$$

$$h_{21}(\bar{x}) = x_{32} - x_{14}x_{48} - x_{18}x_{30} = 0,$$

$$h_{22}(\bar{x}) = 0.333x_1 + x_{15}x_{45} - x_{25} = 0,$$

$$h_{23}(\bar{x}) = 0.333x_1 + x_{15}x_{22} - x_{29} = 0,$$

$$h_{24}(\bar{x}) = 0.333x_1 + x_{15}x_{48} - x_{35} = 0,$$

$$h_{25}(\bar{x}) = 0.333x_2 + x_{10}x_{41} - x_{37} = 0,$$

$$h_{26}(\bar{x}) = 0.333x_2 + x_{10}x_{23} - x_{27} = 0,$$

$$h_{27}(\bar{x}) = 0.333x_2 + x_{10}x_{44} - x_{32} = 0,$$

$$h_{28}(\bar{x}) = 0.333x_3 + x_7x_{21} + x_{11}x_{41} + x_{16}x_{45} + x_{19}x_{46} = 30,$$

$$h_{29}(\bar{x}) = 0.333x_3 + x_7x_{42} + x_{11}x_{23} + x_{16}x_{22} + x_{19}x_{47} = 50,$$

$$h_{30}(\bar{x}) = 0.333x_3 + x_7x_{43} + x_{11}x_{44} + x_{16}x_{48} + x_{19}x_{30} = 30,$$
$$h_{31}(\bar{x}) = x_{33} + x_{34} + x_{36} = 1,$$
$$h_{32}(\bar{x}) = x_{21} + x_{42} + x_{43} = 1,$$
$$h_{33}(\bar{x}) = x_{41} + x_{23} + x_{44} = 1,$$
$$h_{34}(\bar{x}) = x_{38} + x_{39} + x_{40} = 1,$$
$$h_{35}(\bar{x}) = x_{45} + x_{22} + x_{48} = 1,$$
$$h_{36}(\bar{x}) = x_{46} + x_{47} + x_{30} = 1,$$
$$h_{37}(\bar{x}) = x_{43} = 0,$$
$$h_{38}(\bar{x}) = x_{46} = 0,$$

with bounds :

$$0 \leq x_1, ..., x_{20} \leq 150; 0 \leq x_{25}, x_{27}, x_{32}, x_{35}, x_{37}, x_{29} \leq 30;$$
$$0 \leq x_{21}, x_{22}, x_{23}, x_{30}, x_{33}, x_{34}, x_{36}, x_{38}, x_{39}, x_{40}, x_{42}, x_{43}, x_{44}, x_{45}, \leq 1,$$
$$0 \leq x_{46}, x_{47}, x_{48} \leq 1,$$
$$0.85 \leq x_{24}, x_{26}, x_{28}, x_{31} \leq 1$$

2.1.6. Optimal Operation of Alkylation Unit [26]

The main aim of this problem is to maximize the octane number of olefin feed in the presence of acid. The objective function is defined as an alkylating product. The problem is formulated as follows.

Maximize :

$$f(\bar{x}) = 0.035x_1x_6 + 1.715x_1 + 10.0x_2 + 4.0565x_3 - 0.063x_3x_5 \tag{8}$$

subject to :

$$g_1(\bar{x}) = 0.0059553571x_6^2x_1 + 0.88392857x_3 - 0.1175625x_6x_1 - x_1 \leq 0,$$
$$g_2(\bar{x}) = 1.1088x_1 + 0.1303533x_1x_6 - 0.0066033x_1x_6^2 - x_3 \leq 0,$$
$$g_3(\bar{x}) = 6.66173269x_6^2 - 56.596669x_4 + 172.39878x_5 - 10000 - 191.20592x_6 \leq 0,$$
$$g_4(\bar{x}) = 1.08702x_6 - 0.03762x_6^2 + 0.32175x_4 + 56.85075 - x_5 \leq 0,$$
$$g_5(\bar{x}) = 0.006198x_7x_4x_3 + 2462.3121x_2 - 25.125634x_2x_4 - x_3x_4 \leq 0,$$
$$g_6(\bar{x}) = 161.18996x_3x_4 + 5000.0x_2x_4 - 489510.0x_2 - x_3x_4x_7 \leq 0,$$
$$g_7(\bar{x}) = 0.33x_7 + 44.333333 - x_5 \leq 0,$$
$$g_8(\bar{x}) = 0.022556x_5 - 1.0 - 0.007595x_7 \leq 0,$$
$$g_9(\bar{x}) = 0.00061x_3 - 1.0 - 0.0005x_1 \leq 0,$$
$$g_{10}(\bar{x}) = 0.819672x_1 - x_3 + 0.819672 \leq 0,$$
$$g_{11}(\bar{x}) = 24500.0x_2 - 250.0x_2x_4 - x_3x_4 \leq 0,$$
$$g_{12}(\bar{x}) = 1020.4082x_4x_2 + 1.2244898x_3x_4 - 100000x_2 \leq 0,$$
$$g_{13}(\bar{x}) = 6.25x_1x_6 + 6.25x_1 - 7.625x_3 - 100000 \leq 0,$$
$$g_{14}(\bar{x}) = 1.22x_3 - x_6x_1 - x_1 + 1.0 \leq 0.$$

with bounds :

$$1000 \leq x_1 \leq 2000, \; 0 \leq x_2 \leq 100$$
$$2000 \leq x_3 \leq 4000, \; 0 \leq x_4 \leq 100$$
$$0 \leq x_5 \leq 100, \; 0 \leq x_6 \leq 20$$
$$0 \leq x_7 \leq 200.$$

2.1.7. *Reactor Network Design [27]*

A reactor network design problem is solved to design a sequence of two CSTR reactors. The main aim of this problem is to optimize the concentration of the product. This problem is formulated as follows.

Maximize :

$$f(\bar{x}) = x_4 \tag{9}$$

subject to:

$$h_1(\bar{x}) = k_1 x_5 x_2 + x_1 - 1 = 0,$$
$$h_2(\bar{x}) = k_3 x_5 x_3 + x_3 + x_1 - 1 = 0,$$
$$h_3(\bar{x}) = k_2 x_6 x_2 - x_1 + x_2 = 0,$$
$$h_4(\bar{x}) = k_4 x_6 x_4 + x_2 - x_1 + x_4 - x_3 + = 0,$$
$$g_1(\bar{x}) = x_5^{0.5} + x_6^{0.5} \leq 4$$

with bounds :

$$0 \leq x_4, x_3, x_2, x_1 \leq 1,$$
$$0.00001 \leq x_6, x_5 \leq 16.$$

where, $k_3 = 0.0391908$, $k_4 = 0.9k_3$, $k_1 = 0.09755988$, and $k_2 = 0.99k_1$, .

2.2. *Process design and synthesis problems [28, 29]*

Process design and synthesis problems in chemical engineering can be defined as a mixed-integer nonlinear constrained optimization problem.

2.2.1. *Process synthesis problem*

This problem incorporates a non-linear constraint and is defined as follows.

Minimize :

$$f(\bar{x}) = x_2 + 2x_1 \tag{10}$$

subject to :

$$g_1(\bar{x}) = -x_1^2 - x_2 + 1.25 \leq 0,$$
$$g_2(\bar{x}) = x_1 + x_2 \leq 1.6.$$

with bounds :

$$0 \leq x_1 \leq 1.6$$
$$x_2 \in \{0.1\}$$

2.2.2. *Process synthesis and design problem*

This problem incorporates a non-linear constraint and is represented as follows.

Minimize :

$$f(\bar{x}) = -x_3 + x_2 + 2x_1 \tag{11}$$

subject to :

$$h_1(\bar{x}) = -2\exp(-x_2) + x_1 = 0,$$
$$g_1(\bar{x}) = x_2 - x_1 + x_3 \leq 0.$$

with bounds :

$$0.5 \leq x_1, x_2 \leq 1.4,$$
$$x_3 \in \{0, 1\}.$$

2.2.3. *Process flow sheeting problem*

This problem can be formulated as a non-convex constrained optimization problem, which is expressed as follows.

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Minimize :

$$f(\bar{x}) = -0.7x_3 + 0.8 + 5(0.5 - x_1)^2 \tag{12}$$

subject to :

$$g_1(\bar{x}) = -exp(x_1 - 0.2) - x_2 \leq 0,$$

$$g_2(\bar{x}) = x_2 + 1.1x_3 \leq -1.0,$$

$$g_3(\bar{x}) = x_1 - x_3 \leq 0.2.$$

with bounds :

$$-2.22554 \leq x_2 \leq -1,\ 0.2 \leq x_1 \leq 1,$$
$$x_3 \in \{0, 1\}.$$

2.2.4. *Two-reactor problem*

The essential purpose of this problem is to choose one among two reactors to optimize the production cost.

Minimize :

$$f(\bar{x}) = 7.5x_7 + 5.5x_8 + 7x_5 + 6x_6 + 5(x_1 + x_2) \tag{13}$$

subject to :

$$h_1(\bar{x}) = x_7 + x_8 - 1 = 0,$$

$$h_2(\bar{x}) = x_3 - 0.9(1 - \exp(0.5x_5))x_1 = 0,$$

$$h_3(\bar{x}) = x_4 - 0.8(1 - \exp(0.4x_6))x_2 = 0,$$

$$h_4(\bar{x}) = x_3 + x_4 - 10 = 0,$$

$$h_5(\bar{x}) = x_3x_7 + x_4x_8 - 10 = 0,$$

$$g_1(\bar{x}) = x_5 - 10x_7 \leq 0,$$

$$g_2(\bar{x}) = x_6 - 10x_8 \leq 0,$$

$$g_3(\bar{x}) = x_1 - 20x_7 \leq 0,$$

$$g_4(\bar{x}) = x_2 - 20x_8 \leq 0$$

with bounds :

$$0 \leq x_6, x_5, x_4, x_3, x_2, x_1 \leq 100$$
$$x_8, x_7 \in \{0, 1\}.$$

2.2.5. *Process synthesis problem*

This problem provides seven degrees of freedom due to non-linearities in all real variables and binary variables.

Minimize :

$$f(\bar{x}) = (1 - x_4)^2 + (1 - x_5)^2 + (1 - x_6)^2 - \ln(1 + x_7) \\ + (1 - x_1)^2 + (2 - x_2)^2 + (3 - x_3)^2 \tag{14}$$

subject to :

$$g_1(\bar{x}) = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - 5 \leq 0,$$

$$g_2(\bar{x}) = x_6^3 + x_1^2 + x_2^2 + x_3^2 = 5.5 \leq 0,$$

$$g_3(\bar{x}) = x_1 + x_4 - 1.2 \leq 0,$$

$$g_4(\bar{x}) = x_2 + x_5 - 1.8 \leq 0,$$

$$g_5(\bar{x}) = x_3 + x_6 - 2.5 \leq 0,$$

$$g_6(\bar{x}) = x_1 + x_7 - 1.2 \leq 0,$$

$$g_7(\bar{x}) = x_5^2 + x_2^2 - 1.64 \leq 0,$$

$$g_8(\bar{x}) = x_6^2 + x_3^2 - 4.25 \leq 0,$$

$$g_9(\bar{x}) = x_5^2 + x_3^2 - 4.64 \leq 0,$$

with bounds :

$$0 \leq x_2, x_3, x_1 \leq 100, \\ x_7, x_6, x_5, x_4 \in \{0, 1\}.$$

2.2.6. *Process design problem*

It is a minimization problem which can be formulated as following way.

Minimize :

$$f(\bar{x}) = 5.357854x_1^2 + 40792.141 - 37.29329x_4 + 0.835689x_4x_3 \tag{15}$$

subject to :

$$g_1(\bar{x}) = -92 + a_3x_4x_2 + a_1 + a_2x_4x_3 - a_4x_4x_3 \leq 0,$$

$$g_2(\bar{x}) = -110 + a_7x_4x_2 + a_5 + a_6x_5x_3 + a_8x_1^2 \leq 0,$$

$$g_3(\bar{x}) = a_9 + a_{11}x_4x_1 + a_{10}x_4x_3 - 25 + a_{12}x_1x_2 \leq 0$$

with bounds :

$$27 \leq x_3, x_1, x_2 \leq 45, \\ y_1 \in \{78, 79, ..., 102\}, \\ y_2 \in \{33, 34, ..., 45\}.$$

where, parameters a_1 to a_{12} are constant and values of these constants are shown in Table 5

Table 1: Constants for process design problem.

$a_1 = 85.334407$	$a_5 = 80.51249$	$a_9 = 9.300961$
$a_2 = 0.0056858$	$a_6 = 0.0071317$	$a_{10} = 0.0047026$
$a_3 = 0.0006262$	$a_7 = 0.0029955$	$a_{11} = 0.0012547$
$a_4 = 0.0022053$	$a_8 = 0.0021813$	$a_{12} = 0.0019085$

2.2.7. Multi-product batch plant

This is a multi-product batch plant problem with M serial processing stages, where fixed amounts Q_i from N products must be produced. The problem is formulated as follows.

Minimize :

$$f(\bar{x}) = \sum_{j=1}^{M} \alpha_j N_j V_j^{\beta_j} \tag{16}$$

subject to :

$$g_1(\bar{x}) = S_{ij} B_i - V_j \leq 0,$$

$$g_2(\bar{x}) = -H + \sum_{i=1}^{N} \frac{Q_i T_{Li}}{B_i} \leq 0,$$

$$g_3(\bar{x}) = t_{ij} - N_j T_{Li} \leq 0,$$

with bounds :

$$1 \leq N_i \leq N_j^u,$$
$$V_j^l \leq V_j \leq V_j^u,$$
$$T_{Li}^l \leq T_{Li} \leq T_{Li}^u,$$
$$B_j^l \leq B_j \leq B_j^u.$$

where, $N = 2$, $M = 3$, $\alpha_j = 250$, $H = 6000$, $\beta_j = 0.6$, $N_j^u = 3$, $V_j^l = 250$, and $V_j^u = 2500$. The value of other parameters are calculated by

$$T_{Li}^l = \max\left(\frac{t_{ij}}{N_j^u}\right), \tag{17}$$

$$T_{Li}^u = \max\left(t_{ij}\right), \tag{18}$$

$$B_j^l = \frac{Q_i^* T_{Li}}{H}, \tag{19}$$

$$B_j^u = \min\left(Q_i, \min_j \left(\frac{V_j^u}{S_{ij}}\right)\right) \tag{20}$$

Parameters S_{ij} and t_{ij} are given in Table 2

Table 2: Values of S_{ij} and t_{ij}.

S_{ij}				t_{ij}		
2	3	4		8	20	8
4	6	3		16	4	4

2.3. Mechanical Design Problems

In this section, several mechanical element design problems are considered. A brief description of different mechanical design problems is provided in the following subsections.

2.3.1. Weight Minimization of a Speed Reducer [30]

It involves the design of a speed reducer for small aircraft engine. The resulting optimization problem has the following form.

Minimize :

$$f(\bar{x}) = 0.7854x_2^2x_1(14.9334x_3 - 43.0934 + 3.3333x_3^2) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^3 + x_6^3) \tag{21}$$

subject to :

$$\begin{aligned}
&g_1(\bar{x}) = -x_1x_2^2x_3 + 27 \leq 0,\\
&g_2(\bar{x}) = -x_1x_2^2x_3^2 + 397.5 \leq 0,\\
&g_3(\bar{x}) = -x_2x_6^4x_3x_4^{-3} + 1.93 \leq 0,\\
&g_4(\bar{x}) = -x_2x_7^4x_3x_5^{-3} + 1.93 \leq 0,\\
&g_5(\bar{x}) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \leq 0,\\
&g_6(\bar{x}) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \leq 0,\\
&g_7(\bar{x}) = x_2x_3 - 40 \leq 0,\\
&g_8(\bar{x}) = -x_1x_2^{-1} + 5 \leq 0,\\
&g_9(\bar{x}) = x_1x_2^{-1} - 12 \leq 0,\\
&g_{10}(\bar{x}) = 1.5x_6 - x_4 + 1.9 \leq 0,\\
&g_{11}(\bar{x}) = 1.1x_7 - x_5 + 1.9 \leq 0,
\end{aligned}$$

with bounds :

$$\begin{aligned}
&0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 2.6 \leq x_1 \leq 3.6,\\
&5 \leq x_7 \leq 5.5, 7.3 \leq x_5, x_4 \leq 8.3, 2.9 \leq x_6 \leq 3.9.
\end{aligned}$$

2.3.2. Optimal Design of Industrial refrigeration System [31]

The mathematical model of this problem is described in [31] and [30]. This problem can be formulated as non-linear inequality constrained optimization problem and has the following form:

Minimize :

$$\begin{aligned}
f(\bar{x}) = &63098.88x_2x_4x_{12} + 5441.5x_2^2x_{12} + 115055.5x_2^{1.664}x_6 + 6172.27x_2^2x_6\\
&+63098.88x_1x_3x_{11} + 5441.5x_1^2x_{11} + 115055.5x_1^{1.664}x_5 + 6172.27x_1^2x_5\\
&+140.53x_1x_{11} + 281.29x_3x_{111} + 70.26x_1^2 + 281.29x_1x_3 + 281.29x_3^2\\
&+14437x_8^{1.8812}x_{12}^{0.3424}x_{10}x_{14}^{-1}x_1^2x_7x_9^{-1} + 20470.2x_7^{2.893}x_{11}^{0.316}x_1^2
\end{aligned} \tag{22}$$

subject to :

$$g_1(\bar{x}) = 1.524x_7^{-1} \leq 1,$$

$$
\begin{aligned}
&g_2(\bar{x}) = 1.524x_8^{-1} \leq 1,\\
&g_3(\bar{x}) = 0.07789x_1 - 2x_7^{-1}x_9 - 1 \leq 0,\\
&g_4(\bar{x}) = 7.05305x_9^{-1}x_1^2x_{10}x_8^{-1}x_2^{-1}x_{14}^{-1} - 1 \leq 0,\\
&g_5(\bar{x}) = 0.0833x_{13}^{-1}x_{14} - 1 \leq 0,\\
&g_6(\bar{x}) = 47.136x_2^{0.333}x_{10}^{-1}x_{12} - 1.333x_8x_{13}^{2.1195} + 62.08x_{13}^{2.1195}x_{12}^{-1}x_8^{0.2}x_{10}^{-1} - 1 \leq 0,\\
&g_7(\bar{x}) = 0.04771x_{10}x_8^{1.8812}x_{12}^{0.3424} - 1 \leq 0,\\
&g_8(\bar{x}) = 0.0488x_9x_7^{1.893}x_{11}^{0.316} - 1 \leq 0,\\
&g_9(\bar{x}) = 0.0099x_1x_3^{-1} - 1 \leq 0,\\
&g_{10}(\bar{x}) = 0.0193x_2x_4^{-1} - 1 \leq 0,\\
&g_{11}(\bar{x}) = 0.0298x_1x_5^{-1} - 1 \leq 0,\\
&g_{12}(\bar{x}) = 0.056x_2x_6^{-1} - 1 \leq 0,\\
&g_{13}(\bar{x}) = 2x_9^{-1} - 1 \leq 0,\\
&g_{14}(\bar{x}) = 2x_{10}^{-1} - 1 \leq 0,\\
&g_{15}(\bar{x}) = x_{12}x_{11}^{-1} - 1 \leq 0,
\end{aligned}
$$

with bounds :

$$0.001 \leq x_i \leq 5, \; i = 1, ..., 14.$$

2.3.3. Tension/compression spring design [32]

The main objective of this problem is to optimize the weight of tension or compression spring. This problem contains four constraints and three variables are utilized to calculate the weight: the diameter of the wire (x_1), the mean of the diameter of coil (x_2), and the number of active coils (x_3). This problem is defined using following way.
Minimize :

$$f(\bar{x}) = x_1^2x_2(2 + x_3) \tag{23}$$

subject to :

$$
\begin{aligned}
&g_1(\bar{x}) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0\\
&g_2(\bar{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 -{}_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0\\
&g_3(\bar{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0\\
&g_4(\bar{x}) = \frac{x_1 + x_2}{1.5} - 1 \leq 0
\end{aligned}
$$

with bounds :

$$
\begin{aligned}
&0.05 \leq x_1 \leq 2.00\\
&0.25 \leq x_2 \leq 1.30\\
&2.00 \leq x_3 \leq 15.0
\end{aligned}
$$

2.3.4. *Pressure vessel design [32]*

The main objective of this problem is to optimize the welding cost, material, and forming of a vessel. This problem contains four constraints which are needed to be satisfied, and four variables are used to calculate the objective function: shell thickness (z_1), head thickness (z_2), inner radius (x_3), and length of the vessel without including the head (x_4). This problem can be stated as

Minimize :

$$f(\bar{x}) = 1.7781 z_2 x_3^2 + 0.6224 z_1 x_3 x_4 + 3.1661 {z_1}^2 x_4 + 19.84 {z_1}^2 x_3 \tag{24}$$

subject to :

$$g_1(\bar{x}) = 0.00954 x_3 \leq z_2,$$
$$g_2(\bar{x}) = 0.0193 x_3 \leq z_1,$$
$$g_3(\bar{x}) = x_4 \leq 240,$$
$$g_4(\bar{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \leq -1296000.$$

where :

$$z_1 = 0.0625 x_1,$$
$$z_2 = 0.0625 x_2.$$

with bounds :

$$10 \leq x_4, x_3 \leq 200$$
$1 \leq x_2, x_1 \leq 99$ **(integer variables)**.

2.3.5. *Welded beam design [32]*

The main of this problem is to design a welded beam with minimum cost. This problem contains five constraints, and four variables are used to develop a welded beam. The mathematical description of this problem can be defined as follows.

Minimize :

$$f(\bar{x}) = 0.04811 x_3 x_4 (x_2 + 14) + 1.10471 x_1^2 x_2 \tag{25}$$

subject to :

$$g_1(\bar{x}) = x_1 - x_4 \leq 0,$$
$$g_2(\bar{x}) = \delta(\bar{x}) - \delta_{max} \leq 0,$$
$$g_3(\bar{x}) = P \leq P_c(\bar{x})$$
$$g_4(\bar{x}) = \tau_{max} \geq \tau(\bar{x}),$$
$$g_5(\bar{x}) = \sigma(\bar{x}) - \sigma_{max} \leq 0,$$

where,

$$\tau = \sqrt{\tau'^2 + \tau''^2 + 2\tau'\tau''\frac{x_2}{2R}},\ \tau'' = \frac{RM}{J},\ \tau' = \frac{P}{\sqrt{2}x_2 x_1},\ M = p\left(\frac{x_2}{2} + L\right),$$
$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},\ J = 2\left(\left(\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right)\sqrt{2}x_1 x_2\right),$$
$$\sigma(\bar{x}) = \frac{6PL}{x_4 x_3^2},\ \delta(\bar{x}) = \frac{6PL^3}{E x_3^2 X_4},\ P_c(\bar{x}) = \frac{4.013 E x_3 x_4^3}{6L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$
$$L = 14\text{ in},\ P = 6000\text{ lb},\ E = 30.10^6\text{ psi},\ \sigma_{max} = 30,000\text{ psi},$$
$$\tau_{max} = 13,600\text{ psi},\ G = 12.10^6\text{ psi},\ \delta_{max} = 0.25\text{ in},.$$

with bounds :

$$0.1 \leq x_3, x_2 \leq 10$$
$$0.1 \leq x_4 \leq 2$$
$$0.125 \leq x_1 \leq 2$$

2.3.6. Three-bar truss design problem [33]

This optimization problem is taken from civil engineering, which has a problematic constrained space. The main of this problem is to minimize the weight of the bar structures. The constraints of this problem are based on the stress constraints of each bar. The resultant problem is a non-linear objective function with three non-linear constraints. The mathematical description of this problem is given below.

Minimize :

$$f(\bar{x}) = l\left(x_2 + 2\sqrt{2}x_1\right) \tag{26}$$

subject to :

$$g_1(\bar{x}) = \frac{x_2}{2x_2x_1 + \sqrt{2}x_1^2}P - \sigma \leq 0,$$

$$g_2(\bar{x}) = \frac{x_2 + \sqrt{2}x_1}{2x_2x_1 + \sqrt{2}x_1^2}P - \sigma \leq 0,$$

$$g_3(\bar{x}) = \frac{1}{x_1 + \sqrt{2}x_2}P - \sigma \leq 0.$$

with bounds :

$$0 \leq x_1, x_2 \leq 1.$$

2.3.7. Multiple disk clutch brake design problem [34]

The main objective of this problem is to minimize the mass of multiple disk clutch brake. In this problem, five integer decision variables are used which are inner radius (x_1), outer radius (x_2), disk thickness (x_3), force of actuators (x_4), and number of frictional surfaces (x_5). This problem contains nine non-linear constraints. The problem can be defined as follows.

Minimize :

$$f(\bar{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho \tag{27}$$

subject to :

$$g_1(\bar{x}) = -p_{max} + p_{rz} \leq 0,$$

$$g_2(\bar{x}) = p_{rz}V_{sr} - V_{sr,max}p_{max} \leq 0,$$

$$g_3(\bar{x}) = \triangle R + x_1 - x_2 \leq 0,$$

$$g_4(\bar{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \leq 0,$$

$$g_5(\bar{x}) = sM_s - M_h \leq 0,$$

$$g_6(\bar{x}) = T \geq 0,$$

$$g_7(\bar{x}) = -V_{sr,max} + V_{sr} \leq 0,$$

$$g_7(\bar{x}) = T - T_{max} \leq 0,$$

where,

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2} \text{ N.mm},$$

$$\omega = \frac{\pi n}{30} \text{ rad/s},$$

$$A = \pi(x_2^2 - x_1^2) \text{ mm}^2,$$

$$p_{rz} = \frac{x_4}{A} \text{ N/mm}^2,$$

$$V_{sr} = \frac{\pi R_{sr} n}{30} \text{ mm/s},$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} \text{ mm},$$

$$T = \frac{I_z \omega}{M_h + M_f},$$

$$\triangle R = 20 \text{ mm},\ L_{max} = 30 \text{ mm},\ \mu = 0.6,$$

$$V_{sr,max} = 10 \text{ m/s},\ \delta = 0.5 \text{ mm},\ s = 1.5,$$

$$T_{max} = 15 \text{ s},\ n = 250 \text{ rpm},\ I_z = 55 \text{ Kg.m}^2,$$

$$M_s = 40 \text{ Nm},\ M_f = 3 \text{ Nm}.$$

with bounds :

$$60 \leq x_1 \leq 80,\ 90 \leq x_2 \leq 110,\ 1 \leq x_3 \leq 3,$$

$$0 \leq x_4 \leq 1000,\ 2 \leq x_5 \leq 9.$$

2.3.8. Planetary gear train design optimization problem [17]

The main objective of this problem is to minimize the maximum errors in the gear ratio, which is used in automobiles. To minimize the maximum error, the total number of gear-teeth is calculated for an automatic planetary transmission system. This problem contains six integer variables and 11 constraints of different geometric and assembly restrictions. The problem can be defined as follows.

Minimize :

$$f(\bar{x}) = \max|i_k - i_{0k}|,\ k = \{1, 2, .., R\}, \tag{28}$$

where,

$$i_1 = \frac{N_6}{N_4},\ i_{01} = 3.11,\ i_2 = \frac{N_6(N_1 N_3 + N_2 N_4)}{N_1 N_3 (N_6 - N_4)},\ i_{0R} = -3.11,$$

$$I_R = -\frac{N_2 N_6}{N_1 N_3},\ i_{02} = 1.84,\ \bar{x} = \{p, N_6, N_5, N_4, N_3, N_2, N_1, m_2, m_1\}$$

subject to :

$$g_1(\bar{x}) = m_3(N_6 + 2.5) - D_{max} \leq 0,$$

$$g_2(\bar{x}) = m_1(N_1 + N_2) + m_1(N_2 + 2) - D_{max} \leq 0,$$

$$g_3(\bar{x}) = m_3(N_4 + N_5) + m_3(N_5 + 2) - D_{max} \leq 0,$$

$$g_4(\bar{x}) = |m_1(N_1 + N_2) - m_3(N_6 - N_3)| - m_1 - m_3 \leq 0,$$

$$
\begin{aligned}
g_5(\bar{x}) &= -(N_1 + N_2)\sin(\pi/p) + N_2 + 2 + \delta_{22} \le 0,\\
g_6(\bar{x}) &= -(N_6 - N_3)\sin(\pi/p) + N_3 + 2 + \delta_{33} \le 0,\\
g_7(\bar{x}) &= -(N_4 + N_5)\sin(\pi/p) + N_5 + 2 + \delta_{55} \le 0,\\
g_8(\bar{x}) &= (N_3 + N_5 + 2 + \delta_{35})^2 - (N_6 - N_3)^2 - (N_4 + N_5)^2\\
&\quad + 2(N_6 - N_3)(N_4 + N_5)\cos\left(\frac{2\pi}{p} - \beta\right) \le 0,\\
g_9(\bar{x}) &= N_4 - N_6 + 2N_5 + 2\delta_{56} + 4 \le 0,\\
g_{10}(\bar{x}) &= 2N_3 - N_6 + N_4 + 2\delta_{34} + 4 \le 0,\\
h_1(\bar{x}) &= \frac{N_6 - N_4}{p} = \text{integer},
\end{aligned}
$$

where,

$$
\delta_{22} = \delta_{33} = \delta_{55} = \delta_{35} = \delta_{56} = 0.5.
$$

$$
\beta = \frac{cos^{-1}\left((N_4 + N_5)^2 + (N_6 - N_3)^2 - (N_3 + N_5)^2\right)}{2\,(N_6 - N_3)\,(N_4 + N_5)},\ D_{max} = 220,
$$

with bounds :

$$
\begin{aligned}
&p = (3, 4, 5)\,,\\
&m_1 = (1.75, 2.0, 2.25, 2.5, 2.75, 3.0)\,,\\
&m_3 = (1.75, 2.0, 2.25, 2.5, 2.75, 3.0)\,,\\
&17 \le N_1 \le 96, \qquad 14 \le N_2 \le 54, 14 \le N_3 \le 51\\
&17 \le N_4 \le 46, \qquad 14 \le N_5 \le 51, 48 \le N_6 \le 124,\\
&\text{and } N_i = \text{integer}.
\end{aligned}
$$

2.3.9. Step-cone pulley problem [35]

The main objective of this problem is to minimize weight of 4 step-cone pulley using five variables in which four variables are the diameters of each step of the pulley and the last one is the width of the pulley. This problem contains 11 non-linear constraints to assure that the transmit power must be at 0.75 hp. The mathematical formulation of this problem can be defined as follows.

Minimize :

$$
\begin{aligned}
f(\bar{x}) =& \rho\omega\Bigg[d_1^2\left\{11 + \left(\frac{N_1}{N}\right)^2\right\} + d_2^2\left\{1 + \left(\frac{N_2}{N}\right)^2\right\}\\
&+ d_3^2\left\{1 + \left(\frac{N_3}{N}\right)^2\right\} + d_4^2\left\{1 + \left(\frac{N_4}{N}\right)^2\right\}\Bigg]
\end{aligned} \tag{29}
$$

subject to :

$$
\begin{aligned}
h_1(\bar{x}) &= C_1 - C_2 = 0,\\
h_2(\bar{x}) &= C_1 - C_3 = 0,\\
h_3(\bar{x}) &= C_1 - C_4 = 0,\\
g_{i=1,2,3,4}(\bar{x}) &= -R_i \le 2,\\
g_{i=5,6,7,8}(\bar{x}) &= (0.75 \times 745.6998) - P_i \le 0
\end{aligned}
$$

where,

$$C_i = \frac{\pi d_i}{2}\left(1+\frac{N_i}{N}\right)+\frac{\left(\frac{N_i}{N}-1\right)^2}{4a}+2a,\ i=(1,2,3,4),$$

$$R_i = \exp\left(\mu\left\{\pi - 2sin^{-1}\left\{\left(\frac{N_i}{N}-1\right)\frac{d_i}{2a}\right\}\right\}\right),\ i=(1,2,3,4)$$

$$P_i = st\omega\left(1-R_i\right)\frac{\pi d_i N_i}{60},\ i=(1,2,3,4)$$

$$t = 8\text{ mm},\ s = 1.75\text{ MPa},\ \mu = 0.35,\ \rho = 7200\text{ kg/m}^3,\ a = 3\text{ mm}.$$

2.3.10. Robot gripper problem [36]

In this problem, the difference between the minimum and maximum force generated by the robot gripper is used as an objective function. This problem contains seven design variables and six non-linear design constraints associated with the robot. Mathematically, this problem is defined as follows.

Minimize :

$$f(\bar{x}) = -\min_z F_k(x,z) + \max_z F_k(x,z) \tag{30}$$

subject to :

$$g_1(\bar{x}) = -Y_{min} + y(\bar{x}, Z_{max}) \leq 0,$$

$$g_2(\bar{x}) = -y(x, Z_{max}) \leq 0,$$

$$g_3(\bar{x}) = Y_{max} - y(\bar{x}, 0) \leq 0,$$

$$g_4(\bar{x}) = y(\bar{x}, 0) - Y_G \leq 0,$$

$$g_5(\bar{x}) = l^2 + e^2 - (a+b)^2 \leq 0,$$

$$g_6(\bar{x}) = b^2 - (a-e)^2 - (l - Z_{max})^2 \leq 0,$$

$$g_7(\bar{x}) = Z_{max} - l \leq 0$$

where,

$$\alpha = cos^{-1}\left(\frac{a^2+g^2-b^2}{2ag}\right)+\phi,\ g = \sqrt{e^2+(z-l)^2},$$

$$\beta = cos^{-1}\left(\frac{b^2+g^2-a^2}{2bg}\right)-\phi,\ \phi = tan^{-1}\left(\frac{e}{l-z}\right),$$

$$y(x,z) = 2(f+e+c\sin(\beta+\delta)),\ F_k = \frac{Pb\sin(\alpha+\beta)}{2c\cos(\alpha)},\ Y_{min} = 50,$$

$$Y_{max} = 100,\ Y_G = 150,\ Z_{max} = 100,\ P = 100.$$

with bounds :

$$0 \leq e \leq 50,,\ 100 \leq c \leq 200,\ 10 \leq f, a, b \leq 150,$$

$$1 \leq \delta \leq 3.14,\ 100 \leq l \leq 300.$$

2.3.11. Hydro-static thrust bearing design problem [37]

The main objective of this design problem is to optimize bearing power loss using four design variables. These design variables are oil viscosity μ, bearing radius R, flow rate Q, and recess radius R_o. This problem contains seven non-linear constraints associated with inlet oil pressure, load-carrying capacity, oil film thickness, and inlet oil pressure. The problem is defined as follows.

Minimize :

$$f(\bar{x}) = \frac{QP_0}{0.7} + E_f \tag{31}$$

subject to :

$$g_1(\bar{x}) = 1000 - P_0 \leq 0,$$

$$g_2(\bar{x}) = W - 101000 \leq 0,$$

$$g_3(\bar{x}) = 5000 - \frac{W}{\pi(R^2 - R_0^2)} \leq 0,$$

$$g_4(\bar{x}) = 50 - P_0 \leq 0,$$

$$g_5(\bar{x}) = 0.001 - \frac{0.0307}{386.4P_0}\left(\frac{Q}{2\pi Rh}\right) \leq 0,$$

$$g_6(\bar{x}) = R - R_0 \leq 0,$$

$$g_7(\bar{x}) = h - 0.001 \leq 0,$$

where,

$$W = \frac{\pi P_0}{2}\frac{R^2 - R_0^2}{ln\left(\frac{R}{R_0}\right)},\ P_0 = \frac{6\mu Q}{\pi h^3}ln\left(\frac{R}{R_0}\right),$$

$$E_f = 9336Q \times 0.0307 \times 0.5\triangle T,\ \triangle T = 2(10^P - 559.7),$$

$$P = \frac{log_{10}log_{10}\left(8.122 \times 10^6\mu + 0.8\right) + 3.55}{10.04},$$

$$h = \left(\frac{2\pi \times 750}{60}\right)^2 \frac{2\pi\mu}{E_f}\left(\frac{R^4}{4} - \frac{R_0^4}{4}\right)$$

with bounds :

$$1 \leq R \leq 16,\ 1 \leq R_0 \leq 16,$$

$$1 \times 10^{-6} \leq \mu \leq 16 \times 10^{-6},\ 1 \leq Q \leq 16.$$

2.3.12. Four-stage gear box problem

In this problem, the minimization of gearbox weight is considered as an objective where 22 design variables are used. These design variables are discrete in nature which include positions of the gear, positions of pinion, blank thickness, and number of teeth. This problem contains 86 non-linear design constraints associated with the pitch, kinematics, contact ratio, strength of the gears, assembly of gears, and size of gears. The feasible search-space of this problem is in ratio less than 0.0001 with many local solutions. The problem is defined as

Minimize :

$$f(\bar{x}) = \left(\frac{\pi}{1000}\right)\sum_{i=1}^{4}\frac{b_i c_i^2\left(N_{pi}^2 + N_{gi}^2\right)}{\left(N_{pi} + N_{gi}\right)^2},\ \text{where, } i = (1, 2, 3, 4) \tag{32}$$

subject to :

$$g_1(\bar{x}) = \left(\frac{366000}{\pi\omega_1} + \frac{2c_1N_{p1}}{N_{pi}+N_{g1}}\right)\left(\frac{\left(N_{p1}+N_{g1}\right)^2}{4b_1c_1^2N_{p1}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \leq 0,$$

$$g_2(\bar{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1 N_{p1}} + \frac{2c_2N_{p2}}{N_{p2}+N_{g2}}\right)\left(\frac{\left(N_{p2}+N_{g2}\right)^2}{4b_2c_2^2N_{p2}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \leq 0,$$

$$g_3(\bar{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1 N_{p1}N_{p2}} + \frac{2c_3N_{p3}}{N_{p3}+N_{g3}}\right)\left(\frac{\left(N_{p3}+N_{g3}\right)^2}{4b_3c_3^2N_{p3}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \leq 0,$$

$$g_4(\bar{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1 N_{p1}N_{p2}N_{p3}} + \frac{2c_4N_{p4}}{N_{p4}+N_{g4}}\right)\left(\frac{\left(N_{p4}+N_{g4}\right)^2}{4b_4c_4^2N_{p4}}\right) - \frac{\sigma_N J_R}{0.0167WK_oK_m} \leq 0,$$

$$g_5(\bar{x}) = \left(\frac{366000}{\pi\omega_1} + \frac{2c_1N_{p1}}{N_{p1}+N_{g1}}\right)\left(\frac{\left(N_{p1}+N_{g1}\right)^3}{4b_1c_1^2N_{g1}N_{p1}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \leq 0,$$

$$g_6(\bar{x}) = \left(\frac{366000N_{g1}}{\pi\omega_1 N_{p1}} + \frac{2c_2N_{p2}}{N_{p2}+N_{g2}}\right)\left(\frac{\left(N_{p2}+N_{g2}\right)^3}{4b_2c_2^2N_{g2}N_{p2}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \leq 0,$$

$$g_7(\bar{x}) = \left(\frac{366000N_{g1}N_{g2}}{\pi\omega_1 N_{p1}N_{p2}} + \frac{2c_3N_{p3}}{N_{p3}+N_{g3}}\right)\left(\frac{\left(N_{p3}+N_{g3}\right)^3}{4b_3c_3^2N_{g3}N_{p3}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \leq 0,$$

$$g_8(\bar{x}) = \left(\frac{366000N_{g1}N_{g2}N_{g3}}{\pi\omega_1 N_{p1}N_{p2}N_{p3}} + \frac{2c_4N_{p4}}{N_{p4}+N_{g4}}\right)\left(\frac{\left(N_{p4}+N_{g4}\right)^3}{4b_4c_4^2N_{g4}N_{p4}^2}\right) - \left(\frac{\sigma_H}{C_p}\right)^2\left(\frac{\sin(\phi)\cos(\phi)}{0.0334WK_oK_m}\right) \leq 0,$$

$$g_{9-12}(\bar{x}) = -N_{pi}\sqrt{\frac{\sin^2(\phi)}{4} - \frac{1}{N_{pi}} + \left(\frac{1}{N_{pi}}\right)^2} + N_{gi}\sqrt{\frac{\sin^2(\phi)}{4} + \frac{1}{N_{gi}}\left(\frac{1}{N_{gi}}\right)^2} + \frac{\sin(\phi)\left(N_{pi}+N_{gi}\right)}{2} + CR_{min}\pi\cos(\phi) \leq 0,$$

$$g_{13-16}(\bar{x}) = d_{min} - \frac{2c_iN_{pi}}{N_{pi}+N_{gi}} \leq 0,$$

$$g_{17-20}(\bar{x}) = d_{min} - \frac{2c_iN_{gi}}{N_{pi}+N_{gi}} \leq 0,$$

$$g_{21}(\bar{x}) = x_{p1} + \left(\frac{\left(N_{p1}+2\right)c_1}{N_{p1}+N_{g1}}\right) - L_{max} \leq 0,$$

$$g_{22-24}(\bar{x}) = -L_{max} + \left(\frac{\left(N_{pi}+2\right)c_i}{N_{gi}+N_{pi}}\right)_{i=2,3,4} + x_{g(i-1)} \leq 0,$$

$$g_{25}(\bar{x}) = -x_{p1} + \frac{\left(N_{p1}+2\right)c_1}{N_{p1}+N_{g1}} \leq 0,$$

$$g_{26-28}(\bar{x}) = \left(\frac{\left(N_{pi}+2\right)c_i}{N_{pi}+N_{gi}} - x_{g(i-1)}\right)_{i=2,3,4} \leq 0,$$

$$g_{29}(\bar{x}) = y_{p1} + \frac{\left(N_{p1} + 2\right) c_1}{N_{p1} + N_{g1}} - L_{max} \leq 0,$$

$$g_{30-32}(\bar{x}) = -L_{max} + \left(\frac{c_i \left(2 + N_{pi}\right)}{N_{pi} + N_{gi}} + y_{g(i-1)}\right)_{i=2,3,4} \leq 0,$$

$$g_{33}(\bar{x}) = \frac{\left(2 + N_{p1}\right) c_1}{N_{p1} + N_{g1}} - y_{p1} \leq 0,$$

$$g_{34-36}(\bar{x}) = \left(\frac{c_i \left(2 + N_{pi}\right)}{N_{pi} + N_{gi}} - y_{g(i-1)}\right)_{i=2,3,4} \leq 0,$$

$$g_{37-40}(\bar{x}) = -L_{max} + \frac{c_i \left(2 + N_{gi}\right)}{N_{pi} + N_{gi}} + x_{gi} \leq 0,$$

$$g_{41-44}(\bar{x}) = -x_{gi} + \left(\frac{\left(N_{gi} + 2\right) c_i}{N_{pi} + N_{gi}}\right) \leq 0,$$

$$g_{45-48}(\bar{x}) = y_{gi} + \left(\frac{\left(N_{gi} + 2\right) c_i}{N_{pi} + N_{gi}}\right) - L_{max} \leq 0,$$

$$g_{49-52}(\bar{x}) = -y_{gi} + \left(\frac{\left(N_{gi} + 2\right) ci}{N_{pi} + N_{gi}}\right) \leq 0,$$

$$g_{53-56}(\bar{x}) = (b_i - 8.255)(b_i - 5.715)(b_i - 12.70)\left(-N_{pi} + 0.945 c_i - N_{gi}\right)(-1) \leq 0,$$

$$g_{57-60}(\bar{x}) = (b_i - 8.255)(b_i - 3.175)(b_i - 12.70)\left(-N_{pi} + 0.646 c_i - N_{gi}\right) \leq 0,$$

$$g_{61-64}(\bar{x}) = (b_i - 5.715)(b_i - 3.175)(b_i - 12.70)\left(-N_{pi} + 0.504 c_i - N_{gi}\right) \leq 0,$$

$$g_{65-68}(\bar{x}) = (b_i - 5.715)(b_i - 3.175)(b_i - 8.255)\left(0 c_i - N_{gi} - N_{pi}\right) \leq 0,$$

$$g_{69-72}(\bar{x}) = (b_i - 8.255)(b_i - 5.715)(b_i - 12.70)\left(N_{gi} + N_{pi} - 1.812 c_i\right)(-1) \leq 0,$$

$$g_{73-76}(\bar{x}) = (b_i - 8.255)(b_i - 3.175)(b_i - 12.70)\left(-0.945 c_i + N_{pi} + N_{gi}\right) \leq 0,$$

$$g_{77-80}(\bar{x}) = (b_i - 5.715)(b_i - 3.175)(b_i - 12.70)\left(-0.646 c_i + N_{pi} + N_{gi}\right)(-1) \leq 0,$$

$$g_{81-84}(\bar{x}) = (b_i - 5.715)(b_i - 3.175)(b_i - 8.255)\left(N_{pi} + N_{gi} - 0.504 c_i\right) \leq 0,$$

$$g_{85} = \omega_{min} - \frac{\omega_1 \left(N_{p1} N_{p2} N_{p3} N_{p4}\right)}{\left(N_{g1} N_{g2} N_{g3} N_{g4}\right)} \leq 0,$$

$$g_{86} = \frac{\omega_1 \left(N_{p1} N_{p2} N_{p3} N_{p4}\right)}{\left(N_{g1} N_{g2} N_{g3} N_{g4}\right)} - \omega_{max} \leq 0,$$

where,

$$\bar{x} = \{N_{p1}, N_{g1}, N_{p2}, N_{g2} .... b_1, b_2 .... x_{p1}, x_{g1}, x_{g2} .... y_{p1}, y_{g1}, y_{g2} .... y_{g4}\},$$

$$c_i = \sqrt{\left(y_{gi} - y_{pi}\right)^2 + \left(x_{gi} - x_{pi}\right)^2},\ K_0 = 1.5,\ d_{min} = 25,\ J_R = 0.2,\ \phi = 120^o,\ W = 55.9,\ K_M = 1.6,\ CR_{min} = 1.4,$$

$$L_{max} = 127,\ C_p = 464,\ \sigma_H = 3290,\ \omega_{max} = 255,\ \omega_1 = 5000,\ \sigma_N = 2090,\ \omega_{min} = 245.$$

with bounds :

$$b_i \in \{3.175, 12.7, 8.255, 5.715\},$$

$$y_{p1}, x_{p1}, y_{gi}, x_{gi} \in \{12.7, 38.1, 25.4, 50.8, 76.2, 63.5, 88.9, 114.3, 101.6\},$$
$$7 \leq N_{gi}, N_{pi} \leq 76 \in \text{integer}.$$

2.3.13. 10-bar truss optimization with frequency constraints [38]

The main aim of this problem is to minimize the weight of the truss structure with satisfying frequency constraints. The mathematical formulation of this problem can be defined as follows.

Minimize :

$$f(\bar{x}) = \sum_{i=1}^{10} L_i(x_i)\rho_i A_i \tag{33}$$

subject to :

$$g_1(\bar{x}) = \frac{7}{\omega_1(\bar{x})} - 1 \leq 0,$$

$$g_2(\bar{x}) = \frac{15}{\omega_2(\bar{x})} - 1 \leq 0,$$

$$g_3(\bar{x}) = \frac{20}{\omega_3(\bar{x})} - 1 \leq 0,$$

with bounds :

$$6.45 \times 10^{-5} \leq A_i \leq 5 \times 10^{-3},\ i = 1, 2, ..., 10.$$

where,

$$\bar{x} = \{A_1, A_2, ..., A_{10}\},\ \rho = 2770.$$

2.3.14. Rolling element bearing [39]

This problem is formulated to optimize the load-carrying capacity of a rolling element bearing using five design variables and five design parameters. These design variables are pitch diameter (D_m), ball diameter (D_b), outer and inner raceway curvature coefficients (f_o and f_i) and total number of balls (Z). The design parameters are e, ϵ, ζ, K_{Dmax}, and K_{Dmin} appeared in only constraints. These all are considered as variables i.e. five design variables and five design parameters. This problem contains nine non-linear constraints based on manufacturing and kinematic factors.

Maximize :

$$f(\bar{x}) = \begin{cases} f_c Z^{2/3} D_b^{1.8} & \text{, if } D_b \leq 25.4 \text{ mm} \\ 3.647 f_c Z^{2/3} D_b^{1.4} & \text{, otherwise} \end{cases} \tag{34}$$

subject to :

$$g_1(\bar{x}) = Z - \frac{\phi_0}{2\sin^{-1}(D_b/D_m)} - 1 \leq 0,$$

$$g_2(\bar{x}) = K_{Dmin}(D - d) - 2D_b \leq 0,$$

$$g_3(\bar{x}) = 2D_b - K_{Dmax}(D - d) \leq 0,$$

$$g_4(\bar{x}) = D_b -_w \leq 0,$$

$$g_5(\bar{x}) = 0.5\,(D+d) - D_m \le 0,$$
$$g_6(\bar{x}) = D_m - (0.5+e)\,(D+d) \le 0,$$
$$g_7(\bar{x}) = \epsilon D_b - 0.5\,(D - D_m - D_b) \le 0,$$
$$g_8(\bar{x}) = 0.515 - f_i \le 0,$$
$$g_9(\bar{x}) = 0.515 - f_0 \le 0,$$

where,

$$f_c = 37.91\left\{1+\left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i\,(2f_0-1)}{f_0\,(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right\}^{-0.3},\ \gamma=\frac{D_b\cos(\alpha)}{D_m},\ f_i=\frac{r_i}{D_b},\ f_0=\frac{r_0}{D_b},$$

$$\phi_0 = 2\pi - 2\times\cos^{-1}\left(\frac{\{(D-d)/2-3(T/4)\}^2+\{D/2-(T/4)-D_b\}^2-\{d/2+(T/4)\}^2}{2\{(D-d)/2-3(T/4)\}\{D/2-(T/4)-D_b\}}\right)$$
$$T = D - d - 2D_b,\ D = 160,\ d = 90,\ B_w = 30.$$

with bounds :

$0.5(D+d) \le D_m \le 0.6(D+d)$,
$0.15(D-d) \le D_b \le 0.45(D-d)$,
$4 \le Z \le 50$,
$0.515 \le f_i \le 0.6$,
$0.515 \le f_0 \le 0.6$,
$0.4 \le K_{Dmin} \le 0.5$,
$0.6 \le K_{Dmax} \le 0.7$,
$0.3 \le \epsilon \le 0.4$,
$0.02 \le e \le 0.1$,
$0.6 \le \zeta \le 0.85$,

2.3.15. Gas Transmission Compressor Design [40]

The mathematical formulation of this problem can be defined as follows.

Minimize :

$$f(\bar{x}) = 8.61\times10^5 x_1^{1/2}x_2x_3^{-2/3}x_4^{-1/2} + 3.69\times10^4 x_3 + 7.72\times10^8 x_1^{-1}x_2^{0.219} - 765.43\times10^6 x_1^{-1} \tag{35}$$

subject to :

$$x_4x_2^{-2} + x_2^{-2} - 1 \le 0,$$

with bounds :

$20 \le x_1 \le 50$,
$1 \le x_2 \le 10$,
$20 \le x_3 \le 50$,
$0.1 \le x_4 \le 60$.

2.3.16. Tension/compression string design problem (case 2) [41]

The main objective of this problem is to optimize the required volume of steel wire used to build the helical compression spring. There are three design variables in this problem which are the outside diameter (D), a number of spring coils (N), and diameter of the spring wire (d). This problem contains eight non-linear inequality constraints and contains a discreate, an integer, and a continuous variable. This problem can be stated as follows.

Minimize :

$$f(\bar{x}) = \frac{\pi^2 x_2 x_3^2 (x_1 + 2)}{4} \tag{36}$$

subject to :

$$g_1(\bar{x}) = \frac{8000 C_f x_2}{\pi x_3^3} - 189000 \leq 0,$$
$$g_2(\bar{x}) = l_f - 14 \leq 0,$$
$$g_3(\bar{x}) = 0.2 - x_3 \leq 0,$$
$$g_4(\bar{x}) = x_2 - 3 \leq 0,$$
$$g_5(\bar{x}) = 3 - \frac{x_2}{x_3} \leq 0,$$
$$g_6(\bar{x}) = \sigma_p - 6 \leq 0,$$
$$g_7(\bar{x}) = \sigma_p + \frac{700}{K} + 1.05(x_1 + 2)x_3 - l_f \leq 0,$$
$$g_8(\bar{x}) = 1.25 - \frac{700}{K} \leq 0,$$

where,

$$C_f = \frac{4\frac{x_2}{x_3} - 1}{4\frac{x_2}{x_3} - 4} + \frac{0.615 x_3}{x_2},\ K = \frac{11.5 \times 10^6 x_3^4}{8 x_1 x_2^3},\ \sigma_p = \frac{300}{K},\ l_f = \frac{1000}{K} + 1.05(x_1 + 2)x_3.$$

2.3.17. Gear Train Design Problem [42]

The main objective of this problem is to minimize the ratio of gears for the arrangement of the compound gear train. The ratio of gear train is described as the ratio of the angular velocities of the output and input shaft. To generate the desired overall ratio of gears, the compound gear train is assembled using two pairs of gearwheels, $b - f$ and $d - a$. The overall ratio of gears, i_{tot} is defined by the following equation.

$$i_{tot} = \frac{\omega_o}{\omega_i} = \frac{z_d z_b}{z_a z_f} \tag{37}$$

where, z is the total number of teeth on every gearwheel and variables ω_i and ω_o represent angular velocities of the input and output shafts, respectively. The main of this problem is to calculate the total number of teeth for every gearwheel to generate an optimum ratio of gears closer to the desired ratio ($i_{trg} = 1/6.931$). For every gearwheel, the maximum required of teeth is 60 and the minimum is 12. The mathematical formulation of this problem is shown as follows.

Minimize :

$$f(\bar{x}) = (i_{trg} - i_{tot})^2 = \left(\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right)^2 \tag{38}$$

subject to :

$$g_{1-4}(\bar{x}) = 12 - x_i \leq 0,$$
$$g_{5-8}(\bar{x}) = (60 - \bar{x}) \leq 0$$

2.3.18. *Himmelblau's Function [43]*

Himmelblau proposes this problem which is used as common benchmark problem to analyze the non-linear constrained optimization algorithms. This problem contains six nonlinear constraints and five variables. The description of this problem is shown as follows.

Minimize :

$$f(\bar{x}) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \tag{39}$$

subject to :

$$g_1(\bar{x}) = -G1 \leq 0,$$

$$g_2(\bar{x}) = G1 - 92 \leq 0,$$
$$g_3(\bar{x}) = 90 - G2 \leq 0,$$
$$g_4(\bar{x}) = G2 - 110 \leq 0,$$
$$g_5(\bar{x}) = 20 - G3 \leq 0,$$
$$g_6(\bar{x}) = G3 - 25 \leq 0,$$

where,

$$G1 = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5,$$

$$G2 = 80.51249 + 0.0071317_2x_5 + 0.0029955x_1x_2 + 0.0021813x_3^2,$$
$$G3 = 9.300961 + 0.0047026x_3x_5 + 0.00125447x_1x_3 + 0.0019085x_3x_4.$$

with bounds :

$$78 \leq x_1 \leq 102,$$
$$33 \leq x_2 \leq 45,$$
$$27 \leq x_3 \leq 45,$$
$$27 \leq x_4 \leq 45,$$
$$27 \leq x_5 \leq 45.$$

2.3.19. *Topology Optimization [44]*

The main aim of this problem is to optimize the material layout for a provided set of loads, within given design search-space, and constraints associated with the performance of the system. This problem is based on the power-law approach and mathematically, it can be defined as follows.

Minimize :

$$f(\bar{x}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N}(x_e)^p u_e^T k_0 u_0 \tag{40}$$

subject to :

$$h_1(\bar{x}) = \frac{V(\bar{x})}{V_0} - f = 0,$$

$$h_2(\bar{x}) = \mathbf{K}\mathbf{U} - \mathbf{F} = 0,$$

with bounds :

$$0 < \bar{x}_{min} \leq x \leq 1.$$

2.4. Power System Problems

2.4.1. Optimal Sizing of Single Phase Distributed Generation with reactive power support for Phase Balancing at Main Transformer/Grid [45]

Unbalance in practical distribution system is natural phenomenon. The unbalance in distribution system create negative and zero sequence currents leading to inefficient working of rotating machine along with losses in neutral conductors. In balanced system when there is no negative or zero sequence current the neutral current flow is zero. Balanced or assumption of balance in phases of distribution systems the neutral conductor are designed to carry smaller current. Apart from the above said problem due to unbalanced phase currents, one of the major concern are of overloading of main substation transformer. due to unbalance the phase having the maximum loading decides the capacity of the substation transformer. Thus, even if the transformer may be under loaded on the other two phases, it cannot further be loaded to take by the extra load. In present scenario the Distribution Generators (DGs) has been employed in distribution system in good numbers. The DG are of various type and are which of intent in this work is simple phase DGs. A DG in general is suitable generation and therefore no special arrangement are required to switch the DG feeding a phase to another one. The problem of phase balancing can be easily addressed if there are single phase DGs to redistribute the phase currents such that the unbalance is minimized. Single phase DG can be sized to mitigate phase unbalance, thereby reducing non-positive sequence currents at root node. This problem can be formulated as constrained optimization problem, which is as follows.

Minimize :

$$\begin{aligned} f = &\left(I^a_{r,1} + I^b_{r,1} + I^c_{r,1}\right)^2 + \left(I^a_{m,1} + I^b_{m,1} + I^c_{m,1}\right)^2 \\ &+ \left(I^a_{r,1} - 0.5\left(I^b_{r,1} + I^c_{r,1}\right) - 0.5\sqrt{3}\left(I^b_{m,1} - I^c_{m,1}\right)\right)^2 \\ &+ \left(I^a_{m,1} - 0.5\left(I^b_{m,1} + I^c_{m,1}\right) + 0.5\sqrt{3}\left(I^b_{r,1} - I^c_{r,1}\right)\right)^2, \end{aligned} \tag{41}$$

where,

$$I^s_{r,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{sk}_{1,i} V^k_{r,i} - B^{sk}_{1i} V^k_{m,i}\right)$$

$$I^s_{m,1} = \sum_{k\in\{a,b,c\}} \sum_{i=1}^{N} \left(B^{sk}_{1,i} V^k_{r,i} + G^{sk}_{1i} V^k_{m,i}\right)$$

subject to :

$$\begin{aligned} &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} \left(G^{js}_{k,i} V^s_{r,i} - B^{js}_{ki} V^s_{m,i}\right) - \frac{P^j_k V^j_{r,k} + Q^j_k V^j_{m,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0, \\ &\sum_{s\in\{a,b,c\}} \sum_{i=1}^{N} (B^{js}_{ki} V^s_{r,i} + G^{js}_{ki} V^s_{m,i}) - \frac{P^j_k V^j_{m,k} - Q^j_k V^j_{r,k}}{(V^j_{r,k})^2 + (V^j_{m,k})^2} = 0, \\ &P^j_k - P^j_{dg,k} + P^j_{l,k} = 0, \\ &Q^j_k - Q^j_{dg,k} + Q^j_{l,k} = 0, \end{aligned}$$

with bounds :

$$V_{min} \leq V^j_{r,k}, V^j_{m,k} \leq V_{max}$$

$$P_{min} \leq P^j_k \leq P_{max}$$

$$Q_{min} \leq Q^j_k \leq Q_{max}$$

$$P_{dg,min} \leq P^j_{dg,k} \leq P_{dg,max}$$

$$Q_{dg,min} \leq Q^{j}_{dg,k} \leq Q_{dg,max}$$

where $k = 1, 2, ...N$; $j = \{a, b, c\}$; P^j_i and Q^j_i represent the active and reactive injected power, respectively, at i-th bus in j-th phase; $Ybus^{st}_{ij}(= G^{st}_{ij} + 1jB^{st}_{ij})$ is ij-th element of st-th block of admittance matrix; $V^j_i(= V^j_{r,i} + 1jV^j_{m,i})$ is bus voltage at i-th bus in j-th phase; $P^j_{dg,k}$ and $Q^j_{dg,k}$ represent the active and reactive power generation, respectively, at k-th DG in j-th phase; N represents the total number of buses in system.

2.4.2. Optimal Sizing of Distributed Generation for Active Power Loss Minimization

Optimal sizing of both DG is of significance in improving system performance. The aim of this problem is to decide proper size of DG in the network that will offer minimum power loss. This problem can be formulated as constrained optimization problem, which is as follows.

Minimize :

$$f = \sum_{i=1}^{N} P_i \tag{42}$$

subject to :

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\\
&P_k - P_{dg,k} + P_{l,k} = 0,\\
&Q_k + Q_{l,k} = 0,
\end{aligned}$$

with bounds :

$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$

$$P_{min} \leq P_k \leq P_{max}$$

$$Q_{min} \leq Q_k \leq Q_{max}$$

$$P_{min,dg} \leq P_{dg,k} \leq P_{max,dg}$$

where $k = 1, 2, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; $P_{dg,k}$ represents the active power generation of DG at $k-$th bus; N represents the total number of buses in system.

2.4.3. Optimal Sizing of Distributed Generation (DG) and Capacitors for Reactive Power Loss Minimization

System loads such as transformer, induction motors, cables, and transmission lines are usually inductive. These loads consume reactive power and introduce a lagging power factor. Consequently, poor performance and losses are introduced in the system. Shunt capacitors are used to deliver reactive power to improve lagging VAR of the system. Moreover, Distributed Generators (DGs) are better and efficient mean to reduce the active power loss of the system. Further, shunt capacitors (SC) integrate with DG can be utilized to cut down the active and reactive power loss of the distribution network. The optimal sizing of DGs and SCs can be developed as a constrained optimization problem.

Minimize :

$$f = 0.5\sum_{i=1}^{N} P_i + 0.5\sum_{i=1}^{N} Q_i \tag{43}$$

subject to :

$$\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$
$$\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,$$
$$P_k - P_{dg,k} + P_{l,k} = 0,$$
$$Q_k - Q_{sc,k} + Q_{l,k} = 0,$$
$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$
$$P_{min} \leq P_k \leq P_{max}$$
$$Q_{min} \leq Q_k \leq Q_{max}$$
$$P_{min,dg} \leq P_{dg,k} \leq P_{max,dg}$$
$$Q_{min,sc} \leq Q_{sc,k} \leq Q_{max,sc}$$

where $k = 1, 2, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; $P_{dg,k}$ and $Q_{sc,k}$ represent the active power generation of DG and reactive power support from SC, respectively, at $k-$th bus; N represents the total number of buses in system.

2.4.4. Optimal Power flow (Minimization of Active Power Loss)

The optimal power flow (OPF) stands a commonly developed topic among the researchers. The OPF can be prepared as a single-objective constrained optimization problem of emission, voltage deviation, minimizing fuel cost, transmission loss, etc. with constraints based on line capacity, generator potential, power flow balance and bus voltage to be fulfilled. In this case, minimization of active power losses are considered as an objective function and this problem can be prepared as.

Minimize :

$$f = \sum_{i=1}^{N}\left(P_{g,i} - P_{l,i}\right) \tag{44}$$

subject to :

$$P_{g,i} - P_{l,i} - V_i\sum_{j=1}^{N}V_j\left(G_{ij}cos(\delta_i - \delta_j) + B_{ij}sin(\delta_i - \delta_j)\right) = 0,$$
$$Q_{g,i} - Q_{l,i} - V_i\sum_{j=1}^{N}V_j\left(G_{ij}sin(\delta_i - \delta_j) - B_{ij}cos(\delta_i - \delta_j)\right) = 0,$$

with bounds :

$$V_{min} \leq V_k \leq V_{max},$$
$$\delta_{min} \leq \delta_k \leq \delta_{max},$$
$$P_{min} \leq P_{g,k} \leq P_{max}$$
$$Q_{min} \leq Q_{g,k} \leq Q_{max}$$

where $k = 1, 2, ..N$; $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $\mathrm{V}_i(= V_i\angle\delta_i)$ is bus voltage at i-th bus; N represents the total number of buses in system.

2.4.5. Optimal Power flow (Minimization of Fuel Cost)

In this case, the minimization of fuel cost is treated as an objective function. This problem can also be formulated as constrained optimization problem.

Minimize :

$$f = \sum_{i=1}^{N} \left(a_i + b_i P_{g,i} + c_i P_{g,i}^2 \right) \tag{45}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator,

subject to :

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j) \right) = 0,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j) \right) = 0,$$

with bounds :

$$V_{min} \leq V_k \leq V_{max},$$

$$\delta_{min} \leq \delta_k \leq \delta_{max},$$

$$P_{min} \leq P_{g,k} \leq P_{max}$$

$$Q_{min} \leq Q_{g,k} \leq Q_{max}$$

where $i = 1, 2, ...N$; $P_i (= P_{g,i} - P_{l,i})$ and $Q_i (= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij} (= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $\mathrm{V}_i (= V_i \angle \delta_i)$ is bus voltage at i-th bus; N represents the total number of buses in system.

2.4.6. Optimal Power flow (Minimization of Active Power Loss and Fuel Cost)

Minimization of fuel cost and loss are considered as objective functions in this case. By using weight factor, this problem is converted from multi-objective to a single-objectiveconstrained problem. This problem is formulatedas follows.

Minimize :

$$f = \sum_{i=1}^{N} \left(a_i + (b_i + \lambda_p) P_{g,i} + c_i P_{g,i}^2 - \lambda_p P_{l,i} \right) \tag{46}$$

where a_i, b_i, and c_i are the cost coefficient of i-th bus generator and λ_p represents the weight factor

subject to :

$$P_{g,i} - P_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} cos(\delta_i - \delta_j) + B_{ij} sin(\delta_i - \delta_j) \right) = 0,$$

$$Q_{g,i} - Q_{l,i} - V_i \sum_{j=1}^{N} V_j \left(G_{ij} sin(\delta_i - \delta_j) - B_{ij} cos(\delta_i - \delta_j) \right) = 0,$$

with bounds :

$$V_{min} \leq V_k \leq V_{max},$$

$$\delta_{min} \leq \delta_k \leq \delta_{max},$$

$$P_{min} \leq P_{g,k} \leq P_{max}$$

$$Q_{min} \leq Q_{g,k} \leq Q_{max}$$

where $k = 1, 2, ..N$; $P_i(= P_{g,i} - P_{l,i})$ and $Q_i(= Q_{g,i} - Q_{l,i})$ represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $\mathrm{V}_i(= V_i\angle\delta_i)$ is bus voltage at i-th bus; N represents the total number of buses in system.

2.4.7. Microgrid Power flow (Islanded case)

Since the early 1960s, power flow analysis has been an essential research topic for power engineers. For the operational analysis of islanded microgrid, there is a requirement of a suitable power flow tool. In the case of Droop-Based Islanded Microgrids (DBIMGs), sharing of the reactive and active power between the Distribution Generations (DGs) is controlled by the droop controllers. In general, conventional power flow techniques consider four variables to be unknown, such as active power, reactive power, voltage magnitude, and voltage angle for different buses. In the case of PQ bus, the value of voltage angle and voltage magnitude are unknown while reactive power and active power are known. Contrary, in the case of PV bus, voltage magnitude and reactive power are unknown while voltage angle and active power are known. But, in the case of droop bus, all these variables are unknown. Conventional techniques cannot be applied to the power flow problem of islanded MGs as a frequency is not considered constant. In islanded MGs, the operating frequency is operated as an extra unknown variable of the power flow problem. In order to address this issue, this problem can be formulated as constrained optimization problem, which is as follows.

Minimize :

$$\begin{aligned} f = &\sum_{i=1}^{N}\left(P_i - \left(V_{r,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) + V_{m,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j})\right)\right)^2 \\ &+ \sum_{i=1}^{N}\left(Q_i - \left(V_{m,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) - V_{r,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j})\right)\right)^2 \end{aligned} \tag{47}$$

subject to :

$$\begin{aligned} &\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \\ &\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \\ &P_k - Cp_k(w_k^* - w) + P_{l,k} = 0, \\ &Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = 0, \end{aligned}$$

with bounds :

$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$

$$P_{min} \leq P_k \leq P_{max}$$

$$Q_{min} \leq Q_k \leq Q_{max}$$

$$w_{min} \leq w \leq w_{max}$$

where $k = 1, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is operating frequency; N represents the total number of buses in system.

2.4.8. Microgrid Power flow (Grid-connected case)

One of the key challenges in the steady-state power systems analysis is the power flow problem (PFP) of grid-connected microgrid. Since the 1950s, different techniques have been utilized to solve PFP of transmission systems. The developments of these techniques have been essentially done by utilizing numerical techniques which are used to solve non-linear simultaneous equations i.e. Newtons based numerical techniques (NNTs) and their variants. During the solving process of PFP, Jacobian Matrix becomes near singular or singular in gird connected microgrids in case of NNTs. Therefore, they cannot provide solutions in this case. In order to address this issue, PFP can be formulated as an alternative constrained optimization problem, which is as follows.

Minimize :

$$
\begin{aligned}
f = &\sum_{i=2}^{N}\left(P_i - \left(V_{r,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) + V_{m,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j})\right)\right)^2 \\
&+ \sum_{i=2}^{N}\left(Q_i - \left(V_{m,i}\sum_{j=1}^{N}(G_{ij}V_{r,j} - B_{ij}V_{m,j}) - V_{r,i}\sum_{i=1}^{N}(b_{ij}V_{r,j} + G_{ij}V_{m,j})\right)\right)^2
\end{aligned}
\tag{48}
$$

subject to :

$$
\begin{aligned}
&\sum_{i=2}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \\
&\sum_{i=2}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,
\end{aligned}
$$

with bounds :

$$V_{min} \leq V_{r,k}, V_{m,k}...k = 1, 2, ...N \leq V_{max}$$

where $k = 2, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; N represents the total number of buses in system.

2.4.9. Optimal Setting of Droop Controller for Minimization of Active Power Loss in Islanded Microgrids

For the application of an islanded microgrid (IMG), the Distributed Generations (DGs) can distribute the local loads accordingly without crossing acceptable limits of bus voltages and system frequency. Further, the flow of current in lines must be within the bars. In an IMG, several droop control systems have been adopted for power-sharing among the DGs. It is essential to conduct the schemes not entirely stable, but also optimally. In an IMG, tuning of the droop parameters is required to reduce active losses. This problem can be developed as a constrained optimization problem,

Minimize :

$$f = \sum_{i=1}^{N} P_i \tag{49}$$

subject to :

$$
\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0, \\
&P_k - Cp_k(w_k^* - w) + P_{l,k} = 0, \\
&Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = 0,
\end{aligned}
$$

with bounds :

$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$
$$P_{min} \leq P_k \leq P_{max}$$
$$Q_{min} \leq Q_k \leq Q_{max}$$
$$Cp_{min,k} \leq Cp_k \leq Cp_{max,k}$$
$$Cq_{min,k} \leq Cq_k \leq Cq_{max,k}$$
$$w_{min} \leq w \leq w_{max}$$

where $k = 1, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is operating frequency; N represents the total number of buses in system.

2.4.10. Optimal Setting of Droop Controller for Minimization of Reactive Power Loss in Islanded Microgrids

The main aim of this problem is to minimize reactive losses by tuning the droop parameters. Mathematical model of this problem can be expressed as a constrained optimization problem.

Minimize :

$$f = \sum_{i=1}^{N} Q_i \tag{50}$$

subject to :

$$\begin{aligned}
&\sum_{i=1}^{N}(G_{k,i}V_{r,i} - B_{ki}V_{m,i}) - \frac{P_kV_{r,k} + Q_kV_{m,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\\
&\sum_{i=1}^{N}(B_{ki}V_{r,i} + G_{ki}V_{m,i}) - \frac{P_kV_{m,k} - Q_kV_{r,k}}{(V_{r,k})^2 + (V_{m,k})^2} = 0,\\
&P_k - Cp_k(w_k^* - w) + P_{l,k} = 0,\\
&Q_k - Cq_k\left(V_k^* - \sqrt{(V_{r,k})^2 + (V_{m,k})^2}\right) + Q_{l,k} = 0,
\end{aligned}$$

with bounds :

$$V_{min} \leq V_{r,k}, V_{m,k} \leq V_{max}$$
$$P_{min} \leq P_k \leq P_{max}$$
$$Q_{min} \leq Q_k \leq Q_{max}$$
$$Cp_{min,k} \leq Cp_k \leq Cp_{max,k}$$
$$Cq_{min,k} \leq Cq_k \leq Cq_{max,k}$$
$$w_{min} \leq w \leq w_{max}$$

where $k = 1, ...N$; P_i and Q_i represent the active and reactive injected power, respectively, at i-th bus; $Ybus_{ij}(= G_{ij} + 1jB_{ij})$ is ij-th element of admittance matrix; $V_i(= V_{r,i} + 1jV_{m,i})$ is bus voltage at i-th bus; Cp_k and Cq_k represent the active and reactive power droop parameters of controllers, respectively; w is operating frequency; N represents the total number of buses in system.

2.5. Power Electronics: Synchronous Optimal Pulsewidth Modulation

Synchronous optimal pulse-width modulation (SOPWM) is a rising approach to regulatemedium-voltage (MV) drives. It provides a significant reduction of switching frequency without raising the distortion. Consequently, itreduces the switching losses thatenhances the performance of the inverter. Over a single fundamental period, switching angles are calculatedwhile reducing thedistortion of current. SOPWM can be developed as a scalable constrained optimization problem. For different level inverters, the SOPWM problem can be illustrated as follows.

2.5.1. SOPWM for 3-level Invereters [46]

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{\sqrt{\sum_k k^{-4}}} \tag{51}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{f_{s,max}}{f.m} \rfloor$, and $s(i) = (-1)^{i+1}$

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds :

$$0 < \alpha_i < \frac{\pi}{2},\ ,i = 1, 2, ...N.$$

2.5.2. SOPWM for 5-level Inverters [47]

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{2\sqrt{\sum_k k^{-4}}} \tag{52}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{2.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, -1, 1\ , -1, 1, -1, -1$.

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$2m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds :

$$0 < \alpha_i < \frac{\pi}{2},\ ,i = 1, 2, ...N.$$

2.5.3. SOPWM for 7-level Inverters [48]

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{3\sqrt{\sum_k k^{-4}}} \tag{53}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{3.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1\ , -1, -1, 1, 1, -1]$.

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$3m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

$$0 < \alpha_i < \frac{\pi}{2},\ ,i = 1, 2, ...N.$$

2.5.4. *SOPWM for 9-level Inverters [49]*

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{4\sqrt{\sum_k k^{-4}}} \tag{54}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{4.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, 1, -1, 1, -1 - 1, -1, 1, -1, -1]$

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$4m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds :

$$0 < \alpha_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.5.5. *SOPWM for 11-level Inverters [50]*

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{5\sqrt{\sum_k k^{-4}}} \tag{55}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{5.f_{s,max}}{f.m} \rfloor$, and $s = [1, -1, 1, 1, 1, -1, -1, -1, 1, 1, 1, 1]$.

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$5m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds :

$$0 < \alpha_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.5.6. *SOPWM for 13-level Inverters [50]*

Minimize :

$$f = \frac{\sqrt{\sum_k (k^{-4})(\sum_{i=1}^{N} s(i)cos(k\alpha_i))^2}}{6\sqrt{\sum_k k^{-4}}} \tag{56}$$

where, $k = 5, 7, 11, 13.....97$, $N = \lfloor \frac{6.f_{s,max}}{f.m} \rfloor$, and $s = [1, 1, 1, -1, 1, -1, 1, -1, 1, 1, 1, 1]$

subject to :

$$\alpha_{i+1} - \alpha_i - 10^{-5} > 0,\ i = 1, 2, ...N - 1,$$

$$6m - \sum_{i=1}^{N} s(i)cos(\alpha_i) = 0,$$

with bounds :

$$0 < \alpha_i < \frac{\pi}{2},\ , i = 1, 2, ...N.$$

2.6. Livestock Feed Ration Optimization

In the livestock production industry, feed represents a significant part as it accounts for about 60 80% of the product cost depending on stage and race of the animal [51, 52]. Consequently, it is required to provide the most desirable diet at least cost to prune down the operational cost for obtaining added profit. Moreover, the purpose of the feed mix formulation is to determine relevant ingredients and their cost for reducing feed cost while satisfying constraints based on various nutrient necessities [53, 54]. Two type of cattle case viz. beef cattle and dairy cattle are selected and are introduced in below subsections.

2.6.1. Beef cattle case [55]

In case ofbeef cattle, the primary target is to achieve the allocation (x_i) of each availablematerial Y_i with respect to their costs (C_i). The constraints are based on the amount of expected nutrients (in Kg) and weight (w_i). The formulation of this problem can be expressed as follows.

Minimize :

$$f(\bar{x}) = \sum_{i=1}^{n} x_i C_i \tag{57}$$

subject to :

$$h_1(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{DMI}_i - a = 0,$$

$$g_1(\bar{x}) = b - \sum_{i=1}^{n} x_i \mathrm{CP}_i \leq 0,$$

$$g_2(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{CP}_i - c \leq 0,$$

$$g_3(\bar{x}) = d - \sum_{i=1}^{n} x_i \mathrm{TDN}_i \leq 0,$$

$$g_4(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{TDN}_i - e \leq 0,$$

$$g_5(\bar{x}) = f - \sum_{i=0}^{n} x_i \mathrm{Ca}_i \leq 0,$$

$$g_6(\bar{x}) = \sum_{i=0}^{n} x_i \mathrm{Ca}_i - g \leq 0,$$

$$g_7(\bar{x}) = h - \sum_{i=0}^{n} x_i \mathrm{P}_i \leq 0,$$

$$g_8(\bar{x}) = \sum_{i=0}^{n} x_i \mathrm{P}_i - j \leq 0,$$

$$g_9(\bar{x}) = k - \sum_{i=0}^{n} x_i \mathrm{Rhage}_i \leq 0,$$

$$g_{10}(\bar{x}) = \sum_{i=0}^{n} x_i \mathrm{Rhage}_i - l \leq 0,$$

$$g_{11}(\bar{x}) = m - \sum_{i=0}^{n} x_i \mathrm{MC}_i \leq 0,$$

$$g_{12}(\bar{x}) = \sum_{i=0}^{n} x_i \mathrm{MC}_i - o \leq 0,$$

$$g_{13}(\bar{x}) = p - \sum_{i=1}^{n} x_i \mathrm{Conc}_i \leq 0,$$

$$g_{14}(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{Conc}_i - q \leq 0.$$

where DMI, CP, TDN, Ca, P, and Rhage represent Dry Matter Intake, Crude Protein, Total Digestible Nutrients, Calcium, Phosphorous, and Roughages, repectively, in Kg.

2.6.2. Dairy cattle case [55]

In case of dairy cattle, the principal objective is defined asallocation (x_i) of chosen materials (Y_i) multiplied by their costs (C_i). The purpose of this problem is to obtain x_i for reducing the production cost while meeting the problem constraints. The numerical formulation can be expressed as follows.

Minimize :

$$f(\bar{x}) = \sum_{i=1}^{n} x_i C_i \tag{58}$$

subject to :

$$h_1(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{MP}_i - r = 0,$$

$$h_2(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{Lys}_i - s = 0,$$

$$h_3(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{Ca}_i - t = 0,$$

$$h_4(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{P}_i - u = 0,$$

$$h_5(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{ME}_i - v = 0,$$

$$h_6(\bar{x}) = \sum_{i=1}^{n} x_i \mathrm{Met}_i - z = 0,$$

where MP, Lys, Ca, P, ME, and Met represent Metabolizable Protein, Lysine, Calcium, Phosphorous, Metabolizable Energy, and Methionine, respectively in Kg.

2.7. Benchmark Suite

A benchmark suite is created using the above-mentioned real-world constrained problems. A total number of 57 problems is designed from the above-listed problems and is included in the benchmark suite. The details of these problems are reported in Table 3. As shown in Table 3, the number of decision variables vary from 2 to 158, number of equality constraints vary from 0 to 148, and number of equality constraints vary from 0 to 105. The source code of this benchmark suite is available in .

Table 3: Details of the 57 real-world constrained optimization problem. D is the total number of decision variables of the problem, g is the number of inequality constraints and h is the number of equality constraints, $f(\bar{x}^*)$ is best known feasible objective function value.

Prob	Name	D	g	h	$f(\bar{x}^*)$
	Industrial Chemical Processes				
RC01	Heat Exchanger Network Design (case 1)	9	0	8	1.8931162966E+02
RC02	Heat Exchanger Network Design (case 2)	11	0	9	7.0490369540E+03
RC03	Optimal Operation of Alkylation Unit	7	14	0	-4.5291197395E+03
RC04	Reactor Network Design (RND)	6	1	4	-3.8826043623E-01
RC05	Haverly's Pooling Problem	9	2	4	-4.0000560000E+02
RC06	Blending-Pooling-Separation problem	38	0	32	1.8638304088E+00
RC07	Propane, Isobutane, n-Butane Nonsharp Separation	48	0	38	2.1158627569E+00
	Process Synthesis and Design Problems				
RC08	Process synthesis problem	2	2	0	2.0000000000E+00
RC09	Process synthesis and design problem	3	1	1	2.5576545740E+00
RC10	Process flow sheeting problem	3	3	0	1.0765430833E+00
RC11	Two-reactor Problem	7	4	4	9.9238463653E+01
RC12	Process synthesis problem	7	9	0	2.9248305537E+00
RC13	Process design Problem	5	3	0	2.6887000000E+04
RC14	Multi-product batch plant	10	10	0	5.3638942722E+04
	Mechanical Engineering Problem				
RC15	Weight Minimization of a Speed Reducer	7	11	0	2.9944244658E+03
RC16	Optimal Design of Industrial refrigeration System	14	15	0	3.2213000814E-02
RC17	Tension/compression spring design (case 1)	3	3	0	1.2665232788E-02
RC18	Pressure vessel design	4	4	0	5.8853327736E+03
RC19	Welded beam design	4	5	0	1.6702177263E+00
RC20	Three-bar truss design problem	2	3	0	2.6389584338E+02
RC21	Multiple disk clutch brake design problem	5	7	0	2.3524245790E-01
RC22	Planetary gear train design optimization problem	9	10	1	5.2576870748E-01
RC23	Step-cone pulley problem	5	8	3	1.6069868725E+01
RC24	Robot gripper problem	7	7	0	2.5287918415E+00
RC25	Hydro-static thrust bearing design problem	4	7	0	1.6254428092E+03
RC26	Four-stage gear box problem	22	86	0	3.5359231973E+01
RC27	10-bar truss design	10	3	0	5.2445076066E+02
RC28	Rolling element bearing	10	9	0	1.4614135715E+04
RC29	Gas Transmission Compressor Design (GTCD)	4	1	0	2.9648954173E+06
RC30	Tension/compression spring design (case 2)	3	8	0	2.6138840583E+00
RC31	Gear train design Problem	4	1	1	0.0000000000E+00
RC32	Himmelblau's Function	5	6	0	-3.0665538672E+04
RC33	Topology Optimization	30	30	0	2.6393464970E+00
	Power System Problems				
RC34	Optimal Sizing of Single Phase Distributed Generation with reactive power support for Phase Balancing at Main Transformer/Grid	118	0	108	0.0000000000E+00
RC35	Optimal Sizing of Distributed Generation for Active Power Loss Minimization	153	0	148	8.9093896456E-02
RC36	Optimal Sizing of Distributed Generation (DG) and Capacitors for Reactive Power Loss Minimization	158	0	148	7.2066551720E-02
RC37	Optimal Power flow (Minimization of Active Power Loss)	126	0	116	2.1962851478E-02
RC38	Optimal Power flow (Minimization of Fuel Cost)	126	0	116	2.7766131989E+00
RC39	Optimal Power flow (Minimization of Active Power Loss and Fuel Cost)	126	0	116	2.8677165770E+00
RC40	Microgrid Power flow (Islanded case)	76	0	76	0.0000000000E+00
RC41	Microgrid Power flow (Grid-connected case)	74	0	74	0.0000000000E+00
RC42	Optimal Setting of Droop Controller for Minimization of Active Power Loss in Islanded Microgrids	86	0	76	8.6241006360E-02
RC43	Optimal Setting of Droop Controller for Minimization of Reactive Power Loss in Islanded Microgrids	86	0	76	8.0420545897E-02
RC44	Wind Farm Layout Problem	30	105	0	-6.2607000000E+03
	Power Electronic Problems				
RC45	SOPWM for 3-level Invereters	25	24	1	3.8029250566E-02
RC46	SOPWM for 5-level Inverters	25	24	1	2.1215000000E-02
RC47	SOPWM for 7-level Inverters	25	24	1	1.5164538375E-02
RC48	SOPWM for 9-level Inverters	30	29	1	1.6787535766E-02
RC49	SOPWM for 11-level Inverters	30	29	1	9.3118741800E-03
RC50	SOPWM for 13-level Inverters	30	29	1	1.5096451396E-02
	Livestock Feed Ration Optimization				
RC51	Beef Cattle(case 1)	59	14	1	4.5508511497E+03
RC52	Beef Cattle (case 2)	59	14	1	3.3489821493E+03
RC53	Beef Cattle (case 3)	59	14	1	4.9976069290E+03
RC54	Beef Cattle (case 4)	59	14	1	4.2405482538E+03
RC55	Dairy Cattle (case 1)	64	0	6	6.6964145128E+03
RC56	Dairy Cattle (case 2)	64	0	6	1.4748932529E+04
RC57	Dairy Cattle (case 3)	64	0	6	3.2132917019E+03

3. Evaluation of Proposed Benchmark suite

In this section, the proposed benchmark suite is evaluated using three state-of-the-art algorithms viz. IUDE [18], ϵMAgES [19], and iLSHADE$_\epsilon$ [20].

3.1. Improved Unified Differential Evolution Algorithm

To solve constrained optimization problems, an Improved variant of Unified Differential Evolution (IUDE) is proposed in [18]. This algorithm employs three mutation strategies viz. current-to-pbest, current-to-rand, and rand mutation strategy with binomial crossover operator to generate trial solutions. IUDE has been a dual population-based approach where the current population is divided into two sub-population at each generation. The mutation operation utilizes the ranking-based mutation and parameter self-adaptation procedure of SHADE [56]. The constraint handling technique employed in IUDE is a combined approach proposed in C^2oDE [57], where a combination of ϵ-constraint and feasibility based rule approach is applied. For selection, the traditional one-to-one replacement approach is applied in IUDE.

3.2. Matrix Adaptation Evolution Strategy

In [19], a state-of-the-art Evolution Strategy, Matrix Adaptation Evolution Strategy (MA-ES), is proposed to solve real-parameter constrained optimization problems. MA-ES is a computationally efficient version of CMA-ES variant [58]. To handle the constraint, ϵ-constraint is incorporated in MA-ES. In addition, a repair step based on gradient approximation is also employed to deal with equality constraints. This algorithm is named as ϵMAgES.

3.3. LSHADE44 with an Improved ϵ Constraint-handling Method

For solving constrained optimization problems, a differential evolution based variant LSHADE44 is proposed in [20]. Additionally, an improved constrained handling technique, named IEpsilon, is also proposed to deal with complex constraints. In IEpsilon, the ϵ-level is adaptively adjusted according to number of feasible solutions in the current population to provide a balance between infeasible regions and feasible regions during the optimization process. Moreover, a new trail vector generation strategy, DE/randr1*/1, is proposed. This algorithm is named as iLSHADE$_\epsilon$.

3.4. Experimental Setting

All the above-mentioned algorithms have been implemented in MATLAB. Evaluation of the proposed benchmark suite has been done on MATLAB r2017b in a PC having Microsoft Windows 10 operating system with INTEL Core i7 CPU and 8 Gb RAM. The parameter setting of all algorithms is directly taken from their respective papers viz. [18, 19, 20].

To stop the optimization process, a stopping rule based on the number of decision variables is applied in all algorithms. A fixed amount of function evaluations is allotted during the optimization process and after the maximum function evaluations, the optimization process of the algorithms is stopped and result in terms of the best solution is returned. The following criteria are used to decide the maximum function evaluation for each problem of the proposed benchmark suite.

$$Max_{FEs} = \begin{cases} 2 \times 10^5, & \text{if } D \leq 10 \\ 4 \times 10^5, & \text{else if } 10 < D \leq 30 \\ 6 \times 10^5, & \text{else if } 30 < D \leq 50 \\ 8 \times 10^5, & \text{else if } 50 < D \leq 150 \\ 10^6, & \text{else} \end{cases} \tag{59}$$

where Max_{FEs} is maximum allowed function evaluations and D is dimension (number of decision variable) of problem.

3.5. Algorithmic Complexity

The algorithmic complexity of all algorithms is also calculated using the proposed benchmark suite. Following procedures are adopted to calculate the algorithmic complexity.

1. $T_1 = \frac{\sum_{i=1}^{57} t_{1i}}{57}$, where t_{1i} is the computation time required to evaluate function for 100000 times for problem i.
2. $T_2 = \frac{\sum_{i=1}^{57} t_{2i}}{57}$, where t_{2i} is the computation time required by algorithm for 100000 function evaluations for problem i.
3. The algorithmic complexity is evaluated using T_1, T_2, and $\frac{T_2 - T_1}{T_1}$.

Complexity of all algorithms is reported in Table 4. It can be seen from Table 4, IUDE has lower complexity and iLSHADE$_\epsilon$ has higher computational complexity.

Table 4: Computational Complexity

Algorithm	T_1 (sec)	T_2 (sec)	$\frac{T_2-T_1}{T_1}$
IUDE	8.57	9.93	0.16
ϵMAgES	8.57	12.67	0.48
iLSHADE$_\epsilon$	8.57	15.76	0.84

Table 5: Results of Industrial Chemical Processes problems (RC01 -RC07) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
RC01	IUDE	1.91E+02	3.05E+02	2.85E+02	0.00E+00	1.99E+02	0	1.48E+05	0
	ϵMAgES	1.89E+02	1.90E+02	1.90E+02	1.92E+02	7.96E-01	100	0.00E+00	16
	iLSHADE$_\epsilon$	1.89E+02	2.07E+02	1.97E+02	2.03E+02	6.58E+00	32	4.46E-04	4
RC02	IUDE	7.05E+03	7.05E+03	6.93E+03	5.94E+03	3.70E+02	92	9.99E+03	92
	ϵMAgES	7.05E+03	7.05E+03	1.05E+04	2.66E+04	6.54E+03	68	8.29E+01	68
	iLSHADE$_\epsilon$	**7.05E+03**	**7.05E+03**	**7.05E+03**	**7.05E+03**	**5.57E-13**	**100**	**0.00E+00**	**100**
RC03	IUDE	-4.53E+03	-1.43E+02	-6.08E+03	-1.57E+04	5.88E+03	68	3.86E+00	12
	ϵMAgES	-1.43E+02	7.63E+01	3.21E+01	2.49E+02	1.43E+02	100	0.00E+00	0
	iLSHADE$_\epsilon$	-4.53E+03	-1.43E+02	-9.57E+02	4.95E+02	2.04E+03	100	0.00E+00	36
RC04	IUDE	-3.87E-01	-4.70E-01	-5.05E-01	-5.52E-01	7.80E-02	0	8.76E-02	0
	ϵMAgES	-3.88E-01	-3.88E-01	-3.88E-01	-3.86E-01	7.55E-04	100	0.00E+00	84
	iLSHADE$_\epsilon$	**-3.75E-01**	**-3.75E-01**	**-3.75E-01**	**-3.75E-01**	**1.21E-06**	**100**	**0.00E+00**	**100**
RC05	IUDE	-4.00E+02	-4.00E+02	-3.56E+02	-8.30E-03	1.33E+02	100	0.00E+00	72
	ϵMAgES	-4.00E+02	-2.27E+02	-2.44E+02	-1.47E+02	9.30E+01	100	0.00E+00	36
	iLSHADE$_\epsilon$	-4.00E+02	-1.33E-02	-1.01E+02	-6.30E-03	1.57E+02	100	0.00E+00	44
RC06	IUDE	1.88E+00	9.98E-01	1.10E+00	9.98E-01	2.92E-01	0	2.16E+00	0
	ϵMAgES	2.04E+00	2.01E+00	2.06E+00	2.28E+00	1.60E-01	12	8.30E-03	0
	iLSHADE$_\epsilon$	1.31E+00	1.64E+00	1.28E+00	1.22E+00	1.47E-01	0	1.24E-01	0
RC07	IUDE	1.72E+00	2.16E+00	1.65E+00	1.24E+00	3.76E-01	0	2.18E-01	0
	ϵMAgES	2.08E+00	1.67E+00	1.78E+00	1.57E+00	2.14E-01	0	1.86E-02	0
	iLSHADE$_\epsilon$	1.88E+00	1.61E+00	1.77E+00	1.78E+00	1.50E-01	0	7.98E-02	0

3.6. Performance Evaluation Procedure

The problems of the proposed benchmark suite are taken from different engineering applications. Therefore, the difficulty level and complexity of problems have been different from each other. In order to determine the relative difficulty level of each problem of the benchmark suite, the following procedures are adopted in this study.

- The above-mentioned algorithms are implemented independently 25 times on each problem of the benchmark suite.

- The outcomes of algorithms for 25 runs are prepared in terms of the mean objective function (Mean), mean constraint violation (MV), feasibility rate (FR), and success rate (SR).

    1. *Mean Constraint Violation:* Mean constraint violation, $\bar{v}$, is calculated using following equation.

    $$\bar{v} = \frac{\sum_{i=1}^{p} \max\left(g_i(\bar{x}), 0\right) + \sum_{j=p+1}^{m} \max\left(|h_i(\bar{x})| - \varepsilon, 0\right)}{m}, \tag{60}$$

    where ε is set to 0.0001.
    2. *Feasibility Rate:* The ratio of number of runs in which at least one feasible solution is attained within Max_{FEs} and total runs.
    3. *Success Rate:* The ratio of total number of runs in which an algorithm obtained a feasible solution $\bar{x}$ satisfying $f(\bar{x}) - f(\bar{x}^*) \leq 10^{-8}$ within Max_{FEs} and total runs.

- The difficulty level of problems is evaluated using following criteria.

    1. Evaluation of problems based on SR,
    2. Then evaluation of problems based on FR, and
    3. At last, evaluation of problems based on MV.

Table 6: Results of Process Synthesis and Design Problems (RC08 -RC14) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
	IUDE	**2.00E+00**	**2.00E+00**	**2.00E+00**	**2.00E+00**	**1.92E-16**	**100**	**0.00E+00**	**100**
RC08	ϵMAgES	2.00E+00	2.00E+00	2.00E+00	2.00E+00	5.83E-05	100	0.00E+00	96
	iLSHADE$_\epsilon$	**2.00E+00**	**2.00E+00**	**2.00E+00**	**2.00E+00**	**2.36E-16**	**100**	**0.00E+00**	**100**
	IUDE	2.56E+00	2.56E+00	2.60E+00	2.93E+00	1.23E-01	100	0.00E+00	88
RC09	ϵMAgES	**2.56E+00**	**2.56E+00**	**2.56E+00**	**2.56E+00**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	2.56E+00	2.56E+00	2.66E+00	2.69E+00	2.74E-01	88	2.31E-03	64
	IUDE	1.08E+00	1.08E+00	1.10E+00	1.25E+00	5.78E-02	100	0.00E+00	88
RC10	ϵMAgES	**1.08E+00**	**1.08E+00**	**1.08E+00**	**1.08E+00**	**2.36E-16**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	1.08E+00	1.25E+00	1.21E+00	1.25E+00	7.65E-02	100	0.00E+00	76
	IUDE	9.92E+01	9.92E+01	1.02E+02	1.07E+02	4.07E+00	100	0.00E+00	52
RC11	ϵMAgES	9.92E+01	9.92E+01	1.05E+02	6.88E+00	1.13E+02	0	2.96E-06	0
	iLSHADE$_\epsilon$	9.92E+01	1.01E+02	1.03E+02	1.10E+02	4.60E+00	100	0.00E+00	44
	IUDE	2.92E+00	2.95E+00	3.08E+00	4.21E+00	4.21E-01	100	0.00E+00	16
RC12	ϵMAgES	2.92E+00	3.92E+00	3.64E+00	4.63E+00	6.72E-01	100	0.00E+00	20
	iLSHADE$_\epsilon$	**2.92E+00**	**2.92E+00**	**2.92E+00**	**2.92E+00**	**3.85E-08**	**100**	**0.00E+00**	**100**
	IUDE	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
RC13	ϵMAgES	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.69E+04**	**2.69E+04**	**2.69E+04**	**2.69E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
	IUDE	6.19E+04	6.43E+04	6.60E+04	7.36E+04	4.37E+03	100	0.00E+00	0
RC14	ϵMAgES	5.36E+04	5.85E+04	5.78E+04	6.19E+04	2.60E+03	100	0.00E+00	0
	iLSHADE$_\epsilon$	5.85E+04	5.99E+04	6.11E+04	6.57E+04	2.56E+03	100	0.00E+00	0

3.7. Evaluation of Problems of benchmark suite

The above-discussed procedures are adopted on each problem to evaluate the problems of the proposed benchmark suite. The outcomes of all three algorithms on each problem are reported in Tables 5-10. In Table 5, the outcomes of Industrial Chemical Process problems are reported for all algorithms. By analyzing the outcomes of this Table, it can be concluded that four problems (RC06, RC07, RC01, and RC03) are the hardest problems and three problems (RC02, RC04, and RC05) are easiest problems for solving using constrained optimization algorithms. The outcomes of Process Synthesis and Design problems for all algorithms are depicted in Table 6. From this Table, it can be seen that RC14 is the hardest problem, and RC13, RC08, RC10, and RC09 are easiest problems of this group. The difficulty level of the rest of the problems is at a modest level. The detailed outcomes of Mechanical Design problems are outlined in Table 7. Analyzing the results of this Table, it is easy to find out that most of the problems (16 out of 19 problems) of this group are easier to solve. Problems RC25, RC26, and RC22 are the hardest problems of this group. The outcome of Power System problems, Power Electronics problems, and Livestock Feed Ration problems for all algorithms are reported in Tables 8, 9, and 10, respectively. From these Tables, it can be concluded that all the problems of these groups are difficult to solve.

From the above analysis, we can conclude that most of the problems of the proposed benchmark suite are challenging to solve. State-of-the-art algorithms are not able to provide feasible solutions for 14 problems within Max_{FEs} function evaluation. This conclusion suggests that the proposed benchmark suite can be utilized to analyze the performance of constrained optimization algorithms.

4. Conclusion

In this work, a benchmark suite containing 57 real-world constrained optimization problem is proposed for validating the performance and robustness of the constrained optimization algorithms. Three state-of-the-art constrained optimization algorithms have been utilized to show the complexity of the problems proposed in the benchmark suite. The comparative analysis of outcomes of these algorithms concludes that the problems are hard to solve for the recently developed constrained optimization algorithms. Furthermore, these challenging problems can motivate researchers to develop new approaches for handling complex non-linear constraints especially equality constraints.

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Table 7: Results of Mechanical Engineering Problems (RC15 -RC33) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
RC15	IUDE	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**0.00E+00**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.99E+03**	**2.99E+03**	**2.99E+03**	**2.99E+03**	**0.00E+00**	**100**	**0.00E+00**	**100**
RC16	IUDE	**3.22E-02**	**3.22E-02**	**3.22E-02**	**3.22E-02**	**4.91E-18**	**100**	**0.00E+00**	**100**
	ϵMAgES	3.22E-02	3.22E-02	3.40E-02	4.45E-02	4.09E-03	100	0.00E+00	88
	iLSHADE$_\epsilon$	3.22E-02	3.22E-02	3.23E-02	3.25E-02	1.11E-04	100	0.00E+00	76
RC17	IUDE	**1.27E-02**	**1.27E-02**	**1.27E-02**	**1.27E-02**	**1.15E-06**	**100**	**0.00E+00**	**100**
	ϵMAgES	1.27E-02	1.27E-02	1.27E-02	1.27E-02	1.63E-07	100	0.00E+00	96
	iLSHADE$_\epsilon$	1.27E-02	1.27E-02	1.27E-02	1.27E-02	2.22E-05	100	0.00E+00	84
RC18	IUDE	5.89E+03	6.17E+03	6.24E+03	6.86E+03	4.05E+02	100	0.00E+00	24
	ϵMAgES	**5.89E+03**	**5.89E+03**	**5.89E+03**	**5.89E+03**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	5.89E+03	5.89E+03	1.49E+04	5.72E+04	1.71E+04	100	0.00E+00	64
RC19	IUDE	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.67E+00**	**2.08E-16**	**100**	**0.00E+00**	**100**
	ϵMAgES	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.67E+00**	**2.08E-16**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.67E+00**	**1.07E-11**	**100**	**0.00E+00**	**100**
RC20	IUDE	**2.64E+02**	**2.64E+02**	**2.64E+02**	**2.64E+02**	**0.00E+00**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.64E+02**	**2.64E+02**	**2.64E+02**	**2.64E+02**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	2.64E+02	2.64E+02	2.64E+02	2.65E+02	4.47E-01	100	0.00E+00	96
RC21	IUDE	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**0.00E+00**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**2.35E-01**	**2.35E-01**	**2.35E-01**	**2.35E-01**	**0.00E+00**	**100**	**0.00E+00**	**100**
RC22	IUDE	5.26E-01	5.27E-01	5.27E-01	5.30E-01	1.22E-03	100	0.00E+00	36
	ϵMAgES	5.29E-01	5.44E-01	5.76E-01	8.54E-01	1.05E-01	76	3.88E-01	0
	iLSHADE$_\epsilon$	5.26E-01	5.26E-01	5.27E-01	5.31E-01	1.44E-03	100	0.00E+00	52
RC23	IUDE	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.61E+01**	**3.77E-15**	**100**	**0.00E+00**	**100**
	ϵMAgES	**1.61E+01**	**1.61E+01**	**1.61E+01**	**1.61E+01**	**4.74E-14**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	1.61E+01	1.61E+01	1.61E+01	1.61E+01	3.67E-07	100	0.00E+00	92
RC24	IUDE	**2.54E+00**	**2.54E+00**	**2.54E+00**	**2.54E+00**	**4.81E-14**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.55E+00	9.70E+00	3.34E+03	1.00E+04	4.99E+03	100	0.00E+00	0
	iLSHADE$_\epsilon$	2.54E+00	2.54E+00	2.54E+00	2.54E+00	5.14E-06	100	0.00E+00	88
RC25	IUDE	2.51E+03	4.88E+03	4.92E+03	1.05E+04	3.14E+03	56	7.60E-01	0
	ϵMAgES	-4.49E+01	1.63E+03	-3.52E+03	1.63E+03	1.98E+04	100	0.00E+00	0
	iLSHADE$_\epsilon$	1.67E+03	2.04E+03	2.44E+03	1.73E+02	2.14E+03	88	1.73E-05	0
RC26	IUDE	3.54E+01	3.81E+01	3.91E+01	4.56E+01	3.62E+00	100	0.00E+00	24
	ϵMAgES	6.07E+01	2.10E+01	5.78E+01	1.99E+01	6.85E+01	32	2.61E-01	0
	iLSHADE$_\epsilon$	3.65E+01	3.93E+01	4.03E+01	5.42E+01	5.52E+00	100	0.00E+00	32
RC27	IUDE	5.24E+02	5.24E+02	5.24E+02	5.24E+02	1.01E-04	100	0.00E+00	88
	ϵMAgES	5.24E+02	5.24E+02	5.26E+02	5.31E+02	2.70E+00	100	0.00E+00	72
	iLSHADE$_\epsilon$	5.24E+02	5.24E+02	5.24E+02	5.25E+02	9.97E-03	100	0.00E+00	80
RC28	IUDE	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.93E-12**	**100**	**0.00E+00**	**100**
	ϵMAgES	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.93E-12**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.46E+04**	**1.93E-12**	**100**	**0.00E+00**	**100**
RC29	IUDE	**2.96E+06**	**2.96E+06**	**2.96E+06**	**2.96E+06**	**6.59E-10**	**100**	**0.00E+00**	**100**
	ϵMAgES	**2.96E+06**	**2.96E+06**	**2.96E+06**	**2.96E+06**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	2.96E+06	2.96E+06	2.97E+06	2.97E+06	1.23E+03	100	0.00E+00	76
RC30	IUDE	2.61E+00	2.61E+00	2.70E+00	3.07E+00	1.75E-01	100	0.00E+00	64
	ϵMAgES	**2.61E+00**	**2.61E+00**	**2.61E+00**	**2.61E+00**	**5.02E-13**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	2.61E+00	2.63E+00	2.67E+00	2.98E+00	1.17E-01	100	0.00E+00	76
RC31	IUDE	**3.06E-19**	**2.39E-18**	**7.76E-17**	**6.64E-16**	**2.20E-16**	**100**	**0.00E+00**	**100**
	ϵMAgES	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**0.00E+00**	**1.55E-20**	**5.26E-19**	**3.67E-18**	**1.21E-18**	**100**	**0.00E+00**	**100**
RC32	IUDE	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
	ϵMAgES	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**-3.07E+04**	**3.86E-12**	**100**	**0.00E+00**	**100**
RC33	IUDE	**2.64E+00**	**2.64E+00**	**2.64E+00**	**2.64E+00**	**4.44E-16**	**100**	**0.00E+00**	**100**
	ϵMAgES	2.65E+00	2.65E+00	2.65E+00	2.67E+00	8.64E-03	100	0.00E+00	0
	iLSHADE$_\epsilon$	**2.64E+00**	**2.64E+00**	**2.64E+00**	**2.64E+00**	**1.03E-15**	**100**	**0.00E+00**	**100**

Table 8: Results of Power System Problems (RC34 -RC44) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
RC34	IUDE	3.42E+00	4.60E+00	4.54E+00	1.66E+00	1.55E+00	0	4.33E-02	0
	ϵMAgES	**3.99E-01**	**8.90E-01**	**9.50E-01**	**1.83E+00**	**4.17E-01**	**100**	**0.00E+00**	0
	iLSHADE$_\epsilon$	4.33E+00	1.15E+01	8.23E+00	6.26E+00	2.28E+00	0	4.49E-02	0
RC35	IUDE	9.52E+01	8.82E+01	1.02E+02	1.10E+02	9.46E+00	0	8.34E-01	0
	ϵMAgES	1.20E-01	-1.12E+00	-1.69E+00	-7.30E+00	2.47E+00	12	8.26E-02	0
	iLSHADE$_\epsilon$	1.92E+02	1.85E+02	1.68E+02	1.17E+02	2.28E+01	0	1.78E-01	0
RC36	IUDE	7.84E+01	7.79E+01	8.81E+01	9.29E+01	1.07E+01	0	8.69E-01	0
	ϵMAgES	2.46E-01	1.07E+00	1.18E-01	-1.65E+00	7.32E-01	32	5.23E-02	0
	iLSHADE$_\epsilon$	1.54E+02	1.09E+02	1.31E+02	1.94E+02	3.01E+01	0	5.45E-01	0
RC37	IUDE	2.32E+00	1.51E+00	2.10E-01	-1.42E+00	1.79E+00	0	1.24E-01	0
	ϵMAgES	6.55E-01	1.40E+00	8.65E-01	6.68E-01	5.48E-01	12	5.61E-03	0
	iLSHADE$_\epsilon$	3.61E+00	3.66E+00	3.73E+00	3.69E+00	4.69E-01	0	5.71E-02	0
RC38	IUDE	2.03E+00	-7.60E+00	-9.69E+00	-2.48E+01	9.02E+00	0	1.68E-01	0
	ϵMAgES	7.18E+00	6.89E+00	6.49E+00	6.62E+00	7.74E-01	0	4.59E-03	0
	iLSHADE$_\epsilon$	5.07E+00	2.47E+00	3.39E+00	3.25E+00	9.53E-01	0	5.72E-02	0
RC39	IUDE	-5.03E+00	-1.47E+01	-1.69E+01	-3.82E+01	1.31E+01	0	1.84E-01	0
	ϵMAgES	9.25E+00	5.15E+00	7.44E+00	6.19E+00	2.23E+00	0	1.02E-02	0
	iLSHADE$_\epsilon$	4.09E+00	3.25E+00	3.19E+00	1.96E+00	1.42E+00	0	5.94E-02	0
RC40	IUDE	4.78E+01	4.52E+01	8.51E+01	1.97E+02	4.81E+01	0	1.45E+00	0
	ϵMAgES	1.70E-12	4.81E+00	5.69E+01	1.08E+02	8.41E+01	12	2.04E-01	12
	iLSHADE$_\epsilon$	3.45E+01	1.03E+02	1.57E+02	9.94E+01	7.77E+01	0	1.89E+00	0
RC41	IUDE	2.21E+01	1.41E+02	8.18E+01	4.51E+01	9.64E+01	0	1.22E+00	0
	ϵMAgES	**1.25E-19**	**2.80E-19**	**2.52E-19**	**3.53E-19**	**9.08E-20**	**100**	**0.00E+00**	**100**
	iLSHADE$_\epsilon$	5.07E+00	1.22E+02	9.62E+01	2.93E+02	1.10E+02	0	1.18E+00	0
RC42	IUDE	1.33E+02	3.87E+01	-2.38E+01	-5.29E+02	2.07E+02	0	4.99E+00	0
	ϵMAgES	7.64E+01	6.17E+01	6.73E+01	3.64E+01	5.26E+01	0	1.06E+00	0
	iLSHADE$_\epsilon$	-1.26E+00	-9.18E-01	-1.08E+00	-1.25E+00	1.52E-01	0	2.03E+00	0
RC43	IUDE	1.62E+01	9.12E+00	8.95E+00	7.71E+00	9.51E+00	0	2.98E+00	0
	ϵMAgES	1.06E+02	4.04E+01	7.61E+01	1.06E+02	2.71E+01	0	1.16E+00	0
	iLSHADE$_\epsilon$	3.47E+01	3.45E+01	4.50E+01	4.53E+01	7.89E+00	0	2.38E+00	0
RC44	IUDE	-6.15E+03	-6.12E+03	-6.12E+03	-6.07E+03	2.74E+01	100	0.00E+00	0
	ϵMAgES	-6.10E+03	-6.06E+03	-6.05E+03	-5.94E+03	5.22E+01	100	0.00E+00	0
	iLSHADE$_\epsilon$	-6.24E+03	-6.19E+03	-6.20E+03	-6.17E+03	2.70E+01	100	0.00E+00	0

Table 9: Results of Power Electronic Problems (RC45 -RC50) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
RC45	IUDE	5.52E-02	6.56E-02	7.66E-02	1.17E-01	2.41E-02	100	0.00E+00	0
	ϵMAgES	3.80E-02	4.95E-02	5.01E-02	6.42E-02	9.33E-03	100	0.00E+00	4
	iLSHADE$_\epsilon$	7.71E-02	1.13E-01	1.08E-01	1.64E-01	2.97E-02	100	0.00E+00	0
RC46	IUDE	4.31E-02	5.16E-02	5.48E-02	6.71E-02	9.07E-03	100	0.00E+00	0
	ϵMAgES	2.36E-02	3.03E-02	2.91E-02	3.37E-02	4.13E-03	100	0.00E+00	0
	iLSHADE$_\epsilon$	6.67E-02	7.27E-02	9.20E-02	2.04E-01	4.45E-02	100	0.00E+00	0
RC47	IUDE	3.44E-02	3.92E-02	6.44E-02	2.05E-01	5.64E-02	92	7.37E-04	0
	ϵMAgES	1.51E-02	2.06E-02	1.98E-02	2.46E-02	2.99E-03	100	0.00E+00	4
	iLSHADE$_\epsilon$	2.71E-02	4.52E-02	4.57E-02	6.98E-02	1.12E-02	100	0.00E+00	0
RC48	IUDE	4.50E-02	4.69E-02	6.53E-02	2.09E-01	5.38E-02	100	0.00E+00	0
	ϵMAgES	1.68E-02	1.68E-02	1.74E-02	2.24E-02	1.87E-03	100	0.00E+00	4
	iLSHADE$_\epsilon$	4.71E-02	1.96E-01	2.23E-01	4.94E-01	1.62E-01	100	0.00E+00	0
RC49	IUDE	2.85E-02	5.11E-02	5.50E-02	9.58E-02	2.33E-02	100	0.00E+00	0
	ϵMAgES	9.83E-03	2.99E-02	3.06E-02	5.96E-02	1.69E-02	100	0.00E+00	0
	iLSHADE$_\epsilon$	6.78E-02	1.36E-01	1.61E-01	3.20E-01	8.89E-02	100	0.00E+00	0
RC50	IUDE	6.23E-02	2.78E-01	2.53E-01	3.78E-01	1.13E-01	36	2.51E-03	0
	ϵMAgES	1.56E-02	1.66E-02	2.94E-02	7.77E-02	2.34E-02	100	0.00E+00	0
	iLSHADE$_\epsilon$	2.62E-01	3.01E-01	3.07E-01	3.53E-01	3.17E-02	0	4.78E-03	0

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Table 10: Results of Livestock Feed Ration Optimization Problems (RC51 -RC57) using IUDE, ϵMAgES, and iLSHADE$_\epsilon$.

Prob	Algorithm	Best	Median	Mean	Worst	SD	FR	MV	SR
RC51	IUDE	4.55E+03	4.55E+03	4.55E+03	4.55E+03	1.07E-01	0	2.91E-06	0
	ϵMAgES	4.40E+03	4.32E+03	4.17E+03	3.23E+03	3.59E+02	0	4.98E-02	0
	iLSHADE$_\epsilon$	4.55E+03	4.55E+03	4.55E+03	4.56E+03	3.82E+00	0	6.13E-06	0
RC52	IUDE	3.36E+03	3.39E+03	3.39E+03	3.43E+03	2.33E+01	100	0.00E+00	0
	ϵMAgES	3.62E+03	3.84E+03	3.86E+03	4.26E+03	1.79E+02	100	0.00E+00	0
	iLSHADE$_\epsilon$	3.81E+03	3.98E+03	4.01E+03	4.30E+03	1.54E+02	100	0.00E+00	0
RC53	IUDE	5.00E+03	5.04E+03	5.04E+03	5.10E+03	3.38E+01	100	0.00E+00	8
	ϵMAgES	5.57E+03	4.82E+03	5.09E+03	4.54E+03	3.56E+02	12	1.29E-03	0
	iLSHADE$_\epsilon$	5.07E+03	5.15E+03	5.24E+03	5.48E+03	1.59E+02	100	0.00E+00	0
RC54	IUDE	4.24E+03	4.24E+03	4.24E+03	4.24E+03	1.16E+00	100	0.00E+00	20
	ϵMAgES	4.18E+03	3.30E+03	3.33E+03	2.26E+03	5.37E+02	0	5.16E-02	0
	iLSHADE$_\epsilon$	4.24E+03	4.24E+03	4.24E+03	4.24E+03	4.68E-01	100	0.00E+00	36
RC55	IUDE	2.20E+03	2.32E+03	2.16E+03	2.16E+03	1.91E+02	0	9.88E-03	0
	ϵMAgES	6.24E+03	2.57E+03	5.37E+03	5.03E+03	2.59E+03	0	2.46E-01	0
	iLSHADE$_\epsilon$	7.03E+03	6.42E+03	6.54E+03	6.57E+03	2.37E+02	0	2.03E-03	0
RC56	IUDE	1.54E+04	1.08E+04	1.19E+04	1.17E+04	1.49E+03	0	8.69E-03	0
	ϵMAgES	1.48E+04	1.61E+04	1.56E+04	1.37E+04	2.14E+03	0	1.49E-01	0
	iLSHADE$_\epsilon$	1.40E+04	1.34E+04	1.26E+04	1.19E+04	9.72E+02	0	9.28E-03	0
RC57	IUDE	2.57E+03	2.54E+03	2.47E+03	2.84E+03	2.04E+02	0	2.03E-03	0
	ϵMAgES	2.65E+03	2.42E+03	3.42E+03	1.33E+03	5.16E+03	0	1.88E-01	0
	iLSHADE$_\epsilon$	2.65E+03	2.42E+03	3.42E+03	5.16E+03	1.33E+03	0	1.88E-01	0

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